1988 Steele Prizes Awarded at Centennial Celebration in Providence

Three Leroy P. Steele Prizes were awarded at the Society's ninety-first Summer Meeting and Centennial Celebration in Providence, Rhode Island.

The Steele Prizes are made possible by a bequest to the Society by Mr. Steele, a graduate of Harvard College, Class of 1923, in memory of George David Birkhoff, William Fogg Osgood, and William Caspar Graustein.

Three Steele Prizes are awarded each Summer: one for expository mathematical writing, one for a research paper of fundamental and lasting importance, and one in recognition of cumulative influence extending over a career, including the education of doctoral students. The current award is $4,000 for each of these categories.

The recipients of the Steele Prizes for 1988 are SIGURDUR HELGASON for the expository award; GIAN-CARLO ROTA for research work of fundamental importance; and DEANE MONTGOMERY for the career award.

The Steele Prizes are awarded by the Council of the Society, acting through a selection committee whose members at the time of these selections were Frederick J. Almgren, Luis A. Caffarelli, Hermann Flaschka, John P. Hempel, William S. Massey (chairman), Frank A. Raymond, Neil J. A. Sloane, Louis Solomon, Richard P. Stanley and Michael E. Taylor.

The text that follows contains the Committee's citations for each award, the recipients' responses at the prize session in Providence, and a brief biographical sketch of each of the recipients. Professor Montgomery was unable to attend the Summer Meeting to receive the prize in person. He did, however, send a written response to the award.

Expository Writing

Sigurdur Helgason

Citation

The 1988 Steele Prize for expository writing is awarded to SIGURDUR HELGASON for his books *Differential Geometry and Symmetric Spaces* (Academic Press, 1962), *Differential Geometry, Lie Groups, and Symmetric Spaces* (Academic Press, 1978), and *Groups and Geometric Analysis* (Academic Press, 1984).

In 1962 Sigurdur Helgason published a book which has become a classic. The subject matter included central topics in geometry and Lie group theory, with important ramifications for harmonic analysis. More recently this material has been revised and expanded into a two volume treatment.

Proceeding at a leisurely pace, the author first leads the reader through the basic theory of differential geometry, emphasizing an invariant, coordinate-free development. Next is a careful treatment of the foundations of the theory of Lie groups, presented in a manner which since 1962 has served as a model for the treatment of this subject by a number of subsequent authors. The central theme of symmetric spaces is related in a clear fashion to the study of semisimple Lie groups and tools are assembled for the classification of these objects, first into large classes, e.g., compact and noncompact symmetric spaces, Hermitian symmetric spaces, then the fine classification. The last volume covers numerous significant topics in harmonic analysis, from the Radon transform, to invariant differential operators, to Harish-Chandra's *c*-function, ending with a quick overview of harmonic analysis on compact symmetric spaces in terms of the representation theory of compact Lie groups.

The exposition throughout is a model of clarity. Arguments in proofs are very clean, the organization is superb, and the material ranges over a wide vista of important topics of interest to a broad segment of the mathematical community.

Response

I feel deeply grateful and honored to receive the Steele Prize at this Centennial Celebration.

The first book in question, *Differential Geometry and Symmetric Spaces* from 1962, represents my efforts (originating in 1955) at combining Elie Cartan's differential geometric work on symmetric spaces with some of Harish-Chandra's algebraic and analytic work on representation theory of semisimple Lie groups. The ultimate purpose, however, was to develop geometric analysis on symmetric spaces in analogy with Fourier analysis and Radon transforms on R^n and partial differential operators with constant coefficients. My 1984 book, *Groups and Geometric Analysis*, treats the simplest examples and then deals with the first part of the general project.

As this geometric analysis on symmetric spaces has developed, some unexpected feedback in classical analysis has materialized. For example, the familiar Poisson integral formula

$$u(x) = \int_B P(x,b)F(b)db$$

for harmonic functions u in the unit disk D with boundary B becomes a special result in non-Euclidean Fourier analysis on D considered as the hyperbolic plane. This circumstance then suggested that each eigenfunction u of the Laplace-Beltrami operator L on the hyperbolic plane, (say $Lu = c(c-1)u$), should have the form

