Introducing money.

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No role for money in the models we have looked at. Implicitly, centralized markets, with an auctioneer:

- Possibly open once, with full set of contingent markets. (Remember, no heterogeneity, no idiosyncratic shocks. (Arrow Debreu)

- More appealing. Markets open every period.

  Spot markets, based on expectations of the future. For example, market for goods, labor, and one–period bonds. A sequence of **temporary equilibria** (Hicks).

  Still no need for money. An auctioneer. Some clearing house.

So need to move to an economy where money plays a useful role.
The ingredients.

- No auctioneer. Geographically decentralized trades.

- Then, problem of double coincidence of wants. Barter is not convenient. Money, accepted on one side of each transaction, is much more so.

Two types of questions

Foundations

- Why money? What kind of money will emerge?

- Can there be competing monies?

- Fiat versus commodity money?

- Numeraire versus medium of exchange? Should they be the same, or not?

Not just abstract, or history. The rise of barter in Russia in the 1990s. "Natural" dollarization in some Latin American countries. "Units of account" in Latin America.

But most of the time, we can take it as given that money will be used in transactions, that it will be fiat money, and that the numeraire and the medium of exchange will be the same.

If we take these as given, then we can ask another set of questions:

- How different does a decentralized economy with money look like?
- What determines the demand for money, the equilibrium price level, nominal interest rates?
- How does the presence of money affect the consumption/saving choice?
- Steady state and dynamic effects of changes in the rate of money growth.

Start by looking at a benchmark model. Cash in advance.

Then, look at variations on the model; money in the utility function.

Then focus on price and inflation dynamics, especially hyperinflation.

1 A cash in advance model

Think in terms of a decentralized economy (although we shall see that there is an optimization problem which replicates the outcome).

1.1 The optimization problem of consumers/workers

Consumers/workers maximize:

$$E[\sum_{i=0}^{i=\infty} \beta^i U(C_{t+i} \mid \Omega_t]$$

subject to:

$$P_tC_t + M_{t+1} + B_{t+1} = W_t + \Pi_t + M_t + (1 + i_t)B_t + X_t$$

and

$$M_t \geq P_tC_t$$

Note that I ignore:

- **Uncertainty** Because it is not central to the points I want to make. But there is no problem in introducing it in the usual way.
- The **labor/leisure choice**. It would be affected. But I leave it out for simplicity. People supply one unit of labor inelastically.

The notation:

P_t is the price of goods in terms of the numeraire (the price level).

M_t and B_t are holdings of money and bonds at the start of period t.

W_t and Π_t are the nominal wage and nominal profit received by each consumer respectively.

i_t is the nominal interest rate (the interest rate stated in dollars, not goods) paid by the bonds.

X_t is a nominal transfer from the government (which has to be there if and when we think of changes in money as being implemented by distribution of new money to consumers).

Now turn to the assumptions underlying the specification:

- Consumers care only about consumption. They do not derive utility from money.
- The first constraint is the budget constraint. It says that nominal consumption plus new asset holdings must be equal to nominal income—wage income (the labor supply is inelastic and equal to one) and profit

income—plus initial asset holdings, including interest on the bonds, plus nominal government transfers.

- If the only constraint was the first constraint, then people would hold no money: Bonds pay interest, money does not.

  The second constraint explains why people hold money. It is known as the **cash in advance (CIA)** constraint. People must enter the period with enough nominal money balances to pay for consumption.

- One story here. People are composed of a worker and a consumer. The worker goes to work. The consumer goes to buy goods, and must do this before the worker has been paid. So he must have sufficient money balances to finance consumption.

- One can think of more sophisticated, smoother, formulations. For example: The cost of buying consumption goods is decreasing in money balances. I shall return to this below.

Let $\lambda_{t+i}\beta^i$ be associated with the budget constraint, $\mu_{t+i}\beta^i$ be associated with the CIA constraint. Set up the Lagrangian and derive the FOC.

$$C_t : \quad U'(C_t) = (\lambda_t + \mu_t)P_t$$

$$M_{t+1} : \quad \lambda_t = \beta(\lambda_{t+1} + \mu_{t+1})$$

$$B_{t+1} : \quad \lambda_t = \beta(1 + i_{t+1})\lambda_{t+1}$$

Interpretation of each.

