Quiz Solution

Problem #1

This solution takes a step-by-step approach to determining SG tube wall temperature, primary and secondary fluid temperatures, quality and void fraction. Starting with the initial conditions provided at the tube inlet, an energy balance is performed between the primary and secondary.

Neglecting any work, or external energy gains or losses:

$$\Delta E_{primary} + \Delta E_{secondary} = 0 \quad \text{or,}$$

$$-\Delta E_{primary} = \Delta E_{secondary} \tag{1}$$

It will be assumed that the only mode of energy transfer will be convection.

$$\Delta E_i = \pi D \int q'' dz \tag{2}$$

Using Newton's Equation:

$$q''_{pri}(z) = h_{pri}(z)\left(T_P(z) - T_{wall,out}(z)\right) \tag{3}$$
$$q''_{\sec}(z) = h_{\sec}(z)\left(T_{wall}(z) - T_{\sec,in}(z)\right) \tag{4}$$

For this problem, the thermal conductivity of the SG tube is neglected, so there is not temperature drop across the tube thickness (which also is neglected).

$$T_{wall,out}(z) = T_{wall,in}(z) = T_{wall}(z) \tag{5}$$

Since the tube thickness is neglected, equations (2) – (5) can be applied to equation 1 to form a more explicit energy balance:

$$h_{pri}(z)\left(T_{pri}(z) - T_{wall}(z)\right) = h_{\sec}(z)\left(T_{wall}(z) - T_{\sec}(z)\right) \tag{6}$$

The task at hand is to track carefully the fluid conditions in the SG tube, starting from the tube inlet so as to allow equation (6) to remain continually in balance along the axial length of the tube. Note that convective heat transfer is the only mode considered. It is asserted without proof that for pressurized water at the temperature, pressure and flow-rates considered, other heat transfer mechanisms are negligible.

The procedure used in this solution to the problem is as follows:

1. The tube is discretized axially into a number of nodes. For this solution, 200 nodes were used. No analysis was performed to find the numerical sensitivity to the density of axial nodalization. The goal was to have a discretization that is sufficiently fine to allow wall temperature and void fraction to be found and plotted with, in rough terms, reasonable accuracy.
2. At the tube inlet, find the (assumed constant) heat transfer coefficient on the **primary side** of the SG tube. This reduces by one the unknown variables in equation (6). The primary side fluid is assumed to be subcooled, and thus, the heat transfer coefficient will be determined by the **Dittus-Boelter correlation**:

$$Nu_{\infty} = 0.023 \mathrm{Re}^{0.8} \mathrm{Pr}^{0.3} \qquad \text{(Text 2, eq. 10-946)}$$
$$Nu_{\infty} = 0.023(1.485E6)^{0.8}(1.47)^{0.3} = 2235.3$$
$$h_{pri} = \frac{Nu_{\infty} k}{D} = \frac{(2235.5)(0.448)}{(.0071)} = 1.42E5$$

3. **Initially** assume that the **secondary heat transfer** mode will be by subcooled forced convection, with the heat transfer coefficient given by the **Dittus-Boelter**:

$$Nu_{\infty} = 0.023 \mathrm{Re}^{0.8} \mathrm{Pr}^{0.4} \qquad \text{(Text 2, eq. 10-94a)}$$
$$Nu_{\infty} = 0.023(1.44E5)^{0.8}(0.934)^{0.4} = 358.3$$
$$h_{\sec} = \frac{Nu_{\infty} k}{D} = \frac{(358.3)(0.557)}{(.0085)} = 2.35E4$$

Note that it is assumed that the flow can be modeled as fully developed turbulent flow in circular tubes. For this solution, the effects of the SG tube bundle geometry was disregarded.

4. With the above approximations and the given primary and secondary inlet temperature, the inlet wall temperature $T_{wall}(0)$ can be estimated using a re-arrangement of equation (6):

$$T_{wall}(z) = \frac{h_{pri}(z) T_{pri}(z) + h_{\sec}(z) T_{\sec}(z)}{h_{pri}(z) + h_{\sec}(z)} \qquad (7)$$

5. Given the above calculations:

$$T_{wall}(0) = \frac{1.42E5(330) + 2.35E4(230)}{1.42E5 + 2.35E4} = 315.8$$

6. With this estimated wall temperature and heat flux, we must test to determine if nucleate boiling would occur. This is a sort of chicken-and-egg portion of the

problem. The wall temperature (and thus wall-temperature/saturation-temperature delta t) is determined with an assumed mode of heat transfer. If it is found that nucleate boiling would occur with this condition, one must go back and re-determine the heat transfer coefficient with the assumption that nucleate boiling is occurring. These will then, in turn, result in a new heat transfer coefficient, wall temperature and heat flux; different than those used to determine that there is nucleate boiling in the first place. One could wonder then, if one should go back and check (again!) if nucleate boiling should occur. It was decided for this problem that, if nucleate boiling is predicted after the initial computation under the assumption of subcooled forced convection is made, then there is nucleate boiling (period – no re-checking required.)

