MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.04 **Spring Term 2003**

PROBLEM SET 1

Reading: French & Taylor, Chapter 1.

1. **Sizes of atoms** (10 points).

   (a) Given the knowledge that atomic diameters are on the order of 1 Å, estimate the order of magnitude of the number of atoms in: (i) a pin's head, (ii) a human being, (iii) the whole earth.

   (b) The crystal lattice of sodium chloride has ions of sodium (Na^+) and ions of chloride (Cl^+) arranged alternately in a simple cubic array. Thus the crystal volume per ion is d^3, where d is the distance between the centers of adjacent ions. The molecular weight of NaCl is 58.45 and its density is 2.164 g/cm^3.

      i. Calculate the volume of 1 mole of crystalline NaCl.

      ii. Using the fact that 1 mole of NaCl contains Avogadro's number N of sodium ions and N chloride ions, calculate the spacing d of the ions in the lattice. Compare your result with the handbook value of 2.82 Å.

2. **Energies of photons** (15 points).

   What is the energy (in units of eV) of:

   (a) a x-ray of wavelength $\lambda = 1$ Å?

   (b) a photon of visible light, of wavelength $\lambda = 5 \times 10^{-7}$ m?

(c) a radio-wave photon of wavelength $\lambda = 10$ m?

(d) What is the photon flux ($\frac{photons}{cm^2 s}$) at a distance of 1 m from a light bulb emitting 50 W of radiation in the visible domain (with wavelength $\lambda = 5000$ Å)?

3. **Millikan's experiment and quantization of charge** (25 points).

The table below lists data from a run of Millikan's oil drop experiment. The motion of a single drop was followed for a duration of 45 minutes. During that time, the electric field between the capacitor plates in the apparatus was switched on or off causing the charged drop to rise or fall, respectively. The drop occasionally picked up or lost some charge, causing a change in the upward force acting on it when the electric field ($E = 10.60$ statvolts cm^{-1}) was on. The columns labelled T_f and T_r give the time (in seconds) required for the drop to fall or rise, respectively, through a distance $d = 1.021$ cm. The fall times should be the same for all measurements, so differences are related to measurement errors. The rise times also show the effect of measurement errors, however the abrupt changes are due to gain or loss of charge in between measurements.

Rise and fall times of a single oil drop with calculated number of elementary charges on drop†

1 T_f	2 T_r	3 $\frac{1}{T_r'} - \frac{1}{T_r}$	4 n'	5 $\frac{1}{n'}\left(\frac{1}{T_r'} - \frac{1}{T_r}\right)$	6 $\frac{1}{T_f} + \frac{1}{T_r}$	7 n	8 $\frac{1}{n}\left(\frac{1}{T_r} + \frac{1}{T_f}\right)$
11.848	80.708				0.09655	18	0.005366
11.890	22.366	0.03234	6	0.005390	0.12887	24	0.005371
11.908	22.390						
11.904	22.368						
11.882	140.566	0.03751	7	0.005358	0.09138	17	0.005375
11.906	79.600	0.005348	1	0.005348	0.09673	18	0.005374
11.838	34.748	0.01616	3	0.005387	0.11289	21	0.005376
11.816	34.762						
11.776	34.846						
11.840	29.286				0.11833	22	0.005379
11.904	29.236						
11.870	137.308	0.026872	5	0.005375	0.09146	17	0.005380
11.952	34.638	0.021572	4	0.005393	0.11303	21	0.005382
11.860							
11.846	22.104	0.01623	3	0.005410	0.12926	24	0.005386
11.912	22.268						
11.910	500.1	0.04307	8	0.005384	0.08619	16	0.005387
11.918	19.704	0.04879	9	0.005421	0.13498	25	0.005399
11.870	19.668						
11.888	77.630	0.03794	7	0.005420	0.09704	18	0.005390
11.894	77.806						
11.878	42.302	0.01079	2	0.005395	0.10783	20	0.005392
11.880		Means		0.005389	Means		0.005384

Millikan R. A. "Rise and fall times of a single oil drop with calculated number of elementary charges on drop." Data Table. Electrons, Protons, Photons, Mesotrons and Cosmic Rays, (c) University of Chicago Press, 1947.

(a) As mentioned in lecture, the viscous drag force on a spherical droplet is given by $\vec{F}_{drag} = -b\vec{v}$ where $b = 6\pi\eta a$. Here, η (= 1.824×10^{-4} cgs units) is the coefficient of viscosity and a is the radius of the drop. Let ρ (= 0.9432 gm cm^{-3}) be the density of the oil drop. Find an expression for m, the mass of the drop, in terms of T_f, d, η, and ρ. In this experiment, the terminal velocities are reached almost immediately. Calculate m (in gm) using the mean value of T_f.

(b) Write an expression for the total charge ne on the drop in terms of T_f, T_r and the other constants. If an additional charge is picked up between measurements, the rise time changes from T_r to T_r'. Write an expression for the amount of charge gained, $n'e$, in terms of T_r, T_r' and the other constants.

(c) Explain how the values for n' (column 4) and n (column 7) are deduced from the data in the previous columns. Note that n' and n are found to be integers, indicating that the total charge always occurs in multiples of a fundamental unit, e.

(d) Use the mean values recorded at the bottom of the above table to estimate the charge of the electron e.

4. **Rutherford scattering** (25 points).

(a) A beam of α-particles, of kinetic energy $E = 5.30$ MeV and intensity 10^4 particles s^{-1}, is incident normally on a gold foil of density 19.3 g cm^{-3}, atomic weight 197, and thickness 1.0×10^{-5} cm. An α-particle detector of area 1.0 cm^2 is placed at a distance 10 cm from the foil. Let θ be the scattering angle. Use the Rutherford scattering differential cross-section

$$\frac{d\sigma}{d\Omega} = \frac{1}{16} r_0^2 \frac{1}{\sin^4(\theta/2)}$$

(where $r_0 = qQ/E$) to find the number of counts per minute for $\theta = 10^\circ$ and $\theta = 45^\circ$. The charge of the gold nucleus is $79e$ (that is, $Z = 79$). Note that by measuring the counts we can determine the value of the nuclear charge Z.

(b) Although the Rutherford cross-section correctly predicts the scattering rate of 5.3 MeV α-particles from gold foil, serious discrepancies occur when 32 MeV α-particles are scattering through large angles ($\theta \sim 180^\circ$). In order for the Rutherford formula to be correct, the α-particle must not penetrate the nucleus. As a crude indication of the size of a nucleus, calculate the closest approach of a 32 MeV α-particle in a head-on collision with a gold nucleus ($Z = 79$). Neglect the recoil of the gold nucleus. Briefly explain why we may ignore in this calculation the electrons surrounding the gold nucleus.

5. **Collapse of the classical atom** (25 points).

The classical atom has a stability problem. Let's model the hydrogen atom as an electron in a classical circular orbit about a proton. From our knowledge of electricity and magnetism, we know that an accelerating electric charge radiates energy. The power radiated is given by the Larmor formula:

$$P = \frac{2}{3}\frac{q^2}{c^3}a^2$$

in cgs units, where q is the electric charge and a is the magnitude of the acceleration.

(a) Show that the energy lost per revolution is small compared to the electron's kinetic energy. Hence, it is an excellent approximation to regard the orbit as circular at any instant, even though the electron eventually spirals into the proton.

(b) Using the typical size for the atom (1 Å) and the nucleus (10^{-5} Å), calculate how long it would take for the electron to spiral into the proton.