Introducing nominal rigidities.

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In the model we just saw, the price level (the price of goods in terms of money) behaved like an asset price.

$$M/P = CL(i) = CL(r + \pi^e)$$

So any change in the nominal interest rate, from either changes in the equilibrium real interest rate, or in the expected rate of inflation (itself from future changes in the nominal money supply) led to a change in the price level today.

The price level is not an asset price. It is an aggregate of millions of individual prices, each of them set by a price setter, at discrete intervals in time. So, it is unlikely to adjust in the manner above.

- If P adjusts more slowly, then what will happen? If the equation above still holds, then the nominal interest rate will not move in the same way. An increase in M will lead to a decrease in the nominal interest rate, and likely the real rate.

- If the demand for goods is given by the same equations as before, the demand for goods will therefore move differently from before (go back to the FOC for consumers, or the q theory characterization for investment)

- What will happen to output? This depends on how the price (wage) setters decide to respond to shifts in demand. (The older fix price equilibrium line of research–Barro, Grossman, Malinvaud–and why it died).

  If they have monopoly power, they may want to accomodate these shifts so long as price exceeds marginal cost. So movements in demand will have an effect on output.

Much of the work of the last 20 years has gone into looking at the foundations for this story, and the implications for fluctuations, and for monetary and fiscal policy.

I shall proceed in three steps here.

- First, look at a static model, in which these issues can be discussed (Blanchard Kiyotaki). There are enough new steps and concepts that it is better to start this way. First, without nominal rigidities
- Second, with nominal rigidities. Effects of nominal money, and effects on output and welfare.
- Third a dynamic GE version, which has become the workhorse of so called "New Keynesian" models.

The pricing side will remain quite simplistic. So the last topic of the course will look at the behavior of the price level under more realistic assumptions for price setting, and a brief discussion of the implications for fiscal and monetary policy.

1 A one-period model of yeomen farmers

Think of an economy composed of a large number of households, each producing a differentiated good. More specifically, a continuum of households and goods on [01].

Each household produces its good using its own labor (this way we integrate producers and suppliers of labor, and have to keep track only of prices, not wages and prices).

The utility function of a household i is given by:

$$U(C_i, \frac{M_i}{P}, N_i)$$

where:

$$C_i \equiv [\int_0^1 C_{ij}{}^{\sigma-1/\sigma} dj]^{\sigma/(\sigma-1)}$$

$$P = [\int_0^1 P_j{}^{1-\sigma} dj]^{1/(1-\sigma)}$$

The budget constraint is given by:

$$\int_0^1 P_j C_{ij} + M = P_i Y_i + \bar{M}$$

and the production function for producing good i is given by:

$$Y_i = N_i$$

Things to note about the model:

- We set it up as a one-period problem. Also, for the moment, no uncertainty. But will introduce both later on, first uncertainty about $\bar{M}$, and then a dynamic version, with bonds and money.
- Each household enjoys a consumption basket, composed of all goods. It needs money for transactions; this is formalized by putting money in the utility function rather than formalizing the exact structure of transactions and using CIA.
- Each household produces a good using labor and a constant returns technology. If faces a demand curve, which we shall have to derive, which is the demand for the goods by all others.
- The budget constraint is a short cut to a dynamic budget constraint.

It is easy to characterize the equilibrium of the model with a general utility function. But it is even easier to do it with the following utility:

$$U(C_i, \frac{M_i}{P}, N_i) = (\frac{C_i}{\alpha})^{\alpha} (\frac{M_i/P}{1-\alpha})^{1-\alpha} - \frac{N_i{}^{\beta}}{\beta}$$

Among the advantages of this specification will be a very simple relation between consumption and real money balances, and constant marginal utility of income.

To characterize the general equilibrium, proceed in 4 steps;

- Given spending on consumption, derivation of consumption demands for each good by each household.
- Derivation of the relation between aggregate consumption and aggregate real money balances.
- Derivation of the demand curve facing each household, and derivation of its pricing decision
- General equilibrium

For the moment, no nominal rigidities. Could solve all these steps simultaneously, but much less intuitive.

1.1 Demand for individual goods

Suppose household i depends to spend a nominal amount X_i on consumption. So it maximizes:

$$\max C_i \equiv [\int_0^1 {C_{ij}}^{\sigma-1/\sigma} dj]^{\sigma/(\sigma-1)}$$

subject to:

$$\int_0^1 P_j C_{ij} dj = X_i$$

Then, with a bit of algebra, we get:

$$C_{ij} = \frac{X_i}{P}(\frac{P_j}{P})^{-\sigma}$$

where P is the price index we wrote earlier, and C_i, P, X_i satisfy:

$$C_i P = X_i$$

so we can rewrite the consumption demand for good j as;

$$C_{ij} = C_i (\frac{P_j}{P})^{-\sigma}$$

In words, we can think of the consumer taking a two-step decision. First, how much to consume of the consumption basket, at price P. This gives C_i.

