Problem Set 1

Thomas Philippon

April 7, 2002

1 Continuous Time

Consider a non-stochastic continuous time setup. Suppose that the representative consumer maximizes

$$\int_0^\infty e^{-\theta t} u(C_t) dt$$

The capital accumulation is given by

$$\frac{dK}{dt} = ZK_t^{1-\alpha} - \delta K_t - C_t$$

You can use the CES utility function

$$u(C) = \frac{\sigma}{\sigma - 1} C^{\frac{\sigma-1}{\sigma}}$$

1) Derive the Euler equation. What are the steady state levels of consumption and interest rate? What is the maximum sustainable level of consumption and what is the associated interest rate? Why are they different from the optimal ones?

2) Draw the phase diagram. Draw the saddle path. How does the slope of the saddle path depend on σ?

3) What are the effects of an unexpected permanent shock to θ on consumption and investment? Do you think that shocks to θ can by themselves provide a satisfactory account of aggregate fluctuations?

4) What are the effects of an unexpected permanent shock to Z on output, consumption and investment? Suppose that there are only shocks to Z. Would this imply reasonable comovements between output, consumption and investment? Show that, following a positive technology shock, consumption may increase or decrease depending on σ. Explain the intuition.

5) How is this result affected if one considers transitory shocks? Hint: suppose that at time $t = 0$ productivity jumps up and that it is known that it will remain at this higher level until time $t = t_1$. At $t = t_1$, productivity will return to its original level and stay there forever. Where must the economy be at time $t = t_1$? How does it move between $t = 0$ and $t = t_1$

2 Discrete Time Dynamic Programming

Use the discrete time setup of the *RBC* handout (up to section 5) and the MATLAB programs contained in the folder *DynamicProgramming*. Read *tutorial.doc* and the comments in *DP.m*.

1) Derive the Euler equation in discrete time in the model with exogenous labor, first using dynamic programming and then using the Lagrange multiplier method. Compare it to the continuous time Euler equation.

2) Numerical analysis using *DP.m*. Describe the dynamics of the economy when it starts far away from its steady state (say 30% below). What does it tell you about the rapid growth in Europe after the second world war?

3) Numerical analysis using *DP.m*. We have seen in the continuous time setup that the reaction of consumption to a positive technology shock depends on the elasticity σ. Use the policy function estimated by *DP.m* to illustrate this finding. (note: in my program, I define *gamma*, the $CRRA$ instead of σ. How are they related? Why?) Hint: *DP.m* produces 5 plots. The first one is called consumption, and it is 3 dimensional. Consumption is measured on the vertical axis. The two horizontal axis are for K and Z. If you do not see well, you can rotate the picture with the mouse.

4) Using the programs described in the tutorial, explain why iterating on the policy function is much faster than iterating on the value function.

5) Optional. Show numerically how the behavior of consumption is affected by the persistence of the shocks.