Fluctuations. Introducing a Leisure/Labor Choice in the Ramsey Model. RBC models.

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The benchmark model had shocks, uncertainty, but no variation in employment. We want to explore what happens if we allow for a leisure/labor choice.

This class of models is known as the RBC model. It does well at explaining many business cycle facts. Procyclical consumption, investment, and employment.

But the hypotheses appear factually wrong (technological shocks, labor/leisure elasticity). Useless? No. Another step on the path to the relevant model.

Organization:

- Set up and solve the model. First order conditions, special cases, and numerical simulations.
- Evidence on technological shocks, and the nature of technological progress.
- Evidence on movements in non employment: Unemployment versus non participation.

1 The optimization problem

Again look at a planning problem.

$$\max E[\sum_0^\infty \beta^i \, U(C_{t+i}, L_{t+i}) | \Omega_t]$$

subject to:

$$N_{t+i} + L_{t+i} = 1$$

$$C_{t+i} + S_{t+i} = Z_{t+i} F(K_{t+i}, N_{t+i})$$

$$K_{t+i+1} = (1 - \delta) K_{t+i} + S_{t+i}$$

The change from the benchmark. L is leisure and N is work. By normalization, total time is equal to one. Utility is a function of both consumption and leisure.

Again I ignore growth. If growth, then the production function would have Harrod neutral technological progress , so $Z_tF(K_t, A_tN_t)$, with $A_t = A^t, A > 1$ for example.

2 The first order conditions

The easiest way to derive them is again using Lagrange multipliers. Put the three constraints together to get:

$$K_{t+i+1} = (1-\delta)K_{t+i} + Z_{t+i}F(K_{t+i}, 1-L_{t+i}) - C_{t+i}$$

Associate $\beta^i\lambda_{t+i}$ with the constraint at time t:

$E[U(C_t) + \beta U(C_{t+1}) - \lambda_t(K_{t+1} - (1-\delta)K_t - Z_tF(K_t, 1-L_t) + C_t) - \beta\lambda_{t+1}(K_{t+2} - (1-\delta)K_{t+1} - Z_{t+1}F(K_{t+1}, 1-L_{t+1}) + C_{t+1}) + ... \mid \Omega_t]$

The First Order Conditions are therefore given by:

$C_t \; : \; U_C(C_t, L_t) = \lambda_t$
$L_t \; : \; U_L(C_t, L_t) = \lambda_t Z_t F_N(K_t, 1-L_t)$
$K_{t+1} \; : \; \lambda_t = E[\beta\lambda_{t+1}(1-\delta + Z_{t+1}F_K(K_{t+1}, 1-L_{t+1}) \mid \Omega_t]$

Define, as before, $R_{t+1} \equiv 1-\delta + Z_{t+1}F_K(K_{t+1}, 1-L_{t+1})$ and define $W_t = Z_tF_N(K_t, 1-L_t)$, so:

$$U_C(C_t, L_t) = \lambda_t$$

$$U_L(C_t, L_t) = \lambda_t W_t$$

$$\lambda_t = E[\beta \lambda_{t+1} R_{t+1} \mid \Omega_t]$$

Interpretation. Combining the first two:

The **intratemporal condition**:

$$U_L(C_t, L_t) = W_t U_C(C_t, L_t)$$

And the **intertemporal condition**:

$$U_C(C_t, L_t) = E[\beta R_{t+1} U_C(C_{t+1}, L_{t+1}) \mid \Omega_t]$$

Before proceeding, can ask: What restrictions do we want to impose on utility and production so as to have a balanced path in steady state? (Not a totally convincing exercise. Do we really have the same preferences in the short and the long run? But useful to have a balanced path.

