MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.04 **Spring Term 2003**

PROBLEM SET 2

Reading: French and Taylor, Chapter 2.

1. **Time delay in the photo-electric effect** (20 points).

   A beam of ultraviolet light of intensity 10^7 eV s^{-1} is turned on suddenly and falls on a metal surface, ejecting electrons through the photo-electric effect. The beam has a cross-sectional area of 1 cm^2, and the wavelength corresponds to a photon energy of 10 eV. The work function of the metal is 5 eV. How soon after the beam is turned on might one expect photo-electric emission to occur?

   (a) Classically, one can estimate this as the time needed for the work-function energy (5 eV) to be accumulated over the area of one atom (radius $\approx 1\text{Å}$). Calculate how long this would be, assuming the energy of the light beam to be uniformly distributed over its cross-section.

   (b) Actually (as shown by Lord Rayleigh in 1916), the estimate from part (a) is too pessimistic. An atom can present an *effective* area of about λ^2 to light of wavelength λ corresponding to its resonant frequency (i.e. Rayleigh scattering!). Calculate a classical delay time on this basis.

   (c) Under the quantum picture of the process, it is possible for photo-electric emission to begin immediately – as soon as the first photon strikes the emitting surface. But to obtain a time that may be compared to the classical estimates, calculate the average time interval between arrival of successive 10 eV photons. This would also be the average time delay between switching on the beam and getting the first photo-electron.

2. **Determination of Planck's constant** (20 points).

   The clean surface of sodium metal (in a vacuum) is illuminated with monochromatic light. In a series of measurements, various wavelengths are used and the retarding potentials required to stop the most energetic photo-electrons are observed as follows:

Wavelength (Å)	Retarding Potential (V)
2536	2.60
2830	2.11
3039	1.81
3302	1.47
3663	1.10
4358	0.57

Plot these data in such a way as to show that they lie (approximately) along a straight line as predicted by the photo-electric equation, and obtain a numerical value for Planck's constant h.

3. **Compton scattering.** (25 points).

   (a) Show that it is impossible for a free electron to absorb all of the energy of a single photon which collides with it.

   (b) Derive the Compton wavelength shift for a photon scattered from a free, initially stationary electron,

   $$\Delta\lambda = \lambda_1 - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta),$$

   where θ is the photon scattering angle.

   (c) The Compton shift in *wavelength*, $\Delta\lambda$, is independent of the incident photon energy $E_0 = h\nu_0 = hc/\lambda_0$. However, the Compton shift in *energy*, $\Delta E = E_1 - E_0$, is strongly dependent on E_0. Find the expression for the Compton energy shift ΔE. (Be careful to get the sign right – does the photon *gain* or *lose* energy in the collision?) Compute the numerical value of the *fractional* shift in energy for a 10 keV photon and a 10 MeV photon, assuming $\theta = 90°$.

   (d) Why is it much more difficult to observe the Compton effect in the scattering of visible light than in the scattering of X-rays?

   (e) Show that the relation between the directions of motion of the scattered photon and the recoil electron is

   $$\cot\frac{\theta}{2} = \left(1 + \frac{h\nu_0}{m_e c^2}\right)\tan\phi,$$

   where ϕ is the angle for the recoil electron.

4. **The De Broglie wavelengths of visible particles** (15 points).

   If, as De Broglie says, a wavelength can be associated with *every* moving particle, then why are we not forcibly made aware of this property in our everyday experience? In answering, calculate the de Broglie wavelength of each of the following "particles":

(a) an automobile of mass 2 metric tons (2000 kg) traveling at a speed of 50 mph (22 m/sec),

(b) a marble of mass 10 g moving with a speed of 10 cm/sec,

(c) a smoke particle of diameter 10^{-5} cm (and a density of, say, 2 g/cm^3) being jostled about by air molecules at room temperature (27°C = 300 K). Assume that the particle has the same translational kinetic energy as the thermal average of the air molecules:

$$\frac{p^2}{2m} = \frac{3kT}{2},$$

with k = Boltzmann's constant = 1.38×10^{-16} erg/degree K.

5. **Double-slit interference of electrons** (20 points).

   (a) Electrons of momentum p fall normally on a pair of slits separated by a distance d. What is the distance between adjacent maxima of the interference fringe pattern formed on a screen a distance D beyond the slits?

   (b) In the actual experiment performed by Jönsson (see Section 2–6 of French & Taylor), the electrons were accelerated through a 50 kV potential, the slit separation d was 2×10^{-4} cm, and D was 35 cm. Calculate λ and the fringe spacing. You will then appreciate why subsequent magnification using an electron microscope was required!

   (c) What would be the corresponding values of d, D, and the fringe spacing if Jönsson's apparatus were simply scaled up for use with visible light (all dimensions simply multiplied by the ratio of wavelengths)?