Design of Feedback Control Systems for Stable Plants with Saturating Actuators[1]

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ABSTRACT

A systematic control design methodology is introduced for multi-input/multi-output stable open loop plants with multiple saturations. This new methodology is a substantial improvement over previous heuristic single-input/single-output approaches.

The idea is to introduce a supervisor loop so that when the references and/or disturbances are sufficiently small, the control system operates linearly as designed. For signals large enough to cause saturations, the control law is modified in such a way to ensure stability and to preserve, to the extent possible, the behavior of the linear control design.

Key benefits of this methodology are: the modified compensator never produces saturating control signals, integrators and/or slow dynamics in the compensator never windup, the directional properties of the controls are maintained, and the closed loop system has certain guaranteed stability properties.

The advantages of the new design methodology are illustrated in the simulation of an academic example and the simulation of the multivariable longitudinal control of a modified model of the F-8 aircraft.

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1. Introduction

Almost every physical system has maximum and minimum limits or saturations on its control signals. For multivariable systems, a major problem that arises (because of saturations) is the fact that control saturations alter the direction of the control vector. For example, let us assume that there are m control signals with m saturation elements. Each saturation element operates on its input signal independently of the other saturation elements; as we shall show in the performance analysis section, this can disturb the direction of the applied control vector. Consequently, erroneous controls can occur, causing degradation with the performance of the closed loop system over and above the expected fact that output transients will be "slower".

Another performance degradation occurs when a linear compensator with integrators is used in a closed loop system and the phenomenon of reset-windup appears. During the time of saturation of the actuators, the error is continuously integrated even though the controls are not what they should be. The integrator, and other slow compensator states, attain values that lead to larger controls than the saturation limits. This leads to the phenomenon known as reset-windup, resulting in serious deterioration of the performance (large overshoots and large settling times.) Many attempts have been made to address this problem for SISO systems, but a general design process has not been formalized. No research has been found in the literature that addresses and solves the reset-windup problem for MIMO systems.

In practice, the saturations are ignored in the first stage of the control design process, and then the final controller is designed using ad-hoc modifications and extensive simulations. A common classical remedy was to reduce the bandwidth of the control system so that control saturation seldom occurred. Thus, even for small commands and disturbances, one intentionally degraded the possible performance of the system (longer settling times etc.). Although reduction in closed-loop bandwidth by reduction in the loop gain is an "easy" design tool, it clearly is not necessarily the best that could be done. Hence, a new design methodology is desirable which will generate transients consistent with the actuation levels available, but which maintains the rapid speed of response for small exogenous signals (reference commands and disturbances).

One way to design controllers for systems with bounded controls, would be to solve an optimal control problem; for example, the time optimal control problem or the minimum energy problem etc. The solution to such problems usually leads to a bang-bang feedback controller [1]. Even though the problem has been solved completely in principle, the solution to even the simplest systems requires good modelling, is difficult to calculate open loop solutions, or the resulting switching surfaces are complicated to work with. For these reasons, in most applications the optimal control solution is not used.

Because of the problems with optimal control results, other design techniques have been attempted. Most of them are based on solving the Lyapunov equation and getting a feedback which will guarantee global stability when possible or local stability otherwise [2]-[3]. The problem with these techniques is that the solutions tend to be unnecessarily conservative and consequently the performance of the closed loop system may suffer. For example, when global stability is guaranteed, it is often required that the final open loop system is strictly positive-real with all the limitations that such systems possess.

Attempts to solve the reset windup problems when integrators are present in the forward loop, have been made for SISO systems [4]-[10]. Most of these attempts lead to controllers with substantially improved performance but not well understood stability properties. As part of this research, an initial investigation was made on the effects on performance of the reset windups for MIMO systems [11] showing potential for improving the performance of the system. A simple case study was also recently conducted on the effects of saturations to MIMO systems where potential for improvement in the performance was demonstrated [12].

This research brings new advances in the theory concerning the design of control systems with multiple saturations. A systematic methodology is introduced to design control systems with multiple saturations for stable open loop plants. The idea is to design a linear control system ignoring the saturations and when necessary to modify that linear control law. When the exogenous signals are small, and they do not cause saturations, the system operates linearly as designed. When the signals are large enough to cause saturations, the control law is then modified in such a way to preserve ("mimic") to the extent possible the responses of the linear design. Our modification to the linear compensator is introduced at the error via an Error Governor (EG). The main benefits of the methodology are that it leads to controllers with the following properties:

(a) The signals that the modified compensator produces **never cause saturation.** The nonlinear response mimics the shape of the linear one with the difference that its speed of response may be, as expected, slower. Thus the output of the compensator (the controls) are not altered by the saturations.

(b) Possible integrators or slow dynamics in the compensator **never windup.** That is true because the signals produced by the modified compensator never exceed the limits of the saturations.

(c) For closed loop systems with stable plants **finite gain stability** is guaranteed for any reference, disturbance and any modelling error as long as the "true" plant is open loop stable.

(d) The **on-line computation** required to implement the control system **is minimal** and realizable in most of today's microprocessors.

2. Performance Analysis

Without loss of generality one can assume that each element $u_i(t)$ of the control vector $\mathbf{u}(t) = [u_1(t) \ldots u_p(t)]^T$ has saturation limits ± 1 and the saturation operator can be defined as follows:

$$\mathrm{sat}(u_i(t)) = \begin{cases} 1 & u_i(t) \geq 1 \\ u_i(t) & -1 \leq u_i(t) \leq 1 \\ -1 & u_i(t) \leq -1 \end{cases} \tag{2.1}$$

Figure 2.1 shows the closed loop system with the saturation element at the controls. The compensator K(s) is designed using linear control system techniques and it is assumed that the closed loop system without the saturations (the linear system) is stable with "good" properties.

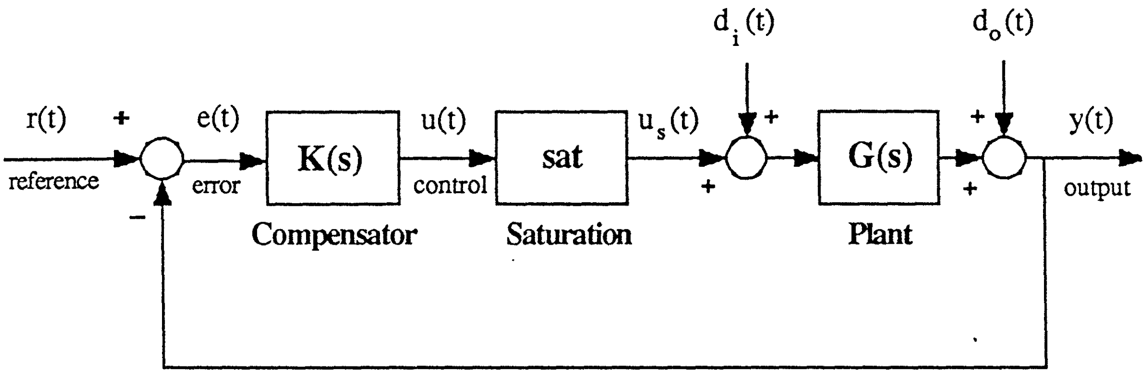

Figure 2.1: The closed loop system

There are well developed methods for defining performance criteria and for designing linear closed loop systems which meet the performance requirements. It would then be desirable, whenever the closed loop system operates in the linear region, to meet the a priori performance constraints (because it easy to define them and easy to design control systems satisfying these constraints). When the system operates in the nonlinear region new performance criteria have to be defined and new ways of achieving the desired performance must be developed.

There are two major problems that multiple saturations can introduce to the performance of the system: (a) the reset windup problem, and (b) the fact that multiple saturations change the direction of the controls.

When the linear compensator contains integrators and/or slow dynamics reset windups can occur. Whenever the controls are saturated the error is continuously integrated and this can lead to large overshoots in the response of the system. It is obvious that if the states of the compensator were such that the controls would never saturate, then reset windups would never appear. See references [8] and [9] for additional discussion of the reset windup problem.

Almost every current design methodology for linear systems inverts the plant and replaces the open loop system with a desired design loop. The inversion is done through the controls with signals at specific frequencies and directions. The saturations alter the direction and frequency of the control signal and thus interfere with the inversion process. The main problem is that although both the compensator and the plant are multivariable highly coupled systems, the saturations operate as SISO systems. Each saturation operates on its input signal *independently* from the other saturation elements.

To see exactly what happens assume as an example that in a two input system the control signal at some time t_0 is $\mathbf{u'}_1 = [\,3 \quad 1.1\,]^T$ the saturated signal will be $\mathbf{u'} = [\,1 \quad 1\,]^T$. Notice that the direction of the $\mathbf{u'}_1$ signal at time t_0 is altered. In fact, any input control signal $\mathbf{u} = [\,u_1 \quad u_2\,]^T$ will be transformed through the saturation to $\mathbf{u_s} = [\,1 \quad 1]^T$ if $u_1 \geq 1$ and $u_2 \geq 1$. Figure 2.2 shows an illustration of four different control directions $\mathbf{u'}_1$, $\mathbf{u'}_2$, $\mathbf{u''}_1$, $\mathbf{u''}_2$ which are mapped at only two directions $\mathbf{u'}$ and $\mathbf{u''}$.

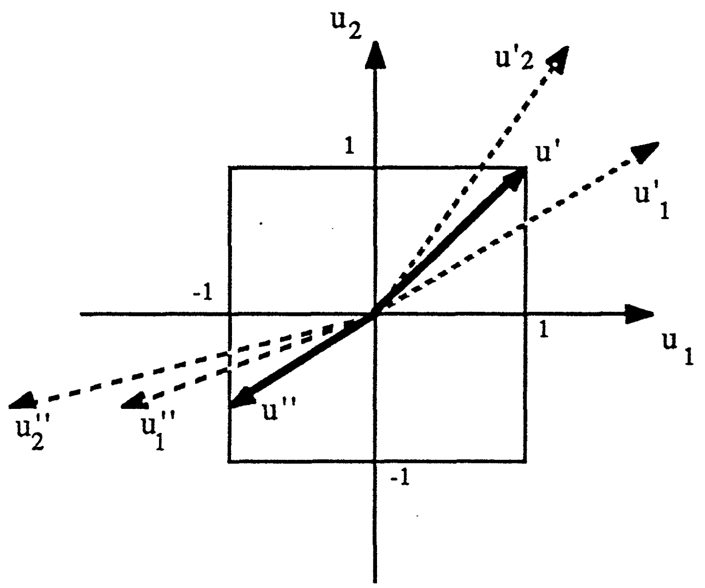

Figure 2.2: Examples of control directions at the input of the saturation $\mathbf{u'}_1$, $\mathbf{u'}_2$, $\mathbf{u''}_1$, $\mathbf{u''}_2$ and at the output of the saturation $\mathbf{u'}$, $\mathbf{u''}$.

Since the saturations can alter the direction of the control signals, and in effect disturb the compensator/plant inversion process, the logical question to ask is, under what conditions the linearly designed compensator that inverts (or partially inverts) the linear plant also inverts the plant when the saturations are present.

To solve the performance problem let us assume that a nonzero operator is added to the system. The operator O_1 is applied to the error signals and for convenience purposes it will be called **Error Governor (EG)**.

$$\mathbf{u} = \mathbf{K}\mathbf{O}_1\mathbf{e} \tag{2.2}$$

The nonzero operator will be chosen, *when possible*, so that the control **u**(t) never saturates, i.e. $\|\mathbf{u}(t)\|_\infty \leq 1$, for any reference and/or disturbances. Figure 2.3 shows the closed loop system with the added operator.

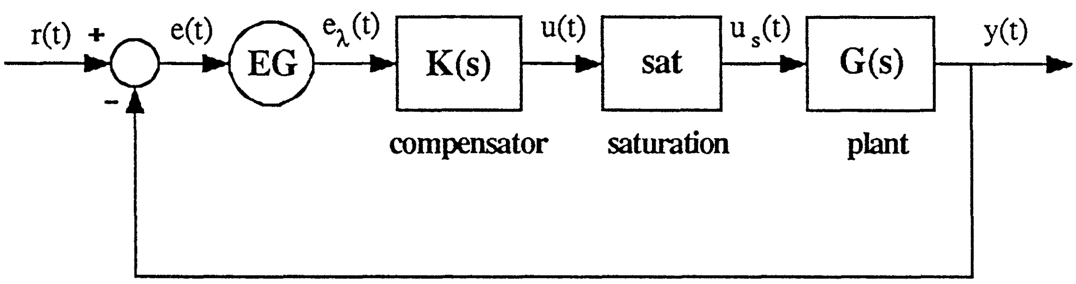

Figure 2.3: General structure for the control system

Effectively, with the introduction of the EG operator, the saturation is transferred from the controls to the errors and it makes the control analysis and design process easier.