- *On the production side, we know that progress has to be Harrod Neutral, say at rate $A > 1$. (Remember we suppressed A_t just for notational convenience.*
- *On the utility side, can use the first order conditions above to derive the restrictions.*

In steady state, leisure is constant. (Empirically: Not quite right. Clearly a large decrease in N over time. But, over the last 40 years in the US, it looks like the substitution and the income effects have roughly cancelled.) Consumption and the wage increase at rate A, so, from the intratemporal condition:

$$\frac{U_L(CA^t, L)}{U_C(CA^t, L)} = WA^t$$

where C, L and W are constant over time, and A increases. This is true for any A^t, so in particular, for $t = 0$ so $A^t = 1$, so

$$\frac{U_L(C,L)}{U_C(C,L)} = W$$

Using the two relations to eliminate the wage, we can write:

$$\frac{U_L(CA^t,L)}{U_C(CA^t,L)} = A^t \frac{U_L(C,L)}{U_C(C,L)}$$

The marginal rate of substitution between consumption and leisure must increase over time at rate A.

This relation holds for any value of the term A^t So use for example $A^t = 1/C$:

$$\frac{U_L(1,L)}{U_C(1,L)} = \frac{1}{C}\frac{U_L(C,L)}{U_C(C,L)}$$

Or, rearranging:

$$\frac{U_L(C,L)}{U_C(C,L)} = C[\frac{U_L(1,L)}{U_C(1,L)}]$$

The rate of substitution must be equal to C times the term in brackets, which is a function only of L. This in turn implies that the utility function must be of the form:

$$u(C\tilde{v}(L))$$

where given the usual restrictions on the original utility function, the function $\tilde{v}(.)$ must be concave.

Now turn to the intertemporal condition. Write it as:

$$U_C(CA^t,L) = (\beta R)U_C(CA^{t+1},L)$$

Or, given the restrictions above:

$$\frac{u'(CA^t\tilde{v}(L))}{u'(CA^{t+1}\tilde{v}(L))} = \beta R$$

For this condition to be satisfied, $u(.)$ must be of the constant elasticity form:

$$u(C\tilde{v}(L)) = \frac{\sigma}{\sigma - 1}(C\tilde{v}(L))^{(\sigma-1)/\sigma}$$

If $\sigma = 1$, then:

$$U(C, L) = \log(C) + v(L)$$

where $v(L) = \log(\tilde{v}(L))$, with $v' > 0, v'' < 0$. (Another way of stating this. If we want to assume separability of leisure and consumption (but there is really no good reason to do that), then the form above is the only one consistent with the existence of a steady state.

Let me use the specification $U(C, L) = \log(C) + v(L)$ and return to the first order conditions.

The **intratemporal condition** becomes:

$$v'(L_t) = W_t/C_t$$

And the **intertemporal condition**:

$$E[\beta R_{t+1}\frac{C_t}{C_{t+1}} \mid \Omega_t] = 1$$

Interpretation. (Note that $U_C = 1/C$ is the marginal value of wealth). So equalize marginal utility of leisure to the wage times the marginal value of wealth. And the Ramsey-Keynes condition for consumption.

So now consider the effects of a favorable technological shock. It increases W and R, both current and prospective.

- Two effects on **consumption**. Smoothing (consumption up) and tilting (consumption down). On net, plausibly up.

- Turn to **leisure/work**. Two effects.

  A substitution effect: Higher W_t leads people to work harder.

  An income/wealth effect. Higher C_t works the other way. As people feel richer (remember that $1/C$ is the marginal value of wealth), they want to consume more and enjoy more leisure.

  Net effect depends on the strength of the two effects. Substitution (elasticity), and wealth (persistence).

- The more transitory the shock, the smaller the increase in C, and so the stronger the substitution effect.

- The more permanent (with C_t increasing as much or more than W_t. Can it? Yes. Think of a permanent shock, plus capital accumulation), the stronger the wealth effect. Employment could decrease.

Another way of looking at the employment effects. An intertemporal condition for leisure (this is the way Lucas and Rapping looked at it):

Replace consumption by its expression from the intratemporal condition. And, just for convenience, use $v(L) = \log(L)$, so $v'(L) = 1/L$. Then:

$$C_t = W_t L_t$$

So, replacing in the intertemporal condition:

$$E[\beta(R_{t+1}\frac{W_t}{W_{t+1}})\frac{L_t}{L_{t+1}} \mid \Omega_t] = 1$$

What is relevant for the leisure decision is the rate of return "in wage units".

