

Problem Set #8

Due December 5, 2001

1. Consider a first price sealed bid auction in which N potential bidders have valuations v_1, v_2, \dots, v_N for a good which are independent draws from a distribution with CDF $F(\cdot)$ on the interval $[0, 1]$. The bidders simultaneously choose bids b_1, \dots, b_N in $[0, 1]$ and the highest bidder gets the good and pays his bid. Show that the unique symmetric equilibrium is

$$b_i^*(v_i) = v_i - \frac{\int_0^{v_i} F(z)^{N-1} dz}{F(v_i)^{N-1}}.$$

Hint: When his opponents use the symmetric equilibrium strategy B , bidder i 's expected payoff when has value v he bids b is $(v - b)(F(B^{-1}(b)))^{N-1}$. Write $\pi(v)$ for the equilibrium utility of a bidder with value v . Use the envelope theorem to show that

$$\frac{d\pi}{dv} = F(v)^{N-1}.$$

Then integrate over v to find $\pi(v)$ and use the fact that in equilibrium a bidder with value v wins with probability $F(v)^{N-1}$ to infer what the bids must be. See Milgrom's JEP article for more help.

2. In a two player model where the player's valuations are independent uniform on $[0, 1]$, find the equilibrium of a variant of a first price sealed bid auction in which the seller announces a reserve price of R and sells the good to the highest bidder only if the winning bid is at least R . What value of R maximizes the seller's expected revenue from the auction?

(Note: If the calculations are difficult just try to describe what you'd think an equilibrium would have to look like.)

3. An environmental group which wants to build a wildlife preserve is bidding against a logging company in a government auction for a tract of land. Because the two parties will use the land for different purposes, assume that the auction can be modeled as one with independent private values. Assume that it is common knowledge that the environmental group's valuation for the land, v_e , is drawn from a uniform distribution on $[0, 2]$, and that the logging company's valuation, v_ℓ is drawn from a uniform distribution on $[1, 2]$.

(a) Find the equilibrium strategies and the probability with which each party wins the tract in a second price sealed bid auction. What is the government's expected revenue? Would you expect a first price sealed bid auction to raise more, less, or the same amount of revenue (in expectation)?

(b) Suppose that the government uses a first price auction, and that before the auction takes place the logging company has the opportunity to install new machinery at a nearby mill which would reduce the marginal cost of processing timber from this tract. Describe how the decision to install the machinery might depend on whether or not the environmental group is able to observe that the new machinery has been installed.

(c) In what ways is the asymmetric auction model described above very different from that used by Hendricks and Porter to model the asymmetries between firms in bidding for offshore oil drainage tracts?

4. In a large portion of the drainage tracts in Hendricks and Porter's sample there is actually more than one "neighbor" firm. What evidence do Hendricks and Porter provide for their contention that it is reasonable to use a model with one informed firm because the informed firms appear to collude? Do you find the evidence convincing?

5. (a) Bajari models the Minnesota paving procurement auctions using a private values framework. Why does he make this choice and does it seem reasonable?

(b) What does Bajari find about the level of prices relative to costs? Given his structural framework could these results have been very different?

(c) How do Bajari's results on the projected revenues from first and second price auctions fit with your intuition from the theoretical models we've discussed?