PROBLEM SET #4

Due: class #8

1. In a perfectly competitive economy there are 2 goods, X and Y, produced using capital, K, and labor L, according to the following production functions:

$$X = \min [K_x, L_x] \text{ and } Y = \min [K_y, L_y/4]$$

where $K_x$, $L_x$ are the inputs of K and L into the production of X, and $K_y$, $L_y$ are defined similarly. There is a fixed total supply of capital (200 units) and of labor (440 units), fully mobile between sectors. All consumers are identical and have preferences represented by the utility function $U = \sqrt{XY}$ .

(a) Describe the conditions for a general equilibrium, paying particular attention to the possibility that one factor may be less than fully employed.

(b) Calculate the equilibrium quantities of X and Y, the relative price $P_x/P_y$ and the factor price ratio w/r.

(c) A regulation is introduced by the government requiring industry X to use an extra 3 units of capital per unit of output to clear up pollution damage. How will this policy affect the equilibrium of the economy? Will both factors be fully employed in the new equilibrium?

2. Consider an economy with a continuum (of unit measure) of identical consumers with utility function

u = (1/3) ln($x_1$) + (2/3) ln(24 + $x_2$), 0 # $x_1$, 0 # 24 + $x_2$ # 24.

Assume that there is a continuum of private firms, all of which have access to a CRTS technology that can convert units of good 2 into units of good 1 with A units of good 1 per unit of good 2.

(a)Find competitive equilibrium. Why didn't I bother to tell you who owns the firms?

Assume that the government has access to a technology that uses B units of good 2 as a fixed cost and then can convert any number of units of good 2 into good 1, with 2A units of good 1 produced per unit of good 2.

(b) Considering only Pareto optima where everyone receives the same allocation, for what values of B should the government's technology be used? How can such an equilibrium be decentralized?

(c) Assuming the government has no access to revenues except from the sale of good 1, for what values of B is it feasible for the government to use its technology? (Do not forget the existence of the competitive suppliers.) Show your answer in an $x_1$ - $x_2$ diagram. For what values of B should the government use its technology?

(d) Explain why the values of B for which the government should use its technology are different in parts (b) and (c).

3. Imagine a three-person economy in which good 1 is gardening services, the consumption of which makes one's yard more beautiful, and good 2 is food. Imagine that consumers 1 and 2 live in adjacent houses, while consumer 3 lives on the other side of a particularly large mountain. Consumption by consumer 3 of gardening services generates no externality for the other consumers, nor does consumer 3 care about the gardens of the other two consumers. Each of the other two consumers generates a positive externality for her neighbor through the consumption of gardening services. To be precise, imagine that consumers 1 and 2 have utility functions of the form

$$V^i(x) = w(x^1{}_1) + w(x^2{}_1) + x^i{}_2$$

where $w : [0,\infty) \to R$ is a strictly increasing, strictly concave, and differentiable function. Note that consumers 1 and 2 get just as much utility out of their neighbor's yard as they do out of their own, and their utility is linear in food. (You are warned that this is a very special setting.) Also imagine that consumer 3 has a utility function of the form $V^3(x) = w(x^3{}_1) + x^3{}_2$. there is an aggregate endowment of gardening services and food.

(a) Suppose the aggregate endowment is allocated evenly among the three consumers. What will be the Walrasian equilibrium (with externalities)?

(b) Characterize the set of Pareto efficient allocations of the social endowment. Is the equilibrium allocation in (a) Pareto efficient? (Kreps, 6.5.)

4. Consider a two-person (Ann and Bob) two-good (x and y) competitive exchange economy with externalities. Ann has utility function

$U^A = 2\text{Min}[x^A, y^A] - x^B$

Bob has utility function

$U^B = 4x^B + 4y^B$

Both have the nonnegative quadrant as a consumption possibility set. Ann's initial endowment is 12 units of x and 12 units of y; and Bob's initial endowment is 12 units of x and no y.

Describe the set of Pareto Optima.

5. There are N fishers in a community. Some of them fish in the ocean. The ocean is so large that each fisher can catch w fish no matter how many fishers go to sea. Some of the fishers fish in a lake. (Lake fish and ocean fish are perfect substitutes in consumption.) If there are x fishers on the lake, each of them catches $x^{-1/2}$ fish (i.e., $x^{1/2}$ fish are caught in total and each worker catches the same number).

(a) If each fisher is free to choose whether to go to the ocean or the lake and no one will go where he expects to catch fewer fish, how many fishers will go to the lake, how many to the ocean, and what will be the average catch?

(b) If the government restricts access to the lake, how many fishers should it allow on the lake to maximize the total catch in the community?

(c) Assume the government charges for a license to fish in the lake and sells as many licenses as fishers want to buy. What size lake fishing license supports this equilibrium? Assume the revenue from the licenses goes completely to consumers who do not fish. Are the fishers better off, worse off, or the same after the introduction of the optimal licenses, as opposed to no government intervention? Might your answer be different with a different technology on the ocean? Explain.

(d) Now assume that all the fish are sold and the demand for fish is:

$$Q = A - BP$$

Compare the price of fish in the free access and efficient allocations.

(e) Now assume that ocean fish and lake fish are <u>not</u> perfect substitutes. The demand for ocean fish is such that each fish is worth $2. The demand for lake fish is

$$Q_L = A' - B'P_L$$

How many lake fishers are there in free access equilibrium? If a positive license fee is charged for fishing in the lake, does the price of lake fish go up or down? What makes this case different from that above?