The selection of the EG operator will be such that the controls will never saturate; and if, for example, the compensator was designed to invert or partially invert the plant, then the inversion process will not be distorted by the saturation and ***GsatK will remain linear and equal to GK***. In the closed loop system with the operator EG the compensator will never cause windups. The integrators and slow dynamics of the compensator will never cause the controls to exceed the limits of the saturation and thus *windups never occur*.

3. Mathematical preliminaries

This section is an introduction to the new design methodology. Some necessary mathematical preliminaries will be given and a basic problem will be introduced. The basic problem will be solved and it's solution will lead to the design of the EG operator that was introduced in section 2. For the proofs of the theorems given in this section see reference [13].

Consider the following linear time invariant system

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) \qquad \mathbf{A} \in \mathbb{R}^{nxn},\ \mathbf{x}(t) \in \mathbb{R}^{n} \tag{3.1}$$

$$\mathbf{x}(0) = \mathbf{x}_0 \tag{3.2}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \qquad \mathbf{C} \in \mathbb{R}^{mxn},\ \mathbf{y}(t) \in \mathbb{R}^{m} \tag{3.3}$$

$$\mathbf{y}(\mathbf{x}_0,t) = \mathbf{C}e^{\mathbf{A}t}\mathbf{x}_0 \tag{3.4}$$

where $e^{\mathbf{A}t}$ is the state transition matrix (matrix exponential) for **A**..

Definition 3.1: The scalar-valued function g(**x**) is defined as follows:

$$g(\mathbf{x}_0):\ \mathbb{R}^{n} \rightarrow \mathbb{R}, \quad g(\mathbf{x}_0) = \|\mathbf{y}(\mathbf{x}_0,t)\|_{\infty} \tag{3.5}$$

Theorem 3.1: Let $\lambda_i(\mathbf{A})$ be an observable mode of (**A**,**C**) and let the multiplicity of $\lambda_i(\mathbf{A})$) be n_i. The function g(**x**) is finite $\forall \mathbf{x} \in \mathbb{R}^{n}$ if and only if

a) $\mathrm{Re}(\lambda_i(\mathbf{A})) \leq 0, \forall i$, and

b) The modes $\lambda_i(\mathbf{A})$ with $\mathrm{Re}(\lambda_i(\mathbf{A})) = 0$ and $n_i > 1$ have independent eigenvectors (i.e. the order of the Jordan blocks associated with the eigenvalues of **A** with $\mathrm{Re}(\lambda_i(\mathbf{A})) = 0$ and $n_i > 1$ is 1.).

The systems that satisfy conditions (a) and (b) of theorem 3.1 are called **neutrally stable.**

Definition 3.2: The set $\mathbf{P_g}$ is defined as:

$$\mathbf{P_g} = \{\ [\mathbf{x},v] : \mathbf{x} \in \mathbb{R}^{n},\ v \in \mathbb{R},\ v \geq g(\mathbf{x})\ \} \tag{3.6}$$

From this definition we see that $\mathbf{P_g}$ is the interior of the graph of the function $g(\mathbf{x})$ in $\mathbb{R}^{n+1}$, as shown in figure 3.1.

Definition 3.3: $\mathbf{B_{A,C}}$ is the set of all $\mathbf{x} \in \mathbb{R}^n$ with $0 \leq g(\mathbf{x}) \leq 1$, i.e.

$$\mathbf{B_{A,C}} = \{\mathbf{x} : 0 \leq g(\mathbf{x}) \leq 1\} \tag{3.7}$$

Suppose that the system (3.1)-(3.4) has an initial condition $\mathbf{x_0} \in \mathbf{B_{A,C}}$. From this definition we see that for such an initial condition the output of the system, $\mathbf{y}(t)$, will satisfy $\|y(t)\|_\infty \leq 1$.

For neutrally stable systems the function $g(\mathbf{x})$, the set $\mathbf{P_g}$ and the set $\mathbf{B_{A,C}}$ have the following properties.

(a) The function $g(\mathbf{x})$ is continuous and even.

(b) The function $g(\mathbf{x})$ is not necessarily differentiable at all points in $\mathbb{R}^n$.

(c) The set $\mathbf{P_g}$ is a convex cone.

(d) The $\mathbf{B_{A,C}}$ set is symmetric with respect to the origin and convex.

The proofs for these properties are given in reference [13].

One might expect that $\mathbf{P_g}$ would be a convex cone from the linearity ($g(\alpha\mathbf{x}) = \alpha g(\mathbf{x})$) of the system (3.1)-(3.4). Figure 3.1 gives a visualization of the function $g(\mathbf{x_0})$ and the sets $\mathbf{B_{A,C}}$ and $\mathbf{P_g}$ in $\mathbb{R}^n$ and $\mathbb{R}^{n+1}$ respectively.

Definition 3.4 [14]: The upper right Dini derivative is defined as

$$D^+f(t_0) = \limsup_{t \to t_0^+} \frac{f(t)-f(t_0)}{t-t_0} \tag{3.8}$$

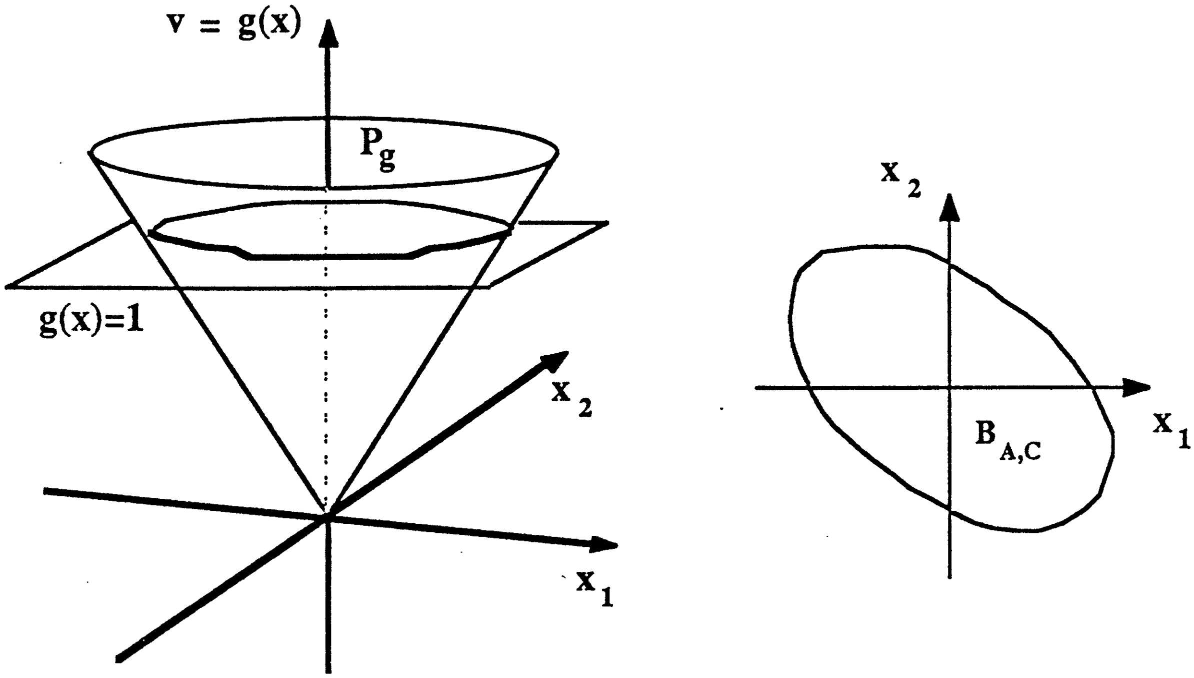

Figure 3.1: Visualization of the function g(x) and the sets P_g and $B_{A,C}$.

Definitions of the lower right, upper left and lower left Dini derivatives are given in reference [14]. In the sequel only the upper right Dini derivative will be used as in definition 3.4. The $D^+f(t_0)$ is finite at t_0 if the function f satisfies the Lipschitz condition locally around t_0 [14]. Note that the function g(x) given in definition 3.1 satisfies the Lipschitz condition locally if the conditions of theorem 3.1 are met. This is obvious because g(x) is the boundary of the cone P_g.

Theorem 3.2 [14]: Suppose that f(t) is continuous on (a,b), then f(t) is nonincreasing on (a,b) iff $D^+f(t) \leq 0$ for every $t \in (a,b)$.

3.1 Design of a Time-Varying Gain such that the Outputs of a Linear System are Bounded

Assume that a linear system is defined by the following equations

$$\dot{x}(t) = Ax(t)+Bu(t) \qquad A \in \mathbb{R}^{nxn},\ B \in \mathbb{R}^{nxm} \qquad (3.9)$$

$$y(t) = Cx(t) \qquad C \in \mathbb{R}^{mxn} \qquad (3.10)$$

and also assume that the linear system is **neutrally stable.** Then, if one were to construct the function g(**x**) (definition 3.1) for the system (3.9)-(3.10) with **B** = 0, the following is true; $g(\mathbf{x}) < \infty, \forall \mathbf{x} \in \mathbb{R}^n$. This follows from theorem 3.1.

The goal here, is to keep the outputs of the linear system (3.9)-(3.10) bounded (i.e. $|y_i(t)| \leq 1, \forall\, t, i$) for any input **u**(t). To achieve our goal, consider the following system with a time-varying scalar gain λ(t)

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\lambda(t)\mathbf{u}(t) \tag{3.11}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \tag{3.12}$$

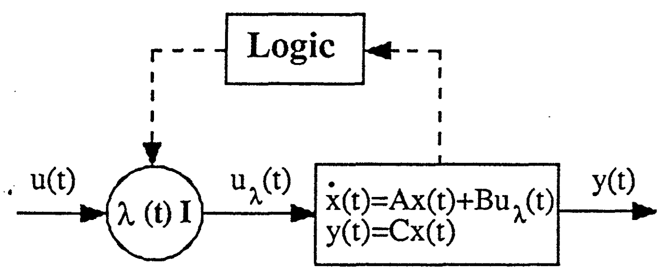

Figure 3.2: The basic system for calculating λ(t).

Figure 3.2 shows the basic system and the location of the time-varying gain λ(t). In this framework a basic problem can be defined.

***The Basic Problem*:**

At time t_0, find the maximum gain $\lambda(t_0)$, $0 \leq \lambda(t_0) \leq 1$, such that $\forall \mathbf{u}(t), t > t_0 \;\exists\; \lambda(t), t > t_0$ such that the output will satisfy $|y_i(t)| \leq 1\ \forall\, i, t > t_0$.

A solution to this problem can be obtained by using a function g(**x**) given in definition 3.1 and by using a set $\mathbf{B}_{A,C}$ given in definition 3.3. To be more specific, for the system (3.11)-(3.12), with **u**(t) = 0, one can define g(**x**) and $\mathbf{B}_{A,C}$ as in eqs. (3.13)-(3.15). The function g(**x**) is finite because the system (3.9)-(3.10) is assumed to be neutrally stable (theorem 3.1).

$$g(\mathbf{x}_0):\ \mathbb{R}^n \rightarrow \mathbb{R}, \quad g(\mathbf{x}_0) = \|\mathbf{y}(\mathbf{x}_0,t)\|_\infty \tag{3.13}$$

$$\mathbf{x}(0) = \mathbf{x}_0 \tag{3.14}$$

$$\mathbf{B}_{A,C} = \{\mathbf{x}:\ g(\mathbf{x}) \le 1\} \tag{3.15}$$

By defining $g(\mathbf{x})$ and $\mathbf{B}_{A,C}$ as in eqs. (3.13)-(3.15) one can construct $\lambda(t)$ as follows:

Construction of $\lambda(t)$:

For every time t choose $\lambda(t)$ as follows

a) if $\mathbf{x}(t) \in \mathrm{Int}\mathbf{B}_{A,C}$ then $\lambda(t) = 1$ (3.16)

b) if $\mathbf{x}(t) \in \mathrm{Bd}\mathbf{B}_{A,C}$ then choose the largest $\lambda(t)$ such that (3.17)

$$0 \le \lambda(t) \le 1 \tag{3.18}$$

$$\lim_{\varepsilon \to 0} \sup \frac{g(\mathbf{x}(t)+\varepsilon[\mathbf{A}\mathbf{x}(t)+\mathbf{B}\lambda(t)\mathbf{u}(t)]) - g(\mathbf{x}(t))}{\varepsilon} \le 0 \tag{3.19}$$

or for the points where $g(\mathbf{x})$ is differentiable choose the largest $\lambda(t)$ such that

$$0 \le \lambda(t) \le 1 \tag{3.20}$$

$$Dg(\mathbf{x}(t))[\mathbf{A}\mathbf{x}(t) + \mathbf{B}\lambda(t)\mathbf{u}(t)] \le 0 \tag{3.21}$$

where $Dg(\mathbf{x}(t))$ is the Jacobian matrix of $g(\mathbf{x}(t))$.

c) if $\mathbf{x}(t) \notin \mathbf{B}_{A,C}$ then choose $\lambda(t)$, $0 \le \lambda(t) \le 1$ such that the expression in (3.19) is minimum.

