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AN ALGORITHM FOR PERFORMANCE OPTIMIZATION IN THE PRESENCE OF NORM BOUNDED STRUCTURED UNCERTAINTY AND ITS APPLICATION TO SYSTEM RECOVERY AFTER A FAILURE[1]

by

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Abstract

A performance criterion given as a bound on the H_∞ norm of some transfer function is known to be equivalent to the stability condition with respect to the auxiliary unstructured uncertainty. Therefore, robust stability and performance of the system in the presence of a block diagonal norm bounded uncertainty with (m) elements is easily transformed into a structured singular value problem with respect to the overall diagonal uncertainty with (m+1) blocks. When the norm bounds on the uncertainty and the performance condition are known and scaled to the value of one, the synthesis consists of the search for a compensator that will make the associated structured singular value smaller than one. This is usually done by the "D-K" iteration introduced by Doyle. In this paper we look at the case where the norm bound on one of the blocks in the structured uncertainty is not known a priori. The problem then becomes one of finding the extreme value of that bound such that there exist a stabilizing compensator that guarantees robust stability and performance. We show that this problem can be solved by a sequence of "D-K" iterations with the value of unknown parameter kept fixed in every iteration and updated only at its end. Unfortunately, this implies that the order of the compensator may significantly increase before the unknown parameter is changed. A recursive algorithm based on altering the standard "D-K" iteration is proposed so that the unknown parameter is updated before the order of the compensator is increased. This algorithm is then evaluated on the closed-loop system containing the augmented model of the F-8 Aircraft. Following a failure in one of the two actuators, the algorithm is used to recover the properties of the original system in the presence of uncertainty associated with the model after the failure.

1. Introduction

This paper deals with the issue of robust stability and performance in the H_∞ norm sense of a feedback system in the presence of a block diagonal, norm bounded uncertainty. Such a system can be represented in its upper fractional transformation, as depicted in Figure 1.

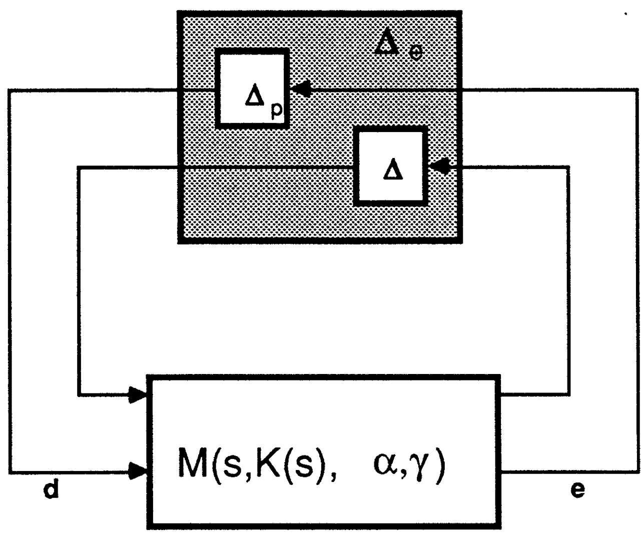

Figure 1. Scaled System with Block Diagonal Uncertainty

M(s), the overall closed loop transfer function, can be scaled so that the norms of the uncertainties are smaller than or equal to one. After scaling, the transfer function takes the form:

$$M(s) = M(s, K(s), \alpha, \gamma) \qquad (1.1)$$

The overall uncertainty Δe is then defined as $\Delta_e = \text{diag}\{ \gamma\Delta_p, \alpha^{-1}\Delta \}$ where $\gamma\Delta_p$ is associate with performance and $\alpha^{-1}\Delta$ with the modelling uncertainty. The set of all admissible uncertainties is, therefore, given as $\Delta_s = \{ \Delta_e \text{ s.t. } \| \Delta_e(jw) \|_\infty < 1 \}$. According to [1], stability and performance of this system are guaranteed if:

$$\mu\{ M(j\omega, K(s), \alpha, \gamma) \} \le 1 \qquad \forall\, \omega \in R_+ \qquad (1.2)$$

where $\mu\{ M \}$ stands for the structured singular value [1].

It is easy to see that if the bounds α and γ are known, the synthesis of a robust system M(s) becomes a search for a compensator K(s) that satisfies (1.2). Unfortunately, there is no existing methodology for minimization of $\mu\{M\}$ over the set of stabilizing compensators that will result in the global optimum. The usual minimization procedure is the so called "D-K" iteration [2].

The problem treated in this paper differs from the standard "μ" synthesis in the fact that the bound on one of the blocks in Δ_e is not given a priori but is left as a variable. Since we don't know how to deal with "μ" minimization directly, we will be looking at the minimization of its upper bound, i.e.

$$\mu_S = \inf_{D,K} \| D\ M(j\omega, K(j\omega), \alpha, \gamma)\ D^{-1} \|_\infty \ge \mu \qquad (1.3)$$

where D is a real block diagonal matrix compatible with Δ_e [2]. We now analyze how a change in α or γ influences individual steps of the "D-K" iteration.

The system $M(j\omega, K, \alpha, \gamma) \in RH_\infty$ can be presented in the (2x2) block form with inputs and outputs corresponding to those of the uncertainties Δ and Δ_p.

With γ fixed we have :

$$M(j\omega, K, \alpha) = \begin{bmatrix} \alpha\, M_{11}(j\omega, K) & M_{12}(j\omega, K) \\ \alpha\, M_{21}(j\omega, K) & M_{22}(j\omega, K) \end{bmatrix} = [\, \alpha m_1 \;\; m_2 \,] =$$

$$= M(K)\, \text{diag}\{ \alpha I_1, I_2 \} \qquad (1.4)$$

where K is an internally stabilizing compensator, M is a transfer function matrix with m_i as its columns and $\alpha \in R_+$. The performance index γ is incorporated in M(K) and is not shown as a variable since it remains constant.

