Problem Set 2

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1 Discrete Time Dynamic Programming

In the folder *DynamicProgramming*, read *tutorial.doc* and the comments in *DP.m*.

1) *DP.m* produces 5 plots. Describe briefly each of them (what they represent and how they are computed. I am not asking for the details of the computations. Simply show me that you understand the architecture of the program).

2) We have seen in the continuous time setup (in problem set 1) that the reaction of consumption to a positive technology shock depends on the elasticity σ. Using *DP.m*, give an estimate of the range of values of σ for which consumption decreases following a positive technology shock. In reality, do you expect σ to be in this range? (Hint: suppose σ is in this range. What would be the elasticity of savings with respect to the risk free interest rate? Is this realistic?)

3) Show numerically how the behavior of consumption is affected by the persistence of the shocks. Hint: Set the elasticity to 100. Find the persistence coefficient. Set it to .95 (I set it to this value originally so it should be .95 unless you've changed it). How is the consumption function? Set the persistence coefficient to 0.5. What happens? Explain the intuition using the answer to question 1.5) of Problem set 1.

4) Optional. (answer only if you have time). Check using the programs that iterating the Bellman equation on the policy function is indeed much faster than iterating on the value function. Why is it so?

2 Linear Difference Models and Rational Expectations. Existence and Uniqueness of the Equilibrium.

The basic reference is Blanchard and Kahn (Econometrica, 1980). Consider the linear model:

$$\begin{bmatrix} E_t\left(c_{t+1}\right) \\ k_{t+1} \end{bmatrix} = A \begin{bmatrix} c_t \\ k_t \end{bmatrix} + Bz_t \tag{1}$$

A is a $2 * 2$ matrix and B is a $2 * 1$ matrix.

$$\begin{aligned} A &= \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \\ B &= \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \end{aligned}$$

This is a system with **one predetermined** variable, k_t, and **one jump** variable c_t.

1) Using Olivier's notes (topic 2, page 7), describe how to obtain equation (1). Assume that z follows the $AR(1)$ process of page 8:

$$z_t = \rho z_{t-1} + \varepsilon_t$$

You do not need to solve for A and B. You need to show how one could obtain them.

2) We are looking for solutions to (1) such that:

$$\begin{aligned} \lim_{\tau \to \infty} E_t \left[c_{t+\tau} \right] &= 0 \\ \lim_{\tau \to \infty} E_t \left[k_{t+\tau} \right] &= 0 \end{aligned} \tag{2}$$

Why is it sensible to restrict ourselves to these solutions?

3) Assume that the matrix A can be decomposed into:

$$A = C^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} C$$

Where C is a $2 * 2$ invertible matrix. Show that λ_1 and λ_2 are the eigenvalues of A.

4) Change the variables from (k_t, c_t) to $(x_{1,t}, x_{2,t})$ defined by:

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} = C \begin{bmatrix} k_t \\ c_t \end{bmatrix}$$

Show that for $i = 1, 2$:

$$E_t \left[x_{i,t+\tau} \right] = (\lambda_i)^\tau \times \left(x_{i,t} + \frac{m_i}{\lambda_i} \times \frac{1 - \left(\frac{\rho}{\lambda_i} \right)^\tau}{1 - \frac{\rho}{\lambda_i}} \times z_t \right)$$

Where are m_1 and m_2 coming from?

5) Assume that $\lambda_1 < 1 < \lambda_2$. Show that there is exactly one solution to (1) satisfying (2).

6) What happens if $\lambda_1 < \lambda_2 < 1$? What happens if $\lambda_1 > \lambda_2 > 1$?

7) Explain intuitively how these results generalize to a linear system with n predetermined variables and m jump variables.