

April, 1994

LIDS- P 2240

Research Supported By:

ARO grant DAAL03-92-G-0115

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Channels

Gallager, R.G.

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Robert G. Gallager
Massachusetts Institute of Technology
Cambridge, MA 02139 USA

Abstract

The effects of multiaccess sources and time varying multipath are considered in analyzing a system in which multiple sources communicate with a fixed base station. We discuss detection and the use of stripping in a multiaccess multipath environment. We finally derive a capacity for these systems. It turns out that CDMA type systems are inherently capable (theoretically) of higher rates than systems such as slow frequency hopping that maintain orthogonality between users.

Dedication

This paper is dedicated to James L. Massey on the occasion of his 60th birthday. His works on coding, on multiaccess, and on information theory have all had an important impact on the ideas here. May his 60's be as much fun and as much an inspiration to all of us as his earlier decades.

1 Introduction

Wireless communication has become a very active research area in recent years. This activity is stimulated partly by the success of the Qualcomm code division multiple access (CDMA) system [GJP91], [Qua91] and the European GSM system [HaS92], partly by the rapidly growing wireless market, and partly by the ability of VLSI technology to implement very sophisticated systems inexpensively. While it is not difficult to learn about the particular systems being implemented today, it is quite difficult to contrast the basic merits of these approaches. This difficulty comes partly from the inherent complexity of these systems and partly from some degree of sales orientation in much of the technical literature in the area. Our purpose in this paper is to provide some perspective on the concepts and theory underlying this area.

The systems of interest here are those with a set of non-stationary subscribers communicating with a fixed base station. The subscriber sets could be in vehicles or be hand held, the communication could be voice or data, and the area of interest could be urban, rural, or inside a building. Systems with multiple base stations, and the inherent problems, first, of handoff from one base station to another and, second, of interference between the cells associated

**This research was supported by the Army Research Office Office under grant ARO DAAL03-92-G-0115 (Center for Intelligent Control).*

with different base stations are important, but they will not be addressed here.

2 Characterization of Fading Multipath Channels

The major distinguishing characteristics of wireless communication are the fading effects and the multiaccess effects. In this section, we discuss the characterization of fading multipath channels in the context of point to point communication; in the next section, we include multiaccess effects. Since the subscriber location is arbitrary and not chosen for its propagation characteristics, multiple propagation paths typically exist from transmitter to receiver. We denote the strength of the i^{th} such propagation path by a_i and the propagation delay by τ_i . These strengths and delays vary with time, and thus can be characterized as $a_i(t)$ and $\tau_i(t)$. If a signal $x(t)$ is transmitted, it is received as

$$y(t) = \sum_i a_i(t)x(t - \tau_i(t)) \quad (1)$$

The observed waveform at the output is then $y(t) + n(t)$ where $n(t)$ is additive noise. It can be seen that the effect of the multipath is the same as the effect of a linear filter (although the filter here is time varying), and thus we can represent the effect of the multipath as a filter impulse response, $g(\tau, t)$ given by

$$g(\tau, t) = \sum_i a_i(t)\delta(\tau - \tau_i(t)) \quad (2)$$

$$y(t) = \int x(t - \tau)g(\tau, t)d\tau \quad (3)$$

Note that $g(\tau, t)$ can be interpreted as the response at time t to an impulse τ seconds earlier at the input. One benefit of representing the multipath as a time varying impulse response $g(\tau, t)$ is that $a_i(t)$ might be frequency selective, and $g(\tau, t)$ can incorporate this effect. The major benefit, however, is that (3) reduces the problem of point to point communication through a multipath channel to a very familiar problem—communication through a linear filter channel with additive noise.

Now let us assume that the input is limited to some band W of frequencies around some center frequency f_0 . Then we can express the input and output to the channel in terms of the baseband complex equivalent waveforms $u(t)$ and $v(t)$,

$$x(t) = \text{Re}[u(t) \exp(j2\pi f_0 t)] = \text{Re}[u(t)] \cos(2\pi f_0 t) - \text{Im}[u(t)] \sin(2\pi f_0 t) \quad (4)$$

$$y(t) = \text{Re}[v(t) \exp(j2\pi f_0 t)] = \text{Re}[v(t)] \cos(2\pi f_0 t) - \text{Im}[v(t)] \sin(2\pi f_0 t) \quad (5)$$

