Problem Set 5

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1 Money in the Utility Function

Consider the standard model with money in the utility function (Topic 6, pp. 10) with log utility:

$$\max \sum \beta^t \left[\log (c_t) + a \log \left(\frac{M_t}{P_t} \right) \right]$$

Subject to

$$P_t C_t + B_{t+1} + M_{t+1} = W_t + \Pi_t + (1 + i_t) B_t + M_t + X_t$$

Keep in mind the timing of the model. At the beginning of the period, consumers hold the stock of money M_t. They also hold the stock of nominal bonds B_t. These are given. Then the price level is determined, consumptions decisions are made and labor is supplied inelastically. At the end of the period, the consumers hold the money supply M_{t+1} that they expect to use tomorrow.

Explain why the cost of holding money from t to $t+1$ is i_{t+1}. Can i_t be negative?

Now think of the effect of inflation. What is the relevant price level when M_{t+1} is bought? What is the relevant price level when M_{t+1} is used? Why is inflation like a tax?

The firms behave as usual (as in the handout 6). Derive and interpret the FOCs. Show that one can separate the real side (C, K, r) from the nominal side (M, i, P) of the model. What particular assumption about the utility function is responsible for this separation?

Suppose that at time $t-1$ the economy is in a steady state with constant money supply M. Suppose that from t on, the money supply is M_t. Show that C, K and r stay constant. What happens to i? What happens to real money balances? Is utility constant?

What would happen if the government did not give the seigniorage revenues back to the consumers? Which equation would be affected? Would this change the steady state?

2 Money in the Production Function

Firms use money (liquidity) as an input in their production function.

$$Y_t = Z\left(\frac{M_t}{P_t}\right) K_t^{1-\alpha} N_t^{\alpha}$$

$Z(.)$ is an increasing, bounded and concave function. $Z(\infty) = 1$, $Z(0) = 0$, $Z'(0) = \infty$. The households maximize:

$$\max \sum \beta^t u(c_t)$$

subject to:

$$P_t C_t + B_{t+1} = W_t + \Pi_t + (1 + i_t) B_t + X_t$$

and they supply labor inelastically:

$$N_t = 1$$

The cash flows of the firms are:

$$\begin{aligned} \Pi_t &= P_t Z\left(\frac{M_t}{P_t}\right) K_t^{1-\alpha} N_t^{\alpha} - W_t N_t - P_t I_t + B_{t+1} + M_t - M_{t+1} - (1 + i_t) B_t \\ K_{t+1} &= (1 - \delta) K_t + I_t \end{aligned}$$

And firms solve the Bellman equation

$$V_t = \max\left\{\Pi_t + \frac{1}{1 + i_{t+1}} V_{t+1}\right\}$$

Derive and interpret the FOCs. Let g be the rate of growth of M. Characterize the steady state. Is money neutral? Superneutral?

3 Seigniorage

3.1 Budget Curve

There is a money demand equation

$$\frac{M}{PY} = \exp(-\alpha \pi^e)$$

There is a deficit that has to be financed by seigniorage

$$\delta = \frac{\partial M}{\partial t} * \frac{1}{PY}$$

Let x be the growth rate of money:

$$x = \frac{1}{M} * \frac{\partial M}{\partial t}$$

Plot the budget curve in the (x, π^e) space.

3.2 Steady State

In steady state, we must have

$$\begin{aligned} \pi &= \pi^e \\ \frac{\partial \pi}{\partial t} &= 0 \end{aligned}$$

Assume that real GDP grows at a rate g (take g as exogenous). Plot the steady state condition. Why can there be multiple steady states for a given δ? What is the maximum deficit compatible with a steady state? How does it depend on g?

3.3 Adaptive Expectations

Assume that expectations evolve according to:

$$\frac{\partial \pi^e}{\partial t} = \beta \left(\pi - \pi^e \right)$$

Do you think that this is a reasonable specification?

Obtain an equation in $\frac{\partial \pi^e}{\partial t}, \pi^e, x, g$. Consider the case where there are two steady states. When is the low inflation state stable? When is the high inflation state stable?

3.4 Rational Expectations

Consider the economy at time $t = 0$, for given x, g, d. Assume that agents cannot expect inflation to be negative. Consider the case where there are two steady states. Show that there are 3 types of dynamic path on which the economy can jump.

1. An equilibrium with low inflation that is expected to remain constant forever
2. A continuum of equilibria with increasing inflation converging to the high inflation steady state
3. A continuum of equilibria with decreasing inflation converging to the high inflation steady state. (and of course there is the particular case where the economy jumps directly on the steady state)