

A DYNAMIC SYNTHESIS OF BASIC MACROECONOMIC THEORY:
IMPLICATIONS FOR STABILIZATION POLICY ANALYSIS

by

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ABSTRACT

Most theoretical analyses of stabilization policy are based on static equilibrium models. The static models suggest that countercyclic demand-management policies stabilize the business cycle. Several studies based on dynamic models, however, have shown that conventional stabilization policies may actually be destabilizing. This study, based on a more sophisticated model, shows how the conflicting results can be reconciled.

A continuous-time, nonlinear, dynamic simulation model of the U.S. economy is developed. The study integrates four models commonly explored in basic macroeconomic theory: the multiplier-accelerator, the IS-LM, the aggregate supply-aggregate demand, and the inventory-adjustment models. Parameters are chosen from readily available sources.

The model produces two cyclic behavior modes. A 4-year business cycle is produced by the interaction of employment and inventory investment. A longer 24-year cycle is produced by the accelerator, multiplier, and capital-stock-adjustment mechanisms. The structural origins of the behavior modes are determined by a new technique for isolating dominant feedback loops using eigenvalue elasticities.

Five standard demand-management policies are examined. They are judged by four different stability criteria measuring both transient response and frequency response. The results show considerable variation with the choice of criteria. By most criteria, the policies destabilize the business cycle, confirming the results of earlier dynamic studies. At the same time, however, the policies stabilize the longer cycle, confirming the results from static analysis. Conventional results from static analysis apply to the longer cycle, which is driven by swings in final demand, while the results from dynamic analysis apply to the shorter business cycle, which is driven by inventory investment. The apparent discrepancy between static and dynamic analysis can, therefore, be resolved.

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CONTENTS

| | |
|--|-----|
| Abstract | 2 |
| Acknowledgments | 3 |
| Contents | 5 |
| Chapter I. Introduction and Literature Review | 7 |
| Chapter II. Background Theory of Dynamic Systems | 18 |
| A. Stability Criteria | 20 |
| 1. Frequency | 20 |
| 2. Damping | 28 |
| 3. Variance | 32 |
| 4. Autospectrum | 37 |
| 5. Frequency Response/Gain Matrix | 48 |
| B. Parameter Sensitivity | 53 |
| 1. Eigenvalue Elasticities | 53 |
| 2. Frequency Response Elasticities | 59 |
| C. Dominant Loop Identification | 62 |
| Chapter III. The Model | 66 |
| A. Structural Assumptions | 66 |
| B. Model Equations | 80 |
| C. Parameter Values | 104 |
| D. Initial Conditions | 120 |

| | |
|---|-----|
| Chapter IV. Model Behavior | 123 |
| A. Response to Noise | 123 |
| B. Eigenvalue Analysis | 132 |
| C. Frequency Response Analysis | 156 |
| D. Summary | 161 |
| Chapter V. Policy Analysis | 163 |
| A. Introduction | 163 |
| B. Policy Implementation and Results | 165 |
| C. Policy Timing | 209 |
| D. Policy Conclusions | 212 |
| Chapter VI. Conclusions and Extensions | 215 |
| Bibliography | 227 |
| Appendices | 235 |
| A. DYNAMO Equations for Simulation | |
| B. DYNAMO Equations for Linearization | |
| C. Variable Definitions | |
| D. Documentor Listing | |
| E. PL/I Program for Linear Analysis | |
| F. Creating PL/I Program from DYNASTAT Output | |

CHAPTER I

INTRODUCTION AND LITERATURE REVIEW

A. INTRODUCTION

For many years, the analysis of economic stabilization policy has been dominated by comparative statics models. The static models show how raising aggregate demand can restore full employment during economic downturns and how curtailing aggregate demand can cool an overheated economy. Based on the static results, countercyclic demand-management policies are commonly thought to stabilize business cycles. Over the past 25 years, however, several authors, using dynamic models, have shown that conventional demand-management policies may actually destabilize the business cycle. (See the literature review section of this chapter.) The authors have had little impact on the conventional wisdom about demand-management policies for at least two reasons:

- 1) The dynamic models they employ are extremely simplistic (usually second- or third-order difference equation models).
- 2) Their analyses provide no way to reconcile the conflicting results from static and dynamic models.

This study attempts to correct both shortcomings.

A reasonably simple (tenth-order) dynamic model of the U.S. economy is formulated from four familiar theoretical models: 1) the multiplier-accelerator model of Samuelson (1939), 2) the inventory-adjustment model of Metzler (1941), 3) the IS-LM model of Hicks (1937), and 4) the aggregate-supply/aggregate-demand model found in current

macroeconomics texts (Dornbusch and Fischer, (1978)). The four models are adapted and drawn together into a continuous-time simulation model of macroeconomic behavior. Parameters for the model are drawn from existing data, empirical studies, and previously published models.

The model generates two distinct cyclic modes of behavior. The first mode is a 4-year cycle very similar to the short-run business cycle in the U.S. economy. The business cycle is produced by the inventory- and employment-adjustment mechanisms. Variations in investment and consumption demand are not part of the endogenous mechanisms that cause the business cycle. Demand shocks can stimulate movements of inventory and employment but do not create the cyclic readjustment pattern. In fact, procyclic variations in final demand have a mild stabilizing influence on the business cycle. The second cyclic mode is a much longer, 24-year cycle. The longer cycle is produced by the multiplier, accelerator, and capital-stock-adjustment mechanisms. The long cycle, unlike the business cycle, is created by variations in consumption and investment demand. The long cycle of the model may not have a counterpart in the U.S. economy. The important point, however, is that endogenous changes in consumption and investment demand are involved in long-term dynamics and are not necessary to generate the business cycle.

The model is used to examine the impact of five conventional macroeconomic stabilization policies. All five are demand-stabilization policies aimed at raising demand when employment and output are low and lowering demand when employment and output are high. The five policies

are the:

- 1) countercyclic transfer payments to consumers,
- 2) countercyclic government spending,
- 3) graduated income tax,
- 4) countercyclic manipulation of money stock, and
- 5) countercyclic manipulation of money-growth.

All five are commonly recommended as antidotes for business-cycle swings in economic activity. The effects of all five policies on both the business cycle and the long cycle are examined. The policies are judged by several different stability criteria. The results by different criteria do not always agree, but, by the most important measures of stability, all five policies destabilize the business cycle and stabilize the long cycle.

Results of the policy tests explain the apparent discrepancy between policy conclusions based on static and dynamic models. The static results are confirmed by the fact that countercyclic demand-management policies do stabilize the demand-driven cycle. The dynamic results are confirmed by the fact that the same countercyclic policies destabilize the business cycle. The static and dynamic results can be reconciled by noting that the business cycle is caused by endogenous mechanisms that do not involve changes in final demand. Variations in demand are important to a much longer cycle created by a separate set of feedback relationships. The two cyclic modes exist simultaneously in the same model, operating through relatively independent mechanisms.

Aside from contributing to the debate over stabilization policy, the study makes two contributions to the field of system dynamics. The work examines the theoretical and practical differences between several alternative stability criteria. The criteria include measures of both transient response and stochastic response. The gain matrix is suggested as the theoretically preferred approach to measuring stability. The effects of a policy on the gain matrix cannot be inferred directly from its effects on transient response. In fact, none of the stability criteria examined consistently yields the same policy conclusions as any other.

The second, and more important, methodological contribution is a technique for identifying important feedback loops. Isolating dominant loops is usually done by trial and error. The experimental approach is time-consuming and error-prone. The new technique is a partial step toward automating the search for important loops. The method involves computing eigenvalue elasticities with respect to changes in structural links of a model. Loops containing links with large-magnitude elasticities are dominant. A more complete method is outlined in the last chapter. The improved technique yields a simple complex number associated with each loop for each eigenvalue. The number shows how important each loop is to each mode of behavior in the system. The complex coordinates separate the effects of the loop on damping and period for oscillatory modes.

B. LITERATURE REVIEW

For several decades, the field of macroeconomic theory has been dominated by static equilibrium models.¹ The IS-LM model and, more recently, the aggregate supply-aggregate demand model have formed the core of most macroeconomics teaching and research. Both models have been used to examine the effects of government stabilization policy. The standard conclusion drawn from these models is that activist management of aggregate demand by government will stabilize the economy. To "stabilize" in this context means to facilitate a rapid and smooth return to equilibrium from a disequilibrium state brought on by some unexpected shock.

Dornbusch and Fischer (1978, p. 270) sum up the conventional wisdom about stabilization policy presented in most macroeconomics texts and courses:

....high unemployment, or a large GNP gap, can be reduced by an expansion of aggregate demand. An increase in aggregate demand can in turn be achieved by expansionary monetary or fiscal policies: an increase in money supply, a reduction in taxes, an increase in government spending or an increase in transfers. Similarly, a boom can be contained by restrictive monetary or fiscal policies.

¹ This section provides a setting for the thesis in the economics literature. Only seminal works in the development and application of dynamic methods for evaluating stabilization policy are cited. The bibliography contains further references. Review articles by Cochrane and Graham (1976) and by Kendrick (1976) would prove useful to those interested in further pursuing the development of dynamic analysis.

Many arguments have been made over the relative effectiveness of monetary vs. fiscal policy and the advantages and disadvantages of automatic vs. discretionary policy, but most macroeconomics students are left with the conclusion that a mixture of these policies can stabilize the economy.

While the idea that government demand-management policies stabilize output and employment still dominates much of macroeconomic theory, doubts are beginning to surface even in basic texts. Dornbusch and Fischer (1978, p. 274) point out that:

...since policy making is difficult, it is entirely possible that the attempts of policy makers to stabilize the economy could be counterproductive.

Similar doubts surface in many other standard texts such as Darby (1979), Gordon (1981), Branson (1979), Edgmand (1979). The reservations expressed about policy effectiveness focus on the difficulty of managing demand in a timely and predictable manner. The authors point out that lags in recognition of economic conditions, policy formulation and implementation can lead to poor timing in demand management. They also point out that the delays may be variable and that the magnitude of policy influence on demand may be unpredictable. They do not, however, question the efficacy of countercyclic demand-management if demand could be properly manipulated.

By contrast, for almost thirty years a thread in the economics literature has questioned the efficacy of conventional stabilization policies even in the absence of both "inside lags" (recognition and action lags) and uncertainty about the strength of policy impact. The

analysts have not used the standard static equilibrium approach; instead, they have recast standard assumptions about economic structure into dynamic models then used them for policy assessment. Most have found that conventional stabilization policies destabilize the business cycle in their models.

Tustin (1953) and Phillips (1954, 1957) were among the first to apply the concepts of feedback control to the problem of economic dynamics and stabilization. Phillips' papers were particularly important in stimulating further work in the area. Phillips used an electronic analog simulator to determine the behavior of a fifth order, continuous-time model of the macroeconomy. His model included multiplier and inventory effects and a government spending rule. He concluded that a simple countercyclic government spending rule (one proportional to the GNP gap) could very easily reduce damping of the 3-year cycle his model produced. The importance of Phillips' papers lay in his use of a simulation approach which was capable of dealing with reasonably complex, nonlinear systems and in his use of damping as a criterion for stability. His finding that conventional stabilization policies might be destabilizing was provocative and stimulated interest in dynamic models.

During the 25 years following Phillips' second article relatively few authors followed his lead. The thrust of his analysis ran counter to the increasing acceptance of activist government stabilization policy during the 1960's. Furthermore, the analog computer was foreign to most economists. Those authors who did continue to look critically at

stabilization policy did so with much simpler models. They backed away from the challenge of building more complex models and analyzing them by simulation. Instead they chose to derive analytically the properties of simple variants on Samuelson's discrete-time multiplier-accelerator model.

William J. Baumol (1961) made the next major contribution to the line of work initiated by Phillips. Baumol investigated the properties of a simple third-order multiplier-accelerator model using isofrequency and isostability maps. He concluded that:

Plausible and reasonable contracyclical policies turn out to be capable of increasing the explosiveness and frequency of economic fluctuations. In fact, none of the possibilities examined was completely harmless in these respects, even in the highly simplified world of the multiplier-accelerator model. There would, therefore, seem to be little ground for confidence in such measures in the far more complex and unpredictable world of reality.

Baumol's analytical tools of isostability and isofrequency plots are interesting but difficult to apply in more complicated models. The surfaces become extremely difficult to calculate and visualize in hyperspace. Baumol's paper made two contributions. First, he introduced the idea that frequency of oscillation might be used to measure stability (higher frequency implies less stability). Second, his analytic tools added legitimacy to the investigation of stabilization policy in dynamic models for those hesitant to accept Phillips' simulation approach.

Smyth (1963) started a protracted exchange in Public Finance and other journals on the potentially destabilizing effects of so-called "automatic stabilizers," particularly graduated income taxes. Smyth (1963), (1966), (1974); Boyes (1975); Ip (1977); Boxer (1977); Delrome (1977); Peel (1979); and Ozmuur (1979). Like Baumol, these authors dealt with extremely simple variants on Samuelson's multiplier-accelerator model. The work is mentioned more for its volume than its significance; it added relatively little in either richness of economic structure or methodology for analysis of stability questions.

The next important step in the development of the dynamic analysis of stabilization policy was taken by Howrey (1967). He investigated policies using variance instead of damping or frequency as the measure of stability. In motivating his paper Howrey states:

[Baumol's] somewhat pessimistic conclusion was suggested by an analysis of the transient response of a deterministic linear system. The question naturally arises whether results similar to those derived by Baumol also hold for stochastic systems.

He used a simple second-order multiplier-accelerator model for which he calculated an isovariance map similar to Baumol's isostability map. He concluded that:

...it is not, in general, valid to assume that the stochastic response and the transient response of a linear system have identical properties with respect to amplitude and periodicity. In fact, it was shown that policies which increase the stability of the system in the sense that they increase the rate at which transient response damps out may actually increase the variance of the time path of income.

Howrey's contribution was to focus discussion on the stochastic properties of a model, particularly variance, as a measure of stability. He showed that a policy which increased damping could also increase variance, making its acceptance or rejection hinge on the choice of transient or stochastic stability criteria.

After Howrey's article, it became clear that variance was too broad a measure of stochastic response. In particular, spectral analysis (decomposition of variance by frequency) suggested that a policy which reduces variance over the entire frequency spectrum might increase variance in a restricted range around the frequency of oscillation targeted for control. It was necessary, therefore, to use a measure of stochastic response which emphasized a particular frequency band of interest. The simplest frequency-dependent measure of stability is a frequency-weighted average of the autospectral density function of the variable or variables to be controlled, with the highest weights lying in a band around the average frequency of the problematic cycle.

Petterson (1973, 1974) was one of the first to use a spectral approach to analyze stabilization policy. He used a variant of the multiplier-accelerator model. Later, Bowden (1977) advocated the use of "spectral utility functions" for designing "optimal" stabilization policies. Wolters (1980) used a spectral approach to measure the impact of policies in an econometric model of the Federal Republic of Germany. Baum and Howrey (1981) compared the effectiveness of monetary and fiscal policy in an econometric model of the U.S. The authors used changes in

the autospectra of output, growth in output, and inflation to judge policy results. The latter two works use large-scale forecasting models rather than theoretical models for policy analysis. The use of forecasting models takes the analysis away from the realm of macroeconomic theory, with its emphasis on structural explanation of behavior, into the realm of practical application.

To summarize, the original work by Phillips used the richest structural model but the least developed stability criterion. Most of the following works emphasized the development of more sophisticated stability criteria but in the context of simple models which could be solved analytically. This study presents a theoretical model two to three times as complex as any of the theoretical models cited above. The effect of five policies on model behavior is assessed according to four stability criteria: damping ratio, frequency, damping time, and frequency response. The frequency-response criterion is an extension of the autospectral criterion introduced by Petterson (1973, 1974).

CHAPTER II

BACKGROUND THEORY OF DYNAMIC SYSTEMS

This chapter provides a brief overview of the theory of linear dynamic systems used to analyze model behavior. The first section explains and compares alternative criteria for judging the stabilizing effects of policy changes. The second section presents the formulas used to calculate parameter sensitivity of model behavior. The third section develops a method for identifying important feedback structures which control model behavior.

General methods for analyzing relative stability and sensitivity in nonlinear systems have not yet been developed. By simulation, it is possible to determine the behavior of an arbitrary nonlinear system for a given set of parameters, initial conditions, and disturbances. In principle, small changes in initial conditions or disturbances could qualitatively alter behavioral properties of a model. For linear systems, on the other hand, general measures of stability and sensitivity have been developed which are independent of both initial conditions and the magnitude of disturbances. Because the methods used to analyze linear systems are so general and powerful, most of the behavioral analyses presented here are performed on a linearized version of the underlying nonlinear simulation model.

Both in theory and in practice, linearization appears to have little impact on model behavior. Any linearized model is an excellent

approximation to its nonlinear parent in the immediate neighborhood of the point around which the linearization is performed. The further variables stray from the linearization point, the poorer becomes the linear approximation. The model used in this study does not stray far from the original linearization point for two reasons. First, the model describes disequilibrium adjustment processes around a stationary equilibrium operating point.² The system does not move to new operating regions where nonlinearities could come into play. Second, stabilization policy is aimed at insulating the economy from shocks and disturbances which are usually too small to push the economy into new operating regions where new dynamic patterns might emerge. At a practical level, simulations were performed to check key results. None of the checks revealed any significant distortions introduced by linearization.

Linearization can be a tedious and error-prone process, but a new computer package called DYNASTAT³ can automatically linearize a nonlinear simulation model. Linearization permits the nonlinear simulation model to be simplified to the form:

$$\dot{\underline{x}} = \underline{Ax} + \underline{Bu} \quad (2.1)$$

$$\underline{y} = \underline{Cx} + \underline{Du} \quad (2.2)$$

² The model deals with business-cycle behavior and does not include long-term growth mechanisms. The movement of model variables can be thought of as changes relative to an underlying trend.

³ See bibliographic reference for DYNASTAT.

where

- $\dot{\underline{x}}$ - (n x 1) vector of state variable derivatives
- \underline{x} - (n x 1) vector of state variables
- \underline{u} - (k x 1) vector of disturbances
- \underline{y} - (m x 1) vector of output variables
- \underline{A} - (n x n) system matrix of real constants

$$\text{where } \underline{A}_{i,j} = \frac{\partial \dot{\underline{x}}_i}{\partial \underline{x}_j}$$

\underline{B} - (n x k) matrix of real constants where $\underline{B}_{i,j} = \frac{\partial \dot{\underline{x}}_i}{\partial \underline{u}_j}$

\underline{C} - (m x n) matrix of real constants where $\underline{C}_{i,j} = \frac{\partial \underline{y}_i}{\partial \underline{x}_j}$

\underline{D} - (m x k) matrix of real constants where $\underline{D}_{i,j} = \frac{\partial \underline{y}_i}{\partial \underline{u}_j}$

DYNASTAT symbolically differentiates the model equations and writes a program for computing the partial derivative matrices \underline{A} , \underline{B} , \underline{C} and \underline{D} for any given set of parameter values. The matrices \underline{A} , \underline{B} , \underline{C} and \underline{D} contain all the information needed to determine the behavioral properties of the linearized model.

A. STABILITY CRITERIA

Frequency Criterion: If a policy change decreases the frequency of oscillation in a system, it is a stabilizer.

Baumol (1961) proposed changes in frequency as a measure of effectiveness for stabilization policy. In a simple model with one dominant cyclic mode, change in frequency can be determined directly from a simulation run of model behavior. Figure 2-1 is a hypothetical simulation run which compares the behavior of an arbitrary output variable, Y , in response to a single disturbance both with and without the policy change being tested. In each case the frequency of oscillation can be calculated as the inverse of the period of oscillation. In Case A, without the policy, the period is shorter and the frequency is higher, than in Case B, with the policy. By the frequency criterion, the policy would be judged as stabilizing.

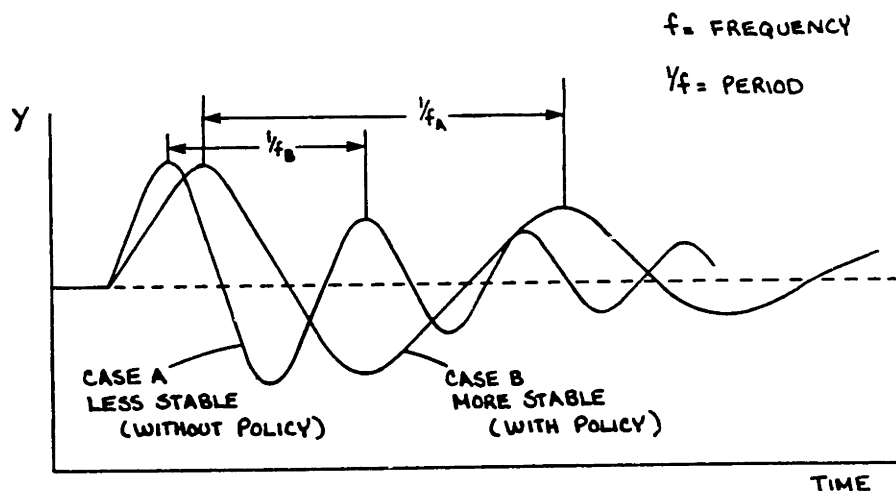


Figure 2-1. Policy Impact by Frequency Criterion

In a complex model with more than one major behavior mode, frequency changes can be difficult to determine from simulation runs. The superposition of several modes distorts individual wave forms to the point where modes are difficult to separate from each other. In such

cases the easiest way to determine changes in frequency is to calculate the eigenvalues of the system matrix, \underline{A} , in equation (2.1).

Eigenvalues describe the modes of behavior inherent in a linear model. Eigenvalues are roots of the characteristic equation of a model or, equivalently, they are values of λ for which the determinant

$$|\lambda \underline{I} - \underline{A}| = 0. \quad (2.3)$$

where λ - scalar eigenvalue

\underline{I} - (n x n) identity matrix

\underline{A} - (n x n) system matrix (from equation 2.1).

If the elements of the system matrix \underline{A} are expressed as functions of the model parameters, the eigenvalues can be very difficult to calculate, especially if the dimension of \underline{A} (the order of the system) is large. On the other hand, if the coefficients in the matrix \underline{A} are numbers which correspond to a particular set of parameter values, then numerous computer routines are available to calculate the numerical values of the system eigenvalues. The computer packages can determine the eigenvalues of very large and complex models at relatively low cost.

The total behavior of a linear model is determined by the superposition (adding up) of simple oscillations and simple exponential growth or decay terms. Each oscillation is described by a complex conjugate pair of eigenvalues. Each exponential term is determined by a single real eigenvalue. Eigenvalues can be plotted on the complex plane as shown in Figure 2-2. The real part of each eigenvalue is measured

along the horizontal axis and the imaginary part is measured along the vertical axis. The figure shows six eigenvalues, each represented as a complex vector. The two real eigenvalues (A and B) correspond to simple exponentials while the two complex conjugate pairs (C and D) correspond to cyclic modes. The eigenvalues with positive real parts (A,C) produce explosive behavior which diverges faster and faster away from equilibrium as time passes. The eigenvalues with negative real parts (B,D) generate behavior which converges to equilibrium.

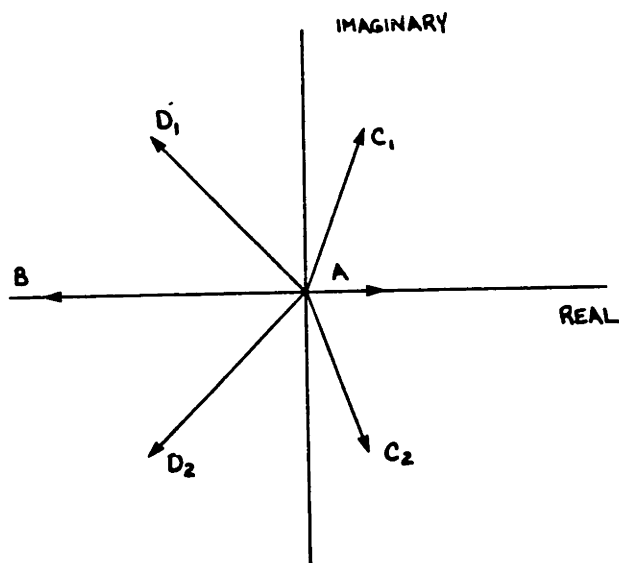
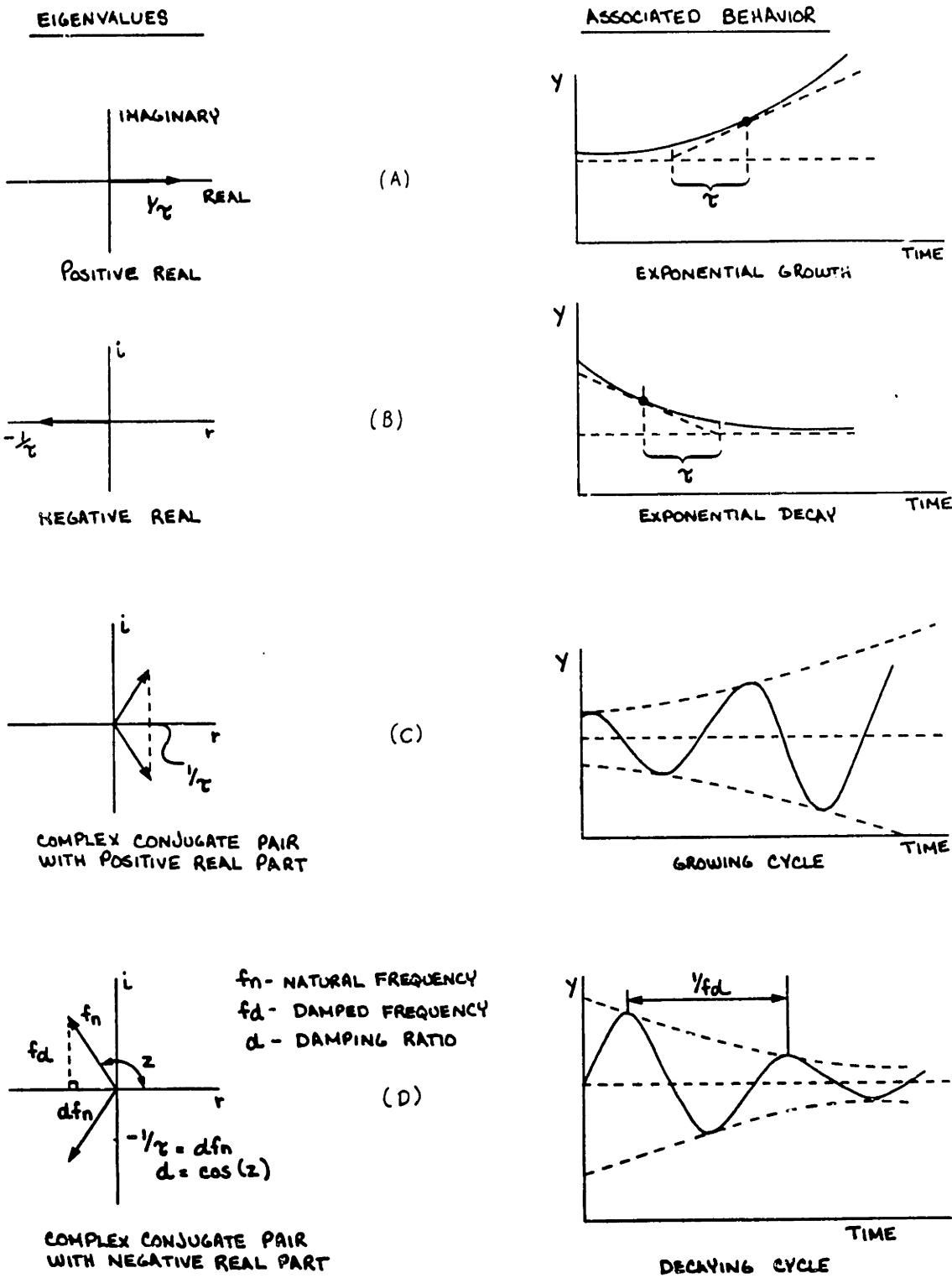


Figure 2-2. Eigenvalues as Complex Vectors

Figure 2-3 illustrates the behavior associated with each of the typical eigenvalues in Figure 2-2. The positive real eigenvalue in part (a) of Figure 2-3 produces exponential growth away from equilibrium. The time constant of growth is equal to the inverse of the eigenvalue. The negative real eigenvalue in part (b) of Figure 2-3 yields exponential decay toward equilibrium. The time constant of exponential decay is

equal to the negative inverse of the eigenvalue. In part (c) of Figure 2-3, the complex conjugate pair of eigenvalues with a positive real part generates explosive cycles. The expanding cycle can be thought of as a pure sinusoid with a frequency equal to the magnitude (length) of the eigenvalues which is then stretched to fit within an envelope of two exponential growth curves diverging from equilibrium. Both envelope growth curves have time constants equal to the inverse of the real part of the complex conjugate pair. Finally, in part (d) of Figure 2-3, the complex conjugate pair of eigenvalues with a negative real part produces cycles which decay to equilibrium. The contracting cycle can be thought of as a pure sinusoid with a frequency equal to the magnitude of the eigenvalue which is then squeezed to fit within an envelope of two exponential decay curves which converge on equilibrium. The exponential decay time of the envelope curves is equal to the negative inverse of the real part of the complex conjugate pair. The frequency of the pure sinusoid is called the natural frequency, f_n . The observed frequency in the decaying cycle, called the damped frequency, f_d , is equal to the absolute value of the imaginary parts of the eigenvalues. The damping ratio, d , is a measure of the attenuation in amplitude from one peak to the next in a decaying cycle. Damping ratio is equal to the cosine of eigenvalue pair near the negative real axis and approaches zero for a pair near the imaginary axis.

The overall behavior of any variable in a linear system can be calculated as a linear combination (weighted sum) of the modes described

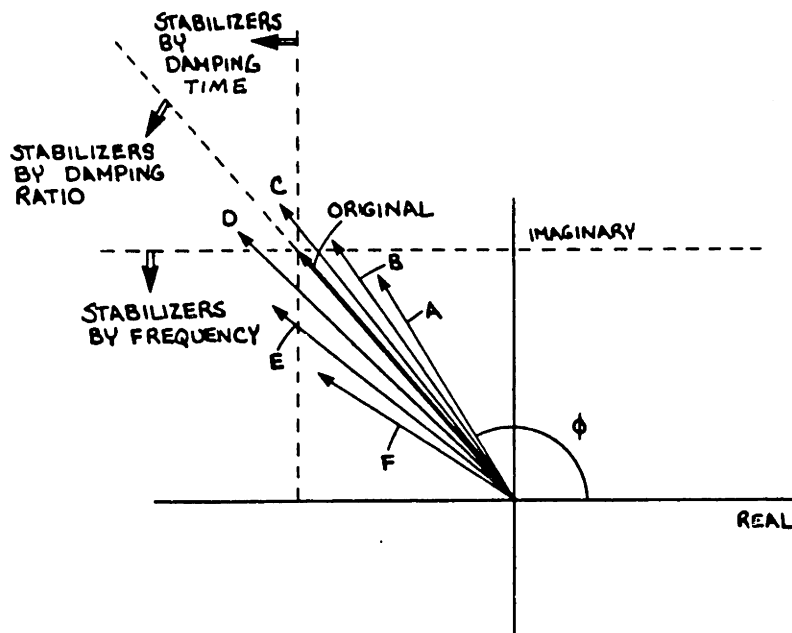


f_n - NATURAL FREQUENCY
 f_d - DAMPED FREQUENCY
 d - DAMPING RATIO

$-1/\tau = d f_n$
 $d = \cos(z)$

Figure 2-3. Eigenvalues with Associated Behavior Over Time

by system eigenvalues. If all eigenvalues have negative real parts, then all dynamics will eventually die away and the system will remain in equilibrium. If any of the eigenvalues has a positive real part, then the eigenvalue(s) with the largest real part will eventually dominate system behavior.



| POLICY | FREQUENCY | DAMPING TIME | DAMPING RATIO |
|--------|-----------|--------------|---------------|
| A | S | D | D |
| B | D | D | D |
| C | D | S | D |
| D | D | S | S |
| E | S | S | S |
| F | S | D | S |

S = STABILIZER
D = DESTABILIZER

Figure 2-4. Effect of Six Policies on an Eigenvalue

In large models, where visual separation of modes in simulation output is difficult, eigenvalues can be used to assess the impact of a policy on frequency. Policies are usually aimed at controlling a particular cyclic tendency that corresponds to a complex conjugate pair of eigenvalues with a negative real part (as in Figure 2-3d). By calculating the eigenvalues of a model before and after a policy intervention, the impact of a policy on the frequency of the target cycle can be determined. The observed (damped) frequency of an oscillatory mode can be measured by the imaginary part of the corresponding complex conjugate pair of eigenvalues. If the imaginary parts of the relevant complex conjugate pair become smaller, then the damped frequency of oscillation is less and the policy is stabilizing. If the imaginary parts are greater, then damped frequency rises, and the policy is destabilizing by the frequency criterion.

Figure 2-4 illustrates how six hypothetical policies (A-F) might shift an eigenvalue of a cyclic mode. The policies can be sorted into stabilizers and destabilizers by the frequency criterion. The frequency criterion says that any policy that decreases the observed (damped) frequency of a cyclic mode is a stabilizer. Since the imaginary part of an eigenvalue is the damped frequency associated with it, any policy that moves the eigenvalue below the horizontal dashed line in Figure 2-4 is a stabilizer by the frequency criterion. Policies A, E and F are therefore classified as stabilizers, while B, C, and D are destabilizers.

As a measure of stability, the frequency criterion suffers two severe problems. First, it is not at all clear why longer, less frequent cycles are preferable to shorter, more frequent cycles. In fact, the contrary argument could be made: more frequent short cycles reduce the chances of any individual's experiencing long, stressful periods of unemployment. Second, a policy that reduces frequency can simultaneously reduce the tendency for cycles to die away. The frequency of a cycle is measured by the imaginary part of a complex conjugate pair of eigenvalues, while the speed of convergence toward equilibrium is measured by the real part. A policy that reduces the imaginary part can simultaneously reduce the real part as with policy A. If reducing frequency means slower convergence, frequency may not be a good measure of "stability". Instead, a measure of convergence such as "damping" might be a better stability criterion.

Damping Criterion: If a policy increases the rate decay of oscillation in a system, it is a stabilizer.

Both Phillips (1954-1957) and Baumol (1951) used a damping criterion to measure stabilization policy effectiveness. Damping measures the speed of convergence of an oscillation to equilibrium. In a simple model with a single major cyclic mode, the effects of a policy on damping can be determined from a simulation plot. Figure 2-5 shows a hypothetical comparison of the behavior of a variable before and after a policy intervention. In both Case A without the policy and Case B with the policy the damped frequency of oscillation happens to be the same.

However, the degree of "damping" differs between the cases. Damping can be measured by taking the ratio of the height of one cyclic peak to the next. The higher the ratio, the more attenuation occurs between successive peaks. In Figure 2-5 the time path shows more damping in Case B, so the policy is judged a stabilizer by the damping criterion.

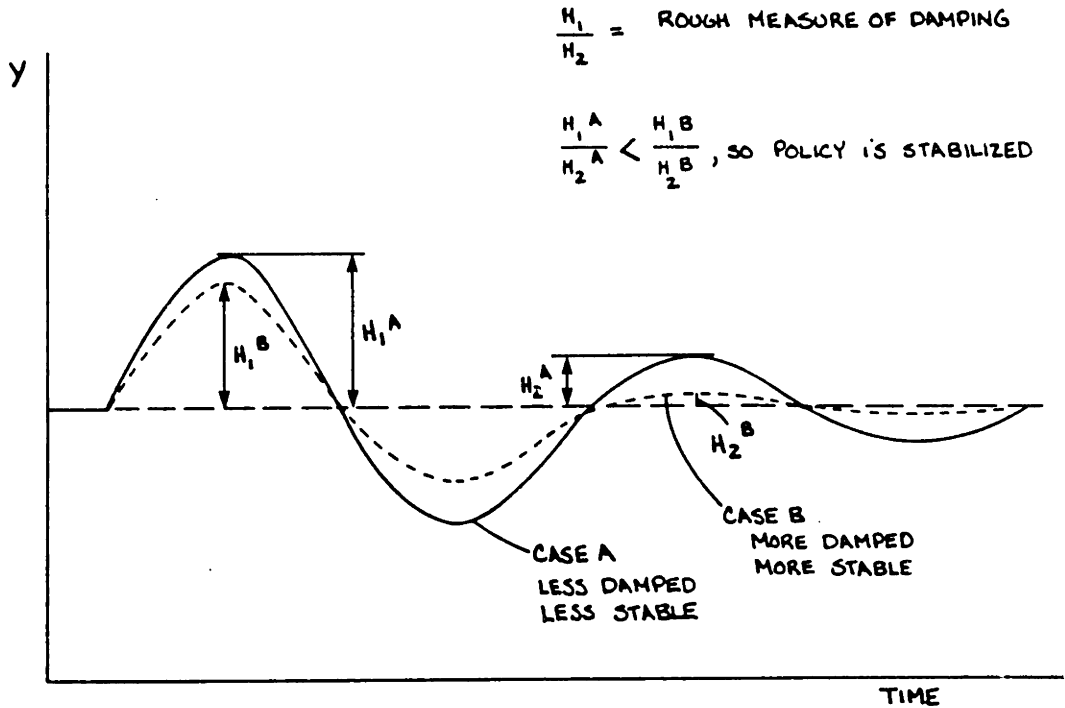


Figure 2-5. Policy Impact by Damping Criterion

In complex models exhibiting multiple modes of behavior, damping can be very difficult to determine visually from a simulation plot. In such cases, damping can be determined quickly and easily by calculating the eigenvalues of the system. Referring back to Figure 2-3d, recall that a damped cyclic mode corresponds to a complex conjugate pair of eigenvalues with negative real parts. Damping can be measured in either of two ways using eigenvalues.

By the first method, damping is defined as the speed of convergence of the envelope exponential curves that contain the oscillation. Convergence of the envelope is directly related to the real part of the complex conjugate pair of eigenvalues. A smaller real part (in absolute value) means slower convergence and less damping, while a larger real part means faster convergence and more damping by the damping-time criterion.

By a second, more common method, damping is the ratio of the speed of convergence to the natural frequency. The damping ratio, so defined, is closely related to the relative height of successive cyclic peaks, which was the measure of damping used for the simulation plot in Figure 2-5. Figure 2-6 shows the time paths corresponding to different eigenvalues with the same natural frequency but with different damping ratios. As shown in Figure 2-3d, the damping ratio of an eigenvalue is

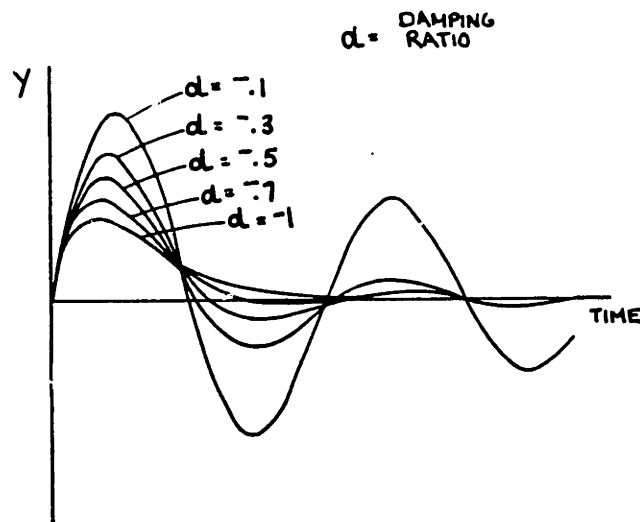


Figure 2-6. Behavior for Different Damping Ratios

calculated as the cosine of the angle in the complex plane from the positive real axis to the eigenvalue (or, equivalently, as the ratio of the real part over the magnitude of the eigenvalue). By the damping ratio-criterion, a greater angle means more damping, while a smaller angle means less damping.

Referring back to Figure 2-4, we can re-sort the six policies into stabilizers by each of the two alternative damping criteria. By the damping-time criterion any policy that increases the real part of the eigenvalue (in absolute value) is a stabilizer. Therefore, if the new eigenvalue lies to the left of the vertical dashed line, it is a stabilizer. By the damping-time criterion, policies A, B, and F are destabilizers, while C, D and E are stabilizers. The results are different using the damping-ratio criterion. By the damping-ratio criterion, any policy that increases the angle z of the eigenvalue is a stabilizer. Therefore, if a policy rotates the eigenvalue toward the negative real axis, it is a stabilizer. By the damping-ratio criterion, policies D, E, and F are stabilizers, while A, B and C are destabilizers.

The primary difficulty with both the frequency and damping criteria is that they measure only the transient response of the system to a single shock. They do not indicate how the system will respond to a stream of shocks from different sources, with varying magnitudes and with erratic timing. The real economy is constantly bombarded by noise and uncertainty, so stabilization policy should not be aimed at controlling transient response but instead at controlling stochastic response. There

is no guarantee that transient response is a good guide to stochastic response. In fact, as will be seen in Chapter 5, stochastic criteria can lead to different conclusions about policy effects.

Variance Criterion: If a policy change reduces the variance of a target variable (or the weighted average of variances of several variables), it is a stabilizer.

Howrey (1967) suggested using variance as a stability criterion. Variance is one of the simplest measures of the stochastic response of a system. The variance of a particular variable is defined as the average of its squared deviation from its mean value. In practical terms, the variance can be determined from a simulation run of model response when all noise and disturbance sources are active. A typical noise response is shown in Figure 2-7. The variance of the plotted variable during the

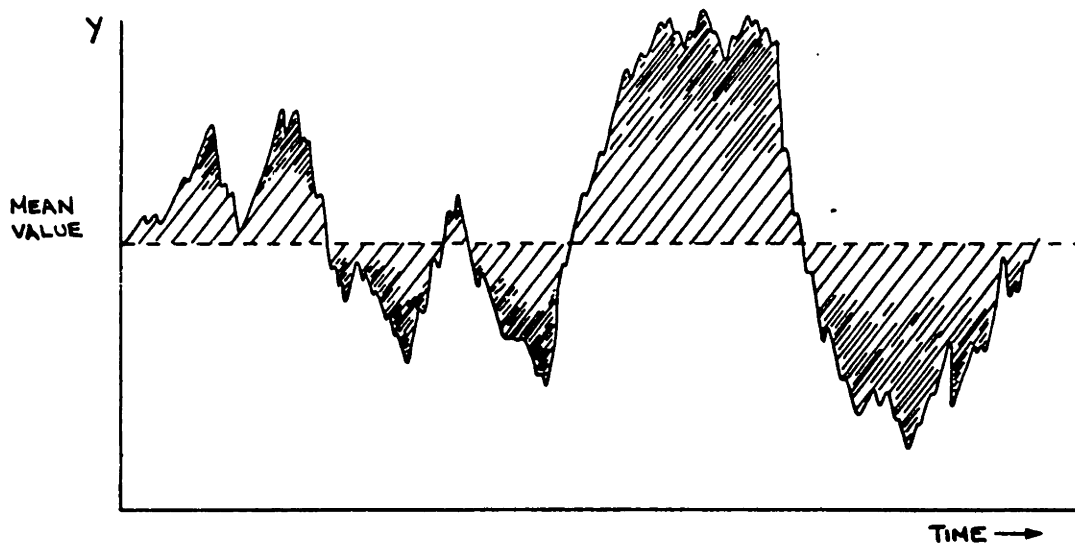


Figure 2-7. Variance as a Weighted Area

period shown is equal to the weighted area between the curve and its mean divided by the length of the period. Each unit of area is weighted by its distance from the mean. In order to obtain a good measure of the "true" (asymptotic) variance of a variable, the simulation run must be very long. If not, then the particular pattern of noise and disturbances that prevailed during the run might not be representative of the underlying stochastic processes, and the calculated variance might be misleading.

Figure 2-8 shows how the variance criterion might be used to judge a policy by changes in the variance of a single variable. Curve A shows the behavior of a variable without the policy, while Curve B shows its behavior with the policy in effect. Curve B clearly deviates less from the mean over the length of the plot, indicating a reduction in variance. The policy would be judged a stabilizer by the variance criterion.

Variance, unlike frequency or damping, is not a measure of behavior of the whole system or all variables. Instead, variance is a measure of the stochastic response of a particular variable to noise and disturbances occurring in the system. The variances of different variables are different. Therefore, in order to judge policies by the variance criterion, one must either choose a particular target variable whose variance is to be controlled or construct an index of several variables weighted according to the relative importance of their respective variances. The value of the index is then computed both with

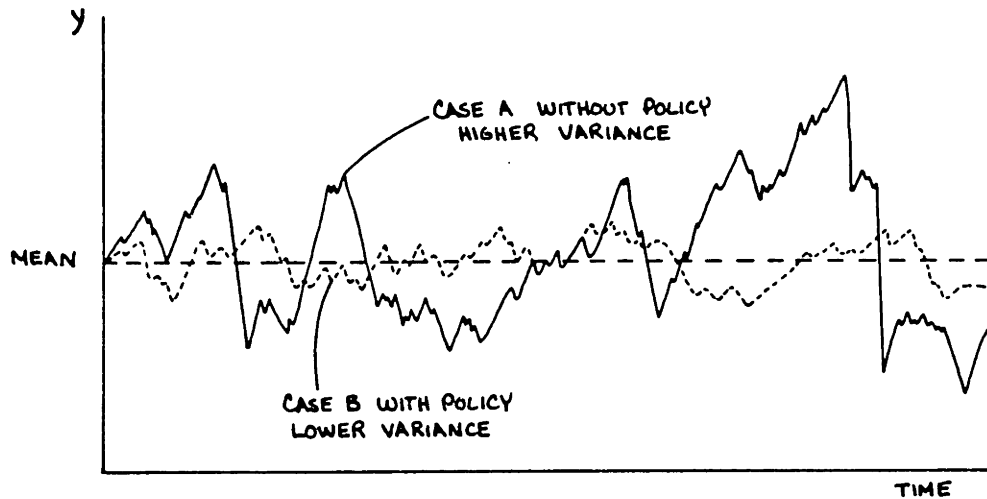


Figure 2-8. Policy Impact by Variance Criterion

and without the policy in effect. If the policy lowers the index value, it is a stabilizing influence. For simplicity, most studies using a variance criterion look at the change in variance of a single variable, usually output.

Variance is a good measure of the tendency for a variable to stray from equilibrium in the face of random disturbances to a system. The variance criterion, however, can lead to a poor choice between policies, because it lumps together the stochastic behavior characteristics at all frequencies. Total variance can be broken down into contributions by frequency.

Such a frequency decomposition of variance is called autospectrum or autospectral density function of a variable. A typical auto

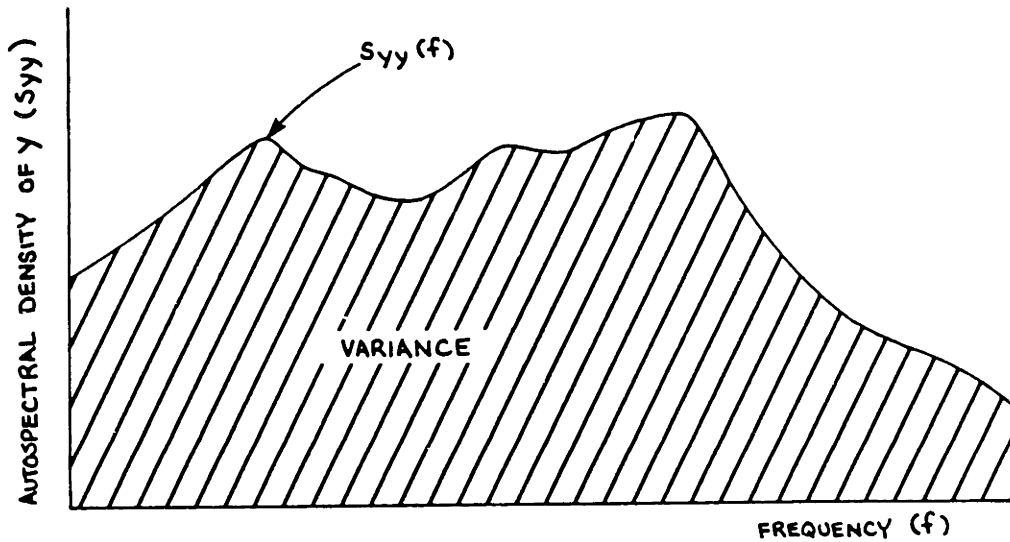


Figure 2-9. Autospectral Density Function (Autospectrum)

spectrum is shown in Figure 2-9. The area under the autospectrum is equal to the total variance of the variable (Bendat and Piersol (1980), p. 51). Peaks in the autospectrum indicate that variance is concentrated at certain frequencies.

Figure 2-10 shows how variance can be a misleading guide to policy selection. The figure shows the autospectrum of a variable before and after a policy is introduced. Before a policy is introduced, the autospectrum shows a peak around frequency f_T . The autospectrum peak shows that the variable tends to oscillate at frequency f_T . The policy to be tested is aimed at reducing the cyclic tendency around f_T . After

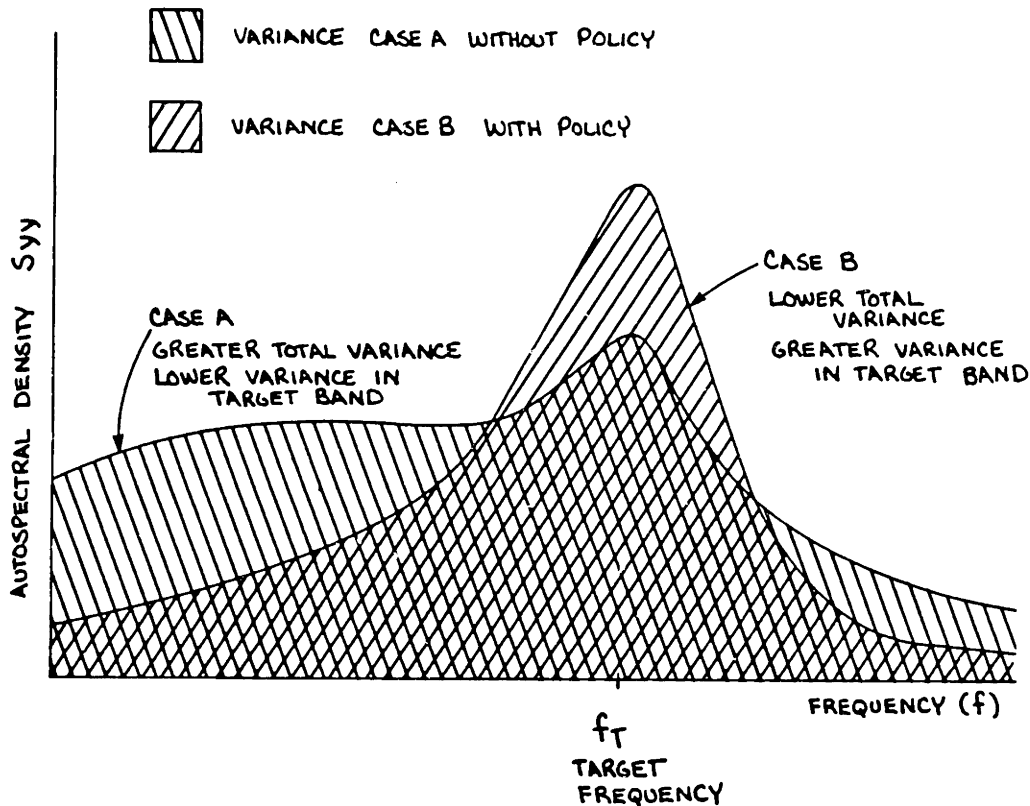


Figure 2-10. Trade-off Between Total Variance and Variance in a Frequency Band

the policy is introduced, the autospectrum changes so that total variance is diminished. The policy would be judged a stabilizing influence by the variance criterion. However, the policy has raised the autospectrum peak around the target frequency f_T . The policy has aggravated the cyclicalities it was intended to control. The policy is, therefore, at best a mixed blessing and, in fact, counterproductive in controlling the problematic cyclicalities. The difficulty with total variance as a stability measure suggests the use of a frequency-weighted measure of variance instead.

Autospectrum Criterion: If a policy reduces variance of a target variable(s) within a target frequency range, it is a stabilizer.

Pettersen (1974), Bowden (1977), Baum and Howrey (1981), and others have suggested measuring stability by changes in the autospectrum of an output variable(s). Changes at particular frequencies can be weighted more heavily than at others. For example, a policy may be aimed at controlling the business cycle, which has a frequency of about .25 cycles per year. The policy might be judged a business-cycle stabilizer if it reduced the autospectrum of important variables in the frequency range of .15-.35 cycles per year. Changes in the autospectrum outside this range would be considered side effects of secondary importance.

Bowden suggested creating a formal stability index based on variance weighted by frequency. Recall that variance is the area under the autospectrum of a variable. A weighting function would specify the relative importance of variance at different frequencies. The index Bowden suggests is the area under a curve that is created by multiplying the original autospectrum curve at each frequency by the appropriate weight from a weighting function. Expressed mathematically, the frequency-weighted variance index is the integral over frequency of the product of the weighting function and the autospectrum.

$$\text{Index} = \int_{0^+}^{\infty} W_y(f) \cdot S_{yy}(f) df \quad (2.4)$$

Figure 2-11 shows graphically how the index is constructed. Curve W_y is the weighting function for variable Y, plotting the

importance of variance at each frequency. Curve S_{yy} is the original autospectrum for variable, Y, showing the decomposition of variance in Y by frequency. Curve $W_y S_{yy}$ is the weighted power spectrum for variable Y equal to the value of curve S_{yy} at each frequency. The shaded area under curve $W_y S_{yy}$ is the frequency-weighted variance index suggested by Bowden. A policy that reduces the index for the target variable is a stabilizer. If stabilizing more than one target variable is important, then a weighted combination of indices for several variables can be used in

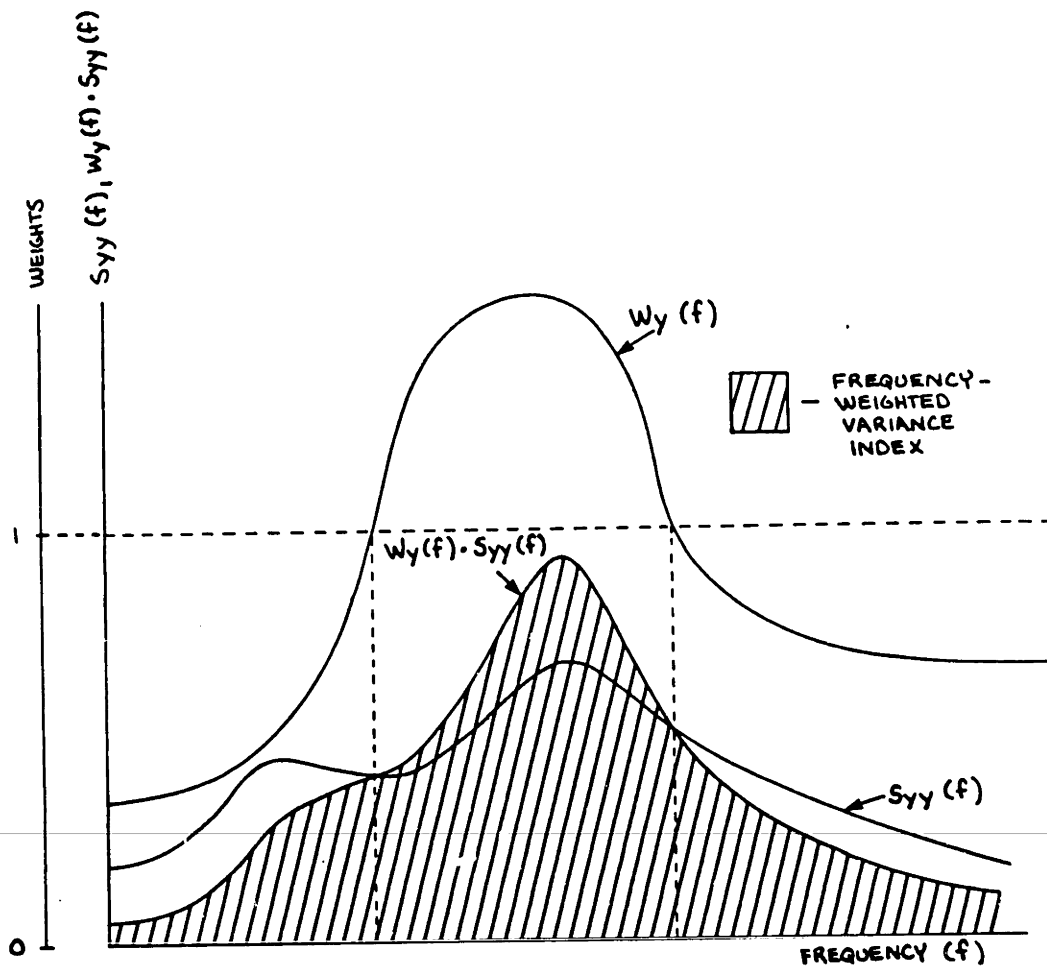


Figure 2-11. Frequency-Weighted Variance Index

place of a single index to measure the stabilizing effect of a policy on the whole system rather than on a single variable.