Substituting (4) and (5) into (1),

$$\begin{aligned} \text{Re}[v(t) \exp(j2\pi f_0 t)] &= \sum_i a_i(t) \text{Re}[u(t - \tau_i(t)) \exp[j2\pi f_0(t - \tau_i(t))]] \\ &= \text{Re} \sum_i a_i(t) \exp[-j2\pi f_0 \tau_i(t)] u(t - \tau_i(t)) \exp[j2\pi f_0 t] \end{aligned} \quad (6)$$

Defining $\alpha_i(t) = a_i(t) \exp[-j2\pi f_0 \tau_i(t)]$ and $h(\tau, t) = \sum_i \alpha_i(t) \delta(\tau - \tau_i(t))$ as the complex baseband equivalent of the path strengths and filter response, we get the baseband equation

$$v(t) = \sum_i \alpha_i(t) u(t - \tau_i(t)) = \int u(t - \tau) h(\tau, t) d\tau \quad (7)$$

The baseband received waveform is then $r(t) = v(t) + z(t)$ where $z(t)$ is the baseband noise, *i.e.*, $n(t) = \text{Re}[z(t) \exp[j2\pi f_0 t]]$ is the noise in the bandwidth W around f_0 .

Note that $\alpha_i(t)$ is complex, and its phase changes with $\tau_i(t)$. As an example of what these equations mean, consider figure 1 in which a vehicle traveling at 60 km/hour is transmitting to a base station behind it, and there is both the direct path and a path with a reflection from a smooth wall in front of the vehicle. Path one, the direct path, is getting longer, and the propagation delay on this path is increasing at 55 nsec per second. Path 2 is getting shorter at 55 nsec per second. Thus the baseband path strengths, $\alpha_i(t) = a_i(t) \exp[-j2\pi f_0 \tau_i(t)]$, are rotating in opposite directions as shown in Figure 1b. These linearly changing phases correspond to Doppler shifts in the received frequency. The important feature here is that different paths have different Doppler shifts, and that the amount of shift depends not only on the speed of the transmitter or receiver but also on the location of the reflector. These different Doppler shifts give rise to frequency selective fading over the transmitted waveform.

It is often useful to view (7) in sampled form. By assumption, the band-pass waveforms have a bandwidth of W , with $W/2$ on each side of the center frequency f_0 . Thus $u(t)$ and $v(t)$ are constrained to bandwidth $W/2$ and are determined by their samples $u(i/W)$ and $v(i/W)$ for integer i . Eq. (7) then becomes

$$\begin{aligned} v\left(\frac{i}{W}\right) &= \sum_k u\left(\frac{i-k}{W}\right) \tilde{h}[k, i] \\ \tilde{h}[k, i] &= \int \frac{\sin(\pi(k - \tau W))}{\pi(k - \tau W)} h(\tau, i/W) d\tau \end{aligned} \quad (8)$$

$\tilde{h}[k, i]$ provides a tapped delay line model for the channel, giving the complex value of the k^{th} tap at time i/W . These taps depend on the bandwidth of the input, but suffice to provide the input output relation on the channel given the bandwidth constraint. Note that the output $v(t)$ can be spread out over a

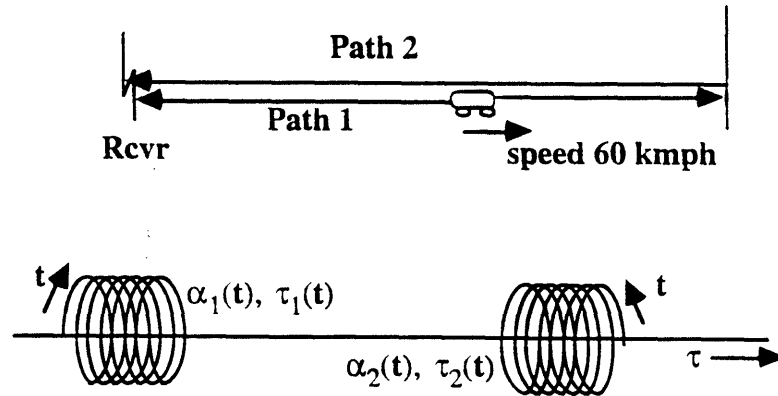


Figure 1: Path Delays and Doppler

somewhat larger bandwidth than the input because of the time varying filter. M. Medard has carried out this sampling argument carefully and shown that W need be no larger than the input bandwidth plus the bandwidth of $h(\tau, t)$ viewed as a function of t (*i.e.*, essentially the largest Doppler shift).