Let us define the following functions as :

$$\text{i) } f(\alpha) = \inf_{K \in Ks} \| M(K) \operatorname{diag}\{ \alpha I_1, I_2 \} \|_\infty \tag{1.5}$$

$$\text{ii) } g(\alpha) = \inf_{D \in \mathbf{D}} \| DM(K)D^{-1} \operatorname{diag}\{ \alpha I_1, I_2 \} \|_\infty ; \quad K(s) \text{ fixed} \tag{1.6}$$

We now check the properties of these functions with respect to a change in α.

Lemma 1.

The $f(\alpha)$ defined in (1.5) is a nondecreasing and continuous function for $\alpha \geq 0$.

Proof :

At the fixed frequency $\omega = \omega_1$, with the same compensator K and with $\varepsilon > 0$, we have

$$M_{\alpha\varepsilon}(j\omega_1, K) = [\, \alpha\varepsilon\, m_1(j\omega_1), m_2(j\omega_1) \,] =$$
$$= [\alpha\, m_1(j\omega_1), m_2(j\omega_1)] \operatorname{diag}\{ \varepsilon I_1, I_2 \} = M_\alpha (j\omega_1, K) \operatorname{diag}\{ \varepsilon I_1, I_2 \} \tag{1.7}$$

Due to the properties of singular values of $M_{\alpha\varepsilon}(j\omega, K)$, the following inequalities hold for the H_∞ norm :

$$\| M_\alpha (j\omega, K) \|_\infty \leq \| M_{\alpha\varepsilon} (j\omega, K) \|_\infty \leq \varepsilon \| M_\alpha (j\omega, K) \|_\infty \quad \text{with} \quad \varepsilon \geq 1 \qquad \text{and}$$

$$\varepsilon \| M_\alpha (j\omega, K) \|_\infty \leq \| M_{\alpha\varepsilon} (j\omega, K) \|_\infty \leq \| M_\alpha (j\omega, K) \|_\infty \quad \text{with} \quad \varepsilon \leq 1 \tag{1.8}$$

Let

$$K_1 = \arg \{ \inf_{K \in Ks} \| M(j\omega, K, \alpha) \|_\infty \}$$

and

$$K_2 = \arg \{ \inf_{K \in Ks} \| M(j\omega, K, \varepsilon\, \alpha) \|_\infty \} \tag{1.9}$$

meaning that these compensators achieve the infimum or that they guarantee the value of the norm to be arbitrarily close to the infimum.

From (1.8) and (1.9) we have:

with $\varepsilon \geq 1$

$$\| M_{\alpha} (j\omega, K_1) \|_{\infty} \leq \| M_{\alpha\varepsilon} (j\omega, K_2) \|_{\infty} \leq \varepsilon \| M_{\alpha} (j\omega, K_1) \|_{\infty}$$

or, equivalently, $$f(\alpha) \leq f(\varepsilon \alpha) \leq \varepsilon f(\alpha) \tag{1.10}$$

and with $\varepsilon \leq 1$

$$\varepsilon \| M_{\alpha} (j\omega, K_1) \|_{\infty} \leq \| M_{\alpha\varepsilon} (j\omega, K_2) \|_{\infty} \leq \| M_{\alpha} (j\omega, K_1) \|_{\infty}$$

or, equivalently, $$\varepsilon f(\alpha) \leq f(\varepsilon \alpha) \leq f(\alpha). \tag{1.11}$$

By letting $\varepsilon \to 1$ in (1.10) and (1.11) we prove that $f(\alpha)$ is continuous for $\alpha \in (0, \infty)$. In order to prove continuity at $\alpha_o = 0$, we look at the following inequalities:

$$0 \leq [f(\alpha) - f(0)] \leq \| M(K_o) \operatorname{diag}\{ \alpha I_1, I_2 \} \|_{\infty} - f(0) ; \tag{1.12}$$

where $f(0) = \| M(K_o) \operatorname{diag}\{ 0 I_1, I_2 \} \|_{\infty}$, $K_o(s)$ is the minimizing solution for $\alpha_o=0$ and $\alpha > 0$.

Furthermore, we have:

$$\| M(K_o) \operatorname{diag}\{ \alpha I_1, I_2 \} \|_{\infty} - f(0) \leq \| M(K_o) \operatorname{diag}\{ \alpha I_1, 0 I_2 \} \|_{\infty} \tag{1.13}$$

$$\| M(K_o) \operatorname{diag}\{ \alpha I_1, 0 I_2 \} \|_{\infty} \leq | \alpha | \, \| M(K_o) \|_{\infty} \tag{1.14}$$

The inequality in (1.12) states that the function is nondecreasing at zero too. The triangular inequality (1.13) and the inequality (1.14) are based on the properties of the infinity norm. From these inequalities we have:

$$0 \leq \lim_{\alpha \to 0} [f(\alpha) - f(0)] \leq 0 \tag{1.15}$$

which completes the proof that the function is continuous at $[0, \infty)$.

■

Lemma 2.

Let the "D" scaling be as introduced in (1.3), and let $\Theta = \operatorname{diag} \{\alpha I_1, I_2\}$ where I_i are compatible with the block uncertainty Δ_e. Then the D and Θ scalings commute, i. e.

$$D\,\Theta\,M(j\omega)\,D^{-1} = \Theta\,D\,M(j\omega)\,D^{-1} \qquad (1.16)$$

The proof is obvious since both matrices are diagonal and their dimensions are compatible.

