### 14.41 Public Economics

Problem Set \#2 Solutions
1)
a) Evaluate at $0 \%, 2 \%, 5 \%$ and $10 \%$.

If you assume that the first payment comes the year after the project is completed:
$\mathrm{NPV}=-300,000+70,000 /(1+r)+70,000 /(1+r)^{2}+70,000 /(1+r)^{3}+70,000 /(1+r)^{4}$ $+70,000 /(1+r)^{5}-30,000 /(1+r)^{6}$

$$
\begin{gathered}
-300000+\sum_{i=1}^{5}\left(\frac{70000}{(1+0)^{i}}\right)-\frac{30000}{(1+0)^{6}}=20000 \\
-300000+\sum_{i=1}^{5}\left(\frac{70000}{(1+.02)^{i}}\right)-\frac{30000}{(1+.02)^{6}}=3303.0 \\
-300000+\sum_{i=1}^{5}\left(\frac{70000}{(1+.05)^{i}}\right)-\frac{30000}{(1+.05)^{6}}=-19323 . \\
-300000+\sum_{i=1}^{5}\left(\frac{70000}{(1+.1)^{i}}\right)-\frac{30000}{(1+.1)^{6}}=-51579 .
\end{gathered}
$$

At a 0\% interest rate, NPV $=20,000$
At a $2 \%$ interest rate, NPV $=3,303$
At a 5\% interest rate, NPV $=-19,323$
At a $10 \%$ interest rate, NPV $=-51,579$
The project should be undertaken at the $0 \%$ and $2 \%$ interest rates, but not at the $5 \%$ or $10 \%$ interest rate.
b) If the government is risk neutral, insert the expected value of the gamble into the PV calculation. The expected value is : . $5 * 10,000+.5 * 30,000=20,000$. The project should be undertaken if $r=0 \%, 2 \%$, but not undertaken if $r=5 \%, 10 \%$.

$$
\begin{gathered}
-300000+\sum_{i=1}^{5}\left(\frac{70000}{(1+0)^{i}}\right)-\frac{20000}{(1+0)^{6}}=30000 \\
-300000+\sum_{i=1}^{5}\left(\frac{70000}{(1+.02)^{i}}\right)-\frac{20000}{(1+.02)^{6}}=12183 \\
-300000+\sum_{i=1}^{5}\left(\frac{70000}{(1+.05)^{i}}\right)-\frac{20000}{(1+.05)^{6}}=-11861 \\
-300000+\sum_{i=1}^{5}\left(\frac{70000}{(1+.1)^{i}}\right)-\frac{20000}{(1+.1)^{6}}=-45934
\end{gathered}
$$

If the government is risk averse, a certain 20,000 is preferred to an outcome with an expected value of 20,000 . This means that the government would be willing to pay a little bit extra to make the cost certain. That is, the government feels indifferent between an expected cost of 20,000 and a certain cost of $20,000+c$, where $c$ is a positive number. This higher, certain cost is called the "certainty equivalent". The actual value of the certainty equivalent depends on the degree of risk aversion. The important point to make in your answer is that the government should first calculate its certainty equivalent of the cost and then take the PDV, using the certainty equivalent, to evaluate this risky project.
2)
a) The marginal valuation of road repair is $(\mathrm{Y} / 10)$-R. For professors, this expression equals 20-R; for students, this expression=10-R.
The marginal cost is 5 for both professors and students.
b) Professors want $20-R=5$, so that road repair $R=15$. Students want $10-R=5$, so that road repair $\mathrm{R}=5$.
c) Brookline provides $R=15$ and the Brookline students are unhappy with the outcome. Boston provides $\mathrm{R}=5$ and the Boston professors are unhappy with the outcome.
d) The Boston professors will move to Brookline and the Brookline students will move to Boston. In equilibrium, there will be 75 professors in Brookline and 75 students in Boston. Everyone is satisfied and the provision of road repair is efficient because everyone in each town values road repair equally.
e) Now, Brookline will provide the quantity of street cleaning that can be paid for with its revenue: $\left(50^{*} 75\right)+\left(25^{*} 25\right)=4375 \rightarrow>=11.67$ in Brookline. Boston will collect revenue equal to $\left(25^{*} 75\right)+\left(50^{*} 25\right)=3125 \rightarrow R=8.33$ in Boston. Now, students will all want to move to Brookline, because they can get more road repair in Brookline without paying higher taxes. The state regulation breaks down the Tiebout sorting mechanism.
3)
a. There are several points to make here:

