**Answer all three questions.**

1. (30 points) Consider a two-person (Ann and Bob) two-good (x and y) competitive exchange economy. Ann has utility function

$U^A = \text{Min}[x^A, y^A]$

Bob has utility function

$U^B = 4x^B + 4y^B$

Both have the nonnegative quadrant as a consumption possibility set. Ann's initial endowment is 12 units of x and 12 units of y; and Bob's initial endowment is 12 units of y and no x.

In answering the following questions, provide brief explanations, including relevant definitions, as to why your answer is correct. You may use algebra or geometry.

a. Describe the set of Pareto Optima. Be precise in your definition of Pareto optimality and careful about corner solutions.

b. Describe the core. Be precise in your definition of the core.

c. Describe the prices and quantities in competitive equilibrium. Is there more than one price ratio that supports competitive equilibrium? Explain.

d. For what other initial endowments are the competitive equilibrium quantities the same as in your answer to part c? Give the equilibrium prices that support the equilibrium for each such initial endowment

2. (35 points) Consider a two-good (apples, denoted x, and apple sauce, denoted y) competitive economy with two agents, Ann and Bob and one firm. Ann has utility function

$U^A = 4\log[x^A] + 4\log[y^A]$.

Bob, who is affected by the apples eaten (noisily) by Ann, has utility function

$U^B = 6\log[x^B] + 6\log[y^B] - 2x^A$ .

Both have the nonnegative quadrant as the consumption possibility set and each has an initial endowment of 4 apples.

There is a single competitive firm that can convert apples into applesauce. If it uses $x^f$ apples (with $x^f > k$), it produces $(x^f - k)^{1/2}$ units of applesauce. The firm is owned by Ann.

a. For what value or values of k does there exist a competitive equilibrium with positive production of applesauce and zero profits? For this value (or these values) of k, determine competitive equilibrium prices and quantities.
b. Is equilibrium Pareto optimal? Explain. Describe a tax policy that can restore Pareto optimality.

3. (35 points) Consider a one-period competitive economy with a complete set of contingent commodity markets. There are two states of nature; with the probabilities of states one and two being 1/3 and 2/3 respectively. There is one input good, z, which must be allocated before the state of nature is known. There are two consumer goods in each state of nature, x and y. There are two consumers, Ann and Bob, who are expected utility maximizers, using the correct probabilities.

Ann's utility function (for expected utility maximization) is:

$U^A = x^A + \log[y^A]$.

Ann's consumption possibility set is the four dimensional space with nonnegative quantities of each good in each state of nature. Ann has an endowment of 12 units of input.

Bob's utility function (for expected utility maximization) is:

$U^B = \log[x^B] + y^B$.

Bob's consumption possibility set is the four dimensional space with nonnegative quantities of each good in each state of nature. Bob owns all four firms that exist in the economy and 12 units of input.

For each unit of input used by firm 1, there are 2 units of good x produced if state 1 occurs and nothing if state 2 occurs. For each unit of input used by firm 2, there are 2 units of good x produced if state 2 occurs and nothing if state 1 occurs. For each unit of input used by firm 3, there are 2 units of good y produced if state 1 occurs and nothing if state 2 occurs. For each unit of input used by firm 4, there are 2 units of good y produced if state 2 occurs and nothing if state 1 occurs.

Derive the complete market competitive equilibrium prices and quantities. Be sure to show the derivation of equations that describe equilibrium. Be careful about corner conditions. Hint: Consider the production side of the economy before analyzing consumers in detail.