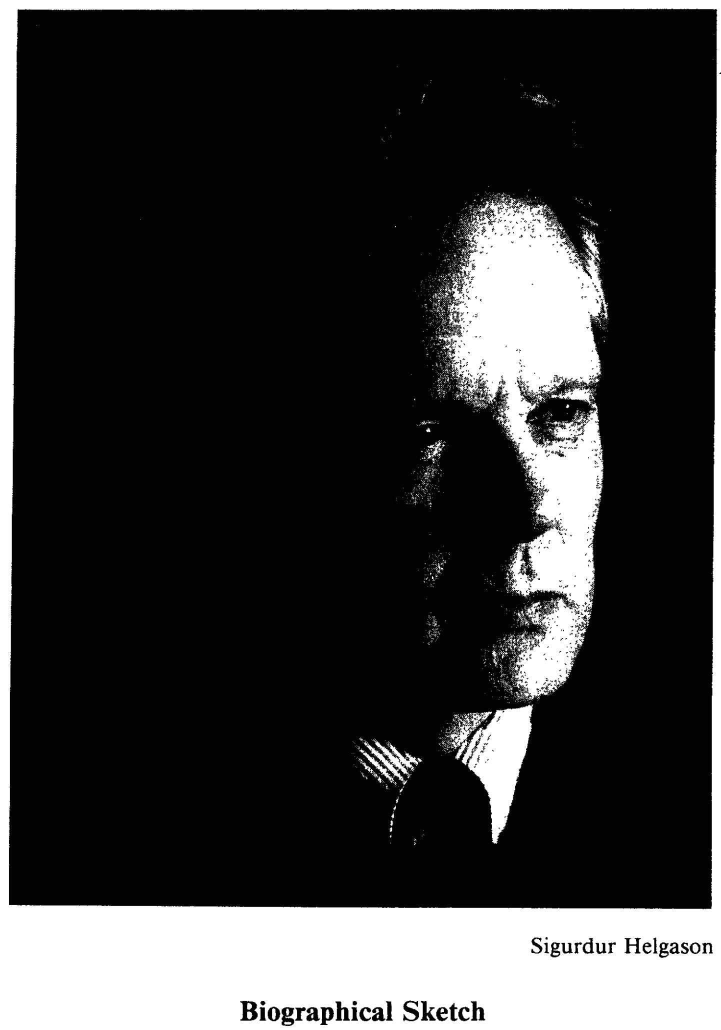
$$u(x) = \int_B P(x,b)^c dT(b)$$

with a certain functional T on the boundary B. *A priori* one would expect that the needed class of functionals T would depend on the eigenvalue $c(c-1)$, but to my surprise I found that the functionals needed were always exactly the hyperfunctions on B, independently of c. Thus hyperfunctions, which at that time (1970) had existed as rather isolated objects outside the mainstream of analysis, showed themselves to be firmly attached to basic analysis on symmetric spaces. This connection has been explored much further in the outstanding work of several Japanese mathematicians.

During the fifties when I embarked on this work, differential geometry had not acquired the great popularity which it enjoys today. Thus I felt compelled in my 1962 book to write an exposition of basic Riemannian geometry, particularly the Hadamard-Cartan's theory of manifolds of negative curvature, and Cartan's theory of symmetric spaces and semisimple Lie groups. It was an interesting experience trying to understand his work in these areas. While his thesis from 1894 was not too difficult to fathom, his papers during the late 1920's on symmetric spaces reflected his accumulated experience with Lie groups, combined with a remarkable geometric intuition; as a result some of his proofs were rather baffling in their informality. When I have taught this material on later occasions I have been embarassed by the clumsiness of some of my proofs. It seems that my exposition of these results was more intended to convince myself that the results were true rather than to explain them to others. In this pursuit I was helped by many mathematicians through personal contact, seminar activity and written papers; here I would like to mention A. Borel, S-S. Chern, J.I. Hano, Harish-Chandra, R. Hermann, A. Korányi, B. Kostant, J. L. Koszul, A. P. Mattuck, G. D. Mostow, K. Nomizu, R. Palais, J. Wolf. I remember this association with deep gratitude.

Harish-Chandra's papers offered an interesting contrast to Cartan's work. While his papers reflected deep originality and accumulated technical power, his proofs were careful in details so that motivation and patience were sufficient for understanding, at least on the local level. It was a source of great satisfaction to me to integrate some of the works of these two great mathematicians in my 1962 book.

The original project, geometric analysis on Riemannian symmetric spaces, is the subject of the 1984 volume and of a further volume in preparation. It is gratifying also to see analysis on nonRiemannian symmetric spaces progressing vigorously in several quarters in recent years.



Sigurdur Helgason

Biographical Sketch

Sigurdur Helgason was born on September 30, 1927 in Akureyri, Iceland. He received his Ph.D. from Princeton University in 1954.