In the construction of $\lambda(t)$ if $\mathbf{x}(t_0) \notin \mathbf{B}_{A,C}$ then the basic problem cannot be solved because there exists a $\mathbf{u}(t_0)$ for $t > t_0$ (i.e. $\mathbf{u}(t) = 0$) where it will lead to $\|\mathbf{y}(\mathbf{x}(t_0),t)\|_\infty > 1$. In such a case, the best that can be done is to find $\lambda(t)$ such that the states $\mathbf{x}(t)$ will be driven into $\mathbf{B}_{A,C}$ as soon as possible.

With the $\lambda(t)$ defined as above let us examine some properties of the system (3.11)-(3.12). To be more specific it will be shown that

(a) There is always exists a $\lambda(t)$ that satisfies all the constraints in the construction of $\lambda(t)$.

(b) If $\lambda(t)$ is constructed as specified above and $\mathbf{x}(t_0) \in \mathbf{B}_{A,C}$, then $\mathbf{x}(t) \in \mathbf{B}_{A,C}$ $\forall t > t_0$ and for all $\mathbf{u}(t)$, $t > t_0$.

(c) The construction of $\lambda(t)$ solves the basic problem when that is possible (i.e. $\mathbf{x}(t) \in \mathbf{B}_{A,C}$ for all t).

Theorem 3.3: For the system given in eqs. (3.11)-(3.12) the following is always true $\forall \mathbf{x} \in \mathbb{R}^n$.

$$\lim_{\varepsilon \to 0} \sup \frac{g(\mathbf{x}(t)+\varepsilon[\mathbf{A}\mathbf{x}(t)]) - g(\mathbf{x}(t))}{\varepsilon} \leq 0 \qquad (3.22)$$

and at the points where $g(\mathbf{x})$ is differentiable

$$Dg(\mathbf{x})\, \mathbf{A}\mathbf{x} \leq 0 \qquad \forall \mathbf{x} \in \mathbb{R}^n \qquad (3.23)$$

where $Dg(\mathbf{x}(t))$ is the Jacobian matrix of $g(\mathbf{x}(t))$.

Proof: Assume that the inequality (3.22) is not true for some $\mathbf{x}(t) = \mathbf{x}_0$. If the $\mathbf{x}_0$ is used as an initial condition to the $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t)$ system then because of theorem 3.2 $\exists t'>0$ such that $g(\mathbf{x}(t')) > g(\mathbf{x}(t))$. But $g(\mathbf{x}_0) = \|\mathbf{C}\mathbf{x}(t)\|_\infty$ so this is a contradiction. Therefore, inequality (3.22) is true $\forall \mathbf{x} \in \mathbb{R}^n$. ////

The construction of $\lambda(t)$ is always possible because of theorem 3.3, namely one can choose $\lambda(t) = 0\ \forall t$ and the inequality (3.19) is always true.

Lemma 3.1: In the system (3.11)-(3.12) if $\mathbf{x}_0 \in \mathbf{B}_{A,C}$ and $\lambda(t)$ is constructed as it was described above, then $\mathbf{x}(t) \in \mathbf{B}_{A,C}$ for all t and for all $\mathbf{u}(t)$.

Proof: The proof of this Lemma follows from the construction of $\lambda(t)$. ////

Theorem 3.4: For the system (3.11)-(3.12) with $\lambda(t)$ constructed as above the following is always true

if $\mathbf{x_0} \in \mathbf{B_{A,C}}$ then $\|y(t)\|_\infty \leq 1$ $\forall$input $\mathbf{u}(t)$

if $\mathbf{x_0} \notin \mathbf{B_{A,C}}$ then $\|y(t)\|_\infty \leq g(\mathbf{x_0})$ $\forall$input $\mathbf{u}(t)$

Proof: If $x_0 \in \mathbf{B_{A,C}}$, then

The construction of $\lambda(t)$ guarantees that $\mathbf{x}(t) \in \mathbf{B_{A,C}}$ $\forall$t. (see Lemma 3.1). It is also true that for any state $\mathbf{x}(t) \in \mathbf{B_{A,C}}$ $\|\mathbf{Cx}(t)\|_\infty \leq 1$. If $\|\mathbf{Cx}(t)\|_\infty > 1$ and $\mathbf{x}(t)$ is used as an initial condition in the system the following will be true, $g(\mathbf{x}(t)) > 1$ and $\mathbf{x}(t) \notin \mathbf{B_{A,C}}$ which is a contradiction. Since $\mathbf{y}(t) = \mathbf{Cx}(t)$ and $\mathbf{x}(t) \in \mathbf{B_{A,C}}$ $\forall$t then $\|\mathbf{y}(t)\|_\infty \leq 1$ $\forall$input $\mathbf{u}(t)$.

If $\mathbf{x_0} \notin \mathbf{B_{A,C}}$, then $g(\mathbf{x_0}) > 1$ and from the construction of $\lambda(t)$ $g(\mathbf{x}(t)) < g(\mathbf{x_0})$ ($g(\mathbf{x})$ is decreasing by theorem 3.2). Thus $\|y(t)\|_\infty \leq g(\mathbf{x}(t)) \leq g(\mathbf{x_0})$. ////

Theorem 3.5: At every time t_0, if $\mathbf{x}(t_0) \in \mathbf{B_{A,C}}$ then the time-varying gain $\lambda(t_0)$ is the maximum possible such gain that $0 \leq \lambda(t_0) \leq 1$ and $\forall \mathbf{u}(t)$, $t>t_0$ $\exists$ $\lambda(t)$, $t > t_0$ such that the output $|y_i(t)| \leq 1$ $\forall$ i, $t>t_0$. If $\mathbf{x}(t_0) \notin \mathbf{B_{A,C}}$ then such a gain $\lambda(t_0)$ does not exist.

Proof: If $\mathbf{x}(t_0) \in \mathbf{B_{A,C}}$, then from the construction of $\lambda(t)$, at any time t_0 the maximum gain $\lambda(t_0)$ is chosen such that $0 \leq \lambda(t_0) \leq 1$ and $\mathbf{x}(t) \in \mathbf{B_{A,C}} \forall t > t_0$. If a greater gain $\lambda(t_0)$ is used then $g(\mathbf{x}(t_0))$ will be increasing (see theorem 3.2) and $\mathbf{x}(t) \notin \mathbf{B_{A,C}} \forall t>t_0$; consequently there exists $\mathbf{u}(t)$ (i.e. $\mathbf{u}(t) = 0$ $t \geq t_0$) where $\|y(t)\|_\infty > 1$.

If $\mathbf{x}(t_0) \notin \mathbf{B_{A,C}}$, then there exists $\mathbf{u}(t)$ (i.e. $\mathbf{u}(t)=0$ $t \geq t_0$) where $\|y(t)\|_\infty > 1$ and thus for any $\lambda(t_0)$ the basic problem does not have a solution. ////

The solution to the basic problem which was given above assumed that $\lambda(t)$ is a scalar. A similar solution can be obtained if a time-varying diagonal matrix $\Lambda(t)$ is employed. The construction of $\Lambda(t)$ and all the properties that were described previously can easily be extended for the matrix case. Similar analysis can be done for systems with a feedforward term from the controls to the outputs [13].

4. Description of the Control Structure with the Operator EG

In section 2 (performance analysis) the need for an operator EG to achieve better control system performance was shown. In section 3, it was shown how to choose a time varying gain $\lambda(t)$, at the inputs of a linear time invariant system, such that the outputs of that system will remain bounded. In this section, we combine the results of sections 2 and 3 to obtain, a control structure with an EG operator (i.e. a time gain-varying gain). This structure will be introduced and analyzed. With the EG operator at the error signal, the system will remain unaltered (linear) when the references and disturbances are such that they don't cause saturation. For "large" reference and disturbance signals the operator EG will ensure that the controls will never saturate. This control structure is useful for feedback systems with stable open loop plants and neutrally stable linear compensators.

The new control structure has inherent good properties (stability, no reset windups etc.) which will be discussed and demonstrated in simulations of two examples. The examples chosen are an academic example (with pathological directional properties) and a model of the F8 aircraft longitudial dynamics.

Consider a feedback control system with a linear plant $\mathbf{G}(s)$, a linear compensator $\mathbf{K}(s)$ and a magnitude saturation at the controls. The plant and the compensator are modelled by the following state space representations:

Plant:
$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}_s(t) \tag{4.1}$$
$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \tag{4.2}$$
$$\mathbf{u}_s(t) = \text{sat}(\mathbf{u}(t)) \tag{4.3}$$

Compensator:
$$\dot{\mathbf{x}}_c(t) = \mathbf{A}_c\mathbf{x}_c(t) + \mathbf{B}_c\mathbf{e}(t) \tag{4.4}$$
$$\mathbf{u}(t) = \mathbf{C}_c\mathbf{x}_c(t) \tag{4.5}$$
$$\mathbf{e}(t) = \mathbf{r}(t) - \mathbf{y}(t) \tag{4.6}$$

where $\mathbf{r}(t)$ is the reference, $\mathbf{u}(t)$ is the control and $\mathbf{y}(t)$ is the output signal.

The compensator can be thought of as an independent linear system with input $\mathbf{e}(t)$ (error signal) and output **u**(t) (control signal). The objective is to introduce a time-varying gain λ(t) (EG operator) at the error, e(t), such that the control, **u**(t), will never saturate. Following the discussion of section 3 the gain, λ(t), is injected at the error signal and the resulting compensator is given by

$$\dot{\mathbf{x}}_c(t) = \mathbf{A}_c\mathbf{x}_c(t) + \mathbf{B}_c\lambda(t)\mathbf{e}(t) \tag{4.7}$$

$$\mathbf{u}(t) = \mathbf{C}_c\mathbf{x}_c(t) \tag{4.8}$$

$$\mathbf{e}(t) = \mathbf{r}(t) - \mathbf{y}(t) \tag{4.9}$$

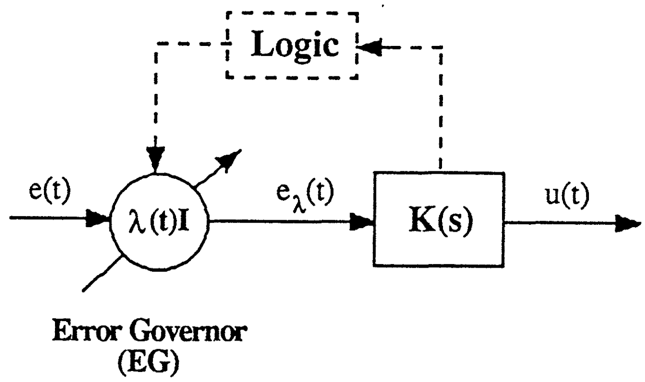

Figure 4.1: The basic system for calculating λ(t).

In analogy to figure 3.2, figure 4.1 shows the basic system for computing λ(t). A function g(**x**) and a set $\mathbf{B}_{A,C}$ are defined and then the construction of λ(t) follows in accordance with the results presented in section 3.

$$g(\mathbf{x}_0):\quad g(\mathbf{x}_0) = \|\mathbf{u}(t)\|_\infty \tag{4.10}$$

where $$\dot{\mathbf{x}}_c(t) = \mathbf{A}_c\mathbf{x}_c(t); \qquad \mathbf{x}_c(0)=\mathbf{x}_0 \tag{4.11}$$

$$\mathbf{u}(t) = \mathbf{C}_c\mathbf{x}_c(t) \tag{4.12}$$

$$\mathbf{B}_{A,C} = \{\mathbf{x}:\ g(\mathbf{x}) \le 1\} \tag{4.13}$$

For g(**x**) to be finite, for all **x**, **the compensator has to be neutrally stable** (theorem 3.1). This is not an overly restrictive constraint because most compensators are usually neutrally stable. With finite g(**x**) the EG operator (λ(t)) is given by

Construction of $\lambda(t)$:

For every time t choose $\lambda(t)$ as follows

a) if $\mathbf{x}_c(t) \in \mathrm{Int}\mathbf{B}_{A,C}$ then $\lambda(t) = 1$ (4.14)

b) if $\mathbf{x}_c(t) \in \mathrm{Bd}\mathbf{B}_{A,C}$ then choose the largest $\lambda(t)$ such that (4.15)

$$0 \leq \lambda(t) \leq 1$$

$$\lim_{\varepsilon \to 0} \sup \frac{g\big(\mathbf{x}_c(t)+\varepsilon[\mathbf{A}_c\mathbf{x}_c(t)+\mathbf{B}_c\lambda(t)e(t)]\big) - g\big(\mathbf{x}_c(t)\big)}{\varepsilon} \leq 0 \tag{4.16}$$

or for the points where $g(\mathbf{x})$ is differentiable choose the largest $\lambda(t)$ such that

$$0 \leq \lambda(t) \leq 1 \tag{4.17}$$

$$Dg(\mathbf{x}_c(t))[\mathbf{A}_c\mathbf{x}_c(t)+\mathbf{B}_c\lambda(t)e(t)] \leq 0 \quad \forall\, t > 0 \tag{4.18}$$

where $Dg(\mathbf{x}_c(t))$ is the Jacobian matrix of $g(\mathbf{x}_c(t))$.

c) if $\mathbf{x}_c(t) \notin \mathbf{B}_{A,C}$ then choose $\lambda(t)$, $0 \leq \lambda(t) \leq 1$ such that the expression (4.16) is minimum.