Can combine them to get:

$$\frac{U'(C_t)}{1+i_t} = \beta\,[\frac{P_t}{P_{t+1}}(1+i_{t+1})]\,\frac{U'(C_{t+1})}{1+i_{t+1}}$$

Note that $P_t/P_{t+1} = 1 + \pi_{t+1}$. If we define the real interest rate as:

$$(1 + r_{t+1}) \equiv \frac{P_t}{P_{t+1}}(1 + i_{t+1})$$

We can rewrite the first order condition as:

$$\frac{U'(C_t)}{1 + i_t} = \beta(1 + r_{t+1})\frac{U'(C_{t+1})}{1 + i_{t+1}}$$

Interpretation.

- Because people have to hold money one period in advance, the effective price of consumption is not 1 but $1 + i$.
- Once we adjust for this price effect, then we get the same old relation, between marginal utility this period, marginal utility next period, and the real interest rate.

Note the role of both the **nominal** and the **real** interest rates. Note that the nominal interest rate is constant, the equation reduces to the standard Euler equation:

$$U'(C_t) = \beta(1 + r_{t+1})U'(C_{t+1})$$

This characterizes consumption. **Consumption behavior** is very similar to that in the non monetary economy. Two differences:

- The relative price effect, if i_t is different from i_{t+1}.
- The fact that the rate of return on total wealth is lower (as some of wealth does not yield interest), so the feasible level of consumption is lower.

Given consumption, the characterization of the demand for money is straightforward. The CIA holds as an equality:

$$\frac{M_t}{P_t} = C_t$$

Pure quantity theory. No interest rate elasticity. Simple, but possibly too simple. Will look at extensions below.

1.2 The optimization problem of firms

Firms produce goods using labor and capital. They pay labor a wage W_t. They buy capital for use in the next period, and they finance these purchases of capital by issuing nominal bonds.

Their nominal cash flow is thus given by:

$$\Pi_t = P_t F(K_t, N_t) - W_t N_t - (1 + i_t) B_t + P_t (1 - \delta) K_t - P_t K_{t+1} + B_{t+1}$$

Cash flow is equal to production minus the wage bill, minus payment of interest and principal on bonds issued last period, plus the value of the remaining capital stock minus the value of the capital purchased, plus bond issues.

The value of a firm is given by the present value of nominal cash flow, discounted by the relevant nominal interest rate.

$$V_t = \Pi_t + (1 + i_{t+1})^{-1} V_{t+1}$$

The three FOC for firms are given by:

$$N_t : \quad P_t F_N(K_t, N_t) = W_t$$

$$B_{t+1} : \quad 1 = 1$$

$$K_{t+1}: \qquad P_t = (1+i_{t+1})^{-1}[P_{t+1}(1-\delta+F_K(K_{t+1},N_{t+1}))]$$

Note the second FOC: It says that the amount of bonds issued by firms is irrelevant. They could finance purchases of capital from current profit, or partly through bond issues, or fully through bond issues. Their decisions would be the same. (But, under our assumption, there are nominal bonds in the economy, which makes it easier to think about the nominal interest rate).

The third FOC can be rewritten as:

$$(1-\delta+F_K(K_{t+1},N_{t+1})) = (1+i_{t+1})\frac{P_t}{P_{t+1}}$$

Or:

$$(1-\delta+F_K(K_{t+1},N_{t+1})) = (1+r_{t+1})$$

Firms purchase capital to the point where the marginal product of capital is equal to the real interest rate.

1.3 The equilibrium and the steady state

To close the model, we have that:

$$N_t = 1$$

Turn to the government budget constraint. Assume that the stock of money is changed through transfers to people:

$$X_t = M_{t+1} - M_t$$

Putting things together, the dynamics of the economy are characterized by the following equations:

$$\frac{U'(C_t)}{1+i_t} = \beta(1+r_{t+1})\frac{U'(C_{t+1})}{1+i_{t+1}}$$

$$(1+i_t) = (1+r_t)(1+\pi_t)$$

$$(1+r_t) = 1-\delta+F_K(K_t,1)$$

$$\frac{M_t}{P_t} = C_t$$

$$K_{t+1} = F(K_t,1)+(1-\delta)K_t - C_t$$

I shall not attempt to look at dynamics, but just focus on steady state: Suppose that the rate of growth of nominal money is equal to x, so

$$\frac{X_t}{P_t} = (1+x-1)\frac{M_t}{P_t} = x\frac{M_t}{P_t}$$

.

