14.472 Handout and **Problem Set**
Spring 2002

Social Security in OLG models with certainty and fixed wage, w, and interest rate, r

This analysis can be extended to the more complicated nonlinear aggregate production function by treating w and r as functions of k, rather than as parameters. The dynamics in the linear case are simpler since there is adjustment to a new steady state equilibrium in one period.

Note there are errors in my BPEA paper - equations 13, 28, and 30.

n ... population growth rate

Consumer choice, inelastic labor supply, no taxes, no social security

1. Max $u[c_1, c_2]$

2. $c_1 + c_2/(1+r) = w$

This gives optimal first period consumption:

3. $c^*[w, r]$

The level of capital per young worker (of the next generation) comes from market clearance:

4. $(1+n)\,k = w - c^*[w, r]$

Now add payroll tax financed social security and a lump sum tax (used to finance the public debt). The budget constraint becomes

5. $c_1 + c_2/(1+r) = w(1-t) - T + b/(1+r)$

First period consumption becomes

6. $c^*[w(1-t) - T + b/(1+r), r]$

Given earmarking, we have government budget balance constraints in the non-social security and social security budgets. Assume government debt outstanding, no other government expenditures.

f=social security funding per young person paying taxes

g=debt per young person in the next cohort

Note that these have different denominators as ratios and so the dollar figures differ by a factor of $(1+n)$.

Lump-sum taxes need to cover the difference between the interest cost on the debt and the part covered by issuing more debt to preserve debt per worker:

7. $T = (r-n)\,g$

Financing for social security benefits come from payroll tax revenues and part of the interest earned by the trust fund:

8. $b = tw\,(1+n) + (r-n)\,f$

Assume that funding is financed by a fixed fraction of revenues:

9. $f = \beta tw$

Then, the social security budget constraint becomes:

10. $b = tw\,(1+n) + (r-n)\,\beta tw = tw(1+(1-\beta)n+\beta r)$

If we have full funding, $\beta = 1$,and $b = tw(1+r)$.

Market clearance

11. $(1+n)\,(k+g) = w\,(1-t) - T - c^*[w(1-t) - T + b/\,(1+r)\,, r] + f$

Problem Set Question 1.
Using the government budget constraints, calculate the derivative effects on capital accumulation and worker utility of (1) using a temporary lump sum tax increase on the young to finance a derivative decrease in the public debt, with the lower level of public debt (per worker) preserved thereafter, and (2) a temporary lump sum tax increase to finance a derivative increase in the level of funding of social security, with the increased funding (per worker) preserved thereafter. Contrast the answers to (1) and (2) and explain.

Problem Set Question 2.
Consider an economy where the fraction α of workers (in every cohort) follow the standard consumer choice problem discussed above, while the fraction $1-\alpha$ of workers do no saving, just consuming their after tax wages and social security benefits.

a. Set up the equations determining the level of capital in the economy.
b. Using the government budget constraints, calculate the derivative effects on capital accumulation and worker utility of (1) using a temporary lump sum tax increase on the young to finance a derivative decrease in the public debt,

with the lower level of public debt (per worker) preserved thereafter, and (2) a temporary lump sum tax increase to finance a derivative increase in the level of funding of social security, with the increased funding (per worker) preserved thereafter. Contrast the answers to (1) and (2) and explain.

Problem Set Question 3.

Using the same setup as in Question 1, assume that the technology is not linear but Leontief. That is, assume that output per worker equals

12. Min $\{y, ak\}$

where a is a parameter such that both labor and capital are fully employed in equilibrium.

a. Set up the equations for equilibrium, recognizing that w and r are now endogenous variables.

b. Using the government budget constraints, calculate the derivative effects on capital accumulation and worker utility of using a temporary lump sum tax increase on the young to finance a derivative decrease in the public debt, with the lower level of public debt (per worker) preserved thereafter.