Now consider a transitory shock, so W_t increases but W_{t+1} does not change much. Then L_t/L_{t+1} will decrease sharply. The increase in the wage will be associated with a strong increase in employment.

Consider a permanent shock: Then W_t/W_{t+1} is roughly constant, and so is L_t/L_{t+1}. (ignoring movements in R). No movement in employment.

3 Solving the model.

The usual battery of methods.

Special cases? The same as before. Assume Cobb Douglas production, assume log log utility. Assume full depreciation.

$$K_{t+1} = Z_t K_t^{\alpha} (1 - L_t)^{1-\alpha} - C_t$$

and

$$U(C_t, L_t) = \log C_t + \phi \log L_t$$

Then, can solve explicitly. And the solution actually looks identical to that of the benchmark model. N is always constant, not by assumption, but by implication now. Substitution and income effects cancel.

$$C_t = (1 - \alpha\beta) Y_t$$

$$N \mid \frac{\phi}{1 - N} = \frac{1 - \alpha}{1 - \alpha\beta} \frac{1}{N}$$

So, nice, but not useful if we want to think about fluctuations in employment.

So need to go to **numerical simulations**. SDP, or log linearization. Campbell gives a full analytical characterization. Thomas has written the Matlab program for the log linear model. ("RBC.m" gives the impulse responses, and the moments, and correlations of the variables with output. Play with it).

The effects different persistence parameters for the technological shocks. See figures from RBC.m for three values of ρ. Can do the same for different elasticities of labor supply. Or different intertemporal elasticities. But in these two cases, need to modify the matrices a bit. You may want to do it.

(See also results from King Rebello. Tables 1, 3. And their figure 7.)

4 Technological shocks. Evidence

A priori, the notion that there would be sharp movements in the production frontier from quarter to quarter, highly correlated across sectors, is not plausible. The diffusion of technology is steady. Breakthroughs are rare, and unlikely to be in all sectors at once.

So second look:

4.1 The measurement of technological shocks

One way to measure technological progress was suggested by Solow. The construction of the Solow residual goes like this:

Suppose the production function is of the form:

$$Y = F(K, N, A)$$

A is the index of technological level, and enters the production function without restrictions. We want to measure the contribution of A to Y.

Differentiate and rearrange to get:

$$\frac{dY}{Y} = \frac{F_K K}{Y}\frac{dK}{K} + \frac{F_N N}{Y}\frac{dN}{N} + \frac{F_A A}{Y}\frac{dA}{A}$$

Suppose now that firms price according to marginal cost. Let W be the price of labor services, and R be the rental price of capital services. Assume no costs of adjustment for either labor or capital. Then:

$$P = \text{MC} = W/F_N = R/F_K$$

Replacing:

$$\frac{dY}{Y} = \frac{RK}{PY}\frac{dK}{K} + \frac{WN}{PY}\frac{dN}{N} + \frac{F_A A}{Y}\frac{dA}{A}$$

Define the Solow residual as $S \equiv (F_A A/Y)(dA/A)$. Let α_K be the share of capital costs in output, and α_N be the share of labor costs in output. Then:

$$S = \frac{dY}{Y} - \frac{dX}{X}$$

where

$$\frac{dX}{X} \equiv \alpha_K \frac{dK}{K} + \alpha_N \frac{dN}{N}$$

The Solow residual is equal to output growth minus weighted input growth, where the weights are shares (and time varying). No need for estimation, or to know anything about the production function.

If we construct the residual in this way:

- Get a highly procyclical Solow residual. Figure 1 from Basu.
- Get a very good fit: From annual data from 1960 to 1998 (different time period from Basu graph):

$$\frac{dY}{Y} = 1.16\,S + 0.36\,S(-1) + \epsilon \qquad \bar{R}^2 = .82$$

Solow used this approach to compute S over long periods of time. Is it reasonable to construct it to estimate technological change from year to year? A number of problems. Among them:

- Costs of adjustment. If costs of adjustment to capital, then the shadow rental cost is higher/lower than the rental price R. Same if costs of adjustment to labor. So shares using prices may not be right.
- Non marginal cost pricing. Firms may have monopoly power, in which case, markup μ will be different from one.
- Unobserved movements in N or K. Effort? Capacity utilization?