In order to apply the concept of a spectral stability criterion, it is necessary to compute the autospectra of relevant output variables. Autospectra can be determined either experimentally by analyzing simulation output or analytically from a linearized version of the model. The simulation approach retains the full complexity of a model but requires the spectral analysis of several variables from long simulation runs. The analytic approach is more compact but involves structural simplifications that may distort the autospectrum for high-amplitude disturbances. Despite its theoretical limitations, only the analytic, linear approach to determining autospectra will be reviewed here.

Figure 2-12 shows schematically a single-disturbance/single-output system. The disturbance, U , excites some variable in the system, which in turn excites dynamic response in all other variables. The response of the output variable, Y , depends on the properties of the disturbance and the structure of the system. In particular, the autospectrum of the output, S_{yy} , is a function of the autospectrum of the disturbance term, S_{uu} , and the gain of the system between the disturbance and the output variable, G_{uy} . The formula for the autospectrum in the single-disturbance/single-output case is (Bendat and Piersol (1980), p. 80):

$$S_{yy}(f) = S_{uu}(f) \cdot [G_{uy}(f)]^2 \quad (2.5)$$

where f is frequency. The meaning of the terms on the right-hand side of equation (2.5) is examined below.

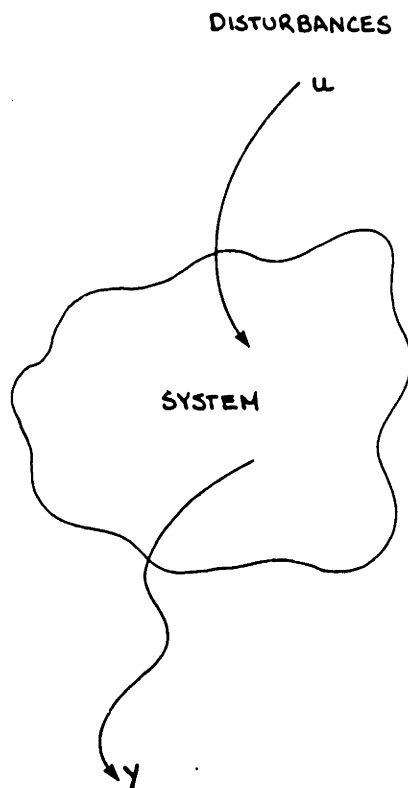


Figure 2-12. Single-Disturbance/Single-Output System

The autospectrum of the disturbance process decomposes the variance of disturbance by frequency. Four possible autospectra for the disturbance term are illustrated in Figure 2-13. Curve A is flat up to a very high frequency then tails away to zero. Over the frequency range where the autospectrum is flat the disturbance is "white" noise or noise without autocorrelation. (Every real-life stochastic process has an upper cutoff frequency above which there is no variance.) Curve B is a rectangular hyperbola that corresponds to the auto spectrum for a random walk (integrated white noise). A random walk has constant power per

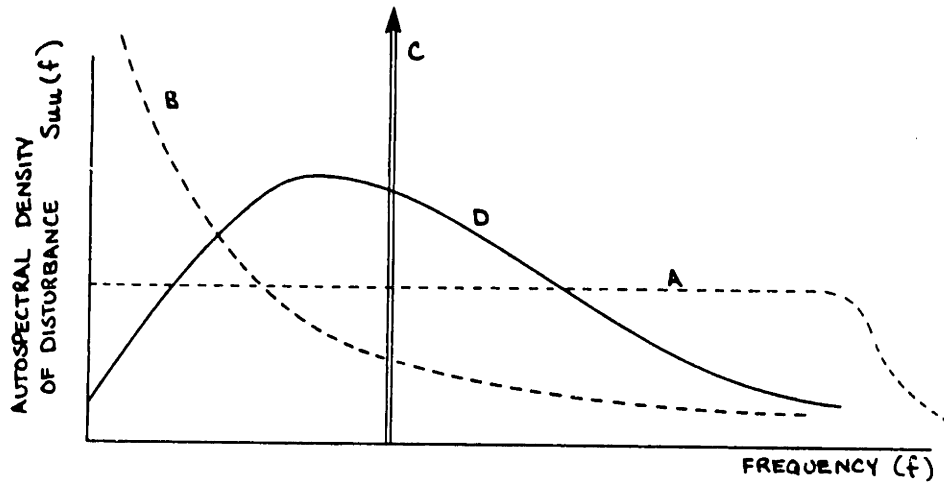


Figure 2-13. Autospectra for Noise: A - White Noise (no autocorrelation), B- Random Walk, C - Sine Wave, D - "Typical" Noise

octave, that is, the variance (area under the autospectrum) between some frequency f and a constant multiple of f is the same for all f . Curve C is a spike with zero width and finite area (a multiple of the delta function). The spike is the autospectrum of a sine wave whose frequency is the frequency where the spike occurs and whose amplitude is the square root of twice the area of the spike. All the variance of the sine wave is concentrated at one frequency. Curve D is a "typical" noise source whose autospectrum rises to a peak then falls off again at higher frequencies. Curve D is probably more realistic than A, B, or C in that it is autocorrelated, variance is concentrated in the midrange frequencies, and it contains some variance at all frequencies.

Referring back to equation (2.5), the second term of the product that determines the autospectrum of the output variable is the gain of

the system between the disturbance and the output variable. The gain is, in general, dependent on the frequency of the disturbance. Disturbances at certain frequencies propagate through the system better than disturbances at other frequencies. Gain at each frequency can be determined by simulating model behavior in response to a sine wave of that frequency. A sine wave is added at the point where the disturbance enters the system. In a linear system with no eigenvalues in the right half of the complex plane, all variables go through a period of transience then

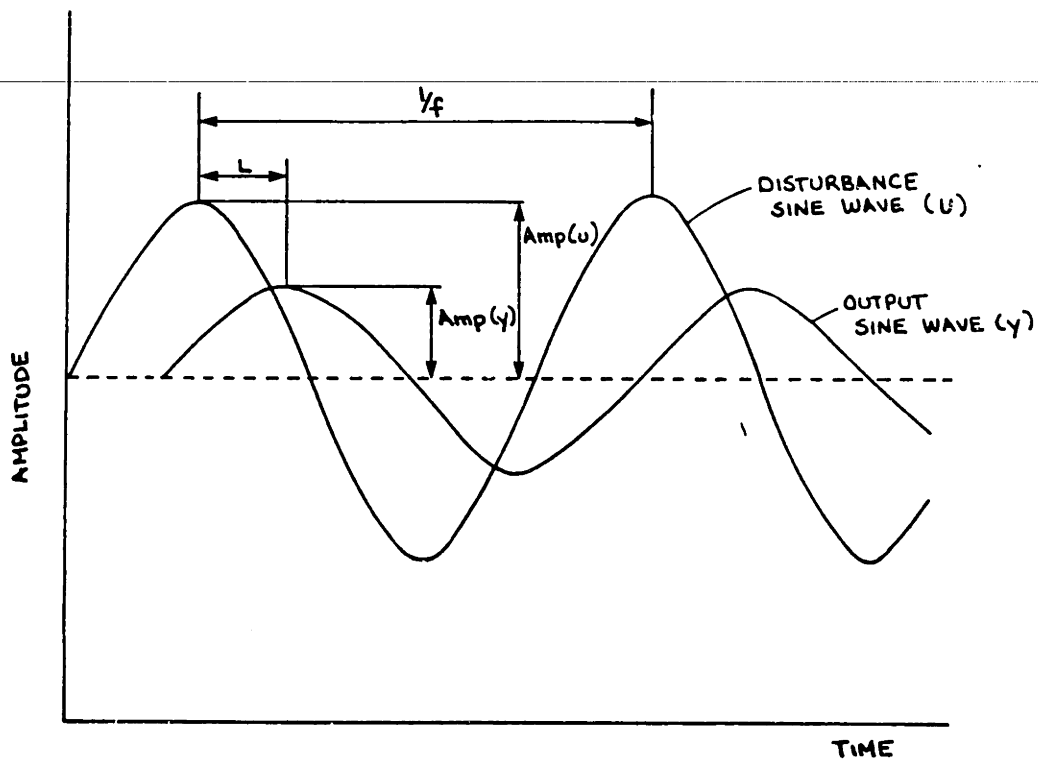


Figure 2-14. Gain and Phase Shift
 $L \cdot f = \text{Phase Lag (in fraction of cycle)}$
 $\frac{\text{Amp}(Y)}{\text{Amp}(U)} = \text{Gain}$

settle down to a sinusoidal movement with exactly the same frequency as the disturbance (Ogata (1970), p. 374). The amplitude of the sine wave of the output variable is then compared with the amplitude of the disturbance to measure gain. Figure 2-14 shows the comparison. Gain (often called closed-loop, steady-state gain) is measured as the ratio of the amplitude of the output sinusoid to the amplitude of the disturbance sinusoid. Gain measures amplification (or attenuation) of the disturbance as it propagates through the system to the output variable. (Note that the output sinusoid is usually shifted in phase relative to the disturbance sinusoid. Phase shift can be measured equally well in fractions of a cycle, degrees, or radians. For example, a 1/4-cycle lag is exactly the same as a 90° lag or a $\pi/2$ -radian lag.)

After simulating model response and calculating gain for several frequencies, the results can be plotted on a graph like that shown in Figure 2-15. The plot shows gain as a function of frequency. The plot illustrates a few "typical" features of the gain curve. First, gain tails away to zero at very high frequencies. Physical systems cannot respond quickly enough to propagate high-frequency signals; instead the signals are filtered out. Second, the gain curve usually shows one or more peaks that indicate resonant frequencies. The system selectively amplifies disturbances containing frequencies at or near the resonant peaks.

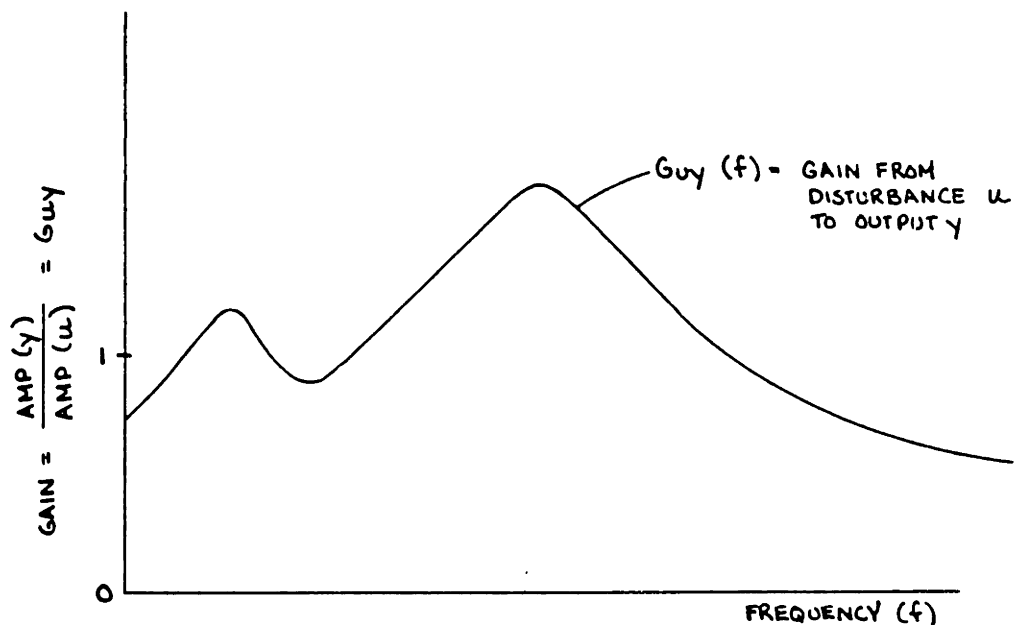


Figure 2-15. Gain vs. Frequency for a Disturbance-Output Pair

Figure 2-16 shows graphically how the autospectrum of the output variable is generated from the autospectrum of the disturbance and the gain curve for a single-disturbance/single-output system. At each frequency the value of the autospectrum of the disturbance is multiplied by the square of the gain; the resulting product is the value of the autospectrum of the output at that frequency. Peaks in the autospectrum of the output variable may or may not correspond to peaks in the gain curve or peaks in the autospectrum of the disturbance, depending on whether the gain and disturbance peaks reinforce or cancel each other.

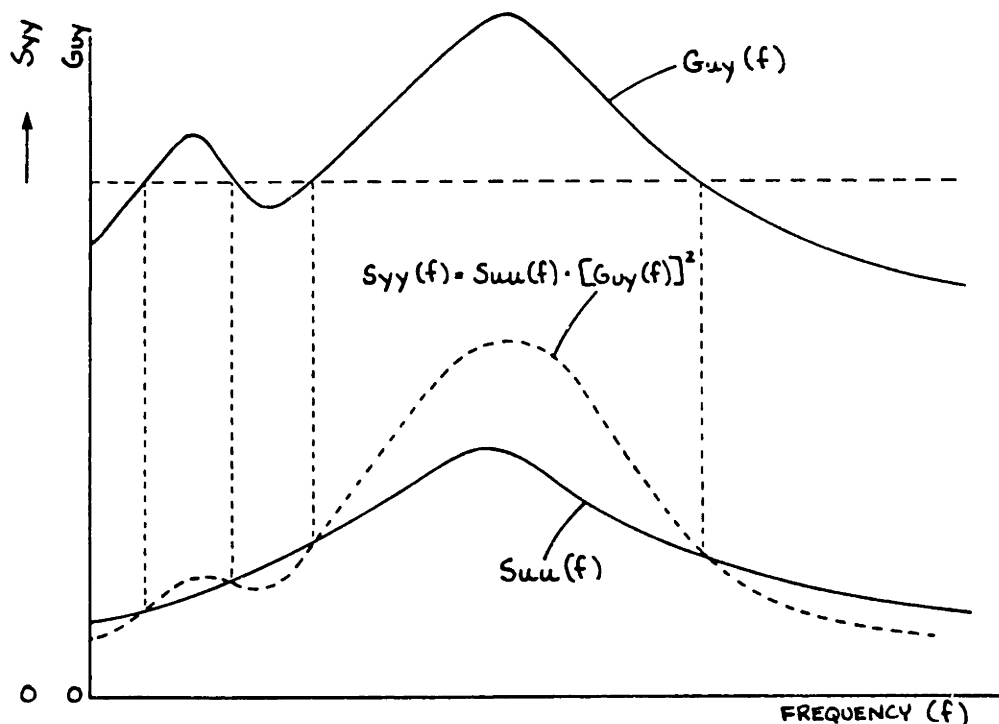


Figure 2-16. Calculating Autospectrum of Output Variable

We will now shift focus from the single-disturbance/single-output systems to ones with several sources of disturbance and several important output variables. Figure 2-17 illustrates schematically a multi-disturbance/multi-output system. We will investigate the case where the disturbance terms may be autocorrelated but are not cross-correlated (cross-spectra of the disturbances are all zero). The formula that determines the autospectrum of an output variable, Y_m , in such a case is (Bendat and Piersol (1980), p. 197):

$$S_{y_m y_m}(f) = \sum_k S_{u_k u_k}(f) \cdot [G_{u_k y_m}(f)]^2 \quad (2.6)$$

The value of the autospectrum of an output variable at a given frequency is the sum, over all disturbances, of the value of the autospectrum of the disturbance multiplied by the squared value of the gain curve relating the disturbance to the output at the given frequency.⁴

⁴ It should be noted that equation (2.6) is a special case of the general function defining an output autospectrum where noise terms are both autocorrelated and cross-correlated (Bendat and Piersol (1980), P. 190):

$$S_{y_m y_m}(f) = \sum_i \sum_n S_{u_i u_n}(f) \cdot T_{u_i y_m}(jf) \cdot [T_{u_n y_m}(jf)]^* \quad (2.6a)$$

where $T_{u_i y_m}$ is the transfer function relating disturbance u_i to output y_m (transfer functions are explained later in this section).

j - square root of -1.

$[]^*$ - denotes complex conjugate of $[]$.

When the cross-spectra, $S_{u_i u_n}(f)$, for $i \neq n$ are all zero (no cross-correlation) equation (2.6a) collapses to (2.6) since

$$T_{u_k y_m}(jf) \cdot [T_{u_k y_m}(jf)]^* = |T_{u_k y_m}(jf)|^2 = [G_{u_k y_m}(f)]^2$$

The analysis presented here could be extended to allow for cross-correlated disturbances at a considerable cost in complexity.

It should also be noted that in order to calculate the autospectra of the output variables one must estimate the autospectra of the disturbances. Such estimates can be obtained by analyzing the residuals in the equations used to estimate system parameters. The disturbance autospectra are not estimated in this work, since the analysis concentrates on changes in the gain curves.

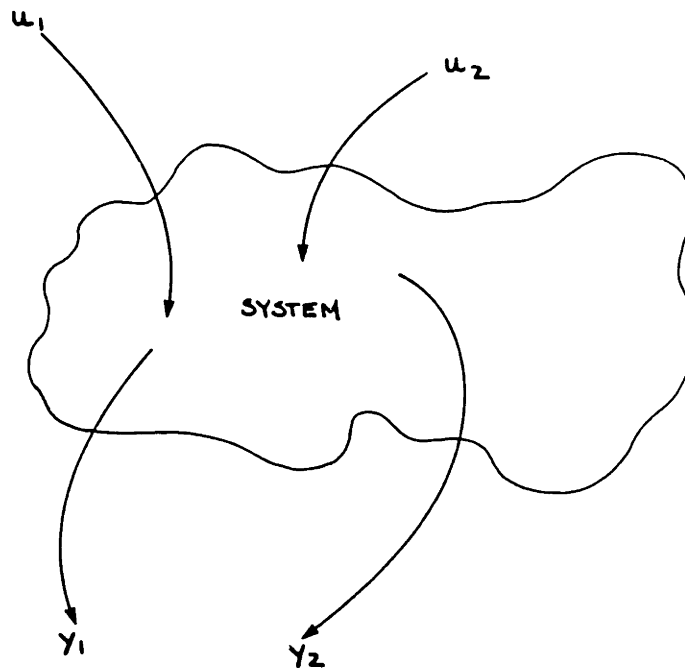


Figure 2-17. Multi-Disturbance/Multi-Output System

The autospectra of the output variables can be collected into a scalar stability index by calculating the frequency-weighted variance of each output, then forming a weighted sum of the weighted variances:

$$\begin{aligned}
 \text{Index} &= \sum_m^m w_m \int_{0^+}^{\infty} W(f) \cdot S_{y_m y_m}(f) df \\
 &= \sum_m^m w_m \int_{0^+}^{\infty} W(f) \cdot \sum_k^k S_{u_k u_k}(f) \cdot [G_{u_k y_m}(f)]^2 df \quad (2.7)
 \end{aligned}$$

A policy would be judged by its impact on the index. An increase in the index means greater sensitivity to noise and hence less stability.

Policies do not affect either the weighting terms or the autospectra of the disturbances. Policies only affect the gain terms that describe the dynamic properties of the system structure.

Using a scalar stability index to measure policy response has a major drawback. The index lumps together all the changes in gain curves resulting from a policy intervention. Only the net effect of a policy is reported; none of the trade-offs involved is revealed. Take, for example, a policy which has no effect on the index because it has no effect on the dynamic properties of the system. By the scalar index, an ineffective policy would be judged the same as a policy with strong positive effects and equally strong negative side-effects. A disaggregated approach must be taken to reveal the trade-offs involved in implementing each policy. A criterion that considers each gain curve separately will avoid the aggregation problem of a stability index.

Frequency-Response (Gain) Criterion: If a policy reduces gain (amplification) in the target frequency range for a particular combination of disturbance source and output variable, it is a stabilizer.

Figure 2-18 illustrates the gain matrix which underlies the frequency-response criterion proposed here. The figure shows the matrix of gain curves for a multi-disturbance/ multi-output system. Each column corresponds to an output variable and each row corresponds to a source of disturbance. Pre-policy gain curves are indicated by solid lines, while dashed lines indicate post-policy gains. Figure 2-18 illustrates three kinds of trade-offs that can be identified. First, a policy may increase the sensitivity of an output variable to one disturbance while decreasing its sensitivity to another disturbance, as seen by comparing curves (a)

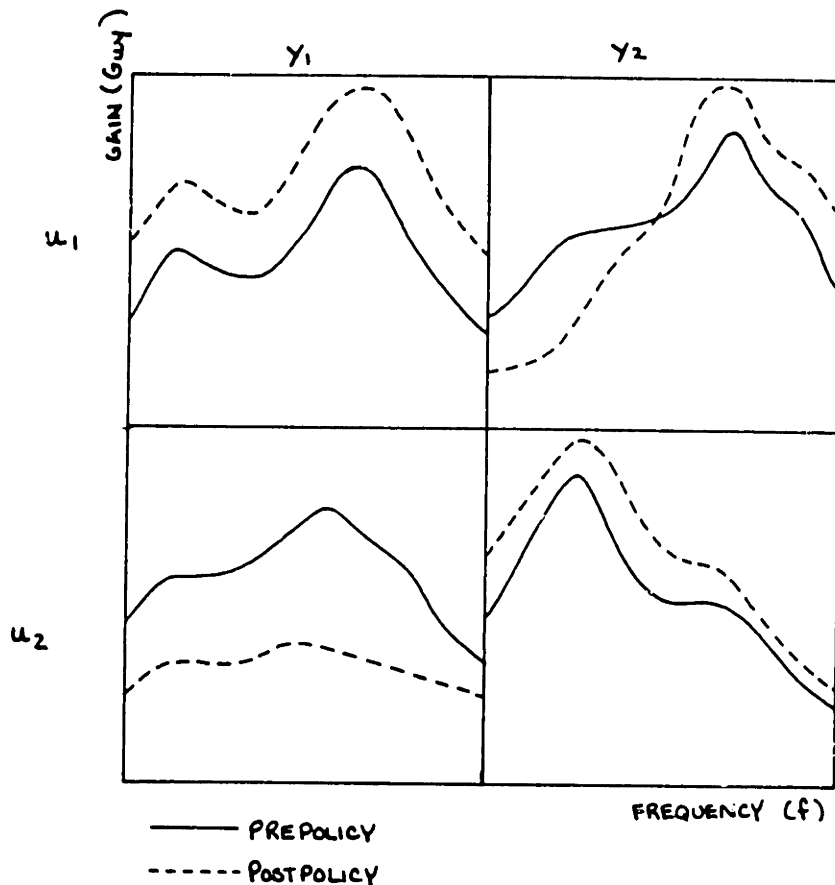


Figure 2-18. Matrix of Gain Curves

and (c). Second, a policy may decrease the sensitivity of one output to a disturbance while increasing the sensitivity of another output to the same disturbance, as seen in curves (c) and (d). Third, a policy may change the sensitivity of any or all outputs to any or all disturbances differently at different frequencies, as shown in curve (b).

Examining changes in the gain-curve matrix allows both the positive and negative features of each policy to be identified. Policies

that are all positive, all negative, or ineffective can be distinguished from policies that entail compromises. If a policy has mostly positive effects but a few negative ones, a set of compensating policies might be found which would offset the undesired side effects once they are discovered.

As discussed before, the gain curves can be obtained through repeated simulation of system response to sinusoidal disturbances at different frequencies. The simulation approach, however, becomes a major computational burden when the number of disturbances and outputs is large. A much more efficient method of calculating the gain curves is to use the transfer-function approach. A transfer function describes the behavior of an output variable in terms of an input signal. Each disturbance-output combination has a different transfer function. Evaluating a transfer function at $j\omega$ (where j is the square root of -1 and ω is a specific frequency) yields a complex number which describes the steady-state response of a particular output variable to a sine wave of frequency ω added to a particular input variable. The complex number is called the "frequency response" and yields exactly the same information about gain and phase shift as the simulation analysis shown previously in Figure 2-14.

The complex number that results from evaluating a transfer function at $j\omega$ can be visualized as a vector in the complex plane, as shown in Figure 2-19. The length of the vector (or modulus or magnitude of the complex number) is equal to the gain (output/input amplitude

ratio) at frequency f . The angle of the vector from the positive real axis is the phase shift between input and output sinusoids at frequency f . In equation form (Ogata (1970), p. 374):

$$\text{Gain} = G_{uy}(f) = |T_{uy}(jf)| \quad (2.8)$$

$$\text{Phase Shift} = \angle T_{uy}(jf) \quad (2.9)$$

where G gain (ratio of output/input amplitude)

T the transfer function

U an input (source of disturbance)

Y an output variable

j square root of -1

f frequency.

$||$ magnitude (modulus)

\angle angle

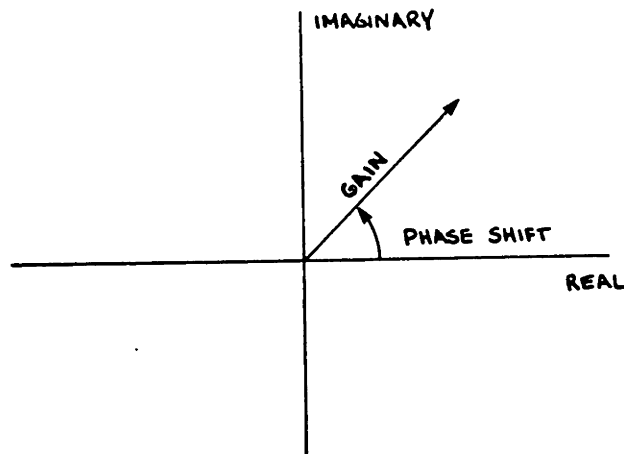


Figure 2-19. Frequency Response as a Complex Vector

A simple formula permits simultaneous computation of frequency response for all disturbance-output combinations at a given frequency. The formula involves the matrices of constant coefficients which describe the linearized model used in equations (2.1) and (2.2) (Ogata (1970), p. 689).

$$\underline{T}(jf) = \begin{bmatrix} \underline{T}_{u_1 y_1}(jf) & \dots & \underline{T}_{u_k y_1}(jf) \\ \vdots & & \vdots \\ \underline{T}_{u_1 y_m}(jf) & \dots & \underline{T}_{u_k y_m}(jf) \end{bmatrix} = \underline{C} (jf \underline{I} - \underline{A})^{-1} \underline{B} + \underline{D} \quad (2.10)$$

Evaluating the right-hand side of equation (2.10) for a given value of f yields a matrix of complex numbers. Each complex number is the frequency response of a particular disturbance-output combination at frequency f . Each row of the matrix corresponds to a different output variable, and each column corresponds to a different disturbance source. By evaluating the expression at several different frequencies, the frequency-response characteristics of all output variables with respect to all disturbances can be determined across the whole frequency spectrum.

Frequency-response information generated by repeated evaluation of equation (2.10) can be used directly to compute the gain curves shown in Figure 2-18. The height of the gain curve for a particular disturbance-output pair at a given frequency, f , is simply the magnitude of the complex number appearing in the corresponding position in the transfer matrix evaluated at jf :

$$\underline{G}_{u_k y_m}(f) = |[\underline{T}(jf)]_{m,k}| = |[\underline{C}(jf \underline{I} - \underline{A})^{-1} \underline{B} + \underline{D}]_{m,k}| \quad (2.11)$$

B. PARAMETER SENSITIVITY

Sensitivity of model behavior to parameter changes will be assessed in two ways. First, sensitivity of the transient response will be determined by calculating eigenvalue elasticities with respect to each parameter. Second, the sensitivity of the stochastic response will be determined by calculating frequency-response elasticities with respect to the parameters. Both elasticities give the ratio of the percentage change in a behavioral measure to the percentage change in a parameter value. The elasticities are accurate only for small changes in the parameter values.

Eigenvalue Elasticities. The elasticity of an eigenvalue with respect to a parameter measures the percentage change in the eigenvalue for a given percentage change in the parameter. The eigenvalue elasticity is defined here as the partial derivative of the eigenvalue with respect to the parameter normalized for the size of the parameter and the size of the eigenvalue. The elasticity of eigenvalue, λ_i , with respect to parameter p_g ; is (Porter and Crossley (1972), p. 22):

$$\frac{\partial \lambda_i}{\partial p_g} \cdot \frac{p_g}{\lambda_i} = \underline{l}_i' \frac{\partial \underline{A}}{\partial p_g} \underline{r}_i \cdot \frac{p_g}{\lambda_i} \quad (2.12)$$

where ∂ = partial derivative (operator)

λ_i = eigenvalue i (scalar)

p_g = parameter g (scalar)

\underline{l}_i = transpose of i^{th} left eigenvector (1 x n vector)

\underline{A} = linear system matrix (n x n)

\underline{r}_i = i^{th} right eigenvector (n x 1)

As shown in equation (2.12) the eigenvalue elasticity can be computed using left and right eigenvectors and the partial derivative of the linear system matrix, \underline{A} , with respect to parameter p_g .

Each eigenvalue has an associated pair of right and left eigenvectors. Eigenvectors are $n \times 1$ vectors of complex numbers which satisfy certain conditions. \underline{l}_i is a left eigenvector of eigenvalue λ_i if it satisfies the equation:

$$\underline{l}_i' \underline{A} = \lambda_i \underline{l}_i' \quad (2.13)$$

Right eigenvectors of λ_i satisfy the condition:

$$\underline{A} \underline{r}_i = \underline{r}_i \lambda_i \quad (2.14)$$

(Right eigenvectors are those referred to by the simple term "eigenvectors" in studies which do not draw a distinction between left and right.) The sets of left and right eigenvectors can be arranged into $n \times n$ left and right eigenvector matrices, where each column is the eigenvector for a particular eigenvalue. The left eigenvector matrix satisfies:

$$\underline{L}' \underline{A} = \underline{\Lambda} \underline{L}' \quad (2.15)$$

where $\underline{\Lambda}$ is the diagonal matrix with the system eigenvalues down the

main diagonal. The right eigenvector matrix satisfies:

$$\underline{A} \underline{R} = \underline{R} \underline{\Lambda} \quad (2.16)$$

It may be noted from equations (2.13 through 2.16) that eigenvectors are uniquely determined only up to a scalar multiple. Eigenvectors can therefore be scaled arbitrarily. The most convenient scaling is a normalization such that:

$$\underline{l}_i' \underline{r}_i = 1 \quad (2.17)$$

If normalization is done according to equation (2.17), then (Porter and Crossley (1972), p. 23):

$$\underline{L}' \underline{R} = \underline{I} \quad (2.18)$$

where \underline{I} is the $n \times n$ identity matrix. Equation (2.18) implies that the matrix of left eigenvectors is the inverse of the matrix of right eigenvalues:

$$\underline{L}' = \underline{R}^{-1} \quad (2.19)$$

Eigenvectors must be normalized in this manner for the eigenvalue elasticity equation (2.12) to hold. Computer routines for computing eigenvalues usually return the right eigenvector matrix. The left eigenvector matrix can be calculated by simply taking its inverse.

The partial derivative of the system matrix, \underline{A} , with respect to a parameter is most easily computed by simply calculating the matrix before and after a small change in the parameter:

$$\frac{\partial \underline{A}}{\partial p_j} = \frac{\underline{A}^* - \underline{A}}{(.01)p_j} \quad (2.20)$$

where \underline{A}^* is the system matrix with one percent greater value of parameter p_j .

Having computed the partial derivative of \underline{A} with respect to each parameter and the normalized left and right eigenvector matrices, the eigenvalue elasticities are computed using equation (2.12). The eigenvalue elasticities are complex numbers that describe the direction and magnitude of change in system eigenvalues. An eigenvalue elasticity can be thought of as a vector in the complex plane, as shown in Figure 2-20. As shown below, the real component of the elasticity measures the impact of a parameter change on the natural frequency of a cyclic mode (or the time constant of an exponential decay mode). The imaginary part of the elasticity measures the impact on the damping ratio of a cyclic mode (the imaginary part is always zero for real eigenvalues). An eigenvalue elasticity in the first quadrant indicates that an increase in the parameter increases the natural frequency and damping of a cyclic mode. An elasticity in the second quadrant indicates lower natural frequency and greater damping. In the third quadrant, an elasticity means lower frequency and lower damping. The fourth quadrant indicates that a parameter increase raises the natural frequency and reduces the damping ratio.

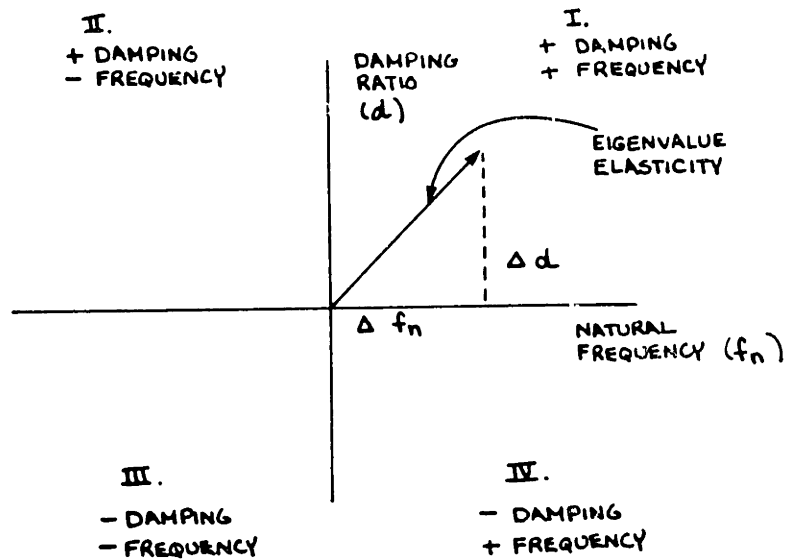


Figure 2-20. Eigenvalue Elasticity as a Complex Vector

Figure 2-21 illustrates why the angle of an eigenvalue elasticity indicates the effect of a parameter change on frequency and damping. Vector A is an eigenvalue with natural frequency equal to its length and damping equal to the cosine of its angle from the real axis. Vector B is the partial derivative of the eigenvalue with respect to a parameter and shows the direction of change in the eigenvalue induced by a change in the parameter. If the vector B has a component parallel to the eigenvalue, then the parameter affects natural frequency. If vector B has a component perpendicular to the eigenvalue, then the parameter affects damping ratio. Recall from equation (2.12) that the computation of an eigenvalue elasticity involves dividing the partial derivative of the eigenvalue (vector B) by the eigenvalue itself. The division involves subtracting the angle of the eigenvalue from the angle of vector

B. Subtracting the angle of the eigenvalue is the same as rotating the axes so that the real axis is parallel to the eigenvalue and the imaginary axis is perpendicular to the eigenvalue. The position of an eigenvalue elasticity with respect to the real and imaginary axes therefore relates directly to the effect of a parameter change on the natural frequency and damping ratio of an eigenvalue.

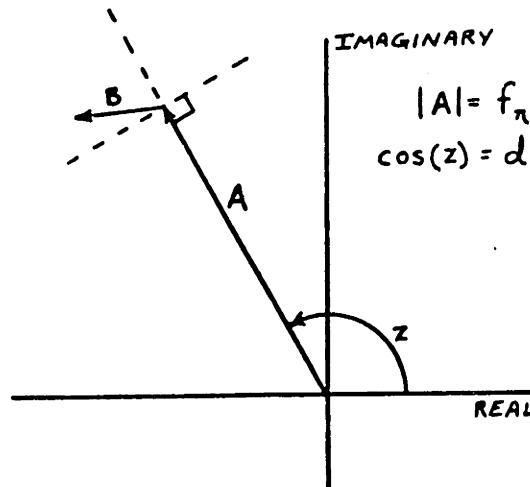


Figure 2-21. Eigenvalue Elasticities From Change in Eigenvalue

Eigenvalue elasticity is a convenient measure of the sensitivity of transient-response sensitivity to parameter changes. The values of the elasticities are scale-invariant (dimensionless), so they can be directly compared with each other. A "larger" elasticity means behavior is more sensitive to a certain percentage change in one parameter than another. A large elasticity suggests that either 1) model structure is deficient in the area around the parameter and should be revised, or 2) extra effort should be expended on obtaining a good estimate of the parameter, or 3) the parameter should be investigated as a possible policy lever.

Frequency-Response Elasticities. The elasticity of frequency response with respect to a parameter change measures the percentage change in frequency response (for a given disturbance/output pair at a given frequency) for a given percentage change in the parameter.

Frequency-response elasticity is defined as the partial derivative of frequency response with respect to a parameter normalized for the size of the parameter and frequency response. The frequency-response elasticity for disturbance, \underline{u}_k and output, \underline{y}_m , at frequency, f , with respect to parameter p_g ; is:

$$\begin{aligned}
 & \frac{\partial [\underline{T}(jf)]_{m,k}}{\partial p_g} \cdot \frac{p_g}{[\underline{T}(jf)]_{m,k}} \\
 = & \frac{\partial [\underline{C} (jf \underline{I} - \underline{A})^{-1} \underline{B} + \underline{D}]_{m,k}}{\partial p_g} \cdot \frac{p_g}{[\underline{C} (jf \underline{I} - \underline{A})^{-1} \underline{B} + \underline{D}]_{m,k}} \\
 = & \left[\begin{aligned} & \left[\frac{\partial \underline{C}}{\partial p_g} (jf \underline{I} - \underline{A})^{-1} \underline{B} \right]_{m,k} - \left[\underline{C} (jf \underline{I} - \underline{A})^{-1} \frac{\partial \underline{A}}{\partial p_g} (jf \underline{I} - \underline{A})^{-1} \underline{B} \right]_{m,k} \\ & + \left[\underline{C} (jf \underline{I} - \underline{A})^{-1} \frac{\partial \underline{B}}{\partial p_j} \right]_{m,k} + \left[\frac{\partial \underline{D}}{\partial p_j} \right]_{m,k} \end{aligned} \right] \frac{p_j}{[\underline{C} (jf \underline{I} - \underline{A})^{-1} \underline{B} + \underline{D}]_{m,k}} \quad (2.21)
 \end{aligned}$$

where ∂ = partial derivative (operator)

$\underline{T}(jf)$ = frequency response matrix

$[\]_{m,k}$ = element m, k of matrix $[\]$

p_g = parameter g

- \underline{I} = $n \times n$ identity matrix
 $\underline{A}, \underline{B}, \underline{C}, \underline{D}$ = linear system matrices from equations (2.1) and (2.2)
 j = square root of -1
 f = frequency

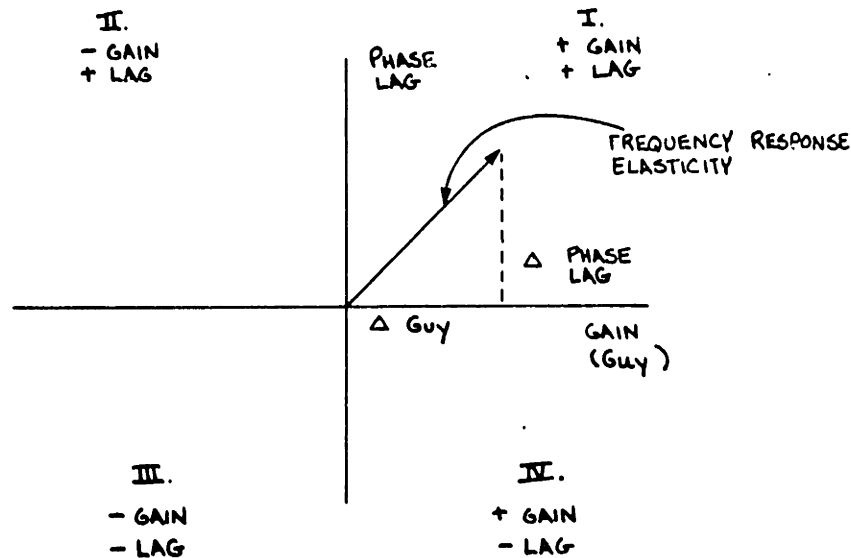


Figure 2-22. Frequency-Response Elasticity as a Complex Vector

Calculating frequency-response elasticity by the formula in equation (2.21) requires only the linear system matrices and their derivatives with respect to the parameters. The matrix derivatives are most easily determined by calculating the change in each matrix before and after a small percentage change in a parameter, then dividing by the parameter change as done in equation (2.20).

Frequency-response elasticities are complex numbers which describe the change in gain and phase shift induced by a parameter change. A frequency-response elasticity can be plotted as a vector in the complex

plane, as shown in Figure 2-22. By an argument similar to that used in Figure 2-21, the real component of a frequency-response elasticity measures change in gain. The imaginary part measures change in phase shift. A frequency-response elasticity in the first quadrant indicates that an increase in a parameter increases gain and phase shift for the particular disturbance-output pair and frequency considered. An elasticity in the second quadrant means gain declines while phase lag increases. An elasticity in the third quadrant indicates falling gain and decreasing lag. Finally, the fourth quadrant corresponds to increasing gain and declining lag.

Frequency response, or sensitivity to disturbance, is measured by gain. Therefore, the gain component (real part) of frequency-response elasticity is most important. Phase lag comes into play only with cross-correlated noise sources. All frequency-response elasticities are directly comparable, since they are normalized, dimensionless measures. If an elasticity has a large positive real part, a small percentage increase in a parameter produces a large percentage increase in gain. A large negative real part means that a small percentage increase in the parameter generates a large percentage drop in gain.

The gain component (real part) of frequency-response elasticity can be plotted as a function of frequency for each disturbance-output combination. A sample gain component plot is shown in Figure 2-23. The figure shows that a parameter increase raises gain at low frequencies slightly while sharply decreasing gain at higher frequencies. An

increase in the parameter, therefore, destabilizes stochastic response at low frequencies and stabilizes it at high frequencies. The effects of a parameter change are, in principle, different for each disturbance-output pair and at each frequency.

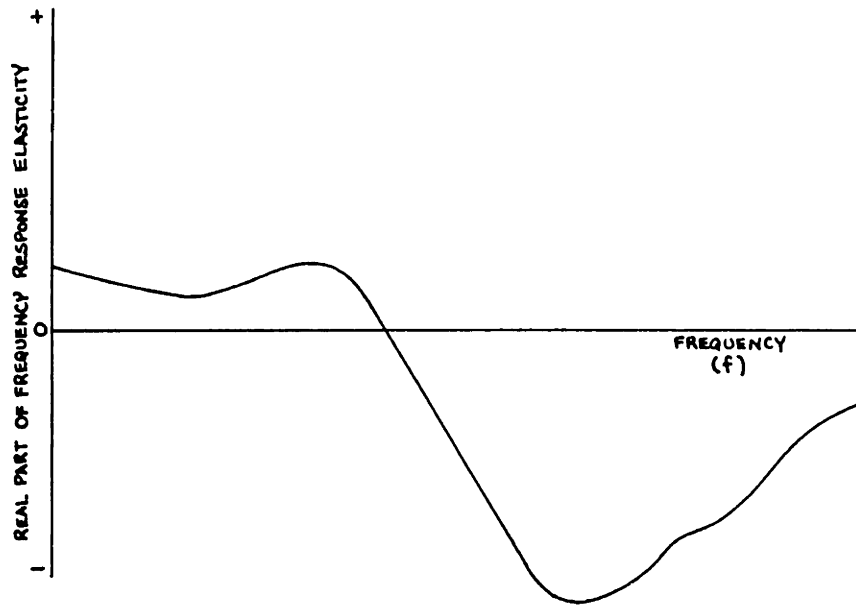


Figure 2-23. Gain Component of Frequency-Response Elasticity

C. DOMINANT LOOP IDENTIFICATION

Most dynamic studies of stabilization policy make no attempt to identify the important causal structures that underlie the cyclic behavior modes. The following section presents a method for identifying the feedback loops which create cyclic behavior (complex eigenvalues).

The system matrix, \underline{A} , in equation (2.1) represents the condensed causal structure of a linearized model. Each non-zero element in matrix \underline{A} is a causal link between state variables. If element a_{ij} is not equal to zero, then the rate of change in state variable x_i depends on the value of state variable x_j . A typical condensed linear structure is shown in Figure 2-24. Each arrow represents the direct effect of a state variable on the rate of change in another. The corresponding system matrix is also shown in the figure. The matrix is relatively sparse, since many of the possible connections between states do not exist in a typical model.

$$A = \begin{bmatrix} 0 & 0 & 0 & a_{14} & a_{15} \\ a_{21} & a_{22} & 0 & 0 & 0 \\ a_{31} & a_{32} & 0 & a_{34} & 0 \\ 0 & a_{42} & 0 & 0 & a_{45} \\ a_{51} & 0 & a_{53} & 0 & 0 \end{bmatrix}$$

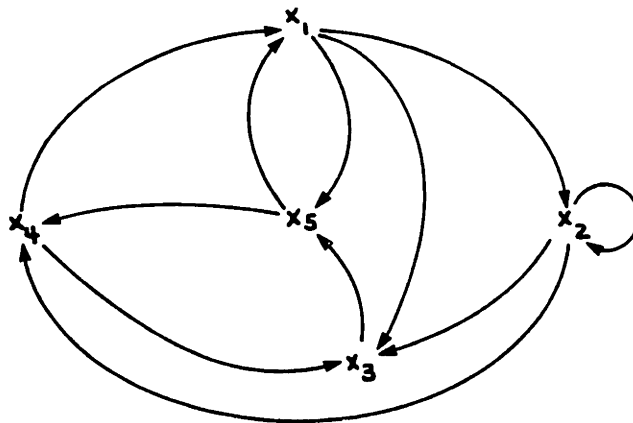


Figure 2-24. Causal Links in Linearized Model

Identifying the dominant structure behind a cyclic behavior mode is done by calculating the elasticity of its eigenvalues with respect to all non-zero elements of the system matrix. The elasticities are based on the same formula as equation (2.12), but the expression simplifies, since the derivative of the \underline{A} matrix with respect to element a_{pq} is an $n \times n$ matrix of zeros except for a 1 in position pq .

$$\frac{\partial \lambda_i}{\partial a_{pq}} \cdot \frac{a_{pq}}{\lambda_i} = \underline{l}_i \cdot \frac{\partial \underline{A}}{\partial a_{pq}} \underline{r}_i \cdot \frac{a_{pq}}{\lambda_i} = \left[\underline{l}_i \right]_p \left[\underline{r}_i \right]_q \frac{a_{pq}}{\lambda_i} \quad (2.22)$$

The elasticities are complex numbers which show the effects of altering the strength of a causal link on damping and the natural frequency of the cyclic mode. The real part gives the effect on natural period, while the imaginary part gives the effect on the damping ratio. The magnitude of the elasticity gives the overall sensitivity of the cyclic mode to a structural link.

Those causal links between states that have large-magnitude eigenvalue elasticities are particularly important. If a small number of elasticities have markedly greater magnitudes than others, then the corresponding links define a dominant subset of model structure. In most cases the dominant links fit together to form one or more interconnected feedback loops. The loops create cyclic tendencies, because they form paths for the propagation of waves. In some cases the eigenvalue elasticities with respect to entries in the system matrix will clearly indicate a small number of loops as dominant in generating a cycle. In other cases the elasticities may not be clearly grouped by magnitude. In

such cases the cutoff between "dominant" and "secondary" feedback loops is arbitrary. An extended version of the loop-identification technique is presented in Chapter VI.

CHAPTER III

THE MODEL

A. STRUCTURAL ASSUMPTIONS

The model integrates the salient processes of four macroeconomic models:

- 1) the multiplier-accelerator model of Samuelson (1939),
- 2) the IS-LM model of Hicks (1939) and his successors,
- 3) the aggregate supply-aggregate demand model now used in most macroeconomics books (Dornbusch and Fischer (1978)), and
- 4) the inventory adjustment model of Metzler (1941).

These four models are the ones most commonly presented in macroeconomics courses on business cycles and stabilization policy. The properties of the four models form the basis of most economists' intuition about stabilization policy. Furthermore, most articles in the literature that deal with the dynamics of stabilization policy draw heavily on these four.

The familiar structures of the four models are integrated into a single dynamic model of the economy. Some of the structure must be modified to form a cohesive model, but only a small number of changes were made. As discussed in Chapter VI, the model could be restructured in many places to make it more sensible, more robust, or more complete.

Such modifications were, however, purposely avoided. Leaving the standard model intact makes the model results easier to compare and contrast with the results that appear in the literature.

The Multiplier-Accelerator Model. Samuelson's (1939) article on the interactions between the multiplier and accelerator phenomena presented one of the first explicitly dynamic models of the macroeconomy. Samuelson's model is only a very rudimentary representation of the economy, but it nevertheless contains several important concepts which form the basis of the model.

The first important concept in the multiplier-accelerator model is the mutual dependency of consumption and output. Consumption demand depends on the level of output, and output responds to the level of demand. Together these produce the multiplier process. Through the multiplier, a disturbance in demand produces a change in output and a proportional change in consumption, which feeds back to further disturb aggregate demand.

The next important concept in the multiplier-accelerator model is that investment depends on demand. Samuelson chose to represent investment as a function of change in consumption, but the underlying concept is that increased demand requires increased capital stock to maintain factor balance in the production process. For the purposes of this study, it is convenient to restate the relationship by expressing investment as a function of desired capital and desired capital as a

function of the expected level of demand. The dependence of investment on demand creates the accelerator process. Through the accelerator, a disturbance in demand produces a change in output and a proportional change in both desired capital and investment, which further disturbs demand.

In order to make Samuelson's model correspond to more recent treatments of a macroeconomic structure, several minor modifications have been made. First, the lag between changes in output and consumption is a simple exponential moving average (or Koyck lag) instead of a discrete lag. The lagged response of consumption corresponds to the permanent-income hypothesis of Friedman (1957). Consumption is a fraction of permanent income. Permanent income is an exponential lag of current disposable income. Current disposable income equals total output less net taxes.

Another change made in the multiplier-accelerator model is that investment depends on the difference between desired and actual capital stock rather than on the rate of change of demand. Investment is determined by a simple stock-adjustment formula for capital. Total capital investment is the sum of depreciation plus a fraction of the discrepancy between desired and actual capital. (Samuelson implicitly assumes that the fraction is one.) The actual capital stock is the accumulated difference between investment and depreciation.

A further alteration of Samuelson's basic model is the introduction of production capacity. In Samuelson's model, output is a one-period lag of demand, regardless of the capacity to produce. The assumption is changed so that output is affected by potential output as well as demand. Potential output is a simple Cobb-Douglas function of two factors, labor and capital. Both labor and capital are adjusted in response to changes in expected demand. Expected demand is a moving average of current demand. Employment responds to short-run changes in demand, while capital responds to long-run changes in demand. The difference between output and potential output is explained by changes in capacity utilization, primarily through use of overtime and undertime.

The final change made to Samuelson's model involved adding an explicit pool of labor. Labor will be referred to hereafter as employment in order to distinguish the concept of the number of people currently employed from the number of people participating in the labor force. The level of employment is assumed to adjust exponentially to desired employment. The net hiring rate is a constant fraction of the difference between desired employment and actual employment. Desired employment is directly proportional to short-run desired output.

None of the above modifications in the multiplier-accelerator model does violence to the theory underlying Samuelson's work. Instead, they simply make more explicit the disequilibrium adjustment processes that produce the multiplier and accelerator phenomena. The modified multiplier-accelerator system is illustrated in Figure 3-1. The diagram

shows each of the major variables and the causal links that connect them. Each of the five state (or stock) variables is represented by a rectangle. The modified multiplier-accelerator model is dynamically richer than the original form, since it is of fifth rather than second order. Yet the new model has added no concepts that were not already implicit.

Figure 3-1 is called a causal-loop diagram. Each arrow in the diagram represents a causal link between one variable and another. A closed chain of causal links represents a feedback loop. The polarity of a feedback loop can be determined by checking the polarity of each causal link that it contains. A positive loop contains zero or an even number of negative links. A negative loop contains an odd number of negative links. A positive loop produces self-reinforcing change, whereas a negative loop produces self-regulating change. The polarity of an individual causal link can be determined by asking the question: does a change in the driving variable produce a change in the same or opposite direction in the driven variable? If a change in the driving variable produces a change in the same direction in the driven variable, then the link is positive. If the change in the driving variable produces a change in the opposite direction in the driven variable, then the link is negative. For example, an increase in output leads to an increase in disposable income. Therefore, the link between output and income is positive. The link between capital and investment, on the other hand, is negative, because an increase in capital stock tends to reduce the rate of investment.

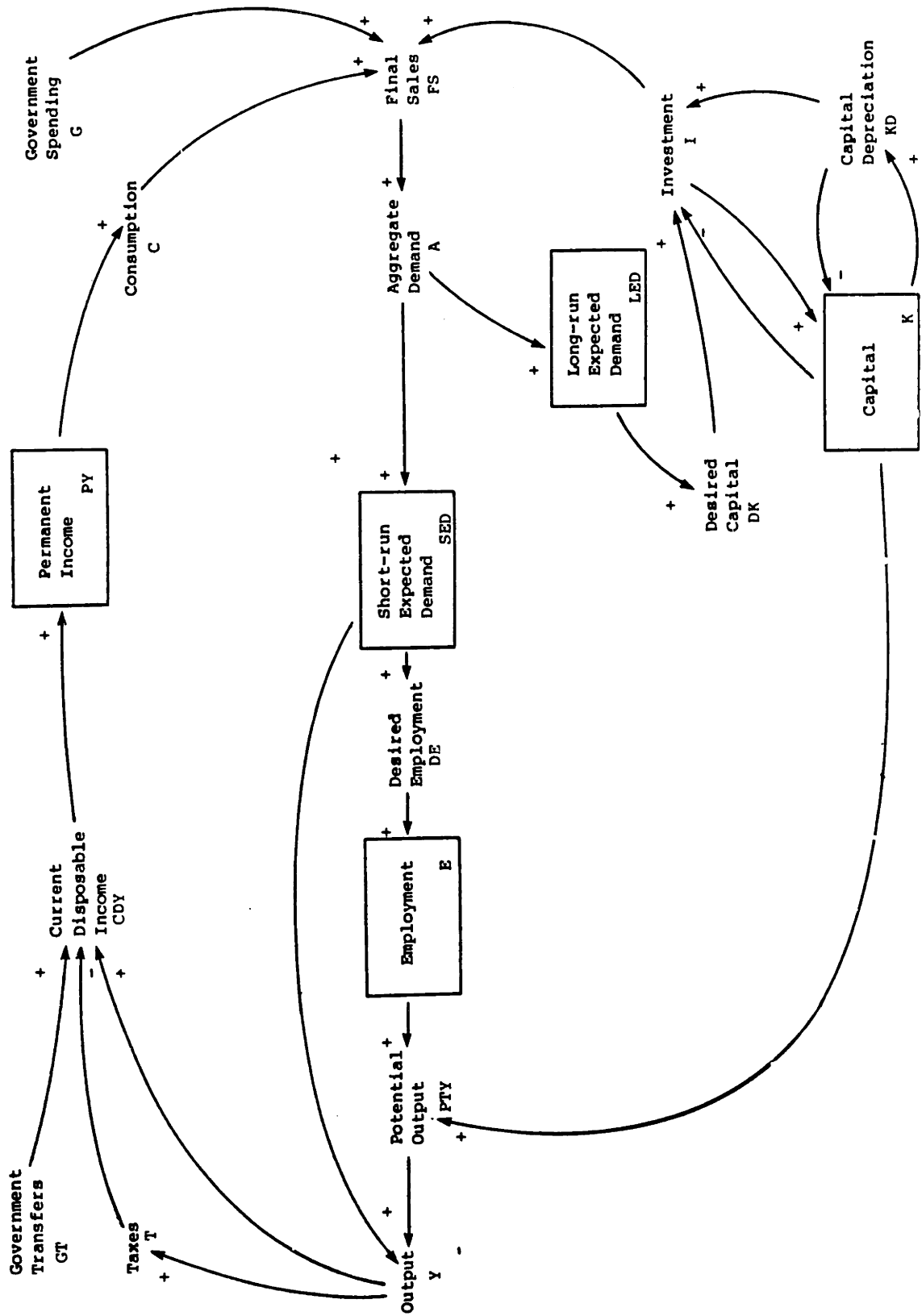


Figure 3-1. Modified Multiplier-Accelerator Model

As can be seen from Figure 3-1, both the multiplier and accelerator mechanisms are positive feedback loops. Both loops are positive since neither contains any negative causal links. The multiplier loop at the top of the figure shows how disturbances in demand can be amplified by changes in disposable income and consumption spending. Assume for the moment that a disturbance occurs in aggregate demand, and trace its effect through the multiplier loop. An increase in aggregate demand will, after a short delay, lead to an increase in short-term expected demand. An increase in short-term expected demand raises desired employment, which leads to an increase in actual output through the addition of employment. A rise in output in turn increases current disposable income and, after a lag, raises consumption spending. The increase in consumption further amplifies the original jump in aggregate demand.