For the example of figure 1, if we take W to be 10^6 and f_0 to be 10^9 , we see that it takes about 18 seconds for a path to move from one tap to the next, and the tap strengths rotate at about 55 H. This means that, relative to input sample times, the taps are rotating slowly and the paths are moving even more slowly relative to the taps.

Typical multipath situations are far more complex than that of figure 1. There are often many more paths, and these paths often separate into a large number of subpaths due to reflection from rough surfaces and other anomalies. The overall delay spread between different paths ranges from less than 100 nsec. to more than 15 msec [Tur80]. The tap gains range from being relatively constant to having Rayleigh, Rician, or Nakagami distributions. As we will see later, the detailed way that taps vary is not critical, since viable communication systems must deal with all these cases, and typically must track the tap gains. The speed with which taps vary is important, but we have already seen how Doppler shifts cause the major part of this variation.

3 Point to Point Detection

Suppose that the transmitter selects the m^{th} of M waveforms, $u_1(t), \dots, u_M(t)$ for transmission in a given interval and suppose there is no multipath. Then the received baseband waveform is $r(t) = u_m(t) + z(t)$, and if the noise is white over the signalling bandwidth, it is well known that the optimal detector consists of

passing $r(t)$ through M matched filters of impulse responses $u_1^*(-t), \dots, u_M^*(-t)$ (assume that the time reference is chosen so that these filters are realizable; the complex conjugates are necessary because the signals are complex baseband signals). The decision is then made from the outputs sampled at time 0.

Next suppose there is multipath but $h(\tau, t)$ is known. This can be viewed as the same detection problem as above, replacing $u_m(t)$ with the filtered signal $v_m(t) = \int u_m(t - \tau)h(\tau, t)d\tau$, $1 \leq m \leq M$, and replacing the matched filters $u_m^*(-t)$ with the matched filters $v_m^*(-t)$. Finally, consider the case where the multipath $h(\tau, t)$ is unknown. The matched filters $v_m^*(-t)$ are then unknown, but $v_m^*(-t)$ is the convolution of $u_m^*(-t)$ with $h^*(-\tau, -t)$. A rake receiver [Pri56], [PrG58] can be viewed as a receiver that first passes $r(t)$ through the matched filters $u_m^*(-t)$, uses the outputs to help estimate $h(\tau, t)$, and then completes the matched filtering with the estimated $h^*(-\tau, -t)$ (see figure 2).

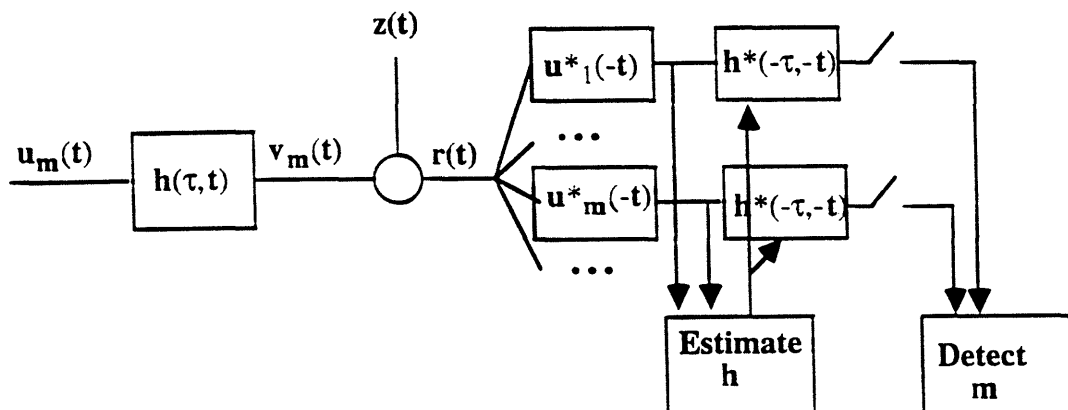


Figure 2: Rake Receiver

This becomes particularly simple in a CDMA system where the signals $u_m(t)$ are broad band noise-like signals. If $u_m(t)$ is transmitted, then the output $w_{m,m'}(t)$ of the matched filter $u_{m'}^*(-t)$ is