Examine the effects of the last two;

Markup pricing

Suppose

$$P = \mu \, \mathrm{MC}$$

Then: $P = \mu W/F_N$ or $F_N = \mu W/P$. Similarly $F_K = \mu R/P$. So:

$$S = \frac{dY}{Y} - \mu \frac{dX}{X}$$

Let measured Solow residual be $\hat{S}$, and true Solow residual be S. Then, if $\mu > 1$ and we construct the Solow residual in the standard way, then we shall overestimate the Solow residual when output growth/input growth is high. :

$$S = \hat{S} - (\mu - 1)\frac{dX}{X}$$

Figure, for $\mu = 1.1, 1.2$. Much less procyclical.

Unobserved inputs

Suppose for example that $N = BHE$, where B is number of workers, H is hours per worker, and E is effort. Going through the same steps as before, leaving markup pricing aside:

$$S \equiv \frac{dY}{Y} - [\alpha_K \frac{dK}{K} + \alpha_N (\frac{dB}{B} + \frac{dH}{H} + \frac{dE}{E})]$$

Suppose we observe B and H but not E, so measure labor (incorrectly) by BH. Then, again, we shall tend to overestimate the Solow residual in booms:

$$S = \hat{S} - \alpha_N \frac{dE}{E}$$

Similar issues with capacity utilization on the capital side.

Are there ways around it?

Suppose that we allow for markup pricing and unobserved effort. Then:

$$S = \frac{dY}{Y} - \mu \frac{dX}{X} - \mu \alpha_N \frac{dE}{E}$$

Or, equivalently:

$$\frac{dY}{Y} = \mu \frac{dX}{X} + \mu \alpha_N \frac{dE}{E} + S$$

Can we estimate it and get a series for the residual? There are two problems:

- Unobservable effort dE/E ? Part of the error term, likely to be correlated with dX/X.

  If firms cost minimize at all margins and can freely adjust effort and hours, then under plausible assumptions, dE/E and dH/H will move together. So will capacity utilization. So can estimate:

$$\frac{dY}{Y} = \mu \frac{dX}{X} + \beta \alpha_N \frac{dH}{H} + S$$

- S correlated with dX/X? Likely as well. Surely under RBC hypotheses. So, need to use instruments: Government spending on defense, oil price, federal funds innovation... Good instruments? Might be easier in a small economy: World GDP.

Results. Basu and Fernald. Find markup around 1, so that correction makes little difference. But the correction for hours makes the estimated Solow residual nearly a-cyclical. See Figure 3.

Role of technological shocks? Variance decomposition of a bivariate VAR in the estimated residual and the usual Solow residual:

Contribution of technological shock to Solow residual, 5% on impact, 38% after a year, 59% after 3 years, 66% after 10 years.

4.2 The role of technological shocks in fluctuations

Having constructed an adjusted series, can look at the dynamic effects on output, employment, and so on. I have not seen this done.

An alternative construction of shocks, and the results. Blanchard Quah, Ramey NBER WP, 2002.

Identify the technological shocks as those shocks with a long term effect on output. (Correct?). Technically:

- Bivariate VAR in $\Delta Y/Y$ and u. Stationary. So no effect of shocks on growth and unemployment rate. But potential effect on level of output.

- Assume two types of shocks. Shocks with permanent effects on level of output. Shocks with no permanent effects on output. This is sufficient for identification.