Then, given that decision, he allocates demand to each good in proportion to its relative price. It is clear that, for later, we need $\sigma > 1$ so the demand curves are sufficiently elastic.

1.2 The choice of money and consumption

Using what we just learned, we can rewrite the problem of the consumer as:

$$\max (\frac{C_i}{\alpha})^{\alpha} (\frac{M_i/P}{1-\alpha})^{1-\alpha} - \frac{N_i}{\beta}^{\beta}$$

subject to:

$$PC_i + M = P_i Y_i + \bar{M}$$

The change is in the budget constraint, where we use the fact that we can think of spending as the product of the consumption basket times its price index, the price level.

Given income and initial money balances, we can solve for optimal consumption and money balances.

Define $I_i \equiv P_i Y_i + \bar{M}$. Then:

$$C_i = \alpha I_i, \qquad \frac{M_i}{P} = (1-\alpha) I_i$$

People allocate their initial wealth in proportion α and $1-\alpha$ to consumption and real money balances. For future use:

- Relation between real money balances and consumption:

$$C_i = \frac{\alpha}{1-\alpha}\frac{M_i}{P}$$

- This implies that the demand for good j by household i is given by:

$$C_{ij} = C_i(\frac{P_j}{P})^{-\sigma} = \frac{\alpha}{1-\alpha}\frac{M_i}{P}(\frac{P_j}{P})^{-\sigma}$$

- Replacing C_i and M_i/P in the utility function gives an indirect utility function of the form:

$$\frac{P_i}{P}Y_i - (1/\beta){N_i}^{\beta} + \frac{\bar{M}_i}{P}$$

  This is where the special form helps a bit. It basically implies constant marginal utility of income.

1.3 Pricing and output decisions

Household i then chooses the price and the level of output of good i. To do so, it maximizes:

$$\max \frac{P_i}{P}Y_i - (1/\beta){Y_i}^{\beta} + \frac{\bar{M}_i}{P}$$

where I have used the fact that $N_i = Y_i$.

Integrating over households, the demand for good i is given by:

$$Y_i = \int_0^1 C_{ji}\, dj = \frac{\alpha}{1-\alpha}\frac{M}{P}(\frac{P_i}{P})^{-\sigma}$$

where $M = \int_0^1 M_j dj$. Using the fact that, in equilibrium, the money balances households want to hold must be equal to the nominal money stock, so $M = \bar{M}$, then:

$$Y_i = \frac{\alpha}{1-\alpha}\frac{\bar{M}}{P}(\frac{P_i}{P})^{-\sigma}$$

Solving the maximization problem gives:

$$\frac{P_i}{P} = \frac{\sigma}{\sigma - 1} Y_i^{(\beta-1)}$$

Price equals marginal cost times a markup. Solving for Y_i gives:

$$\frac{P_i}{P} = [\frac{\sigma}{\sigma - 1} X^{(\beta-1)}]^{1/(1+\sigma(\beta-1))}$$

where

$$X \equiv \frac{\alpha}{1-\alpha}\frac{\bar{M}}{P}$$

An increase in $\bar{M}/P$ leads to an increase in the relative price. The effect depends on β and σ. The closer β is to unity, the smaller the effect on the relative price.

Can characterize the equilibrium graphically. Demand is a function of relative price, and real money balances. Marginal revenue as well. Marginal cost is increasing in output. Draw marginal cost, marginal revenue and demand. Figure 8-1 in BF.

1.4 General equilibrium

In general equilibrium, the relative price must be equal to 1. So, output for each household must be such that this holds:

$$1 = \frac{\sigma}{\sigma - 1} Y^{(\beta-1)}$$

Not the same equilibrium than under competition, but only a small modification, for the presence of a markup. Output is lower.

The price level must be such that the real money stock generates the right level of demand:

$$Y = \frac{\alpha}{1-\alpha}\frac{\bar{M}}{P} \Rightarrow P = \frac{\alpha}{1-\alpha}\frac{\bar{M}}{Y}$$

So this would seem like little progress. Output determined by marginal cost plus markup equals price. Nominal money neutral. But in fact, much closer:

- First, a model with aggregate demand. An effect of real money balances. Clearly simplistic, but we know how to extend it.
- Second, price setters. So we can look at how they set prices, and what determines the price level.
- Some intuition for price level determination. Consider an increase in nominal money, from M to M'.