In steady state, $C_t, K_t, r_t, i_t, \pi_t$ are constant, so:
From the FOC of the consumer, and the demand for capital by firms:

$$(1+r) = 1+F_K(K,1)-\delta = 1/\beta$$

This is the same rule as without money: The modified golden rule.
In steady state, real money balances must be constant, so:

$$\pi = x$$

Inflation is equal to money growth. And so, $i = \pi + r = x + r$. This one for one effect of money growth on the nominal interest rate is known as the **Fisher effect**.

Using these relations in the budget constraint of the consumer gives:

$$C = F(K, 1) - \delta K$$

So, on the real side, the economy looks the same as before. In addition people hold money. And inflation proceeds at the same rate as money growth. The fact that, in steady state, **money growth** has no effect on the real allocation is refered to as the **superneutrality** of money.

Is this superneutrality a general result? I now explore an alternative formalization.

2 Money in the utility function

The CIA constraint is too tight. One can clearly maintain a lower level of real money balances is one is willing to go to the ATM machine more often. More reasonable to assume that

- The higher the level of real money balances one holds, the lower the transaction costs, so the higher the level of output net of transaction costs,
- Or the higher the level of utility, again net of transaction costs.

One can formalize this explicitly, A dynamic Baumol Tobin model. This is what is done by Romer (see original article or BF). Very useful, but a bit heavy for here.

One can take short cuts. Real money balances in the production function, or in the utility function.

See effects of putting money in the utility function. (Sidrauski model).

So the optimization problem of consumers/workers is:

$$E[\sum \beta^i U(C_{t+i}, \frac{M_{t+i}}{P_{t+i}}) \mid \Omega_t]$$

subject to:

$$P_t C_t + M_{t+1} + B_{t+1} = W_t + \Pi_t + M_t + (1 + i_t)B_t + X_t$$

where, plausibly $U_m > 0$ and $U_{mc} \geq 0$ (why?).

Let $\lambda_{t+i}\beta^i$ be the lagrange multiplier associated with the constraint. Then the FOC are given by:

$$C_t : \qquad U_c(C_t, \frac{M_t}{P_t}) = \lambda_t P_t$$

$$B_{t+1} : \qquad \lambda_t = \lambda_{t+1}\beta(1 + i_{t+1})$$

$$M_{t+1} : \qquad \lambda_t = \beta\lambda_{t+1} + \frac{1}{P_{t+1}} U_m(C_{t+1}, \frac{M_{t+1}}{P_{t+1}})$$

Interpretation. Can rewrite as:

An **intertemporal condition**:

$$U_c(C_t, \frac{M_t}{P_t}) = \beta(1 + r_{t+1})U_c(C_{t+1}, \frac{M_{t+1}}{P_{t+1}})$$

An **intratemporal condition**

$$U_m(C_t, \frac{M_t}{P_t})/U_c(C_t, \frac{M_t}{P_t}) = \beta\ i_t$$

Interpretation. Note that the second says that the ratio of marginal utilities has to be equal to the opportunity cost of holding money, so i, the nominal interest rate.

If for example,

$$U(C, M/P) = \log(C) + a\log(M/P)$$

Then,

$$\frac{M_t}{P_t} = (a/\beta)\frac{C_t}{i_t}$$

This gives us an *LM* relation. (Indeed you can think of the first condition as giving us a simple *IS* relation, this giving us an *LM* relation. More on this in the next lectures).

The demand for money is a function of the level of transactions, here measured by consumption, and the opportunity cost of holding money, i.

Turn to steady state implications. (firms' side is the same as before).

$$1 + r = 1/\beta$$

$$C = F(K, 1) - \delta K$$

$$U_m(C, \frac{M}{P})/U_c(C, \frac{M}{P}) = \beta(x + r)$$

So, same real allocation again. And a level of real money balances inversely proportional to the rate of inflation, itself equal to the rate of money growth.

Dynamic effects? Yes. But nothing very exciting. Can make it more exciting by modelling trips to the bank and having people come at different times. Then, distribution effects. But does not seem to capture much of what we actually observe.

So, bottom line: Money as a medium of exchange, without nominal rigidities gives us a way of thinking about the economy, the price level, the nominal interest rate, but not much in the way of explaining fluctuations.

Very useful however when money growth and inflation become high and variable. Turn to this.

3 Money growth, inflation, seignorage

Start with the money demand we just derived:

$$\frac{M_t}{P_t} = C_t \ L(r_t + \pi_t^e)$$

If money growth and inflation are high and variable, M, P and π^e will move a lot relative to C and r. So assume, for simplicity, that $C_t = C$, and $r_t = r$, so:

$$\frac{M_t}{P_t} = C \ L(r + \pi_t^e)$$

This gives a relation between the price level and the expected rate of inflation. The higher expected inflation, the lower real money balances, the higher the price level.