- Most, if not all, returns to college are private. So individuals internalize the benefits of going to college and make the correct choice. No role for
government subsidization.
- However, there may be a failure in capital markets (can't get loans for college because no collateral). In this case, the government should provide loans, not grants.
- This program is inequitable: no grants if family's income is under $\$ 20,000$.
- It also matters whether you believe the human capital model or screening/signaling model of education. If you believe signaling, then this policy may dilute the value of the signal (by sending less smart people to college), forcing others to get more education to send a signal. This is actually a negative externality resulting from the scholarship program.
b. Again, there were several points to make here:
-Elementary education generates positive externalities (more educated voters, less crime, benefits from all knowing the same language, etc.), so there is justification for government spending here.
- But the Peltzman point is relevant: higher spending on public elementary education may lead some parents to move their kids from private to public schools. Total spending on education may be lower due to this crowdout effect.
- Also, this program is more equitable than the Hope Scholarship because it does not exclude kids from poor families from benefitting.
c. This is a question about Tiebout:
- Tiebout says that voters move with their feet and sort into homogeneous communities, so that they have exactly level of public goods spending they desire.
- The idea of Tiebout sorting has numerous limitations (imperfect mobility, moving costs, must live near job, imperfect information, not enough communities, many public goods factor into decision, positive spillovers, economies of scale in public goods production, ability to zone).
d.

The block grant is like an increase in income, so there is a parallel shift out of the BC, except that the question specifies that the grant can only be spent on education, so the most you can spend on other goods is your original income. The matching grant lowers the price of education, so the BC rotates outward (the new slope is flatter). With the block grant, there is an income effect, so you spend more on education. With the matching grant, there is an income effect and also a substitution effect (the relative price of education is lower, so choose more education). Since the matching grant has the income and substitution effects working in the same direction, the matching grant leads to a larger increase in education (although if start at a very low level of education and block grant is large, block grant could lead to larger increase).
e. Again, there are several issues:

- Lowering state tuition has crowdout problem (Peltzman point): some people decide to switch from private to public school and total education spending may decrease.
- Vouchers lead to more competition among schools.
- Vouchers help the inframarginal person (the rich kid who would have gone to private school anyway now receives some funding from the government).

4) 

(a)
costs:

$$
\sum_{i=0}^{6}\left(\left(\frac{-45000000}{(1+.05)^{i}}\right)+\left(\frac{-20000000}{(1+.05)^{i}}\right)+\left(\frac{-30 * 15 * 100000}{(1+.05)^{i}}\right)\right)=-6.6833 \times 10^{8}
$$

benefit:
((infinite stream of benefits starting at time $=0)$ - (stream of benefits from time $t=0$ to time $t=6)$ ) $=$

$$
\left(\frac{(1+.05)}{.05} *((35 * 15 * 100000)+(40 * 100000))\right)-\left(\sum_{i=0}^{6}\left(\left(\frac{35 * 15 * 100000}{(1+.05)^{i}}\right)+\left(\frac{40 * 100000}{(1+.05)^{i}}\right)\right)=\right.
$$

$$
8.4322 \times 10^{8}
$$

Yes, proceed with the project.
(b)
cost:

$$
\sum_{i=0}^{6}\left(\left(\frac{-45000000}{(1+.05)^{i}} * .5\right)+\left(\frac{-100000000}{(1+.05)^{i}} * .5\right)+\left(\frac{-20000000}{(1+.05)^{i}}\right)+\left(\frac{-30 * 15 * 100000}{(1+.05)^{i}}\right)\right)=
$$

$$
-8.3541 \times 10^{8}
$$

benefit:

$$
\left(\frac{(1+.05)}{.05} *((35 * 15 * 100000)+(40 * 100000))\right)-\left(\sum_{i=0}^{6}\left(\left(\frac{35 * 15 * 100000}{(1+.05)^{i}}\right)+\left(\frac{40 * 100000}{(1+.05)^{i}}\right)\right)=\right.
$$

Yes, proceed with the project.
(c)
cost:

$$
\begin{aligned}
& \text { cost equivalent }+\sum_{i=0}^{6}\left(\left(\frac{-20000000}{(1+.05)^{i}}\right)+\left(\frac{-30 * 15 * 100000}{(1+.05)^{i}}\right)\right)-> \\
& -450000000+\sum_{i=0}^{6}\left(\left(\frac{-20000000}{(1+.05)^{i}}\right)+\left(\frac{-30 * 15 * 100000}{(1+.05)^{i}}\right)\right)=-8.4492 \times 10^{8}:
\end{aligned}
$$

benefit:

$$
\left(\frac{(1+.05)}{.05} *((35 * 15 * 100000)+(40 * 100000))\right)-\left(\sum_{i=0}^{6}\left(\left(\frac{35 * 15 * 100000}{(1+.05)^{i}}\right)+\left(\frac{40 * 100000}{(1+.05)^{i}}\right)\right)=\right.
$$

$$
8.4322 \times 10^{8}
$$

No, do not proceed. The costs are larger than the benefits given the risk aversion.

