MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.04 **Spring Term 2003**

PROBLEM SET 5

Reading: French & Taylor, Chapter 3.

1. **Gaussian quantum wave function** (25 points).

   A particle of mass m is in the state

   $$\Psi(x,t) = Ae^{-a[(mx^2/\hbar)+it]},$$

   where A and a are positive real constants.

   (a) Find A.

   (b) Find the potential energy function $V(x)$ for which Ψ satisfies the Schrödinger equation. What is this potential called?

   (c) Calculate the expectation values of x, x^2, p, and p^2. Find σ_x and σ_p. Is their product consistent with the Heisenberg uncertainty principle, $\sigma_x \sigma_p \geq \frac{\hbar}{2}$? Comment on what this says about Gaussian wave functions.

   [*Hint:* $\int_{-\infty}^{\infty} e^{\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$, and applying $\frac{d}{d\alpha}$ leads to another useful relation.]

2. **Probability current** (25 points).

   A particle is in a state described by the wavefunction $\Psi(x,t)$. Let $P_{ab}(t)$ be the probability of finding the particle in the range $(a < x < b)$ at time t. Show that

   $$\frac{dP_{ab}}{dt} = J(a,t) - J(b,t),$$

   where

   $$J(x,t) \equiv \frac{i\hbar}{2m}\left(\Psi\frac{\partial\Psi^*}{\partial x} - \Psi^*\frac{\partial\Psi}{\partial x}\right).$$

   [*Hint:* Calculate $\frac{\partial}{\partial t}|\Psi|^2$.]

   What are the units of $J(x,t)$?

   J is called the *probability current*, because it tells you the rate at which probability is "flowing" past the point x. If $P_{ab}(t)$ is increasing, then more probability is flowing into the region at one end than flows out at the other.