$$w_{m,m'}(t) = \int r(t-\tau)u_{m'}^*(-\tau)d\tau = \int \left[\int u_m(t-\tau-\phi)h(\phi, t-\tau)d\phi + z(t-\tau) \right] u_{m'}^*(-\tau)d\tau$$

$$\approx \int R_{m,m'}(t-\phi)h(\phi, t)d\phi + z_{m'}(t)$$

where $R_{m,m'}(t) = \int u_m(t-\tau)u_{m'}^*(-\tau)d\tau$ is the cross-correlation of the input signals and $z_{m'}(t)$ is the response of the matched filter to the noise $z(t)$. The approximation here is in assuming that $h(\tau, t)$ is slowly varying in t over the

multipath spread. For CDMA signals, this cross correlation function is approximately zero for $m \neq m'$. For $m = m'$, it is approximately zero for $t \neq 0$ and is equal to S , the energy of the signal at $t = 0$. Thus $w_{m,m}(\phi) \approx Sh(\phi, t) + z_m(\phi)$, and for $m \neq m'$, $w_{m,m'} \approx z_{m'}(\phi)$. Assuming one of these M signals is transmitted each signalling interval, and assuming that $h(\tau, t)$ changes slowly with t , the outputs of the matched filters can be used to update the estimate of the tap values $\hat{h}(\tau, t)$, and this can be used for the final part of the matched filter, as shown in figure 2.

The optimality of the matched filter above is predicated on one of M signals being transmitted in isolation. If the interval between successive signals is greater than the multipath spread, then successive signal outputs from the matched filters $u_m^*(-t)$ are isolated. Conversely, if the interval between signals is less than the multipath spread, one should either use a different set of signals on adjacent signalling intervals or should somehow combine the above receiver with adaptive equalization [Qur85].

The receiver described above is called a coherent rake receiver. One must estimate both the phase and magnitude of each tap of \tilde{h} . Since the phases typically change faster than the magnitudes, it is sometimes better to use an incoherent rake receiver in which one estimates the magnitude of \tilde{h} and uses the corresponding matched filter on the magnitude of the outputs of the matched filters $u_m^*(-t)$.

For narrow band signals, successive signals are separated by only one or two sampling intervals. These sampling intervals are typically long with respect to the multipath spread so the sample tap channel \tilde{h} typically has one large tap. Unfortunately other taps are large enough to cause significant intersymbol interference, necessitating adaptive equalization. In slow frequency hopping systems, this is usually accomplished by sending a known string of symbols in the middle of a hop and training the equalizer on those symbols. It appears that the broad band system is able to "resolve" the channel better than the narrow band system since it evaluates a large number of taps and is able to add the responses from these taps coherently. We look at this point of view more carefully later, and show that it is slightly misleading.

Although the rake approach to measuring a channel is very different from the narrow band approach, we see that each, if successful, succeeds in measuring the response of the channel to signals in the given bandwidth and then uses the measured response as if it were the true channel impulse response. To the extent that this measurement can be done precisely, the channel is simply a time varying, but known, channel with additive white Gaussian noise. In what follows, except for occasional comments and warnings, we assume that the channel can be measured precisely. This is a reasonable assumption for a channel with multipath spread on the order of $10\mu\text{sec}$ or less and Doppler shifts of 100 H or less, but it appears that such measurements are difficult to make in practice. One possible approach to improving the channel measurement is to take advantage of the fact that the phase rotation of individual paths is due to

path length variations, which often tend to be approximately linear in time.

4 Multiuser Detection

Suppose now that there are K different subscribers using the same bandwidth W to communicate with a base station. We will ignore the reverse problem of a base station communicating with multiple subscribers; this reverse problem is important practically, but is less interesting conceptually since there is no interesting problem of interference between the different transmissions. We also restrict attention to broad band (CDMA) transmission here, since it would not make much sense to share a narrow band between several subscribers at the same time. Let $u^{(k)}(t)$ denote the transmitted waveform from the k^{th} source. Let $h^{(k)}(\tau, t)$ be the output response at time t to an impulse from source k that was sent τ seconds earlier. Then the received waveform at the base station is given by

$$r(t) = \sum_k v^{(k)}(t) + z(t), \quad \text{where } v^{(k)}(t) = \int u^{(k)}(t - \tau)h^{(k)}(\tau, t)dt \quad (9)$$

Optimal detection is quite difficult in such a system, even in the absence of multipath [Ver86]. Figure 3 illustrates the difficulty. Suppose that source 1 is sending a sequence of data carrying waveforms, c_1, c_2, \dots , and source 2 is sending another sequence d_1, d_2, \dots with an offset in timing. Detection of c_1 is improved by knowledge about d_1 and d_2 , but knowledge about d_1 is similarly improved by knowledge about c_0 and c_1 , and knowledge about d_2 is improved by c_1 and c_2 . Thus detection of c_1, c_2, \dots and d_0, d_1, \dots are all linked together.