■

Lemma 3.

For the fixed compensator K(s), $\alpha \geq 0$, and $\varepsilon \in \mathbb{R}_+$, the function $g(\alpha)$ defined in (1.6) is nondecreasing and continuous.

Proof : It is analogous to the proof of Lemma 1. For the details see [4].

■

The above mentioned properties of the functions $f(\alpha)$ and $g(\alpha)$ imply that an increase in α can be performed by a sequence of full "D-K" iterations, each of them corresponding to a fixed value of α. If the obtained minimum value of the "μ_S" function from the performed "D-K" iteration with fixed α is less than one, then, according to lemmas 1÷3, we can either find a larger value of α that makes μ_S=1 or we could increase α up to some previously set upper limit. This search for α can be performed by bisection with a suitable step size. Unfortunately, the full fledged "D-K" iteration involves a sequence of approximations of the "D" scaling with a real-rational stable function and potentially results in substantial increase in the order of the compensator before the value of α is changed. Therefore, the idea is to modify this approach in such a way that the varying parameter α is increased as much as possible before the "D" scaling is changed. A way to do this is to increase α in the first step of the "D-K" iteration while the scaling "D" used in designing the compensator is kept fixed.

2. Algorithm Outline

The algorithm is based on lemmas 1÷3 and on the properties of the "D-K" iteration [1], including the fact that the obtained solution will usually correspond to a local optimum only. Since there is no difference in applying the algorithm for increasing α or $1/\gamma$, without loss in generality we can focus our attention to one case only. The objective of the algorithm can then be stated as follows :

- for a given γ, find the largest $\alpha \in R_+$ s.t.

$$\mu_S = \inf_{D,K} \| D\, M(j\omega, K(j\omega), \alpha, \gamma)\, D^{-1} \|_\infty \leq 1 \tag{2.1}$$

The outline of the algorithm is shown in Figure 2 where "D_F" is the diagonal scaling kept fixed in each step A). As it can be seen, the algorithm has two main steps. In the first step the objective is to find a pair $[\alpha, K]$ as the solution to the constrained max-min problem. The constraint is given in the form of the "μ_S" function associated with the system $M(j\omega,K,\alpha)$ which is not allowed to exceed the value of one over the frequency. The cost function $\|D_F\, M(K,\alpha) D_F^{-1}\|_\infty$ is the upper bound for $\mu_S\{M(j\omega,K,\alpha)\}$ defined in (2.1) and, therefore, it can take a value greater than one. Furthermore, this cost function is convex in K [1] and in α separately but not jointly convex in both variables.

The constrained problem in Step A) is solved iteratively. The procedure that we will refer to as "K-α" iteration, consists of three substeps A1, A2, and A3. To be able to trace changes of α and other variables of interest throughout the algorithm, we assign a double index (i,j) to them where $i \geq 1$ and $j \geq 0$. The index "i" will show how many times step A) has been executed. The "j" index will trace the increase of α within step A) from the initial value $\alpha_{1,0}$. The variables of interest that remain constant within step A) will have the index "i" only. A choice of the initial conditions and stopping of the algorithm will be discussed once the latter is introduced in detail.

The detailed presentation of both steps is given as follows :

STEP A) D_{F_i} is fixed, the initial choice of $\alpha_{i,0}$ is available, and the upper bound $\overline{\alpha}$ is set

A1) start with the value of $\alpha = \alpha_{i,j}$, j=0,1,2,.. and find $K_{i,j+1}$ that makes

$$\| D_{F_i}\, M(j\omega, K_{i,j+1}, \alpha_{i,j})\, D_{F_i}^{-1} \|_\infty = \inf_{K \in Ks} \| D_{F_i}\, M(j\omega, K, \alpha_{i,j})\, D_{F_i}^{-1} \|_\infty \tag{2.2}$$

A2) with $K_{i,j+1}$ from above, find μ_S and $D(\omega)$ s.t.

$$\mu_S = \sup_{\omega} \inf_{D(\omega) \in \mathbf{D}} \overline{\sigma}\{D\, M(j\omega, K_{i,j+1}, \alpha_{i,j})\, D^{-1}\} \geq \sup_{\omega} \mu\{M(j\omega, K_{i,j+1}, \alpha_{i,j})\} \tag{2.3}$$

A3) **if $\mu_s \leq 1$,** then by bisection find the largest $\alpha = \alpha_{i,j+1}$ that makes

$$\sup_{\omega} \inf_{D \in \mathbf{D}} \bar{\sigma}\{ D\, M(j\omega, K_{i,j+1}, \alpha_{i,j+1})\, D^{-1} \} \leq 1 \qquad (2.4)$$

- if $\alpha_{i,j} < \alpha_{i,j+1} < \bar{\alpha}$, then save $K_f = K_{i,j+1}$ and go back to A1) with $\alpha_{i,j+1}$
- if $\alpha_{i,j+1} = \alpha_{i,j}$, then go to step (**B**) with : D from (2.4), $\alpha^{(i)}=\alpha_{i,j}$, K_f, and $\mu_{s_i} = \mu_s$
- if $\alpha_{i,j+1} \geq \bar{\alpha}$, then save $\alpha_{max} = \bar{\alpha}$, $K_f = K$ and STOP the algorithm

if $\mu_s > 1$, then go to step (**B**) with : D from (2.3), $\alpha^{(i)}=\alpha_{i,j}$, K_f, and $\mu_{s_i}= \mu_s$

STEP B) if $\alpha^{(i)} = \alpha^{(i-1)}$ and $\mu_{s_i} = \mu_{s_{i-1}}$, save $\alpha_{max} = \alpha^{(i)}$, and the compensator K_f and STOP.