  Requires a proportional increase in P, no change in relative prices.

  But nobody in charge of the price level. Try to adjust relative prices. If β not too far above 1, then relative prices increase only a little. And then a bit more, and so on, until the price level has adjusted.

Suggests adjustment may be slow. Now ready to introduce nominal rigidities.

2 Yeomen farmers and nominal rigidities

Think of the households having to set nominal prices. Two arguments for why they may want to do this at discrete intervals.

- Menu costs. (Akerlof Mankiw) Small changes in prices have only a second order effect on profit.

But a small change in the price level has a first order effect on output and welfare. Why? Because of the initial wedge created by monopoly power. Back to diagram.

- Desired change in relative price may be small. Go back to the equation for P_i/P earlier. If marginal cost is relatively flat, then want to change the relative price by little.

So modify the model as follows. Each household chooses the price of its product before knowing the realization of nominal money this period. Consumption decisions, and thus demand, are taken after observing the realization.

So return to the choice of the relative price by households.

$$\max E[\frac{P_i}{P}Y_i - \frac{1}{\beta}{Y_i}^{\beta}]$$

subject to:

$$Y_i = \frac{\alpha}{1-\alpha}\frac{\bar{M}}{P}(\frac{P_i}{P})^{-\sigma}$$

The difference is that $\bar{M}$ is now a random variable. The FOC is given by:

$$E[X(1-\sigma)(\frac{P_i}{P})^{-\sigma} + \sigma X^{\beta}(\frac{P_i}{P})^{-\beta\sigma-1}] = 0$$

Or, rearranging:

$$\frac{P_i}{P} = [\frac{\sigma}{\sigma-1}\frac{E[X^{\beta}]}{E[X]}]^{1/(1+\sigma(\beta-1)}$$

The only difference from before is the presence of the expectation. But the principle is the same. The higher expected nominal money, the higher the relative price.

2.1 General equilibrium

In general equilibrium, all price setters must set prices so that the relative price is equal to 1. So, the price level is implicitly determined by:

$$1 = [\frac{\sigma}{\sigma - 1}\frac{E[X^{\beta}]}{E[X]}]^{1/(1+\sigma(\beta-1)}$$

where $X = (\alpha/(1-\alpha))\bar{M}/P$.

This gives us our basic set of results:

- Given the predetermined price level, $\bar{M}/P$ moves with $\bar{M}$ and so does consumption.
- Movements in nominal money affect real money balances one for one and so affect consumption one for one.
- Demand affects output, so long as marginal cost is less than price—so suppliers willing to supply. Back to diagram.
- No systematic movement in relative prices (in real wages in a model with a labor market). Fits the data well.
- Welfare goes up and down with demand. Indeed, higher than expected money is good. This again has many implications. Temptation to increase welfare by unexpectedly increasing money.

The log linear version of the model gives us the simplest macro model:

$$p = Em$$

$$y = m - p = m - Em$$

Simple... but a rich story behind. Still: Many issues. Here, one period. Transmission of changes in real money to output through interest rates?

More realistic price setting. So look at a dynamic version.

3 A dynamic GE model of yeomen farmers

One would like to construct a dynamic GE model which had:

- Non trivial investment and consumption decisions, as in the model examined in topic 4. (A rich IS)
- A rich description of how monetary policy determines the short term nominal interest rate, along the lines of topic 6 (A rich LM)
- A theory of price determination, which expanded on the model we have just seen. (A rich AS).

A model which did all this could be constructed. But at some pain, and clearly requiring numerical simulations. So, need a simpler benchmark model. Here is one, variations of which can be found in the literature.

3.1 The optimization problem

The economy is composed of yeomen farmers, who maximize the following objective function:

$$\max E[\sum_{0}^{\infty} \beta^k (U(C_{it+k}) + V(\frac{M_{it+k+1}}{P_{t+k}}) - Q(N_{it+k})\ )|\ \Omega_t]$$

subject to:

$$C_{it} \equiv [\int_0^1 C_{ijt}{}^{\sigma-1/\sigma} dj]^{\sigma/(\sigma-1)} \qquad P_t = [\int_0^1 P_{jt}{}^{1-\sigma} dj]^{1/(1-\sigma)}$$

$$\int_0^1 P_{jt} C_{ijt} + M_{it+1} + B_{it+1} = P_{it} Y_{it} + (1+i_t) B_{it} + M_{it} + X_{it}$$

$$Y_{it} = N_{it}$$

where k now denotes time, and the rest of the notation is standard.