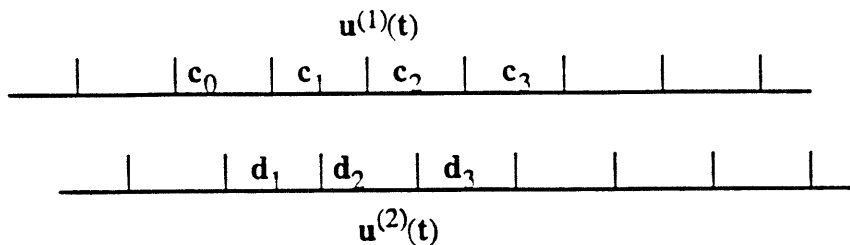


Figure 3: Asynchronous Sources

In CDMA systems, it is common, when detecting source 1, to model the reception due to source 2 as white Gaussian noise over the band W . This

makes a certain amount of sense, since pseudo-noise waveforms are used to send data. Unfortunately, if, say, source 2 has much more power than source 1, this conventional approach is quite inferior. A more powerful approach is first to decode the symbols of source 2, treating source 1 as noise. Then (still assuming no multipath), $u^{(2)}(t)$ can be subtracted (stripped) from the received waveform $r(t)$, and the weak source can then be decoded with no interference from the strong source. This is somewhat counter-intuitive, since it asserts that a source can be detected better in the presence of a very strong interfering source than in the presence of a weaker interfering source. Techniques such as stripping that allow weak data sources to be decoded in the presence of known strong data sources are called near-far resistant.

The situation becomes more complicated in the presence of multipath. Assuming that the multipath can be measured exactly, one can determine $v^{(2)}(t)$ from knowledge of $h^{(2)}(\tau, t)$ and from decoding the symbols in $u^{(2)}(t)$. However, under the assumption that $v^{(2)}(t)$ has much higher power than $v^{(1)}(t)$, small errors in measuring $h^{(2)}(\tau, t)$ can leave significant interference in the received signal $r(t)$ after stripping the estimate of $v^{(2)}(t)$ from $r(t)$.

It is unclear from a practical point of view, and even from a theoretical point of view, how effective stripping high power sources from $r(t)$ is in the presence of multipath. [Var93] gives one indication that something akin to stripping might be effective by showing that an incoherent receiver in Gaussian noise can achieve a form of near-far resistance.

5 Multiuser Information Theory

The idea of decoding strong sources and then stripping their effect from the received waveform is the crucial idea in the multiaccess coding theorem [Ahl71], [Lia72]. In order to avoid concealing the central ideas in what follows, the development will be heuristic. First, assume that each baseband source waveform $u^{(k)}(t)$ goes through a linear time invariant filter with impulse response $h^{(k)}(\tau)$, so that the received baseband waveform is $r(t) = \sum_k \int u^{(k)}(t - \tau)h(\tau)d\tau + z(t)$ where $z(t)$ is a sample function of white Gaussian noise of spectral density $N_0/2$. The multiaccess coding theorem then asserts that if source k , $1 \leq k \leq K$ ($K \geq 2$) has rate R_k and is constrained to a power spectral density S_k/W over the given band of width W , then arbitrarily small error probability can be achieved for all sources if for each subset A of the integers $\{1, 2, \dots, K\}$,

$$\sum_{k \in A} R_k < \int_{-W/2}^{W/2} \ln \left[1 + \frac{\sum_{k \in A} S_k |H^{(k)}(f)|^2}{WN_0} \right] df \quad (10)$$

where $H^{(k)}(f)$ is the Fourier transform of $h^{(k)}(t)$ (see [Gal68] and [Gal85]). If the left side of (10) is larger than the right for any subset A , then arbitrarily small error probability can not be achieved. It is also true that the encoders

need not know the impulse responses $h^{(k)}(t)$, although the decoders do need this knowledge. The quantity on the right hand side of (10) is the conditional average mutual information per unit time, $I(A)$, between the inputs in A and the output, conditional on the inputs not in A ; this assumes that the inputs are independent stationary white Gaussian noise (WG) processes over the bandwidth W .