In a similar manner, trace the effects of a disturbance in aggregate demand through the accelerator loop. An increase in demand raises long-run expected demand, which pushes up desired capital. A rise in desired capital increases the rate of investment, which further increases aggregate demand. Both the multiplier and accelerator loops tend to amplify demand disturbances.

The modified multiplier-accelerator model is very similar to the model proposed by Low (1976). Low, however, did not go on to incorporate interest rate, price, and inventory as done below.

The IS-LM Model. Hicks (1937) first presented the IS-LM model. Since Hicks' original article, the IS-LM model has become the standard analytical framework used in most macroeconomics textbooks. The IS-LM model can be viewed as an extension of the multiplier-accelerator model in that it adds a rudimentary financial system. In its dynamic version, the IS-LM model adds one new variable and a major negative feedback loop to the model. The new variable is the interest rate, which changes the holding cost of capital and therefore becomes an argument in the determination of the desired capital stock. Increased interest rates lead to higher capital costs, which reduce desired capital stock and suppress the rate of investment.

The interest rate itself is a function of output by a slightly more complicated argument. Assuming a fixed price level and constant desired money velocity, an increase in output leads to an increase in money demand. More money is needed in order to support a higher flow of transactions. The money supply, however, is assumed to be exogenously fixed. The interest rate is the mechanism through which possible discrepancies between money demand and money supply are reconciled. A rise in the interest rate increases the opportunity cost of holding money and therefore suppresses the demand for money. The money market is assumed to clear very quickly, so, for simplicity, the assumption is made that the interest rate instantaneously takes on whatever value is necessary to equate money demand with money supply.

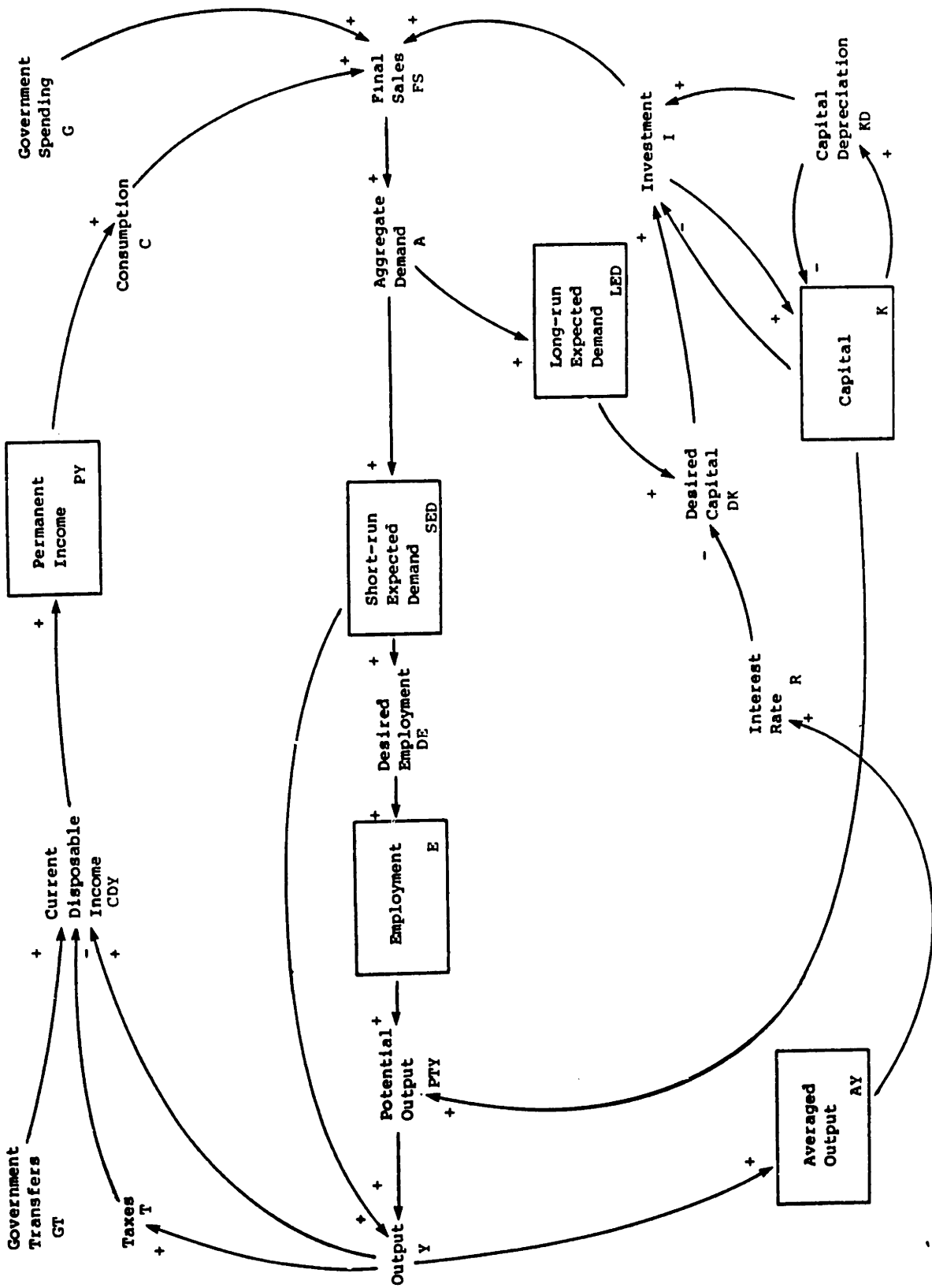


Figure 3-2. IS-LM Model (Interest Rate Added)

The causal links between output, the interest rate, and desired capital complete a new negative feedback loop in the basic multiplier-accelerator model as shown in Figure 3-2. An increase in aggregate demand raises output. A rise in output leads to an increase in average (perceived) output. An increase in average output would raise money demand, but instead the interest rate rises to maintain equilibrium in the money market. The increased interest rate raises the holding cost of capital, lowering the desired capital stock and suppressing investment. The drop in investment tends to counteract the original disturbance in aggregate demand. As can be seen in Figure 3-2, the new interest rate loop is negative since it contains a single negative causal link between interest rate and desired capital stock. The new negative loop produces compensating feedback to hold demand and output at the level which can be supported by the existing money stock and price level.

The Aggregate Demand-Aggregate Supply Model. Dornbusch and Fischer (1978) and many other authors work with the aggregate-demand model, which is a relatively recent extension of the IS-LM framework. The aggregate supply-aggregate demand model has now become standard in most macroeconomics texts at the intermediate and advanced levels. The principal new ingredient is price change. One negative feedback loop is added.

The new structure added by the aggregate supply-aggregate demand model is highlighted in Figure 3-3. In discussing the new causal links, it is perhaps best to start with the desired capital and work backward.

Desired capital is now a function of the interest rate. The interest rate is determined in much the same way as interest rate in the IS-LM model. The only difference is that price now enters the formulation as well as output and money stock. Price multiplied by output gives the nominal value of transactions ($P \cdot T = M \cdot V$). The interest rate varies so that money demand exactly equals the money supply at the current rate of nominal transactions.

The price level is determined by a Phillips curve. The Phillips curve relates price change to the unemployment rate. As the unemployment rate falls, the rate of increase in wage costs rises, forcing up prices. At high levels of unemployment, wages are bid down, reducing costs and allowing prices to fall. The unemployment rate is that fraction of a constant total labor force which is not employed. The Phillips-curve effect on price change assumes that real wage is constant. Any change in wage due to excess supply or demand for labor is passed on to price. None of the four basic macroeconomic models deals with changes in real wages.

The impact of the new negative loop can now be examined. An increase in aggregate demand leads to higher expected demand and employment. An increase in employment reduces unemployment, which, through the Phillips curve, causes price to rise. A price rise raises money demand, which raises the interest rate. The higher interest rate lowers desired capital stock and chokes off investment demand. Lower investment finally reduces aggregate demand, counteracting the original increase.

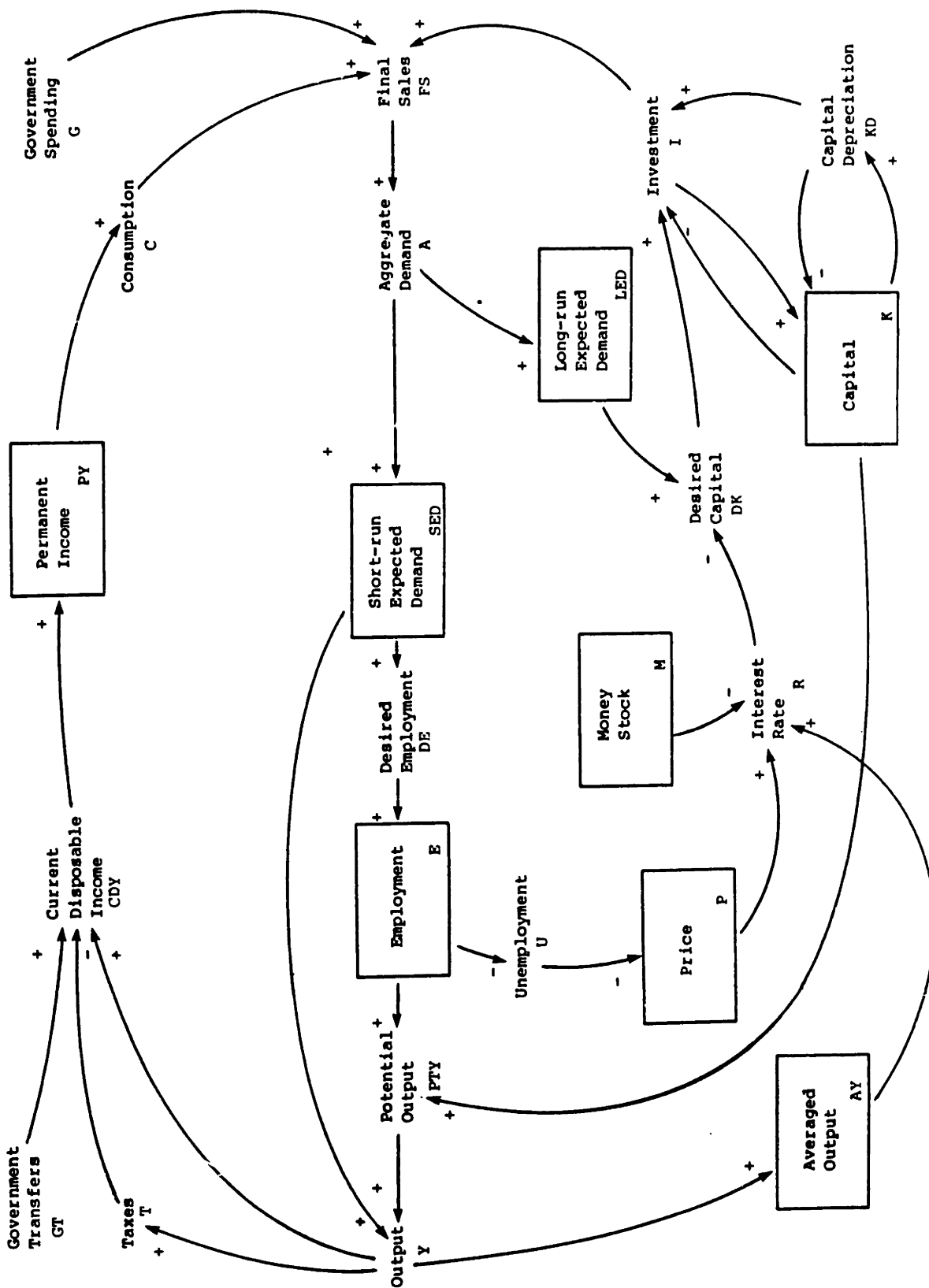


Figure 3-3. Aggregate Supply - Aggregate Demand Model (Variable Price)

4. The Inventory-Adjustment Model. Metzler (1941) outlined a series of simple inventory-adjustment models. His last and most complex model was adapted for use in this thesis. Inventory is treated as the stock of finished goods, which is augmented by output and depleted by final sales. Inventory is the integral of production minus sales. Inventory feeds back to output through desired inventory investment and expected demand. Before introducing inventories, demand was made up of only final sales for consumption, investment, and government purchases. With inventories added, aggregate demand also includes a component for desired inventory investment. Desired inventory investment is equal to a fraction of the discrepancy between desired and actual inventory. Desired inventory is proportional to long-run expected demand.

The new inventory-adjustment structure is highlighted in Figure 3-4. The most important feature added to the model is a major negative feedback loop involving inventory and employment. The effects of the new loop can be traced starting at aggregate demand. An increase in aggregate demand raises short-run desired output, which pushes up desired employment. The increase in desired employment raises hiring and the level of employment. Higher employment leads to greater output and the accumulation of inventory. Rising inventory then suppresses desired inventory investment, closing the negative feedback loop. As will be seen in the analysis of model behavior, the inventory-employment loop plays a major role in generating a business cycle in the model.

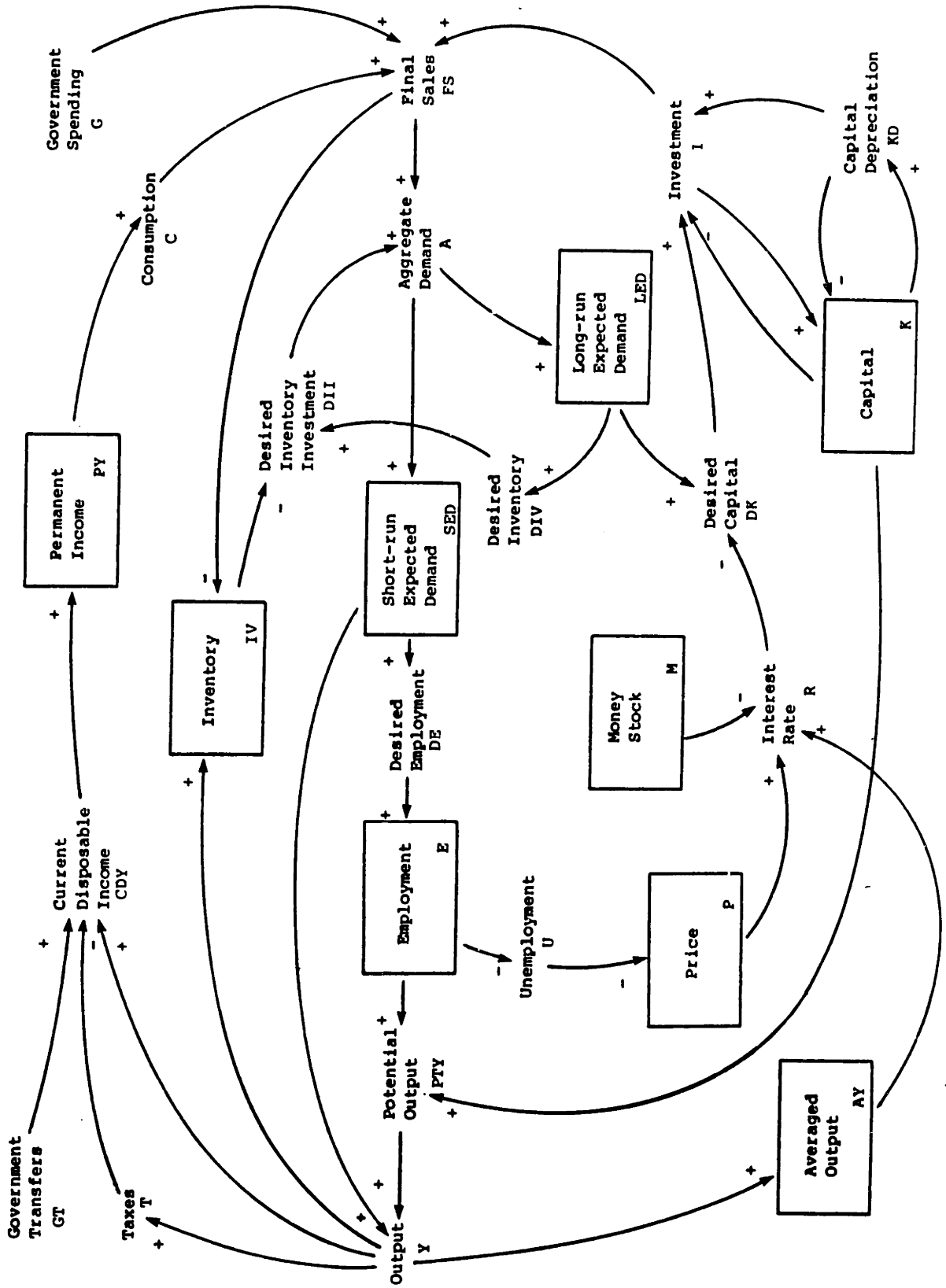


Figure 3-4. Complete Model (Inventory Added)

The model shown in Figure 3-4, with inventories, is superficially similar to that of Mass (1975). The choice of variables is much the same. The fine structure of the model, however, is quite different. Mass invoked many assumptions not generally associated with the four theoretical models discussed here.

Three features of the thesis model should be noted. First, the model is written in continuous time, not discrete time, as were the original models of Samuelson and Metzler. The continuous-time format permits the use of smooth lag functions, whose dynamic properties do not depend on discrete steps between successive computations. Second, the model deals with the dynamics of macroeconomic variables around a stationary equilibrium operating point. Growth dynamics has been omitted from the model. The implicit assumption has been made that the dynamics of growth and fluctuation are separable, that growth simply produces a trend in the equilibrium operating point. Third, the model is "demand-constrained." The supplies of labor and capital are not limited; either is available if required.

B. MODEL EQUATIONS

The base model contains twenty-seven equations. Since there are only ten state variables, the model could be simplified to ten first-order differential equations, or one tenth-order differential equation. The extra equations were added to clarify the underlying economic concepts. All twenty-seven equations are listed below, along

with the definitions of all variables and constants. A causal-loop diagram of the complete model is provided in Figure 3-5. (The only difference between Figures 3-4 and 3-5 is the inclusion of the tenth state variable, lagged unemployment, which is used only in policy analysis.)

The equations appearing in this chapter are written in simple differential equation format. A few conventions should be noted:

- 1) Variables are represented by upper-case abbreviations;
- 2) Constants are represented by lower-case abbreviations;
- 3) A dot over a variable represents its derivative with respect to time;
- 4) Multiplication is represented by an asterisk, division by a slash.

The actual equations used for simulation are written for the DYNAMO compiler and have been omitted here to avoid any possible confusion, due to special syntax, over the economic content of the model. A complete listing of equations for the DYNAMO simulation model appears in Appendix A. Appendix C lists all the DYNAMO equations with variable names and definitions interspersed.

Basic Model Equations

$$Y = PTY * (1 - fcu) + SED * fcu \quad (1)$$

$$PTY = ey * \left(\frac{E}{ee}\right)^{1-\alpha} * \left(\frac{K}{ek}\right)^{\alpha} \quad (2)$$

$$\dot{E} = (DE - E)/tae \quad (3)$$

$$DE = (1-\alpha) * SED/rw \quad (4)$$

$$\dot{SED} = (A - SED)/tssd \quad (5)$$

$$A = FS + DII \quad (6)$$

$$FS = C + I + G \quad (7)$$

$$DII = (DIV - IV)/tai \quad (8)$$

$$DIV = nic * LED \quad (9)$$

$$\dot{IV} = Y - FS \quad (10)$$

$$C = PY * apc \quad (11)$$

$$\dot{PY} = (CDY - PY)/tsy \quad (12)$$

$$CDY = Y - (T - GT) \quad (13)$$

$$I = KD + (DK - K)/tak \quad (14)$$

$$\dot{K} = I - KD \quad (15)$$

$$KD = K/alk \quad (16)$$

$$DK = \alpha * LED / (1/alk + R) \quad (17)$$

$$\dot{LED} = (A - LED) / tsld \quad (18)$$

$$R = lr * [M * (\frac{ey * ep}{eyvm})^{-1} * (\frac{AY}{ey})^{-yem} * (\frac{P}{ep})^{-1}]^{1/iem} \quad (19)$$

$$\dot{AY} = (Y - AY) / tsay \quad (20)$$

$$\dot{P/P} = spc * ((nru/U) - 1) \quad (21)$$

$$\dot{M} = 0 \quad (22)$$

$$U = (\frac{ee}{1 - nru} - E) / (\frac{ee}{1 - nru}) \quad (23)$$

$$T = tr * Y \quad (24)$$

$$GT = egt \quad (25)$$

$$G = egs \quad (26)$$

$$\dot{LU} = (U - LU) / tsu \quad (27)$$

Variable Definitions

| | | |
|-----|---|------------------------------|
| A | - | Aggregate Demand |
| AY | - | Averaged Output |
| C | - | Consumption |
| CDY | - | Current Disposable Income |
| DE | - | Desired Employment |
| DII | - | Desired Inventory Investment |
| DIV | - | Desired Inventory |
| DK | - | Desired Capital Stock |
| E | - | Employment |
| FS | - | Final Sales |
| G | - | Government Spending |
| GT | - | Government Transfers |
| I | - | Capital Investment |
| IV | - | Inventory |
| K | - | Capital Stock |
| KD | - | Capital Depreciation |
| LED | - | Long-run Expected Demand |
| LU | - | Lagged Unemployment |
| M | - | Money |
| P | - | Price Level |
| PTY | - | Potential Output |
| PY | - | Permanent Income |
| R | - | Interest Rate |
| SED | - | Short-run Expected Demand |
| T | - | Taxes |
| U | - | Unemployment |
| Y | - | Output |

Constant Definitions

| | | |
|----------|---|---|
| α | - | Coefficient on Capital in Cobb-Douglass Production Function |
| alk | - | Average Life of Capital |
| apc | - | Average Propensity to Consume |
| ee | - | Equilibrium Employment |
| egs | - | Equilibrium Government Spending |
| egt | - | Equilibrium Government Transfers |
| ek | - | Equilibrium Capital Stock |
| ep | - | Equilibrium Price Level |
| ey | - | Equilibrium Output |
| eyvm | - | Equilibrium Income Velocity of Money |
| fcu | - | Flexibility of Capacity Utilization |
| iem | - | Interest Elasticity of Money Demand |
| lr | - | Long-term Interest Rate |
| nic | - | Normal Inventory Coverage |
| nru | - | Natural Rate of Unemployment |
| rw | - | Real Wage |
| spc | - | Slope of Phillips Curve |
| tae | - | Time to Adjust Employment |
| tai | - | Time to Adjust Inventory |
| tak | - | Time to Adjust Capital Stock |
| tr | - | Tax Rate |
| tsay | - | Time to Smooth Average Output |
| tsld | - | Time to Smooth Long-run Demand |
| tsu | - | Time to Smooth Unemployment |
| tsy | - | Time to Smooth Income |
| tssd | - | Time to Smooth Short-run Demand |
| yem | - | Income Elasticity of Money Demand |

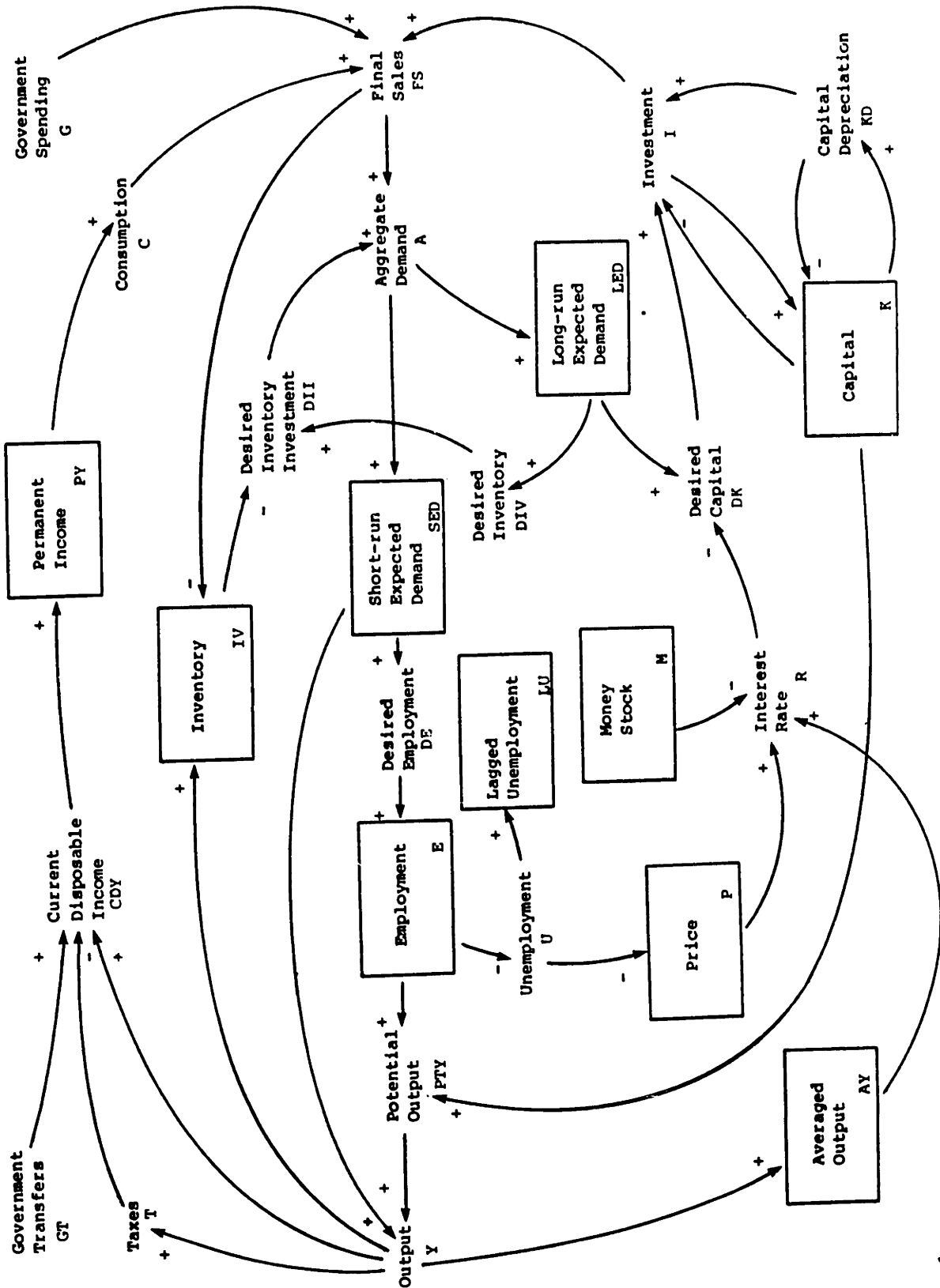


Figure 3-5. Causal Diagram of Model

Each of the model equations is discussed below starting with output and working backward through the causal chains. Each equation is repeated in the text along with the definitions and units of measure of all variables and constants which appear in it.

Output (Y). Output is a weighted average of potential output and short-run expected demand. The weighting parameter is the flexibility of capacity utilization. Higher flexibility means that output can vary more strongly with expected demand and is constrained less by potential output. Increased production in the absence of new capacity is accomplished through the use of overtime, extra shifts, and more intensive use of capital plant. Equation 3.1 defines output:

$$Y = PTY * (1 - fcu) + SED * fcu \quad (3.1)$$

Y - Output (units/year)
 PTY - Potential Output (units/year)
 fcu - Flexibility of Capacity Utilization (dimensionless)
 SED - Short-run Expected Demand (units/year)

Potential Output (PTY). Potential output is determined by the stocks of labor and capital according to a Cobb-Douglas production function, which is by far the most commonly used production function in simple macroeconomic models. The Cobb-Douglas function is mathematically convenient and exhibits most of the desirable properties of a production function. It has constant returns to scale and a diminishing marginal product to the production factors. The equilibrium share of total output (income) accruing to each production factor is constant and equal to the

factor's exponent in the production function. The equilibrium share of income to labor is equal to the exponent on labor, and the share of income to capital is equal to the exponent on capital. The constancy of equilibrium factor shares will be derived below in the discussion of desired employment. Equation (3.2) defines potential output:

$$\text{or:} \quad \text{PTY} = e_y * \left(\frac{E}{e_e}\right)^{(1-\alpha)} * \left(\frac{K}{e_k}\right)^\alpha \quad (3.2)$$

| | | |
|----------|---|-------------------------------------|
| PTY | - | Potential Output (units/year) |
| e_y | - | Equilibrium Output (units/year) |
| E | - | Employment (persons) |
| e_e | - | Equilibrium Employment (persons) |
| K | - | Capital Stock (units) |
| e_k | - | Equilibrium Capital Stock (units) |
| α | - | Exponent on Capital (dimensionless) |

The equilibrium constants e_y , e_e , and e_k are the initial values of output, employment, and capital when the model is started. The initial conditions are chosen so that the model is in full economic equilibrium at the start. Derivation of these equilibrium conditions is given in the discussion of parameter values in the last section of this chapter.

Employment (E). Employment is defined as total man-hours dedicated to production. The level of employment is determined by a simple exponential stock-adjustment formula. The stock adjustment formula can be represented as a first-order differential equation, where the net rate of change in the stock is equal to a fraction of the discrepancy between desired and actual stocks. The rate of change in employment is equal to the discrepancy between desired and actual

employment divided by the time to adjust employment. If desired employment remained constant, then actual employment would move exponentially toward desired employment with a time constant equal to the time to adjust employment.

$$\dot{E} = (DE - E)/\text{tae} \quad (3.3)$$

| | | |
|-----------|---|---|
| \dot{E} | - | Rate of Change in Employment (persons/year) |
| DE | - | Desired Employment (persons) |
| E | - | Employment (persons) |
| tae | - | Time to Adjust Employment (years) |

Desired Employment (DE). Desired employment, defined below in Equation (3.4), is proportional to short-run expected demand. Desired employment is equal to the exponent on labor in the production function multiplied by short-run expected demand divided by the real wage. The formula for desired employment is based on the standard neoclassical assumption of profit-maximizing behavior in the acquisition of production factors. In this simple model, profit is total revenues (output times price of output) less the holding cost of capital (capital times the price of capital times the depreciation rate plus the interest rate) less the cost of labor (employment times the wage rate).

$$\text{Profit} = Y * P_o - K * P_k * (1/\text{alk} + R) - E * (\text{rw} * P_o)$$

The profit-maximizing level of employment is obtained by setting the partial derivative of profit with respect to employment equal to zero and solving for employment. The result is a static equilibrium target for

employment that does not respond to changes in capital stock. The lack of cross-coupling between production factors is justifiable, because the supplies of both labor and capital are not constrained in the model. Factor imbalance resulting from unavailability of one or the other cannot occur.

$$0 = \frac{\partial \text{Profit}}{\partial E} = \frac{\partial Y}{\partial E} * P_o - (rw * P_o) = \frac{(1 - \alpha) * Y * P_o}{E} - (rw * P_o)$$

or

$$E = \frac{(1 - \alpha) * Y * P_o}{rw * P_o} = \frac{(1 - \alpha) * Y}{rw}$$

The profit-maximizing stock of capital is similarly calculated by setting the partial derivative of profit with respect to capital equal to zero and solving for capital.

$$\frac{\partial \text{Profit}}{\partial K} = \frac{\partial Y}{\partial K} * P_o - P_k * (1/alk + R) = \frac{\alpha * Y * P_o}{K} - P_k * (1/alk + R)$$

or:

$$K = \frac{\alpha * Y * P_o}{P_k (1/alk + R)} = \frac{\alpha * Y}{(1/alk + R)}$$

The cancellation of price of output P_o and the price of capital P_k occurs when capital units are scaled so that they are the same "size" as any other unit of output.

The expressions for desired capital and labor can be used to show that the Cobb-Douglas production function implies that a constant share of physical output accrues to each production factor. The total amount of real output accruing to a production factor in equilibrium is

equal to its real holding cost multiplied by the desired factor stock. For the case of capital, holding cost multiplied by desired stock equals the exponent on capital multiplied by real output. Therefore, the capital exponent is the constant fraction of output accruing to capital.

$$(1/\text{alk} + R) * \frac{\alpha * Y}{(1/\text{alk} + R)} = \alpha Y$$

In a similar manner, multiplying real wage by desired labor shows that the labor exponent is the constant fraction of output accruing to labor.

$$\text{rw} * \frac{(1 - \alpha) * Y}{\text{rw}} = (1 - \alpha) * Y$$

In conformity with the above derivation for equilibrium labor stock, desired employment is equal to the exponent on employment multiplied by expected demand, divided by the real wage. In this model real wage is a constant, since the fractional rates of change in wage and price are assumed to be equal.

$$\text{DE} = (1 - \alpha) * \text{SED}/\text{rw} \quad (3.4)$$

- DE - Desired Employment (persons)
- α - Exponent on Capital in Production Function (dimensionless)
- SED - Short-run Expected Demand (units/year)
- rw - Real Wage Rate (units/person-year)

Short-run Expected Demand (SED). Firms are assumed to have adaptive sales expectations. Short-run expected demand is therefore an exponential average of current aggregate demand. The rate of change in expected demand is proportional to the difference between current

aggregate demand and expected demand. The constant of proportionality is the inverse of the time to smooth short-run demand.

$$\dot{SED} = (A - SED)/tssd \quad (3.5)$$

\dot{SED} - Rate of Change in Short-run Expected Demand
(units/year/year)

SED - Short-run Expected Demand (units/year)

A - Aggregate Demand (units/year)

tssd - Time to Smooth Short-run Demand (years)

Aggregate Demand (A). Total aggregate demand is the sum of final sales of goods plus desired inventory investment. Final Sales is the sum of consumption, investment, and government demands on the assumption that all demands are satisfied. For the economy as a whole, demand for goods to build up inventories is indistinguishable from demand for final sales, so the two are added together.

$$A = FS + DII \quad (3.6)$$

A - Aggregate Demand (units/year)

FS - Final Sales (units/year)

DII - Desired Inventory Investment (units/year)

Final Sales (FS). Final Sales is defined as the sum of consumption demand, gross fixed capital investment, and government expenditure. Consumers, producers, and government all order goods and services from the same production sector. No attempt was made to disaggregate production into consumer, capital, and government sectors. The assumption of an aggregated production sector follows in the spirit of most simple macroeconomic models.

$$FS = C + I + G \quad (3.7)$$

FS - Final Sales (units/year)
 C - Consumption (units/year)
 I - Investment (units/year)
 G - Government Spending (units/year)

Desired Inventory Investment (DII). The desired rate of investment in inventory is determined by a simple stock-adjustment formula. A fraction of the discrepancy between desired and actual inventory is the desired inventory investment rate. The fractional rate of change is equal to the reciprocal of the time to adjust inventory. Desired and actual inventory investment will differ if production is different for aggregate demand.

$$DII = (DIV - IV)/tai \quad (3.8)$$

DII - Desired Inventory Investment (units/year)
 DIV - Desired Inventory (units)
 IV - Inventory (units)
 tai - Time to Adjust Inventory (years)

Desired Inventory (DIV). Desired inventory is assumed to be proportional to short-run expected demand. The implicit assumption being made is that a constant number of weeks' or months' worth of inventory must be held in order to cover the production and distribution delays before final sales. Many motives for holding inventory have been discussed in the literature, but most imply that desired inventory rises more or less proportionally with the sales rate (Blinder (1980)).

$$\text{DIV} = \text{nic} * \text{SED} \quad (3.9)$$

DIV - Desired Inventory (units)
 nic - Normal Inventory Coverage (years)
 SED - Short-run Expected Demand (units/year)

Inventory (IV). The rate of change in inventory is, by definition, the difference between output and final sales. Output is the only flow into inventory, and final sales is the only flow out. Inventory losses and obsolescence are ignored for simplicity. All demand is assumed to be satisfied. Supply does not constrain demand; inventory availability does not affect sales. The independence of sales from supply is a simplifying assumption which is valid in a restricted range around the equilibrium operating point of the system. Wide variations in inventory were not encountered in dynamic analysis of model behavior, so the assumption should be permissible.

$$\dot{\text{IV}} = \text{Y} - \text{FS} \quad (3.10)$$

$\dot{\text{IV}}$ - Rate of Change in Inventory (units/year)
 Y - Output (units/year)
 FS - Final Sales (units/year)

Consumption (C). The consumption function is based on the permanent income hypothesis of Friedman (1957). Consumption is equal to a constant fraction of permanent income. The constant fraction of disposable income spent on consumption goods equals the average propensity to consume. Permanent income is an exponential average of current disposable income. A drop in income does not cause an immediate drop in consumption. Instead, consumption falls slowly as the change in income comes to be regarded as permanent. The partial adjustment of

consumption to changes in income means that the short-run marginal propensity to consume will be less than the long-run average propensity to consume.

$$C = PY * apc \quad (3.11)$$

C - Consumption (units/year)
 PY - Permanent Income (units/year)
 apc - Average Propensity to Consume (dimensionless)

Permanent Income (PY). Expectations of permanent income are assumed to be adaptive. Friedman (1957) also made the assumption of adaptive expectations. Permanent income is therefore an exponential average of current disposable income. The rate of change in permanent income is a fraction of the discrepancy between current disposable income and permanent disposable income. The exponential time constant of adjustment for permanent disposable income is the time to smooth income.

$$\dot{PY} = (CDY - PY)/tsy \quad (3.12)$$

\dot{PY} - Rate of Change in Permanent Income
 (units/year/year)
 PY - Permanent Income (units/year)
 CDY - Current Disposable Income (units/year)
 tsy - Time to Smooth Income (years)

Current Disposable Income (CDY). Current disposable income is defined as total output less net taxes. Net taxes are the difference between taxes and government transfers.

$$CDY = Y - (T - GT) \quad (3.13)$$

CDY - Current Disposable Income (units/year)
 Y - Output (units/year)
 GT - Government Transfers (units/year)
 T - Taxes (units/year)

Investment (I). Fixed capital investment is given by a simple stock-adjustment formula. The base rate of investment is equal to physical capital depreciation. The base rate is modified by the need to expand or contract the capital stock. Investment is therefore equal to capital depreciation plus a fraction of the discrepancy between desired and actual capital stocks. The exponential time constant on the adjustment is the time to adjust capital.

$$I = KD + (DK - K)/tak \quad (3.14)$$

I - Investment (units/year)
 KD - Capital Depreciation (units/year)
 DK - Desired Capital (units)
 K - Capital (units)
 tak - Time to Adjust Capital (years)

Capital (K). The stock of capital is augmented by investment and depleted by capital depreciation. Therefore, the rate of change in capital is equal to the difference between investment and capital depreciation.

$$\dot{K} = I - KD \quad (3.15)$$

\dot{K} - Rate of Change in Capital (units/year)
 I - Investment (units/year)
 KD - Capital Depreciation (units/year)

Capital Depreciation (KD). Physical capital is assumed to have a constant average service life. Capital depreciation is therefore proportional to capital stock. The constant exponential decay rate on capital is the average service life of capital. The assumption of a depreciation rate based on the total capital stock ignores the importance of capital vintaging.

$$KD = K/alk \quad (3.16)$$

KD - Capital Depreciation (units/year)
 K - Capital (units)
 alk - Average Life of Capital (years)

Desired Capital (DK). The desired stock of capital was calculated along with the desired stock of employment in the discussion of Equation (3.4). Capital, unlike labor, is a long-lived asset for the production sector. Capital should respond more slowly than labor to changes in demand and inventory conditions. Desired capital is, therefore, a function of long-run expected demand, not short-run expected demand, as was labor.

$$DK = \alpha * LED / (1/alk + R) \quad (3.17)$$

DK - Desired Capital (units)
 α - Production Function Exponent on Capital
 (dimensionless)
 LED - Long-run Expected Demand (units/year)
 alk - Average Life of Capital (years)
 R - Interest Rate (fraction/year)

Long-run Expected Demand (LED). Long-run expected demand is an exponential average of aggregate demand. Expectations of long-run demand are assumed to be adaptive, just like expectations of short-run demand in

Equation (3.5). The only difference between long- and short-run expected demand is the length of the smoothing time. The time to smooth long-run demand is greater, reflecting the longer average life of capital plant as compared with labor.

$$\dot{LED} = (A - LED)/tsld \quad (3.18)$$

\dot{LED} - Rate of Change in Long-run Expected Demand
(units/year/year)

A - Aggregate Demand (units/year)

LED - Long-run Expected Demand (units/year)

tsld - Time to Smooth Long-run Demand (years)

Interest Rate (R). The interest rate is assumed to instantaneously equilibrate supply and demand in the money market. The primary determinants of money demand are the needs for transactions and precautionary balances. Transactions balances are needed to smooth expenditures in the face of lumpy income receipts. Precautionary balances are needed to reduce the probability of being caught short of cash by unexpected changes in income or expenditure patterns. Transactions balances and precautionary balances can and do overlap. Money held for one purpose can be used for another. The demand for money is usually thought of as a function of three variables: real output, price level, and interest rate. Money demand rises with real output since a higher level of real transactions requires greater real money balances. Money demand should change in direct proportion to the price level to avoid the possibility of money illusion. Money demand should vary inversely with the interest rate. An increase in nominal interest rate raises the opportunity cost of holding money. Higher money holding

costs should lead to a shorter payment period and therefore lower transactions balances. Higher holding costs also reduce precautionary balances since the expense of insuring against unforeseen cash requirements is increased. The speculative demand for money is ignored since the model does not contain sufficient detail to track wealth.

Money demand can be written as equilibrium money demand multiplied by the effects of output, price, and interest rate. Equilibrium money demand is equal to equilibrium output times equilibrium price divided by the equilibrium income velocity of money. The effect of output on money demand is current output divided by equilibrium output all raised to a constant power. The exponent on output is the income elasticity of money demand. The effect of price on money demand is current price divided by equilibrium price. The implicit exponent on price is one, so money demand rises proportionally with the price level (no money illusion). The effect of the interest rate on money demand is given by the current interest rate divided by long-run (equilibrium) interest rate all raised to a constant power. The exponent on the interest rate is the interest elasticity of money demand, which is negative.

$$\text{Money Demand} = \frac{e_y * e_p}{e_{ym}} * \left(\frac{Y}{e_y}\right)^{y_{em}} * \left(\frac{P}{e_p}\right) * \left(\frac{R}{l_r}\right)^{i_{em}}$$

The supply of money is constant in the base model. In later analysis the supply of money will be manipulated by the government in response to economic conditions. The nominal interest rate changes

instantaneously to equate money demand with current money supply. Fluctuations in the interest rate thus immediately compensate for changes in money supply, price, or output. The equilibrium interest rate can be calculated by substituting money supply for money demand in the money demand equation then solving for interest rate.

$$R = lr * [M * (\frac{ey * ep}{eyvm})^{-1} * (\frac{AY}{ey})^{-yem} * (\frac{P}{ep})^{-1}]^{1/iem} \quad (3.19)$$

- R - Interest Rate (fraction/year)
- eyvm - Equilibrium Income Velocity of Money (1/years)
- ep - Equilibrium Price (dollars/unit)
- ey - Equilibrium Output (units/year)
- lr - Long-run Interest Rate (fraction/year)
- M - Money Supply (dollars)
- AY - Average Output (units/year)
- iem - Income Elasticity of Money Demand (dimensionless)
- P - Price (dollars/unit)
- yem - Interest Elasticity of Money Demand (dimensionless)

Average Output (AY). The demand for money is assumed to be based not on the instantaneous level of output but on an exponential average of output. The assumption is very similar to the permanent income hypothesis for consumption. The rate of change in average output is proportional to the difference between current and average output. The constant of proportionality is the time to smooth average output.

$$\dot{AY} = (Y - AY)/tsay \quad (3.20)$$

- AY - Average Output (units/year)
- Y - Output (units/year)
- tsay - Time to Smooth Average Output (years)

Price (P). The rate of change in price level is a function of the change in wage costs. The cost-push effect on price change comes from excess demand in the labor market forcing up wage costs. The Phillips curve used here is a rectangular hyperbola which passes through zero price change at the natural rate of unemployment and rises to infinity as unemployment rate approaches zero. The curve is asymmetric around the natural rate of unemployment. Price falls very slowly at high unemployment and rises very rapidly at low unemployment.

$$\dot{P}/P = \text{spc} * ((\text{nru}/U) - 1) \quad (3.21)$$

\dot{P} - Rate of Change in Price (dollars/unit/year)
 P - Price Level (dollars/unit)
 spc - Slope of Phillips Curve (fraction/year)
 nru - Natural Rate of Unemployment (fraction)
 U - Unemployment Rate (fraction)

Money (M). The stock of money is assumed to be fixed exogenously by government policy. In the base model, monetary policy is not present, so the money supply is constant and its rate of change is zero.

$$\dot{M} = 0 \quad (3.22)$$

\dot{M} - Rate of Growth in Money (dollars/year)

Unemployment Rate (U). The unemployment rate is defined as the fraction of total labor force participants who are not currently employed. Unemployment rate equals the number of persons in the total labor force minus the number employed all divided by total labor force participants. The size of the labor force can be deduced from the level of equilibrium employment and the natural rate of unemployment.

$$U = \left(\frac{ee}{1 - nru} - E \right) / \left(\frac{ee}{1 - nru} \right) \quad (3.23)$$

U - Unemployment Rate (dimensionless)
 ee - Equilibrium Employment (persons)
 E - Employment (persons)
 nru - Natural Rate of Unemployment (dimensionless)

Taxes (T). Only income taxes are considered in this model.

Taxes are proportional to total output. All income is subject to taxation at a constant percentage rate. In later policy runs, the income tax will be changed to a graduated form, where a larger and larger percentage is taken as income rises.

$$T = tr * Y \quad (3.24)$$

T - Taxes (units/year)
 tr - Tax Rate (dimensionless)
 Y - Output or Income (units/year)

Government Transfers (GT). Government transfers are a constant.

In later policy runs, transfers are made a function of unemployment to represent countercyclical transfer payments.

$$GT = egt \quad (3.25)$$

GT - Government Transfers (units/year)
 egt - Equilibrium Government Transfers (egt)

Government Spending (G). Government spending on goods and services in the base model is a constant number of units per year. In the policy runs of Chapter 5, government spending is made a function of the unemployment rate to represent a countercyclical spending rule.

$$G = \text{egs} \quad (3.26)$$

G - Government Spending
egs - Equilibrium Government Spending

Lagged Unemployment (LU). The tenth and final state variable is lagged unemployment. The variable is a first-order exponential average of the unemployment rate. Lagged unemployment is only used in a special set of policy tests to examine the importance of "inside lags" in policies that are driven by the unemployment rate. Lagged unemployment plays no role in the dynamics of the basic model.

$$\dot{LU} = (U - LU)/\text{tsu} \quad (3.27)$$

LU - Lagged Unemployment (dimensionless)
U - Unemployment Rate (dimensionless)
tsu - Time to Smooth Unemployment (years)

Budget Constraints. Neither the model presented here nor any of the simple macro models from which it was constructed explicitly considers budget constraints. Households, businesses, and government are able to spend according to desired expenditure patterns whether or not their incomes equal their desired expenditure streams. In the real economy, an excess of expenditure over income in any sector must be met by borrowing from another. (A deficit in the government sector can also be met by the creation of money.) However, this apparent shortcoming is not a major limitation. According to the IS-LM and aggregate-supply/aggregate-demand models, the stock of bonds and rate of borrowing have no bearing on interest rates. Only the money supply and money demand modulate interest rates. The debt-financing of deficits, therefore, has no effect on the

variables of the model. All sectors can be assumed to finance deficits (surpluses) by borrowing (lending) without altering model structure. Government deficit financing by money creation could have been represented explicitly by modulating money supply as a function of the difference between government expenditures and receipts but was not, so as to preserve model simplicity.

As mentioned at the beginning of this section, the actual simulation model equations are listed in Appendix A. The equations in Appendix A include not only the 27 equations listed above, but also equations describing government policy rules, parameter values, initial conditions, and special equations needed to set up a simulation run and obtain output. While the structural equations of Appendix A differ in format, they contain exactly the same economic assumptions as the equations discussed above.

C. PARAMETER VALUES

Wherever possible, the parameter values were deduced from data published in statistical abstracts for the U.S. economy. In other cases, values were taken from empirical studies where parameters were estimated statistically. Where no better source was found, parameter values were taken directly from published economic modeling efforts where the author made informed guesses as to the appropriate values. No new parameter estimates were made as a part of this study. The research effort has

deemphasized parameter estimation and focused on methods for analyzing stabilization policy in a simple macromodel. The model is not used to draw firm conclusions about the effects of particular policies in the real system. If firm conclusions were to be drawn about stabilization policy, much more attention would have to be paid to improving structural formulation and estimating parameter values.

Three of the model parameters are arbitrary. They define the size of the three independent units of measure. Equilibrium employment determines the size of a labor unit. Equilibrium output determines the size of an output unit. Equilibrium price determines the size of a money unit. The values of equilibrium employment, output, and price were chosen so that labor, output, and money units correspond to the natural units of measure for the U.S. economy. Labor is measured in number of persons (or person-equivalents working an average number of hours per week). Output is measured in constant dollars' worth of production. Money is measured in dollars. The equilibrium values of employment, output, and price are chosen so that the equilibrium operating point of the model corresponds to the state of the U.S. economy in 1978. Changing the values of the arbitrary parameters does not affect model behavior.

(ee) Equilibrium Employment. The equilibrium level of employment is equal to the number of persons employed in the U.S. economy in 1978. Total employment passed 100 million during 1978, so equilibrium employment is set at 100 million people.

(ey) Equilibrium Output. Total GNP in the U.S. economy (measured in current dollars) passed the \$2 trillion mark in 1978. Equilibrium output is measured in constant dollars per year. If 1978 is taken as the base year for defining a constant dollar, then equilibrium output can be rounded to \$2 trillion 1978 dollars per year.

(ep) Equilibrium Price. Output could have been defined in terms of tons or person-years' worth of effort or some other arbitrary measure of real production. However, since output is defined in terms of dollars' worth of production, price of a unit of output must equal 1 to be consistent. One dollars worth of output must cost \$1.00. Equilibrium price therefore is equal to 1.

Many of the parameters discussed below are available either directly from U.S. Government statistical abstracts or can be inferred directly from such data, assuming that the model structure correctly represents the economy. The sources are three Department of Commerce publications: the 1980 Statistical Abstract of the U.S. (SAUS); the Historical Statistics of the United States: Colonial Times to 1970, Parts 1 and 2 (HSUS); and the Long-Term Economic Growth, 1860-1970 (LTEG). The numbers of the particular data series used to obtain each parameter are given along with the source abbreviation.

Natural Rate of Unemployment (nru). For the purposes of this model it is not necessary to have an exact estimate of the level of unemployment that corresponds to a zero rate of change in wages.

Instead, it is important to determine the approximate relative sizes of the pools of unemployed and employed persons. The natural rate of unemployment can therefore be set equal to the average rate of unemployment. The average rate of unemployment from World War II until the early 1970s was close to 4% (HSUS #86). From 1970 to 1980 the unemployment rate has averaged closer to 6% (SAUS #682). Since the recent increase in unemployment rates may or may not be permanent, the intermediate value of 5% was chosen as the natural rate of unemployment.

(egs) Equilibrium Government Spending. In the base model, government spending is a constant, so it is treated here as a parameter. Later in the policy analysis section, government spending becomes a variable which responds to changing conditions in the model. Data show that over the last decade total government spending at the federal, state, and local levels has averaged about 20% of gross national product (SAUS #724). Therefore, government spending is set at 1/5 of initial equilibrium output, or 400 billion units per year.

(eyvm) Equilibrium Income Velocity of Money. The changing structure of financial markets makes measurement of the income velocity of money difficult. During the period from 1960 to 1979, the income velocity of M1 has increased from approximately 3.6 to 6.1. Over the same period the velocity of M2 has remained relatively stable, between 1.5 and 1.6. At the same time, the income velocity of a yet broader measure of liquidity, L, has declined from 1.25 to 1.1. The pattern of changing velocities indicates that money holdings have shifted away from

non-interest bearing demand deposits to short-run interest bearing accounts and Treasury securities (SAUS #902). Most macroeconomic models use M1 or some close variant to measure the money supply. M1 is the monetary aggregate under closest control of the Federal Reserve system. The convention of using M1 for money supply will be maintained in this model even though it is losing importance as a form of storing liquid financial assets. The 1978 velocity of M1 was 5.9. The parameter eyvm is therefore set to 6.

(egt) Equilibrium Government Transfers. In the base model, government transfers are constant. Transfers are therefore treated as a parameter. In the section on policy testing, transfer payments become a variable which depends on the unemployment rate. In 1978, total transfer payments to persons from all levels of government equaled approximately 10% of gross national product (SAUS #732). Government transfers are therefore set at 10% of equilibrium output or 200 billion units per year.

(tr) Tax Rate. Taxes in the base model are a constant fraction of net national product. In the chapter on policy testing, the tax rate increases with the level of income. In equilibrium, the tax rate must provide a revenue stream exactly equal to total government outlays. In reality, taxes are some 2% less than expenditures, reflecting continuing deficits (SAUS #654). These relatively small deficits were ignored in determining model parameters. The only government outlays treated by the model are government expenditures on goods and services and transfer payments. Taxes must therefore equal government spending plus transfers.

$$(\text{Income}) * \text{tr} = \text{egs} + \text{egt}$$

$$\text{or: } (\text{ey}) * \text{tr} = \text{egs} + \text{egt}$$

$$\text{or: } \text{tr} = \frac{\text{egs} + \text{egt}}{\text{ey}}$$

(Government spending was set at 20% of equilibrium output and government transfers at 10% for a total of 30%. In 1978, total government taxes equaled approximately 37% of GNP (SAUS #480). The 7% discrepancy between taxes in the model and taxes in the real economy is accounted for by categories of government expenditure which are not treated in the model.)

(nic) Normal Inventory Coverage. For the purposes of this model, the proper measure of inventory coverage is total inventories divided by total GNP. The inventory-sales ratio for individual companies or industries is generally less than total inventory coverage of the economy due to sales of intermediate goods. The total level of inventories held by government, business, farms, and households is some 40-45% of GNP (SAUS #790). This inventory figure includes all tangible assets except land, structures, and long-life equipment. This definition of inventories is somewhat broader than appropriate for the model, since it includes household and government holding of nondurable goods after final sale. Another statistic available is the holding of non-farm business inventories, which has averaged slightly less than 25% of GNP between 1975 and 1979 (SAUS #941). Excluded are farm inventories. A

proper measure of inventories that included all finished and semi-finished goods before final sale, including agricultural goods, would probably equal approximately 30% of GNP. Normal inventory coverage is therefore at 0.3.

