

14.41 Public Economics
Fall 2002
Problem Set #1 Solutions

1)

a) The socially optimal level of pollution abatement will occur where the marginal cost of abatement (MC) equals the marginal benefit of abatement (MB). MB is constant at 300. Calculate the MC for each firm by taking the derivative of each firm's total cost function:

$$MC_A = 3x_A^2$$

$$MC_B = 2x_B$$

Set $MC = MB$ to obtain the socially optimal level of abatement for each firm.

$$3x_A^2 = 300; x_A^* = 10$$

$$2x_B = 300; x_B^* = 150$$

The total level of optimal abatement is 160, with firm A abating 10 units and firm B abating 150 units.

b) Under this type of quantity regulation, the optimal *level* of abatement, 160, will be obtained. However, this is not socially optimal. Under the socially optimal abatement described in a), the total cost of abatement is

$$x_A^3 + x_B^2 = (10)^3 + (150)^2 = 23500$$

. Under the quantity regulation, the total cost is :

$$x_A^3 + x_B^2 = (80)^3 + (80)^2 = 5.184 \times 10^5$$

The command and control achieves the optimal *level* of abatement, but it does so at a much higher total cost than the socially optimal abatement of x_A^*, x_B^* . (Note: You can also show this using a graph similar to the one used to illustrate the Hyatt/Polaroid example in lecture).

c) Each firm will engage in abatement until its marginal cost of pollution abatement equals

the subsidy of \$300. For example, if the marginal cost of abatement is a , where $a < 300$, then the firm will abate a unit of pollution and earn $(300 - a)$ by doing so. When the MC of abatement equals 300, it is no longer profitable to engage in abatement and the firm ceases to abate any more pollution. The calculations are identical to a). The pigouvian tax (or subsidy in this case) causes the firms to internalize the externality - the socially optimal pollution abatement is achieved.

d) Each firm will seek to minimize its cost of pollution abatement. The market for permits is perfectly competitive. The price of permits is therefore parametric - i.e. the firms consider the price fixed. The only choice variable for the firms is the number of permits to buy (or sell).

Set up the firm's cost minimization problem formally:

Let a =permits, p = price of permits

Firm A:

$\min_a [(100 - a_A)^3 + pa_A]$ is equivalent to

$\max_a [-(100 - a_A)^3 - pa_A]$ yields

$$(100 - a_A)^2 = p/3$$

solving for a_A yields firm A's permit demand function :

$$\left\{ a_A = 100 + \frac{1}{3} \sqrt{3} \sqrt{p} \right\}, \left\{ a_A = 100 - \frac{1}{3} \sqrt{3} \sqrt{p} \right\}$$

Firm B:

$\min_a [(60 - a_B)^2 + pa_B]$

$\max_a [-(60 - a_B)^2 - pa_B]$

$$(60 - a_B) = p/2$$

solving for a_B yields firm B's permit demand function

$$\left\{ a_B = -\frac{1}{2}p + 60 \right\}$$

In order for the market to clear, $a_A = -a_B$:

$$100 - \frac{1}{3}\sqrt{3}\sqrt{p} = -(-\frac{1}{2}p + 60)$$

Solve for p , the market clearing price, and then plug p back into the permit demand functions.

Solution:

$$p = 300$$

$$a_A = 100 - \frac{1}{3}\sqrt{3}\sqrt{300} = 90$$

$$a_B = -\frac{1}{2}300 + 60 = -90$$

Firm A purchases 90 permits from firm B at a market clearing price of 300 and engages in 10 units of pollution abatement. Firm B sells 90 permits to firm A and engages in 150 units of pollution abatement. These values represent the social optimum. Both the subsidy in c) and the permit market are capable of producing the social optimum.

(Note: If you use the $a_A = 100 + \frac{1}{3}\sqrt{3}\sqrt{p}$ solution for a_A , you will end up with firm A increasing pollution, i.e. engaging in negative abatement. This violates the stated assumption that no firm can increase its pollution level.)

In practice the market will not be perfectly competitive when there are only two participants. This may prevent the market from functioning properly. Another potential problem is political credibility. The market will only function if the participants believe the government will honor the property rights conveyed by the permits. If the firms believe the president will not be re-elected and a new president will not honor the property rights conveyed by the permits, the market will not function properly.

