14.41 Public Economics, 2002
Problem Set # 3 Solutions

1)
a)
BC:

$$C_1 + \frac{C_2}{1+.05} = 100 + \frac{50}{(1+.05)} \quad (1)$$
$$\frac{C_2}{1.05} = 147.62 - C_1 \quad (2)$$
$$C_2 = 155 - 1.05 * C_1 \quad (3)$$

$$max[log(C1) + log(155 - 1.05 * C_1)] \quad (4)$$
$$\frac{\partial}{\partial C_1} = \frac{1}{C_1} + \frac{-1.05}{155 - 1.05 * C_1} = 0 \quad (5)$$
$$C_1 = 73.810 \quad (6)$$
$$C_2 = 155 - 1.05 * 73.810 = 77.5 \quad (7)$$
$$S = 100 - C_1 = 26.19 \quad (8)$$

b)
New BC:

$$C_1 + \frac{C_2}{1.05} = (100 - 10) + \frac{50}{1.05} + 10 * \frac{1.05}{1.05} \quad (9)$$
$$C_1 + \frac{C_2}{1.05} = 100 + \frac{50}{1.05} \quad (10)$$

In the new BC, you are taxed \$10 in the first period, but get 10*(1+.05) in the second period (which is discounted by 1+.05 because you get it in period 2. The BC can be rewritten to show that it is the same as the BC in part a. Since there is no effect on the BC, there is no effect on the outcome.

What happens is that public savings crowd out private savings dollar-for-dollar. Individuals "undo" the social security system by reducing their private savings by the amount that the government saves for them. New private savings are \$16.19 per person, public savings are \$10 per person and national (or total) savings are \$26.19 per person and therefore are unchanged from part a. This is called a funded social security system. (Note that if the tax was greater than the original amount of savings and the individual could not borrow against second period income, he could not completely undo the government's action.)

c)
New BC:

$$C_1 + \frac{C_2}{1.05} = 90 + 10 * \frac{1.05}{1.05} \quad (11)$$
$$C_2 = 1.05 * (100 - C_1) \quad (12)$$

$$\max[\log C_1 + \log(1.05 * (100 - C_1))] \quad (13)$$
$$\frac{\partial}{\partial C_1} = \frac{1}{C_1} - \frac{1.05}{1.05 * (100 - C_1)} = 0 \quad (14)$$
$$C_1 = 50 \quad (15)$$
$$C_2 = 52.5 \quad (16)$$

Private savings is now higher than in parts a and b. The reason is that now your retirement period is longer, so you have to save more during your working life to consume during retirement. So while a simple SS system will crowd out private savings 1-for-1 if there is no change in labor supply, savings may increase if the SS system leads people to retire earlier.

d)

Social security may distort retirement decisions. The key is that the program exists to insure you against the event of being too old to work, but since retirement is in your control and you have to be retired to get the benefits, you may retire to become eligible. In particular, if the benefit to you of waiting (higher benefit in the future) is not enough to offset your loss of benefits today, you will be more likely to retire early.

There has been a big decline in the labor force participation of older men since WWII, during the period that social security has become more generous. But it is hard to say if this relationship is causal; a lot of other stuff has also changed. More convincing is the fact that lots of people retire right at 62 when they first become eligible. In general, the microdata evidence from the US suggests that perhaps 1/3 of the decline in labor force participation is due to social security. International evidence suggests that incentives do matter; the higher the "tax" on work due to lost social security benefits, the more likely that people will be retired.

2)

a) No, the fact that people who do not receive UI are unemployed for a shorter period of time does not prove that UI causes longer durations of unemployment. There are several differences between those who receive UI and those who don't:

1) only people who are laid off are eligible for UI, not those who are fired or quit;

2) of those who are eligible for UI, only $\frac{2}{3}$ decide to take up benefits. One alternative explanation for Bush's table is that people who know they can find another job easily don't bother to apply for UI. The key is that receiving UI is not a random experiment with a treatment and control group, so there could be other differences between the groups that account for the difference in outcomes.

Better evidence about the effect of UI on unemployment durations can be found in the work of economist Bruce Meyer. In one study of state law changes (a natural experiment where some people were affected by the law change and others weren't), he found that a 10% increase in benefits was associated with an 8% increase in durations. In another study where he looked at the probability of going back to work each week conditional on being unemployed for that amount of time (the hazard rate), he found that lots of people found jobs at 26 weeks, when their unemployment benefits were running out. Both these studies suggest that UI benefits do affect the duration of unemployment.

b) No, longer durations of unemployment could lead to better job matches, which is a benefit to society. We do not want brain surgeons working at McDonald's just because it takes a little while to find a new brain surgeon position. Relatively generous benefits allow people to take the time they need to make a good job match. However, the Meyer study of hazard rates and evidence showing that people who are unemployed longer do not get higher wages suggests that this is not a very important consideration in reality.

c)

(i) With individual perfect experience rating, firms don't pay any of the cost of layoffs, so we would expect more layoffs than under a policy of firm perfect experience rating, where firms pay the cost. (If firms are able to pass the cost of unemployment benefits through to the worker in the form of lower wages, then individuals may pay under both policies.) With individual perfect experience rating, we would expect unemployment durations to be shorter; since individuals have to pay back all the benefits they receive while unemployed, they stay unemployed only long enough to find a good job match.