From the results in section 3 it can be proven that if, at time t = 0, the compensator states, $\mathbf{x}_c(t)$, belong in the $\mathbf{B}_{A,C}$ set, then the EG operator exists and the signal $\mathbf{u}(t)$ remains bounded for any signal e(t). Hence, the controls will never saturate for any reference, any input disturbance, and any output disturbance.

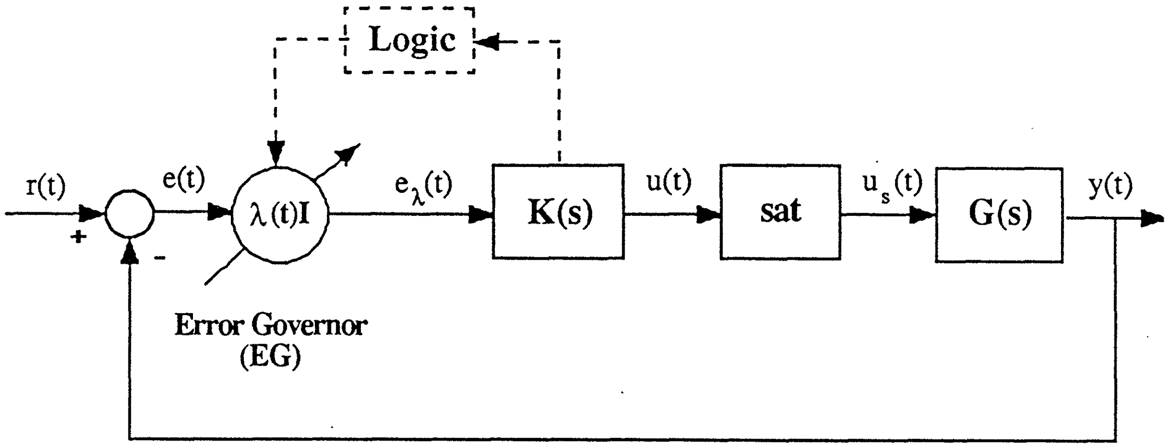

Figure 4.2: Control structure with the EG operator.

Figure 4.2 shows the control structure obtained with the operator EG at the error signal. With this control structure the feedback system will never suffer from the reset windup problems which occur when open loop integrators or "slow" poles are present. The reason for the absence of reset windups is that the Error Governor will prevent any states associated with integrators or the "slow" poles from reaching a value which will cause the controls to exceed the saturation limits.

Another important property of the new control structure, is that the saturation does not alter either the direction of the control vector or the magnitude of the controls. Thus, if the compensator inverts part of the plant the saturation does not alter the inversion process.

4.1 Stability Analysis for the Control System with the EG

When the plant is stable and the compensator includes the EG operator the following theorem can be proven.

Theorem 4.1: The feedback system with a stable plant given by eqs. (4.1)-(4.3) and a compensator given by eqs.(4.7)-(4.9) is finite gain stable.

Proof: $\exists r_0 \ni \|r\|_\infty \leq r_0 \Rightarrow \|u\|_\infty \leq 1$

if $\|r\|_\infty \le r_0$ then $\lambda(t) = 1$ and the linear system is stable, thus finite gain stable

$\exists y_0 \ni \|y\|_\infty \le y_0 \; \forall r(t)$ because $\mathbf{G}(s)$ is stable with bounded inputs

if $\|r\|_\infty > r_0$ then $\|y\|_\infty \le (\|r\|_\infty / r_0) y_0$ and $\|y\|_\infty \le (y_0/r_0)\|r\|_\infty$

Thus, for $k = (y_0/r_0)$ then $\|y\|_\infty \le k\|r\|_\infty$ ////

Every stable system $\mathbf{G}(s)$ with bounded inputs is BIBO stable because the outputs are always bounded. The system in figure 4.2 is finite gain stable because in addition to being BIBO stable it is known that there exists a class of "small" inputs, $\|\mathbf{r}(t)\|_\infty \le r_0$, for which the system remains linear.

For unstable plants one cannot guarantee closed loop stability because when $\lambda(t) = 0$ the system operates open loop. This is the reason why the control structure with the EG should be used for feedback systems with stable open loop plants. Another control structure can be used for systems with open loop unstable plants [13]. This problem will be addressed separately in a future publication .

For stable plants the closed loop system remains finite gain stable in the presence of any input and/or output disturbance. This is true because the controls never saturate for any input and/or output disturbance. In addition, it is easy to see that the closed loop system will remain finite gain stable for any stable unmodelled dynamics. In fact, the controls will never saturate if the model is replaced by the "true" stable plant; thus, integrator windups and/or control direction problems cannot occur.

4.2 Simulation of the Academic Example #1

The purpose of this example is to illustrate how the saturation can disturb the directionality of the controls and alter the compensator inversion of the plant. The "academic" plant $\mathbf{G}(s)$ has two zeros with low damping which the designed compensator $\mathbf{K}(s)$ cancels. Consider the following state space representation of the plant $\mathbf{G}(s)$

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -1.5 & 1 & 0 & 1 \\ 2 & -3 & 2 & 0 \\ 0 & .5 & -2 & 1 \\ 1 & -1.5 & 0 & -5 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 1 \\ 0 & 1.8 \end{bmatrix} \mathbf{u}_s(t) \tag{4.19}$$

$$\mathbf{y}(t) = \begin{bmatrix} 0 & 2.4 & -3.1 & 1 \\ 1 & 6 & -.5 & -2.8 \end{bmatrix} \mathbf{x}(t) \tag{4.20}$$

$$\mathbf{u}_s(t) = \text{sat}(\mathbf{u}(t)) \tag{4.21}$$

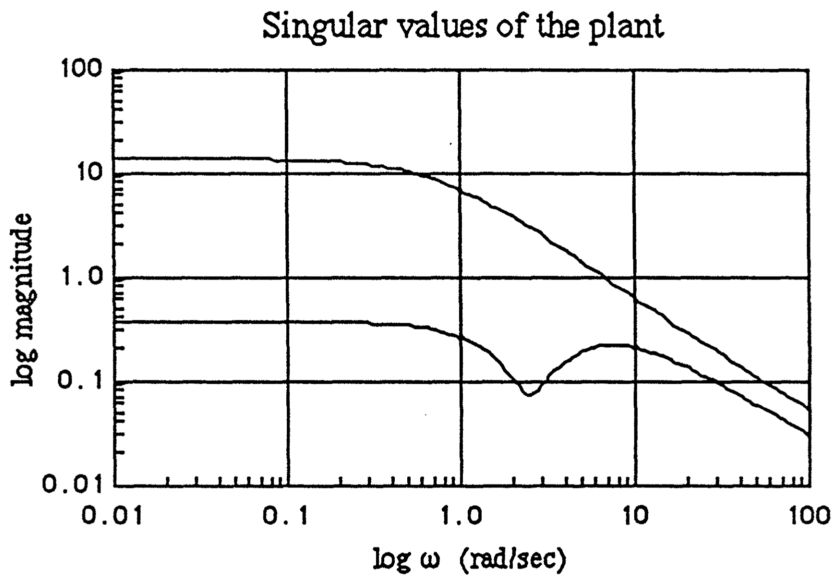

Figure 4.3: Singular values of the plant in the academic example #1.

Figure 4.3 shows the singular values of the open loop plant. Notice the effect of the two resonant zeros of the plant in the singular values at approximately 2.5 rad/sec. A compensator was designed to cancel the two resonant zeros of the plant. The compensator state space representation is given by the following model

$$\dot{x}_c(t) = \begin{bmatrix} -2.6093 & 1.4180 \\ -7.1476 & 1.5213 \end{bmatrix} x_c(t) + \begin{bmatrix} -29.8308 & 2.989 \\ -68.7543 & 10.8387 \end{bmatrix} \lambda(t)\, e(t) \quad (4.22)$$

$$u(t) = \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix} x_c(t) \quad (4.23)$$

The compensator has two states with poles at **-.544 ± j2.422.** The eigenvectors of the poles are collinear with the control direction of the transmission zero of the plant and thus, the compensator cancels the zeros of the plant.

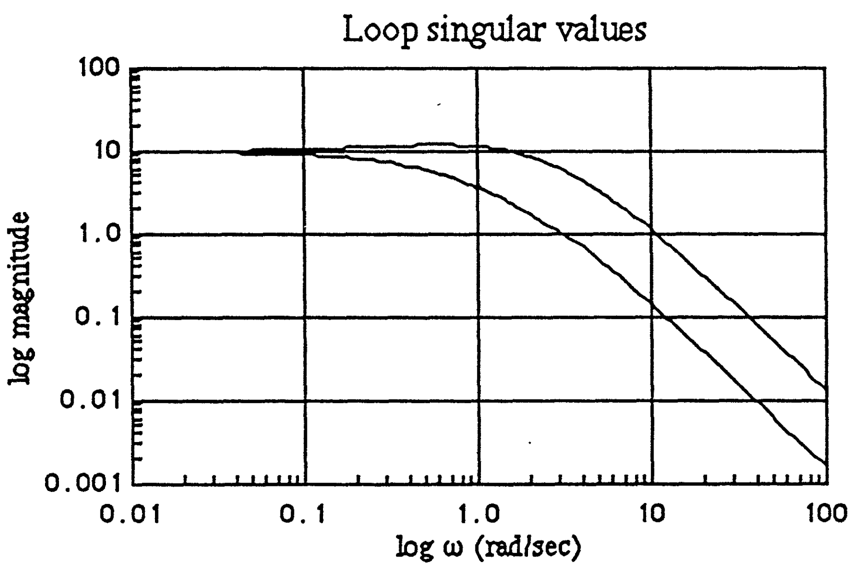

Figure 4.4: Singular values of the loop transfer function in the academic example #1.

Figure 4.4 shows the singular values of the **G**(s)**K**(s) transfer function matrix. Since the compensator cancels the poorly damped zero the antiresonance present in figure 4.3 is not present in figure 4.4.

In this example, the saturation can disturb the cancellation of the plant zeros by the compensator. Since both the plant and the compensator are stable the control structure with the operator EG can be used to correct the problem. Three simulations were performed for the closed loop system, these different simulations are as follows:

1) In the first simulation $\lambda(t) = 1$ and $\mathbf{u}(t) = \mathbf{u}_s(t)$. This is a simulation for a linear time invariant closed loop system and is referred to as the simulation for the *linear system.*

2) In the second simulation $\lambda(t) = 1$ and $\mathbf{u}_s(t) = sat(\mathbf{u}(t))$. This is a simulation where the saturation element is added to the linear system without any other modification. This simulation is referred to as the simulation for the *system with saturation.*

3) In the third simulation $\mathbf{u}_s(t) = sat(\mathbf{u}(t))$, and $\lambda(t)$ served as the EG operator. This type of simulation is referred to as the simulation of the *system with saturation and the EG.*

Figure 4.5 shows the state trajectory of the compensator states for the simulation of the linear system. Note that the states of the compensator do not remain within the $\mathbf{B}_{A,C}$ set so there is a potential for the controls to saturate.

Figures 4.6 and 4.7 show the linear response of the outputs $\mathbf{y}(t)$ and the controls $\mathbf{u}(t)$ respectively. The controls satisfy $\|\mathbf{u}(t)\|_\infty > 1$ at certain times and saturation is expected. It is assumed that the output responses meet the specifications. Thus, we would like the outputs to retain the relative shapes of figure 4.6 when we introduce the nonlinear saturations.

Figure 4.8 shows the state trajectory of the compensator states for the simulation of the system with saturation, it is clear that he states of the compensator do not remain within the $\mathbf{B}_{A,C}$ set. When the controls are saturated the direction of the controls is disturbed and the state trajectory changes dramatically (compare figures 4.5 and 4.8).

Figures 4.9 and 4.10 show the response of the outputs and the controls respectively. The controls have magnitude greater than one and consequently are saturating. In this example, when saturation occurs, the direction of the controls is altered in such a way that even though the original reference is $[.3 \quad .3]^T$, the control direction at saturation drives the system towards $[.3 \quad -.3]^T$ resulting in oscillatory behavior. The compensator does not have any integrators to cause windups and the problems in the performance of the system are solely due to the effects of the saturation upon the direction of the control vector.

