

14.41 Public Economics  
Problem Set #1, revised  
Due: Friday, September 20, 2002

(1) The newly elected president of an island nation has pledged to reduce air pollution. The nation has no close neighbors – the only source of air pollution are the two domestic chemical plants run by firm A and firm B. Firm A has been in operation for 50 years and has a pollution abatement (i.e. reduction) cost of  $x^3$ , where  $x$  is a unit of pollution. Firm B, which operates a 6-month old plant has a pollution abatement cost of  $x^2$ . Assume that neither firm is initially engaging in pollution abatement. The per unit benefit to a unit of pollution abatement experienced by the island's citizens is constant at \$300.

- a. What is the socially optimal level of pollution abatement? How is the socially optimal level of abatement split between the two firms?
- b. The president considers engaging in command and control style quantity regulation and declares that each firm must engage in 80 units of pollution abatement. Is this socially optimal? Why or why not?
- c. Alternatively, the president considers providing a subsidy of \$300 per unit of pollution abatement. What is the per firm and total level of pollution abatement? Is this socially optimal?
- d. The president also considers issuing pollution permits and establishing a market for these permits. For reasons associated with the relative generosity of the firms to his recent election campaign, firm A is given permits such that it must engage in 100 units of pollution abatement if it fails to enter the permit market. Firm B is given permits such that it must engage in 60 units of pollution abatement if it fails to enter the market. Each permit allows the firm holding it to engage in one unit of pollution. Assume the market for pollution permits is perfectly competitive. Also assume that no firm may increase the amount of pollution it is engaged in (i.e. negative abatement is not allowed). This assumption restricts firm A to buying 100 or fewer permits and firm B to buying 60 or fewer permits.<sup>1</sup> What is the market clearing price for pollution permits? What level of pollution abatement does each firm engage in? (If you cannot solve this explicitly, you may receive some partial credit by explaining the process that would occur.) Is this socially optimal? How does the level of abatement compare here with part c? Dropping the perfect competition assumption, what problems are likely to arise in practice in the permit market?

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<sup>1</sup> Mathematically, this problem has 2 solutions, one of which is nonsensical in the context of the problem. This assumption rules out the nonsensical solution.

2) 100 commuters need to use a strip of highway to get to work. They all drive alone and prefer to drive in big cars - it gives them more prestige and makes them feel safer. Bigger cars cost more per mile to operate, however, since their gas mileage is worse. Worse yet, bigger cars cause greater permanent damage to roads.

More specifically, if  $w$  is the weight of the car, suppose that the benefits from driving are  $5w$ , while the costs are  $(1/2)*w^2$ . The damage to roads is  $(2/3)*w^3$ . Assume that individuals have utility functions of the form:  $U = x$ , where  $x$  are the net benefits from driving a car of a given size.

- i) What car weight will be chosen by drivers?
- ii) What is the optimal car weight? If this differs from (a), why?
- iii) Can you design a toll system which causes drivers to choose the optimal car weight?

b) Suppose that tastes change and drivers no longer care about their car weight. They are now only interested in minimizing the costs of driving. The costs of driving in the absence of government regulation are a function of the number of axles on a car - the gas costs per 100 miles driven is  $(1/6)*A^2$ , where  $A$  is the number of axles.

The government reestimates the damage to roads from driving and finds that damage is reduced by having more axles on a car. If a car has  $A$  axles, the damage to the road per 100 miles driven is  $20/A$ . Using more gas on the many axle cars also causes more pollution, so that the total pollution for a 100 mile trip is  $(1/3)*A^2$ .

- (i) What is the socially optimal number of axles?
- (ii) How can the government enforce this social optimum?

3) The small 'city' of Middleville has been hit by a crime wave. In response to this crime wave, a police department will be formed. The city has two residents : Mac and Gloria. Each of the two residents has a utility function over private goods ( $X$ ) and total policemen ( $M$ ), of the form:  $U = 2*\log(x) + 1*\log(M)$ . The total provision of policemen hired,  $M$ , is the sum of the number hired by each of the two persons:  $M = M_G + M_M$ . Mac and Gloria both have income of 100, and the price of both the private good and a fireman is 1. They are limited, for the purposes of this problem, to providing between 0 and 100 policemen.

a) How many policemen are hired if the government does not intervene? How many are paid for by Mac? By Gloria?

b) What is the socially optimal number of policemen? If your answer differs from (a), why?

c) Suppose the government is not happy with the private equilibrium, and it decides to provide 10 more policemen. It taxes Mac and Gloria equally to pay for the new hires. What is the new total number of policemen? How does your answer compare to (a)? Have we achieved the social optimum? Why or why not?

d) Suppose now that the government is still not happy, so it decides to provide 35 new policemen. It taxes Gloria 10 to pay for them, and it taxes Mac 25. What is the new total number of policemen? How many are provided by Mac, and how many by Gloria? How does this compare to the level of provision in (c), and why?

4) For each of the examples below, answer the following three questions:

i) Is there an externality? If so, describe it, including references to whether it is positive or negative, and whether it is a consumption or production externality. If you are uncertain, describe the conditions under which there may, or may not, be an externality

ii) If there is an externality, does it seem likely that private markets will arise that allow this externality to be internalized?

iii) If there is an externality, contrast the effectiveness of each of the following corrective measures that the government can use to address the problem:

- 1) direct quantity regulation
- 2) taxes/subsidies
- 3) permits with trading

For each of (1)-(3), discuss the advantages and disadvantages, and conclude as to which would be best. Make use of the Weitzman uncertainty framework where applicable; diagrams are not necessary.

- a) Smoking by individuals
- b) Nuclear waste production by nuclear power plants
- c) Research and development by a high-tech firm
- d) The owner of a house clearing the snow from the sidewalk in front of his home