- if not, approximate $D(j\omega)$ with the real rational, stable invertible $Dr(j\omega) \in RH_\infty$, and go back to step (**A**) with $D_{F_{i+1}} = Dr$ and $\alpha_{i+1,0} = \alpha^{(i)}$. (2.5)

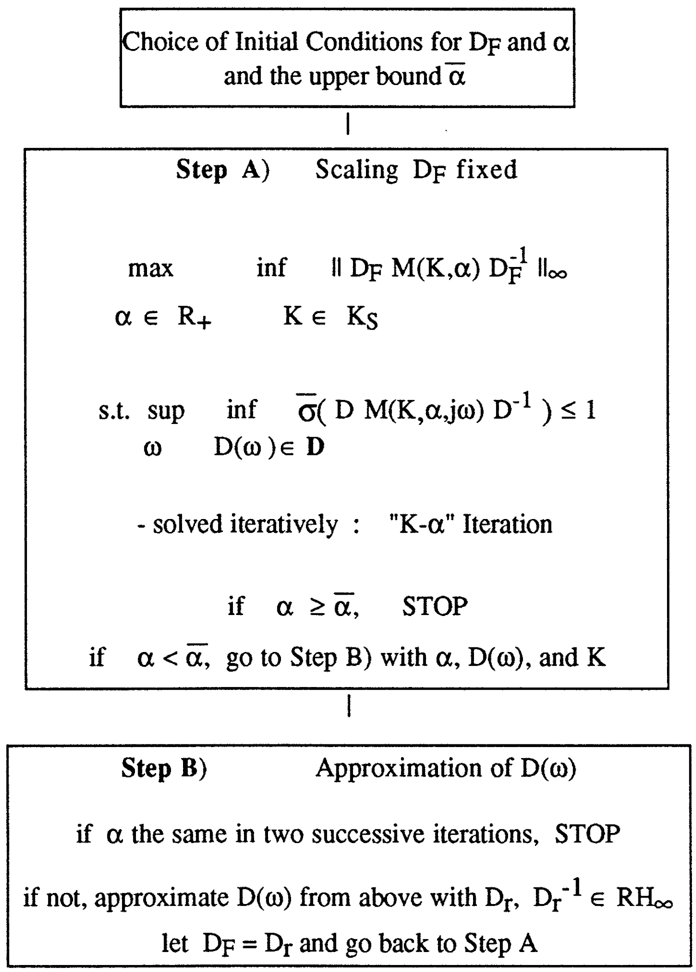

Figure 2. Flowchart of the Algorithm

We now analyze the convergence of the algorithm and discuss its properties.

Property #1) The sequence { α(i) }, consisting of the maximum bounds on the uncertainty obtained in substep A3) in each "i" iteration, is nondecreasing, meaning that $\alpha^{(i)} \leq \alpha^{(i+1)}$. This property is guaranteed by the construction of the algorithm. For further details see [4].

Property #2) The obtained sequence { $\alpha^{(i)}$ } is bounded from above.
From the previous discussion on the algorithm construction, it is obvious that the obtained $\alpha_{max} \leq \overline{\alpha}$. Since the sequence is nondecreasing, all its elements are bounded by $\overline{\alpha}$.

Property #3) The obtained sequence { $\alpha^{(i)}$ } converges.
This is the consequence of the first two properties. Any nondecreasing sequence that is bounded from above is convergent [3].

Property #4) In each step A), the algorithm increases α as much as possible without changing the block diagonal scaling D_F and, therefore, without increasing the order of the compensator.

Property #5) A possible initial condition for the algorithm is a sufficiently small α_{10} such that μ_s { $M(j\omega; K_{1,1} ; \alpha_{10})$ } ≤ 1.

Property #6) The algorithm depends on the initial conditions and the obtained result corresponds to a local optimum.

The algorithm inherits the properties of the standard D-K iteration [2]. Therefore, the resulting compensator and the bound on the uncertainty α_{max} will in general correspond to a local optimum. In other words, if the algorithm doesn't obtain a compensator that guarantees robustness properties for some $\alpha=\alpha_{1,0}$, this doesn't necessarily mean that such a compensator does not exist.

3. Algorithm Application to a Robust Model Matching Example

The algorithm introduced in the previous section is evaluated herein. A benchmark for this study is a modified linearized model of the longitudinal dynamics of the F-8 Aircraft. This two input - two output model is augmented with the dynamics of actuators present at both control channels. The example discussed herein corresponds to the situation after a failure in one of the two actuators.

Let the control system before the failure be presented as in Figure 3. The state space representation of the F-8 aircraft and the actuators Fe and Ff are given in Appendix 1. The stabilizing compensator Kp(s) is designed by the LQG/LTR methodology [5] to match the singular values at both low and high frequencies.