(α) Exponent on Capital in the Production Function. As was demonstrated earlier in this chapter, the exponent on capital is equal to the equilibrium share of income accruing to capital, if producers are assumed to maximize profits under perfect competition. While the exponent on capital is difficult to measure directly, it can be inferred from the distribution of national income. In the national income accounts, capital-consumption allowances are an estimate of the cost of replacing depreciating capital. Capital consumption allowances amount to approximately 11% of GNP (SAUS #732) over the 1970s. Profits are an additional 7% of GNP (SAUS #732). Profits are part of the net return to owners of capital plant and equipment. Another component of their return is interest paid on borrowed financial capital. The sum of capital consumption allowances, profits, and relevant interest payments is certainly greater than 18%. Another measure of income accruing to capital is the residual left after deducting compensation of employees. Employee compensation amounted to approximately 75% of national income in 1978 (SAUS #737). By this alternative method, the exponent on capital would be .25. Both measures of the capital exponent are admittedly quite rough, but do indicate that it lies in a restricted range, above 0.2. The exponent on capital in the production function is therefore set at the value of 0.25.

(lr) Long-run Interest Rate. The long-run interest rate can be estimated by taking the long-term average rate of interest charged to businesses and deducting the long-term average rate of inflation. The average rate of interest charged by banks on loans to businesses from 1919 to 1965 was 4.3% (HSUS #X466). The average compound rate of increase in the GNP deflator over the same period was 1.5% (LTGE #B62). The difference between these two figures implies an underlying real interest rate of 2.8% per year. The exact rate of interest obtained from such a calculation depends on the particular interest rate and price data used and the time period over which the calculation is made. A rate of 2.8% agrees quite closely with the rule of thumb used by many bankers that real interest rates have averaged approximately 3% since Biblical times. The long-term interest rate, therefore, is set at 3%.

(alk) Average Life of Capital. The average life of capital is a compromise between the long lifetime of plant and the shorter lifetime of machinery and equipment. The average life of capital used in national income accounting can be determined from the capital consumption allowance and the capital-output ratio. The capital-output ratio can be calculated by dividing total reproducible assets, less inventories and consumer durables, by total GNP. During the 1970s the capital-output ratio has averaged almost exactly 2 (SAUS #790). The capital-consumption allowance has equaled approximately 11% of GNP over the same period. Dividing the capital-output ratio by the fraction of GNP to capital consumption yields an estimate of the average life of capital equal to 18 years. Another estimate of the average life of capital can be obtained

from the equation for desired capital. Desired capital is set equal to actual capital and the previously determined parameters are substituted. Solving the resulting equation for average life of capital yields an estimate of approximately 11 years.

$$\frac{\alpha * Y}{(1/alk + R)} = K$$

$$\frac{.25 * Y}{(1/alk + .03)} = 2 * Y$$

$$alk = 1/((.25/2 - .03)) = 11$$

The parameter average life of capital is set at 14 years in the model as a compromise between the estimates of 11 and 18 years obtained above.

(apc) Average Propensity to Consume. In equilibrium, output must equal final sales. Under the equilibrium assumption, the average propensity to consume can be derived in terms of other parameters in the model.

$$Y = C + I + G$$

$$C = Y - I - G$$

$$PY * apc = Y - DK/alk - G$$

$$apc = \left[Y - \frac{\alpha * Y}{(1/alk + lr)} * \frac{1}{alk} - egs \right] * \frac{1}{(Y * (1 - tr) + egt)}$$

$$= \left[ey - \frac{\alpha * ey}{(1 + lr * alk)} - .2 ey \right] * \frac{1}{(ey * (1 - tr) + .1 ey)}$$

$$= 1 - \frac{.25}{(1 + .03 * 14) - .2} * \frac{1}{((1 - .3) + .1)}$$

$$= .62 * 1/.8 = .78$$

The average propensity to consume can be used to calculate the multiplier for government spending. The multiplier is the eventual increase in output resulting from a unit of additional government spending. Output is set equal to final sales, then the equation is solved for output in terms of the exogenous rate of government expenditure.

$$Y = FS = C + I + \bar{G}$$

$$= PY * apc + DK/alk + \bar{G}$$

$$= (Y * (1 - tr) + \overline{GT}) * apc + \frac{\alpha * Y}{(1/alk + lr) * alk} + \bar{G}$$

$$= Y[(1 - tr) * apc + \frac{\alpha}{(1 + lr * alk)}] + \overline{GT} * apc + \bar{G}$$

$$Y = (\overline{GT} * apc + \bar{G}) * \frac{1}{[1 - (1 - tr) * apc + \frac{\alpha}{(1 + lr * alk)}]}$$

$$= \overline{GT} * apc + \bar{G} * \frac{1}{[1 - (1 - .3) * .78 - \frac{.25}{(1 + .03 * 14)}]}$$

$$= \overline{GT} * apc + \bar{G}) * 3.6$$

So the multiplier for government spending is 3.6, which is similar to that found in other studies.

(ek) Equilibrium Capital. The equilibrium stock of capital is now fully determined by the values of α , lr , and alk . In equilibrium, ek must equal desired capital. Substituting the previously determined parameters into the equation for desired capital shows that equilibrium capital is 2.58 times output 5.16 trillion units.

$$ek = DK = \frac{\alpha * Y}{(1/alk + lr)} = \frac{.25 * ey}{(1/14 + .03)} = 2.46 * ey$$

(rw) Real Wage. The equilibrium real wage is chosen so that equilibrium employment equals desired employment. Substituting the proper expression for desired employment and solving for real wage shows that real wage is a function of previously determined parameters α , ey , and ee . The value of the real wage is therefore equal to 15 thousand units (constant dollars) per employee per year.

$$ee = DE = \frac{(1 - \alpha) * Y}{rw} = \frac{(1 - .25)ey}{rw}$$

$$rw = \frac{.75ey}{ee}$$

The values for the next five parameters are taken from previous empirical studies. The appropriate values cannot be determined directly from readily available data sources.

(yem) Income Elasticity of Money Demand. The simple optimal-inventory model of transactions demand for money can be written:

$$M_d = \sqrt{\frac{B * Y}{i}} = B^{.5} Y^{.5} i^{-.5}$$

- M_d - money demand
 B - transaction cost
 Y - nominal income
 i - interest rate (opportunity cost)

Assuming constant transactions costs, the model implies that the elasticity of money demand with respect to real income is one-half. A 1% rise in real income produces a 1% rise in nominal income and a .5% rise in money demand. Transaction costs, however, are, in part, a function of the opportunity cost of the time required to complete transactions. The opportunity cost of a person's time should rise with real income. The effect of real income on transaction cost will raise the income elasticity of money demand above one-half. Goldfeld (1973) estimated the long-run income elasticity of money demand at 0.68. While Goldfeld's money demand equation is more sophisticated than that of the thesis model, it is sufficiently close in concept to provide an acceptable measure of the parameter η_{em} . The income elasticity of money demand is therefore set at 0.7.

(iem) Interest Elasticity of Money Demand. The optimal-inventory model described in the preceding paragraph implies that the interest elasticity of money demand should equal minus one-half. Estimates by Goldfeld (1973) indicate that money demand is considerably less responsive to changes in the short-run interest rate than the inventory model would indicate. Goldfeld estimates the long-run interest

elasticity of money demand to short-run interest rates at -0.22 . The interest rate in the model affects capital investment and should behave like the long-term bond rate. Assuming that the long-term bond rate is about one-fifth as volatile as Goldfeld's interest rate, the parameter iem is set to -1 .

(tsy) Time to Smooth Income and (tsay) Time to Smooth Average Output. According to the permanent income hypothesis, consumption expenditures change slowly in response to changes in disposable income. Consumers spend on the basis of smoothed, not current, disposable income. In his book on the consumption function, Friedman (1957) concludes (page 229):

Permanent income for the community as a whole can be regarded as a weighted average of current and past measured incomes, adjusted upwards by a steady secular trend and with weights declining as one goes farther back in time. The average time span between the measured incomes averaged and current permanent income is about 2.5 years.

The time to smooth permanent income is therefore set at 2.5 years. The time to smooth average output is also set at 2.5 years on the assumption that the smoothing before consumption demand is similar to the smoothing before money demand.

(tak) Time to Adjust Capital. The time to adjust capital represents the entire delay between recognition of a change in capital needs and actual change in capital stock. The adjustment time reflects planning and delivery delays plus purposeful smoothing of investment activity. Senge (1978) performed a detailed empirical analysis of

investment behavior. He found a one-half year planning delay and a one-year delivery delay. Mass (1975), in his model of economic cycles, uses a two-year delivery delay. The time to adjust capital is set to a slightly higher value to account for intentional smoothing. The parameter τ_k is set to 3 years.

(spc) Slope of Phillips Curve. Phillips (1958) found a very stable relationship between the rate of change in money wages and the rate of unemployment in the United Kingdom. The relationship Phillips estimated indicated that a drop in unemployment from 5% to 4% would produce an increase in the inflation rate of 0.37%. A value for spc of 0.015 yields exactly the same relationship between unemployment and wage inflation in the model. Attempts to estimate a Phillips curve for the U.S. economy have been disappointing. A commonly used rule of thumb, however, has developed. The informal rule states that inflation falls one-third of a percentage point for each year that unemployment remains 1% above the natural rate. The rule of thumb implies a Phillips-curve slope of about 0.020. As a compromise between these two values, the slope of the Phillips curve is set at 0.0175. The Phillips curve used in the model is graphed in Figure 3-6.

(tai) Time to Adjust Inventory. The time to adjust inventory must be short enough that unexpected increases in inventory do not lead to prolonged, higher carrying costs and unexpected inventory decreases do not lead to erosion of market share. Inventory adjustment time must not be so short that minor variations in inventory lead to major disturbances

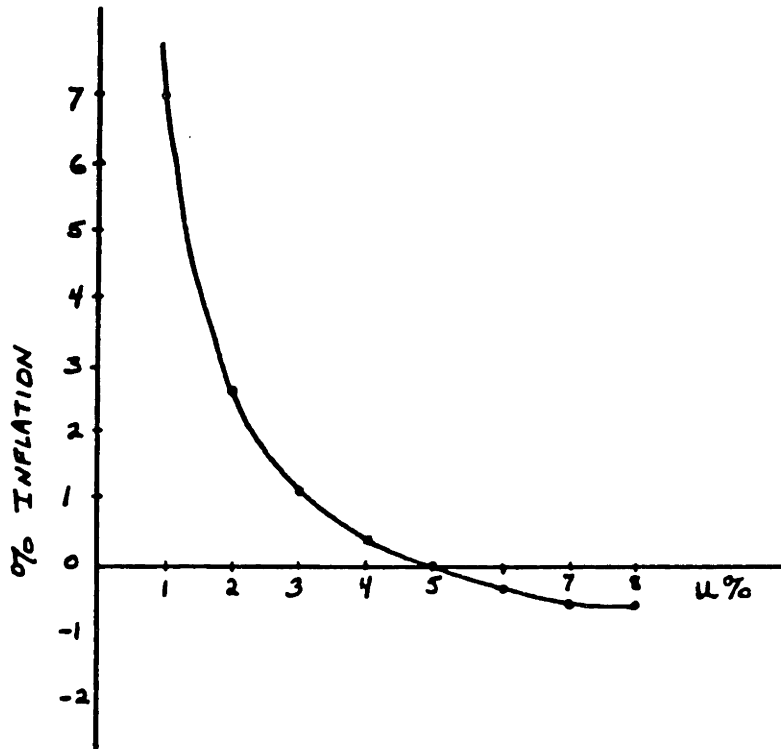


Figure 3-6. Assumed Phillips Curve

in production and employment. Senge (1978) estimated that the time to adjust inventory in nondurable manufacturing was 0.43 years. Phillips (1957) used a value of 0.5 years in his model of inventory and output dynamics. A value of 0.4 years was selected for the time to adjust inventory.

(tae) Time to Adjust Employment. Adjusting the number of employees involves two delays. First is the physical delay required to advertise job openings, select, and train a new worker. Second is a conscious policy to adjust labor slowly in order to avoid personal dislocation of employees and to avoid the costs associated with firing

and rehiring of workers. The total time to adjust employment is the sum of the physical delay in acquiring labor plus any additional smoothing due to firms' employment policies. Mass (1975) uses a delay in filling vacancies of 0.25 years and an additional time to adjust labor of 0.5 years for a total of 0.75 years. Senge (1978) uses a planning delay of 0.1 years and acquisition delay of 0.1 years, and a time to correct labor of 0.3 years, for a total of 0.5 years. Phillips (1957) uses a production adjustment lag of 0.25 years. A time to adjust employment of 0.4 years was selected as a compromise.

(tssd) Time to Smooth Short-run Demand. The time to smooth short-run demand must be long enough to smooth out minor shifts in demand but short enough to permit recognition of business-cycle variations in demand. The smoothing time must be approximately one year or less to track business-cycle fluctuations. Senge (1978) estimates the averaging time on sales for manufacturers of nondurable goods as slightly more than one-half year. Phillips (1957) averages demand over three quarters to determine desired inventory levels. Mass (1975) uses production smoothed over one year as the basis for making decisions on changes in employment and inventory. A time to average aggregate demand of 0.5 years was assumed in the model.

(tsld) Time to Smooth Long-run Demand. The time to smooth long-run demand insulates capital investment from short-run swings in demand. Because capital has such a long life, it should be adjusted only to relatively long-run changes in demand. The smoothing time should be

long enough to filter out most business-cycle fluctuations but short enough to respond to longer-term changes. A value of 3 years was chosen for the time to smooth long-run demand. The smoothing time for long-run demand is also set at 3 years.

(fcu) Flexibility of Capacity Utilization. The parameter for flexibility of capacity utilization controls the under- and overutilization of plant and undertime and overtime for labor. Mass (1975) used a function with an equivalent fcu value of .4. The parameter fcu is set at .5.

D. INITIAL CONDITIONS

The model is initialized in the full equilibrium. The equilibrium conditions can be calculated easily from the structural equations and parameters of the model. The initial values for each of the ten state variables of the model are discussed briefly below.

(SED) Short-run Expected Demand, (LED) Long-run Expected Demand, and (AY) Average Output. In equilibrium, expected demand must equal actual demand, which in turn must equal output. Also, average output must equal output. Therefore, in equilibrium, long-run expected demand, short-run expected demand, and average output are equal to equilibrium output.

$$SED = A = Y = ey$$

$$LED = A = Y = ey$$

$$AY = Y = ey$$

(E) Employment. The equilibrium level of employment has already been specified as an arbitrary parameter of the model.

$$E = ee$$

(P) Price. The equilibrium price level was also specified as an arbitrary parameter of the model.

$$P = ep$$

(K) Capital. In equilibrium, capital is equal to desired capital, which is a function of the exponent on capital, equilibrium output, the average life of capital, and the long-run real interest rate.

$$K = DK = \frac{\alpha * ey}{(1/\alpha k + lr)}$$

(M) Money. The initial value of the money supply must be chosen so that the initial interest rate equals the long-run interest rate of 3%. From the equation for the interest rate it can be seen that equilibrium money supply must equal equilibrium output times equilibrium price divided by equilibrium income velocity of money.

$$M = e_y * e_p / e_{yvm}$$

(IV) Inventory. Inventory must equal desired inventory in full equilibrium. Desired inventory is expected demand times normal inventory coverage. In equilibrium, expected demand equals equilibrium output, so inventory must equal equilibrium output times the normal inventory coverage.

$$IV = DIV = SED * nic = e_y * nic$$

(PY) Permanent Income. In equilibrium, permanent income must equal current disposable income, which is defined as output less net taxes. Net taxes equal taxes minus government transfers. Taxes are spending plus transfers, so net taxes equal equilibrium government spending.

$$PY = CDY = Y - (T - GT) = e_y - e_{gs}$$

(LU) Lagged Unemployment. Lagged unemployment is an exponential moving average of the unemployment rate used in tests of policy timing. In equilibrium, lagged unemployment must equal the natural rate of unemployment.

$$LU = U = nru$$

CHAPTER IV
MODEL BEHAVIOR

This chapter examines the behavior of the model outlined in Chapter 3. The chapter is divided into three sections. The first section analyzes a 100-year simulation run of model behavior. The periodicity and phase relationships between model variables correspond to those observed in the U.S. economy over the business cycle. The second section uses eigenvalue analysis to identify and explain the origin of behavior modes present in the model. The eigenvalues reveal the existence of a heavily damped, longer cycle that is not evident in the simulation runs. Eigenvalue elasticities demonstrate that the business cycle is caused by inventory and employment interactions, while the longer cycle is created by the multiplier and accelerator mechanisms. The long cycle of the model may not correspond to a mode of behavior in the U.S. economy; the important point is that consumption and investment spending are not part of the endogenous mechanism producing business cycles. The third section examines the frequency response of the model. Gain curves for selected disturbance-output pairs are analyzed.

A. RESPONSE TO NOISE

When stimulated by noise, the model exhibits a business-cycle mode of behavior. The cycle closely resembles the business cycle in the U.S. economy. The peak-to-peak period of the cycle over a 100-year simulation run varies between two and eight years with an average value

of 3.9 years. The lead-lag relationships between variables change from one cycle to another but, on average, correspond closely to those observed in U.S. data. Similarities between behavior of the model and behavior of the U.S. economy are examined in detail below.

Initial conditions for the ten state variables are chosen so that the model starts in equilibrium. Small amounts of random noise are input to the endogenously calculated values of output and aggregate demand. The noise terms represent supply and demand shocks unaccounted for by model structure. The two noise terms are independent, normally distributed, random variables with zero mean and a variance of 1% of equilibrium output. New values for the two random terms are drawn four times per year. Each quarterly drawing is independent of those preceding it, so the two noise processes are neither autocorrelated nor cross-correlated.

The model was run for 100 years subject to noise in output and demand. The first 15 years of the simulation are shown in Figure 4-1. The remaining 85 years are qualitatively similar to the first 15 and have been omitted to increase resolution in the figure. The model generates a series of erratic "cycles." The behavior is not exactly periodic, but most of the variables fluctuate in consistent patterns, producing swings of slightly smaller magnitude than in postwar U.S. business cycles. The unemployment rate varies between 3.5% and 7%, while output varies from 97% to 103% of its normal value.

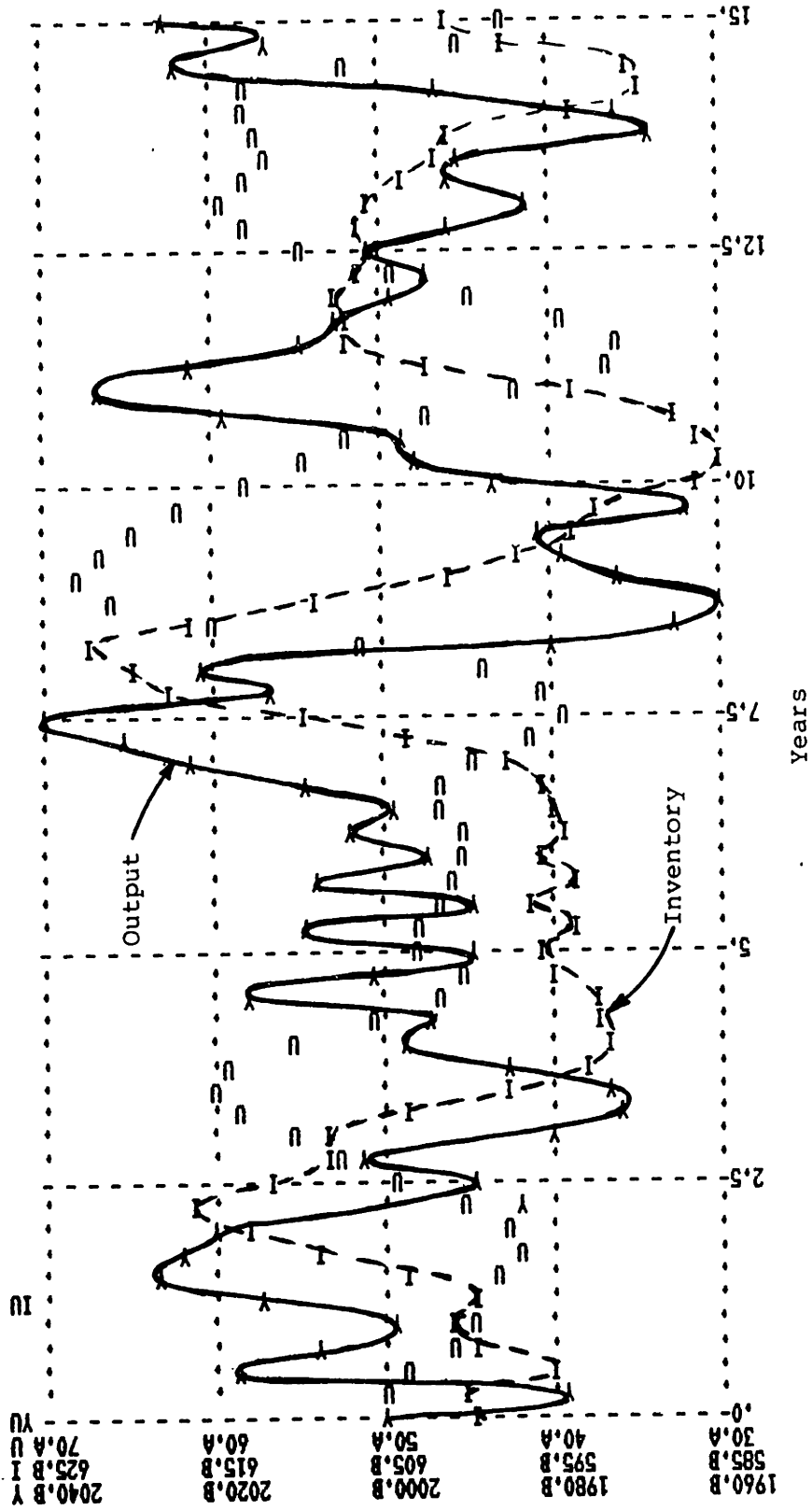


Figure 4-1. Noise Run

Periodicity. Over the 100-year noise run, the peak-to-peak period of fluctuation in output varies from 2 to 8.25 years. The mean period of oscillation is between 3.7 and 3.9 years, depending on exactly how many peaks are identified. The observed mean period of fluctuation is less than the natural, or resonant, period seen later in the eigenvalue and frequency-response sections of this chapter. The noise produces a shorter period because uncorrelated noise contains much higher power at shorter periods (higher frequencies) than at the natural or resonant period. The observed periodicity in the simulation run reflects the combined effects of noise power and amplification by the system.

The average periodicity of the model corresponds quite closely to the estimated average period of the business cycle. Burns and Mitchell (1946) estimate the average length of a business cycle at just over 4 years. In a more recent work, Moore (1980) estimates the average length of a growth cycle (a business cycle with trend growth removed) at a little over 3 years in the era since the close of World War II. Exact measures of periodicity are difficult to obtain both in the real economy and in a noise run of model behavior. Average period depends on the number of major turning points identified and is subject to error. Turning points created by large random disturbances are difficult to distinguish from turning points created by the underlying disequilibrium adjustment processes. In any case, the apparent periodicity of model output falls well within the range of uncertainty as to business-cycle period in the economy.

Moore (1980) observes marked irregularity in period from one cycle to the next:

Even a cursory examination of Table 1 attests to the validity of one aspect of the definition of business cycles cited above: business cycles are recurrent but not periodic. The variation in their duration since 1834 is obvious whether one considers the contraction phase alone, the expansion phase alone, or both phases together.... There is, however, more of a central tendency than these ranges suggest.... Half of the full cycle periods occupy the span of three to four and one-half years. But these ranges have never been tight enough for one to predict with reasonable confidence when the next turn in the cycle would come merely from knowledge from when the last turn had been reached. Business cycles are not periodic.

The irregularity of cycle period is also evident in the model simulation run. Over the 100-year run, 52% of the cycles have a period in the range from 3 to 4.5 years. The shortest is 2 years and the longest is 8.25 years.

Lead-Lag Relationships. In its study of U.S. business cycles, the National Bureau of Economic Research (NBER) has compiled lists of leading, lagging, and coincident indicators. The leading indicators are those time series which tend to peak before total output. Coincident indicators peak simultaneously with output, and lagging indicators peak after output. A few, but by no means all, of the NBER indicators correspond to variables in the model. Figure 4-2 shows the correspondence between certain NBER indicators and the related model variables. Three model variables correspond to leading indicators. Net hiring rate corresponds to the accession rate of new employees in manufacturing. Capital investment should show roughly the same movements

| <u>NBER Indicator</u> | <u>Model Variable</u> | Phase Relation To Output Y in Model | |
|--|---|--|--------------|
| | | <u>Average</u> | <u>Range</u> |
| <u>Leading:</u> | | | |
| Accession of new employees | Change in Employment (E) | +2.8 | +6 to 1 |
| Orders for Producer Durables/Commercial Building | Capital Investment (I) | +1.2 | +4 to -1 |
| Rate of Change in Business Inventories | Rate of Change in Inventories (IV) | +3 | +4 to -1 |
| <u>Coincident:</u> | | | |
| Employment in Non-agricultural Establishments | Employment (E) | -.2 | +3 to -2 |
| Unemployment | Unemployment Rate (U) | -.2 | +3 to -2 |
| GNP (constant \$) | Output (Y) | 0 | 0 |
| Personal Income | Output less Capital Depreciation (Y - KD) | 0 | 0 |
| <u>Lagging:</u> | | | |
| Manufacturers' Inventories | Inventory (IV) | -3.1 | -1 to -9 |
| Bank Interest Rate to Businesses | Interest Rate (R) | -3.3 | -1 to -8.5 |

Figure 4-2. Leading/Lagging Indicators

as orders for durable producer goods and commercial and industrial building contracts. The rate of change in inventories is essentially the same as change in business inventories. The model contains four variables which correspond to NBER coincident indicators. The level of employment should correspond closely to employment in nonagricultural establishments. The unemployment rate in the model is essentially the same as the NBER unemployment indicator. Total output corresponds to gross national product measured in constant dollars. Output less capital depreciation is reasonably close to personal income measured in dollars. Two model variables correspond to NBER lagging indicators. The level of inventories should have approximately the same timing as manufacturer's inventories. The interest rate should correspond to the NBER bank interest rate on business loans.

Figure 4-2 shows the phase relationship between the several model variables and output over the 100-year noise run. The third column in the figure gives the mean lead or lag of each variable with respect to output over 26 cycles. The fourth column gives the range of extreme leads or lags seen over the same 100-year simulation. The variability of lead-lag relationships in the economy was observed by Dow (1968):

Although nearly always some leaders lagged and some laggards lead, the important thing to note is that on the whole they performed quite well--that is, as a group leaders lead and the laggards lagged.

The exact numerical values should not be given undue weight for two reasons. First, the model is extremely aggregated and so cannot be expected to behave exactly like the economy. Second, the simulation results depend on the number and type of noise sources active in the

model and on the particular set of random drawings made during the simulation. Additional simulation runs would yield qualitatively similar but numerically different results.

All three model variables corresponding to leading indicators lead output on average over the simulation. Change in employment leads output an average of 2.8 quarters. Change in employment leads output by as much as 6 quarters and as little as 1 quarter. Capital investment leads output by a little more than 1 quarter on average, sometimes leading by 4 quarters, sometimes lagging a quarter. The rate of change in inventory leads output very little over the simulation. On average, change in inventory is only about 1 month ahead of output. Since change in inventory is the derivative of inventory, it leads inventory by one-fourth of a cycle. The fact that change in inventory leads output so little is a reflection of the fact that inventory lags output a little more in the model than it does in the economy. The discrepancy can be explained by the fact that model inventories are treated as finished goods, while economic data includes in-process inventories. The in-process inventories peak before finished goods and so cause the inventory data series for the economy to peak earlier than the model equivalent.

All four model variables which correspond to NBER coincident indicators peak at about the same time as output. Personal income and, of course, output move exactly in phase with output. Employment and unemployment lag output very slightly on average. Employment can lead by

as much as 3 quarters and lag by 2 quarters but is generally very close to output. Unemployment in the model is exactly 180° out of phase with employment and so troughs as output peaks and peaks as output troughs.

Both model variables which correspond to lagging indicators consistently lag output. Inventory peaks, on the average, 3.1 quarters after output. Inventory shows considerable variability in its phase relationship with output. In the model, the inventory lag can be as little as 1 quarter or as long as 9 quarters. Interest rate lags output by an average of 3.3 quarters. Interest rate lags by as little as 1 quarter and as much as 8.5 quarters. The interest rate used in the model is a long-term bond rate, not a short-term rate, such as the NBER bank interest rate to business. The difference between the long and short rates is one of reduced volatility rather than phase shift, so the lagging interest rate of the model is consistent with observed data.

In summary, we can conclude that the model exhibits a mode of behavior closely resembling the U.S. business cycle in four respects. First, the model shows fluctuations with approximately the same average periodicity as the business cycle; the average peak-to-peak period is between 3.7 and 3.9 years. Second, the fluctuations are not highly regular; the period varies from 2 years to 8.5 years. Third, the lead and lag relationships between major economic variables in the real economy correspond to the lead-lag relationships of the model on average. Fourth, the lead-lag relationships show considerable variation from one cycle to the next.

B. EIGENVALUE ANALYSIS

Eigenvalue analysis permits a more compact and comprehensive investigation of model behavior than does the simulation approach. Multiple behavior modes can be more clearly and easily separated. Eigenvalue sensitivity analysis readily identifies important parameters and feedback loops controlling each of the different modes. Eigenvalue analysis requires linearization of the nonlinear simulation model. Linearization is a simplification of the original model structure, which could, in principle, yield misleading results. However, linearization does not appear to distort the model behavior significantly. First, on a theoretical level, the business-cycle dynamics investigated in this thesis involve small deviations in the variables from their equilibrium values. The linearized model is a very good approximation to the full nonlinear structure in the neighborhood of the equilibrium point where the linearization is performed. Second, on a practical level, nonlinear simulations were performed to check key results based on eigenvalue analysis. In no case were the results of the linearized model inconsistent with results of the full nonlinear model.

The nonlinear simulation model described in Chapter 3 was linearized using the DYNASTAT computer package. (DYNASTAT is available from Pugh-Roberts Associates, Inc., 5 Lee Street, Cambridge, MA.) The linearization was done around the original equilibrium operating point. As discussed in Chapter 2, linearization yields a system matrix which relates the rate of change in each state variable to the values of the

states. The system matrix is square and has dimension equal to the number of states. The system has exactly as many eigenvalues as state variables. The model has ten state variables and, therefore, ten eigenvalues. While the eigenvalues and states are equal in number, each eigenvalue is not associated with a particular state. The eigenvalues describe modes of behavior present in the structure of the linearized model. Real eigenvalues correspond to simple modes of exponential growth or decay, while complex conjugate pairs of eigenvalues correspond to cyclic behavior modes.

Figure 4-3 displays information about the eigenvalues of the model. Five different characteristics of each of the ten eigenvalues are

| EIGENVALUES | | | | | |
|-------------|------------------|---------------------|-----------------|------------------|-------------------|
| | DAMPING RATIO | DAMPED FREQUENCY | DAMPING TIME | DAMPED PERIOD | NATURAL PERIOD |
| (1) | -1.00 | 0.000 | 0.0 | 000000.0 | 0.0 |
| (2) | -1.00 | 0.000 | 2.0 | 000000.0 | 2.0 |
| (3) | -0.36 | 0.234 | 11.0 | 4.3 | 4.0 |
| (4) | -0.36 | 0.234 | 11.0 | 4.3 | 4.0 |
| (5) | -0.61 | 0.033 | 39.3 | 30.0 | 23.8 |
| (6) | -0.61 | 0.033 | 39.3 | 30.0 | 23.8 |
| (7) | -1.00 | 0.000 | 15.8 | 000000.0 | 15.8 |
| (8) | -1.00 | 0.000 | 283.2 | 000000.0 | 283.2 |
| (9) | -1.00 | 0.000 | 15.7 | 000000.0 | 15.7 |
| (10) | 0.00 | 0.000 | 000000.0 | 000000.0 | 000000.0 |

Figure 4-3. Eigenvalues of Model

displayed in the figure. The five characteristics are: damping ratio, damped frequency, damping time, damped period, and natural period.

Damping ratio is the cosine of the angle of the eigenvalue. The damping ratio is -1 for stable exponential decay modes (negative real eigenvalues). Damping ratio is between zero and -1 for stable oscillatory

modes (complex conjugate eigenvalue pairs with negative real parts). The closer the damping ratio is to -1 the more stable is an oscillating mode by the damping-ratio criterion. Damped frequency is the imaginary part of complex eigenvalues and is nonzero only for oscillating modes. Lower damped frequency means increased stability by the frequency criterion. Damping time is the inverse of the real component of an eigenvalue and, for cyclic modes, corresponds to the time constant of convergence of the envelope curves that contain the oscillation. A shorter damping time indicates greater stability. Damped periods are included because most people find it easier to think in terms of periodicity than in terms of frequency; they are not used as measures of stability. Damped period is the inverse of the imaginary part of the eigenvalue (inverse of the damped frequency) and corresponds to the observed period of oscillation in simulation output. The natural period is the inverse of the magnitude of the eigenvalue, and it corresponds to the time constant of a real root or the natural period of oscillation of complex roots.

The eigenvalues in Figure 4-3 can now be examined. Five of the ten eigenvalues (1, 2, 7, 8, 9) have negative real values (a damping ratio of -1) and, so, describe stable exponential decay modes. The tenth eigenvalue is zero, because one of the state variables, money supply, is inactive except when monetary policy is in effect. The remaining four eigenvalues form two complex conjugate pairs (3-4 and 5-6), which describe two stable oscillatory modes. The third and fourth eigenvalues describe an oscillation with a natural period of 4 years and a damping ratio of $-.36$. The 4-year oscillation corresponds to the business-cycle

mode seen in the noise run. The damping ratio of $-.36$ is high enough that the fluctuation dies away in less than two cycles if left undisturbed. (Refer back to Figure 2-6 for a graph of behavior at different damping ratios.) The fifth and sixth eigenvalues describe an oscillation with a natural period just under 24 years and a damping ratio of $-.61$. The 24-year cycle, or "long cycle," is so heavily damped that it dies away before one cycle is complete. The damping is so great that the mode is invisible in the noise run.

The remainder of this section explains the structural origins of the two cyclic behavior modes. Five different techniques are employed in identifying the dominant feedback loops that generate the business cycle and the long cycle. Eigenvalue elasticities with respect both to parameters and structural links between variables are used to isolate the structures most likely to be involved in the two modes. Once potentially important structures are identified, the secondary loops are disconnected, and the eigenvalues are recomputed to confirm the continued presence of the relevant behavior mode. A simulation of model response to a single demand shock is then used to develop a description of the oscillatory mechanism. Finally, the oscillation is explained by the tendency of the dominant structure to propagate waves.

Eigenvalue Elasticities. Chapter 2 explained a technique using eigenvalue elasticities to identify the important structural links involved in creating different behavior modes. The method involves calculating the elasticities of the eigenvalues with respect to changes

in the strength of structural links between state variables. Large eigenvalue elasticities indicate important structural links. Links with small elasticities can be ignored. When the elasticities are expressed in polar coordinates, the magnitude shows the overall importance of the link, and the angle shows whether it affects primarily damping or periodicity. If the angle is near $+90^\circ$, the link primarily affects damping. If the angle is near 0 or $+180^\circ$, the link primarily affects periodicity.

Figure 4-4 lists the links that have eigenvalue elasticities with a magnitude greater than .2 for the business-cycle mode. The links

| EIGENVALUE ELASTICITIES WRT CAUSAL LINKS | | |
|--|------|--------|
| MODE-(3) - 20% CUTOFF - POLAR COORDINATES | | |
| SED > SED | 0.67 | 0.269 |
| IV > SED | 0.76 | -0.061 |
| SED > E | 0.34 | -0.173 |
| E > E | 0.35 | 0.224 |
| SED > IV | 0.43 | -0.063 |
| E > IV | 0.33 | -0.167 |

Figure 4-4. Important Links for Business Cycle

interconnect inventory, employment, and short-run expected demand. The important links form a simple feedback-loop structure, shown in Figure 4-5. The structure responsible for the business cycle is the inventory-adjustment loop of Metzler (1941). Interestingly, neither the multiplier nor the accelerator mechanism appears to play a major role in the business cycle.

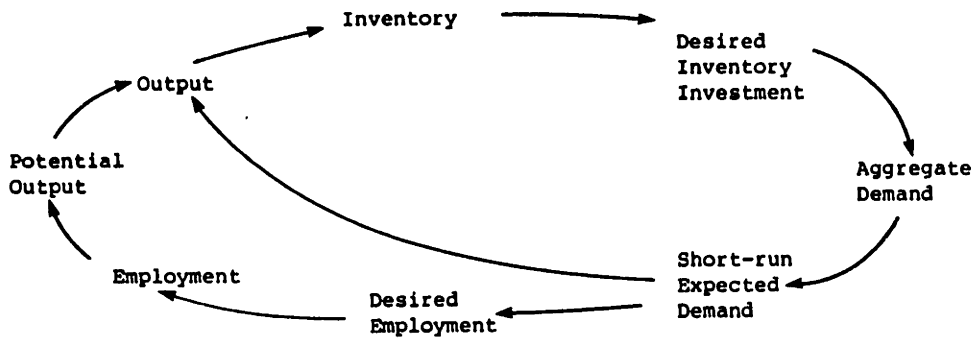


Figure 4-5. Inventory Loop

The sensitivity of the business-cycle mode to parameter changes can help confirm the dominant role of inventory adjustment in the business cycle. Figure 4-6 shows the elasticities of each of the ten eigenvalues with respect to each of the independent parameters of the model. The third and fourth columns give the elasticities for the business cycle. The two columns are complex conjugates of each other, so the information they contain is redundant. The elasticities are expressed in rectangular coordinates. As explained in Chapter 2, if the imaginary part of the elasticity is large, then the parameter strongly influences the damping ratio. If the real part is large, then the parameter strongly influences periodicity of the eigenvalue. The parameters which are most important to the business cycle can be found by scanning down the third column for elasticities with large real and/or imaginary parts.

Figure 4-6 shows that the four most influential parameters for the business-cycle mode are:

EIGENVALUE ELASTICITIES WRT PARAMETERS

| PARAM: | MODE | | | | | | | | | |
|--------|-------|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| | (1) | | (2) | | (3) | | (4) | | (5) | |
| TAE | 0.000 | 0.000 | -0.656 | -0.000 | -0.210 | -0.044 | -0.210 | 0.044 | 0.043 | 0.012 |
| TAI | 0.000 | 0.000 | -0.083 | 0.000 | -0.454 | 0.334 | -0.454 | -0.334 | 0.008 | -0.035 |
| EYVM | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LR | 0.000 | 0.000 | 0.002 | 0.000 | 0.021 | -0.040 | 0.021 | 0.040 | 0.036 | 0.273 |
| EGS | 0.000 | 0.000 | 0.001 | -0.000 | -0.005 | -0.036 | -0.005 | 0.036 | 0.175 | 0.113 |
| NRU | 0.000 | 0.000 | -0.003 | 0.000 | 0.026 | 0.040 | -0.026 | -0.040 | -0.237 | -0.105 |
| EGT | 0.000 | 0.000 | 0.000 | -0.000 | -0.002 | -0.014 | -0.002 | 0.014 | 0.070 | 0.045 |
| TAK | 0.000 | 0.000 | -0.002 | -0.000 | 0.130 | 0.037 | 0.130 | -0.037 | -0.417 | -0.154 |
| IEM | 0.000 | 0.000 | -0.003 | -0.000 | 0.024 | 0.072 | 0.024 | -0.072 | -0.184 | -0.243 |
| ALK | 0.000 | 0.000 | 0.004 | 0.000 | -0.105 | -0.053 | -0.105 | 0.053 | 0.365 | -0.158 |
| ALFA | 0.000 | 0.000 | -0.045 | 0.000 | -0.193 | 0.048 | -0.193 | -0.048 | 0.429 | -0.392 |
| YEM | 0.000 | 0.000 | 0.001 | 0.000 | 0.001 | -0.034 | 0.001 | 0.034 | -0.041 | 0.144 |
| TSY | 0.000 | 0.000 | 0.002 | -0.000 | 0.011 | -0.106 | 0.011 | 0.106 | -0.234 | 0.419 |
| TSSD | 0.000 | 0.000 | -0.242 | 0.000 | -0.460 | -0.244 | -0.460 | 0.244 | 0.084 | 0.059 |
| NIC | 0.000 | 0.000 | -0.005 | 0.000 | -0.058 | -0.036 | -0.058 | 0.036 | 0.065 | -0.045 |
| SFC | 0.000 | 0.000 | 0.003 | -0.000 | -0.025 | -0.039 | -0.025 | 0.039 | 0.228 | 0.101 |
| TSLD | 0.000 | 0.000 | 0.007 | -0.000 | 0.171 | -0.002 | 0.171 | 0.002 | -0.750 | 0.248 |
| TSAY | 0.000 | 0.000 | -0.001 | 0.000 | -0.010 | 0.034 | -0.010 | -0.034 | 0.120 | 0.026 |
| FCU | 0.000 | 0.000 | -0.197 | 0.000 | 0.209 | 0.137 | 0.209 | -0.137 | -0.132 | 0.012 |
| SGYT | 0.000 | 0.000 | 0.001 | -0.000 | -0.006 | -0.043 | -0.006 | 0.043 | 0.211 | 0.136 |
| SCGS | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| SCGT | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| SCMG | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| SCMS | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

EIGENVALUE ELASTICITIES WRT PARAMETERS

| PARAM: | MODE | | | | | | | | | |
|--------|--------|--------|---------|--------|--------|--------|---------|--------|-------|-------|
| | (6) | | (7) | | (8) | | (9) | | (10) | |
| TAE | 0.043 | -0.012 | 0.006 | 0.000 | 0.002 | -0.000 | -0.000 | -0.000 | 0.000 | 0.000 |
| TAI | 0.008 | 0.035 | 0.004 | -0.000 | -0.010 | 0.000 | -0.000 | 0.000 | 0.000 | 0.000 |
| EYVM | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| LR | 0.036 | -0.273 | 0.050 | 0.000 | 0.142 | 0.000 | -0.000 | -0.000 | 0.000 | 0.000 |
| EGS | 0.175 | -0.113 | 0.046 | 0.000 | -0.588 | -0.000 | 0.000 | -0.000 | 0.000 | 0.000 |
| NRU | -0.237 | 0.105 | -0.014 | -0.000 | -0.593 | -0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| EGT | 0.070 | -0.045 | 0.018 | -0.000 | -0.236 | -0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| TAK | -0.417 | 0.154 | -0.450 | -0.000 | 0.047 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| IEM | -0.184 | 0.243 | -0.136 | -0.000 | -0.520 | -0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| ALK | 0.365 | 0.158 | 0.197 | 0.000 | -0.395 | 0.000 | 0.000 | -0.000 | 0.000 | 0.000 |
| ALFA | 0.429 | -0.392 | 0.103 | 0.000 | 0.299 | 0.000 | 0.000 | -0.000 | 0.000 | 0.000 |
| YEM | -0.041 | -0.144 | 0.124 | 0.000 | -0.044 | -0.000 | 0.000 | -0.000 | 0.000 | 0.000 |
| TSY | -0.234 | -0.419 | -24.790 | 0.000 | 0.096 | 0.000 | 24.156 | -0.000 | 0.000 | 0.000 |
| TSSD | 0.084 | -0.059 | 0.015 | -0.000 | -0.002 | -0.000 | -0.001 | -0.000 | 0.000 | 0.000 |
| NIC | 0.065 | 0.045 | 0.004 | -0.000 | -0.013 | 0.000 | -0.001 | 0.000 | 0.000 | 0.000 |
| SFC | 0.228 | -0.101 | 0.015 | 0.000 | 0.569 | 0.000 | -0.001 | -0.000 | 0.000 | 0.000 |
| TSLD | -0.750 | -0.248 | 0.161 | -0.000 | 0.012 | 0.000 | -0.001 | 0.000 | 0.000 | 0.000 |
| TSAY | 0.120 | -0.026 | 23.940 | -0.000 | -0.003 | -0.000 | -25.137 | 0.000 | 0.000 | 0.000 |
| FCU | -0.132 | -0.012 | 0.049 | -0.000 | 0.466 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| SGYT | 0.211 | -0.136 | 0.055 | -0.000 | -0.707 | -0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| SCGS | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| SCGT | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| SCMG | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| SCMS | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

Figure 4-6. Parameter Sensitivities

- 1) Time to adjust employment, TAE,
- 2) Time to adjust inventory, TAI,
- 3) Time to smooth short-run demand, TSSD, and
- 4) Flexibility of capacity utilization, FCU.

For example, the second row of column (3) indicates that increasing the time to adjust inventory reduces the natural frequency (real component) and also increases damping (imaginary component). All four parameters are embedded in the inventory-adjustment loop shown in Figure 4-5.

One of the best ways to verify the structural origin of the business cycle is to disconnect all feedback loops in the model except the inventory-adjustment mechanism. If the business cycle remains active, then inventory adjustment would seem to be the key element. Figure 4-7 shows the eigenvalues for the linearized system with only the inventory loop active: The multiplier, accelerator, interest rate, and price loops have all been deactivated. (Severing the other loops is accomplished by setting SPC to zero and the parameters TSY, TSAY, and TAK to extremely large values.) The long cycle has disappeared, but the business cycle is still present. The business cycle is slightly less damped and has a somewhat shorter period than in the full model, but it is clearly still active. The influence of the other loops on the business cycle will be examined later in this chapter. For the moment, it is sufficient to conclude that they do not play a critical role in creating the business cycle.

Metzler (1941) and Mass (1975) and others have explained in detail how the inventory-control process can lead to oscillation. It may be useful, however, to examine how the business cycle arises in the model

EIGENVALUES

| | DAMPING RATIO | DAMPED FREQUENCY | DAMPING TIME | DAMPED PERIOD | NATURAL PERIOD |
|------|------------------|---------------------|-----------------|------------------|-------------------|
| (1) | -1.00 | 0.000 | 0.0 | 000000.0 | 0.0 |
| (2) | -1.00 | 0.000 | 2.0 | 000000.0 | 2.0 |
| (3) | -0.31 | 0.269 | 11.6 | 3.7 | 3.5 |
| (4) | -0.31 | 0.269 | 11.6 | 3.7 | 3.5 |
| (5) | -1.00 | 0.000 | 23.2 | 000000.0 | 23.2 |
| (6) | -1.00 | 0.000 | 002816.0 | 000000.0 | 002816.0 |
| (7) | -1.00 | 0.000 | 475472.0 | 000000.0 | 475472.0 |
| (8) | -0.50 | 0.000 | 000000.0 | 000000.0 | 000000.0 |
| (9) | 0.00 | 0.000 | 000000.0 | 000000.0 | 000000.0 |
| (10) | 0.00 | 0.000 | 000000.0 | 000000.0 | 000000.0 |

Figure 4-7. Eigenvalues for Inventory Loop

presented here. The mechanism is easiest to understand by tracing the response of the model to a single shock in consumption demand. Figure 4-8 shows a 7.5-year run of the full simulation model. The model is disturbed from equilibrium after one year by a sudden burst of demand. The figure plots the behavior of variables in the inventory loop.

The immediate impact of the demand shock is to draw down inventory. The reduction in inventory raises desired inventory investment. Both the original increase in demand and higher desired inventory investment add to short-run expected demand. Higher expected demand leads to greater employment. The increases in both expected demand and employment raise output, which begins to replenish inventory. The increase in employment and output continue until inventory nears its original level. With inventory back near equilibrium, desired inventory investment falls, lowering aggregate demand. As demand falls, expected demand and employment also begin to fall but are still at high levels. High employment and expected demand keep output up and drive inventory

past its equilibrium value. The further inventory goes above equilibrium, the further desired inventory investment falls, and the faster expected demand and employment fall. About two years after the disturbance in demand, output has fallen enough that final sales exceed output, and inventory begins to fall. Inventory, however, is still high and continues to drive down employment and output. By the time inventory declines back to its equilibrium level, output is below final demand, and inventory continues to decline. The system has then completed one cycle. The oscillation continues, but the amplitude declines rapidly. The second peak is much smaller than the first.

The oscillation stems from the inventory-correction process rather than from changes in final demand. Figure 4-8 shows that swings in final sales are small and 90° out of phase, with swings in total aggregate demand. The movements in aggregate demand are dominated by changes in desired inventory investment rather than changes in consumption, investment, or government spending.

The inventory loop would not oscillate if it were not for the delays in perceiving changes in demand and in adjusting employment. The delays keep output out of balance with final demand when inventory returns to its equilibrium value. The imbalance causes inventory to overshoot and undershoot, which perpetuates the cycle.

The importance of the delays can also be seen from another, more technical, perspective. The inventory loop will create business cycles

if it will propagate waves at the business-cycle frequency. The ability of a loop to propagate a wave depends on the phase shift and change in amplitude experienced by the wave as it passes through the loop. If the wave returns in phase with the original wave and with significant amplitude, then the loop tends to generate oscillations. In particular, if the phase shift is 360° (or any integer multiple of 360°) and the gain (ratio of output/input amplitude) is 1, then the loop is a perfect oscillator at the frequency in question. The inventory loop will create business-cycle oscillations to the extent that its phase shift approaches 360° and its gain approaches unity for waves with a 4-year period. (The idea of oscillatory tendencies stemming from loop phase and gain is a common one in engineering control theory, Ogata (1970), and has been elaborated by Graham (1977).)

The incremental effects of adding a new loop to a structure that has an oscillatory tendency is shown in Figure 4-9. The figure gives the qualitative effect of an additional loop on both periodicity and damping ratio as a function of the phase shift of the new loop at the natural period of the oscillator. The strength of the effect depends on the gain of the new loop. Adding a loop with a phase shift near 0° or 360° (or any integer multiple of 360°) tends to reduce damping without altering periodicity. Adding a loop with a phase shift near 90° (or 90° plus an integer multiple of 360°) tends to lengthen the period of oscillation without changing damping. Adding a loop with a phase shift near 180° (or 180° plus an integer multiple of 360°) tends to increase damping without affecting periodicity. Adding a loop with a phase shift near 270° (or

270° plus an integer multiple of 360°) tends to shorten periodicity without altering the damping ratio.

| Phase Shift | Effect on Damping | Effect on Period |
|--------------------------|-------------------|------------------|
| 90° | 0 | + |
| 180° | + | 0 |
| 270° | 0 | - |
| 0° or 360° | - | 0 |

Figure 4-9. Effect of New Loops on an Oscillator

The total phase shift of a loop structure is obtained by adding the phase shift from each variable to the next going around the loop. Phase shift is produced either by integration (passing through a state variable) or by an inverse relationship (sign reversal). Gain around a loop is calculated by multiplying the gains from one variable to the next. Figure 4-10 combines the two loops in Figure 4-5 into a single loop. Figure 4-10 shows the phase and gain for each link in the inventory loop and for the loop as a whole at a period of 4 years. Phase shift is expressed in degrees and gain in decimal fractions. The phase shift of an inverse relationship, such as that between inventory and desired inventory investment, is 180° . The phase shift through a pure integration, such as inventory, is always 90° . The phase shift through an exponential delay or a stock-adjustment process, depends on the ratio of the delay (adjustment) time to the period of the wave (see Forrester (1961), Appendix G):

$$\text{Phase shift} = \tan^{-1} \left(2 \pi \frac{\text{Delay Time}}{\text{Period}} \right)$$

For example, the phase shift from aggregate demand to short-run expected demand is:

$$\tan^{-1} \left(2 \pi \frac{\text{TSSD}}{4} \right) = \tan^{-1} \left(2 \pi \frac{.5}{4} \right) = 38^\circ.$$

The gain through a link without integration is simply the partial derivative of the dependent variable with respect to the independent variable (the corresponding element of the linear system matrix). The gain through a pure integration equals the period of the wave divided by 2π . The gain through an exponential smoothing or simple stock adjustment is:

$$\text{Gain} = \left[\left(2 \pi \frac{\text{Delay Time}}{\text{Period}} \right)^2 + 1 \right]^{-1/2}$$

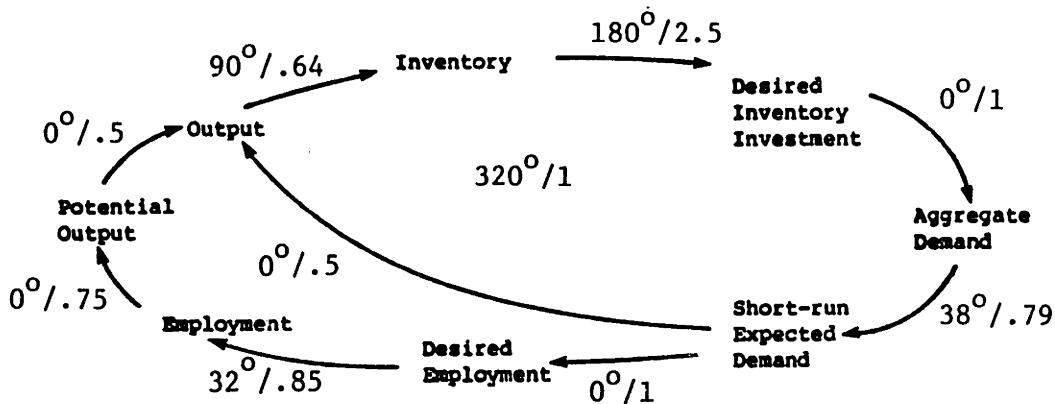


Figure 4-10. Phase and Gain for Inventory Loop

(See Forrester (1961), Appendix G.) For example, the gain through short-run expected demand is:

$$\left(\left(2 \pi \frac{\text{TSSD}}{4} \right)^2 + 1 \right)^{-1/2} = \left(\left(2 \pi \frac{.5}{4} \right)^2 + 1 \right)^{-1/2} = .79.$$

The total phase shift and gain for the inventory-adjustment loop is computed by adding the phase shifts and multiplying the gains of the individual links. The two parallel paths between short-run expected demand and output, as seen in Figure 4-5, are combined by first computing the phase shift and gain of each path separately, then adding the resulting vectors together. The path through employment has a phase shift of 32° and a gain of $.85 * .75 * .5 = .32$, while the direct path has a phase shift of 0° and a gain of $.5$. The combined (additive) effect of the two paths is a phase shift of 12° and a gain of $.79$.

The total phase shift through the inventory-adjustment loop in Figure 4-10 is 320° , which is fairly close to 360° . Gain through the loop is unity. The combination of relatively high gain and a phase shift close to one full cycle is sufficient to produce a strong oscillatory tendency. The delays in smoothing short-run expected demand and in adjusting employment add 50° of phase shift to the loop. The extra lag from the delays takes total phase shift for the loop from a neutral 270° degrees to a wave-propagating 320° . Without the two delays the loop would not produce oscillations.

Long Cycle. Having established that inventory adjustment creates the business cycle, we can now focus on causes of the long cycle. The long cycle was not visible in the noise run but did appear as a heavily damped mode in the system eigenvalues. Recall that the long cycle disappeared from the system when the multiplier, accelerator, interest rate, and price loops were cut. Presumably, the cause of the cycle lies somewhere in the loops which do not involve inventory.

