MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.04 **Spring Term 2003**

PROBLEM SET 4

Reading: Liboff, Chapter 3.
Gasiorowicz, Chapter 2 contains a useful section on uncertainty relations.

1. **Particle diffraction and the uncertainty principle.** (20 points).

   A well-collimated beam of particles with sharply defined x-momentum p_x falls normally on a screen in which is cut a slit of width d. (The screen lies along the y-axis.) On the far side of the slit, the particles travel with a slight spread of directions, $\theta \approx \lambda/d$, as described by the diffraction of their De Broglie waves with $\lambda = h/p_x$. Show that this diffraction spreading angle can also be (roughly) characterized in terms of the uncertainty product $(\Delta y)(\Delta p_y)$ for the direction y transverse to the initial direction of the beam.

2. **The uncertainty principle and precision experiments.** (25 points).

   A child on top of a ladder of height H is dropping marbles of mass m to the floor and trying to hit a crack in the floor. To aim, the child is using equipment of the highest possible precision. Assume that the effects of air resistance and breezes are entirely negligible. Show that the marbles will miss the crack by a typical distance of order $(\hbar/m)^{1/2}(2H/g)^{1/4}$, where g is the acceleration due to gravity. Using reasonable values of H and m, evaluate this distance numerically.

3. **Triangular wave packet and uncertainty relation.** (35 points).

   Let's take the triangular pulse from the previous problem set to be a wave function.

$$\psi(x) = \begin{cases} N(1 + \frac{x}{b}) & -b \le x \le 0 \\ N(1 - \frac{x}{b}) & 0 < x \le b \\ 0 & \text{elsewhere.} \end{cases}$$

   (a) Find N using the normalization condition.

   (b) The expectation value of position $\langle x \rangle$ is given by:

$$\langle x \rangle = \int_{-\infty}^{\infty} x \ |\psi(x)|^2 dx.$$

   The expectation value of momentum $\langle p \rangle$ is given by:

$$\langle p \rangle = \int_{-\infty}^{\infty} \hbar k \ |\tilde{\psi}(k)|^2 dk.$$

Calculate $\langle x \rangle$ and $\langle p \rangle$. (*Hint:* Take advantage of symmetry.)

(c) The uncertainty in position Δx may be calculated from the relation $(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2$. A similar relation holds for the uncertainty in momentum Δp. Calculate Δx and Δp. Compare the product $\Delta x \Delta p$ with the minimum value allowed by the Heisenberg Uncertainty Principle.

Hint: You may find the following mathematical formulas useful:

$$1 - \cos(x) = 2\sin^2(x/2).$$

$$\int_0^\infty \frac{1}{x^2} \sin^4(x) dx = \frac{\pi}{4}.$$

4. **Parseval's Theorem** (20 points).

Prove the following theorem (known as Parseval's theorem): If $\tilde{\psi}(k)$ is the Fourier transform of $\psi(x)$, then they have the same normalization. That is,

$$\int_{-\infty}^{\infty} |\tilde{\psi}(k)|^2 dk = \int_{-\infty}^{\infty} |\psi(x)|^2 dx.$$

(Note: The proof is not very long.)