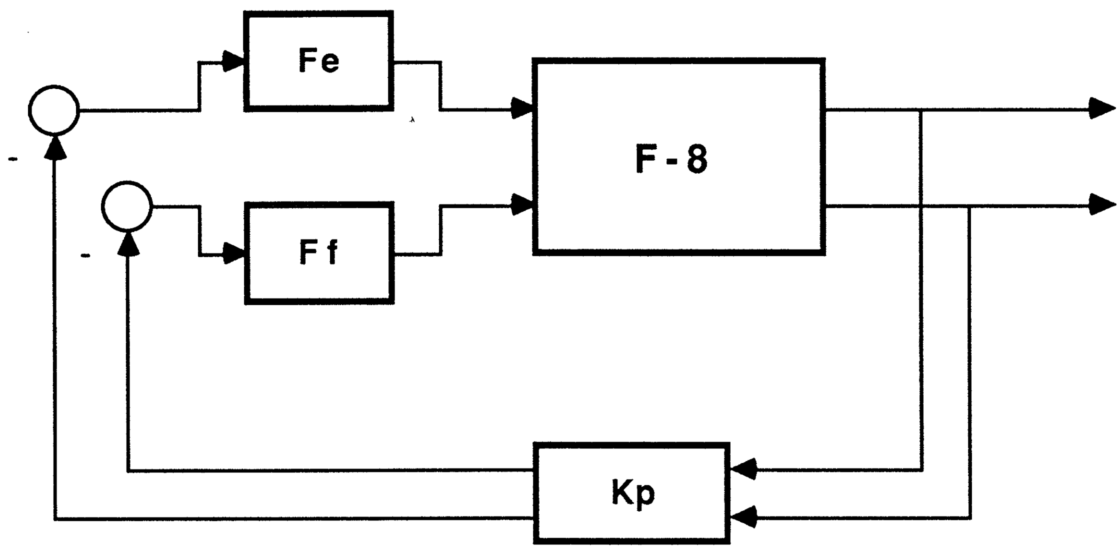

Figure 3. Block Diagram Representation of the Original System

We assume that the actuator in the second control channel has changed its dynamics due to a failure, and that its new model is only partially known. The available description of the "failed" actuator is given by the nominal model $Ff_1(s)$ and by the accompanying norm-bounded additive uncertainty Δ_1, as depicted in Figure 4. The frequency dependence of the uncertainty is given by the weighting function $W_1(s)$. The corresponding transfer functions and the bound on the uncertainty are:

$$F_{f1} = 3\,[\,s + 12\,]^{-1} \qquad W_1(s) = (s+20)^{-1} \qquad \|\Delta\|_\infty < \alpha \tag{3.1}$$

The original actuator dynamics were given by :

$$F_{f1} = 15\,[\,s + 15\,]^{-1} \tag{3.2}$$

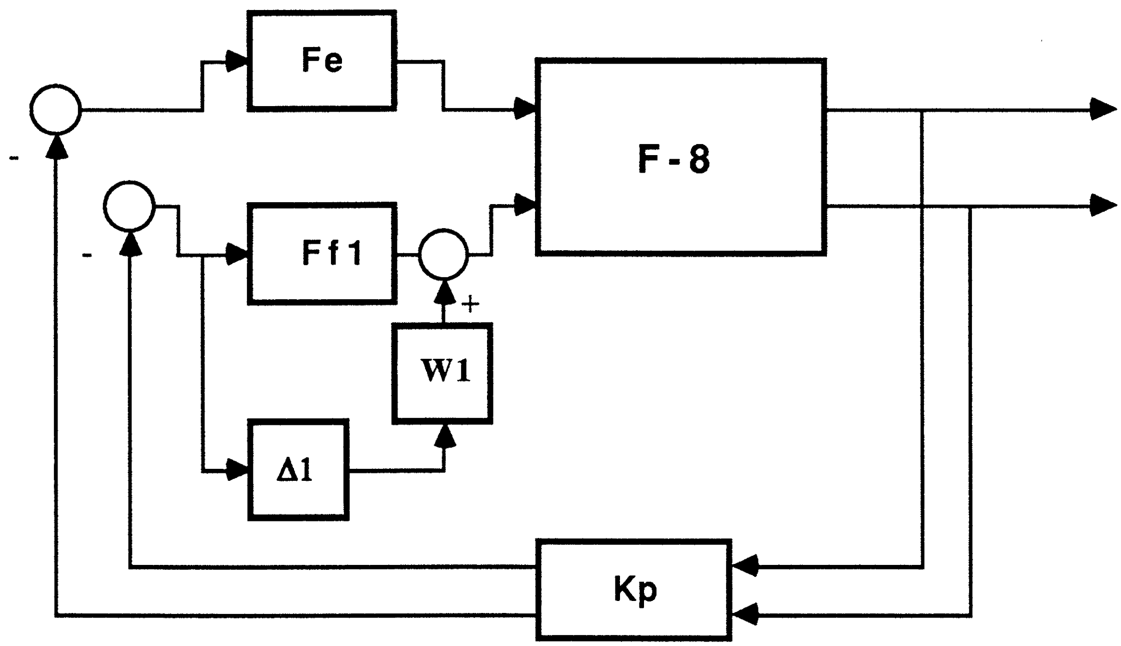

Figure 4. Block Diagram of System with Failed Actuator and Original Controller

The failure in the second actuator has changed the behavior of the system significantly. This is obvious from the singular value plots of the sensitivity transfer function of the original and the nominal postfailure systems presented in figures 5 and 6. They are described as follows :

$$Sp(s,Kp) = [\,I + F\text{-}8(s)\ \mathrm{diag}\{\,Fe(s)\,,\,Ff(s)\}Kp(s)\,]^{-1} = [I + Pp(s)\,Kp(s)]^{-1} \tag{3.3}$$

$$So(s,Kp) = [\,I + F\text{-}8(s)\ \mathrm{diag}\{\,Fe(s)\,,\,Ff_1(s)\}Kp(s)\,]^{-1} = [I + Po(s)\,Kp(s)]^{-1} \tag{3.4}$$