The first technique for isolating the important structure is identifying the structural links which have large eigenvalue elasticities for the long-cycle behavior mode. Figure 4-11 lists the links which have elasticities greater than .2 in magnitude. The list is quite long and contains most of the links present in the model, including all those of the inventory loop. The structure controlling the long cycle can be isolated more clearly, but the link elasticities are not a great help in this case. The problem can be traced to the way the elasticities are calculated. Recall that the links are structural connections between state variables; each link corresponds to an element in the system matrix

EIGENVALUE ELASTICITIES WRT CAUSAL LINKS
MODE-(5) - 20% CUTOFF - POLAR COORDINATES

| | |
|-----------|------------|
| PY > PY | 0.74-0.023 |
| SED > PY | 0.32-0.383 |
| E > PY | 0.26-0.397 |
| PY > SED | 0.50 0.103 |
| SED > SED | 0.80-0.257 |
| K > SED | 0.47-0.489 |
| P > SED | 0.26 0.375 |
| IV > SED | 0.84-0.460 |
| LED > SED | 1.27 0.076 |
| SED > E | 0.41 0.203 |
| E > E | 0.43-0.311 |
| K > K | 0.76 0.372 |
| P > K | 0.33 0.236 |
| AY > K | 0.20 0.464 |
| LED > K | 0.85-0.064 |
| E > P | 0.25 0.066 |
| PY > IV | 0.63-0.423 |
| SED > IV | 0.50 0.217 |
| E > IV | 0.40 0.203 |
| K > IV | 0.69-0.015 |
| P > IV | 0.32-0.151 |
| LED > IV | 0.83-0.450 |
| PY > LED | 0.48-0.377 |
| K > LED | 0.44 0.031 |
| P > LED | 0.25-0.105 |
| IV > LED | 0.79 0.060 |
| LED > LED | 0.44-0.405 |

Figure 4-11. Important Links for Long Cycle

of the linearized model. The links of the linear model do not correspond to links in the simulation model, because the nonstate variables have to be simplified out. Links in the full and linearized models will match only in cases where the variables between states do not involve branching. In general, changing the strength of a link in the full model corresponds to changing several links in the linear model in different degrees. The eigenvalue link elasticities can therefore be misleading in cases where the important loops contain branching at nonstate variables (at investment rate, for example). The problem can be solved by extending the link elasticity approach, as discussed in Chapter 6, but the technique was not utilized here for lack of appropriate software.

Eigenvalue elasticities with respect to the parameters reveal more than the link elasticities do about the causes of the long cycle. The elasticities of the long-cycle eigenvalues with respect to the parameters were given previously in columns 5 and 6 of Figure 4-6. The five most important parameters are:

- 1) Time to adjust capital, TAK,
- 2) Average life of capital, ALK,
- 3) Exponent on capital, ALFA,
- 4) Time to smooth long-run demand, TSLD, and
- 5) Time to smooth income, TSY.

All five parameters are involved in the accelerator and multiplier loops, but not in the inventory loop.

The Accelerator Loop. Adding the accelerator and multiplier structures to the basic inventory-adjustment mechanism confirms their

role in producing the long cycle. Figure 4-12 shows the eigenvalues of the system when the accelerator is added to the basic inventory loop. The multiplier, interest rate, and price loops remain disconnected (SPC = 0, and both TSY and TSAY are extremely large). Comparing the eigenvalues of Figure 4-12 to those in Figure 4-7 shows that activating the accelerator loop causes the long cycle to emerge. The business cycle remains active and is only slightly modified by the addition of the accelerator loop. The business-cycle period increases slightly from 3.5 to 3.9 years, and the damping ratio increases from $-.31$ to $-.34$. The long-cycle period is a little over 19 years, and the damping ratio is a strong $-.64$.

| EIGENVALUES | | | | | |
|-------------|------------------|---------------------|-----------------|------------------|-------------------|
| | DAMPING RATIO | DAMPED FREQUENCY | DAMPING TIME | DAMPED PERIOD | NATURAL PERIOD |
| (1) | -1.00 | 0.000 | 0.0 | 000000.0 | 0.0 |
| (2) | -1.00 | 0.000 | 2.0 | 000000.0 | 2.0 |
| (3) | -0.34 | 0.239 | 11.7 | 4.2 | 3.9 |
| (4) | -0.34 | 0.239 | 11.7 | 4.2 | 3.9 |
| (5) | -0.64 | 0.040 | 30.3 | 25.1 | 19.3 |
| (6) | -0.64 | 0.040 | 30.3 | 25.1 | 19.3 |
| (7) | -1.00 | 0.000 | 210176.0 | 000000.0 | 210176.0 |
| (8) | -1.00 | 0.000 | 000000.0 | 000000.0 | 000000.0 |
| (9) | 0.00 | 0.000 | 000000.0 | 000000.0 | 000000.0 |
| (10) | 0.00 | 0.000 | 000000.0 | 000000.0 | 000000.0 |

Figure 4-12. Eigenvalues for Inventory-Accelerator System

Analyzing the phase shift and gain of the accelerator mechanism shows why it produces a long cycle. Figure 4-13(a) shows the accelerator mechanism as it appears in the full model. Figure 4-13(b) shows the same structure with the various links connecting desired capital to investment

condensed into one. Figure 4-13(b) also shows the phase shift and gain of each link in the accelerator loop for a wave with a 24-year period (the natural period of the long cycle in the full model). The total phase shift around the accelerator loop is almost exactly 360° . If its gain were high, the loop would be a nearly perfect oscillator. The gain, however, is only .41. The accelerator mechanism therefore exhibits only a mild tendency to oscillate.

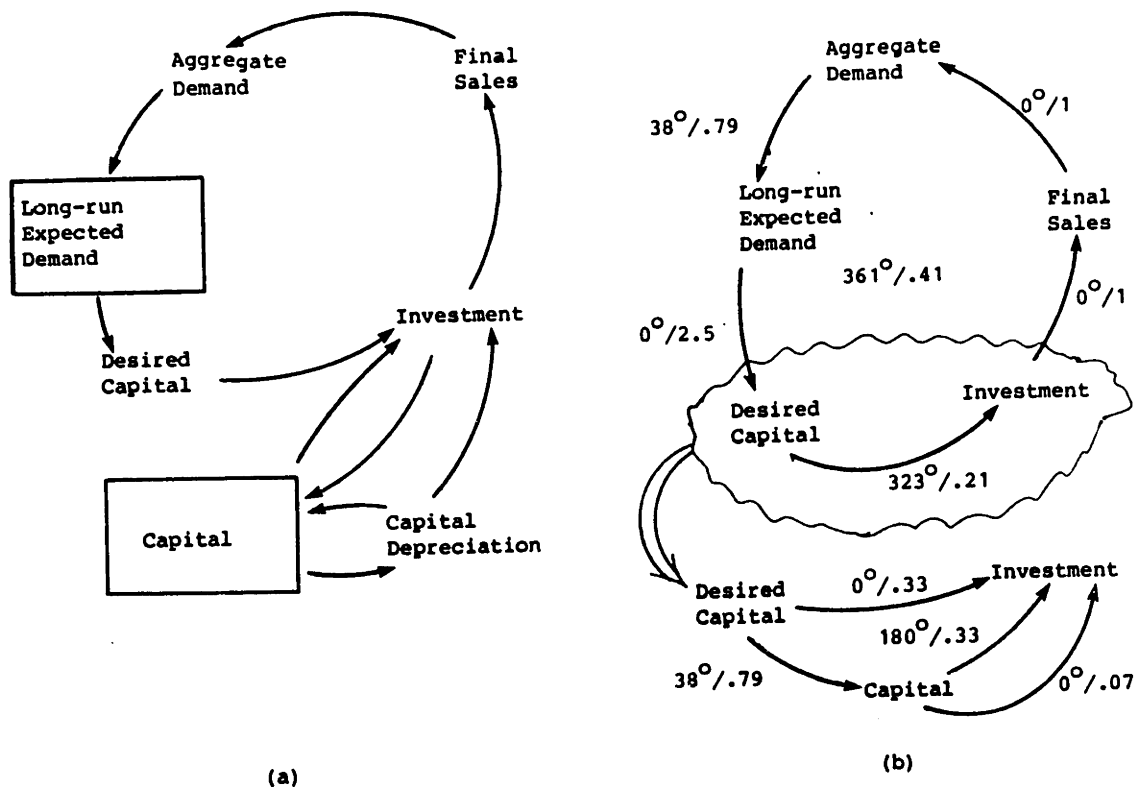


Figure 4-13. Accelerator Loop

The Multiplier loop. The impact of the multiplier loop on both the business cycle and the long cycle is shown in Figure 4-14. The figure gives the eigenvalues of the inventory-accelerator system with the multiplier loop activated. The interest rate and price loops are still disconnected (SPC = 0, and TSAY is extremely large). The multiplier stabilizes the business cycle and does not change its periodicity. The business-cycle damping ratio increases from $-.34$ to $-.41$, while the natural period remains at 3.9 years. The multiplier destabilizes the long cycle and lengthens its period. The long-cycle damping ratio drops from $-.64$ to $-.42$, while the natural period increases from 19.3 to 32.2 years. The multiplier therefore has a pronounced effect on the long cycle and only a mild interaction with the business cycle.

| EIGENVALUES | | | | | |
|-------------|------------------|---------------------|-----------------|------------------|-------------------|
| | DAMPING RATIO | DAMPED FREQUENCY | DAMPING TIME | DAMPED PERIOD | NATURAL PERIOD |
| (1) | -1.00 | 0.000 | 0.0 | 000000.0 | 0.0 |
| (2) | -1.00 | 0.000 | 2.0 | 000000.0 | 2.0 |
| (3) | -0.42 | 0.235 | 9.1 | 4.3 | 3.9 |
| (4) | -0.42 | 0.235 | 9.1 | 4.3 | 3.9 |
| (5) | -0.41 | 0.028 | 78.6 | 35.3 | 32.2 |
| (6) | -0.41 | 0.028 | 78.6 | 35.3 | 32.2 |
| (7) | -1.00 | 0.000 | 17.5 | 000000.0 | 17.5 |
| (8) | -0.65 | 0.000 | 000000.0 | 000000.0 | 000000.0 |
| (9) | 0.00 | 0.000 | 000000.0 | 000000.0 | 000000.0 |
| (10) | 0.00 | 0.000 | 000000.0 | 000000.0 | 000000.0 |

Figure 4-14. Eigenvalues for Inventory-Accelerator-Multiplier System

The impact of the multiplier loop can be explained by its phase shift and gain at business-cycle and long-cycle frequencies. Figure 4-15 shows the inventory and multiplier loops with the phase shifts and gains

for a 4-year cycle. The multiplier adds a loop that connects output to final sales, passing through permanent income and consumption. The multiplier and inventory loops share the links from aggregate demand through expected demand and employment to output. The new multiplier loop has a total phase shift of 126° and a gain of .09 at the business-cycle period of 4 years. The phase shift is closer to 180° than to a multiple of 360° , so the loop has a stabilizing effect. The stabilizing effect is small, because the gain is low.

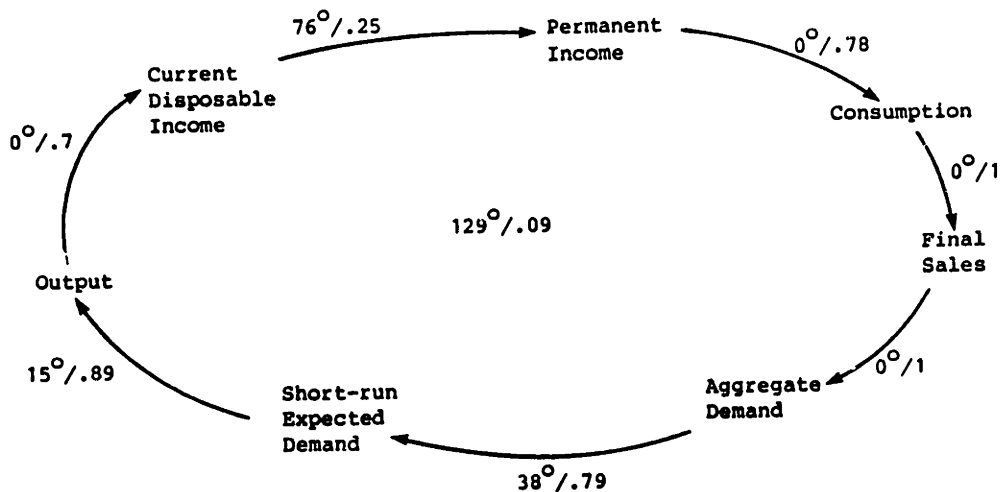


Figure 4-15. Phase and Gain at Business-cycle Period

Figure 4-16 shows the accelerator loop with the multiplier loop added. (The figure also contains the interest rate and price loops to avoid repetition of the diagram when they are added.) The phase shift and gain of all the links are given at the long-cycle period of 24 years.

The multiplier loop through output, permanent income, and aggregate demand has a phase shift of 44° and a gain of .38. The multiplier loop adds phase shift closer to zero than 180° , so it accentuates the long cycle, reducing its damping ratio. Furthermore, since the loop has a shift between 0° and 90° , it tends to lengthen the period. The multiplier, then, has opposite effects on the business cycle and long cycle. The different effects arise from the changes that occur in the phase-shift and amplitude characteristics of the multiplier loop between the business-cycle and long-cycle periodicities.

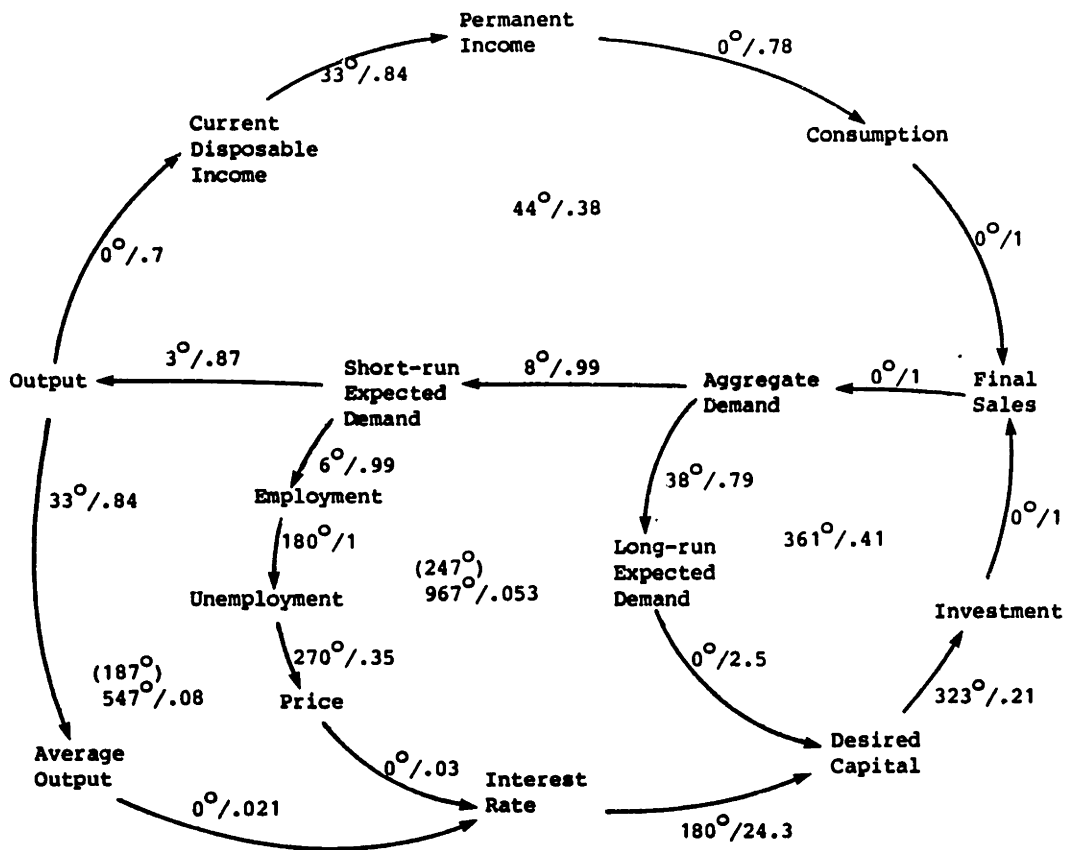


Figure 4-16. Phase and Gain at Long-cycle Period

The IS-LM/Interest-Rate Loop. Having noted that the multiplier stabilizes the business cycle and destabilizes the long cycle, we can turn our attention to the effect on IS-LM/interest-rate loop. Figure 4-17 gives the eigenvalues for the inventory-accelerator-multiplier model with interest rate active. Only the price loop remains disconnected (SPC = 0). The business cycle is almost unaffected by the addition of interest rate; neither damping nor period changes significantly. Interest rate does, however, affect the long cycle. The damping ratio increases from $-.41$ to $-.55$. The period of the long cycle is unchanged.

| EIGENVALUES | | | | | |
|-------------|------------------|---------------------|-----------------|------------------|-------------------|
| | DAMPING RATIO | DAMPED FREQUENCY | DAMPING TIME | DAMPED PERIOD | NATURAL PERIOD |
| (1) | -1.00 | 0.000 | 0.0 | 000000.0 | 0.0 |
| (2) | -1.00 | 0.000 | 2.0 | 000000.0 | 2.0 |
| (3) | -0.40 | 0.237 | 9.8 | 4.2 | 3.9 |
| (4) | -0.40 | 0.237 | 9.8 | 4.2 | 3.9 |
| (5) | -0.55 | 0.026 | 57.4 | 38.1 | 31.8 |
| (6) | -0.55 | 0.026 | 57.4 | 38.1 | 31.8 |
| (7) | -1.00 | 0.000 | 16.0 | 000000.0 | 16.0 |
| (8) | -1.00 | 0.000 | 15.7 | 000000.0 | 15.7 |
| (9) | 0.00 | 0.000 | 000000.0 | 000000.0 | 000000.0 |
| (10) | 0.00 | 0.000 | 000000.0 | 000000.0 | 000000.0 |

Figure 4-17. Eigenvalues for Inventory-Accelerator-Multiplier-Interest Rate System

The impact of the interest-rate loop on damping in the long cycle can be explained by its phase and gain characteristics at the long-cycle period. Figure 4-16 showed the phase shift and gain of each link in the interest-rate loop. The total phase shift is 547° or,

equivalently, 187° . The near 180° shift gives the IS-LM/interest-rate loop a stabilizing effect on the long cycle without changing its periodicity. The gain of .08 is low enough that the damping effect is minor.

The Price Loop. The last loop to examine is the price loop, added by the aggregate supply - aggregate demand model. The eigenvalues of the inventory-accelerator-multiplier-interest rate model with the price loop active are the eigenvalues of the full model already shown in Figure 4-3. All loops are now active. The business cycle shows a very slight decrease in damping, from $-.40$ to $-.36$, and an insignificant increase in period, from 3.9 to 4.0 years. The business cycle is almost unaffected by the price loop. The long cycle, however, is more strongly influenced by the price loop. Long cycle damping increases a little from $-.55$ to $-.61$, while the period drops sharply from 31.8 to 23.8 years. The effects can be explained by the phase shift and gain of the price loop at the long-cycle period. The phase and gain of the loop were shown in Figure 4-16. The total phase shift of the loop was 607° or, equivalently, 247° . The phase shift lies between the damping shift of 180° and the period-shortening shift of 270° , though closer to 270° . Not surprisingly, the loop increases damping slightly and shortens the period of oscillation significantly. The gain of the loop is only .05, so its effect is not overpowering.

C. FREQUENCY-RESPONSE ANALYSIS

This section presents information about the frequency response of the model. Frequency response measures the tendency for a system to amplify disturbances at different frequencies. Gain curves show how sensitive a particular output variable is to disturbances from a particular source at different frequencies. A peak in the gain curve for a particular disturbance-output pair means that the output variable is especially sensitive to disturbances with frequencies in a band around the peak. The system will selectively amplify frequencies near the peak. Most stochastic processes contain some power at all frequencies, so a system will tend to oscillate at or near the frequencies where it has maximum gain. Changes in gain do not necessarily correspond to changes in damping or other measures of transient response. The theory of gain curves was presented in Chapter 2 and should be reviewed if necessary.

In principle, the gain curve is different for each pair of disturbance source and output variable. This study examines gain curves for two disturbances and four output variables. The two disturbances most commonly discussed as sources of random shocks in the economy are supply shocks and demand shocks. Supply disturbances are represented by a noise term in the equation for output. Demand disturbances are represented by a noise term in the equation for aggregate demand. (Note that the disturbance in demand is not added into final sales and, so, corresponds to a disturbance in desired inventory investment. An equally plausible formulation, with potentially different results, would add the

disturbance into final sales.) The four output variables include the two most commonly discussed targets of stabilization policy, employment and disposable income, as well as the level of inventory and the rate of capital investment. The 2 disturbances and 4 outputs form 8 disturbance-output combinations and, so, a matrix of 8 gain curves. Additional disturbance sources or output variables could have been selected but were not, in order to keep the analysis relatively simple.

Figure 4-18 shows the matrix of eight gain curves for the full linearized model. Each row corresponds to a disturbance source. Each column corresponds to an output variable. The disturbance-output combination for each curve is shown vertically to the left of the curve. The first row corresponds to disturbances in supply; the second to disturbances in demand. The columns correspond to the output variables (in order left to right): current disposable income, employment, inventory, and investment. Most of the gain curves show a peak around the business-cycle frequency of .25 cycles per year. Several of the curves show a resonant peak at the long-cycle frequency of about .05 cycles per year. The long-cycle peaks appear to be smaller and narrower than the business-cycle peaks. The long-cycle peaks are smaller because the long cycle is heavily damped and so does not amplify disturbances between most sources and outputs as much as the business cycle. The long-cycle peaks appear narrower than the business-cycle peaks because the frequency scale is linear. Many more gain values are calculated in the range of the business cycle than the long cycle. A logarithmic scale would facilitate visual comparison; however, the increased resolution at high frequencies is useful for evaluating business-cycle stabilization policy.

Most of the gain curves in Figure 4-18 show a broad peak around the business-cycle frequency. All the output variables will exhibit one tendency to fluctuate at the business-cycle frequency in response to noise in output and/or demand. If the noise is wide-band (that is, contains significant power across many frequencies), then the resulting business cycles will have an erratic period. The gain curves show considerable amplification between the frequencies of .1 and .4 cycles per year. The corresponding range of periodicities is 10 to 2.5 years.

The differences between gain curves depend on where the disturbances enter the structure and what outputs are measured. Disturbances which enter the structure in different places will excite the different feedback loops in varying degrees. Disturbances from different sources are filtered differently by the system structure before they reach any particular feedback loop. The choice of output variables also affects gain. A variable which lies in a loop that receives little or no stimulation will show little gain. For example, the gain curves for investment are lower than for other output variables around the business-cycle frequency, because investment is not strongly coupled to the inventory-adjustment mechanism.

A detailed explanation of the differences between gain curves for different disturbance-output pairs is not necessary to stabilization-policy analysis. Instead, an explanation must be developed for changes in the gain curves that occur when policies are activated. Chapter 5 compares the gain curves before and after activating each policy. A drop

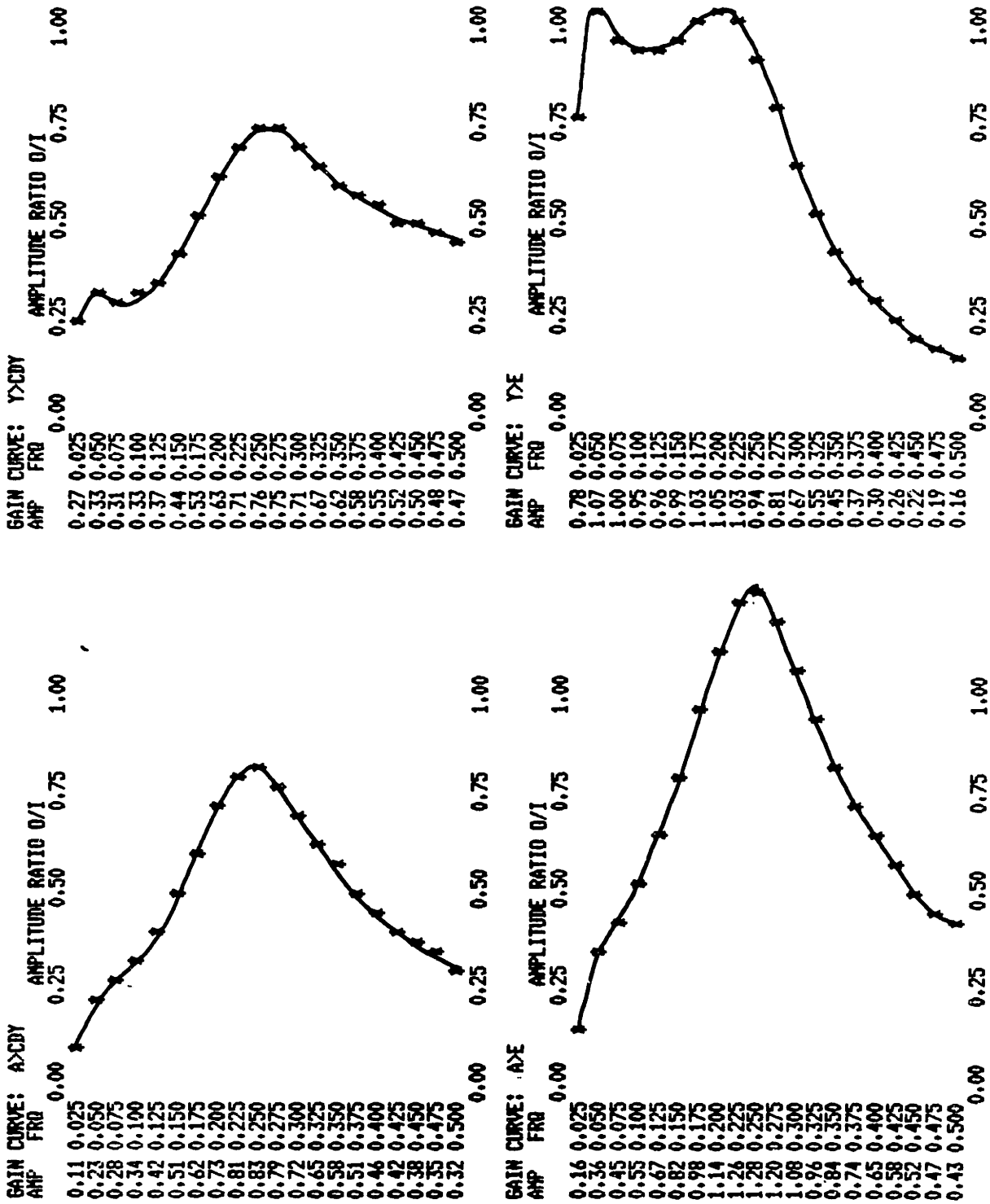


Figure 4-18(a). Gain Curves of Model

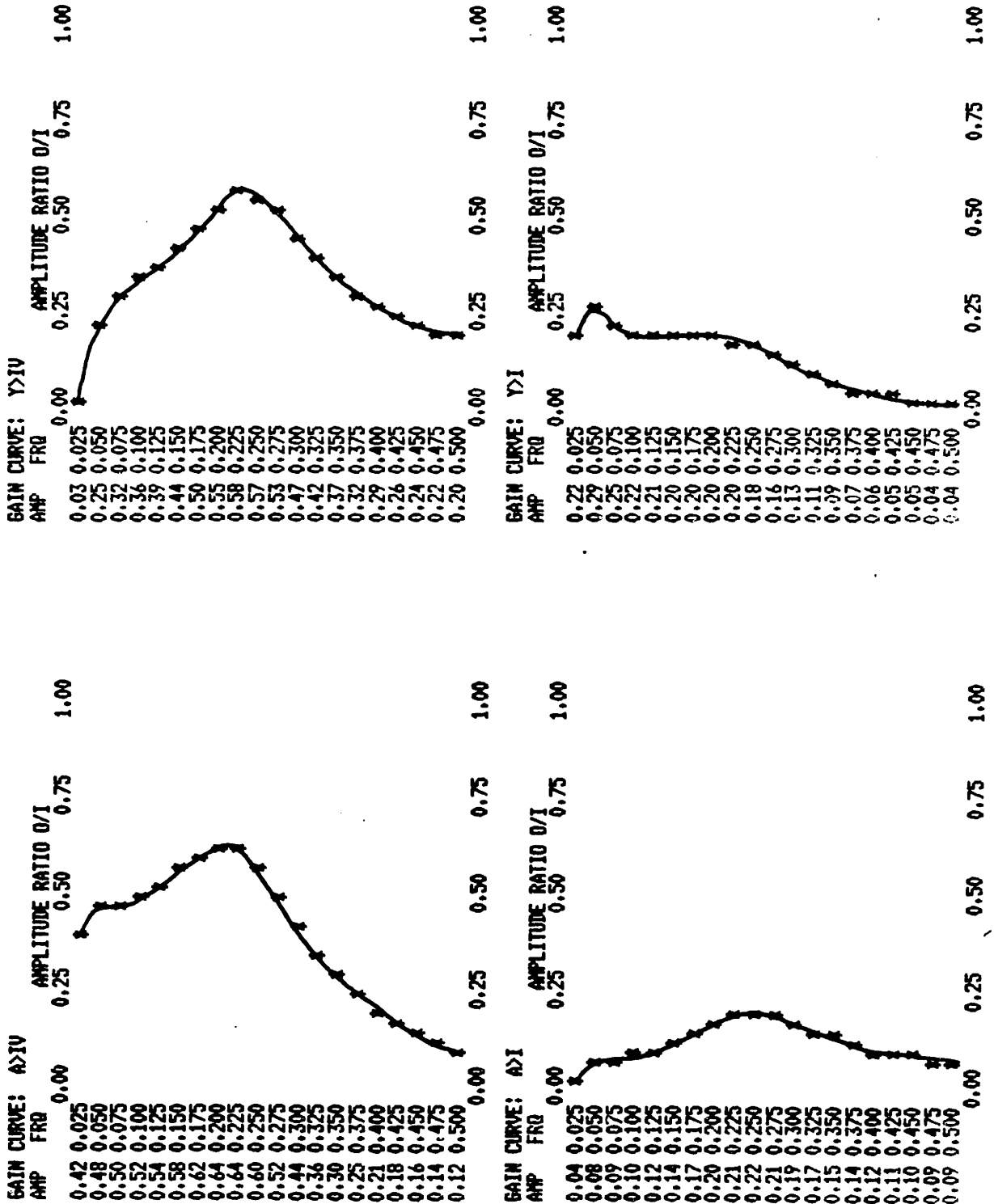


Figure 4-18(b). Gain Curves of Model

in gain indicates a stabilizing influence, while an increase in gain means that a policy heightens sensitivity to disturbances. The section on policy interpretation explains why some policies have different effects on different curves in the gain matrix.

D. SUMMARY

In analyzing the behavior of the model, several points have come to light.

- 1) The model contains two cyclic behavior modes:
 - a) a 4-year business cycle, and
 - b) a 24-year "long" cycle.
- 2) The business-cycle mode exhibits a periodicity and phase relationships between variables which are characteristic of business cycles in the U.S. economy.
- 3) The business cycle is created by the inventory-adjustment process discussed by Metzler (1941). The business cycle is not driven by changes in final demand.
- 4) The multiplier, accelerator, and capital-stock adjustment mechanisms do not cause the business cycle. Instead they generate a much longer cycle.
- 5) The interest-rate loop of the IS-LM model and the price loop of the aggregate supply - aggregate demand model do not interact with the business cycle but play minor roles in damping the long cycle.
- 6) Eigenvalue elasticities are a useful tool for identifying the feedback loops that create different behavior modes in the model.
- 7) The phase shift and gain characteristics of the major feedback loops explain their impact on behavior.

- 8) The frequency-response analysis shows that the model selectively amplifies disturbances in a frequency range around the business-cycle and long-cycle frequencies.
- 9) The gain curves show that different variables have different sensitivities to disturbances from different sources.

CHAPTER V
POLICY ANALYSIS

A. INTRODUCTION

This chapter examines how the model responds to various business-cycle stabilization policies. Stabilization policies are usually broken into two general categories: automatic stabilizers and discretionary policies. Automatic stabilizers are policies embedded in system structure that respond automatically to changing economic conditions. They require no overt government action to take effect. Discretionary policies, on the other hand, are general rules for government action which require recognition of changes in conditions, followed by overt action. Of the four stabilization policies most commonly discussed, two are automatic stabilizers and two are discretionary policies. The two automatic stabilizers are a graduated personal income tax and a system of countercyclic transfer payments such as unemployment insurance. The two discretionary policies are countercyclic government spending and countercyclic monetary policy. Monetary policy is tried in two versions: a money-growth rule and a money-stock rule.

The policies could be implemented in the model in many different ways. In each case, the simplest formulation was chosen. All policies, both automatic and discretionary, respond instantly to changing economic conditions. "Inside lags" (lags between change in conditions and change

in policy action) are assumed to be zero in all cases. In reality, discretionary policy requires time to recognize changing conditions, time to decide on a policy action, and time to act on the decision. The assumption of no inside lags was made in order to test each policy under "ideal" conditions where lags are very short. The policies cannot, however, avoid the "outside lags." Outside lags (lags between policy action and effect) are set by the structure of the system and are necessarily encountered as policy changes propagate through the system. To further ensure comparability between policy experiments, each policy has roughly the same "size." The first-order effects of each policy on aggregate demand are of comparable magnitude. The exact structural changes needed to implement each policy are discussed along with the results of each policy experiment.

The effects of each policy are evaluated by several different stability criteria. The policies are judged by three different measures of transient response and by one measure of frequency response. The effects of a policy on transient response are measured by changes in damping ratio, damped frequency, and damping time. The effects of a policy on frequency response are measured by changes in the gain curves of the model before and after policy implementation. The policy results by the different criteria are compared and contrasted.

The chapter is arranged in three sections. The first section shows how each policy is implemented, then how and why it affects model behavior. In the second section, tests are performed to find the "best"

and "worst" timing for demandmanipulation rules for stabilizing the business cycle. The last section summarizes the policy implications of the modeling effort.

B. POLICY IMPLEMENTATION AND RESULTS

Countercyclical Government Transfers. A system of countercyclical transfer payments is one of the most commonly discussed automatic stabilizers. The transfer scheme implemented in the model corresponds to a simple unemployment insurance program. In the original model, transfers were constant. Under the countercyclical transfer-payment policy, transfers are a linear function of the unemployment rate. Transfers are high when unemployment is high and low when unemployment is low. The policy action is automatic, requiring no overt government action to vary the transfer stream. Changes in the unemployment rate produce immediate changes in transfers; the policy action involves no "inside lags." The policy raises disposable income during downturns in employment. The object of the policy is to keep consumption demand from falling during recessions. The policy is based on the assumption that increasing demand during recessions (and lowering demand during booms) will stabilize business-cycle fluctuations.

The transfer-payment policy is implemented by adding a term to the otherwise constant stream of government transfers:

$$\begin{aligned} GT &= egt + CGT \\ CGT &= egt * scgt * (U - nru) \end{aligned}$$

GT -- government transfers (units/year)
 CGT -- change in government transfers (units/year)
 egt -- equilibrium government transfers (units/year)
 scgt -- strength of countercyclic government transfers
 (dimensionless)
 U -- unemployment rate (dimensionless)
 nru -- natural rate of unemployment (dimensionless)

The parameter scgt controls the strength of the countercyclical transfers policy. When the scgt is set to zero, transfers are constant as in the original model. The policy becomes more exaggerated as scgt is set to larger values. The parameter scgt is set to 2.67 in the policy test. Figure 5-1 shows graphically how the transfer policy is formulated.

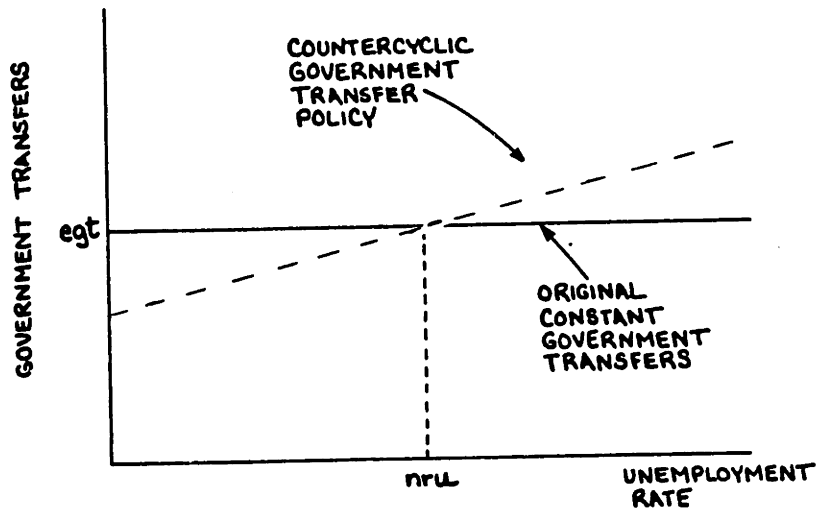


Figure 5-1. Government Transfer Policy

A value for the strength of countercyclic government transfers of 2.67 means that a rise of one percentage point in the unemployment rate will create a .2% rise in aggregate demand. The following computation derives the equilibrium change in aggregate demand as a

function of a change in unemployment rate. In making the calculation, the time delay through smoothing of permanent income is ignored.

$$\begin{aligned}
 \Delta A &= \Delta C = \text{apc} * \Delta PY = \Delta CDY = \\
 &= \text{apc} * \Delta GT = \text{apc} * \Delta CGT = \text{apc} * (\text{egt} * \text{scgt} * \Delta U) \\
 &= .75 * .1\text{ey} * 2.67 * .01 \\
 &= .002 \text{ey} = .002 A
 \end{aligned}$$

The other stabilization policies, examined later in this section, are given strengths that also lead to a .2% change in aggregate demand for a 1% change in the unemployment rate.

Figure 5-2 shows the loop that is added to the model when the transfer policy is activated. The loop actually has two channels between final sales and aggregate demand, which technically makes it two loops. One channel goes directly to aggregate demand, while the other passes through inventory and desired inventory investment. For convenience, the two links are collapsed into one which represents the combined effect of both.

The behavioral effects of the new loop can be anticipated by its phase-and-gain characteristics. Each link in the loop is shown with its phase shift and gain at both the business-cycle and long-cycle periods. In those cases where phase shift and/or gain varies with period, the figures for the long cycle are given in parentheses. By adding the phase shifts and multiplying the gains, phase-and-gain characteristics of the

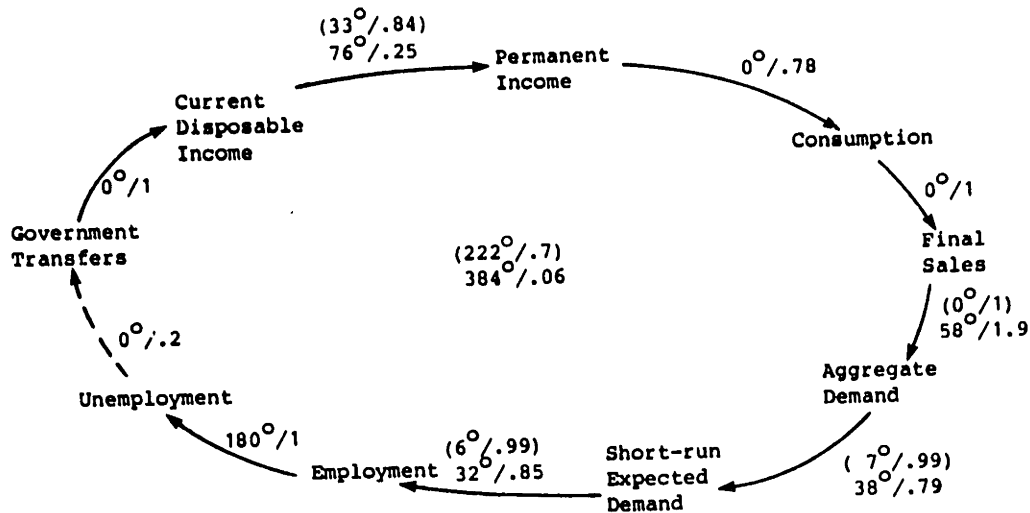


Figure 5-2. Transfer-Policy Loop

loop can be calculated. The loop produces a phase shift of 384° and a gain of .06 at the business-cycle period of 4 years. The phase shift is only 222° , and the gain is .17 at the long-cycle period of 24 years.

Figure 4-9 showed how adding a new loop affects period and damping of a cyclic structure. The phase shift at the business-cycle period is a little greater than 360° , so the loop should reduce damping and lengthen the period slightly. The effects on the business cycle will be relatively small, however, because the gain is low. The phase shift of the loop at the long-cycle period is a little more than 180° , so it should increase damping and shorten the period. The new loop shares most of its links (from CDY to E) with the multiplier. The transfer policy adds gain with a 180° phase shift between E and CDY. The policy essentially reduces the gain (strength) of the multiplier. Recall that the multiplier stabilized the business cycle and destabilized the long cycle. It is not surprising that effectively weakening the multiplier

should have the reverse impact of destabilizing the business cycle and stabilizing the long cycle.

Figure 5-3 shows the eigenvalues of the system with countercyclic government transfer policy activated. Comparing the eigenvalues in Figure 5-3 with those of the original model in Figure 4-3 reveals the effect of countercyclical transfers on the transient response of the model. The comparison reveals that the transfer policy affects both the business cycle and the long cycle. Effects on the business cycle will be examined first.

| EIGENVALUES | | | | | |
|-------------|------------------|---------------------|-----------------|------------------|-------------------|
| | DAMPING RATIO | DAMPED FREQUENCY | DAMPING TIME | DAMPED PERIOD | NATURAL PERIOD |
| (1) | -1.00 | 0.000 | 0.0 | 000000.0 | 0.0 |
| (2) | -1.00 | 0.000 | 2.0 | 000000.0 | 2.0 |
| (3) | -0.32 | 0.233 | 12.6 | 4.3 | 4.1 |
| (4) | -0.32 | 0.233 | 12.6 | 4.3 | 4.1 |
| (5) | -1.00 | 0.000 | 391.8 | 000000.0 | 391.8 |
| (6) | -0.72 | 0.034 | 28.4 | 29.1 | 20.3 |
| (7) | -0.72 | 0.034 | 28.4 | 29.1 | 20.3 |
| (8) | -1.00 | 0.000 | 14.3 | 000000.0 | 14.3 |
| (9) | -1.00 | 0.000 | 16.7 | 000000.0 | 16.7 |
| (10) | 0.00 | 0.000 | 000000.0 | 000000.0 | 000000.0 |

Figure 5-3. Eigenvalues with Transfer Policy

The transfer policy reduced the damping ratio of the business cycle from $-.36$ to $-.33$ and, so, is a destabilizer by the damping-ratio criterion. The policy does not alter the damped frequency significantly. Damped frequency drops from $.234$ to $.233$, indicating a neutral to slightly stabilizing influence by the frequency criterion. The policy increases the damping time from 11.0 to 12.6 years and so is a destabilizer by the damping-time criterion. The three different transient

response criteria do not lead to the same conclusion about the stabilizing or destabilizing impact of transfers. Inconsistencies between the three measures of transient response will be seen repeatedly.

The transfer policy increases the damping ratio of the long cycle from $-.61$ to $-.69$. Transfers, therefore, stabilize the long cycle by the damping-ratio criterion. The policy slightly increases the damped frequency of the long cycle from $.033$ to $.034$ and so is destabilizing by the frequency criterion. Transfers shorten the damping time from 39.3 to 30.5 years and, so, stabilize the long cycle by the damping-time criterion. Again, the three transient responses do not lead to the same policy conclusion. The effects of the transfer policy on the business cycle and the long cycle are exactly reversed. The policy destabilizes the business cycle by two of the transient response criteria (damping ratio and damping time) and stabilizes the long cycle by the same two criteria.

Figure 5-4 shows the matrix of gain curves for the model when a countercyclic transfer policy is activated. The dashed curves are the original gain curves before the policy was introduced. In all 8 graphs, gain is reduced at the low-frequency end of the spectrum. The policy is unequivocally a stabilizing influence in the frequency range of the long cycle. The transfer policy decreases the tendency for noise to stimulate long cycles in the system. The drop in gain at low frequencies is consistent with increased damping for the long cycle.

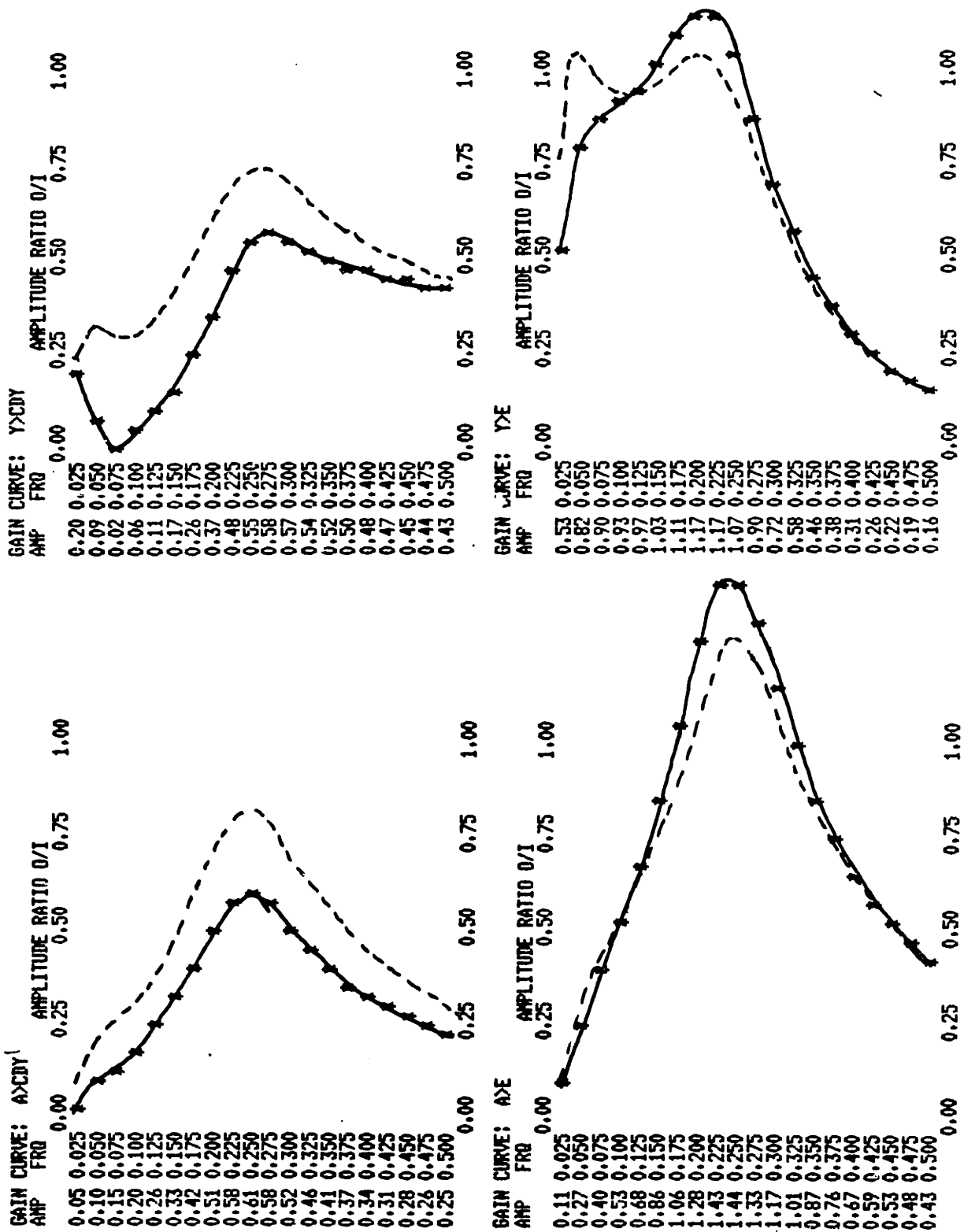


Figure 5-4a. Gain Curves With Transfer Policy

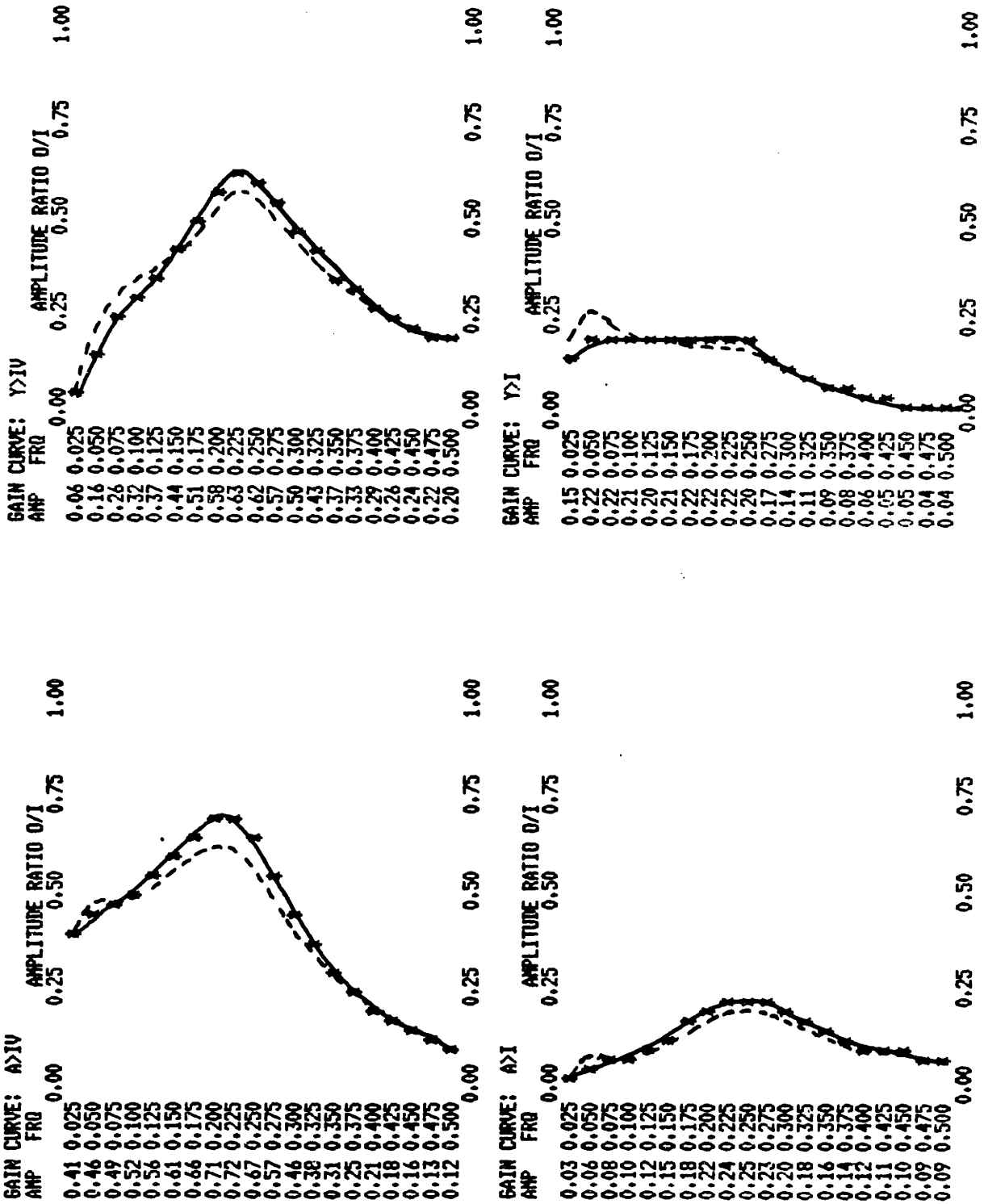


Figure 5-4b. Gain Curves With Transfer Policy

Figure 5-4 shows that a transfer policy increases gain in the frequency range of the business cycle for all but two disturbance-output pairs. The policy would, therefore, be classified as a destabilizer of the business cycle. The increase in gain at frequencies near .25 cycles per year is consistent with reduced damping for the business cycle.

Two of the gain curves in Figure 5-4 show a drop in gain at the business-cycle frequency, while all the others show an increase. Both gain curves involve current disposable income as an output variable. The inconsistency can be explained by the fact that transfers act directly on current disposable income. Transfers hold disposable income up when it would otherwise fall due to decreased employment and output. The direct effect of transfers on disposable income more than offsets the increased instability of employment and output over the business cycle. The figure shows that the sensitivity of some output variables to disturbances can be increased by a policy, while the sensitivity of other outputs to the same disturbances declines. A policy may involve a trade-off between increased sensitivity of one output and decreased sensitivity of another to the same disturbance.

Countercyclical Government Spending. In the original model, government spending was a constant. The countercyclical government spending policy increases government purchases of goods and services when unemployment is unusually high. The policy is, in principle, discretionary and could involve significant delays in recognizing changes in unemployment, passing appropriate legislation, and changing the actual

government purchase rate. Such delays, however, have been assumed away. Actual government spending instantaneously changes with variations in the rate of unemployment. The object of the policy is much like that of transfers. The countercyclical spending rule raises final demand for goods when the economy is weak and unemployment is high. The idea is that higher demand during recessions will stimulate the economy and bring employment back up to its normal level faster, thereby reducing the tendency toward business-cycle fluctuations. The spending policy differs from the transfer policy in that it acts immediately on final demand rather than passing through permanent income and consumption.

The countercyclical spending policy is implemented by adding a new term into the rate of government spending:

$$G = egs + CG$$

$$CG = egs * scgs * (U - nru)$$

G -- government spending (units/year)
 CG -- change in government spending (units/year)
 egs -- equilibrium government spending (units/year)
 scgs -- strength of countercyclic government spending
 (dimensionless)
 U -- unemployment rate (dimensionless)
 nru -- natural rate of unemployment (dimensionless)

The parameter scgs controls the strength of the countercyclical spending policy. When scgs is set to zero, government spending is constant as in the original model. A value of scgs greater than zero means that the government follows a countercyclic spending rule. The parameter is set at 1 for the policy test. The spending policy is illustrated in Figure 5-5.

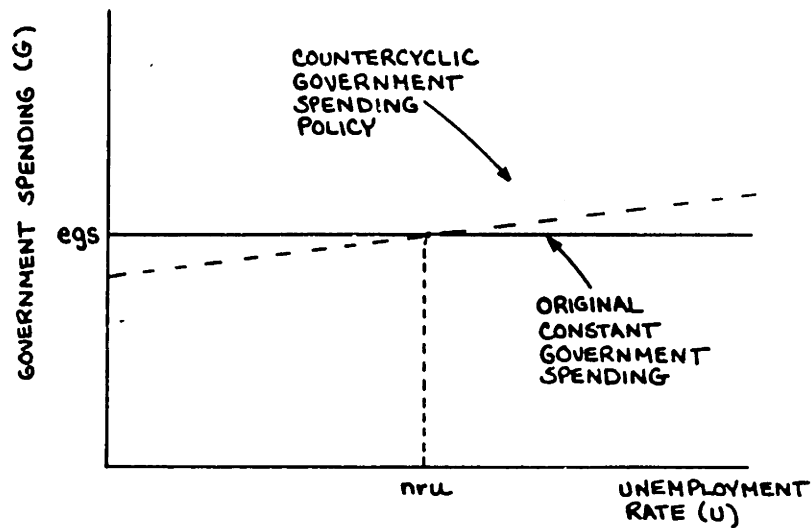


Figure 5-5. Government Spending Policy

A value for the strength of countercyclic government spending of 1 means that a change of one percentage point in the unemployment rate will cause a change of .2% in aggregate demand:

$$\begin{aligned}
 \Delta A &= \Delta G = \Delta CG = egs * scgs * (\Delta U) \\
 &= .2 ey * 1 * (.01) \\
 &= .002 ey = .002 A
 \end{aligned}$$

The spending policy, therefore, has the same effect on aggregate demand as the transfer policy for a one-point rise in the unemployment rate.