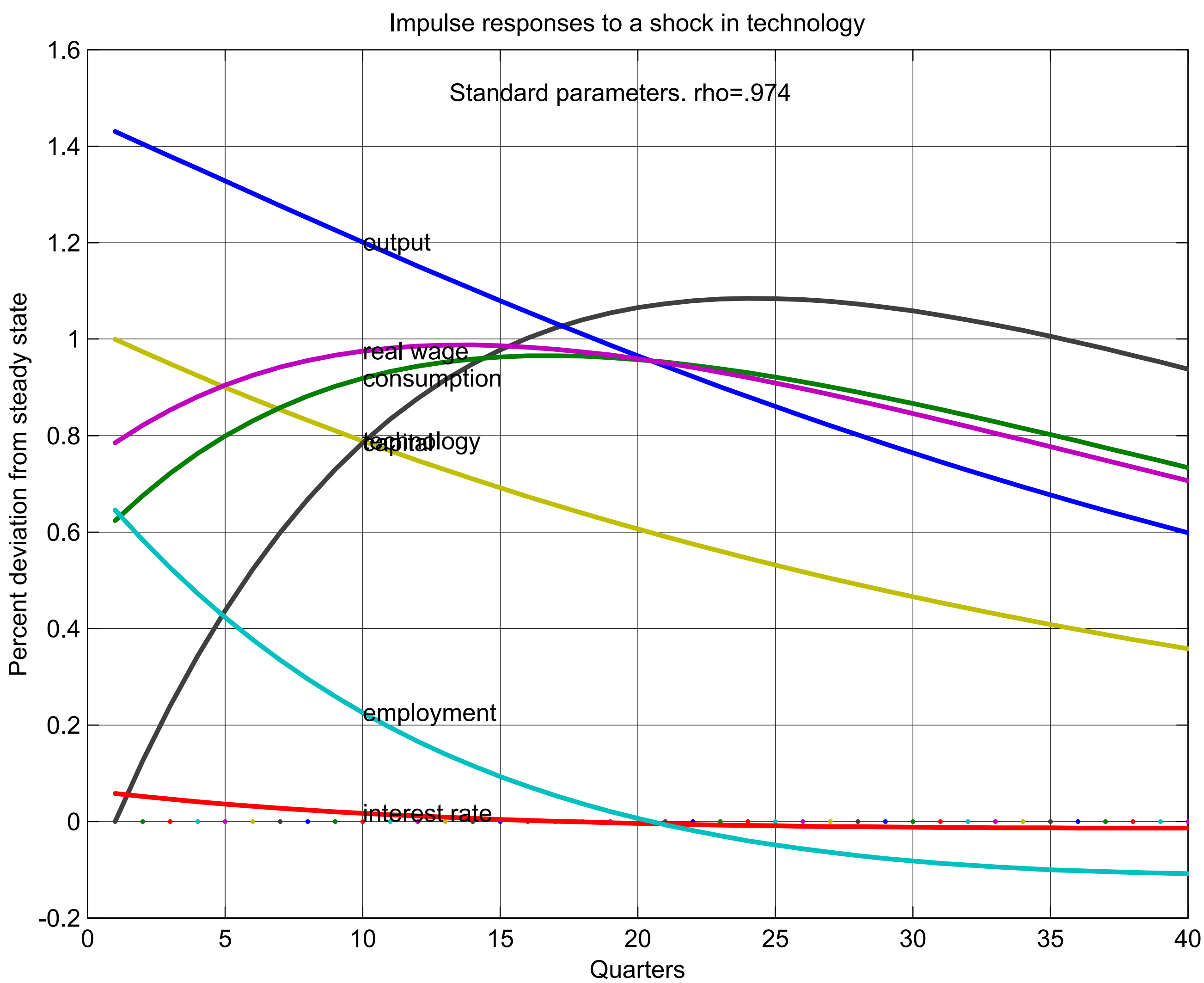
- Call the first technological shocks. Impulse responses. Figures 3 to 6 in BQ. Can one make sense of these effects: negative on employment? Yes, if demand matters.

- Variance decomposition. Tables 2 to 2B. Due to technological shocks: 1% to 16% at one quarter, 20 to 50% at 8 quarters.

4.3 Movements in employment and the labor/leisure choice

Are movements in unemployment plausibly due to leisure/labor choice? Important to distinguish between non participation and unemployment.

Decision not to participate plausibly reflects work at home/work choice. Unemployment is a different decision. The two move partly together, but not identical.