Next, suppose the baseband linear filters are time varying with impulse responses $h^{(k)}(\tau, t)$ as before. Let T_0 be the multipath spread of the channels; *i.e.*, for all k and t , $h^{(k)}(\tau, t) = 0$ for $\tau < 0$ and for $\tau > T_0$. Let W_0 be the Doppler spread of the channels and assume that $W_0 \ll 1/T_0$. Consider a bandwidth W in the range $W_0 \ll W \ll 1/T_0$. Such a W was referred to above as narrow band, but W is also large enough that we can neglect the effect of in band signals moving out of band because of Doppler shifts. Let $H^{(k)}(f, t) = \int h^{(k)}(\tau, t) \exp(-j2\pi f\tau) d\tau$. Since W is much less than the coherent bandwidth, $1/T_0$, of each filter, we can approximate $H^{(k)}(f, t)$ as being constant for $-W/2 \leq f \leq W/2$. With this approximation, we can simply evaluate the right hand side of (10) as a rate per unit time of generation of average conditional mutual information, $I(A, t)$,

$$I(A, t) = W \ln \left[1 + \frac{\sum_{k \in A} S_k |H^{(k)}(0, t)|^2}{WN_0} \right] \quad (11)$$

We now want to view $H^{(k)}(0, t)$ as a stochastic process in t , but must recall that (11) is then conditioned on $H^{(k)}$ for each k , and $I(A, t)$ is a random variable that depends on the sample values of $H^{(k)}(0, t)$ for each k . We assume that the processes $\{H_k(0, t); -\infty < t < \infty\}$ are ergodic. We discuss this somewhat unrealistic assumption later. We now claim that if

$$\sum_{k \in A} R_k < E \left\{ W \ln \left[1 + \frac{\sum_{k \in A} S_k |H^{(k)}(0, t)|^2}{WN_0} \right] \right\} \quad (12)$$

for all sets A , then reliable communication is possible at the rates R_1, \dots, R_K . The argument is that for large enough T , the inequalities

$$\sum_{k \in A} R_k < \frac{1}{T} \int_0^T W \ln \left[1 + \frac{\sum_{k \in A} S_k |H^{(k)}(0, t)|^2}{WN_0} \right] dt \quad (13)$$

will all be satisfied with high probability, and codes exist over this block length for which the error probability is then small. In the same way, if one of the inequalities in (12) is reversed, reliable communication is not possible. We now use (12) to compare two scenarios. In the first, K users each use a separate band of width W . In the second, K users each use all K bands, again each of width W . Thus, in the second scenario, each user uses the entire bandwidth KW . We assume that $W \ll 1/T_0$, but not that $KW < 1/T_0$. Let $H^{(k)}(i, t)$ denote the response of the i^{th} of these bands to user k . For scenario 1, suppose

user k uses channel k , so that (12) becomes

$$R_k < E \left\{ W \ln \left[1 + \frac{S_k |H^k(k, t)|^2}{WN_0} \right] \right\} \quad (14)$$

Reliable communication for each user is not possible (given the assumed use of bandwidth and power) if each of these inequalities is reversed. Eq. (14) is equivalent to the larger set of equalities over all subsets A :

$$\sum_{k \in A} R_k < E \left\{ W \sum_{k \in A} \ln \left[1 + \frac{S_k |H^k(k, t)|^2}{WN_0} \right] \right\} \quad (15)$$

For scenario 2, assuming that user k uses power spectral density S_k/KW in each band, (12), summed over all bands, becomes

$$\sum_{k \in A} R_k \leq E \left\{ \sum_i W \ln \left[1 + \frac{\sum_{k \in A} S_k |H^{(k)}(i, t)|^2}{KWN_0} \right] \right\} \quad (16)$$

Assuming that each process $\{H^{(k)}(i, t); t \geq 0\}$ is statistically identical over all bands i , this can be rewritten as

$$\sum_{k \in A} R_k \leq E \left\{ KW \ln \left[1 + \frac{\sum_{k \in A} S_k |H^{(k)}(0, t)|^2}{KWN_0} \right] \right\} \quad (17)$$