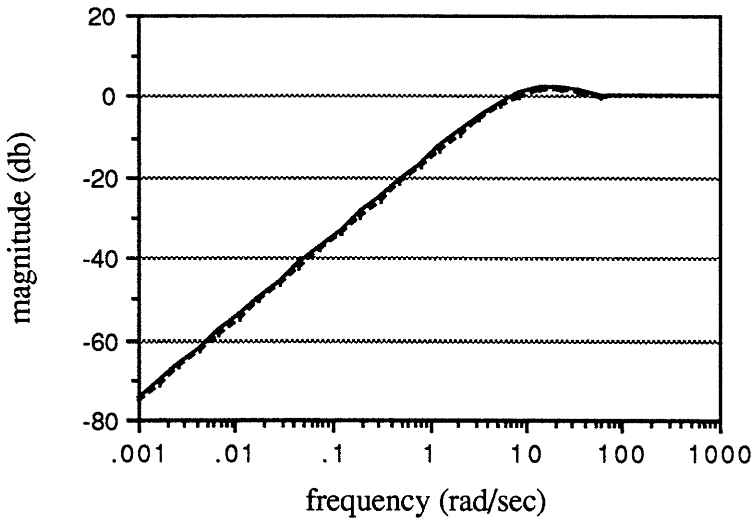

Figure 5. Sensitivity Transfer Function of the Original System - S.V. Plot

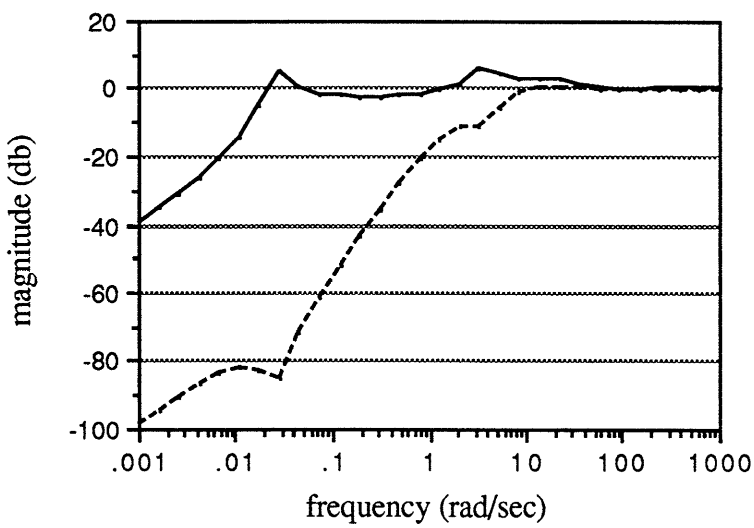

Figure 6. Sensitivity T.F. of Nominal System with Kp(s) and $F_{f1}(s)$ - S.V. Plot

In order to recover the characteristics of the original system, we must design a new compensator K(s) for the postfailure plant. This has to be done taking into account the modelling uncertainty Δ. The setting for this minimization is given in Figure 7. The compensator K(s) has to stabilize the "postfailure" system and minimize, or keep smaller than the given bound γ, the difference between the postfailure and the original systems in the sense of the error signal $e(t) \in L_2$ when they are both subjected to the same input $d(t) \in L_2$. This, in fact, represents the bound on the H_∞ norm of the weighted difference between Sp(s,Kp) and S(s,K,Δ) in the presence of uncertainty Δ, $\|\Delta\|_\infty < \alpha$. The weighting function W(s) reflects the importance of the change in performance over the range of frequencies. It is given by the following transfer function:

$$W(s) = (s+5)(s+50)\,[\,(s+10^{-5})(s+100)\,]^{-1}\,\mathrm{diag}\{\,1,\,1\,\} \tag{3.5}$$

and it is chosen to resemble the inverse of Sp(s).