Comparing the outputs, i.e. figures 4.6 and 4.9, we see that the shapes of the outputs in figure 4.9 do not match those desired and shown in figure 4.6. Thus, in this case the impact of saturation has produced an unacceptable output response.

Figure 4.11 shows the compensator state trajectory for the simulation of the system with saturation and the EG operator. The states of the compensator do remain within the $\mathbf{B}_{A,C}$ set so control saturation is not expected. In fact, the state trajectory remains on the boundary of the $\mathbf{B}_{A,C}$ set for a long period of time which implies that the controls will stay at their maximum level for a long period of time.

Figures 4.12 and 4.13 show the response of the outputs and the controls respectively. Note that the controls (the inputs to the saturation operator) do not cause saturation. Also note that when u_2 reaches the value of -1, the control u_1 is reduced to the appropriate level so that both controls will drive the output towards $[.3 \quad .3]^T$ as desired. In effect, it is like having a "smart multivariable saturation" instead of the SISO saturations in each channel. The net effect can be seen easier in the output responses. Comparison of figure 4.12 with figure 4.6, shows that the outputs have similar shapes (as desired), except that the outputs in figure 4.12 are "slower" because the control magnitudes are smaller than those in the linear case (compare figures 4.7 and 4.13).

Figure 4.14 shows the real-time behavior of the gain $\lambda(t)$. At the beginning, $\lambda(t)$ is 1 and the system is linear. When the states of the compensator are such that they may lead the controls to saturate, $\lambda(t)$ becomes zero preventing the large errors to be driven by the compensator. The controls at the same time remain at their maximum possible level ($\|\mathbf{u}(t)\|_\infty = 1$). Eventually, $\lambda(t)$ allows the compensator to accept more and more error, while at the same time the controls are kept at maximum level. At the end, $\lambda(t)$ becomes 1 and the system becomes linear time invariant again.

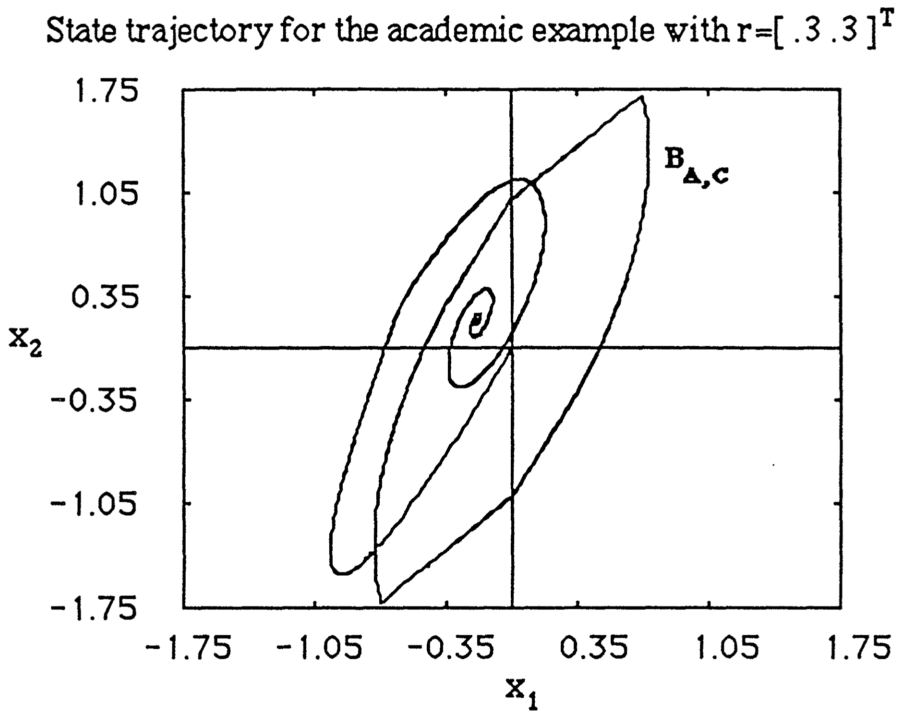

Figure 4.5: State trajectory of the compensator states in the linear system, ($\mathbf{r}$ = [.3 .3]^T).

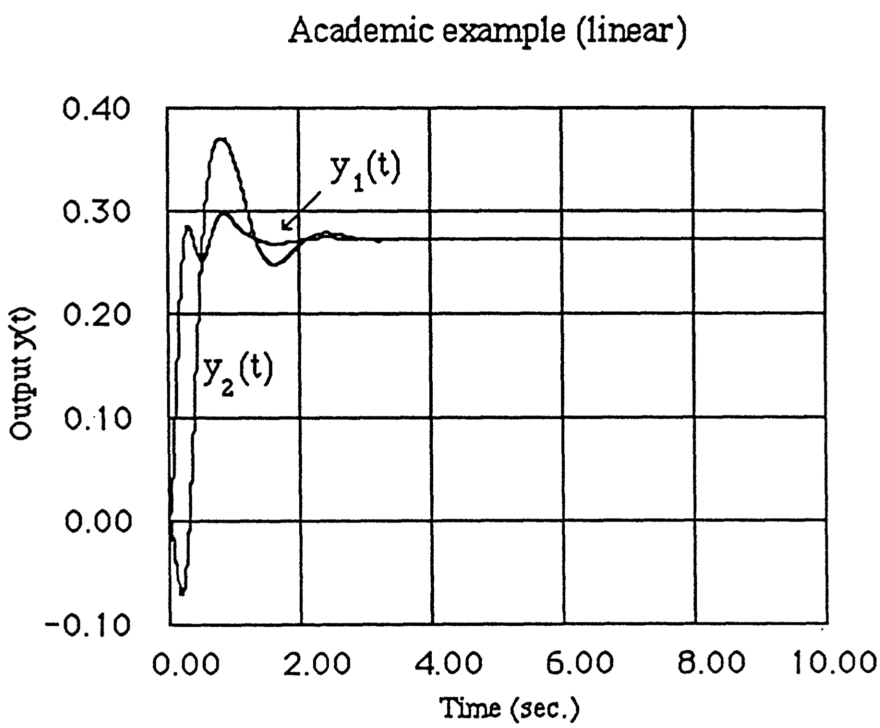

Figure 4.6: Output response for the linear system, ($\mathbf{r}$ = [.3 .3]^T).

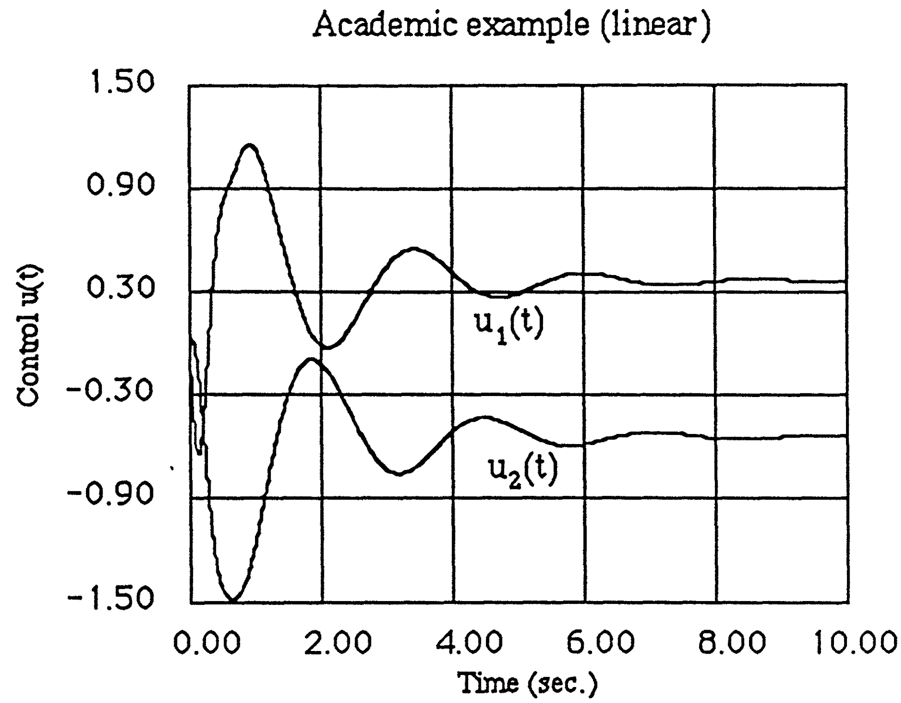

Figure 4.7: Controls in the linear system, ($\mathbf{r} = [.3 \quad .3]^T$).

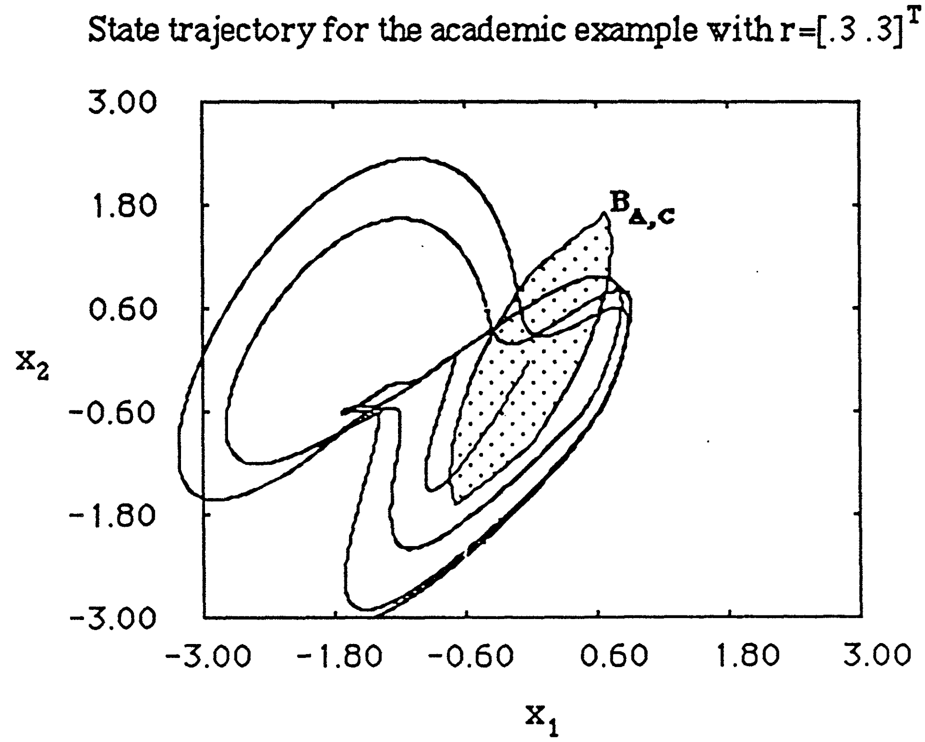

Figure 4.8: State trajectory of the compensator states in the system with saturation, ($\mathbf{r} = [.3 \quad .3]^T$).

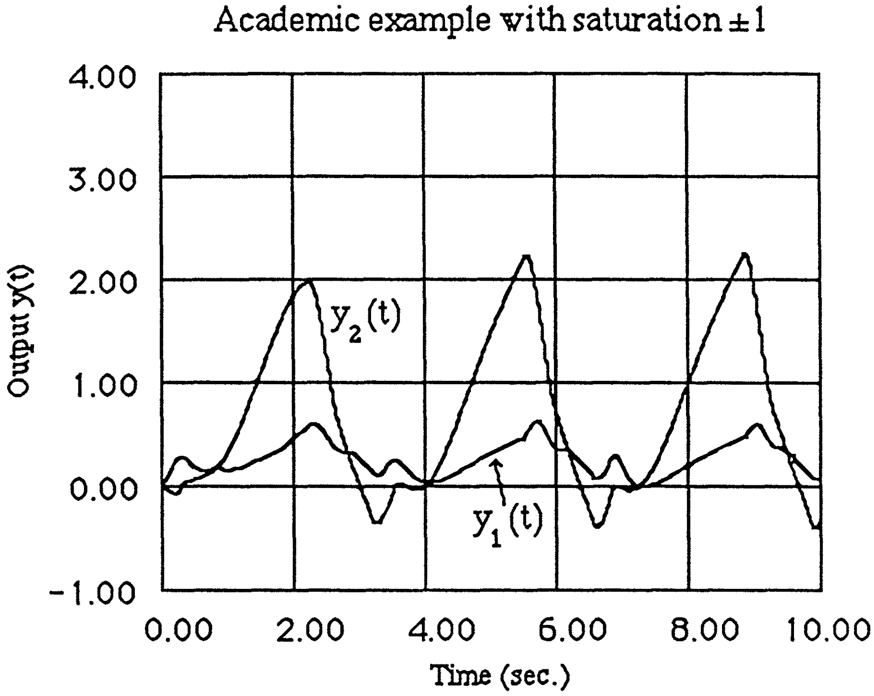

Figure 4.9: Output response for the system with saturation, ($\mathbf{r} = [.3 \quad .3]^T$).