To check for the onset of nucleate boiling, the **Davis-Anderson correlation** is used

$$q''_{IB} = \left(\frac{k_L H_{fg} \rho_g}{8\sigma T_{sat}}\right)(T_w - T_{sat})^2 \qquad \text{(Text 1, eq. 4-3)}$$

Close inspection of this equation reveals that it is dimensionally inconsistent (British Units are used) – and a conversion factor of 778 (lbf-ft/BTU) is required. Additionally, it is better to re-arrange this equation to give the $T_{wall} - T_{sat}$ to indicate that incipient boiling would occur at a given heat flux. Performing this re-arrangement:

$$T_{wall} - T_{sat} = \left(\frac{q'' 8\sigma T_{sat}}{778 k_L H_{fg} \rho_g}\right)^{0.5} \quad \text{in British units this is:}$$

$$\Delta T = \left(\frac{(6.39E6)8(0.0011)(554.87)}{778(0.3219)(632.6)(2.47)}\right)^{0.5} = 2.87$$

For the previously assumed conditions of subcooled forced convection: $\Delta T = 45.1$ degrees F, therefore, **nucleate boiling must occur at the inlet**.

7. Heat transfer correlations in **the nucleate boiling region** were computed using the **Thom correlation**:

$$q'' = \frac{\exp\left(2p/8.7\right)}{(22.7)^2}(T_{wall} - T_{sat})^2 \qquad \text{(Text 2, eq. 12-28b)}$$

To obtain an expression for the heat transfer coefficient, the above expression is divided by the quantity $T_{wall} - T_{sec}$. This is needed because the heat flux will depend upon wall temperature. The wall temperature is, in turn determined by the heat transfer coefficient. (As compared to the primary side heat transfer coefficient, and the respective primary and secondary bulk temperatures.) This requires an iterative procedure. The algorithm used in this solution is provided in the included Matlab® files.

8. The heat flux has now been determined. The heat flux is a quantity that describes the amount of heat that flows from the primary coolant to the secondary water in the SG. It is assumed that $q''(z)$ is sufficiently smooth so as to be accurately approximated by a piecewise constant function. Therefore:

$$\dot{m}_{pri} c_{p,pri} \left(T_{pri}(z_{i+1}) - T_{pri}(z_i)\right) = \pi D q''(z_i) = \pi D \left\{h_{pri}(z_i)\left[T_{pri}(z_i) - T_{wall}(z_i)\right]\right\} \quad (8)$$

   The primary coolant mass flow rate and specific heat are assumed to be constant and given. The only unknown in equation (8) is: $T_{pri}(z_{i+1})$.

9. Similarly, on the secondary side, the analogous heat balance is performed. Since the secondary water enters the SG in a subcooled state, the water specific heat is used in conjunction with the secondary mass flow rate to determine the secondary temperature until the water reaches a saturated state. At that time, enthalpy is used to account for the transfer of energy from the primary coolant to the secondary water.

$$\dot{m}_{\sec} c_{p,\sec} \left(T_{\sec}(z_{i+1}) - T_{\sec}(z_i)\right) = \pi D q''(z_i) = \pi D \left\{h_{\sec}(z_i)\left[T_{wall}(z_i) - T_{\sec}(z_i)\right]\right\} \quad (9)$$

or:

$$= \dot{m}_{\sec}\left(h_s(z_{i+1}) - h_s(z_i)\right)$$

In a similar fashion, the secondary quality is tracked for all axial levels.

10. When it is determined that equilibrium quality is equal to zero, the correlation for secondary heat transfer coefficient is shifted to the **Chen correlation** as given in NS Vol I equations 12-29 through 12-32. The details of this correlation are also provided in the accompanying Matlab® script. It is notable that this equation also requires iteration since the heat transfer coefficient depends of the difference between wall temperature and saturation temperature – with the wall temperature itself depending on the heat transfer coefficient.
11. This process is carried out for each axial level in the channel. It was determine using the Biasi CHF correlation that the Dryout CHF condition occurs at approximately 4 meters up the channel. It is expected that for a real SG, the thermal resistance of the SG tubes would cause the axial location of CHF to occur further up the channel.
12. The void fraction is computed using the **real quality**

$$x(z) = x_e(z) - x_e(Z_D)\exp\left[\frac{x_e(z)}{x_e(Z_D)} - 1\right] \qquad \text{(Text 2, eq. 12-22)}$$

$$\{\alpha(z)\} = \frac{1}{1 + \dfrac{1 - x(z)}{x(z)}\dfrac{\rho_v}{\rho_\ell} S} \qquad \text{(Text 2, eq. 5-55)}$$

and the given expression for slip as related to quality and (other) water properties:

$$\frac{u_v}{u_\ell} = .4 + (1 - .4)\left[\frac{\rho_\ell/\rho_v + .4(1/x(z) - 1)}{1 + .4(1/x(z) - 1)}\right]^{1/2}$$