In other words: Each household produces a differentiated product, using labor. It derives disutility from work, and utility from a consumption basket, and from real money balances.

It can save either in the form of bonds, or in the form of money. Bonds pay interest. Money does not.

A number of remarks

- Utility is separable in consumption, money balances, and leisure.

- Utility of money depends on end of period money balances, divided by the price level this period.

  Would look less strange if, as in some papers, we denoted end of period balances by M_t rather than M_{t+1}, so utility would depend on M_t/P_t rather than M_{t+1}/P_t.

  But the assumption would be the same. It role is to deliver a relation between the demand for nominal money, the current price level, and the interest rate (M_{t+1}, P_t, i_{t+1}). The formalization we saw earlier gives a relation between the demand for nominal money, the price level **next period**, and the interest rate $(M_{t+1}, P_{t+1}, i_{t+1}$.

  (The problem is not deep. It would go away in continuous time, where people would continuously rebalance their portfolios)

- There is no capital in the model. (Constant returns to labor). So, demand will be equal to consumption. Bonds are nominal bonds. They can be thought as inside bonds (in zero net supply, and so equal to zero in equilibrium), or government bonds, perhaps introduced in open market operations.

- It is easy to introduce uncertainty, which here will come from nominal money, but could come from other shocks as well.

The structure of the solution is very much the same as before.

- Given spending on consumption, derivation of consumption demands for each good by each household.
- Derivation of consumption, real money balances and bond holdings. The relation between aggregate consumption and aggregate real money balances.
- Derivation of the demand curve facing each household, and derivation of its pricing decision
- General equilibrium

3.2 Demand for individual goods

Going through the same steps as in the static model gives the demand by household i for good j in period t:

$$C_{ijt} = C_{it} \, (\frac{P_{jt}}{P_t})^{-\sigma}$$

where, as before:

$$\int_0^1 P_{jt} C_{ijt} = P_t C_{it}$$

So that, for later use, aggregating over households, the demand for good j in period t is given by:

$$Y_{jt} = C_t (\frac{P_{jt}}{P_t})^{-\sigma}$$

3.3 Consumption and real money balances

Using the results above, the problem of the household can be rewritten as:

$$\max E[\sum_0^\infty \beta^k (U(C_{it+k}) + V(\frac{M_{it+k+1}}{P_{t+k}}) - Q(N_{it+k})\ )|\ \Omega_t]$$

subject to the budget constraint:

$$P_t C_{it} + M_{it+1} + B_{it+1} = P_{it} Y_{it} + (1+i_t) B_{it} + M_{it} + X_{it}$$

and the demand and production functions:

$$Y_{it} = C_t (\frac{P_{it}}{P_t})^{-\sigma} \qquad\qquad Y_{it} = N_{it}$$

Let $\lambda_{t+k}\beta^k$ be the Lagrange multiplier associated with the budget constraint at $t+k$. (Replace N_{it} by Y_{it} in the objective function, and Y_{it} by the expression for demand, in the budget constraint, so only one constraint is left).

Look first at the FOC associated with the choices for consumption, real money balances, :

$$C_{it}\ :\ U'(C_{it}) = \lambda_t P_t$$

$$M_{it+1}\ :\ V'(\frac{M_{it+1}}{P_t}) = (\lambda_t - \beta E[\lambda_{t+1}\ |\Omega_t]) P_t$$

$$B_{it+1}\ :\ \lambda_t = \beta(1+i_{t+1}) E[\lambda_{t+1}\ |\ \Omega_t]$$

which we can reduce to two conditions (this should be familiar by now):

An intertemporal condition

$$U'(C_{it}) = E[\ \beta\ (1 + r_{t+1} U'(C_{it+1})\ |\ \Omega_t]$$

An intratemporal condition

$$V'(\frac{M_{t+1}}{P_t})/U'(C_{it}) = \frac{i_{t+1}}{1+i_{t+1}}$$

The interpretation is as before:

- The tilting smoothing condition for consumption, and the role of the real interest rate.
- The choice between real money balances and consumption, which depends on the nominal interest rate.

There is one FOC left, for the choice of the relative price, and the associated level of output and employment. Let's turn to it.