2)

a)

define $U(w)$ as individual utility and $S(w)$ as societal utility

(i)

$$\text{Max}_w U(W) = \max_w (5W - (\frac{1}{2})W^2)$$

$$\frac{\partial U_w}{\partial w} = 5 - w$$

F.O.C.: $5 - W = 0 - >$

$W_i^* = 5$

(ii)

$Max_w S(W) = \max_w (U(W) - \text{Road Damage}) =$
 $\max_w (5W - (\frac{1}{2})W^2 - \frac{2}{3}W^3)$

F.O.C.: $5 - W - 2W^2 = 0$

Solution is: $\{W = -\frac{1}{4} + \frac{1}{4}\sqrt{41}\}, \{W = -\frac{1}{4} - \frac{1}{4}\sqrt{41}\}$

A negative weight does not make sense, therefore

$$W_s^* = -\frac{1}{4} + \frac{1}{4}\sqrt{41} = 1.3508$$

$W_i^* > W_s^*$ since there is a negative externality.

(iii)

Charge a Pigouvian toll: the marginal (road) damage that drivers impose, $2W^2$, evaluated at the social optimum. In this case that turns out to be a tax rate of $2*(1.3508)^2 = 3.6493$ per W.

b)

(i)

$Min_A C(A) = \min(\text{Road Damage} + \text{Pollution} + \text{Cost of gas})$
 $= \min((20/A) + (1/3)A^2 + (1/6)A^2)$

F.O.C.:

$$\frac{-20}{A^2} + \frac{2}{3}A + \frac{1}{3}A = 0$$

$$A_s^* = 20^{\frac{1}{3}}$$

:

(ii) Charge Pigouvian tax: marginal road damage + marginal pollution damage evaluated at the social optimum, A_S^*

$$tax = \frac{-20}{A_S^{*2}} + \frac{2}{3}A_S^*$$

$$tax = -\frac{1}{3}(20^{\frac{1}{3}})$$

:

A negative tax is a subsidy.

3)

(a)

In the absence of government intervention, the total level of policemen provided will be decided by a Nash bargaining game between Mac and Gloria. We can solve for the outcome of this game by maximizing each individual's utility, taking the action of the other individual as given. For example, for Gloria:

$$\begin{aligned} \text{Max}U &= 2 * \log(X) + 1 * \log(P_G + P_M) \\ \text{s.t. } X + P_G &= 100 \end{aligned}$$

$$\text{Lagrangian: } L = 1 * \log(X) + 1 * \log(P_G + P_M) - \lambda * (X + P_G - 100)$$

F.O.C.:

$$4/X = \lambda$$

$$2/(P_G + P_M) = \lambda$$

Combining these F.O.C., and using the budget constraint, we obtain:

$$\frac{2 * (P_G + P_M)}{100 - P_G} = 1 \rightarrow P_G = \frac{100 - 2P_M}{3}$$

Using the symmetry of the problem, we obtain the same first order condition for Mac:

$$P_M = \frac{100 - 2P_G}{3}$$

Combining these, we can solve the optimal level for both individuals:

$$P_G = P_M = 20$$

b)

The socially optimal number of policemen is that which sets the sum of the marginal rates of substitution equal to marginal rate of transformation:

$$MRS_G = \frac{MU_P}{MU_X} = \frac{\frac{2}{P}}{\frac{4}{X}} = \frac{\frac{2}{P}}{\frac{4}{100-P_G}} = \frac{100-P_G}{2P}$$

$$MRS_M = \frac{100-P_M}{2P}$$

$$MRT = \frac{PRICE_P}{PRICE_X} = 1$$

$$\text{sum of MRS} = \frac{200-P}{2P} = 1 = MRT \longrightarrow P^* = 66.6$$

The socially optimal number of policemen is higher than the equilibrium amount in (a), since the free rider problem has led the individuals to underprovide in the private provision equilibrium.

c)

The government provides 10 more policemen, and pays for this by taxing Gloria and Mac 5 each. Thus, for Mac, the problem becomes:

$$\begin{aligned} \text{Max } U &= 2\log(X) + 1\log(P_G + P_M + 10) \\ \text{s.t. } X + P_G &= 95 \end{aligned}$$

Solving this problem for Mac, and then using the symmetry to get the first order conditions for Gloria, and combining, we find that:

$$P_G = P_M = 15$$

So the new total number of policemen is 40 (15 each from Gloria and Mac, and 10 from the government). This is the same as in part (a)! The government provision has been fully crowded out by cutbacks in private provision. This is because the government is providing less than either individual desired to provide ex-ante, and funding it by equal taxation on the two individuals. Thus, each person can get back to their original (free market) optimum by simply cutting back on their provision by exactly the amount they are taxed for government provision. Thus, we have fallen short of the social optimum.