(ii) Individual perfect experience rating provides individuals with good incentives to find a job quickly, but does not provide them any insurance against becoming unemployed (they bear the full cost of the layoff). Firm perfect experience rating provides individuals with insurance against layoffs, but does not provide any insurance for firms (though it gives the firm the right incentives about whether to lay off a worker). As always with social insurance, there is a tradeoff between incentives and insurance because of moral hazard.

3.)

a) Write down the expected utility function of the worker if she purchases insurance.

$$E(U) = (1-q) * \log[W-p] + q * \log[b] \tag{17}$$

b)

Use the no profit condition implied by the perfect competition in the insurance market.

$$E(\pi) = (1-q)*p - q*b = 0 \quad (18)$$

$$b = \frac{(1-q)*p}{q} \quad (19)$$

Given this budget constraint, we can go back and maximize expected utility. Plug in b from equation 19

$$E(U) = (1-q)*log(W-p) + q*log(\frac{(1-q)*p}{q}) \quad (20)$$

$$\max_p E(U) \quad (21)$$

$$\frac{\partial E(U)}{\partial p} = \frac{-(1-q)}{W-p} + \frac{q}{\frac{(1-q)*p}{q}} * \frac{1-q}{q} = 0 \quad (22)$$

$$\frac{\partial E(U)}{\partial p} = \frac{-(1-q)}{W-p} + \frac{q}{p} = 0 \quad (23)$$

$$p^* = W*q \quad (24)$$

c)
Plug equation 24 back into the utility function.

$$E(U) = (1-q)*\log[W-p] + q*\log[\frac{(1-q)*p}{q}] \quad (25)$$

$$E(U) = (1-q)*\log[W-Wq] + q*\log[\frac{(1-q)*Wq}{q}] \quad (26)$$

$$E(U) = (1-q)*\log[W-Wq] + q*\log[(1-q)*W] \quad (27)$$

$$E(U) = (1-q)*\log[(1-q)W] + q*\log[(1-q)*W] \quad (28)$$

$$E(U) = \log[(1-q)W] \quad (29)$$

The worker desires full insurance.

d)
The insurance market will function as long as the type s workers participate in the market. If only the type i workers participate, the insurance companies will go bankrupt and the market will collapse. The type s workers participate if their expected utility in the insured state exceeds their expected utility in the uninsured state.

$$E_1(U) = (1-q_s)*log(W-p) + q_s*log(\frac{1-q_a}{q_a}*p) \quad (30)$$

$$E_2(U) = (1-qs)*log(W) + q_s*log(0) \quad (31)$$

Worker's of type s participate it $E_1(U) > E_2(U)$. This will always hold because the log of 0 = - ∞. No matter how actuarially unfair the insurance is, the type s workers will purchase it. This is because of the nature of their utility functions. You could think of this utility function as indicating the worker's starve to death if they become disabled and are uninsured.

4)

(a) (i) Expected utility of seller 1 and 2 :

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$$E_1(U) = (\frac{1}{2} * (64)^{\frac{1}{2}}) + (\frac{1}{2} * (0)^{\frac{1}{2}}) = 4 \tag{32}$$

$$E_1(U) = (\frac{1}{2} * (0)^{\frac{1}{2}}) + (\frac{1}{2} * (64)^{\frac{1}{2}}) = 4 \tag{33}$$

(ii) The key here is that utility is maximized when income is smoothed over the two periods. Thus, an arrangement can clearly be struck which will make both sellers better off - the seller which gains from the weather outcome can agree to pay something to the seller that loses. Since there is diminishing marginal utility of income, this will make both parties better off by smoothing their income.

The insurance arrangement which will maximize the total societal utility is that the sellers will agree that whoever wins from the weather the next day will give one-half of his proceeds to the other seller. This will give them each a utility of $(32)^{\frac{1}{2}}$, which is 5.66. In total, social utility will increase from 8 in part (i) to 11.32. You should be able to see that any other division of income leaves you with lower social utility.

(iii) If it is announced that it will be sunny, then there will be no market for insurance. This is because the sun tan lotion vendor knows he will get all of the income the next day, so there is no need for him to enter into an insurance arrangement with the jacket salesman. This gives the sun tan lotion vendor an expected utility of 8, and the jacket salesman an expected utility of 0. Total social utility is the same as in (i), but it has fallen from (ii). This is because an efficient insurance market has been destroyed by the introduction of this information.

(b) (i) Expected utility of seller 1 and 2

$$E_1(U) = (\frac{1}{2} * \frac{1}{2} * 64) + (\frac{1}{2} * \frac{1}{2} * 0) = 16 \tag{34}$$

$$E_1(U) = (\frac{1}{2} * \frac{1}{2} * 0) + (\frac{1}{2} * \frac{1}{2} * 64) = 16 \tag{35}$$

(ii) There are no gains from the private insurance market here. Any redistribution of income from state A to state B leaves both individuals equally well off.

This is because there is no longer diminishing marginal utility of income. With linear utility, there are no gains from insurance. So any insurance arrangement which transfers income will leave society equally well off.

(iii) For the reasons in (ii), introducing the information about the weather does not affect the total social utility; it remains at 32. The only difference is that now the sun tan lotion vendor has all the money and the jacket salesman has none. Once again, since constant marginal utility meant that the introduction of insurance didn't make us any happier, it means that destroying the insurance market doesn't make us less happy.