To find the location of Z_D, the **Saha and Zuber criterion** was used:

For Pe < 7x10^4 :

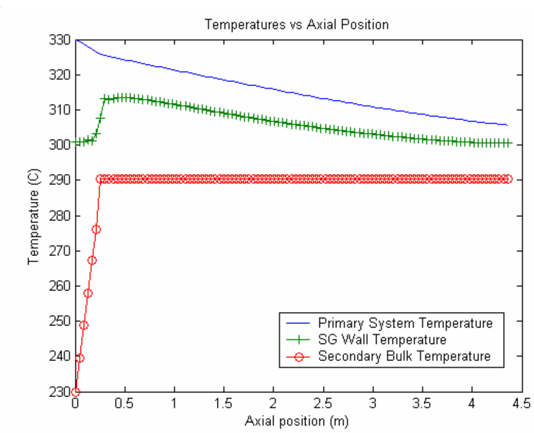
$$(Nu)_{Dep} = 455 \quad \text{or} \quad T_{sat} - T_{bulk}(z) = 0.0022\left(\frac{q''(z)D_e}{k_\ell}\right) \qquad \text{(Text 2, eq. 12-21a)}$$

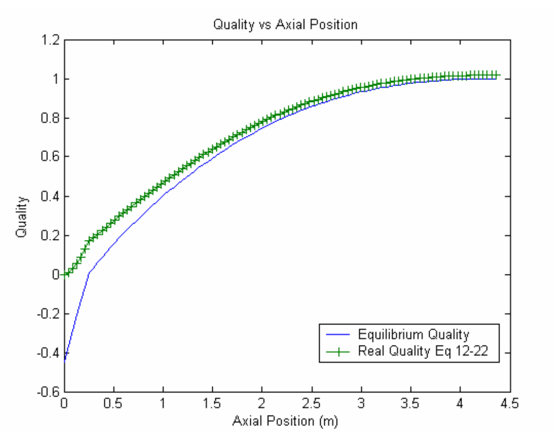
For Pe > 7 x 10^4:

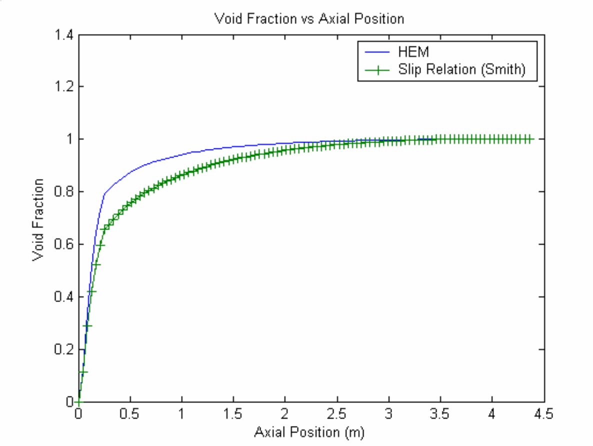
$$(St)_{Dep} = 0.0065 \quad \text{or} \quad T_{sat} - T_{bulk}(z) = 154\left(\frac{q''(z)}{Gc_{p\ell}}\right) \qquad \text{(Text 2, eq. 12-21b)}$$

For this problem, the Peclet number for the secondary fluid was ~ 134,000 , so use equation 12-21b. For this problem, heat flux and bulk temperature was tracked up the axial length of the tube. At the location where the right hand side of 12-21b is greater than or equal to the left hand side, the criteria was satisfied, and the point of bubble departure Z_D was found also to occur at the channel entrance.

Using the above computed information, the following **plots** were produced:







Part B:

As a simple mathematical relationship, if the secondary side heat transfer coefficient approaches zero – as can be seen by equation (7):

$$T_{wall}(z) = \frac{h_{pri}(z)T_{pri}(z) + (0)T_{\sec}(z)}{h_{pri}(z) + (0)} = \frac{h_{pri}(z)T_{pri}(z)}{h_{pri}(z)} = T_{pri}(z)$$

Physically this can be equated to an insulation of the secondary water. Insulation prevents the passage of heat flux, and thus, on the primary side, where there is a small heat flux (but a constant heat transfer coefficient) the temperature drop is small. Conversely, from the secondary side where the heat transfer coefficient is vanishingly small, even a small heat flux requires a large temperature drop as a driving force.