d)

If the government provides 35 new policemen, and funds this by taxing Mac 25 and Gloria only 10, then the problem has lost its symmetry, and we have to do the calculations separately for the two individuals. Solving the problem for Gloria first, we have:

$$\begin{aligned} \text{Max } U &= 2\log(X) + 1\log(P_G + P_M + 35) \\ \text{s.t. } X + P_G &= 90 \end{aligned}$$

Finding the first order conditions, and using the budget constraint, we get:

$$P_G = \frac{20-2P_M}{3}$$

For Mac, the problem is:

$$\begin{aligned} \text{Max } U &= 2\log(X) + 1\log(P_G + P_M + 35) \\ \text{s.t. } X + P_M &= 75 \end{aligned}$$

Finding the first order conditions, and using the budget constraint, we get:

$$P_M = \frac{5-2P_G}{3}$$

Solving the two simultaneously, we obtain:

$$P_G = 10 ; P_M = -5$$

However, there is no way for Mac to provide negative policemen. Thus, Mac provides zero; he is at a *corner solution*. As a result, Gloria must re-evaluate the number of policemen that he will provide:

$$P_G = \frac{20-2P_M}{3} = \frac{20-0}{3} = 6 \frac{2}{3}$$

The total number of policemen provided is $41 \frac{2}{3}$, which is above that in (c). The reason is that now the government has actually done something, which is to force Mac to provide more than he wants to. This means that we don't just get crowding out.

4)

a)

(i) There is an externality from second hand smoke and possibly from your paying my health care costs if I am uninsured or elderly. This is a negative consumption externality. It is only present if second hand smoke actually has a negative effect on you, and if all my increased health care costs are not borne simply by me.

(ii) It seems unlikely that the private market will internalize this externality, because of the lack of property rights over "air", and because of the transactions costs involved in negotiating over the corrective price that would be paid.

(iii) Direct quantity regulation is clearly dominated here by either taxes or trading. Trading would have difficulties because of the transactions costs involved in negotiating over each

cigarette. Taxes therefore seem optimal.

b)

(i) There is an externality from the fact that the firms who produce the waste do not account for the long run effects that disposing of it might have on people. They will be tempted to just dump it in the river, since this is the cheapest, and they don't see the cost to others. This is a negative production externality.

(ii) Given the diffuse nature of the parties injured by toxic waste dumping, it seems unlikely that they could get together and negotiate with the firm. There is also the problem that the harm of toxic waste may happen years in the future, to individuals who cannot currently negotiate

(iii) In the Weitzman uncertainty framework, this is clearly a case where the marginal benefit of reducing dumping is quite steep, which suggests quantity regulations over taxation. Trading would offer efficiency gains over direct quantity regulation.

c)

(i) There may be an externality here from the fact that my firm's R & D can benefit other firms. This would be a positive production externality. However, it is only present if my R & D is general enough to be of use to others.

(ii) It is quite possible that a market could arise to internalize this externality, with other firms paying me to use my R & D. The key is whether I can stop firms who don't pay from getting access to the information anyhow.

(iii) Since there is tremendous uncertainty about both the costs and benefits of R & D, this is a clear case for the Weitzman uncertainty framework. It seems here that benefits are quite flat, so that taxes/subsidies would be preferred to quantity regulation in the Weitzman framework. It is difficult to envision trading permits for a positive externality; the permits would have to be for not doing R & D.

d)

(i) There is a positive production externality to the home owner clearing the sidewalk. Individuals not part of the home owner's household benefit from the clear sidewalk. It is easier to walk and the probability of injury is reduced.

(ii) It seems unlikely that a market can arise. The sidewalk is owned collectively - there are no firm property rights. The transaction costs would be prohibitive given the large number of individuals using the sidewalk and the small size of the payments that would be involved.

(iii) There is little uncertainty in this case, so there is no need for the Weitzman framework. Both the cost and benefits could easily be assessed. Permits will not work for a positive externality. Subsidies would involve high transaction costs. Subsidies are a possibility however. Direct quantity regulation - i.e. mandating that all home owners clear the sidewalks in front of their homes - is the most viable solution.