Impulse responses to a shock in technology
Standard parameters. rho=.974
output
real wage
consumption
technology
capital
employment
interest rate
Percent deviation from steady state
Quarters
1.6
1.4
1.2
1
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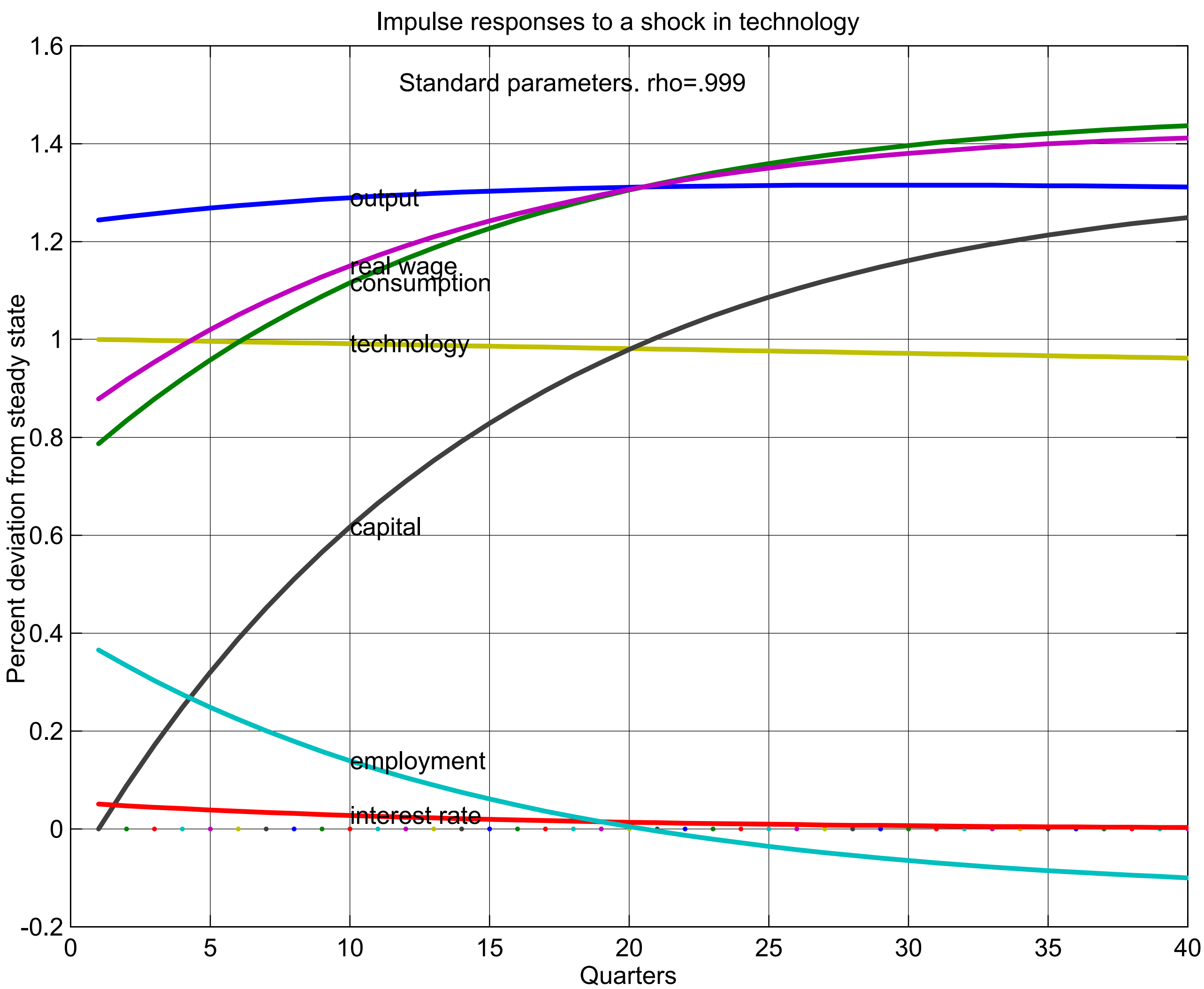
35
40