Figure 5-6 shows the loop that the spending policy adds to the model. The phase shift and gain of the loop is given at both the

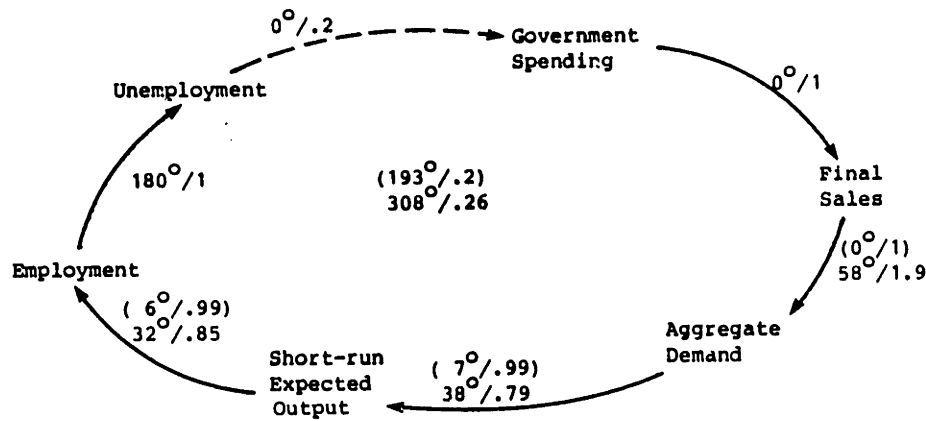


Figure 5-6. Spending-Policy Loop

business-cycle and long-cycle (in parentheses) periodicities. The business-cycle phase shift is 308° , and the gain is .26. The long-cycle phase shift is 193° , and the gain is .2. The shift at the business-cycle period is between 270° and 360° , so the new loop should destabilize the business cycle and decrease its period. The gain of the spending loop is more than 4 times as great as the gain of the loop of the transfer policy, because it does not pass through the smoothing at permanent income or through consumption. The destabilizing effect of spending, however, will not be 4 times as great as that of transfers, because the spending loop has a phase shift further away from 360° . At the long-cycle period, the loop has a phase shift slightly greater than 180° , so it should primarily damp the long cycle and shorten its period slightly.

The eigenvalues of the system with the countercyclic government spending policy in operation are shown in Figure 5-7. Comparing the new

EIGENVALUES

| | DAMPING RATIO | DAMPED FREQUENCY | DAMPING TIME | DAMPED PERIOD | NATURAL PERIOD |
|------|------------------|---------------------|-----------------|------------------|-------------------|
| (1) | -1.00 | 0.000 | 0.0 | 000000.0 | 0.0 |
| (2) | -1.00 | 0.000 | 2.0 | 000000.0 | 2.0 |
| (3) | -0.32 | 0.272 | 11.0 | 3.7 | 3.5 |
| (4) | -0.32 | 0.272 | 11.0 | 3.7 | 3.5 |
| (5) | -1.00 | 0.000 | 383.8 | 000000.0 | 383.8 |
| (6) | -0.72 | 0.030 | 32.4 | 33.1 | 23.1 |
| (7) | -0.72 | 0.030 | 32.4 | 33.1 | 23.1 |
| (8) | -1.00 | 0.000 | 15.7 | 000000.0 | 15.7 |
| (9) | -1.00 | 0.000 | 15.9 | 000000.0 | 15.9 |
| (10) | 0.00 | 0.000 | 000000.0 | 000000.0 | 000000.0 |

Figure 5-7. Eigenvalues With Spending Policy

eigenvalues to those of the original model in Figure 4-3 shows the impact of the policy on transient response. The spending policy destabilizes the business cycle by the damping-ratio criterion. The policy reduces the damping ratio from $-.36$ to $-.32$. Countercyclic spending also destabilizes the business cycle by the frequency criterion. Damped frequency rises from $.234$ to $.277$ cycles per year. By contrast, the policy does not alter stability by the damping-time criterion. Damping time remains at 11.0 years after introduction of countercyclic spending. As with countercyclic transfer policy, the three transient response criteria do not yield the same conclusion. In this case, damping time, not damped period, is the inconsistent measure. Countercyclical spending also has another important effect on the business cycle. The policy sharply reduces the natural period of the business cycle from 4.0 to 3.5 years.

The spending policy affects not only the business cycle but also the long cycle. Figure 5-7 shows that the long cycle becomes more stable

by the damping criterion. The damping ratio rises from $-.61$ to $-.72$. The long cycle is also more stable by the frequency criterion. Damped frequency drops from $.033$ to $.030$ cycles per year. Countercyclic spending also stabilizes the long cycle by the damping-time criterion. Damping time falls from 39.3 to 32.4 years. In this case, all three transient response criteria lead to the same conclusion: Countercyclic spending is a stabilizing influence on the long cycle.

Figure 5-8 shows the matrix of gain curves before and after introducing the countercyclic spending policy. All 8 curves show reduced gain in the low-frequency (long-cycle) range below $.2$ cycles per year. All 8 curves also show an increase in gain at the high-frequency end of the spectrum above $.325$ cycles per year. In the intermediate business-cycle range between $.2$ and $.325$ cycles per year, the new gain curves cross the originals indicating a neutral policy effect. The lower gain at low frequencies corresponds to greater damping of the long cycle. The higher gains at high frequencies are consistent with reduced damping of the business cycle and an increase in business-cycle frequency.

Perhaps the most interesting feature of Figure 5-8 is the height of the peaks in gain. Peak gain at business-cycle frequencies drops for all but two of the disturbance-output pairs. The spending policy is, therefore, a stabilizing influence on the business cycle by the frequency-response criterion. The gain curves show that stability conclusions based on the frequency-response criterion can contradict conclusions based on the damping criterion.

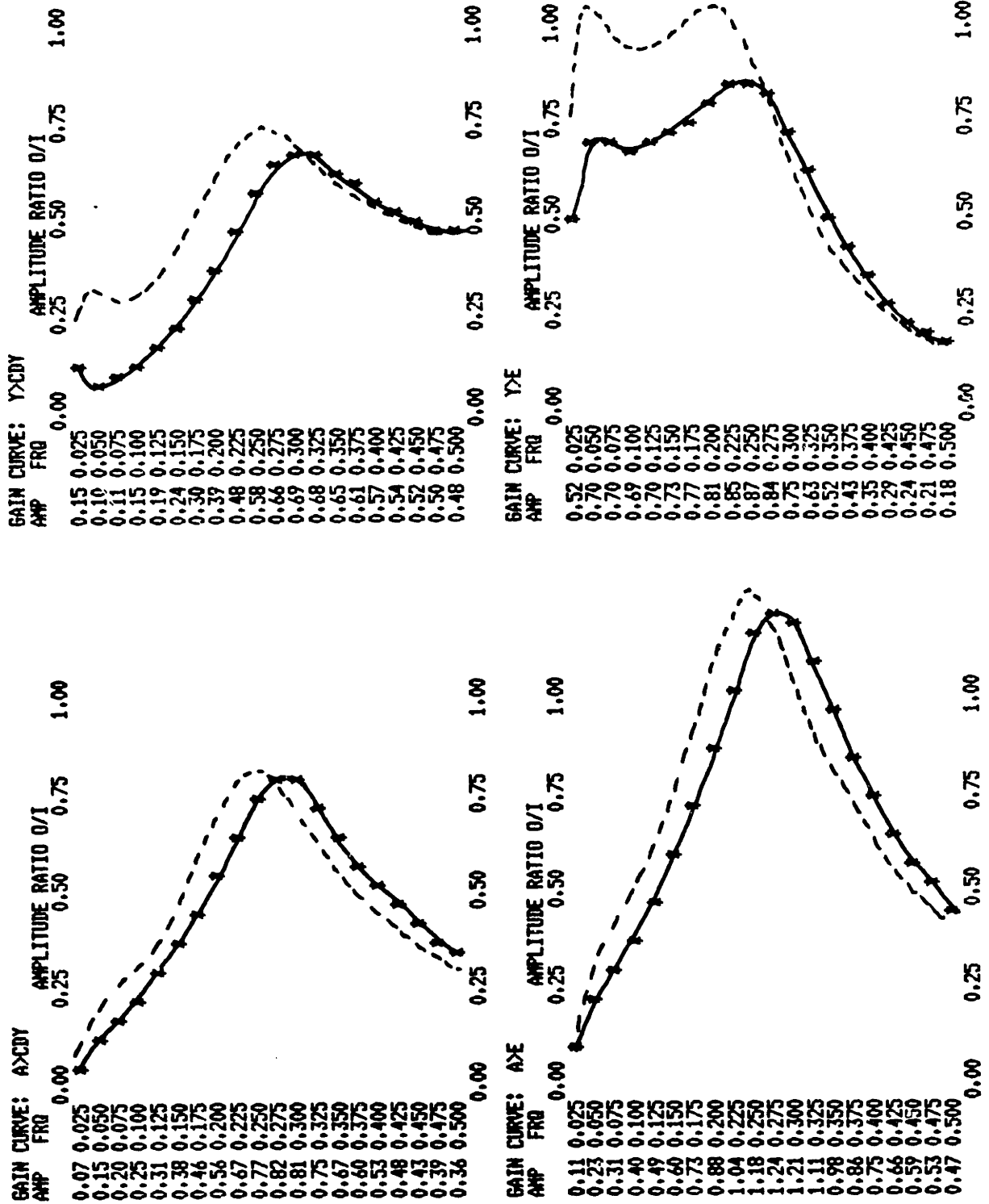


Figure 5-8a. Gain Curves With Spending Policy

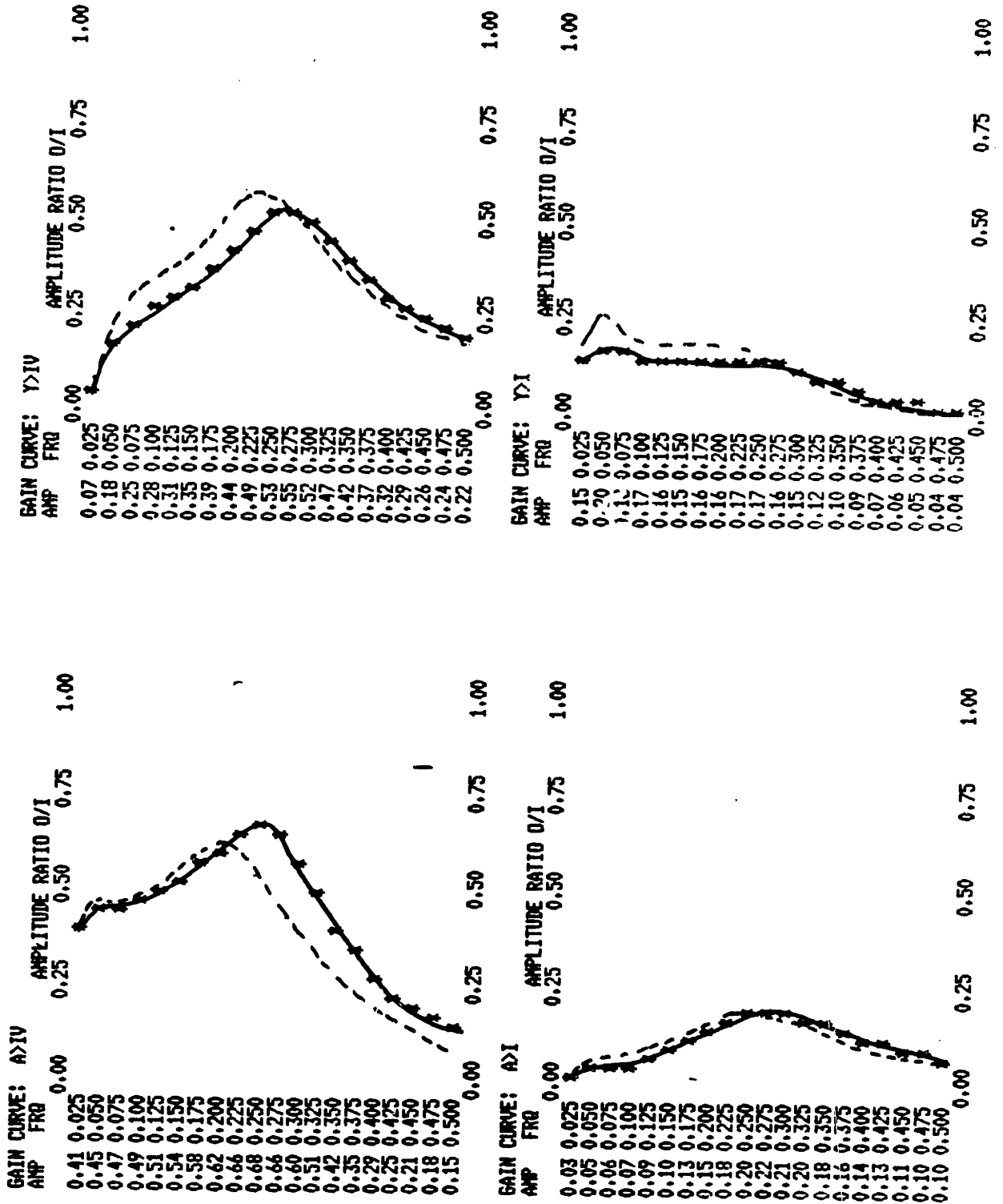


Figure 5-8b. Gain Curves With Spending Policy

Figure 5-9 provides a simplified explanation of the discrepancy between damping and frequency-response results. Gain at each frequency can be decomposed into multiplicative contributions from each of the eigenvalues and from the dynamic properties of the propagation paths between the disturbance source and output variable. (In the terminology of control theory, the properties of the propagation paths are the "zeros" of the system.) The contributions of the eigenvalues are the same for every disturbance-output pair. Contributions to gain from the propagation paths is different for each pair. The general tendency for gain peaks to drop in Figure 5-8 can be explained by the effects of the spending policy on the eigenvalues of the system.

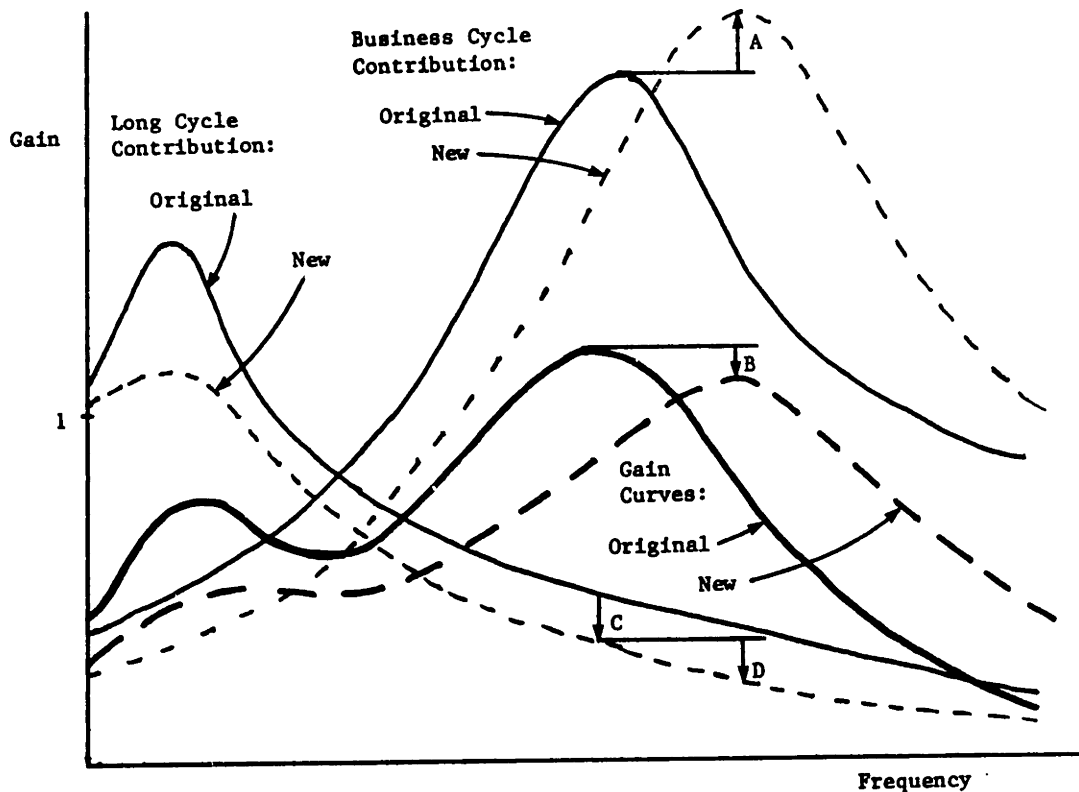


Figure 5-9. Gain Depends on All Eigenvalues

Assume that the solid curve in Figure 5-9 is the original gain curve for some disturbance-output pair. The gain curve can be decomposed into contributions from the two pairs of oscillatory eigenvalues. The contributions of the complex eigenvalues are shown as finer solid curves. Multiplying the two components yields the original gain curve. (Real eigenvalues are ignored for simplicity.) Now assume that a policy damps the long cycle, lowering the gain contribution of the long cycle. Also assume that the policy reduces damping and period of the business cycle, shifting the gain contribution of the business cycle higher and to the right. The new contribution curves can be multiplied to obtain the new gain curve.

The peak of the new gain curve can be lower than the peak of the original curve if the contribution of the long cycle drops by more than the contribution of the business cycle rises. The contribution of the long cycle at the new business-cycle peak declines for two reasons. First, the long-cycle contribution drops at all frequencies due to an increase in damping of the long cycle. Second, the business-cycle peak has moved to a higher frequency, where the contribution to gain from the long cycle is lower. The effect on gain from the long cycle can dominate the effect from the business cycle. The dominance will be strongest when the policy adds a feedback loop that has a phase shift near 270° for the business cycle and near 180° for the long cycle. A phase shift near 270° for the business cycle will primarily increase its frequency without reducing damping very much. A phase shift near 180° for the long cycle strongly will increase its damping without altering its frequency.

Recall that the phase shift of the spending loop is close to the conditions for maximum dominance; the shift is 308° for the business cycle and 193° for the long cycle. Phase shift for the transfer policy was not close to the condition for dominance; the phase shift was 384° for the business cycle and 222° for the long cycle.

Two of the curves in Figure 5-8 do not show a drop in peak gain at the business-cycle frequencies ($A > IV$, $A > I$). In both cases, gain at the low-frequency end of the spectrum is almost unaffected by the spending policy. Apparently, the propagation paths (zeros) from demand to inventory and from demand to investment cancel the effect of greater long-cycle damping on the gain contribution of the long cycle for those pairs. If the gain contribution of the long cycle does not drop, then decreased stability of the business cycle will raise the peak of the gain curve. A detailed structural explanation of the canceling effect of the zeros has not been developed. It is sufficient to note that the gain from one disturbance to an output variable can decline while, at the same frequency, the gain from another disturbance to the same output variable can increase. That is, a policy may involve a trade-off in sensitivities of the same output variable to different disturbances.

Graduated Income Tax. A graduated income tax, like countercyclic transfers, is an automatic stabilizer. In the original model, income tax was a constant fraction of personal income. Under a graduated income tax policy, the fraction of income taken by government rises with the level of income. During periods of high employment,

output, and income, the tax bite increases. During recessionary periods average tax rate declines. The policy is designed to keep current disposable income from rising during booms or falling during recessions by as much as it would without the policy. The policy is predicated on the idea that holding disposable income more constant will hold consumption more constant and keep demand variations from producing business cycles.

The graduated income tax policy is implemented in the model by changing the equation for taxes. The new tax equation becomes:

$$T = (egs + egt)(1 - sgyt) + Y * ntr * sgyt$$

$$ntr = (egs + egt)/ey$$

T -- taxes (units/year)
 egs -- equilibrium government spending (units/year)
 egt -- equilibrium government transfers (units/year)
 sgyt -- strength of graduated income tax (dimensionless)
 Y -- output (units/year)
 ntr -- normal tax rate (dimensionless)
 ey -- equilibrium output (units/year)

The parameter sgyt varies the strength of the graduated tax. When the parameter is set to one, the equation for taxes collapses to the equation for proportional taxes in the original model. When the parameter is greater than one, the income tax becomes graduated. For the policy test, sgyt is set at 2.2 so that the marginal tax rate is 120% greater than the average tax rate. Figure 5-10 graphs the graduated tax as a function of personal income.

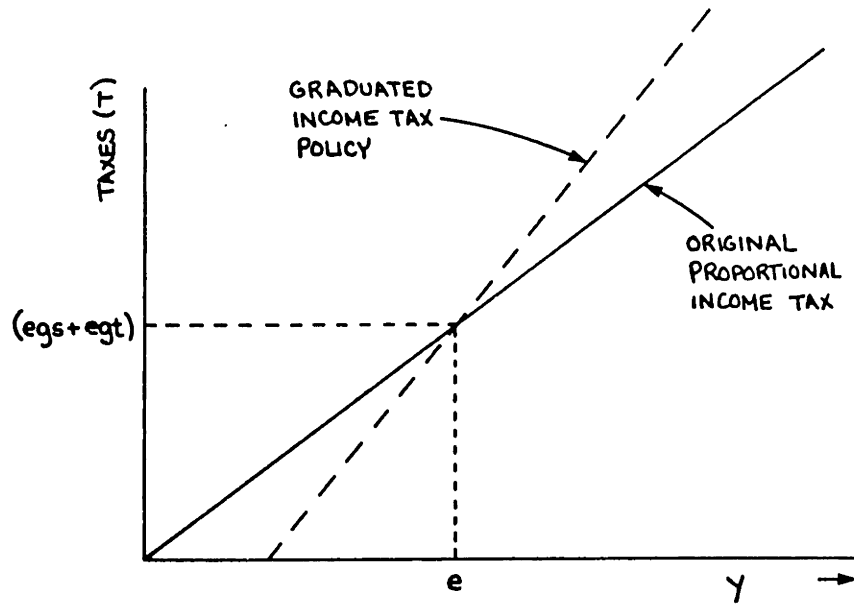


Figure 5-10. Graduated Income Tax Policy

A value for the strength of graduated income tax of 2.2 means that a rise of one percentage point in the unemployment rate will create a .2% increase in aggregate demand through consumption. In calculating the impact on consumption, the time delay through permanent income is ignored just as it was in the case of transfers.

$$\begin{aligned}
 \Delta A &= \Delta C = apc * \Delta PY = apc * \Delta CDY \\
 &= -apc * \Delta T = -apc * (\Delta Y * ntr * (sgyt - 1)) \\
 &= -apc * ((1 - \alpha) * \frac{\Delta E}{ee} * ey * ntr * (sgyt - 1)) \\
 &= -apc * ((1 - \alpha) * (-\Delta U) * ey * ntr * (sgyt - 1)) \\
 &= -.75 * (.75 * -.01 * ey * .3 * (2.2 - 1)) \\
 &= .002 ey = .002 A
 \end{aligned}$$

The graduated income tax policy has the same impact on aggregate demand as both the spending and transfers policies.

Figure 5-11 illustrates the loop formed when a graduated income tax is added to the model. The loop is similar to the one created by the transfer policy. Both share most of their links with the multiplier. The income-tax loop has a phase shift of 364° and a gain of .08 at the business-cycle period. The loop has a phase shift of 222° and a gain of

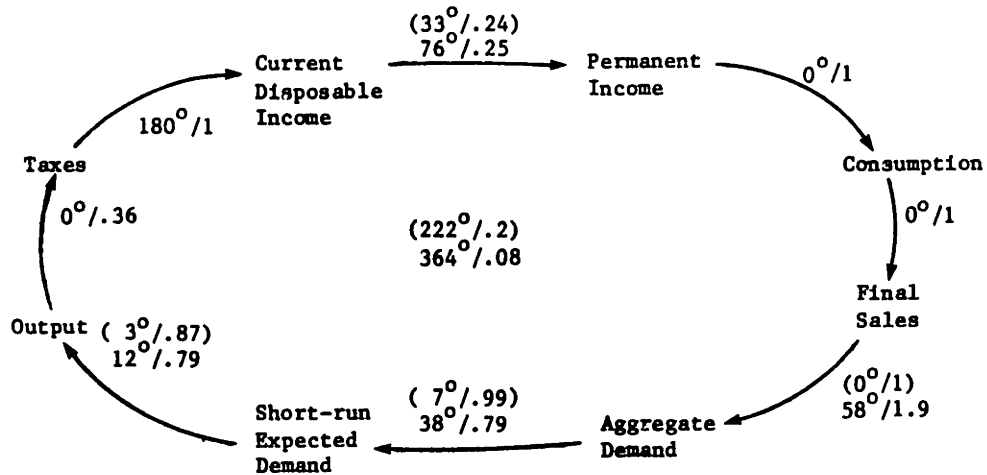


Figure 5-11. Tax-Policy Loop

.2 at the long-cycle period. The business-cycle shift is very close to 360° , so the loop should decrease damping without affecting the period of the business cycle. The long-cycle shift is between 180° and 270° , so the loop should stabilize the long cycle and shorten its period. The income-tax policy works in much the same way as the transfer policy in weakening the multiplier loop. The multiplier mechanism damped the

business cycle and destabilized the long cycle, so weakening it will, of course, have the reverse effect.

Figure 5-12 shows the eigenvalues of the system when the graduated income tax is activated. Comparing the new eigenvalues with those of the original model in Figure 4-3 shows the effect of the graduated income tax on transient response. The tax policy destabilizes the business cycle by the damping criterion. The damping ratio drops from $-.36$ to $-.32$. The tax has a very slight destabilizing effect on the business cycle by the frequency criterion. The policy is nearly neutral but does raise the damped frequency from $.234$ to $.238$ cycles per year.

EIGENVALUES

| | DAMPING RATIO | DAMPED FREQUENCY | DAMPING TIME | DAMPED PERIOD | NATURAL PERIOD |
|------|------------------|---------------------|-----------------|------------------|-------------------|
| (1) | -1.00 | 0.000 | 0.0 | 000000.0 | 0.0 |
| (2) | -1.00 | 0.000 | 2.0 | 000000.0 | 2.0 |
| (3) | -0.31 | 0.238 | 12.8 | 4.2 | 4.0 |
| (4) | -0.31 | 0.238 | 12.8 | 4.2 | 4.0 |
| (5) | -0.72 | 0.035 | 27.5 | 28.6 | 19.8 |
| (6) | -0.72 | 0.035 | 27.5 | 28.6 | 19.8 |
| (7) | -1.00 | 0.000 | 14.6 | 000000.0 | 14.6 |
| (8) | -1.00 | 0.000 | 625.4 | 000000.0 | 625.4 |
| (9) | -1.00 | 0.000 | 15.7 | 000000.0 | 15.7 |
| (10) | 0.00 | 0.000 | 000000.0 | 000000.0 | 000000.00 |

Figure 5-12. Eigenvalues With Tax Policy

The damping-time criterion, like the damping-ratio criterion, indicates that the income tax is a destabilizer. The damping-time rises from 11.0 to 12.8 years. The only discrepancy between the three criteria is that damped frequency indicates that the tax policy is only a very mild destabilizing influence, while the other two criteria indicate a stronger

effect. The three criteria come closer to agreeing with each other for the income tax policy than for either the transfers or spending policies.

Figure 5-12 shows that the income tax increases the damping ratio of the long cycle from $-.61$ to $-.72$, and, so, is a stabilizer by the damping criterion. Taxes are a slight destabilizer of the long cycle by the frequency criterion. Damped frequency rises from $.033$ to $.035$ cycles per year. By the damping-time criterion, the tax is a long-cycle stabilizer, lowering damping time from 39.3 to 27.5 years. Note also that the natural period of the long cycle dropped from 23.8 to 19.8 years after introduction of the graduated tax. The overall effects of the graduated income tax on the transient response of the model are very similar to the effects of countercyclical transfers.

Gain curves for the system under a graduated income tax policy are shown in Figure 5-13. The changes in frequency response produced by the tax policy are almost identical to those produced by transfers. Gain declines for all disturbance-output pairs at the low-frequency end of the spectrum. Gain rises in the business-cycle frequency band for all pairs except those involving current disposable income as the output variable. Just as in the case of transfers, the direct stabilizing effect of the tax policy on disposable income more than offsets the destabilizing influence of the policy on gross (pretax) income. The graduated income tax provides another example where a policy involves a trade-off between stability effects on different output variables. The policy decreases the sensitivity of one output variable to disturbances, while increasing the sensitivity of others over the business-cycle frequency range.

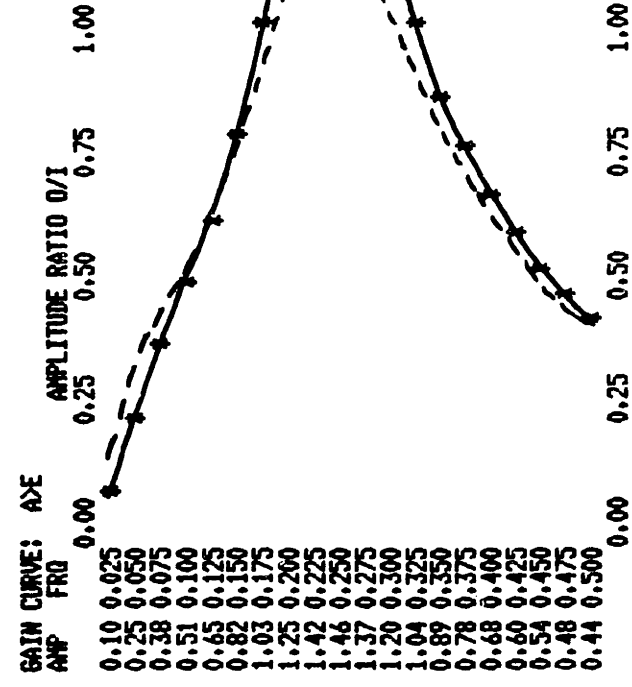
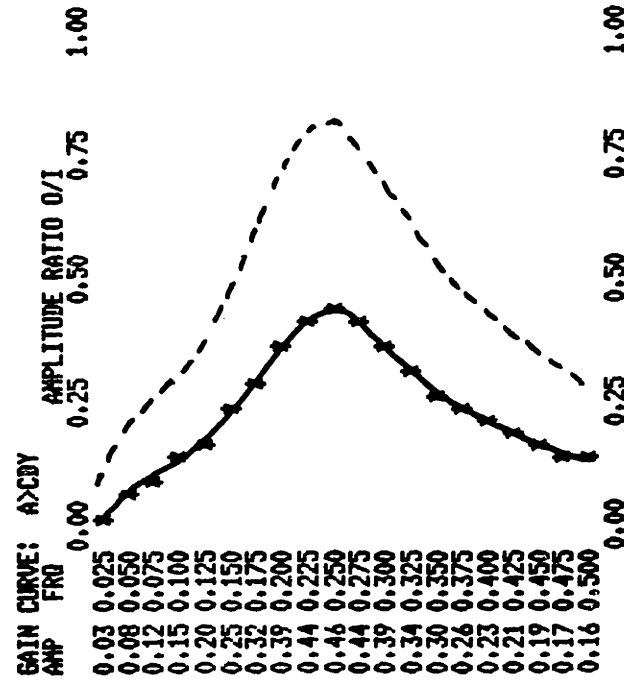
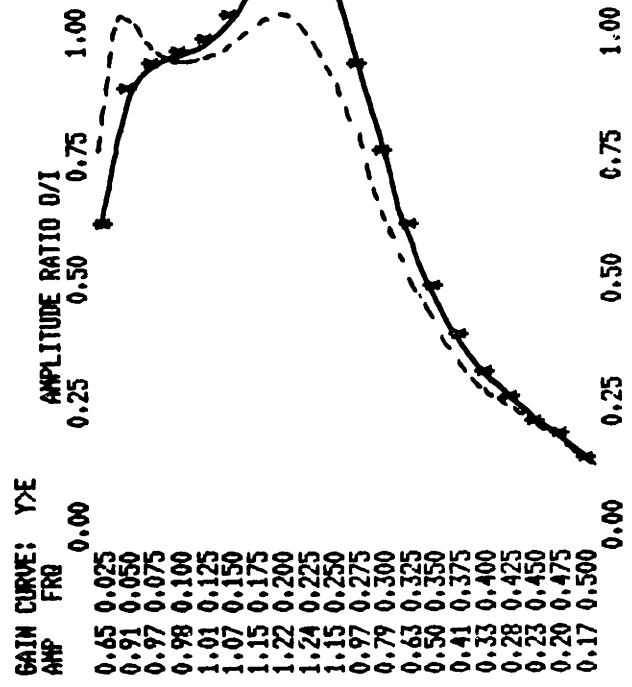
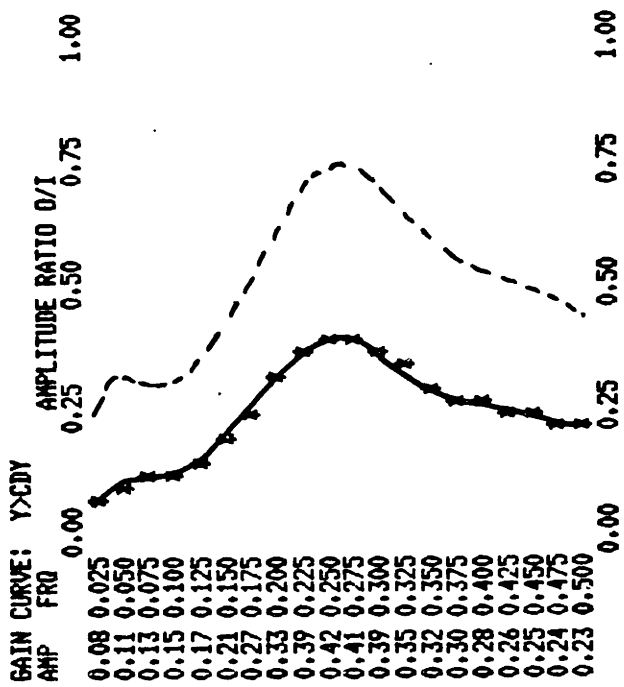


Figure 5-13a. Gain Curves With Tax Policy

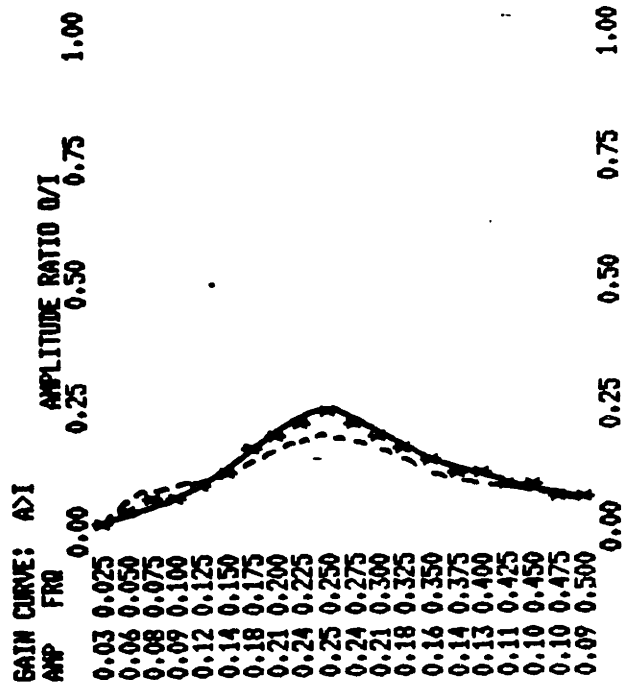
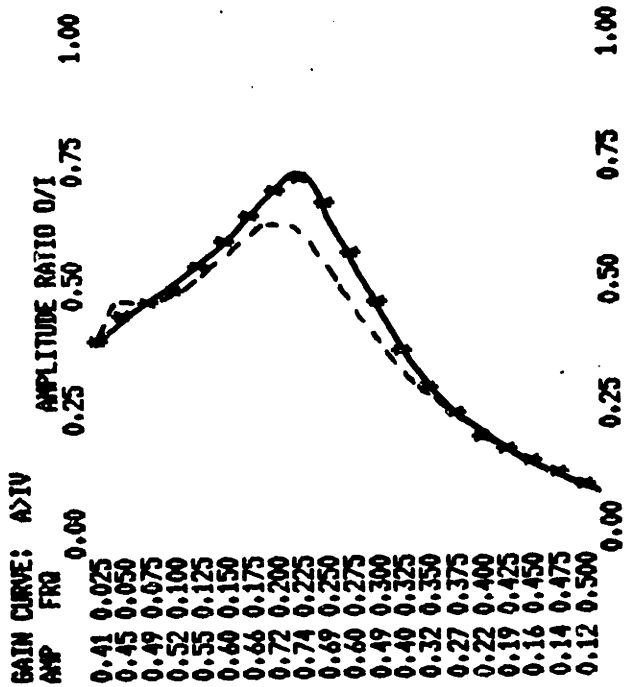
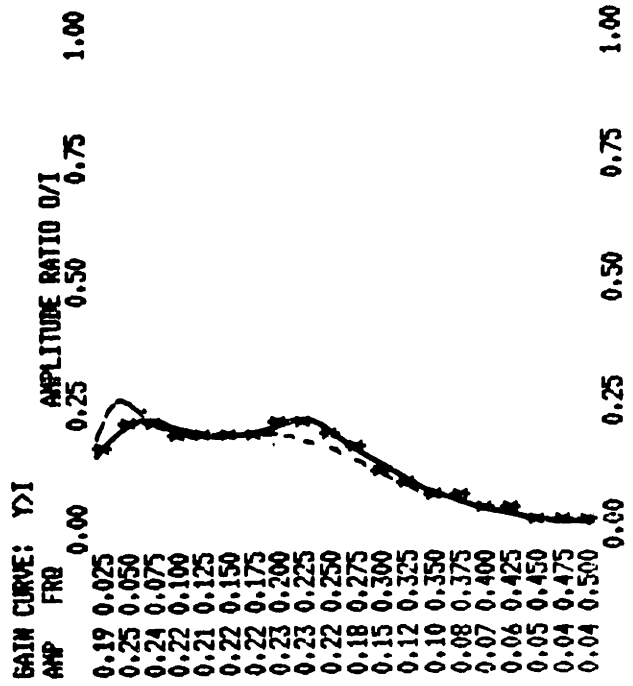
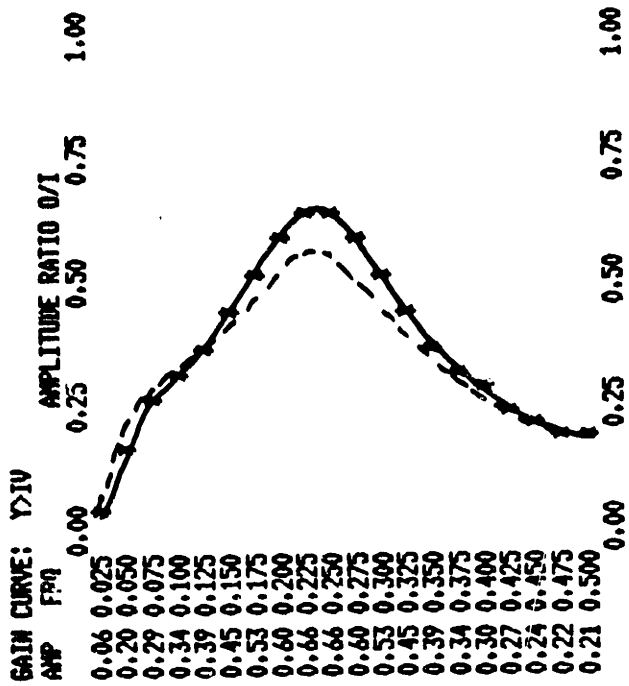


Figure 5-13b. Gain Curves With Tax Policy

Countercyclical Money-Stock Rule. In the original model, money stock was assumed to be constant. Under a money-stock rule the quantity of money rises and falls with the unemployment rate. Money moves exactly out of phase with employment. The mechanism behind the movements is assumed to be the following: the Federal Reserve sets a money-stock target, which moves linearly with the unemployment rate. The Fed then buys or sells securities in open-market operations to manipulate bank reserves. The banks then respond immediately to changes in reserves by calling or issuing loans so that they just meet reserve requirements. The scenario clearly assumes a high degree of coordination and control on the part of the Fed. The policy was not designed to be realistic, but rather to illustrate the consequences of a countercyclic money stock in the absence of "inside lags." The money-stock policy will be contrasted with a countercyclic money-growth rule in the next section.

The money-stock rule is designed to increase investment demand during recessions and thereby stimulate the economy, causing an early recovery. During a recession, as unemployment rises, money also increases. The rise in money stock momentarily creates excess money. The excess money is then presumably used to buy bonds, which drives up the price of bonds and drives down bond yields and other interest rates. The bond-purchase mechanism is not explicitly included in the model. Instead, a rise in money supply is assumed to automatically and immediately cause a decrease in interest rates. As interest rates fall, the holding cost of capital also declines, which increases the desired capital stock. An increase in desired capital raises the investment

rate, which adds to aggregate demand. In short, an increase in unemployment causes an increase in money, a fall in interest rate, and a rise in investment spending. The additional investment demand is supposed to stimulate the slack economy and thereby stabilize the business cycle.

The money-stock policy is implemented in the model by changing the equation for the rate of change in money from a constant set at zero to a first-order exponential lag of the target-money stock. The target-money stock is then a linear function of the unemployment rate:

$$RCM = (TMS - M) / tam$$

$$TMS = em + em * scms * (U - nru)$$

RCM -- rate of change in money (\$/year)
 TMS -- target-money stock (\$)
 M -- money supply (\$)
 tam -- time to adjust money (years)
 em -- equilibrium money stock (\$)
 scms -- strength of countercyclical money-stock policy
 (dimensionless)
 U -- unemployment rate (dimensionless)
 nru -- natural rate of unemployment (dimensionless)

The parameter scms controls the strength of the countercyclical policy. When scms is set to zero, the policy is deactivated and money stock remains constant. The larger scms is set, the more money stock varies for a given change in unemployment. The time to adjust money is set to an extremely small value, so there is essentially no delay between a change in the target-money stock and a change in the actual money supply.

The parameter $scms$ is set to .8 and tam is set to .001 for the policy trial. The money-stock policy is shown graphically in Figure 5-14.

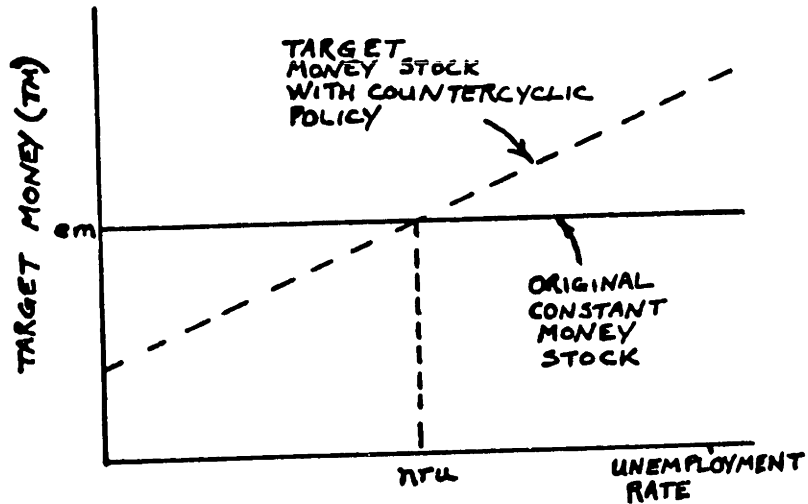


Figure 5-14. Money-Stock Policy

The strength of the countercyclic money-stock policy is calculated to have the same effect on aggregate demand as the previous policies:

$$\begin{aligned}
 \Delta A &= \Delta I = \Delta [(DK - K)/tak] \approx \Delta DK * 1/tak \\
 &= \Delta R * \frac{\partial DK}{\partial R} * \frac{1}{tak} = \Delta M * \frac{\partial R}{\partial M} * \frac{\partial DK}{\partial R} * \frac{1}{tak} \\
 &= U * \frac{\partial M}{\partial U} * \frac{\partial R}{\partial M} * \frac{\partial DK}{\partial R} * \frac{1}{tak} \\
 &= .01 * [\cancel{em} * scms] * \left[\frac{1r}{(iem * \cancel{em})} \right] * \left[\frac{-\alpha ey}{(1/alk + 1r)^2} \right] * \frac{1}{tak}
 \end{aligned}$$

$$\begin{aligned}
 &= .01 * [.8] * [-.03] * [-24.3 \text{ ey}] * \frac{1}{3} \\
 &= .002 \text{ ey} = .002 \text{ A}
 \end{aligned}$$

A rise of one percentage point in the unemployment rate leads to an increase of .2% in aggregate demand through increased investment. The stimulus would not persist if unemployment remained high, because investment would cause the gap between desired and actual capital stock to close. Investment would eventually return to nearly its original level. Changes in capital stock are relatively slow, however, so, at least over the business cycle, the money-stock policy should give the same degree of demand manipulation as the other policies.

Figure 5-15 shows the new loop added to the model when the countercyclical money-stock policy is activated. The loop follows many of the links in the price loop of the original model. The difference is that the connection between unemployment and interest rate passes through money supply instead of through price. The loop has a phase shift of 298° and a gain of .24 at the business-cycle period and a phase shift of 156° and a gain of .12 at the long-cycle period. The business-cycle shift is between 270° and 360° (nearer 270°), so the loop should primarily shorten the period of the business cycle but also reduce its damping slightly. The relatively high gain means the absolute change in damping should approach that of the transfer and tax policies, which had more phase shift and less gain. At the long-cycle period, the loop has a phase shift slightly less than 180° , so it should dampen the long cycle and increase its period slightly. The phase shift and gain of the

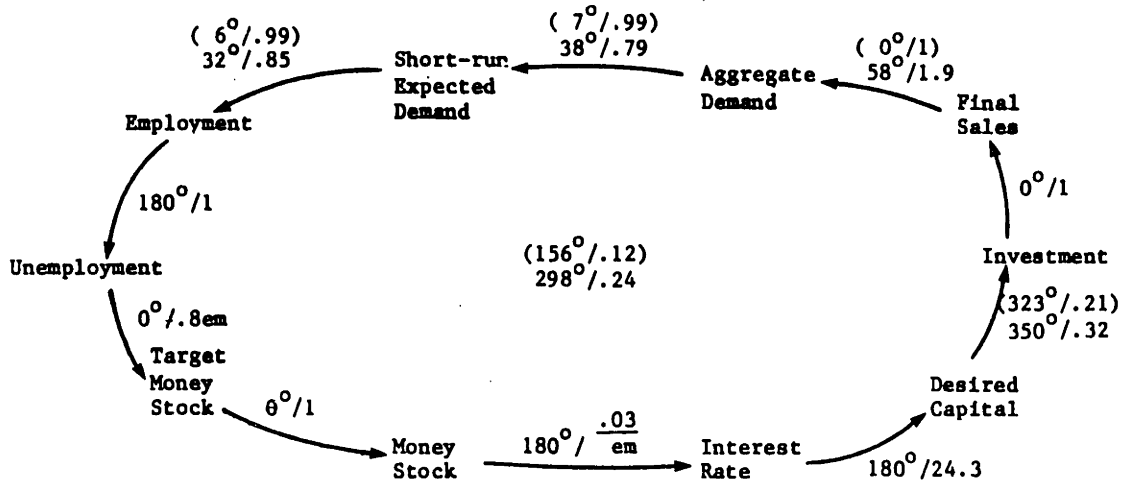


Figure 5-15. Money-Stock-Policy Loop

money-stock loop are very close to those of the government-spending loop seen earlier.

Figure 5-16 displays the eigenvalues of the system with the countercyclical money-stock rule in operation. The policy destabilizes the business cycle by the damping-ratio criterion. The damping ratio drops from $-.36$ to $-.33$. The policy is also a destabilizer by the frequency criterion. The damped frequency rises from $.234$ to $.273$ cycles per year. By the damping-time criterion, however, the policy stabilizes the business cycle. The damping time falls from 11.0 to 10.4 years. Note that the natural frequency of the business cycle drops sharply from 4.0 to 3.5 years, just as it did for the government-spending policy.

| EIGENVALUES | | | | | |
|-------------|------------------|---------------------|-----------------|------------------|-------------------|
| | DAMPING RATIO | DAMPED FREQUENCY | DAMPING TIME | DAMPED PERIOD | NATURAL PERIOD |
| (1) | -1.00 | 0.000 | 0.0 | 000000.0 | 0.0 |
| (2) | -1.00 | 0.000 | 0.1 | 000000.0 | 0.1 |
| (3) | -1.00 | 0.000 | 2.0 | 000000.0 | 2.0 |
| (4) | -0.33 | 0.273 | 10.4 | 3.7 | 3.5 |
| (5) | -0.33 | 0.273 | 10.4 | 3.7 | 3.5 |
| (6) | -1.00 | 0.000 | 289.5 | 000000.0 | 289.5 |
| (7) | -0.66 | 0.028 | 40.5 | 35.5 | 26.7 |
| (8) | -0.66 | 0.028 | 40.5 | 35.5 | 26.7 |
| (9) | -1.00 | 0.000 | 15.7 | 000000.0 | 15.7 |
| (10) | -1.00 | 0.000 | 16.0 | 000000.0 | 16.0 |

Figure 5-16. Eigenvalues With Money-Stock Policy

Figure 5-16 also shows the effect of the money-stock rule on the long cycle. The policy is a stabilizer of the long cycle by the damping-ratio criterion. The damping ratio increases from $-.61$ to $-.66$. The policy is also a stabilizer by the frequency criterion. Damped frequency drops from $.033$ to $.028$ cycles per year. By contrast, the money-stock policy destabilizes the long cycle by the damping-time criterion. Damping time rises from 39.3 to 40.5 years. Note that the policy increases the natural period of the long cycle from 23.8 to 26.7 years.

Figure 5-17 shows the effect of the money-stock policy on the gain curves of the system. In all cases, gain falls in the low-to-middle frequency range. The fall corresponds to the increased damping of the long cycle seen in the eigenvalues. The business-cycle peak in gain shifts to the right (to a higher frequency) in every case. The shift corresponds to the shorter natural period of the business cycle seen in the eigenvalues. In all but two cases, gain increases at higher frequencies. The increase corresponds to lower damping and shorter

period in the business cycle. The two exceptions both involve investment as the output variable. Investment, remember, is the immediate target of the money-stock policy. The policy effectively counteracts movements in investment over the business cycle.

Figure 5-17 shows that peak gain at business-cycle frequencies is lower in all cases except for the demand-inventory combination. The money-stock policy therefore stabilizes the business cycle by the frequency-response criterion. Recall that the policy destabilizes the business cycle by the damping criterion. The discrepancy between results by the frequency-response and damping criteria can be explained by the effect of the policy on the long cycle. The policy increases damping of the long cycle enough to more than offset the reduction of damping it produces in the business cycle. Overall gain in the business-cycle frequency range is reduced. A similar result was previously seen in the case of a countercyclic government spending policy.

The results of the money-stock policy are, by and large, very similar to the results of the countercyclical government-spending policy. For both policies, damping of the business cycle declines while peak gain at the business-cycle frequencies also falls. The policies also produce a marked drop in the period of oscillation of the business cycle. The difference between the two policies lies primarily in their effects on gain between the two disturbances (A and Y) and investment. The difference is not surprising in view of the fact the monetary policy has a direct and immediate countercyclic influence on investment spending.

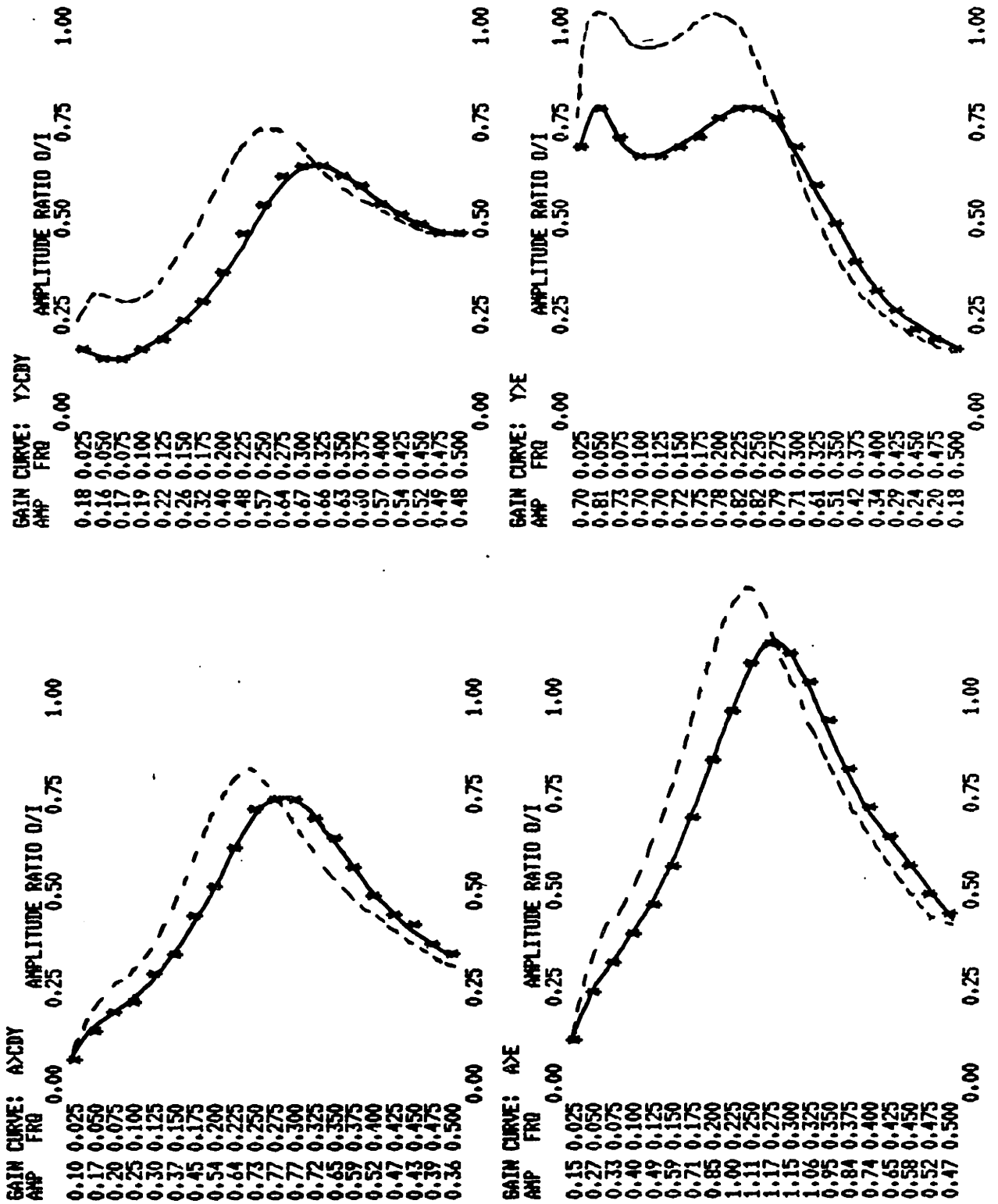


Figure 5-17a. Gain Curves With Money-Stock Policy

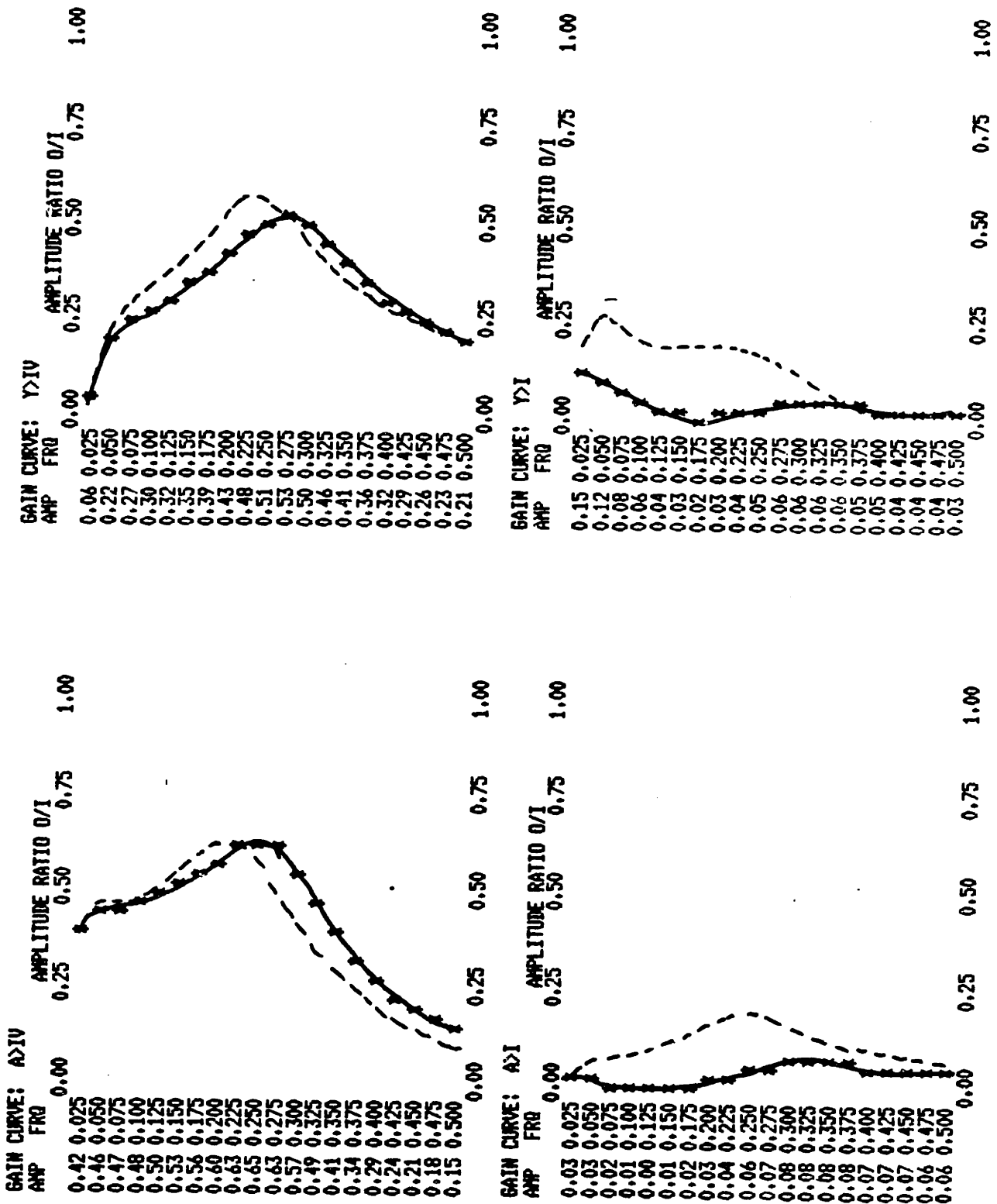


Figure 5-17b. Gain Curves With Money-Stock Policy

Countercyclical Money Growth. The last policy to be examined is a countercyclical money-growth rule. The policy is a variant of the money-stock rule of the preceding section. The money-growth rule causes the rate of growth in money supply, rather than the money supply itself, to rise and fall with the unemployment rate. Money growth moves countercyclically with employment. In order to follow the policy rule, the Federal Reserve would make open-market purchases (increasing reserves) when employment is low. Banks, it is assumed, would adjust outstanding loans in response to changes in reserves and thereby adjust the money supply exactly as the Fed adjusts reserves. The policy is designed to make interest rates drop when the economy is in a recession and rise when the economy is especially strong. The movement of interest rates causes investment to rise during recessions and fall during peaks in economic activity. The changes in investment are supposed to stabilize the economy by raising investment demand during bad times and lowering demand in good times. The difference between the two monetary policies is one of timing. The effect of the growth rule lags the effect of the stock rule by one-quarter cycle.