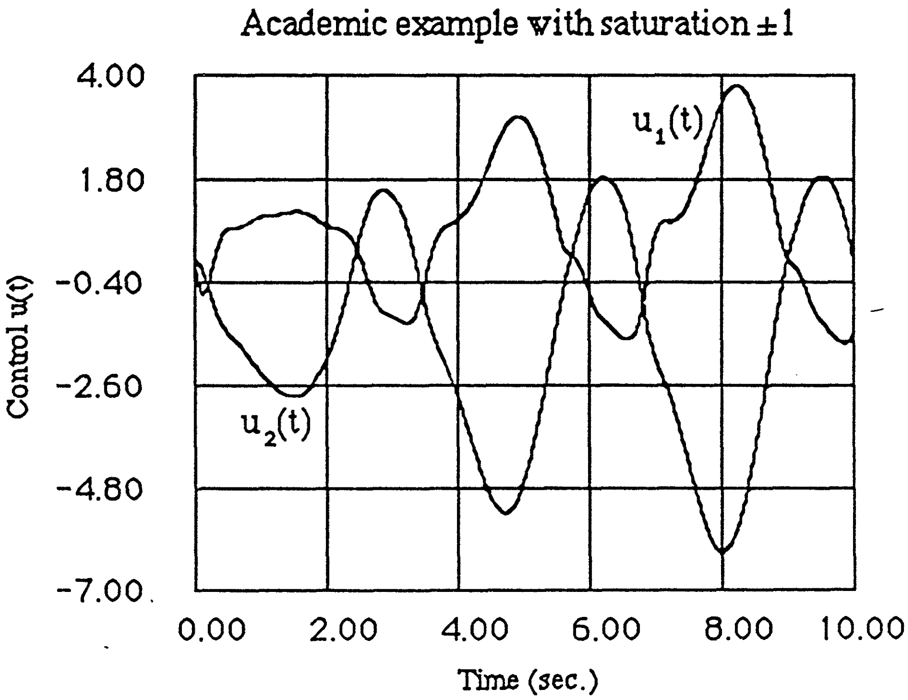

Figure 4.10: Controls in the system with saturation, ($\mathbf{r} = [.3 \quad .3]^T$).

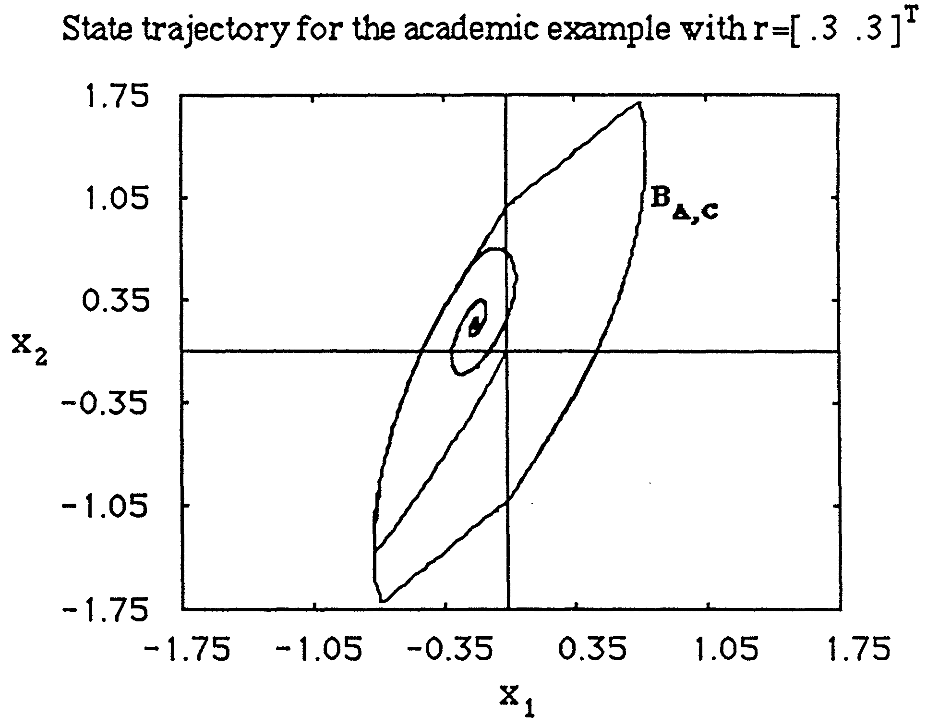

Figure 4.11: State trajectory of the compensator states in the system with saturation and the EG, ($\mathbf{r} = [.3 \quad .3]^T$).

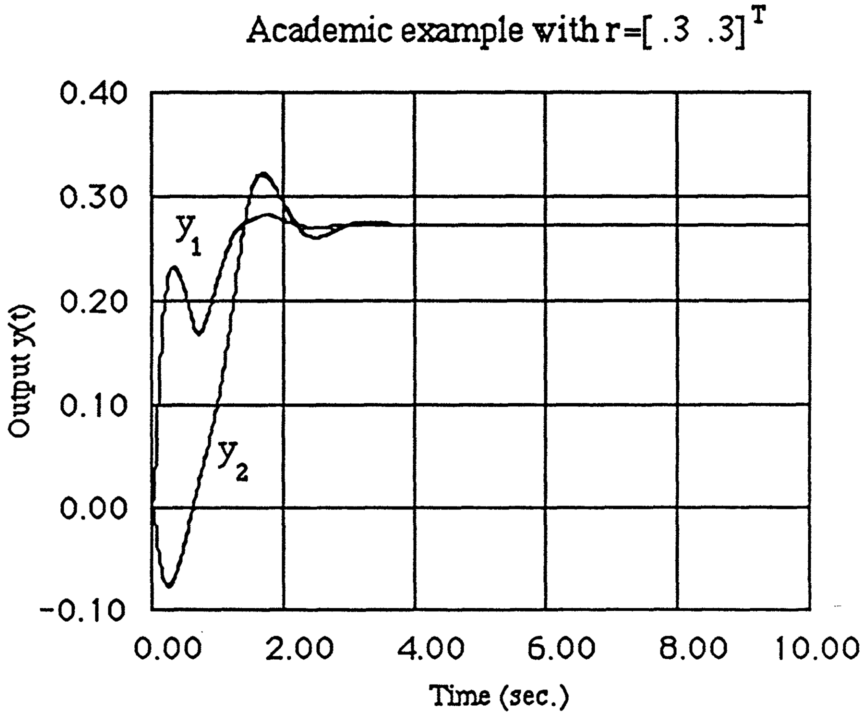

Figure 4.12: Output response for the system with saturation and the EG, ($r = [.3 \quad .3]^T$).

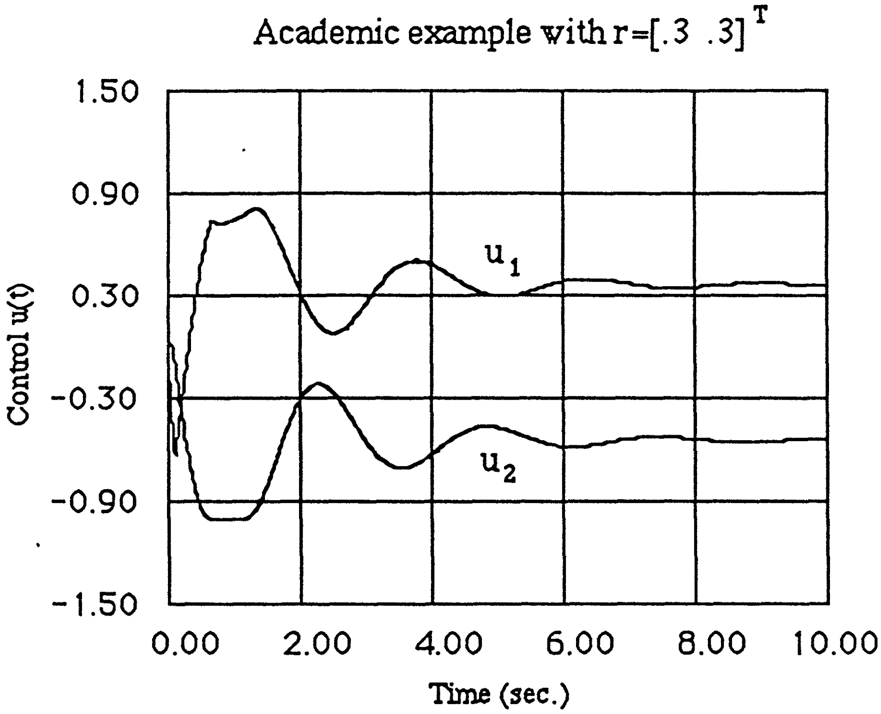

Figure 4.13: Controls in the system with saturation and the EG, ($\mathbf{r}$ = [.3 .3]^T).

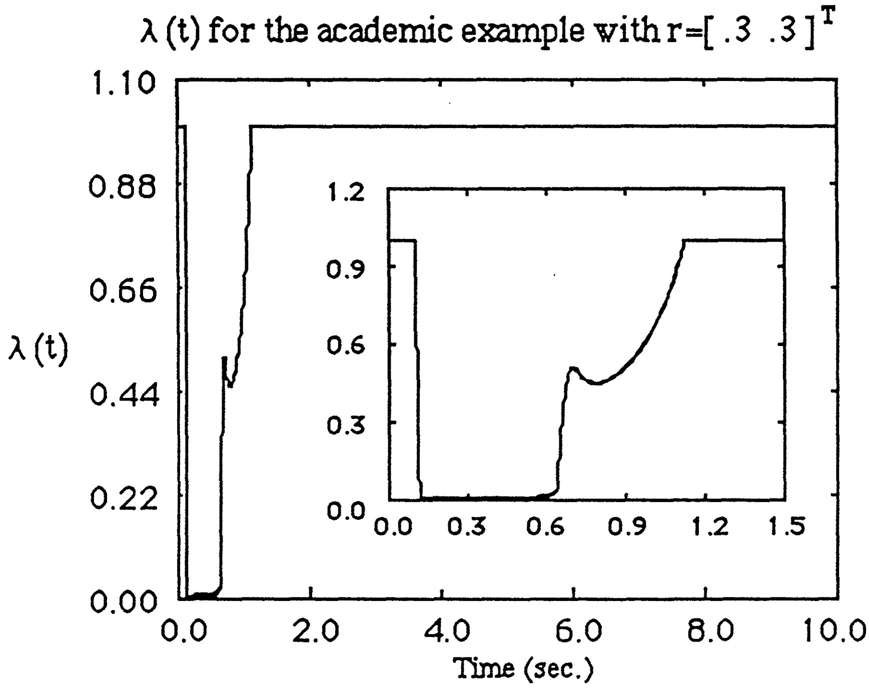

Figure 4.14: $\lambda(t)$ in the system with saturation and the EG, ($\mathbf{r}$ = [.3 .3]^T).

Insert: Blowup with $0 \leq t \leq 1.5$ sec.

4.3 Simulation of a Model of the F8 Aircraft

The purpose of this example is to illustrate the effects of multiple saturations on the directions of the controls and consequently on the response of the control system and the integrator windup phenomenon. The simulation confirms our claim that the integrators in the control system with the EG never windup, and that the saturation does not effect the direction of the controls when the EG operator is used.