Again, reliable communication, given the assumed allocation of power to frequency bands, is possible if (16) and thus (17), is satisfied for all A ; reliable communication is impossible if any of the inequalities is reversed. Now denote the right hand side of (15) for a given A as $I_{NB}(A)$ and denote the right hand side of (17) as $I_{WB}(A)$. Because of the concavity of the logarithm, we have

$$I_{NB}(A) \leq E \left\{ |A| W \ln \left[1 + \frac{\sum_{k \in A} S_k |H^{(k)}(k, t)|^2}{|A|WN_0} \right] \right\} \quad (18)$$

where $|A|$ is the number of sources in A . The inequality is strict unless $S_k |H^{(k)}(k, t)|^2$ is the same for all k with probability 1. Since the sources are assumed to have separate paths, we assume these terms are not all the same, and thus we assume that (18) is satisfied with strict inequality for all A such that $|A| \geq 2$. Since $x \ln(1 + b/x)$ is increasing in x for any $b > 0$, (18) can be further bounded by

$$I_{NB}(A) < E \left\{ KW \ln \left[1 + \frac{\sum_{k \in A} S_k |H^{(k)}(k, t)|^2}{KWN_0} \right] \right\} \quad (19)$$

with strict inequality unless $|A| = K$. Finally, since the channels are statistically identical,

$$I_{NB}(A) < E \left\{ KW \ln \left[1 + \frac{\sum_{k \in A} S_k |H^{(k)}(0, t)|^2}{KWN_0} \right] \right\} = I_{WB}(A) \quad (20)$$

Thus $I_{NB}(A)$, for each A , is strictly less than $I_{WB}(A)$. Thus the rates achievable under the wide band power assumptions of (17) are strictly larger than those achievable under the narrow band assumptions of (15).

Now we observe that slow frequency hopping, hopping between bands of width W , is also constrained by (15) because of the assumption that the different frequency bands are statistically identical. That is, frequency hopping achieves diversity, allowing an individual user to obtain averaging over channels, but does nothing to enhance average mutual information, which is necessary to achieve high data rates.

Conversely, CDMA is constrained by (17). It also appears, although some proof would be required, that reliable communication is possible using CDMA (in conjunction with long constraint length, low rate, error correcting codes) if (17) is satisfied. In other words, CDMA is theoretically capable of higher data rates than slow frequency hopping.

This result is rather surprising. To make it a little more understandable, we look at a simple example with $K = 2$, $S = 1$, $W = 1$, $N_0 = 1$. We also take $H^k(0, t)$ to be 0 or 1, each with probability 1/2, so that in scenario 1, the received signal power on a single channel is 0 or 1 with equal probability. For scenario 2, the received signal power on a single channel is 0 with probability 1/4 (if both sources are faded), is 1/2 with probability 1/2 (if one source is faded) and is 1 with probability 1/4 (if neither source is faded). The right hand side of (15) is then $\ln 2 = 0.69$, and the right hand side of (16) is $\ln(3/2) + (1/2)\ln(2) = 0.75$. Since the fading is independent between sources, the use of both sources together tends to partially average the received signal power, which is the effect that increases average mutual information.

One peculiar effect of this result is that K users, distributed in space, can send more data to a base station than a single user with K times as much power. Another peculiar effect is that if one achieves orthogonality in any way between the K users, this lowers the achievable data rate over that possible without orthogonality.

One should not assume that this result implies that CDMA is "better" than slow frequency hopping. In order to approach the rates promised by this result, one must code, or interleave, over a long time period, and one must jointly decode all the sources. The usual technique of viewing the other sources as noise in decoding a given source might give up more than what has been gained here. It is possible that stripping could be used to achieve joint decoding, but as pointed out before, it is not clear that the channel can be measured well enough to make this effective. The best power levels for stripping have been worked out in [Vit90], but multipath effects were not taken into account. With the relatively rapid changes in channel strength due to multipath, one might guess that even if one attempted to keep received power levels the same, actual differences might be enough to make use of stripping. We have also neglected inter-cell interference from our model, and this is an important aspect for practical systems.

We have assumed that the channel can be accurately measured, and it is

not clear how dependent our result is on that assumption. Finally, we have assumed in that there is no feedback from base station to sources. If feedback were available, a source could in principle transmit only when the channel is good (or choose a good channel to transmit on). Current systems use feedback to try to maintain constant received signal power from each source, but do not attempt to transmit only when the channel is good, so that the assumptions here are more or less reasonable for these systems.

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