The money-growth rule is implemented in the model by making the rate of change in money a linear function of the unemployment rate:

$$RCM = M * scmg * (U - nru)$$

RCM -- rate of change in money supply (\$/year)
M -- money supply (\$)
scmg -- strength of countercyclic monetary policy (1/years)
U -- unemployment rate (dimensionless)
nru -- natural rate of unemployment (dimensionless)

The parameter $scmg$ varies the strength of the countercyclic money-growth policy. A value of zero is equivalent to the original assumption that money supply never changes. A positive value creates countercyclic money growth and investment demand. The parameter $scmg$ is set to 1.3 for the policy test. Figure 5-18 illustrates the monetary rule.

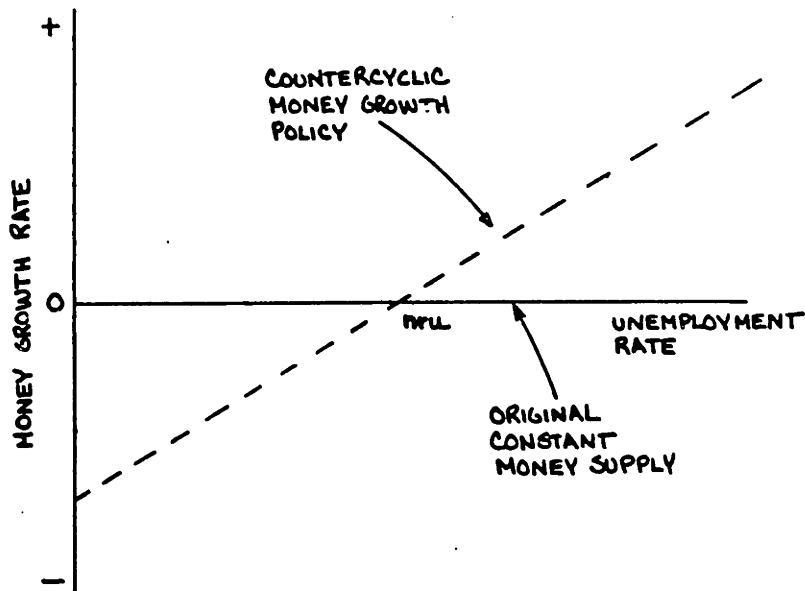


Figure 5-18. Money-Growth Policy

A value of 1.3 for the parameter $scmg$ means that a business-cycle fluctuation in unemployment rate with an amplitude of one percentage point (peak-to-average value) will create a fluctuation in investment rate with an amplitude equal to .2% of aggregate demand. In this sense, the money-growth policy has the same magnitude as the other policies with respect to the business cycle. (Note, in going from the first to second lines, that capital is a first-order stock adjustment of

desired capital. At a 4-year period and with a 3-year time to adjust capital, capital stock lags desired capital by 78° and has an amplitude .21 times as great as that of desired capital. The difference between the two curves ($(0^\circ, 1) - (78^\circ, .21)$) has an amplitude .98 times as great as that of desired capital. Note, in going from the second to third lines, that money (M) is a pure integrator of the rate of change in money (RCM) and so has an amplitude equal to that of RCM multiplied by the period of the oscillation in question (4 years) divided by 2π .)

$$\begin{aligned}
 \text{Amp (A)} &= \text{Amp (I)} = \text{Amp} ((DK - K)/\text{tak}) = \\
 &= .98/\text{tak} * \text{Amp(DK)} = .98/\text{tak} * \frac{\partial R}{\partial M} * \frac{\partial DK}{\partial R} * \text{Amp (M)} \\
 &= .98/\text{tak} * \frac{\partial R}{\partial M} * \frac{\partial DK}{\partial R} * .64 * \text{Amp(RCM)} \\
 &= .98/\text{tak} * \frac{\partial R}{\partial M} * \frac{\partial DK}{\partial R} * .64 * \epsilon_m * \text{scmg} * \text{Amp(U)} \\
 &= .98/\text{tak} * \left[\frac{1r}{(i\epsilon_m * \epsilon_n)} \right] * \left[\frac{-\alpha \epsilon_y}{(1/\text{alk} + 1r)^2} \right] * .64 * \epsilon_m * \text{scmg} * \text{Amp(U)} \\
 &= .98/3 * (-.03) * (-24.3 \epsilon_y) * .64 * 1.3 * .01 \\
 &= .002 \epsilon_y = .002 A
 \end{aligned}$$

The money-growth policy is not comparable to the other policies at all frequencies, because the gain at each integration (at money supply and capital stock) depends on frequency. The policy does have approximately the same effect on aggregate demand as the other policies at the business-cycle frequency. The money-growth policy is, however, much stronger than the others at low frequencies.

Figure 5-19 illustrates the new loop formed when the money-growth rule is activated. The loop is the same as the money-stock loop except for the links between unemployment and money stock. The new links add 90° of phase shift at both the business-cycle and long-cycle periods. Gain at the long-cycle period is also greatly increased. Phase shift is 388° , and gain is .25 at the business-cycle period. Phase shift is 246° , and gain is .74 at the long-cycle period. The business-cycle shift is a little greater than 360° , so the loop should destabilize the business cycle and lengthen its period. The phase shift is similar to that of the transfer loop and the spending loop, but the gain is much greater because the money-growth policy is not filtered through permanent income or consumption. The higher gain means that a money-growth rule

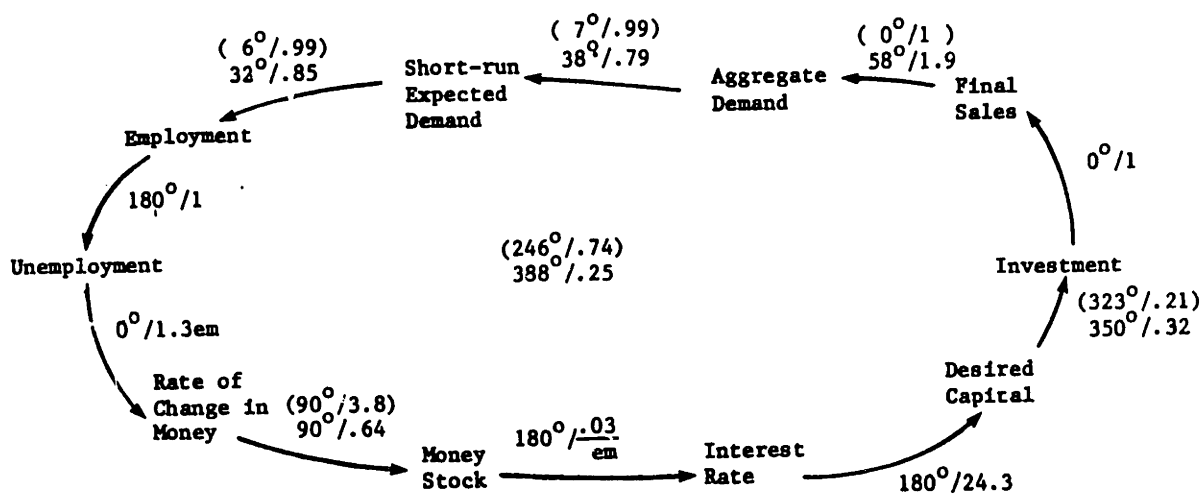


Figure 5-19. Money-Growth-Policy Loop

will have very strong effects. At the long-cycle period, the loop has a phase shift between 180° and 270° , so it will stabilize the long cycle and shorten its natural period. The effects on the long cycle will be

exaggerated because of the high gain through the link from money growth to money stock at low frequencies.

Figure 5-20 shows the eigenvalues of the system when the countercyclical money-growth policy is activated. The policy destabilizes the business cycle by the damping criterion. The damping ratio drops very sharply from $-.36$ to $-.19$. The policy has a neutral effect by the frequency criterion. Damped frequency changes insignificantly from $.234$ to $.236$ cycles per year. By the damping-time criterion, the policy strongly destabilizes the business cycle. The damping time jumps from

| EIGENVALUES | | | | | |
|-------------|------------------|---------------------|-----------------|------------------|-------------------|
| | DAMPING RATIO | DAMPED FREQUENCY | DAMPING TIME | DAMPED PERIOD | NATURAL PERIOD |
| (1) | -1.00 | 0.000 | 0.0 | 000000.0 | 0.0 |
| (2) | -1.00 | 0.000 | 1.9 | 000000.0 | 1.9 |
| (3) | -0.19 | 0.236 | 21.9 | 4.2 | 4.2 |
| (4) | -0.19 | 0.236 | 21.9 | 4.2 | 4.2 |
| (5) | -0.93 | 0.023 | 17.1 | 43.8 | 15.9 |
| (6) | -0.93 | 0.023 | 17.1 | 43.8 | 15.9 |
| (7) | -1.00 | 0.000 | 12.6 | 000000.0 | 12.6 |
| (8) | -1.00 | 0.000 | 156.2 | 000000.0 | 156.2 |
| (9) | -1.00 | 0.000 | 15.7 | 000000.0 | 15.7 |
| (10) | 1.00 | 0.000 | 903088.0 | 000000.0 | 903088.0 |

Figure 5-20. Eigenvalues With Money Growth

11.0 to 21.9 years. Just as in the case of transfers and income taxes, the money-growth policy shows a destabilizing influence on both the damping ratio and damping time, but a neutral effect on the damped frequency of the business cycle. Note that the natural period of the business cycle increased slightly from 4.0 to 4.2 years.

Figure 5-20 also shows the effects of the money-growth policy on the long cycle. The policy is strongly stabilizing by the damping criterion. The damping ratio of the long cycle rises from $-.61$ to $-.93$. The long cycle is also stabilized by the frequency criterion. The damped frequency falls from $.033$ to $.023$. By the damping-time criterion, money growth is also a stabilizing influence. The damping time drops from 39.3 to 17.1 years. The very large drop in damping time is produced by both the increase in damping and a marked decrease in the natural period of the long cycle from 23.8 to 15.9 years.

Figure 5-21 shows gain curves for the system when the counter-cyclic money-growth rule is activated. All but two of the gain curves are lower in the low-frequency range. The two curves which increase both involve investment as the output variable. The increased sensitivity of investment to disturbances is not surprising, because the policy acts directly on investment to manipulate demand. The direct destabilizing effect of the policy on investment more than outweighs the indirect stabilizing effect on the long cycle. All the curves show a strong increase in gain around the business-cycle frequency. The rise in gain corresponds to reduced damping in the business cycle.

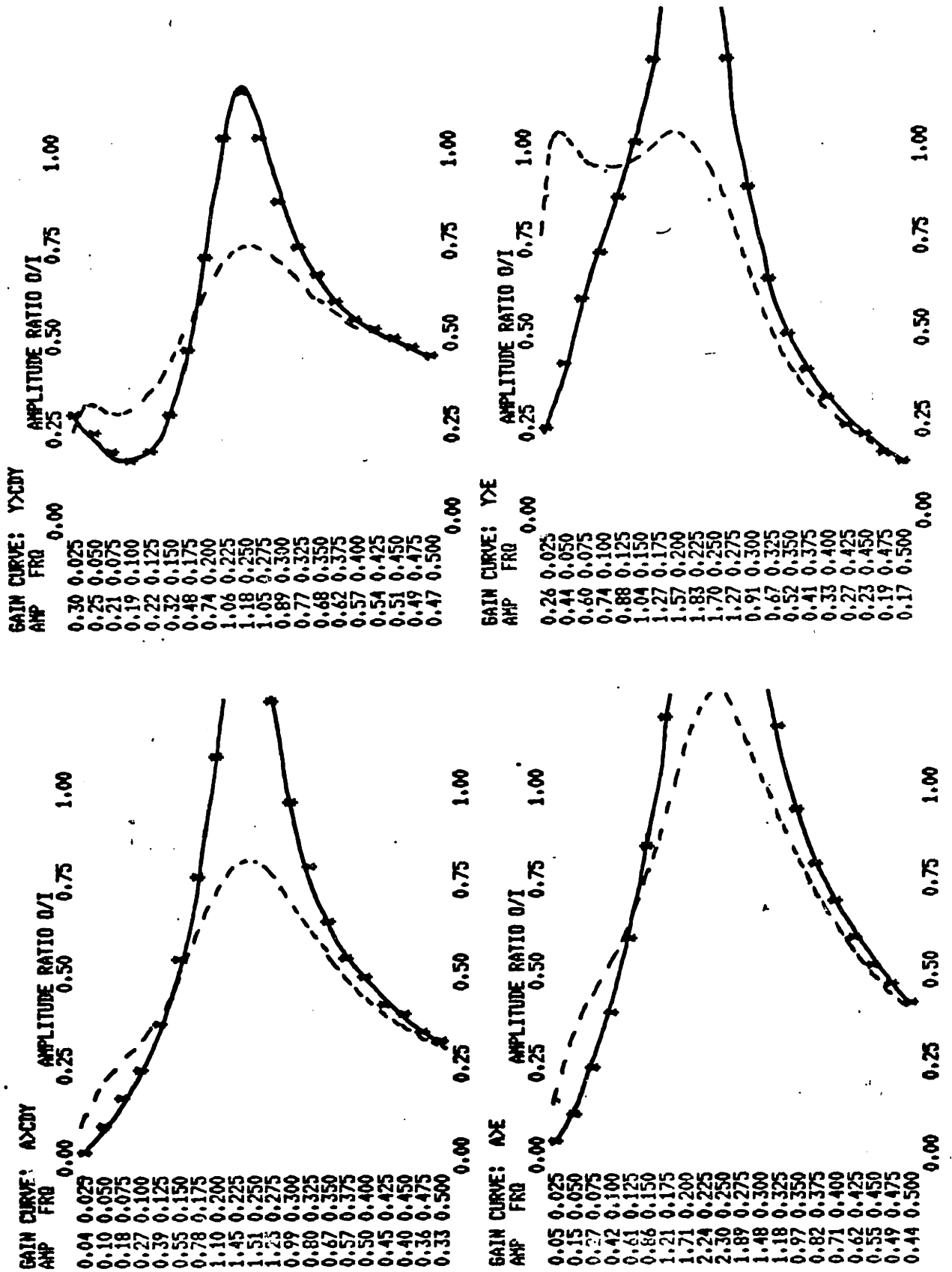


Figure 5-21a. Gain Curves With Money Growth

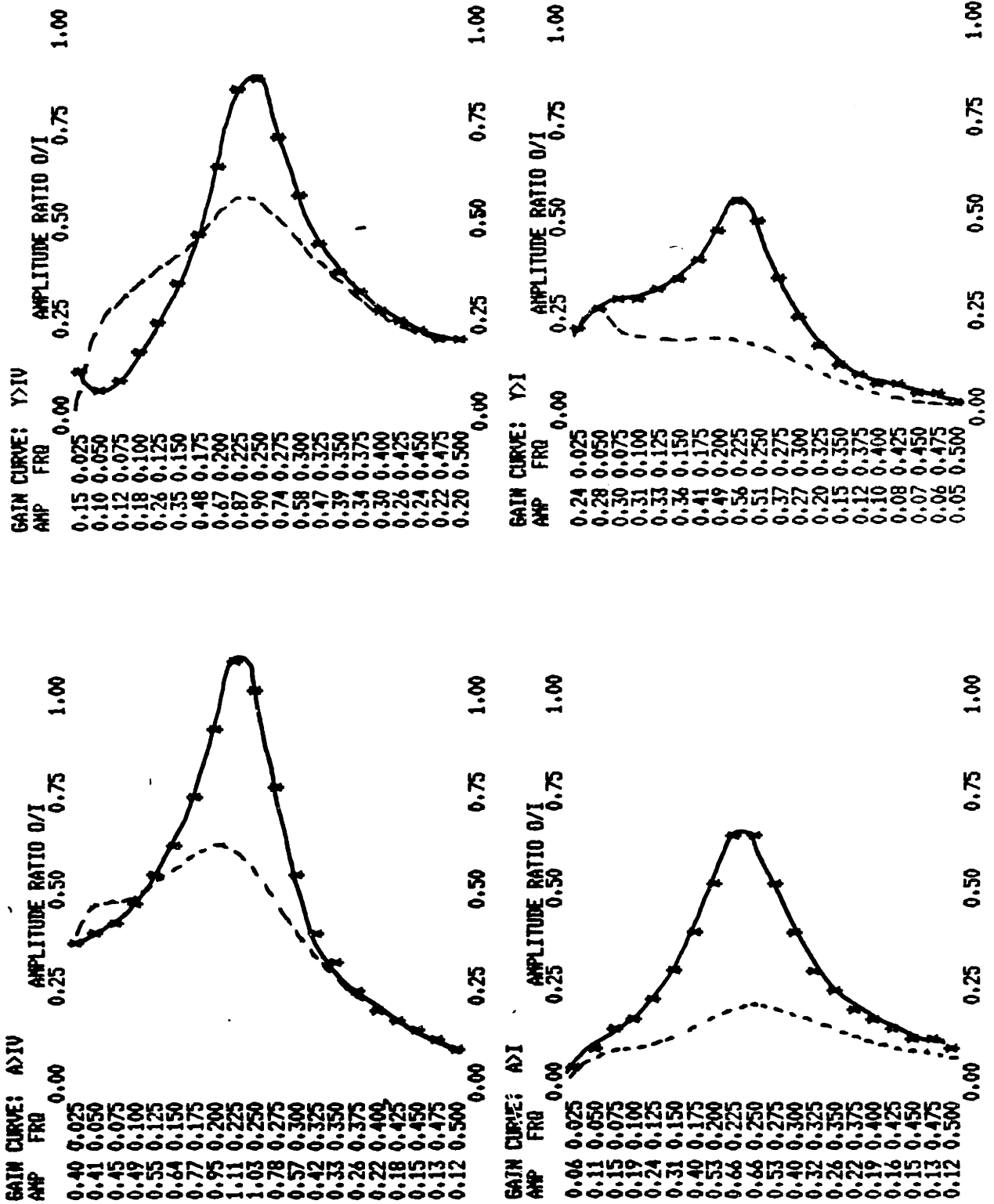


Figure 5-21b. Gain Curves With Money Growth

Summary Figure. The chart in Figure 5-22 summarizes the impact of the five policies on both transient response and frequency response of the model.

| | SCGT | SCGS | SGYT | SCMS | SCMG |
|--|-----------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| <u>Transient Response</u> | | | | | |
| Business Cycle: | | | | | |
| Phase/Gain | 384 ⁰ /.06 | 308 ⁰ /.26 | 364 ⁰ /.08 | 298 ⁰ /.24 | 388 ⁰ /.25 |
| Damping Ratio | D | D | D | D | D |
| Damped Frequency | N | D | D | D | N |
| Damping Time | D | N | D | S | D |
| Period | (+) | - | N | - | + |
| Long Cycle: | | | | | |
| Phase/Gain | 222 ⁰ /.17 | 193 ⁰ /.2 | 222 ⁰ /.2 | 156 ⁰ /.12 | 246 ⁰ /.74 |
| Damping Ratio | S | S | S | S | S |
| Damped Frequency | D | S | D | S | S |
| Damping Time | S | S | S | D | S |
| Period | - | N | - | + | - |
| <u>Frequency Response</u> (Gain Curves) | | | | | |
| Frequencies: | | | | | |
| Low | | | | | |
| (f < .2) | S | S | S | S | S ⁽³⁾ |
| Middle | | | | | |
| (.2 < f < .35) | D ⁽¹⁾ | N | D ⁽¹⁾ | N | D |
| High | | | | | |
| (f > .35) | N ⁽¹⁾ | D | N ⁽¹⁾ | D ⁽³⁾ | N ⁽³⁾ |
| Peak Gain | D ⁽¹⁾ | S ⁽²⁾ | D ⁽¹⁾ | S ⁽⁴⁾ | D |
| <u>Key:</u> | | | | | |
| (1) except A > CDY, Y > CDY | | | S - Stabilizing | | |
| (2) except A > IV, A > I | | | D - Destabilizing | | |
| (3) except A > I, Y > I | | | N - Neutral | | |
| (4) except A > IV | | | - - Shorter | | |
| | | | + - Longer | | |

Figure 5-22. Summary of Policy Effects

C. POLICY TIMING

All five policies tested in the preceding section reduced damping of the business cycle. All five are standard countercyclical demand-stabilization policies often discussed in the economics literature. The perverse effects cannot be attributed to "inside lags" of policy decision making. None of the policies involves an inside lag. Three of the policies encounter "outside lags" between policy action and its impact on demand. The three policies that encounter lags have a stronger adverse effect on damping than the other two policies. The results suggest two questions: 1) If a short lag increases the destabilizing impact of a policy, how much lag creates the worst possible effect on business-cycle damping? 2) If all the countercyclic policies destabilize the business cycle, would procyclic demand policies be stabilizing?

How much lag has the most destabilizing effect? As seen in the preceding section, the stronger effect of lagged policies is explained by phase shift. The lagged policies create loops with phase shift very near 360° , while the policies which encounter no outside lags create loops with a phase shift close to 300° . Phase analysis suggests that adding an inside lag to the policies that have no outside lags would accentuate their destabilizing influence, because the phase shift of the loops they add would increase toward 360° . The result was confirmed by adding a six-month lag between unemployment and the policy action for both the government-spending and money-stock policies. Both policies reduced

damping more than when no inside lag was included. The reverse test was also done. The policies were made to lead unemployment, and they reduced damping less than before. The effects of leads and lags were also tested on policies that do encounter outside lags. Neither a lead nor a lag significantly affected damping for either the transfer or the money-growth policies. To generalize the findings: A countercyclic demand policy with no inside or outside lags appears to have a mild destabilizing influence on the business cycle. A countercyclic demand policy with inside and/or outside lags totaling about 60° at the business-cycle frequency has the maximum destabilizing effect on the business cycle.

Are procyclic demand policies stabilizing? Countercyclic policies create loops with phase shift between 300° and 390° . The shifts fall well within the range of destabilization, which lies between 270° and 450° (90°). A procyclic policy would reverse the direction of policy action, adding another 180° of phase shift to the new loops. The phase shift would then lie between 480° (120°) and 570° (210°), which is well within the range of stabilization, between 90° and 270° . Procyclic demand policies would appear to be business-cycle stabilizers. The theory was tested by running each of the policies with its sign reversed. Indeed, all five of the policies increased damping of the business cycle. The most effective stabilizers were those procyclic policies with an outside lag between policy action and the impact on demand.

The effects of procyclic and countercyclic policies are simply reversed. It would be imprudent, however, to conclude that the government should institute procyclic demand policies. The procyclic policies have a reversed effect not only on the business cycle but also on the long cycle. Four out of the five procyclic policies tend to destabilize the long cycle. While a small amount of procyclic demand manipulation could help stabilize the business cycle, a firm procyclic rule might accentuate any longer cycles present in the economy. A more fruitful approach to business-cycle stabilization policy is to look within the inventory-employment structure that creates the business cycle.

For example, Figure 4-6 indicates that increasing the flexibility of capacity utilization has a strong stabilizing effect on the business cycle without affecting the stability of the long cycle. Macroeconomic policies aimed at increasing the use of overtime and undertime, therefore, might help stabilize the business cycle. Such policies might include taxing fringe benefits so that they become a smaller share of total compensation. Lower fringe benefits would reduce the cost of undertime to employers. Many other policies might also be devised that would affect the management of employment, inventory, and production and thereby stabilize the business cycle.

D. POLICY CONCLUSIONS

Several conclusions can be drawn from the preceding analysis of standard demand-stabilization policies. The conclusions are broken down into economic insights and theoretical insights.

Economic Insights. Variations in final demand are often improperly thought to cause business cycles. Instead, the structure that amplifies and propagates business cycles is a set of feedback loops involving inventory, inventory investment, expected demand, employment, and output. Procyclic movements in final demand, created by the multiplier mechanism, actually help stabilize the business cycle. Standard, countercyclic demand-management policies destabilize the business cycle, because they weaken or reverse the procyclic swings in final demand. The perverse effect of countercyclic policy suggests that procyclic demand-management might stabilize the business cycle. Procyclic demand policies do stabilize the business cycle but also destabilize the long cycle. The long cycle, unlike the business cycle, is indeed driven by variations in final demand that propagate through the multiplier and accelerator mechanisms. Demand-management may indeed be an appropriate tool for stabilizing the demand-driven long cycle but is probably not an effective method for controlling the business cycle. More appropriate business-cycle stabilization policies lie within the inventory-employment structure that creates the cycle. The use of overtime and undertime is one example of an effective business-cycle stabilization policy that does not have adverse effects on the longer mode.

Theoretical Insights. Different stability criteria often yield qualitatively different policy conclusions. The three measures of transient response used in this study seldom yielded the same result. A decline in damping ratio can coexist with a decline in damped frequency or a decline in damping time. Of the three measures of transient response, damping ratio appears to be a better guide to frequency response than the other two. A decline in damping, however, does not necessarily mean that gain increases. Gain depends on all the eigenvalues of the system. An increase in damping of other eigenvalues, coupled with increased frequency separation, can cause gain to drop even at frequencies where damping has declined for the closest eigenvalues.

Three trade-offs are important in judging stabilization policy by the frequency-response (gain) criterion. Policies can reduce gain at some frequencies and increase gain at others. Policies can reduce gain between a disturbance and one output variable while increasing gain between the same disturbance and another output variable. Policies can reduce gain from one disturbance to an output variable while increasing gain from another disturbance to the same output variable. The choice of disturbance, output, and frequency is important to policy assessment by the frequency-response criterion.

Finally, most of the effects that policies have on damping and periodicity can be anticipated by phase-and-gain analysis. Each policy activates one or more feedback loops with phase-and-gain characteristics that vary with frequency. The effect of a policy on different modes is

determined by calculating its phase and gain at the period of the cyclic mode of interest. If the phase shift is near 180° , the policy is stabilizing; if the shift is nearer 360° , it is destabilizing; if the shift is near 90° or 270° , the policy tends to lengthen or shorten the period of the cycle without affecting the damping ratio.

CHAPTER VI

CONCLUSIONS AND EXTENSIONS

A. CONCLUSIONS

Four elementary models from macroeconomic theory can be integrated into a single, closed-loop, dynamic model of the U.S. economy. The multiplier-accelerator, IS-LM, aggregate supply - aggregate demand, and inventory-adjustment models are combined into one. The dynamic adjustment mechanisms underlying some of the static equilibrium assumptions must be made explicit, but the required changes do not alter the fundamental economic concepts. Most of the parameters needed for the model are readily available from statistical abstracts and previously published empirical studies. The behavior of the model is plausible and reasonably robust under minor modifications of both structure and parameters.

The model produces a 4-year fluctuation which closely resembles the U.S. business cycle. Both the periodicity and the phase relationships between variables in the model are very similar to those observed in U.S. data. The business-cycle mode of behavior is produced by the interaction of employment and inventory investment. Fluctuations in final sales for consumption, investment, and government spending are not responsible for the business cycle. In fact, procyclical movements in consumption that stem from the multiplier mechanism actually help stabilize the cycle. Price, interest rate, and capital investment have little involvement with business cycle.

The model also produces a longer, 24-year cycle. Unlike the business cycle, the long cycle is indeed created by variations in final sales. The accelerator and capital stock-adjustment mechanisms produce long swings in investment demand. The multiplier mechanism compounds the cycle by creating parallel swings in consumption demand. Interest rate and price changes exert a mild stabilizing influence on the long cycle by damping investment swings. No attempt was made to relate the long cycle of the model to real economic behavior. The important point is not any conclusion about long cycles in the economy, but, rather, that consumption and investment are tied to a much longer behavior mode than the business cycle.

The model was used to test five standard stabilization policies. All five are countercyclic demand-management policies, which lower demand during periods of high output and raise demand during periods of low output. Each of the five policies was tested by four different stability criteria. All five policies destabilize the business cycle by the damping-ratio criterion and by at least one other measure of transient response. Three of the five policies also destabilize the business cycle by the frequency-response criterion for most disturbance-output pairs examined. The policies with outside lags have the strongest destabilizing effects, but even those policies that involve no lags at all tend to destabilize the business cycle. All five policies tend to stabilize the long cycle by most stability criteria. The lagged policies have the strongest stabilizing influence on the long cycle.

The destabilizing effect of countercyclic demand management suggests that a procyclic policy might stabilize the business cycle. In fact, procyclic policies do stabilize the business cycle but destabilize the long cycle. Demand management is probably not the most effective means of controlling the business cycle. Variations in final demand simply do not produce the cyclic adjustment pattern. More effective policies may lie within the inventory-adjustment mechanism that is responsible for the business cycle. For example, policies that promote more flexible capacity utilization help stabilize the business cycle without destabilizing the long cycle.

In one sense, the policy results support conventional wisdom that countercyclic demand management can help stabilize the economy. Standard policies stabilize the model's long cycle, which is created by the multiplier and accelerator mechanisms. The study does not, however, support the idea that conventional policies help stabilize the business cycle. By most stabilizing criteria, standard policies are destabilizing. Authors who have warned that demand-management policies might be destabilizing appear to be correct. Several of the authors, however, based their warnings on analysis of multiplier-accelerator models. This study indicates that they may have incorrectly diagnosed the structural origin of the perverse effects of conventional policies. The model, with its two distinct cycles, provides a framework for reconciling the conventional wisdom based on static analysis with the apparently contradictory results based on dynamic analysis. The two schools may both be correct with respect to one behavior mode, but not the other.

From a methodological point of view, the study shows that the choice of stability criteria can strongly affect conclusions about stabilization policy. The three measures of transient response lead to the same policy conclusion in only one out of five cases. Furthermore, results by the damping-ratio criterion, the most common measure of transient response, conflict with results by the frequency-response criterion in two out of five cases. Last, even within the frequency-response criterion, policy conclusions depend on the choice of frequency range, disturbance source, and output variable.

From a theoretical standpoint, frequency response is the stability criterion of choice. The object of stabilization policy is to insulate the economy from random shocks and disturbances. Damping ratio measures the tendency only for a single cyclic mode to die away after a single shock. Frequency response directly measures the sensitivity of different variables to streams of random disturbances from different sources. The richness of information available from frequency-response analysis is unparalleled by other stability criteria. Criteria such as damping ratio, damped frequency, damping time, variance, and autospectral densities may be useful shortcuts to frequency response but do not appear to be reliable substitutes.

The development of new computer packages allows the quick and easy conversion of nonlinear simulation models into linear models. The dynamic properties of the linear models can be analyzed using eigenvalue and frequency response techniques. Eigenvalue analysis permits clear

separation of behavior modes in large models where superposition of multiple modes complicates the analysis of simulation output. Frequency response analysis permits the computation of stochastic response without making long simulation runs with noise inputs. Eigenvalue and frequency response elasticities permit direct computation of parameter sensitivity. Perhaps the most important technique introduced here is the use of eigenvalue elasticities to isolate dominant feedback loops. Eigenvalue elasticities with respect to changes in the structural links between variables can indicate which feedback loops are most active in each mode of behavior. The loop-identification technique used in this study is fairly crude; an improved technique is outlined in the following section.

B. EXTENSIONS

Structural Improvements. The structure of the model used in this study could be extended in many ways. Such extensions were not undertaken, in order to preserve as much fidelity as possible with the underlying theoretical models.

1) The model locks wage and price together so that real wage is a constant. In a more elaborate formulation, wage could be a function of supply and demand in the labor market and the cost of living. Price could be a function of supply and demand for goods and the cost of production. Resulting variations in the real wage rate could introduce new dynamics.

2) The model contains no budget constraints for government, business, or households. A simple budget constraint would set net borrowing (or lending) by each sector equal to the difference between its current expenses and current income. Interest rate could then be a function of total net demand for funds for the whole economy. A more elaborate set of constraints would keep track of separate stocks of money, savings, and debt for each sector.

3) The model contains no supply constraints on hiring, consumption, or capital investment. A more refined model would make hiring a function not only of demand for labor but also unemployment, which represents the supply of unused labor. As unemployment falls, additional labor becomes increasingly difficult to obtain. Consumption and investment spending could be made a function of inventory availability. As inventory levels fall, goods become increasingly difficult to obtain. Supply constraints would require the addition of order backlogs to keep track of unsatisfied demand. The addition of constraints could affect damping in the business cycle by limiting the movements of employment and inventory away from equilibrium.

4) The model implicitly assumes that all consumption goods are perishable. Goods are not stocked. Consumption could instead feed a stock of goods in the household. Consumption could then be a function of the discrepancy between actual and desired stocks of goods. Adding an explicit stock of goods could create new dynamic behavior.

5) In the base model, labor supply is a constant. Labor supply could be made a function of real wage, stock of household goods, relative supply and demand for assets, and other variables. The change can be made by adding a household utility function.

6) The model assumes all productive effort is concentrated in a single, homogenous production sector. At the cost of a quantum leap in complexity, the model could be disaggregated into two or more production sectors. The first step might be to divide the production sector into two blocks, one producing long-lived goods such as capital plant, equipment, housing, and consumer durables and another producing all other goods and services. Such a division might reveal the very long, 50-year Kondratieff cycle. Forrester (1976) has traced the Kondratieff cycle in his model to interactions between capital and consumer goods sectors.

The list of structural improvements and additions is endless. The preceding list suggests only a few relatively small structural changes that can be made without altering the spirit of a simple theoretical model.

Parameter Estimation. Another possible extension of the study is formal estimation of model parameters from economic time series. Relatively little effort has gone into assembling the parameters used in the base model. Some of the suggested structural improvements should probably be made before investing the time and effort needed for careful estimation. A by-product of the parameter estimation process would be

estimates of noise characteristics in the system. Spectral analysis of the residuals in estimated equations could generate the autospectral density functions of the noise sources. The density functions can be used in conjunction with frequency-response analysis to determine how policies affect the variance and autospectrum of important variables. Of course, the accuracy of these estimates in general would be no greater than the accuracy of the parameter estimates themselves, which depend on the structural assumptions of the model.

Policy Testing. Two extensions involve additional policy analysis. First, effects of the five standard stabilization policies on new disturbance-output combinations can be examined. The results might yield additional insight into the differences between damping ratio and frequency response as measures of stability. Second, new stabilization policies might be tested. The study strongly suggests that demand-management is not a proper approach to business-cycle stabilization policy. Instead, policies could be tried that act directly on production and inventory-management decisions.

Dominant Loop Identification. The method for loop identification outlined in Chapter 2 suffers from two deficiencies. First, the method isolates important links between state variables in a condensed linear model where all nonstate variables have been eliminated. The links in the condensed model do not correspond directly to links in the original model. Changing a link in the original model would require changing several links in the condensed model to different degrees.

Finding important links in the condensed model may help locate important links in the original model when the correspondence is close, but it can also be misleading, as seen in analyzing the long cycle. Important links of the original model can be isolated directly, but the new approach requires more computation and a different linearization routine. The approach was not tried in this study, because appropriate software for linearization has not been developed.

The linearized model can be written in the "descriptor" form, which preserves the original structure between nonstate variables. Each causal link in the original model corresponds to an element of one of the four subscripted matrices in equation (6.1).

$$\begin{bmatrix} \dot{\underline{x}} \\ \underline{z} \end{bmatrix} = \begin{bmatrix} \underline{A}_1 & \underline{A}_2 \\ \underline{A}_3 & \underline{A}_4 \end{bmatrix} \begin{bmatrix} \underline{x} \\ \underline{z} \end{bmatrix} \quad (6.1)$$

or:

$$\begin{aligned} \dot{\underline{x}} &= \underline{A}_1 \underline{x} + \underline{A}_2 \underline{z} \\ 0 &= \underline{A}_3 \underline{x} + \underline{A}_4^* \underline{z} \end{aligned}$$

where $\dot{\underline{x}}$ -- vector of state variable derivatives

\underline{x} -- vector of state variables

\underline{z} -- vector of non-state variable

$\underline{A}_1, \underline{A}_2, \underline{A}_3, \underline{A}_4, \underline{A}_4^*$ -- matrices of constant coefficients.

The importance of a link to a particular eigenvalue equals the eigenvalue elasticity with respect to the corresponding element of the matrices \underline{A} , \underline{A}_2 , \underline{A}_3 , or \underline{A}_4 . The elasticity is calculated by the parameter-sensitivity formula in equation (2.23). The formula involves the partial derivative of the system matrix with respect to the link. The partial derivative can be computed from the change in the system matrix, \underline{A} , before and after a change in an element of \underline{A}_1 , \underline{A}_2 , \underline{A}_3 , or \underline{A}_4 . The relationship between \underline{A} and the subscripted matrices is given in equation (6.2).

$$\underline{A} = \underline{A}_1 - \underline{A}_2 (\underline{A}_4^* - \underline{I})^{-1} \underline{A}_3 \quad (6.2)$$

The elements of \underline{A}_1 , \underline{A}_2 , \underline{A}_3 , and \underline{A}_4^* are actually easier to compute than the elements of \underline{A} , but computation of \underline{A} by equation (6.2) involves matrix inversion, which, if done repeatedly to obtain partial derivatives with respect to all structural links, can consume a significant amount of computer time. A by-product of using the descriptor form is that not only eigenvalue elasticities but also frequency-response elasticities with respect to the structural links can be computed. The dominant loops controlling frequency response may not be the same as those controlling the eigenvalues.

A second major deficiency of the loop-identification technique in Chapter 2 is the ad hoc criterion for loop "importance." A loop is considered important if it contains only links with large eigenvalue elasticities. As shown below, the criterion can be misleading. Note

that the sum of the eigenvalue elasticities of all links coming into a variable equals the sum of the eigenvalue elasticities of all links leaving the variable. (This property was observed in the model used here but was not proven.) The property implies that a certain numerical value for the eigenvalue elasticity passes from one link to another around each feedback loop. The elasticity of any link is the sum of the elasticities of all the loops that pass through the link. A link could lie in two different feedback loops with opposite effects on an eigenvalue. The elasticities of the two loops would cancel, leaving the link with a small elasticity. Both loops might be considered unimportant, because they contained a link with a small elasticity.

The problem of cancellation can be overcome by solving a system of simultaneous linear equations for the eigenvalue elasticity associated with each loop. To make the calculation, first identify all feedback loops in the model and all the links they pass through. Then set up and solve a set of simultaneous equations, where each equation sets the eigenvalue elasticity of a structural link equal to the sum of eigenvalue elasticities of all the loops that contain the link. In general, the set of equations will be overdetermined yet will have a unique solution. Many of the equations will be linearly dependent. Solving for the loop elasticities yields a single number associated with each loop showing its relative contribution to the behavior mode in question. The approach was not applied to the model in this study for lack of available software to search for all possible loops.

The final and most useful extension of this work would be to consolidate all the techniques of linear analysis used and suggested here into an integrated, user-friendly computer package. The ability to switch readily from nonlinear simulation modeling to linear analysis and back again would greatly increase the efficiency of work on complex models. Linear analysis alone is not sufficient but can be an extremely powerful guide to the origins of complex behavior patterns and to the probable effects of policy intervention.

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```

A      DK,K=ALFA*LED,K/(1/ALK+R,K)
L      LED,K=LED,J+(DT)(A,J-LED,J)/TSLD
N      LED=EY
C      TSLD=4
NOTE
NOTE      CONSUMPTION
NOTE
A      C,K=APC*PY,K
N      APC=(PY-KD)/PY
L      PY,K=PY,J+(DT)(CDY,J-PY,J)/TSY
N      PY=ET-(T-GT)
C      TSY=2.5
A      CDY,K=Y,K-(T,K-GT,K)
NOTE
NOTE      INTEREST RATE
NOTE
A      R,K=LR*EXP(-1/IEM*LOGN(EY*P,K/(M,K*EYVM)))*EXP(-YEM/IEM*LOGN
X      (AY,K/EY))
C      LR=.03
C      YEM=.7
C      EYVM=5
C      IEM=-1
L      AY,K=AY,J+(DT)(Y,J-AY,J)/TSAY
N      AY=EY
C      TSAY=2.5
L      M,K=M,J+(DT)(RCM,J)
N      M=EM
N      EM=EY*EP/EYVM
NOTE
NOTE      PRICE LEVEL
NOTE
L      P,K=P,J+(DT)(P,J*SPC*(NRU/U,J-1))
N      P=EP
C      EP=1
C      SPC=.0175
NOTE
NOTE      GOVERNMENT
NOTE
A      G,K=EGS+CGS,K
C      EGS=400E9
A      CGS,K=EGS*PT,K*SCGS
C      SCGS=0
A      T,K=(EGS+EGT)(1-SGYT)+Y,K*NTR*SGYT
C      SGYT=1
N      NTR=(EGS+EGT)/EY
A      GT,K=EGT+CGT,K
C      EGT=200E9
A      CGT,K=EGT*SCGT*PT,K
C      SCGT=0
A      RCM,K=M,K*PT,K*SCMG+STM*(TMS,K-M,K)/TAM
C      TAM=.0625

```

```

C      SCMG=0
C      STM=0
A      TMS,K=EM*(1+SCMS*PT,K)
C      SCMS=0
A      FT,K=(LU,K-NRU)+SDC*(E,K-DE,K)/(TAE*E,K)
C      SDC=0
L      LU,K=LU,J+(DT)*(U,J-LU,J)/TSU
N      LU=U
C      TSU=.0625
NOTE
NOTE          CONTROL CONSTANTS
NOTE
C      IT=.0625
C      LENGTH=12.5
C      PLTPER=.25
C      PRTPER=0
S      RCI,K=Y,K-C,K-IVST,K-G,K
S      RCE,K=((DE,K-E,K)/TAE)/E,K
S      PERY,K=Y,K-KD,K
PLOT Y=Y/IV=V/IVST=I/R=R/RCI=C/RCE=H/U=U/PERY=P
NOTE PLOT Y=Y,FS=S,A=A(1980E9,2020E9)/JV=I/U=U/E=E/SED=D/DII=V
PRINT  E,DE,U,K,DK,IVST,KD,R
PRINT  Y,A,SED,LED,C,G,T,GT,FY,CDY,AY
PRINT  P,M,TMS,RCM,CGS,CGT,LU,PT,PTY
PRINT  IV,DIV,DII,PERY,RCI,RCE,FS
EOF:

```

TOF:

*
*
*

APPENDIX B

DYNAMO EQUATIONS FOR LINEARIZATION

NOTE

NOTE

INVENTORY

NOTE

L $IV.K=Y.J-FS.J$
 N $IV=EY*NIC$
 C $NIC=.3$
 A $Y.K=PTY.K*(1-FCU)+SED.K*FCU$
 C $FCU=.5$
 A $PTY.K=EY*EXP((1-ALFA)*LOGN(E.K/EE))*EXP(ALFA*LOGN(K.K/EK))$
 X $+NOISE(YNSE.K)$
 D $YNSE=1,.1$
 C $EY=1$
 C $ALFA=.25$
 A $A.K=FS.K+DII.K+NOISE(ANSE.K)$
 A $FS.K=C.K+IVST.K+G.K$
 D $ANSE=1,.1$
 A $DII.K=(DIV.K-IV.K)/TAI$
 C $TAI=.4$
 A $DIV.K=NIC*LED.K$
 L $SED.K=(A.J-SED.J)/TSSD$
 N $SED=EY$
 C $TSSD=.5$

NOTE

NOTE

LABOR

NOTE

L $E.K=(DE.J-E.J)/TAE$
 N $E=EE$
 C $EE=1$
 C $TAE=.4$
 A $DE.K=(1-ALFA)*SED.K/RW$
 N $RW=(1-ALFA)*EY/EE$
 A $U.K=(EE/(1-NRU)-E.K)/(EE/(1-NRU))$
 N $U=NRU$
 C $NRU=.05$

NOTE

NOTE

CAPITAL

NOTE

L $K.K=(IVST.J-KD.J)$
 N $K=EK$
 N $EK=ALFA*EY/(1/ALK+LR)$
 A $KD.K=K.K/ALK$
 C $ALK=14$
 A $IVST.K=(KD.K+(DK.K-K.K)/TAK)$
 C $TAK=3$
 A $DK.K=ALFA*LED.K/(1/ALK+R.K)$
 L $LED.K=(A.J-LED.J)/TSLD$
 N $LED=EY$
 C $TSLD=4$

NOTE
NOTE
NOTE

CONSUMPTION

A $C.K = APC * PY.K$
 N $APC = (PY.K - KD) / PY$
 L $PY.K = (CDY.J - PY.J) / TSY$
 N $PY = EY - (T - GT)$
 C $TSY = 2.5$
 A $CDY.K = Y.K - (T.K - GT.K)$

NOTE
NOTE
NOTE

INTEREST RATE

A $R.K = LR * \exp(-1 / IEM * \text{LOGN}(EY * P.K / (M.K * EYVM))) * \exp(-YEM / IEM * \text{LOGN}(AY.K / EY))$
 X
 C $LR = .03$
 C $YEM = .7$
 C $EYVM = 5$
 C $IEM = -1$
 L $AY.K = (Y.J - AY.J) / TSAY$
 N $AY = EY$
 C $TSAY = 2.5$
 L $M.K = RCM.J$
 N $M = EM$
 N $EM = EY * EP / EYVM$

NOTE
NOTE
NOTE

PRICE LEVEL

L $P.K = P.J * SPC * (NRU / U.J - 1)$
 N $P = EP$
 C $EP = 1$
 C $SPC = .0175$

NOTE
NOTE
NOTE

GOVERNMENT

A $G.K = EGS + CGS.K$
 C $EGS = .2$
 A $CGS.K = EGS * PT.K * SCGS$
 C $SCGS = 0$
 A $T.K = (EGS + EGT) * (1 - SGYT) + Y.K * NTR * SGYT$
 C $SGYT = 1$
 N $NTR = (EGS + EGT) / EY$
 A $GT.K = EGT + CGT.K$
 C $EGT = .1$
 A $CGT.K = EGT * SCGT * PT.K$
 C $SCGT = 0$
 A $RCM.K = M.K * PT.K * SCMG + STM * (TMS.K - M.K) / TAM$
 C $TAM = .01$
 C $SCMG = 0$
 C $STM = 0$
 A $TMS.K = EM * (1 + SCMS * PT.K)$
 C $SCMS = 0$