Consider a model of the longitudinal dynamics of the F8 aircraft. A flaperon has been added which does not exist in the F8 prototype. The state equations are given by

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -0.8 & -.0006 & -12 & 0 \\ 0 & -.014 & -16.64 & -32.2 \\ 1 & -.0001 & -1.5 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} -19 & -3 \\ -.66 & -.5 \\ -.16 & -.5 \\ 0 & 0 \end{bmatrix} \mathbf{u}_s(t) \qquad (4.24)$$

$$\mathbf{y}(t) = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \mathbf{x}(t) \qquad (4.25)$$

$$\mathbf{u}_s(t) = \text{sat}(\mathbf{u}(t)) \qquad (4.26)$$

and in compact form

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}_s(t) \qquad (4.27)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \qquad (4.28)$$

where

$$\text{Controls } \mathbf{u}(t) = \begin{bmatrix} \delta_e(t) & \text{elevator angle (deg)} & \text{limit at } 25^{\circ} \\ \delta_f(t) & \text{flaperon angle (deg)} & \text{limit at } 25^{\circ} \end{bmatrix} \qquad (4.29)$$

$$\text{Outputs } \mathbf{y}(t) = \begin{bmatrix} \theta(t) & \text{pitch angle (rad)} \\ \gamma(t) & \text{flight path angle (rad)} \end{bmatrix} \tag{4.30}$$

$$\text{States } \mathbf{x}(t) = \begin{bmatrix} q(t) & \text{pitch rate (rad/sec)} \\ v(t) & \text{forward velocity (ft/sec)} \\ \alpha(t) & \text{angle of attack (rad)} \\ \theta(t) & \text{pitch angle (rad)} \end{bmatrix} \tag{4.31}$$

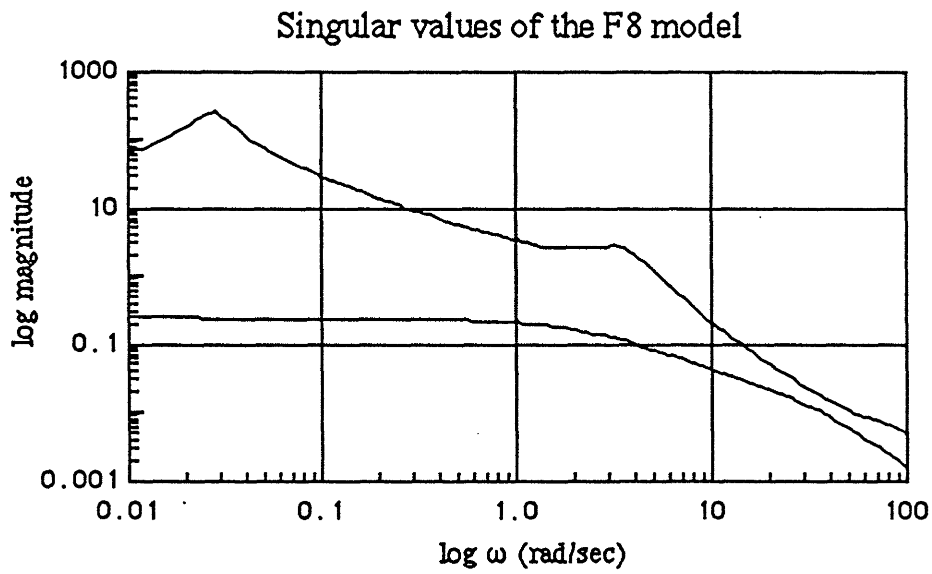

Figure 4.15: Singular values of the F8 model.

Figure 4.15 shows the singular values of the F8 linear model. Assume that a closed loop system has to be designed for the F8 model to follow pitch and flight path angle commands. Also assume that zero steady state error is required for step commands. The control system to be designed, should be thought as a semi-realistic MIMO controller so as to test the new design methodology introduced in this section.

The design process is the following. First, linear control theory will be used to design the closed loop system. Then the linear compensator will be modified with the EG operator. Finally, simulations of the closed loop system will be performed to assess the benefits of the new design methodology.

To obtain the required linear control system the saturation is ignored ($\mathbf{u_s}(t) = \mathbf{u}(t)$) and, two integrators were added at the controls. The augmented system (sixth order) is given by the following

$$\dot{\mathbf{x}}_{\mathbf{a}}(t) = \mathbf{A_a x}(t) + \mathbf{B_a u_a}(t) \tag{4.32}$$

$$\mathbf{y}(t) = \mathbf{C_a x}(t) \tag{4.33}$$

$$\mathbf{u}(t) = \frac{\mathbf{I}}{s}\,\mathbf{u_a}(t) \tag{4.34}$$

where $\mathbf{A_a} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{B} & \mathbf{A} \end{bmatrix} \quad \mathbf{B_a} = \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \quad \mathbf{C_a} = \begin{bmatrix} \mathbf{0} & \mathbf{C} \end{bmatrix}$

Next, a linear compensator was designed for the augmented system to control the pitch angle and flight path angle. The LQG/LTR methodology was used to design the compensator which is computed as follows:.

$$\mathbf{K}(s) = \mathbf{G}[\, s\mathbf{I}\text{-}\mathbf{A_a}\text{-}\mathbf{B_a G}\text{-}\mathbf{HC_a}\,]^{-1}\,\mathbf{H} \tag{4.35}$$

$$\mathbf{K_a}(s) = \frac{\mathbf{I}}{s}\,\mathbf{K}(s) \tag{4.36}$$

where

$$\mathbf{H} = \begin{bmatrix} -.844 & .819 \\ -11.54 & 13.47 \\ -.86 & .25 \\ -47.4 & 15 \\ 4.68 & -4.8 \\ 4.82 & .14 \end{bmatrix} \qquad \mathbf{G} = \begin{bmatrix} -52.23 & -3.36 & 73.1 & -.0006 & -94.3 & 1072 \\ -3.36 & -29.7 & -2.19 & -.006 & 908.9 & -921 \end{bmatrix}$$

The LQG/LTR compensator **K**(s) cancels part of the F8 dynamics. From now on we assume that the **G**(s)$\mathbf{K}_a$(s) is the desired forward loop transfer matrix, and that we would like to mimic (to the extent possible) the transient response of this linear feedback system even in the presence of saturations. Figure 4.16 shows the singular values of the resulting loop transfer function matrix **G**(s)$\mathbf{K}_a$(s).

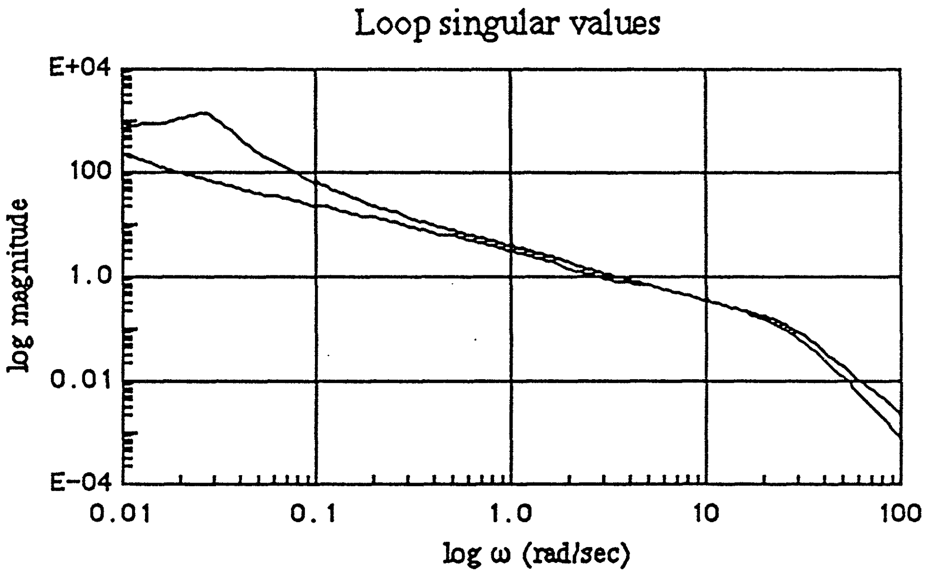

Figure 4.16: Singular values of the loop transfer function in the F8 closed loop system

To prevent control saturations, the Error Governor (the λ(t) time-varying gain) is added to the feedback system at the error signal **e**(t). The construction of λ(t) is possible because the compensator **K**(s) is neutrally stable and finite gain stability is guaranteed because in addition the plant **G**(s) is stable.

The result is a multivariable control system with integrators in the forward loop. In the presence of saturation, and without the EG operator, integrator windups would be expected and the direction of the control vector would be distorted. Three simulations were performed to show the integrator windup problem and how the problem is resolved by the operator EG.

First, the closed loop system was simulated with reference vector **r** = $[\,10 \quad 10]^T$. Figures 4.17 and 4.18 show the linear output and control responses. As expected from the singular values of $G(s)K_a(s)$, both outputs behave similarly and it is assumed that this type of an output response satisfies the posed constraints. Note that the controls have "impulsive" action at the beginning, and they violate the $\pm 25^\circ$ limit; thus saturation is expected.

Figures 4.19 and 4.20 show the outputs and controls of the system with saturation. From the oscillations in the output response it can be inferred that the integrators windup. In addition, the direction of the output is disturbed and the outputs are "not matched" any more (compare figures 4.17 and 4.19).

Figures 4.21 and 4.22 show the output and control responses of the system with saturation and the EG operator. Compare figures 4.17 and 4.21 and notice how the outputs are similar in shape (as it was desired), in addition to the fact that there are no integrator windups. The output response has of course slower rise time, since we must use smaller controls, but the nature of the response is similar to the linear one. The controls $\mathbf{u}(t)$ in figure 4.22 never exceed the limits of the saturation; and when the flaperon $\delta_f(t)$ reaches 25° the elevator $\delta_e(t)$ remains almost constant until $\delta_f(t)$ unsaturates. The direction of the controls during that period of time is such that drives the plant output towards the command $[10 \quad 10]^T$. The system behaves like having "a smart multivariable saturation".

Figure 4.23 shows the $\lambda(t)$. Note that the error is almost completely "turned-off" at about .05 seconds. The gain $\lambda(t)$ then increases slowly towards unity and the system operates linearly again.

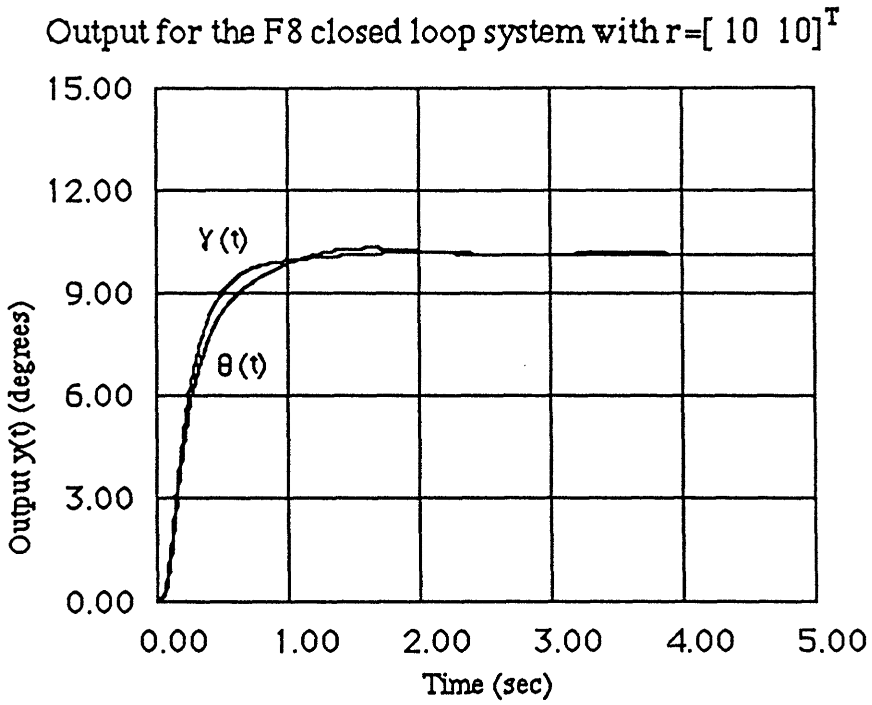

Figure 4.17: Output response for the F8 linear system, ($\mathbf{r}$ = [10 10]^T).

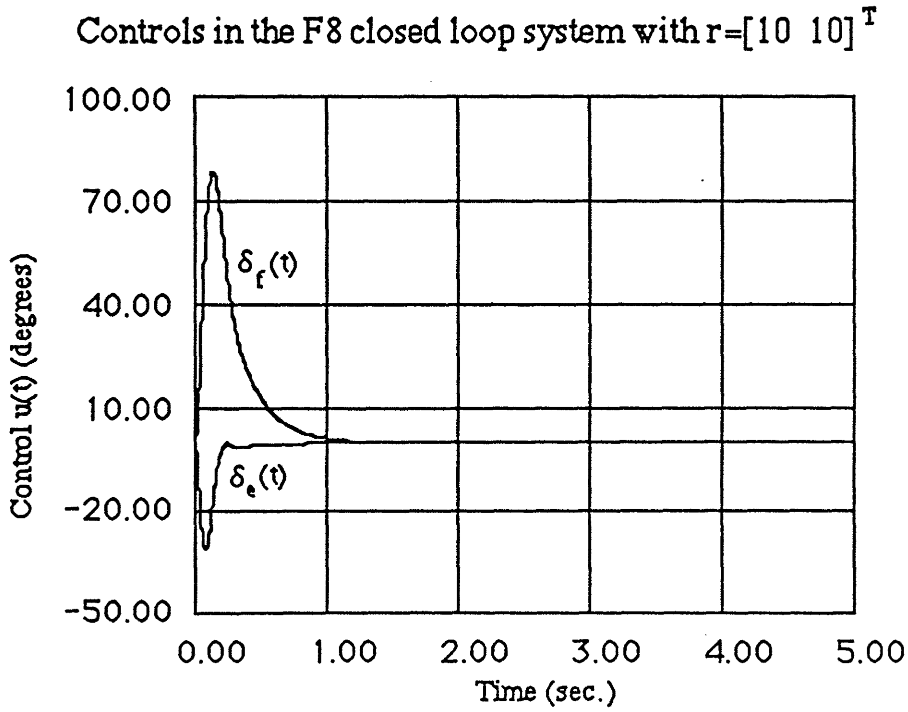

Figure 4.18: Controls in the F8 linear system, ($\mathbf{r}$ = [10 10]^T).