Impulse responses to a shock in technology
Standard parameters. rho=.999
output
real wage
consumption
technology
capital
employment
interest rate
Percent deviation from steady state
Quarters
1.6
1.4
1.2
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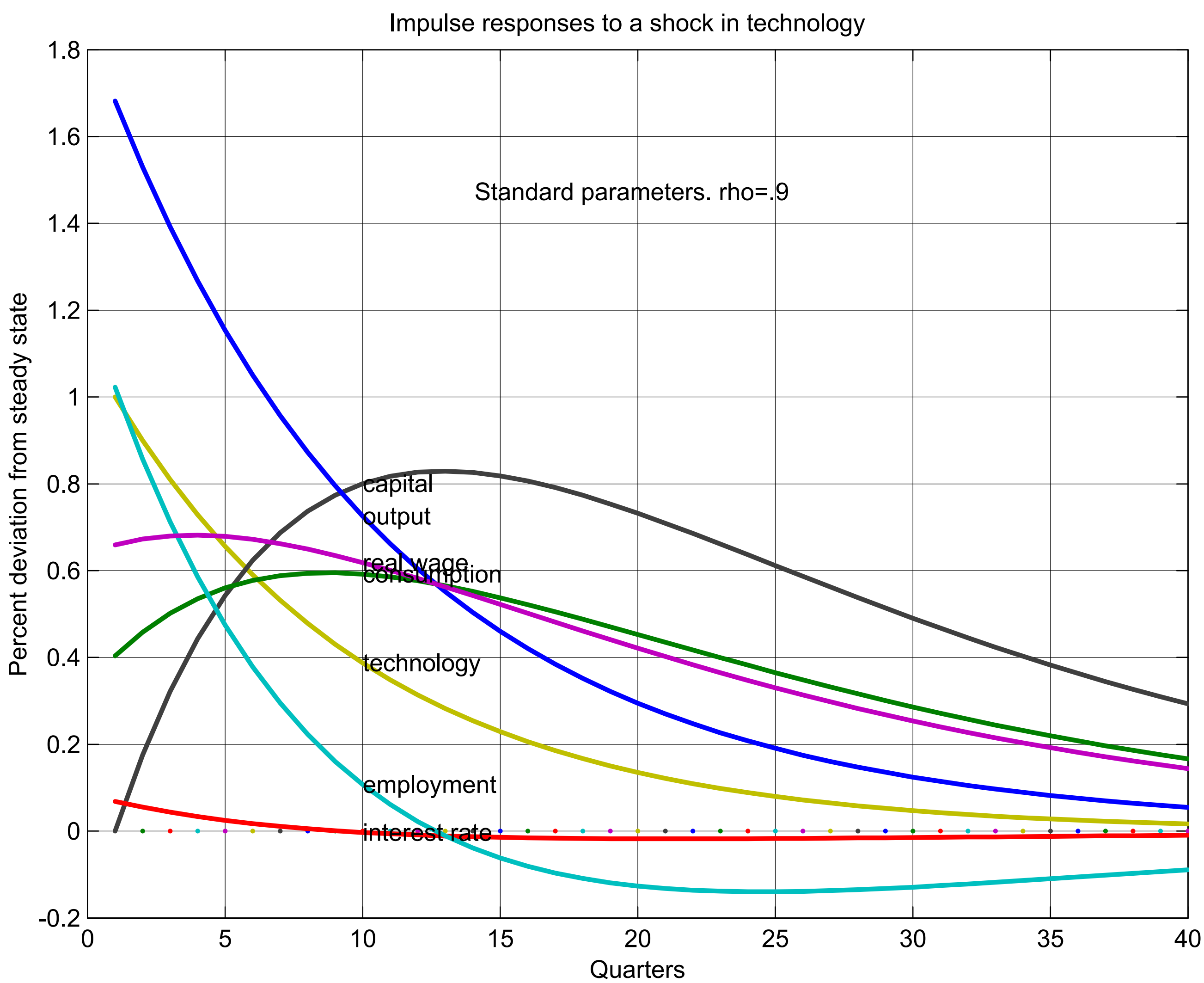
Impulse responses to a shock in technology
Standard parameters. rho=.9
capital
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real wage
consumption
technology
employment
interest rate
Percent deviation from steady state
Quarters
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