MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.04 **Spring Term 2003**

PROBLEM SET 10

Reading: French & Taylor, 9.10 and 5.1 – 5.7. Liboff, 10.1 – 10.2.

1. **Alpha decay of heavy elements.** (20 points)

   Following the WKB approximation and Gamow theory of alpha decay discussed in lecture, calculate the lifetimes of the U^{238} and Po^{212} nuclei.

   Hints: Since the density of nuclear matter is relatively constant (i.e., the same for all nuclei), you may assume the empirical relation $R_{\text{nucleus}} \approx A^{1/3}$ fm, where 1 fm $= 10^{-13}$ cm and A is the atomic weight (the number of protons plus neutrons). The energy of the emitted alpha particle is given by Einstein's formula $E = mc^2$:

   $$E = m_p c^2 - m_d c^2 - m_\alpha c^2,$$

   where m_p is the mass of the parent nucleus, m_d is the mass of the daughter nucleus, and m_α is the mass of the alpha particle (that is, the He^4 nucleus). A few recommended atomic masses are (http://csnwww.in2p3.fr/AMDC/web/masseval.html):

   mass of He^4 = 4.00260325 amu

   mass of Pb^{208} = 207.9766359 amu

   mass of Po^{212} = 211.9888518 amu

   mass of Th^{234} = 234.0435955 amu

   mass of U^{238} = 238.0507826 amu

   To estimate the effective velocity v of the alpha particle inside the nucleus, use $E = (1/2)m_\alpha v^2$. This ignores the (negative) potential inside the nucleus and underestimates v, but is a reasonable first approximation nonetheless. For comparison, the experimentally determined lifetimes are 4.5×10^9 yr and 3.0×10^{-7} s, respectively (http://nucleardata.nuclear.lu.se/NuclearData/toi/perchart.htm).

2. **Center of mass and relative coordinates.** (20 points)

   An isolated system consists of two particles which interact via a force represented by a potential that depends only on the distance between them. In three-dimensions, the Hamiltonian is

   $$H = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + V(\vec{r}_1, \vec{r}_2) = -\frac{\hbar^2}{2m_1}\nabla_1^2 - \frac{\hbar^2}{2m_2}\nabla_2^2 + V(|\vec{r}_1 - \vec{r}_2|)$$

(a) Perform a change of variables to center-of-mass and relative coordinates

$$\vec{R} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} \quad , \quad \vec{r} = \vec{r}_1 - \vec{r}_2$$

and show that the Hamiltonian may be written as

$$H = -\frac{\hbar^2}{2M}\nabla^2_{\mathbf{R}} - \frac{\hbar^2}{2\mu}\nabla^2_{\mathbf{r}} + V(r),$$

where the total mass $M = m_1 + m_2$ and the reduced mass $\mu = m_1 m_2/(m_1 + m_2)$. Calculate the reduced mass of a system consisting of a proton and an electron (in MeV/c^2).

(b) Show that solutions of

$$H\Psi(\vec{r}_1, \vec{r}_2) = E\Psi(\vec{r}_1, \vec{r}_2)$$

may be found with the form

$$\Psi(\vec{r}_1, \vec{r}_2) = \phi(\vec{R})\psi(\vec{r})$$

by using the separation of variables technique. Show that you get the Schrodinger equation for a free particle in the variable $\vec{R}$ and solve it.

3. **Energy degeneracy in a cubical box.** (20 points)

   French & Taylor, Problem 5-4

4. **Spherical step-well.** (20 points)

   French & Taylor, Problem 5-7

5. **Inadequacies in the Bohr theory of hydrogen.** (20 points)

   French & Taylor, Problem 5-9