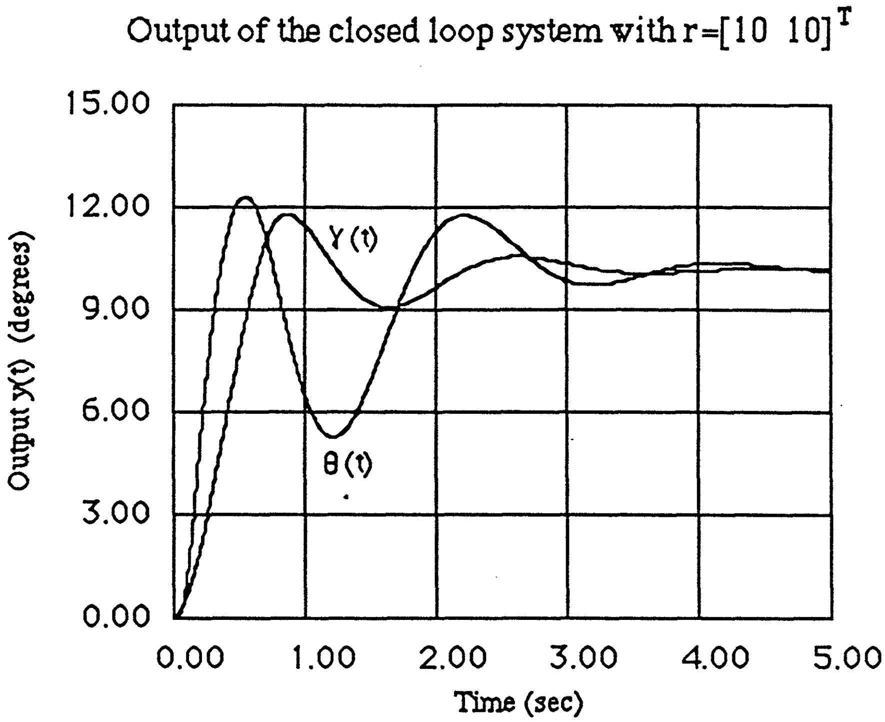

Figure 4.19: Output response for the F8 system with saturation, ($\mathbf{r}$ = [10 10]^T).

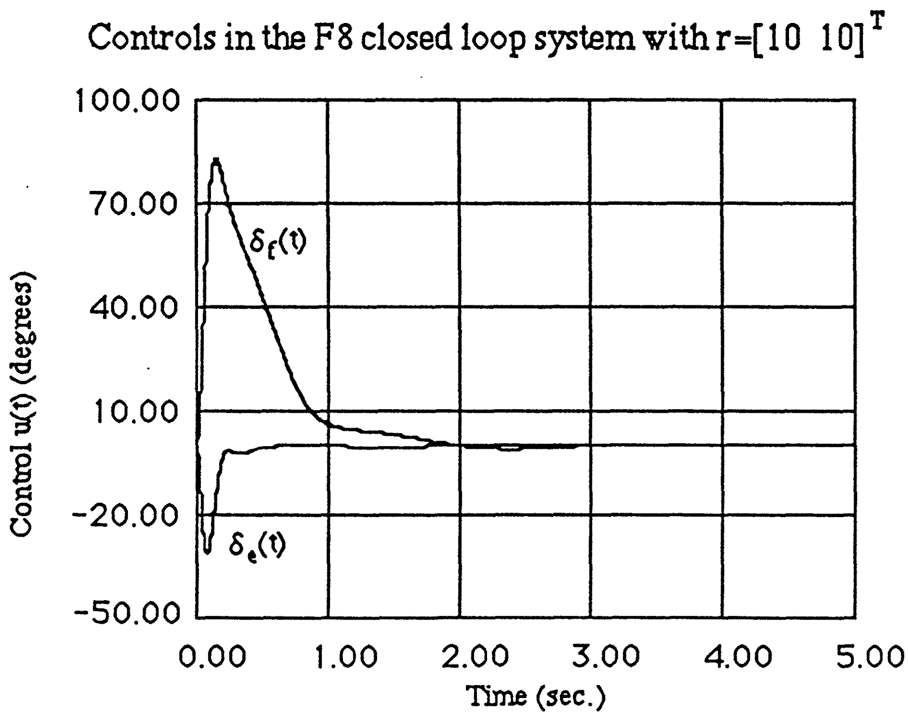

Figure 4.20: Controls in the F8 system with saturation, ($\mathbf{r}$ = [10 10]^T).

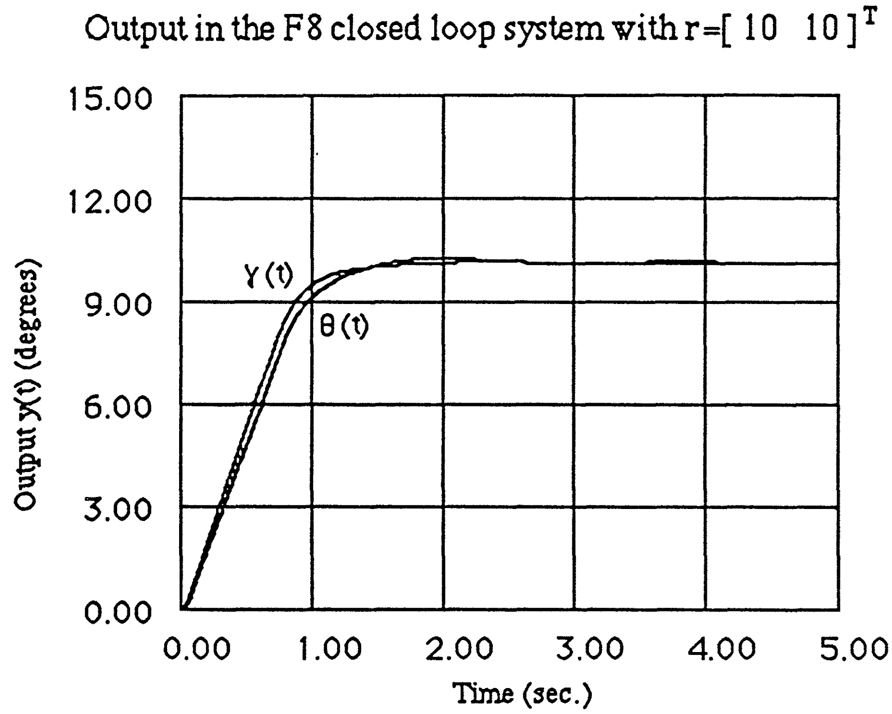

Figure 4.21: Output response of the F8 system with saturation and the EG, ($\mathbf{r}$ = [10 10]^T).

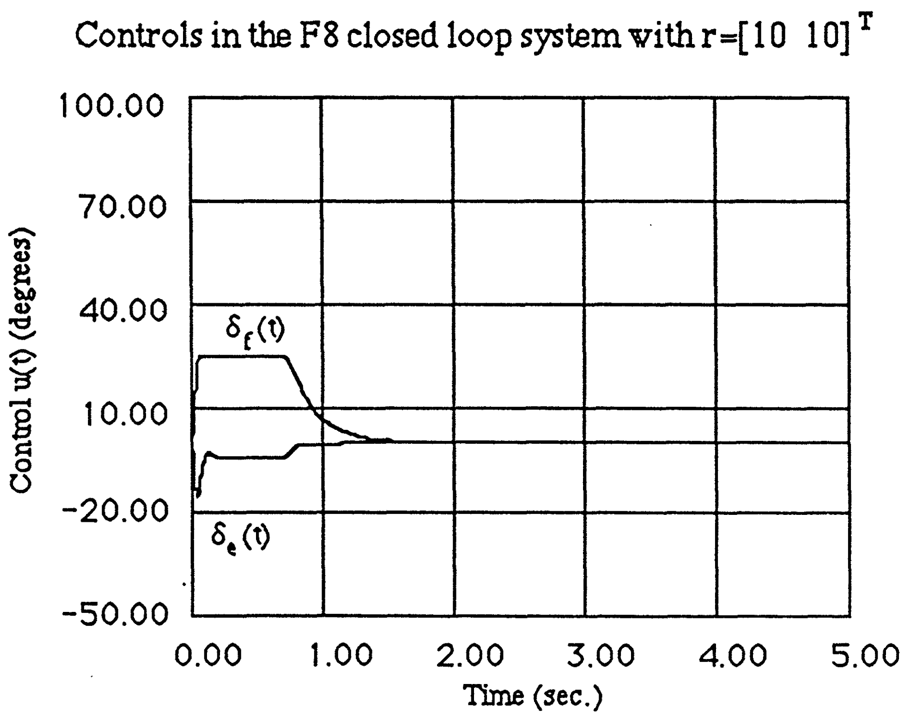

Figure 4.22: Controls in the F8 system with saturation and the EG, ($\mathbf{r}$ = [10 10]^T).

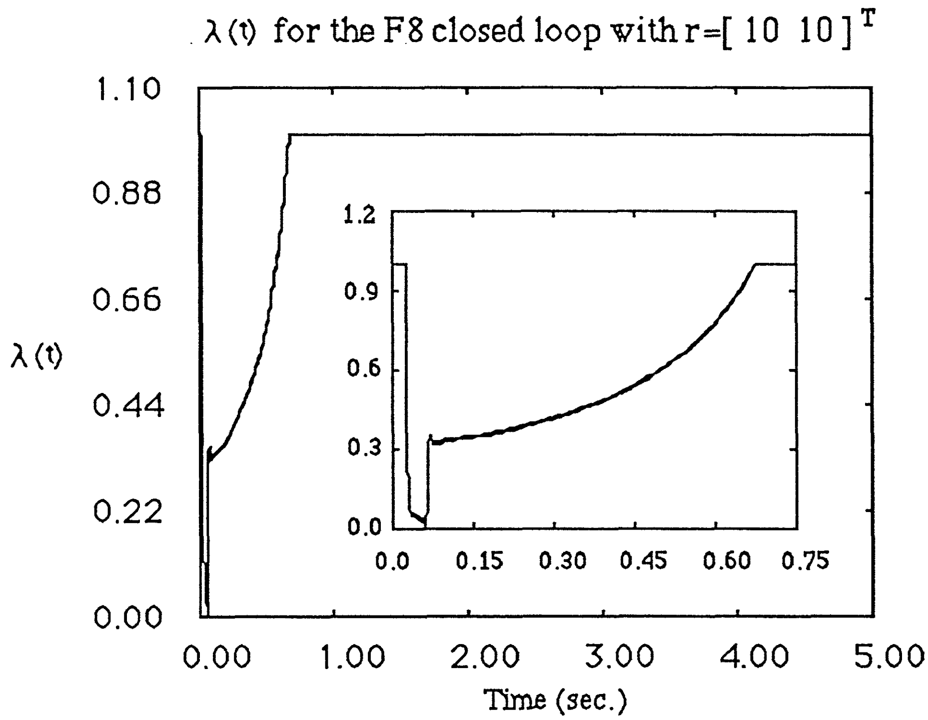

Figure 4.23: $\lambda(t)$ in the F8 system with saturation and the EG, ($\mathbf{r}$ = $[10 \quad 10]^T$).

Insert: Blowup with $0 \leq t \leq .75$ sec.

5. Conclusion

Saturations exist in almost every physical system. In this research, the effects of multiple saturations present in a closed loop control system were studied extensively. In the presence of saturations the performance of a linear control system can suffer. For example, a linear control system that is closed loop stable can become unstable when saturations are present for certain references and disturbances. Saturations can also affect the performance of the control system by introducing reset windups and by changing the direction of the control signal. Large overshoots and oscillatory outputs are the consequence.

A systematic methodology was introduced for the design of control systems with multiple saturations. The idea was to introduce a supervisor loop; and when the references and/or disturbances are "small" enough so as not to cause saturations, the system operates linearly as designed. When the signals are large enough to cause saturations, then the control law is modified in such a way to preserve, to the extent possible, the behavior of the linear control design.

The main benefits of the methodology are that it leads to controllers with the following properties:

(a) The signals that the modified compensator produces never cause saturation.

(b) Possible integrators or slow dynamics in the compensator never windup.

(c) The closed loop system has inherent stability properties.

(d) The on-line computation required to implement the control system is feasible.

These properties were demonstrated in simulations of the F8 aircraft (stable) model and an academic example.

Extensions of the methodology can be made to address the class of systems with open loop unstable plants [13]. Future publication will cover this problem in detail.

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