# PROBLEM SET #5

Due: class #11

1. Consider the following variant of question 3 on Problem Set #4:

Imagine a three-person economy in which good 1 is gardening services, the consumption of which makes one's yard more beautiful, and good 2 is food. Imagine that consumers 1 and 2 live in adjacent houses, while consumer 3 lives on the other side of a particularly large mountain. Consumption by consumer 3 of gardening services generates no externality for the other consumers, nor does consumer 3 care about the gardens of the other two consumers. Each of the other two consumers generates a positive externality for her neighbor through the consumption of gardening services. To be precise, imagine that consumers 1 and 2 have utility functions of the form

$$V^i(x) = w(x_1^1) + w(x_1^2) + x_2^i$$

where $w:[0,\infty) \to R$ is a strictly increasing, strictly concave, and differentiable function. Note that consumers 1 and 2 get just as much utility out of their neighbor's yard as they do out of their own, and their utility is linear in food. (You are warned that this is a very special setting.) Also imagine that consumer 3 has a utility function of the form $V^3(x) = w(x_1^3) + x_2^3$. Instead of endowments of both goods, there is an endowment of food and a continuum of firms that can convert food into gardening services one-for-one.

a. Assume each household has the same endowment of food. Find competitive equilibrium assuming $w'(0) > 1 > w'(e)$ where e is each household's endowment of food. Is equilibrium Pareto optimal? Explain.

b. Derive the equations for the allocation that maximizes the sum of utilities assuming that some food is eaten. What is a sufficient condition for the optimum to have this property?

c. How can your answer to question b. be decentralized?

d. Derive the optimum if the government must tax or subsidize gardening services at the same rate for everyone. Assume that everyone gets positive food consumption in the optimum. Can you imagine a very different optimum without this assumption?

2. Consider a competitive economy with 2 individuals, Ann and Bob. There is one non produced good (labor) in the economy and each individual has an endowment of 3 units of labor. There are 2 firms in the economy -- firm $F_A$ and $F_B$. Ann owns firm $F_A$ while Bob owns firm $F_B$. Both firms convert labor ($L$) into good $x$. The production functions are as follows:

$$\text{Firm } F_A: \quad x_A = L_A$$
$$\text{Firm } F_B: \quad x_B = 2\sqrt{L_B}$$

Also, production by firm $F_B$ creates pollution where pollution $z = x_B$ (i.e. output of firm $F_B$). Finally, utility of each is $U^i\left(L^i, x^i\right) = \left(3 - L^i\right)^{1/2} \cdot \left(x^i\right)^{1/2} - z/8; \quad i = A, B$ (note: $L^i$ represents $i$'s labor supply).

a. Derive the competitive equilibrium of this economy (i.e., equilibrium prices, production and consumption plans). [Restrict your attention to an interior allocation where (i) both firms are producing positive amounts of good $x$ and (ii) consumer choices are interior.]

b. Is the equilibrium a Pareto Optimum? Explain.

c. Assume the government fixes a limit z* on the amount of pollution emitted by firm $F_B$. As a function of z* solve for the resulting "regulated competitive equilibrium". [Assume that z* is set at a value less than that obtained in (a)].

d. Assume the government taxes pollution at the rate t per unit and gives the tax revenue to the two consumers equally per capita. As a function of the tax rate derive the equations for the resulting competitive equilibrium.

e. Discuss the differences in allocation between your answers in c) and d).

3. Consider an economy with many identical consumers. The economy lasts two periods. The consumers have (identical) endowments of trees (T). Each tree yields $f_1$ units of fruit in period 1 and $f_2$ units of fruit in period 2. Each consumer has utility function $u\left(x_1, x_2\right)$, where $x_i$ is fruit eaten in period $i$. There are competitive markets in trees, fruit in period 1 and fruit in period 2 that all clear before period 1. Derive the equation for the price of trees relative to the price of fruit in terms of the preferences and endowment. The fruit is not storable.

4. Consider a competitive economy in continuous time with a continuum of unit measure of identical households. Assume that all consumers have the utility function

$$u = \int_0^T e^{-rs} \log \left(c(s)\right) ds$$

where $T$ is the (known) end of time.

Assume that the economy begins with a (nondepreciating, nonaugmentable) stock $A$ of consumer goods owned equally by all consumers. Derive the equations for competitive equilibrium per capita consumption as a function of time. Derive the price of a unit of consumer goods as a function of time.

5. Consider a competitive economy in continuous time with a continuum of unit measure of identical households. Assume that all consumers have the utility function

$$u = \int_0^T e^{-rs} \log \left(c(s)\right) ds$$

where $T$ is the (known) end of time.

Assume that the economy begins with a stock $A$ of (edible) bushes owned equally by all consumers. At time $t$ each bush is of size $f(t)$, with $f(0) > 0$, $f'(t) > 0$, $f''(t) < 0$. No new bushes can be planted. Derive the equations for competitive equilibrium per capita consumption as a function of time. Derive the value of a standing bush as a function of time. (Hint: make use of the fact that some bushes are cut at each moment of time.)