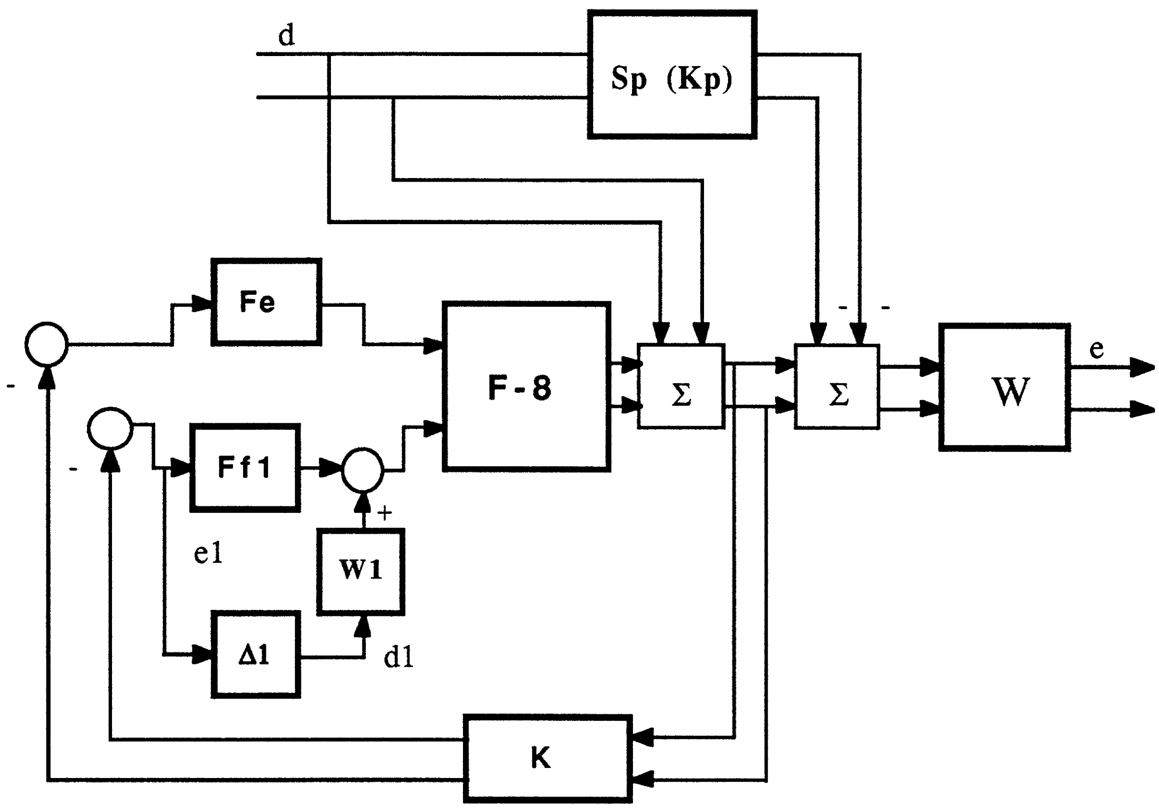

Figure 7. Block Diagram Setting for Compensator Redesign

In this example γ is fixed and the goal is to maximize the stability margin with respect to Δ, i.e. to maximize α by the appropriate choice of the stabilizing compensator K(s). Therefore, the optimization problem we want to solve can be formulated as

$$\max_{R_+} \alpha \quad \text{s.t.} \quad \inf_{K \in K_{S\Delta}} \| W(s)\,[S(K,\Delta) - Sp(Kp)] \|_\infty \le \gamma \;, \quad \forall \; \|\Delta\|_\infty < \alpha \tag{3.6}$$

where $K_{S\Delta}$ defines the family of stabilizing compensators for the plant in the presence of the unstructured uncertainty Δ.

Since in this example we are dealing with the single norm bounded uncertainty Δ, the overall block diagonal uncertainty will have two elements, i.e. Δ_e= diag { Δ_p, Δ} where Δ_p corresponds to the performance specification γ. Therefore, the diagonal weighting function will also have only two blocks and it can be presented as D_F= diag { I_1, ε I_2} with ε>0. The dimension of I_1 and I_2 is compatible with those of Δ_p and Δ. The H_∞ norm minimization step in the algorithm will be executed by the two Riccati equation approach of Doyle et al [1]. The scaled open loop system corresponding to the problem formulated in (3.6) is depicted in Figure 8.

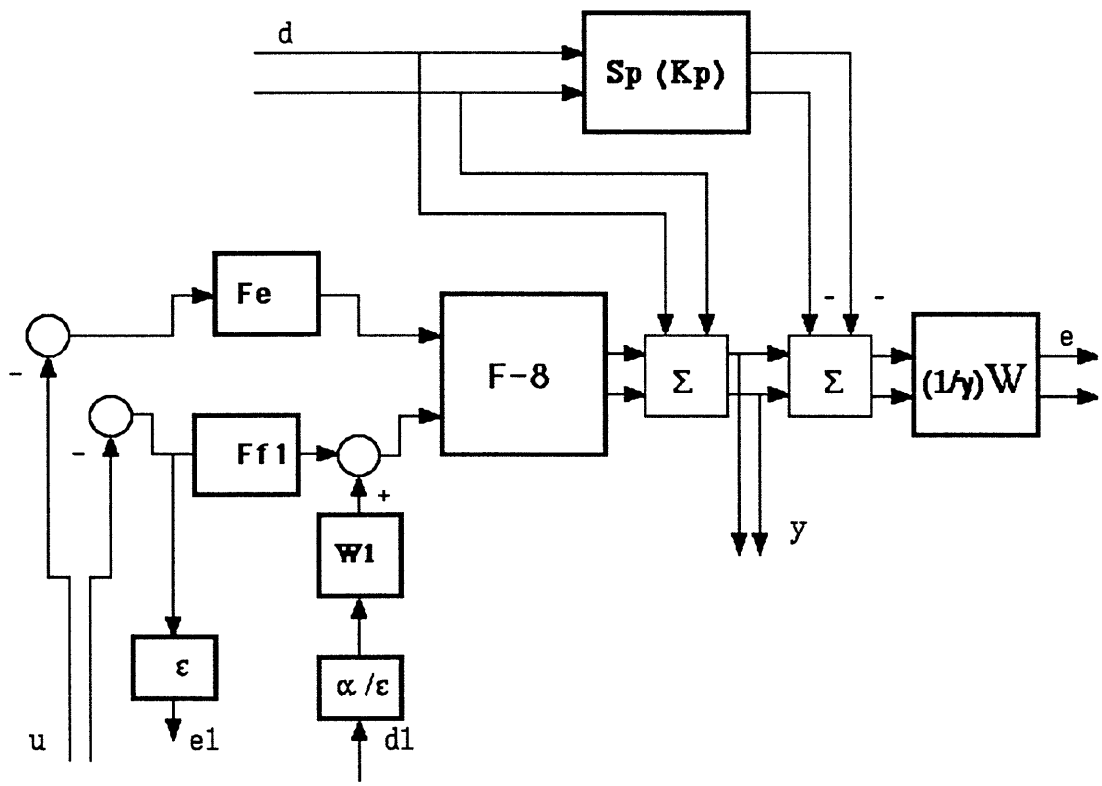

Figure 8. Scaled Open Loop System for the Algorithm Implementation

3.1 Compensator Redesign with $\gamma = 5$, $\gamma = 1$, and $\gamma = 0.5$

We now design a compensator that maximizes the stability margin α, with the fixed value of the performance index $\gamma = 5$. We will show how the compensator $K_{i,j}$ and $\alpha_{i,j}$ change through the "i" and "j" iterations that were described in section 2. The diagonal scaling D_{F_i} remains the same throughout the entire execution of step A. Therefore, it is indexed according to the current value of "i".