3.4 Pricing and output decisions

Replacing Y_{it} by the demand function in the budget constraint, differentiating with respect to P_{it}, and using the fact that $\lambda_t U'(C_{it}/P_t$, gives:

$$\frac{P_{it}}{P_t} = \frac{\sigma}{\sigma - 1}\frac{Q'(Y_i t)}{U'(C_{it})}$$

Each household sets the price of its product as a markup over marginal cost. The markup is equal to $\sigma/(1-\sigma)$. The marginal cost is equal to the disutility of work, divided by marginal utility.

3.5 General equilibrium

In symmetric general equilibrium:

$$Y_{it} = C_{it} = C_t = Y_t$$

So collecting equations:

$$IS: \qquad U'(Y_t) = E[\,\beta(1+r_{t+1})U'(Y_{t+1}) \mid \Omega_t]$$

$$LM: \qquad V'(\frac{M_{t+1}}{P_t})/U'(Y_t) = \frac{i_{t+1}}{1+i_{t+1}}$$

$$AS: \qquad 1 = \frac{\sigma}{\sigma-1}\frac{Q'(Y_t)}{U'(Y_t)}$$

A nice characterization in terms of an IS relation, an LM relation, and an AS (aggregate supply) relation. But not much action. Get full dichotomy.

- AS determines $Y_t = Y$. (What would happen if we allowed for technological shocks, say $Y_{it} = Z_{it}N_{it}$?)
- IS determines $r_{t+1} = r = 1/\beta$.
- LM determines the price level, as a function of current and future nominal money.

Now introduce nominal rigidities. Assume prices chosen before the realization of money. What is changed? Only the third equation:

The individual price setting equation becomes:

$$\frac{P_{it}}{P_t} = \frac{\sigma}{\sigma-1}\frac{E[Q'(Y_{it})C_t]}{E[U'(C_{it})C_t]}$$

Note the expectations. At the time the price decisions are taken, aggregate consumption, individual consumption, individual output, are not known. So their covariance matters.

In general equilibrium, the relative price must be equal to zero, $Y_{it} = C_{it} = Y_t = C_t$ so:

$$1 = \frac{\sigma}{\sigma-1}\frac{E[Q'(Y_t)Y_t]}{E[U'(Y_t)Y_t]}$$

This determines the expected level of output, and by implication, the price level, call it $\bar{P}_t$, which supports this allocation. (Not so easy to characterize this equilibrium price level here.)

3.6 The implied IS-LM-AS model

Now we truly have an IS-LM-AS model. Collecting equations once more:

$$IS: \qquad U'(Y_t) = E[\ \beta(1 + r_{t+1})U'(Y_{t+1}) \mid \Omega_t]$$

$$LM: \qquad V'(\frac{M_{t+1}}{P_t})/U'(Y_t) = \frac{i_{t+1}}{1 + i_{t+1}}$$

$$AS \qquad \bar{P}_t \mid 1 = \frac{\sigma}{\sigma - 1}\frac{E[Q'(Y_t)Y_t]}{E[U'(Y_t)Y_t]}$$

Look at it more closely:

- The IS relation gives current demand today,as a function of the expected real rate of interest rate, and income next period.
- The LM relation determines the nominal interest rate, and given the predetermined price level, shows how changes in nominal money affect the nominal rate.
- The AS relation implies that the price level is predetermined this period, but that expected to deliver the flex price equilibrium in future periods.

Can represent it informally in the output–nominal interest rate $(Y_t, i_{t+1}$ space.

- The IS implies that Y_t depends on the expected real interest rate and the expected income next period (this plays fast and loose with the expectation. The covariation of the two matters). So downward sloping, for given expected inflation, with position depending on EY_{t+1}
- The LM is the usual upward sloping relation, with position determined by $M/\bar{P}$. So, expectations of good/bad things in the future, and inflation expectations both shift the IS.

Effect of anticipated technological shocks in the future. Of fiscal policy changes in the future. Of higher nominal money in the future. Work each one out, using intuition. (or, if more ambitious, log linearize the model, and work it out explicitly).

The log linearization:

$$y_t = -a(i_{t+1} - Ep_{t+1} + p_t) + Ey_{t+1}$$

$$m_{t+1} - p_t = by_t - ci_{t+1}$$

$$\bar{p}_t \mid Ey_t = 0$$

The reason why expected output is constant (so the deviation from the steady state is zero in the third equation) is that I have not introduced supply shocks. If there were supply shocks, the price level would be set for in expected value, the level of output is set at the flexible price level.

Move (briefly) to the last two topics. Extensions, for more realistic price setting and dynamics of prices. And applications to fiscal and monetary policy.