During his academic career, Professor Helgason has served as Moore Instructor of Mathematics at the

Massachusetts Institute of Technology (1954-1956) and Louis Block Lecturer at the University of Chicago (1957-1959). At MIT, he moved from Assistant Professor of Mathematics to Associate Professor of Mathematics (1959-1965). He held visiting positions at Princeton University (1956-1957) and at Columbia University (1959-1960). Since 1965, he has been Professor of Mathematics at MIT. He has also been, on leave, at the Institute for Advanced Study (1964-1966, 1974-1975, and Fall 1983), and at the Institut Mittag-Leffler (1970-1971).

Professor Helgason has been a member of the American Mathematical Society for 35 years and has given the following addresses: Invited Address, Summer Meeting, Boulder, August 1963; Summer Institute on Harmonic Analysis on Homogeneous Spaces, Williamstown, July 1972; Invited Address, Annual Meeting, Washington, D.C., January 1975; Special Session on Representations of Lie groups, Washington, D.C., October 1979. He gave an Invited Address at the 1970 International Congress of Mathematicians in Nice. He also served on the Organizing Committee for the 1972 Summer Research Institute and the 1984 AMS Summer Research Conference on Integral Geometry.

Professor Helgason received the Gold Medal of the University of Copenhagen in 1951 and held a Guggenheim Fellowship at the Institute for Advanced Study in 1964-1965. He was awarded a *Doctor Honoris Causa* from the University of Iceland in 1986 and from the University of Copenhagen in 1988. He is a member of the Icelandic Academy of Sciences, the Royal Danish Academy of Sciences and Letters, and the American Academy of Arts and Sciences.

Professor Helgason's research interests include Lie groups and differential geometry, integral geometry, and harmonic analysis and differential equations on Lie groups and coset spaces.

Fundamental Paper

Gian-Carlo Rota

Citation

The 1988 Steele Prize for a paper which has proved to be of fundamental or lasting importance in its field is awarded to GIAN-CARLO ROTA for his paper:

On the foundations of combinatorial theory, I. Theory of Möbius functions.
Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gebiete, 2 (1964), pages 340-368.

Only 25 years ago the subject of combinatorics was regarded with disdain by "mainstream" mathematicians, who considered it as little more than a bag of *ad hoc* tricks. Now, however, the new subject of "algebraic combinatorics" is a highly active and universally accepted discipline. Two of its most prominent features are its unifying techniques which bring together a host of previously disparate topics, and its deep connections with other branches of mathematics, such as algebraic topology, algebraic geometry, commutative algebra, and representation theory. The single paper most responsible for bringing on this revolution is the paper of Rota cited above. It showed how the theory of Möbius functions of a partially ordered set, as developed earlier by L. Weisner, P. Hall, and others, could be used to unify and generalize a wide selection of combinatorial results. Moreover, it hinted at connections with algebra, topology, and geometry which were later to be extensively developed by Rota and his followers. Today the theory of Möbius functions occupies a central position within algebraic combinatorics and has found many applications outside combinatorics. Perhaps more importantly, Rota's paper has inspired many mathematicians to develop systematic techniques for solving combinatorial problems and to apply them to problems outside combinatorics.

Response

I feel deeply honored by the Steele Prize which the Society has voted to award me this year, and I am delighted to accept it.

The generalization of the Möbius function of number theory to locally finite partially ordered sets is an idea whose time has come. The fact that I should have been the one to first point out the timeliness of this idea is a historical accident.

I am sure that some combinatorialists of the early part of this century who leafed through Dickson's *History of the Theory of Numbers* had realized that many of the identities collected in that book relating to the number-theoretic Möbius function depended only on the divisibility partial order on the integers. Hans Rademacher once told me that he had been struck by this fact, and admitted that he had not been able to carry through a proper generalization. What he missed was an insight that came almost simultaneously to Louis Weisner and to Philip Hall in the thirties. They realized that the generalization could be carried out using functions of *two* variables on a partially ordered set, rather than using analogs of the arithmetic functions of number theory. Functions of two variables on a partially ordered set (under certain restrictions) form an algebra, which in my paper I called the *incidence algebra*. This algebra can be viewed as a generalization of the algebra of upper triangular matrices.

Applications of the Möbius inversion formula on a partially ordered set keep cropping up. We may recall T. P. Speed's theory of statistical cumulants, the generalization to all finite group actions of the Moreau-Witt formula for the number of primitive necklaces, Zaslavsky's