Problem #2

In general for liquid and vapor entering a vertical tube and rising

$$j_v = u_b + j_\ell$$

Where the rise velocity of the inertia controlled slug bubble u_b is given as

$$u_b = 0.35(gd)^{1/2} \left(\frac{\rho_\ell - \rho_v}{\rho_\ell}\right)^{1/2}$$

In this case only vapor enters the bottle so:

$$j_V = 0.35(gd)^{1/2}\left(\frac{\rho_\ell - \rho_V}{\rho_\ell}\right)^{1/2}$$

Now

$$V = \frac{\pi d^2}{4} j_V\, t$$

so

$$t = \frac{4V}{\pi d^2}\frac{1}{0.35(gd)^{1/2}}\left[\frac{\rho_\ell}{\rho_\ell - \rho_V}\right]^{1/2}$$

for V = 10^{-3}m^3, d = 0.02m, $\rho_\ell = 10^3\,\mathrm{kg/m^3}$, $\rho_g = 1.25\,\mathrm{kg/m^3}$

t = 20.5 seconds (compared to 15.84 sec for flooding controlled homework case)

Problem #3

1. The space above beer in a capped bottle while predominately filled with CO_2 also contains water vapor (since beer is mostly water). When the bottle is opened, the gasses/vapor in the neck expand rapidly into the surroundings causing the temperature in the neck to drop rapidly. The cooler temperature causes the water molecules in the neck to move so sluggishly that by chance several combine to form small embryos that serve as sites for further condensation. Particles in the neck region at the time of bottling likely have settled to the surfaces by gravity (large ones) or by diffusion (small ones). In a word the observed phenomena is homogeneous nucleation.

2. Overwhelmingly nucleation sites on the bottom and sides of the glass.

3. Dissolved carbon dioxide in the supersaturated beer diffuses into the gas in these sites. It also diffuses into the rising bubbles (see question 4).

4a. Observed bubble size 0.2 mm.
From figure 3.4 (corrected) from Whalley given in class, the rise velocity is then 2 cm/s.

4b. As the bubble rises it expands: Its diameter change is due to decreasing external pressure and increasing CO_2 mass, which is diffused from the beer through the bubble wall. Analytic determination of bubble diameter with axial position requires simultaneous solution of the ideal gas law, bubble mechanical energy balance and diffusion law for dissolved CO_2 into the bubble. The important parameters are:

- surface tension
- gas constant
- external beer pressure
- beer temperature
- CO_2 diffusion coefficient and concentration

I believe bubble growth is dominated by diffusion of dissolved carbon dioxide in the supersaturated beer into the rising bubbles since the external pressure change over the glass height is small.

To predict change in bubble diameter we must know the time available for diffusion. If we take the rise distance as 1.5 inches form and initial rise velocity of 2cm/s, then the first estimate of that time is about 1.9 seconds (somewhat long)

$$\frac{1.5\,(2.54)}{2} = 1.9 \text{ seconds}$$

However, with this time one could start to solve the CO_2 diffusion kinetics and calculate the increase in mass of CO_2 in the bubble. The problem needs an iterative solution since per fig 3.4, Whalley, the velocity increases dramatically with diameter.

Estimate the bubble diameter increases by 3X over the rise distance of 1.5 inches. At a diameter of 0.6mm, from fig 3.4, the rise velocity is of order 40 cm/s. For this velocity a bubble could traverse the 1.5” distance in 0.1 sec (a more satisfying duration).

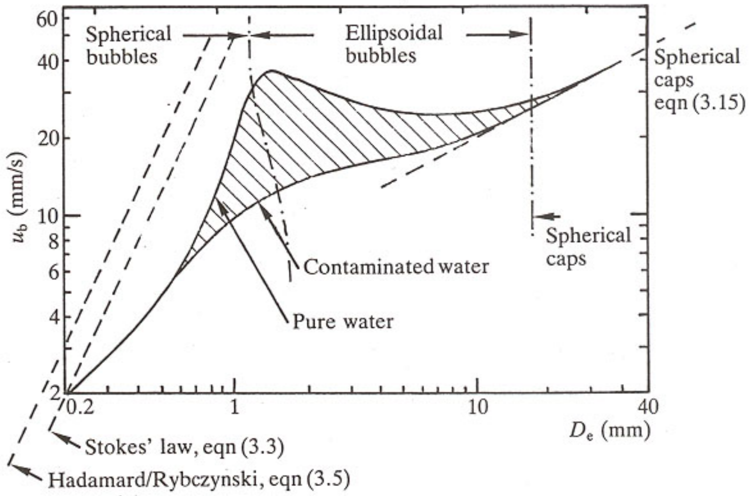


Fig. 3.4. Rising velocity of single air bubbles in water. (Whalley, 1987)

Note: Units of u_b should be cm/s versus mm/s