This relation, together with an assumption about money growth, and the formation of expectations, allows us to think about the behavior of inflation. This is what Cagan did. Looking at hyperinflations, he asked;

- Was hyperinflation the result of money growth, and only money growth?
- Why was money growth so high? Did it maximize seignorage. And if not, then why?

Now have a quick look at his model (Read the paper, written in 1956. It is a great read, even today). Also, read BF4-7, and BF10-2. What follows is just a sketch.

Continuous time, more convenient here. Assume a particular form for the demand for money:

$$M/P = \exp(\ -\alpha\pi^e)$$

So, in logs:

$$m - p = -\alpha\, \pi^e$$

Log real money balances are a decreasing function of expected inflation. Or differentiating with respect to time:

$$x - \pi = \alpha\, d\pi^e/dt$$

Assume that people have adaptive expectations about expected inflation. (In an environment such as hyperinflation, this assumption makes a lot of sense. More on rational expectations below).

$$d\pi^e/dt = \beta\, (\pi - \pi^e)$$

Money growth and inflation

Suppose money growth is constant, at x. Will inflation converge to $\pi = x$? To answer, combine the two equations above and eliminate $d\pi^e/dt$ between the two, to get:

$$x - \pi = -\alpha\beta(\pi - \pi^e)$$

This is a line in the (π, π^e) space. For a given x, $d\pi^e/d\pi = -(1-\alpha\beta)/\alpha\beta$, so if $\alpha\beta < 1$ the line is downward sloping. If $\alpha\beta > 1$ upward sloping.

- If $\alpha\beta < 1$, then the equilibrium is stable. Start with $x > 0$, and $\pi = 0$. Then converge to $\pi = \pi^e = x$.
- If $\alpha\beta > 1$, then not. Why?

Cagan estimated α and β, found $\alpha\beta < 1$. Hyperinflation was the result of money growth, not a bubble.

Seignorage

What is the maximum revenue the government can get from money creation (called **seignorage**:

$$S \equiv \frac{dM/dt}{P} = \frac{dM/dt}{M}\frac{M}{P} = x\ \exp(-\alpha\pi^e)$$

So, in steady state:

$$S = x\ \exp(-\alpha x)$$

So $x^* = 1/\alpha$

Much lower than the growth rates of money observed during hyperinflation.

But just a steady state result. Can clearly get more in the short run, when π^e has not adjusted yet. This suggests looking at different dynamics: Given seignorage, dynamics of money growth and inflation.

Seignorage, money growth and inflation

Start from:

$$S = x\exp(-\alpha\pi^e)$$

For a given S, draw the relation between π^e and x in π^e, x space. Concave. Can cross the 45 degree line twice, once if tangent, not at all if no way to generate the required seignorage in steady state.

Which equilibrium is stable? Using the equation for adaptive expectations and the money demand relation in derivative form:

$$d\pi^e/dt = \beta(\pi - \pi^e) = \beta(x + \alpha d\pi^e/dt - \pi^e)$$

Or:

$$d\pi^e/dt = 1/(1 - \alpha\beta)\ (x - \pi^e)$$

If two equilibria, lower one is stable. Start from it, and suppose S increases so no equilibrium.

Then, money growth and inflation will keep increasing. This appears to capture what happens during hyperinflations.

Some other issues

- Adaptive or rational expectations? (see BF 5-1)
- Fiscal policy, and the effects of inflation on the need for seignorage. (See Dornbusch et al)
- Unpleasant monetarist arithmetic? (see BF 10-2)

From Cagan:

Seven Hyperinflations of the 1920s and 1940s

Country	Beginning	End	P_T/P_0	Average Monthly Inflation rate (%)	Average Monthly Money Growth (%)
Austria	Oct. 1921	Aug. 1922	70	47	31
Germany	Aug. 1922	Nov. 1923	1.0×10^{10}	322	314
Greece	Nov. 1943	Nov. 1944	4.7×10^{6}	365	220
Hungary 1	Mar. 1923	Feb. 1924	44	46	33
Hungary 2	Aug. 1945	Jul. 1946	3.8×10^{27}	19,800	12,200
Poland	Jan. 1923	Jan. 1924	699	82	72
Russia	Dec. 1921	Jan. 1924	1.2×10^{5}	57	49

P_T/P_0: Price level in the last month of hyperinflation divided by the price level in the first month.