Let the initial conditions for $i = 1$ and $j = 0$ be:

$$D_{F_1} = \text{diag}\{1,1,\varepsilon\} = I \qquad I \in R^{3x3} \quad ; \qquad \alpha_{1,0} = 0.2 \tag{3.7}$$

where $\alpha_{1,0}$ was chosen as stated.

By following the steps of the algorithm we have the following :

j=1, D_{F1} ; $\alpha_{1,0} = 0.2 \quad \Rightarrow \quad \alpha_{1,1} = 2.5$

$\alpha_{1,2} = 3.8$

$\alpha_{1,3} = 4.2$

$\alpha_{1,4} = 4.2$

j=2, D_{F2} ; $\alpha_{2,0} = 4.2 \quad \Rightarrow \quad \alpha_{2,1} = 4.32$

$\alpha_{2,2} = 4.32$

j=3, D_{F3} ; $\alpha_{2,0} = 4.32 \quad \Rightarrow \quad \alpha_{3,1} = \boldsymbol{\alpha_{max}}\ (\gamma=5) = 4.32$

The diagonal scaling D_{Fj} was updated every time when there was no opportunity for further increase of α with a fixed scaling. The algorithm stopped when even the change of scaling didn't give the freedom for increasing the bound uncertainty. The first compensator that achieves this bound is $K_{2,1}(s)$ and the corresponding nominal sensitivity transfer function $S_p(s,K_{2,1})$ is shown in Figure 9.

The same procedure was repeated for $\gamma=1$ and $\gamma=0.5$. The obtained stability margins were equal to 2.52 and 0.22 consequently. This decreasing pattern was expected because more emphasis was put on the performance condition than on the robustness properties of the system.

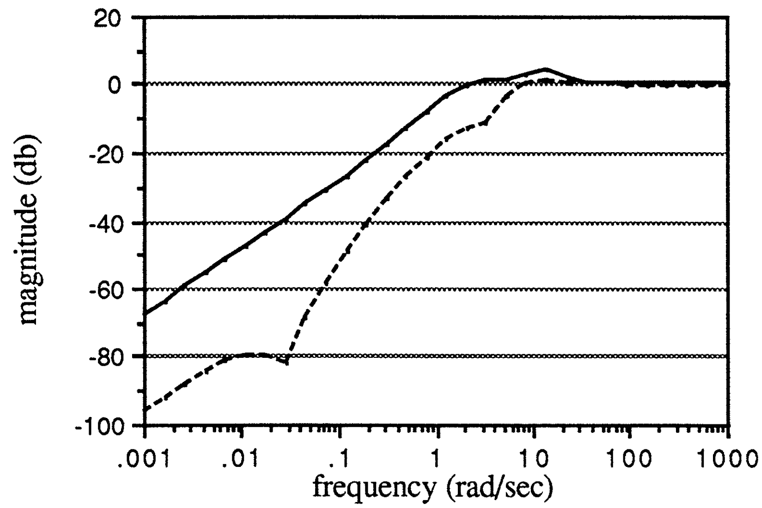

Figure 9. Sensitivity T.F. of the Nominal System with $K_{2,1}$ - S.V. Plot

The maximum singular value plot in Figure 3.7 is very similar to the singular value plot of the original sensitivity Sp(s,Kp) presented in Figure 5.

4. Conclusions

In this paper we discussed stability and performance robustness in the presence of the block diagonal uncertainty. The individual elements of the latter were assumed to be frequency dependent and norm bounded. The case where one of the bounds is not known a priori is shown herein to give freedom for the optimization problem of finding its extreme value such that there exist a stabilizing compensator that guarantees robust stability and performance. An iterative algorithm for increasing the varying bound was introduced. It is obtained by altering the standard "D-K" iteration procedure for the "μ" synthesis. The algorithm has two major steps with the first step consisting of an iterative procedure that has three substeps. It was shown that convergence is guaranteed by the construction of the algorithm. The result obtained depends on the initial choice of α, which is the varying parameter, and on the block diagonal scaling D_F and, therefore, it represents a local optimum.

The algorithm stops either when in two successive iterations the bound on the uncertainty and the values of corresponding "μ_S" functions remain unchanged or when the previously set upper bound $\overline{\alpha}$ is reached.

Furthermore, the control redesign algorithm was evaluated on the augmented model of the F-8 Aircraft. A case is treated where one of the existing two actuators partially fails. Its postfailure dynamics are assumed to be given by the nominal model plus the associated unstructured uncertainty. The algorithm was run with three different values of the performance index γ. The obtained values of the uncertainty bound were discussed. It was shown that they follow the expected trend with respect to the successive increasing or decreasing of γ. It was noted that the results depend on the initial conditions and the quality of approximating the block diagonal scaling "D".

References

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Appendix 1.

$F\text{-}8(s) = C\,(sI - A)^{-1}B$ where

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1.5 & -1.5 & 0 & 0.0057 \\ -12 & 12 & -0.8 & -0.0344 \\ -0.8524 & 0.2904 & 0 & -0.014 \end{bmatrix} \qquad (A1.1)$$

$$B' = \begin{bmatrix} 0 & 0.16 & -19 & -0.0115 \\ 0 & 0.6 & -2.5 & -0.0087 \end{bmatrix} \qquad (A1.2)$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad (A1.3)$$

$$Fe(s) = Ff(s) = 15\,[\,s + 15\,]^{-1} \qquad (A1.4)$$