```

A      PT,K=(LU,K-NRU)+SDC*(E,K-DE,K)/(TAE*E,K)
C      SDC=0
L      LU,K=(U,J-LU,J)/TSU
N      LU=U
C      TSU=.001
NOTE
NOTE          CONTROL CONSTANTS
NOTE
R      TIME,KL=TIME,JK+DT
S      MP,K=P,K+NOISE(QNSE,K)
N      MP=P+NOISE(.001)
D      QNSE=1,.1
L      OVCDY,K=CDY,J
N      OVCDY=CDY
L      OVE,K=E,J
N      OVE=E
L      OVIV,K=IV,J
N      OVIV=IV
L      OVI,K=IVST,J
N      OVI=IVST
C      DT=.0625
C      LENGTH=15
C      PLTPER=.5
C      PRTPER=15
PLOT   Y=Y/E=E/A=A/U=U/IV=V/IVST=I/C=C/P=P
PRINT  E,DE,U,K,DK,IVST,KD,R
PRINT  Y,A,SED,LED,C,G,T,GT,PY,AY,CDY
PRINT  P,M,TMS,RCM,CGS,CGT
PRINT  IV,DIV,MP,OVCDY,OVE,OVIV,OVI

```

APPENDIX C - VARIABLE DEFINITIONS

| | |
|--------|---|
| A | AGGREGATE DEMAND (UNITS/YEAR) |
| ALFA | CAPITAL EXPONENT IN PRODUCTION FUNCTION (DIMENSIONLESS) |
| ALK | AVERAGE LIFE OF CAPITAL (YEARS) |
| APC | AVERAGE PROPENSITY TO CONSUME (DIMENSIONLESS) |
| AY | AVERAGE OUTPUT (UNITS/YEAR) |
| C | CONSUMPTION (UNITS/YEAR) |
| CDY | CURRENT DISPOSIBLE INCOME (UNITS/YEAR) |
| CGS | COUNTERCYCLIC GOVERNMENT SPENDING (UNITS/YEAR) |
| CGT | COUNTERCYCLIC GOVERNMENT TRANSFERS (UNITS/YEAR) |
| DE | DESIRED EMPLOYMENT (PERSONS) |
| DII | DESIRED INVENTORY INVESTMENT (UNITS/YEAR) |
| DIV | DESIRED INVENTORY (UNITS) |
| DK | DESIRED CAPITAL (UNITS) |
| DT | SOLUTION INTERVAL (YEARS) |
| E | EMPLOYMENT (PERSONS) |
| EE | EQUILIBRIUM EMPLOYMENT (PERSONS) |
| EGS | EQUILIBRIUM GOVERNMENT SPENDING (UNITS/YEAR) |
| EGT | EQUILIBRIUM GOVERNMENT TRANSFERS (UNITS/YEAR) |
| EK | EQUILIBRIUM CAPITAL (UNITS) |
| EM | EQUILIBRIUM MONEY (DOLLARS) |
| EP | EQUILIBRIUM PRICE (DOLLARS/UNIT) |
| EXP | EXPONENTIAL FUNCTION |
| EY | EQUILIBRIUM OUTPUT (UNITS/YEAR) |
| EYVM | EQUILIBRIUM INCOME VELOCITY OF MONEY (1/YEARS) |
| FCU | FLEXIBILITY OF CAPACITY UTILIZATION (DIMENSIONLESS) |
| FS | FINAL SALES (UNITS/YEAR) |
| G | GOVERNMENT SPENDING (UNITS/YEAR) |
| GT | GOVERNMENT TRANSFERS (UNITS/YEAR) |
| IEM | INTEREST ELASTICITY OF MONEY DEMAND (DIMENSIONLESS) |
| IV | INVENTORY (UNITS) |
| IVST | INVESTMENT (UNITS/YEAR) |
| K | CAPITAL (UNITS) |
| KD | CAPITAL DEPRECIATION (UNITS/YEAR) |
| LED | LONG-RUN EXPECTED DEMAND (UNITS/YEAR) |
| LENGTH | LENGTH OF SIMULATION RUN (YEARS) |
| LOGN | NATURAL LOGARYTHM FUNCTION |
| LR | LONG-RUN INTEREST RATE (FRACTION/YEAR) |
| LU | LAGGED UNEMPLOYMENT RATE (DIMENSIONLESS) |
| M | MONEY (DOLLARS) |
| NIC | NORMAL INVENTORY COVERAGE (YEARS) |
| NORMRN | NORMAL RANDOM NOISE FUNCTION |
| NRU | NATURAL RATE OF UNEMPLOYMENT (DIMENSIONLESS) |
| P | PRICE (DOLLARS/UNIT) |
| PERY | PERSONAL INCOME (UNITS/YEAR) |
| PLSH | PULSE HEIGHT (DIMENSIONLESS) |
| PLST | PULSE TIME (YEARS) |
| PLTPER | PLOTTING PERIOD (YEARS) |
| PRTPER | PRINTING PERIOD (YEARS) |

FT POLICY TRIGGER (DIMENSIONLESS)
 FTY POTENTIAL OUTPUT (UNITS/YEAR)
 FY PERMANENT INCOME (UNITS/YEAR)
 RCE RATE OF CHANGE IN EMPLOYMENT (FRACTION/YEAR)
 RCI RATE OF CHANGE IN INVENTORY (UNITS/YEAR)
 RCM RATE OF CHANGE IN MONEY (DOLLARS/YEAR)
 R INTEREST RATE (FRACTION/YEAR)
 RW REAL WAGE (UNITS/PERSON-YEAR)
 SAMPLE SAMPLE FUNCTION
 SCGS STRENGTH OF COUNTERCYCLIC GOVERNMENT SPENDING (DIMENSIONLESS)
 SCGT STRENGTH OF COUNTERCYCLIC GOVERNMENT TRANSFERS (DIMENSIONLESS)
 SCMG STRENGTH OF COUNTERCYCLIC MONEY GROWTH (DIMENSIONLESS)
 SCMS STRENGTH OF COUNTERCYCLIC MONEY STOCK (DIMENSIONLESS)
 SDC STRENGTH OF DERIVATIVE CONTROL (DIMENSIONLESS)
 SDVA STANDARD DEVIATION IN AGGREGATE DEMAND (DIMENSIONLESS)
 SDVY STANDARD DEVIATION IN OUTPUT (DIMENSIONLESS)
 SED SHORT-RUN EXPECTED DEMAND (UNITS/YEAR)
 SGYT STRENGTH OF GRADUATED INCOME TAX (DIMENSIONLESS)
 SPC SLOPE OF PHILLIPS CURVE (FRACTION/YEAR)
 STM SWITCH ON TARGET MONEY (DIMENSIONLESS)
 T TAXES (UNITS/YEAR)
 TAE TIME TO ADJUST EMPLOYMENT (YEARS)
 TAI TIME TO ADJUST INVENTORY (YEARS)
 TAK TIME TO ADJUST CAPITAL (YEARS)
 TAM TIME TO ADJUST MONEY (YEARS)
 TMS TARGET MONEY STOCK (DOLLARS)
 NTR NORMAL TAX RATE (DIMENSIONLESS)
 TSAY TIME TO SMOOTH AVERAGE OUTPUT (YEARS)
 TSLD TIME TO SMOOTH LONG-RUN DEMAND (YEARS)
 TSNA TIME TO SAMPLE NOISE IN AGGREGATE DEMAND (YEARS)
 TSNY TIME TO SAMPLE NOISE IN OUTPUT (YEARS)
 TSSD TIME TO SMOOTH SHORT-RUN DEMAND (YEARS)
 TSU TIME TO SMOOTH UNEMPLOYMENT (YEARS)
 TSY TIME TO SMOOTH INCOME (YEARS)
 U UNEMPLOYMENT RATE (DIMENSIONLESS)
 Y OUTPUT (UNITS/YEAR)
 YEM INCOME ELASTICITY OF MONEY DEMAND (DIMENSIONLESS)

APPENDIX D -- DOCUMENTOR LISTING

$IV.K = IV.J + (DT)(Y.J - FS.J)$ 1, L
 $IV = EY * NIC$ 1.1, N
 $NIC = .3$ 1.2, C
 IV - INVENTORY (UNITS)
 DT - SOLUTION INTERVAL (YEARS)
 Y - OUTPUT (UNITS/YEAR)
 FS - FINAL SALES (UNITS/YEAR)
 EY - EQUILIBRIUM OUTPUT (UNITS/YEAR)
 NIC - NORMAL INVENTORY COVERAGE (YEARS)

$Y.K = PTY.K * (1 - FCU) + SED.K * FCU$ 2, A
 $FCU = .5$ 2.1, C
 Y - OUTPUT (UNITS/YEAR)
 PTY - POTENTIAL OUTPUT (UNITS/YEAR)
 FCU - FLEXIBILITY OF CAPACITY UTILIZATION
 (DIMENSIONLESS)
 SED - SHORT-RUN EXPECTED DEMAND (UNITS/YEAR)

$PTY.K = EY * EXP((1 - ALFA) * LOGN(E.K / EE)) * EXP(ALFA * LOGN(K.K / EK)) + SAMPLE(NORMRN(0, SDVY * EY), TSNY, 0)$ 3, A
 $EY = 2E12$ 3.2, C
 $ALFA = .25$ 3.3, C
 $SDVY = 0$ 3.4, C
 $TSNY = .25$ 3.5, C
 PTY - POTENTIAL OUTPUT (UNITS/YEAR)
 EY - EQUILIBRIUM OUTPUT (UNITS/YEAR)
 EXP - EXPONENTIAL FUNCTION
 ALFA - CAPITAL EXPONENT IN PRODUCTION FUNCTION
 (DIMENSIONLESS)
 LOGN - NATURAL LOGARYTHM FUNCTION
 E - EMPLOYMENT (PERSONS)
 EE - EQUILIBRIUM EMPLOYMNET (PERSONS)
 K - CAPITAL (UNITS)
 EK - EQUILIBRIUM CAPITAL (UNITS)
 SAMPLE - SAMPLE FUNCTION
 NORMRN - NORMAL RANDOM NOISE FUNCTION
 SDVY - STANDARD DEVIATION IN OUTPUT
 (DIMENSIONLESS)
 TSNY - TIME TO SAMPLE NOISE IN OUTPUT (YEARS)

$DII.K = (DIV.K - IV.K) / TAI$ 4, A
 $TAI = .4$ 4.1, C
 DII - DESIRED INVENTORY INVESTMENT (UNITS/YEAR)
 DIV - DESIRED INVENTORY (UNITS)
 IV - INVENTORY (UNITS)
 TAI - TIME TO ADJUST INVENTORY (YEARS)

$DIV.K = NIC * LED.K$ 5, A
 DIV - DESIRED INVENTORY (UNITS)
 NIC - NORMAL INVENTORY COVERAGE (YEARS)
 LED - LONG-RUN EXPECTED DEMAND (UNITS/YEAR)

$SED.K = SED.J + (DT)(A.J - SED.J) / TSSD$ 6, L
 $SED = EY$ 6.1, N
 $TSSD = .5$ 6.2, C
 SED - SHORT-RUN EXPECTED DEMAND (UNITS/YEAR)
 DT - SOLUTION INTERVAL (YEARS)
 TSSD - TIME TO SMOOTH SHORT-RUN DEMAND (YEARS)
 EY - EQUILIBRIUM OUTPUT (UNITS/YEAR)

$A.K = FS.K + DII.K + SAMPLE(NORMRN(0, SDVA * EY), TSNA, 0)$ 7, A
 $SDVA = 0$ 7.1, C
 $TSNA = .25$ 7.2, C
 FS - FINAL SALES (UNITS/YEAR)
 DII - DESIRED INVENTORY INVESTMENT (UNITS/YEAR)
 SAMPLE - SAMPLE FUNCTION
 NORMRN - NORMAL RANDOM NOISE FUNCTION
 SDVA - STANDARD DEVIATION IN AGGREGATE DEMAND
 (DIMENSIONLESS)
 EY - EQUILIBRIUM OUTPUT (UNITS/YEAR)
 TSNA - TIME TO SAMPLE NOISE IN AGGREGATE DEMAND
 (YEARS)

$FS.K = C.K + IVST.K + G.K + PULSE(PLSH * EY / DT, PLST, 1000)$ 8, A
 $PLSH = .01$ 8.1, N
 $PLST = 1000$ 8.2, C
 FS - FINAL SALES (UNITS/YEAR)
 C - CONSUMPTION (UNITS/YEAR)
 IVST - INVESTMENT (UNITS/YEAR)
 G - GOVERNMENT SPENDING (UNITS/YEAR)
 PLSH - PULSE HEIGHT (DIMENSIONLESS)
 EY - EQUILIBRIUM OUTPUT (UNITS/YEAR)
 DT - SOLUTION INTERVAL (YEARS)
 PLST - PULSE TIME (YEARS)

LABOR

$E.K = E.J + (DT)(DE.J - E.J) / TAE$ 9, L
 $E = EE$ 9.1, N
 $EE = 100E6$ 9.2, C
 $TAE = .4$ 9.3, C
 E - EMPLOYMENT (PERSONS)
 DT - SOLUTION INTERVAL (YEARS)
 DE - DESIRED EMPLOYMENT (PERSONS)
 TAE - TIME TO ADJUST EMPLOYMENT (YEARS)
 EE - EQUILIBRIUM EMPLOYMENT (PERSONS)

$DE.K = (1 - ALFA) * SED.K / RW$ 10, A
 $RW = (1 - ALFA) * EY / EE$ 10.1, N
 DE - DESIRED EMPLOYMENT (PERSONS)
 ALFA - CAPITAL EXPONENT IN PRODUCTION FUNCTION
 (DIMENSIONLESS)
 SED - SHORT-RUN EXPECTED DEMAND (UNITS/YEAR)
 RW - REAL WAGE (UNITS/PERSON-YEAR)
 EY - EQUILIBRIUM OUTPUT (UNITS/YEAR)
 EE - EQUILIBRIUM EMPLOYMENT (PERSONS)

| | |
|--|---------|
| $U.K = (EE / (1 - NRU) - E.K) / (EE / (1 - NRU))$ | 11, A |
| $U = NRU$ | 11.1, N |
| $NRU = .05$ | 11.2, C |
| U - UNEMPLOYMENT RATE (DIMENSIONLESS) | |
| EE - EQUILIBRIUM EMPLOYMENT (PERSONS) | |
| NRU - NATURAL RATE OF UNEMPLOYMENT (DIMENSIONLESS) | |
| E - EMPLOYMENT (PERSONS) | |
| CAPITAL | |
| $K.K = K.J + (DT)(IVST.J - KD.J)$ | 12, L |
| $K = EK$ | 12.1, N |
| $EK = ALFA * EY / (1 / ALK + LR)$ | 12.2, N |
| K - CAPITAL (UNITS) | |
| DT - SOLUTION INTERVAL (YEARS) | |
| IVST - INVESTMENT (UNITS/YEAR) | |
| KD - CAPITAL DEPRECIATION (UNITS/YEAR) | |
| EK - EQUILIBRIUM CAPITAL (UNITS) | |
| ALFA - CAPITAL EXPONENT IN PRODUCTION FUNCTION (DIMENSIONLESS) | |
| EY - EQUILIBRIUM OUTPUT (UNITS/YEAR) | |
| ALK - AVERAGE LIFE OF CAPITAL (YEARS) | |
| LR - LONG-RUN INTEREST RATE (FRACTION/YEAR) | |
| $KD.K = K.K / ALK$ | 13, A |
| $ALK = 14$ | 13.1, C |
| KD - CAPITAL DEPRECIATION (UNITS/YEAR) | |
| K - CAPITAL (UNITS) | |
| ALK - AVERAGE LIFE OF CAPITAL (YEARS) | |
| $IVST.K = (KD.K + (DK.K - K.K) / TAK)$ | 14, A |
| $TAK = 3$ | 14.1, C |
| IVST - INVESTMENT (UNITS/YEAR) | |
| KD - CAPITAL DEPRECIATION (UNITS/YEAR) | |
| DK - DESIRED CAPITAL (UNITS) | |
| K - CAPITAL (UNITS) | |
| TAK - TIME TO ADJUST CAPITAL (YEARS) | |
| $DK.K = ALFA * LED.K / (1 / ALK + R.K)$ | 15, A |
| DK - DESIRED CAPITAL (UNITS) | |
| ALFA - CAPITAL EXPONENT IN PRODUCTION FUNCTION (DIMENSIONLESS) | |
| LED - LONG-RUN EXPECTED DEMAND (UNITS/YEAR) | |
| ALK - AVERAGE LIFE OF CAPITAL (YEARS) | |
| R - INTEREST RATE (FRACTION/YEAR) | |
| $LED.K = LED.J + (DT)(A.J - LED.J) / TSLD$ | 16, L |
| $LED = EY$ | 16.1, N |
| $TSLD = 4$ | 16.2, C |
| LED - LONG-RUN EXPECTED DEMAND (UNITS/YEAR) | |
| DT - SOLUTION INTERVAL (YEARS) | |
| TSLD - TIME TO SMOOTH LONG-RUN DEMAND (YEARS) | |
| EY - EQUILIBRIUM OUTPUT (UNITS/YEAR) | |

CONSUMPTION

$C.K = APC * PY.K$ 17, A
 $APC = (PY - KD) / PY$ 17.1, N
 C - CONSUMPTION (UNITS/YEAR)
 APC - AVERAGE PROPENSITY TO CONSUME
 (DIMENSIONLESS)
 PY - PERMANENT INCOME (UNITS/YEAR)
 KD - CAPITAL DEPRECIATION (UNITS/YEAR)

$PY.K = PY.J + (DT)(CDY.J - PY.J) / TSY$ 18, L
 $PY = EY - (T - GT)$ 18.1, N
 $TSY = 2.5$ 18.2, C
 PY - PERMANENT INCOME (UNITS/YEAR)
 DT - SOLUTION INTERVAL (YEARS)
 CDY - CURRENT DISPOSIBLE INCOME (UNITS/YEAR)
 TSY - TIME TO SMOOTH INCOME (YEARS)
 EY - EQUILIBRIUM OUTPUT (UNITS/YEAR)
 T - TAXES (UNITS/YEAR)
 GT - GOVERNMENT TRANSFERS (UNITS/YEAR)

$CDY.K = Y.K - (T.K - GT.K)$ 19, A
 CDY - CURRENT DISPOSIBLE INCOME (UNITS/YEAR)
 Y - OUTPUT (UNITS/YEAR)
 T - TAXES (UNITS/YEAR)
 GT - GOVERNMENT TRANSFERS (UNITS/YEAR)

INTEREST RATE

$R.K = LR * EXP(-1 / IEM * LOGN(EY * P.K / (M.K * EYVM))) *$ 20, A
 $EXP(-YEM / IEM * LOGN(AY.K / EY))$
 $LR = .03$ 20.2, C
 $YEM = .7$ 20.3, C
 $EYVM = 5$ 20.4, C
 $IEM = -1$ 20.5, C
 R - INTEREST RATE (FRACTION/YEAR)
 LR - LONG-RUN INTEREST RATE (FRACTION/YEAR)
 EXP - EXPONENTIAL FUNCTION
 IEM - INTEREST ELASTICITY OF MONEY DEMAND
 (DIMENSIONLESS)
 LOGN - NATURAL LOGARYTHM FUNCTION
 EY - EQUILIBRIUM OUTPUT (UNITS/YEAR)
 P - PRICE (DOLLARS/UNIT)
 M - MONEY (DOLLARS)
 EYVM - EQUILIBRIUM INCOME VELOCITY OF MONEY (1/
 YEARS)
 YEM - INCOME ELASTICITY OF MONEY DEMAND
 (DIMENSIONLESS)
 AY - AVERAGE OUTPUT (UNITS/YEAR)

$AY.K = AY.J + (DT)(Y.J - AY.J) / TSAY$ 21, L
 $AY = EY$ 21.1, N
 $TSAY = 2.5$ 21.2, C
 AY - AVERAGE OUTPUT (UNITS/YEAR)
 DT - SOLUTION INTERVAL (YEARS)
 Y - OUTPUT (UNITS/YEAR)
 TSAY - TIME TO SMOOTH AVERAGE OUTPUT (YEARS)
 EY - EQUILIBRIUM OUTPUT (UNITS/YEAR)

$M.K = M.J + (DT)(RCM.J)$ 22, L
 $M = EM$ 22.1, N
 $EM = EY * EP / EYVM$ 22.2, N
 M - MONEY (DOLLARS)
 DT - SOLUTION INTERVAL (YEARS)
 RCM - RATE OF CHANGE IN MONEY (DOLLARS/YEAR)
 EM - EQUILIBRIUM MONEY (DOLLARS)
 EY - EQUILIBRIUM OUTPUT (UNITS/YEAR)
 EP - EQUILIBRIUM PRICE (DOLLARS/UNIT)
 EYVM - EQUILIBRIUM INCOME VELOCITY OF MONEY (1/YEARS)

PRICE LEVEL

$P.K = P.J + (DT)(P.J * SPL * (NRU / U.J - 1))$ 23, L
 $P = EP$ 23.1, N
 $EP = 1$ 23.2, C
 $SPC = .0175$ 23.3, C
 P - PRICE (DOLLARS/UNIT)
 DT - SOLUTION INTERVAL (YEARS)
 SPC - SLOPE OF PHILLIPS CURVE (FRACTION/YEAR)
 NRU - NATURAL RATE OF UNEMPLOYMENT (DIMENSIONLESS)
 U - UNEMPLOYMENT RATE (DIMENSIONLESS)
 EP - EQUILIBRIUM PRICE (DOLLARS/UNIT)

GOVERNMENT

$G.K = EGS + CGS.K$ 24, A
 $EGS = 400E9$ 24.1, C
 G - GOVERNMENT SPENDING (UNITS/YEAR)
 EGS - EQUILIBRIUM GOVERNMENT SPENDING (UNITS/YEAR)
 CGS - COUNTERCYCLIC GOVERNMENT SPENDING (UNITS/YEAR)
 $CGS.K = EGS * PT.K * SCGS$ 25, A
 $SCGS = 0$ 25.1, C
 CGS - COUNTERCYCLIC GOVERNMENT SPENDING (UNITS/YEAR)
 EGS - EQUILIBRIUM GOVERNMENT SPENDING (UNITS/YEAR)
 PT - POLICY TRIGGER (DIMENSIONLESS)
 SCGS - STRENGTH OF COUNTERCYCLIC GOVERNMENT SPENDING (DIMENSIONLESS)

$T.K = (EGS + EGT)(1 - SGYT) + Y.K * NTR * SGYT$ 26, A
 $SGYT = 1$ 26.1, C
 $NTR = (EGS + EGT) / EY$ 26.2, N

T - TAXES (UNITS/YEAR)
 EGS - EQUILIBRIUM GOVERNMENT SPENDING (UNITS/YEAR)
 EGT - EQUILIBRIUM GOVERNMENT TRANSFERS (UNITS/YEAR)
 SGYT - STRENGTH OF GRADUATED INCOME TAX (DIMENSIONLESS)
 Y - OUTPUT (UNITS/YEAR)
 NTR - NORMAL TAX RATE (DIMENSIONLESS)
 EY - EQUILIBRIUM OUTPUT (UNITS/YEAR)

$GT.K = EGT + CGT.K$ 27, A
 $EGT = 200E9$ 27.1, C
 GT - GOVERNMENT TRANSFERS (UNITS/YEAR)
 EGT - EQUILIBRIUM GOVERNMENT TRANSFERS (UNITS/YEAR)
 CGT - COUNTERCYCLIC GOVERNMENT TRANSFERS (UNITS/YEAR)

$CGT.K = EGT * SCGT * PT.K$ 28, A
 $SCGT = 0$ 28.1, C
 CGT - COUNTERCYCLIC GOVERNMENT TRANSFERS (UNITS/YEAR)
 EGT - EQUILIBRIUM GOVERNMENT TRANSFERS (UNITS/YEAR)
 SCGT - STRENGTH OF COUNTERCYCLIC GOVERNMENT TRANSFERS (DIMENSIONLESS)
 PT - POLICY TRIGGER (DIMENSIONLESS)

$RCM.K = M.K * PT.K * SCMG + STM * (TMS.K - M.K) / TAM$ 29, A
 $TAM = .0625$ 29.1, C
 $SCMG = 0$ 29.2, C
 $STM = 0$ 29.3, C
 RCM - RATE OF CHANGE IN MONEY (DOLLARS/YEAR)
 M - MONEY (DOLLARS)
 PT - POLICY TRIGGER (DIMENSIONLESS)
 SCMG - STRENGTH OF COUNTERCYCLIC MONEY GROWTH (DIMENSIONLESS)
 STM - SWITCH ON TARGET MONEY (DIMENSIONLESS)
 TMS - TARGET MONEY STOCK (DOLLARS)
 TAM - TIME TO ADJUST MONEY (YEARS)

$TMS.K = EM * (1 + SCMS * PT.K)$ 30, A
 $SCMS = 0$ 30.1, C
 TMS - TARGET MONEY STOCK (DOLLARS)
 EM - EQUILIBRIUM MONEY (DOLLARS)
 SCMS - STRENGTH OF COUNTERCYCLIC MONEY STOCK (DIMENSIONLESS)
 PT - POLICY TRIGGER (DIMENSIONLESS)

$PT.K=(LU.K-NRU)+SDC*(E.K-DE.K)/(TAE*E.K)$ 31, A
 $SDC=0$ 31.1, C
 FT - POLICY TRIGGER (DIMENSIONLESS)
 LU - LAGGED UNEMPLOYMENT RATE (DIMENSIONLESS)
 NRU - NATURAL RATE OF UNEMPLOYMENT (DIMENSIONLESS)
 SDC - STRENGTH OF DERIVATIVE CONTROL (DIMENSIONLESS)
 E - EMPLOYMENT (PERSONS)
 DE - DESIRED EMPLOYMENT (PERSONS)
 TAE - TIME TO ADJUST EMPLOYMENT (YEARS)

$LU.K=LU.J+(DT)(U.J-LU.J)/TSU$ 32, L
 $LU=U$ 32.1, N
 $TSU=.0625$ 32.2, C
 LU - LAGGED UNEMPLOYMENT RATE (DIMENSIONLESS)
 DT - SOLUTION INTERVAL (YEARS)
 U - UNEMPLOYMENT RATE (DIMENSIONLESS)
 TSU - TIME TO SMOOTH UNEMPLOYMENT (YEARS)

CONTROL CONSTANTS

$DT=.0625$ 32.6, C
 $LENGTH=12.5$ 32.7, C
 $PLTPER=.25$ 32.8, C
 $PRTPER=0$ 32.9, C
 DT - SOLUTION INTERVAL (YEARS)
 LENGTH - LENGTH OF SIMULATION RUN (YEARS)
 PLTPER - PLOTTING PERIOD (YEARS)
 PRTPER - PRINTING PERIOD (YEARS)

$RCI.K=Y.K-C.K-IVST.K-G.K$ 33, S
 RCI - RATE OF CHANGE IN INVENTORY (UNITS/YEAR)
 Y - OUTPUT (UNITS/YEAR)
 C - CONSUMPTION (UNITS/YEAR)
 IVST - INVESTMENT (UNITS/YEAR)
 G - GOVERNMENT SPENDING (UNITS/YEAR)

$RCE.K=((DE.K-E.K)/TAE)/E.K$ 34, S
 RCE - RATE OF CHANGE IN EMPLOYMENT (FRACTION/YEAR)
 DE - DESIRED EMPLOYMENT (PERSONS)
 E - EMPLOYMENT (PERSONS)
 TAE - TIME TO ADJUST EMPLOYMENT (YEARS)

$PERY.K=Y.K-KD.K$ 35, S
 PERY - PERSONAL INCOME (UNITS/YEAR)
 Y - OUTPUT (UNITS/YEAR)
 KD - CAPITAL DEPRECIATION (UNITS/YEAR)

PLOT $Y=V/IV=U/IVST=I/R=R/RCI=C/RCE=H/U=U/PERY=P$ 35.1

Y - OUTPUT (UNITS/YEAR)
 IV - INVENTORY (UNITS)
 IVST - INVESTMENT (UNITS/YEAR)
 R - INTEREST RATE (FRACTION/YEAR)
 RCI - RATE OF CHANGE IN INVENTORY (UNITS/YEAR)
 RCE - RATE OF CHANGE IN EMPLOYMENT (FRACTION/YEAR)
 U - UNEMPLOYMENT RATE (DIMENSIONLESS)
 PERY - PERSONAL INCOME (UNITS/YEAR)

PRINT E,DE,U,K,DK,IVST,KD,R 35.3

E - EMPLOYMENT (PERSONS)
 DE - DESIRED EMPLOYMENT (PERSONS)
 U - UNEMPLOYMENT RATE (DIMENSIONLESS)
 K - CAPITAL (UNITS)
 DK - DESIRED CAPITAL (UNITS)
 IVST - INVESTMENT (UNITS/YEAR)
 KD - CAPITAL DEPRECIATION (UNITS/YEAR)
 R - INTEREST RATE (FRACTION/YEAR)

PRINT Y,A,SED,LED,C,G,T,GT,PY,CDY,AY 35.4

Y - OUTPUT (UNITS/YEAR)
 SED - SHORT-RUN EXPECTED DEMAND (UNITS/YEAR)
 LED - LONG-RUN EXPECTED DEMAND (UNITS/YEAR)
 C - CONSUMPTION (UNITS/YEAR)
 G - GOVERNMENT SPENDING (UNITS/YEAR)
 T - TAXES (UNITS/YEAR)
 GT - GOVERNMENT TRANSFERS (UNITS/YEAR)
 PY - PERMANENT INCOME (UNITS/YEAR)
 CDY - CURRENT DISPOSIBLE INCOME (UNITS/YEAR)
 AY - AVERAGE OUTPUT (UNITS/YEAR)

PRINT P,M,TMS,RCM,CGS,CGT,LU,PT,PTY 35.5

P - PRICE (DOLLARS/UNIT)
 M - MONEY (DOLLARS)
 TMS - TARGET MONEY STOCK (DOLLARS)
 RCM - RATE OF CHANGE IN MONEY (DOLLARS/YEAR)
 CGS - COUNTERCYCLIC GOVERNMENT SPENDING (UNITS/YEAR)
 CGT - COUNTERCYCLIC GOVERNMENT TRANSFERS (UNITS/YEAR)
 LU - LAGGED UNEMPLOYMENT RATE (DIMENSIONLESS)
 PT - POLICY TRIGGER (DIMENSIONLESS)
 PTY - POTENTIAL OUTPUT (UNITS/YEAR)

PRINT IV,DIV,DII,PERY,RCI,RCE,FS 35.6

IV - INVENTORY (UNITS)
 DIV - DESIRED INVENTORY (UNITS)
 DII - DESIRED INVENTORY INVESTMENT (UNITS/YEAR)
 PERY - PERSONAL INCOME (UNITS/YEAR)
 RCI - RATE OF CHANGE IN INVENTORY (UNITS/YEAR)
 RCE - RATE OF CHANGE IN EMPLOYMENT (FRACTION/YEAR)
 FS - FINAL SALES (UNITS/YEAR)

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/*                      APPENDIX E                      */
/*                      */
/*                      FL/1 PROGRAM FOR LINEAR ANALYSIS  */
/*                      */
/* GPSIE USER PROGRAM */
MODEL ;      PROCEDURE OPTIONS(MAIN) REORDER ;
DEFAULT RANGE(A;I,K;Z) STATIC BINARY FLOAT,
RANGE(J) STATIC BINARY FIXED;
DCL (D1,DU,DZ,DX,DP,DF,DXZ,DU1)BIN FIXED(31);
D1=1 ; DU=2 ; DZ=4 ; DX=10 ; DP=24 ; DF=20 ; DXZ=DX+DZ; DU1=DU+D1;
DCL (FTIL(14,14),XHAT(14),IALPHA(3),IDALPHA(3),
PMX(24))BINARY FLOAT,
ANAME(3) CHAR(15),
NOISE      EXTERNAL ENTRY RETURNS (BINARY FLOAT),
(EXP_,LOG_) EXTERNAL ENTRY RETURNS (BINARY FLOAT) ;
DCL EIGRF ENTRY((10,10)BIN FLOAT,BIN FIXED(31),BIN FIXED(31),
BIN FIXED(31),(10)CPLX FLOAT DEC(6),(10,10)CPLX FLOAT DEC(6),
BIN FIXED(31),(120)FLOAT DEC(6),BIN FIXED (31)) EXT
OPTIONS(FORTRAN INTER);
DCL LEQTIC ENTRY((10,10)CPLX FLOAT DEC(6),BIN FIXED(31),BIN FIXED(31),
(10,10)CPLX FLOAT DEC(6),BIN FIXED(31),BIN FIXED(31),BIN FIXED(31)
,(120)FLOAT DEC(6),BIN FIXED(31)) EXT OPTIONS(FORTRAN INTER) ;
DCL (RV(10,10),LVT(10,10),SI(20),EV(10),PF(10,10),WCK(20),TEMP1(10,10)
,TEMP2(10,10),TEMP3(10,10),TEMPI(4,2),TEMPB(10,2),TEMPC(4,10)
,IAI(10,10,20),EVEP(24,10),EVEL(10,10,10),FRM(4,2,20),FREF
(4,2,20,24),TEMPI(10),TEMPC2(4,10),COMPN,
AMX(10,10),BMX(10,2),CMX(4,10),DMX(4,2),PDAF(10,10,24),
PDBP(10,2,24),PDCF(4,10,24),PDDP(4,2,24))CPLX FLOAT DEC(6) ;
DCL (D(8),XV(10),ZV(4),I,J,H,F,L,JOB0,JOB2,IER)BIN FIXED(31) ;
DCL (S(20),
IDEN(10,10),WK(120),CT,SC)FLOAT DEC(6) ;
DCL (XN(10),PN(24),ZN(4),UN(2))CHAR(6) ;
IDEN=0;
DO I=1 TO DX;
  IDEN(I,I)=1 ;
END ;
JOB2=2 ;
JOB0=0 ;
XN(1)='PY' ;
XN(2)='SED ' ;
XN(3)='E ' ;
XN(4)='K ' ;
XN(5)='M ' ;
XN(6)='P ' ;
XN(7)='IV ' ;
XN(8)='AY ' ;
XN(9)='PU ' ;
XN(10)='LED ' ;
PN(1)='TAE' ;
PN(2)='TAI' ;

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FN(3)='EYVM' ;
FN(4)='LR' ;
FN(5)='EGS' ;
FN(6)='NRU' ;
FN(7)='EGT' ;
FN(8)='TAK' ;
FN(9)='IEM' ;
FN(10)='ALK' ;
FN(11)='ALFA' ;
FN(12)='YEM' ;
FN(13)='TSY' ;
FN(14)='TSSD' ;
FN(15)='NIC' ;
FN(16)='SFC' ;
FN(17)='TSLD' ;
FN(18)='TSAY' ;
FN(19)='FCU' ;
FN(20)='SGYT' ;
FN(21)='SCGS' ;
FN(22)='SCGT' ;
FN(23)='SCMG' ;
FN(24)='SCMS' ;
ZN(1)='CDY' ;
ZN(2)='E' ;
ZN(3)='IV' ;
ZN(4)='I' ;
UN(1)='A' ;
UN(2)='Y' ;
XV(1)=1 ; XV(2)=12 ; XV(3)=4 ; XV(4)=6 ; XV(5)=9 ; XV(6)=10 ; XV(7)=11 ;
XV(8)=13 ; XV(9)=5 ; XV(10)=7 ;
ZV(1)=2 ; ZV(2)=14 ; ZV(3)=8 ; ZV(4)=3 ;
DCL TAE DEF PMX(1),
    TAI DEF PMX(2),
    EYVM DEF PMX(3),
    LR DEF PMX(4),
    EGS DEF PMX(5),
    NRU DEF PMX(6),
    EGT DEF PMX(7),
    TAK DEF PMX(8),
    IEM DEF PMX(9),
    ALK DEF PMX(10),
    ALFA DEF PMX(11),
    YEM DEF PMX(12),
    TSY DEF PMX(13),
    TSSD DEF PMX(14),
    NIC DEF PMX(15),
    SFC DEF PMX(16),
    TSLD DEF PMX(17),
    TSAY DEF PMX(18),
    FCU DEF PMX(19),
    SGYT DEF PMX(20) ,

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      SCGS DEF PMX(21) ,
      SCGT DEF PMX(22) ,
      SCMG DEF PMX(23) ,
      SCMS DEF PMX(24) ;
  DECLARE QNSE DEF IALPHA(1),YNSE DEF IALPHA(2),ANSE DEF IALPHA(3),FY
  DEF XHAT(1),OVCDY DEF XHAT(2),OVI DEF XHAT(3),E DEF XHAT(4),LU
  DEF XHAT(5),K DEF XHAT(6),LED DEF XHAT(7),OVIV DEF XHAT(8),M
  DEF XHAT(9),P DEF XHAT(10),JV DEF XHAT(11),SED DEF XHAT(12),AY
  DEF XHAT(13),OVE DEF XHAT(14) ;
CALL CON ; D=0 ;
LP: CALL CHANGE ; CALL IC ; CALL UPDATE ;
FTIL=0 ; TEMPR=0 ; TEMPC=0 ; TEMPD=0 ;
AMX=0 ; BMX=0 ; CMX=0 ; DMX=0 ;
CALL F_Q_TIL ;
DO I=1 TO DX ;
DO J=1 TO DX ;
  AMX(I,J)=FTIL(XV(I),XV(J)) ;
END ; END ;
DO I=1 TO DZ ;
DO J=1 TO DX ;
  CMX(I,J)=FTIL(ZV(I),XV(J)) ;
END ; END ;
BMX=TEMPR ;
DMX=TEMPD ;
TEMP1=AMX ;
CALL EIGRF(TEMP1,DX,DX,JOB2,EV,RV,DX,WK,IER) ;
PUT SKIP(3) EDIT('EIGENVALUES')(A(11)) ;
PUT SKIP(2) EDIT('DAMPING','DAMPED','DAMPING','DAMPED','NATURAL')
  (COL(7),A(7),COL(19),A(6),COL(31),A(7),COL(43),A(6),COL(55),A(7)) ;
PUT SKIP EDIT('RATIO','FREQUENCY','TIME','PERIOD','PERIOD')
  (COL(7),A(5),COL(19),A(9),COL(31),A(9),COL(43),A(6),COL(55),A(6)) ;
DO I=1 TO DX ;
PUT SKIP EDIT('(',I,')',REAL(EV(I))/(ABS(EV(I))+1.E-30),ABS(IMAG(EV(I)))
  )/6.283,6.283/(ABS(REAL(EV(I)))+1.E-30),6.283/(ABS(IMAG(EV(I)))+
  1.E-30),6.283/(ABS(EV(I))+1.E-30))(A(1),F(2),A(1),COL(7),F(5,2),
  COL(19),F(5,3),COL(29),F(8,1),COL(41),F(8,1),COL(53),F(8,1)) ;
END ;
IF D(1)=1ID(5)=1ID(6)=1 THEN DO ;
DO H=1 TO DP ;
IF PMX(H)>0IFPMX(H)<0 THEN DO ;
  PMX(H)=PMX(H)*(1.01) ; TEMPR=0 ; TEMPC=0 ; TEMPD=0 ; FTIL=0 ;
CALL IC ; CALL UPDATE ; CALL F_Q_TIL ;
DO I=1 TO DX ;
DO J=1 TO DX ;
  TEMP1(I,J)=FTIL(XV(I),XV(J)) ;
END ; END ;
DO I=1 TO DZ ;
DO J=1 TO DX ;
  TEMPC(I,J)=FTIL(ZV(I),XV(J)) ;
END ; END ;
PDAP(*,*,H)=(TEMP1-AMX)/(PMX(H)/100) ;

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PDBF(*,*,H)=(TEMPB-BMX)/(PMX(H)/100) ;
PDCP(*,*,H)=(TEMPC-CMX)/(PMX(H)/100) ;
PDDF(*,*,H)=(TEMPE-DMX)/(PMX(H)/100) ;
PMX(H)=PMX(H)/(1.01) ;
END ;
ELSE ;
END ;
END ;
ELSE ;
IF D(1)=1 AND D(2)=1 THEN DO ;
LVT=IDEN ;
TEMP1=RV ;
CALL LEQT1C(TEMP1,DX,DX,LVT,DX,DX,JOBO,WK,IER) ;
END ;
ELSE ;
IF D(1)=1 THEN DO ;
DO I=1 TO DP ;
DO J=1 TO DX ;
TEMPR=0 ;
DO H=1 TO DX ;
DO F=1 TO DX ;
TEMPR(H)=TEMPR(H)+LVT(J,F)*PDAP(F,H,I) ;
END ; END ;
COMPEN=0 ;
DO H=1 TO DX ;
COMPEN=COMPEN+TEMPR(H)*RV(H,J) ;
END ;
EVEP(I,J)=COMPEN*PMX(I)/(EV(J)+1.E-30) ;
END ; END ;
DO H=1 TO 2 ;
PUT SKIP(3) EDIT('EIGENVALUE ELASTICITIES WRT PARAMETERS')(A(50)) ;
PUT SKIP EDIT('MODE')(COL(39),A(4)) ;
PUT SKIP EDIT('PARAM:')(A(6)) ;
DO J=(H-1)*5+1 TO (H-1)*5+5 ;
PUT EDIT(' ','(',J,')','')(A(5),A(1),F(2),A(1),A(5)) ;
END ;
DO I=1 TO DP ;
PUT SKIP EDIT(PN(I))(A(5)) ;
DO J=(H-1)*5+1 TO (H-1)*5+5 ;
PUT EDIT(REAL(EVEP(I,J)),IMAG(EVEP(I,J)))(5(F(8,3),F(6,3))) ;
END ; END ;
END ; END ;
ELSE ;
IF D(2)=1 THEN DO ;
DO I=1 TO DX ;
DO J=1 TO DX ;
DO H=1 TO DX ;
EVEL(I,J,H)=LVT(H,I)*RV(J,H)*AMX(I,J)/(EV(H)+1.E-30) ;
END ; END ; END ;
AGAIN: CALL EVELS ;
END ;

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ELSE GO TO NEXT ;
EVELS: PROCEDURE ;
CT=10;
PUT SKIP(2) LIST('ENTER I=MODE CT=%CUTOFF. END WITH SEMICOLON. I=0 TO
SKIP') ;
GET DATA(I,CT) ;
IF I=0 THEN GO TO NEXT ;
ELSE DO ;
PUT SKIP(3) LIST('EIGENVALUE ELASTICITIES WRT CAUSAL LINKS') ;
PUT SKIP EDIT('MODE--(' ,I,') - ' ,CT, '%CUTOFF - POLAR COORDINATES')
(A(6),F(2),A(4),F(3),A(30)) ;
PUT SKIP LIST(' ') ;
DO H=1 TO DX ;
DO J=1 TO DX ;
IF (ABS(EVEL(H,J,I))) > (CT/100) THEN DO;
  PUT SKIP EDIT(XN(J), '>', XN(H), ABS(EVEL(H,J,I)), ATAN(IMAG(EVEL(H,J,
  I)), REAL(EVEL(H,J,I))+1.E-30)/6.283)
  (A(4),A(2),A(3),F(8,2),F(6,3)) ;
END;
ELSE ;
END; END;
GO TO AGAIN ;
END ;
END;
NEXT: IF D(3)=1|D(4)=1|D(5)=1|D(6)=1 THEN DO ;
S(1)=.025 ;
DO I=2 TO DF ;
S(I)=S(I-1)+.025 ;
END;
DO I=1 TO DF ;
REAL(SI(I))=0 ;
IMAG(SI(I))=6.283*S(I) ;
END ;
DO I=1 TO DF ;
TEMP1=IDEN ;
TEMP3=SI(I)*TEMP1-AMX ;
TEMP2=IDEN ;
CALL LEQT1C(TEMP3,DX,DX,TEMP2,DX,DX,JOBO,WK,IER) ;
IAI(*,*,I)=TEMP2 ;
END ;
DO I=1 TO DF ;
TEMPC=0 ;
DO F=1 TO DZ ;
DO L=1 TO DX ;
DO Q=1 TO DX ;
  TEMPC(F,L)=TEMPC(F,L)+CMX(F,Q)*IAI(Q,L,I) ;
END; END; END;
TEMPD=0 ;
DO F=1 TO DZ ;
DO L=1 TO DU ;
DO Q=1 TO DX ;

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    TEMPD(F,L)=TEMPD(F,L)+TEMPC(F,Q)*BMX(Q,L) ;
END; END; END;
FRM(*,*,I)=(TEMPD+DMX) ;
END;
END;
ELSE ;
IF D(3)=1 THEN DO ;
PUT SKIP(3) LIST('GAIN RATIOS' INPUT > OUTPUT ') ;
PUT SKIP(2) EDIT('FREQ')(A(6)) ;
DO I=1 TO DU ;
DO H=1 TO DZ ;
PUT EDIT(' ',UN(I),'>',ZN(H))(A(3),A(1),A(1),A(3)) ;
END ; END ;
DO J=1 TO DF ;
PUT SKIP EDIT(S(J))(F(5,3)) ;
DO H=1 TO DZ ;
DO I=1 TO DU ;
PUT EDIT(ABS(FRM(H,I,J)))(F(8,2)) ;
END; END; END;
END;
ELSE ;
IF D(4)=1 THEN DO ;
SC=.5 ;
PUT SKIP(2) LIST('ENTER SC=SCALE. END WITH SEMICOLON.') ;
GET DATA(SC) ;
DO I=1 TO DU ;
DO H=1 TO DZ ;
PUT SKIP(2) EDIT('GAIN CURVE:',UN(I),'>',ZN(H))
(A(13),A(1),A(1),A(3)) ;
PUT SKIP EDIT('AMP FRQ', 'AMPLITUDE RATIO O/I')(A(9),COL
(23),A(20)) ;
PUT SKIP EDIT('')(A(4)) ;
DO CT=0 TO 4 ;
PUT EDIT(CT*SC)(5(F(10,2))) ;
END ;
DO F=1 TO DF ;
PUT SKIP EDIT(ABS(FRM(H,I,F)),S(F),'*')
(F(4,2),F(6,3),COL(12+10*ABS(FRM(H,I,F))/SC),A(1)) ;
END ;
PUT SKIP EDIT('')(A(4)) ;
DO CT=0 TO 4 ;
PUT EDIT(CT*SC)(5(F(10,2))) ;
END ;
END; END;
END;
ELSE ;
IF D(5)=1 AND D(6)=1 THEN DO ;
DO I=1 TO DF ;
TEMPC=0 ;
DO F=1 TO DZ ;
DO L=1 TO DX ;

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```

DO Q=1 TO DX ;
  TEMPC(F,L)=TEMPC(F,L)+CMX(F,Q)*IAI(Q,L,I) ;
END; END; END;
TEMPB=0 ;
DO F=1 TO DX ;
DO L=1 TO DU ;
DO Q=1 TO DX ;
  TEMPB(F,L)=TEMPB(F,L)+IAI(F,Q,I)*BMX(Q,L) ;
END; END; END;
DO J=1 TO DP ;
TEMPC2=0 ;
DO F=1 TO DZ ;
DO L=1 TO DX ;
DO Q=1 TO DX ;
  TEMPC2(F,L)=TEMPC2(F,L)+TEMPC(F,Q)*PDAP(Q,L,J) ;
END; END; END;
TEMPD=0 ;
DO F=1 TO DZ ;
DO L=1 TO DU ;
DO Q=1 TO DX ;
  TEMPD(F,L)=TEMPD(F,L)+TEMPC2(F,Q)*TEMPB(Q,L) ;
END; END; END;
FREP(*,*,I,J)=TEMPD ;
TEMPD=0 ;
DO F=1 TO DZ ;
DO L=1 TO DU ;
DO Q=1 TO DX ;
  TEMPD(F,L)=TEMPD(F,L)+TEMPC(F,Q)*PDBP(Q,L,J) ;
END; END; END;
FREP(*,*,I,J)=FREP(*,*,I,J)+TEMPD ;
TEMPD=0 ;
DO F=1 TO DZ ;
DO L=1 TO DU ;
DO Q=1 TO DX ;
  TEMPD(F,L)=TEMPD(F,L)+PDCP(F,Q,J)*TEMPB(Q,L) ;
END; END; END;
FREP(*,*,I,J)=(FREP(*,*,I,J)+TEMPD+PDDP(*,*,J))*PMX(J)/FRM(*,*,I);
END ; END ;
END ;
ELSE;
IF D(5)=1 THEN DO ;
REPEAT; PUT SKIP(2) LIST('ENTER F=PARAMETER. END WITH SEMICOLON.
  F=0 TO SKIP.') ;
GET DATA(F) ;
IF F=0 THEN GO TO PART6 ;
ELSE DO ;
PUT SKIP(2) EDIT ('GAIN COMPONENT OF FREQUENCY RESPONSE ELASTICITIES WR
T PARAMETER:',PN(F))(A(65),A(6)) ;
PUT SKIP(2) EDIT('FREQ')(A(6)) ;
DO I=1 TO DU ;
DO H=1 TO DZ ;

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PUT EDIT (' ',UN(I),'>',ZN(H))(A(3),A(1),A(1),A(3)) ;
END ; END ;
DO J=1 TO DF ;
PUT SKIP EDIT(S(J))(F(5,3)) ;
DO H=1 TO DZ ;
DO I=1 TO DU ;
PUT EDIT(REAL(FREP(H,I,J,F)))(F(8,2)) ;
END ; END ; END ; END ;
GO TO REPEAT ;
END ;
ELSE ;
PART6: IF D(6)=1 THEN DO ;
DENOVO: SC=.25 ;
PUT SKIP(2) LIST('ENTER J=PARAMETER SC=SCALE.') ;
PUT SKIP LIST('J=0 TO SKIP, END WITH SEMICOLON.') ;
GET DATA(SC,J) ;
IF J=0 THEN GO TO LP ;
ELSE DO ;
DO I=1 TO DU ;
DO H=1 TO DZ ;
PUT SKIP(2) LIST('GAIN COMPONENT OF FREQUENCY RESPONSE ELASTICITY') ;
PUT SKIP EDIT(' ', 'WRT PARAMETER:', PN(J), ' ', UN(I), '>', ZN(H)
)(A(16),A(15),A(4),A(6),A(1),A(1),A(3)) ;
PUT SKIP EDIT(' REAL FRQ')(A(10)) ;
DO F=1 TO 5 ;
PUT EDIT((F-3)*SC)(5(F(10,2))) ;
END ;
DO F=1 TO DF ;
PUT SKIP EDIT(REAL(FREP(H,I,F,J)),S(F))(F(5,2),F(6,3)) ;
IF (REAL(FREP(H,I,F,J))) > (-2.5*SC) THEN DO ;
IF (REAL(FREP(H,I,F,J))) < (3*SC) THEN DO ;
PUT EDIT('*')(COL(38+10*REAL(FREP(H,I,F,J))/SC),A(1)) ;
END ;
ELSE ;
END ;
ELSE ;
END ;
PUT SKIP EDIT(' ')(A(10)) ;
DO F=1 TO 5 ;
PUT EDIT((F-3)*SC)(5(F(10,2))) ;
END ;
END ; END ;
END ;
GO TO DENOVO ;
END ;
ELSE GO TO LP ;
CON : PROCEDURE ;
QNSE=1.E-0 ; IDALPHA(1)=.1E-0 ; ANAME(1)='QNSE' ;
YNSE=1.E-0 ; IDALPHA(2)=.1E-0 ; ANAME(2)='YNSE' ;
ANSE=1.E-0 ; IDALPHA(3)=.1E-0 ; ANAME(3)='ANSE' ;
_TSSD=.5E-0 ;

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```
ALFA=.25E-0 ;
EGS=.2E-0 ;
SIC=0.E-0 ;
TAM=10.E-3 ;
CMPPER=0.E-0 ;
SPC=17.5E-3 ;
TSU=1.E-3 ;
TAI=.4E-0 ;
SCMS=0.E-0 ;
LENGTH=15.E-0 ;
FCU=.5E-0 ;
IEM=-1.E-0 ;
TAK=3.E-0 ;
EGT=.1E-0 ;
LR=30.E-3 ;
SCMG=0.E-0 ;
SCGT=0.E-0 ;
SCGS=0.E-0 ;
PRTPER=15.E-0 ;
EP=1.E-0 ;
TSY=2.5E-0 ;
TSLD=4.E-0 ;
NRU=50.E-3 ;
YEM=.7E-0 ;
SAVPER=0.E-0 ;
SPRPER=0.E-0 ;
TAE=.4E-0 ;
ALK=14.E-0 ;
STM=0.E-0 ;
TSAY=2.5E-0 ;
PLTPER=.5E-0 ;
NIC=.3E-0 ;
SGYT=1.E-0 ;
EY=1.E-0 ;
EE=1.E-0 ;
EYVM=5.E-0 ;
PMX(1)=TAE ;
PMX(2)=TAI ;
PMX(3)=EYVM ;
PMX(4)=LR ;
PMX(5)=EGS ;
PMX(6)=NRU ;
PMX(7)=EGT ;
PMX(8)=TAK ;
PMX(9)=IEM ;
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PMX(10)=ALK ;
PMX(11)=ALFA ;
PMX(12)=YEM ;
PMX(13)=TSY ;
PMX(14)=TSSD ;
PMX(15)=NIC ;
PMX(16)=SFC ;
PMX(17)=TSLD ;
PMX(18)=TSAY ;
PMX(19)=FCU ;
PMX(20)=SGYT ;
PMX(21)=SCGS ;
PMX(22)=SCGT ;
PMX(23)=SCMG ;
PMX(24)=SCMS ;
  RETURN ;
END ;
UPDATE ; PROCEDURE ;
  DE=((1.E-0-ALFA)*SED)/RW ;
  PT=LU-NRU+(SDC*(E-DE))/(TAE*E) ;
  CGT=EGT*SCGT*PT ;
  GT=EGT+CGT ;
  CGS=EGS*PT*SCGS ;
  PTY=EY*EXP((1.E-0-ALFA)*LOG(E/EE))*EXP(ALFA*LOG(K/EK))
  ;
  Y=PTY*(1.E-0-FCU)+SED*FCU ;
  T=(EGS+EGT)*(1.E-0-SGYT)+Y*NTR*SGYT ;
  CDY=Y-(T-GT) ;
  TMS=EM*(1.E-0+SCMS*PT) ;
  DIV=NIC*LED ;
  DII=(DIV-IV)/TAI ;
  R=LR*EXP((-1.E-0/LEM)*LOG((EY*P)/(M*EYVM)))*EXP((-YEM/LEM)*LOG(AY/
  EY)) ;
  DK=(ALFA*LED)/((1.E-0/ALK)+R) ;
  KD=K/ALK ;
  IVST=KD+(DK-K)/TAK ;
  C=APC*PY ;
  G=EGS+CGS ;
  FS=C+IVST+G ;
  A=FS+DII ;
  RCM=M*PT*SCMG+(STM*(TMS-M))/TAM ;
  U=((EE/(1.E-0-NRU))-E)/(EE/(1.E-0-NRU)) ;
  RETURN ;
END ;
F_Q_TIL ; PROCEDURE ;
  C_=APC*1.E-0 ; /*2*/
  FS_=C_ ; /*2*/
  A_=FS_ ; /*2*/
  FTIL(1,1)=-1.E-0/TSY ; /*2*/
  FTIL(7,1)=A_/TSLD ; /*2*/
  FTIL(11,1)=-FS_ ; /*2*/

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FTIL(12,1)=A_/TSSD ; /*2*/
FTIL(2,2)=0.E-0 ;
FTIL(3,3)=0.E-0 ;
PT_=((SDC*1.E-0)/(TAE*E))-(SDC*(E-DE)*(TAE*1.E-0))/(TAE*E*(TAE*E)) ;
/*3*/
CGT_=EGT*SCGT*PT_ ; /*3*/
GT_=CGT_ ; /*3*/
CGS_=EGS*PT_*SCGS ; /*3*/
PTY_=EY*EXP_((1.E-0-ALFA)*LOG(E/EE),(1.E-0-ALFA)*LOG_(E/EE,1.E-0/
EE))*EXP(ALFA*LOG(K/EK)) ; /*3*/
Y_=PTY_*(1.E-0-FCU) ; /*3*/
T_=Y_*NTR*SGYT ; /*3*/
CDY_=Y_-(T_-GT_) ; /*3*/
TMS_=EM*(SCMS*PT_) ; /*3*/
G_=CGS_ ; /*3*/
FS_=G_ ; /*3*/
A_=FS_ ; /*3*/
RCM_=M*PT_*SCMG+(STM*TMS_)/TAM ; /*3*/
U_=-1.E-0/(EE/(1.E-0-NRU)) ; /*2*/
FTIL(1,4)=CDY_/TSY ; /*3*/
FTIL(2,4)=CDY_ ; /*3*/
FTIL(4,4)=-1.E-0/TAE ; /*2*/
FTIL(5,4)=U_/TSU ; /*2*/
FTIL(7,4)=A_/TSLD ; /*3*/
FTIL(9,4)=RCM_ ; /*3*/
FTIL(10,4)=P*SFC*-((NRU*U_)/(U*U)) ; /*3*/
FTIL(11,4)=Y_-FS_ ; /*3*/
FTIL(12,4)=A_/TSSD ; /*3*/
FTIL(13,4)=Y_/TSAY ; /*3*/
FTIL(14,4)=1.E-0 ; /*1*/
PT_=1.E-0 ; /*1*/
CGT_=EGT*SCGT*PT_ ; /*2*/
GT_=CGT_ ; /*2*/
CGS_=EGS*PT_*SCGS ; /*2*/
CDY_=-(-GT_) ; /*2*/
TMS_=EM*(SCMS*PT_) ; /*2*/
G_=CGS_ ; /*2*/
FS_=G_ ; /*2*/
A_=FS_ ; /*2*/
RCM_=M*PT_*SCMG+(STM*TMS_)/TAM ; /*3*/
FTIL(1,5)=CDY_/TSY ; /*2*/
FTIL(2,5)=CDY_ ; /*2*/
FTIL(5,5)=-1.E-0/TSU ; /*2*/
FTIL(7,5)=A_/TSLD ; /*2*/
FTIL(9,5)=RCM_ ; /*3*/
FTIL(11,5)=-FS_ ; /*2*/
FTIL(12,5)=A_/TSSD ; /*2*/
PTY_=EY*EXP_((1.E-0-ALFA)*LOG(E/EE))*EXP_(ALFA*LOG(K/EK),ALFA*LOG_
(K/EK,1.E-0/EK)) ; /*3*/
Y_=PTY_*(1.E-0-FCU) ; /*3*/
T_=Y_*NTR*SGYT ; /*3*/

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CDY_=Y_-T_ ; /*3*/
KD_=1.E-0/ALK ; /*2*/
IVST_=KD_+-1.E-0/TAK ; /*2*/
C_=0,E-0 ;
FS_=IVST_ ; /*2*/
A_=FS_ ; /*2*/
FTIL(1,6)=CDY_/TSY ; /*3*/
FTIL(2,6)=CDY_ ; /*3*/
FTIL(3,6)=IVST_ ; /*2*/
FTIL(6,6)=IVST_-KD_ ; /*2*/
FTIL(7,6)=A_/TSLD ; /*2*/
FTIL(11,6)=Y_-FS_ ; /*3*/
FTIL(12,6)=A_/TSSD ; /*2*/
FTIL(13,6)=Y_/TSAY ; /*3*/
DIV_=NIC*1.E-0 ; /*2*/
DII_=DIV_/TAI ; /*2*/
DK_=(ALFA*1.E-0)/((1.E-0/ALK)+R) ; /*3*/
IVST_=DK_/TAK ; /*3*/
FS_=IVST_ ; /*3*/
A_=FS_+DII_ ; /*3*/
FTIL(3,7)=IVST_ ; /*3*/
FTIL(6,7)=IVST_ ; /*3*/
FTIL(7,7)=(A_-1.E-0)/TSLD ; /*3*/
FTIL(11,7)=-FS_ ; /*3*/
FTIL(12,7)=A_/TSSD ; /*3*/
FTIL(8,8)=0.E-0 ;
R_=LR*EXP_((-1.E-0/LEM)*LOG((EY*P)/(M*EYUM)),(-1.E-0/LEM)*LOG_((EY
*P)/(M*EYUM),-((EY*P*(1.E-0*EYUM))/(M*EYUM*(M*EYUM)))))*EXP((-YEM/
LEM)*LOG(AY/EY)) ; /*3*/
DK_-=((ALFA*LED*R_)/(((1.E-0/ALK)+R)*((1.E-0/ALK)+R))) ; /*3*/
IVST_=DK_/TAK ; /*3*/
FS_=IVST_ ; /*3*/
A_=FS_ ; /*3*/
RCM_=1.E-0*PT*SCMG+(STM*-1.E-0)/TAM ; /*3*/
FTIL(3,9)=IVST_ ; /*3*/
FTIL(6,9)=IVST_ ; /*3*/
FTIL(7,9)=A_/TSLD ; /*3*/
FTIL(9,9)=RCM_ ; /*3*/
FTIL(11,9)=-FS_ ; /*3*/
FTIL(12,9)=A_/TSSD ; /*3*/
R_=LR*EXP_((-1.E-0/LEM)*LOG((EY*P)/(M*EYUM)),(-1.E-0/LEM)*LOG_((EY
*P)/(M*EYUM),(EY*1.E-0)/(M*EYUM)))*EXP((-YEM/LEM)*LOG(AY/EY)) ;
/*3*/
DK_-=((ALFA*LED*R_)/(((1.E-0/ALK)+R)*((1.E-0/ALK)+R))) ; /*3*/
IVST_=DK_/TAK ; /*3*/
FS_=IVST_ ; /*3*/
A_=FS_ ; /*3*/
FTIL(3,10)=IVST_ ; /*3*/
FTIL(6,10)=IVST_ ; /*3*/
FTIL(7,10)=A_/TSLD ; /*3*/
_FTIL(10,10)=1.E-0*SPC*((NRU/U)-1.E-0) ; /*3*/

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FTIL(11,10)=-FS_ ; /*3*/
FTIL(12,10)=A_/TSSD ; /*3*/
DII_=-1.E-0/TAI ; /*2*/
A_=DII_ ; /*2*/
FTIL(7,11)=A_/TSLD ; /*2*/
FTIL(8,11)=1.E-0 ; /*1*/
FTIL(11,11)=0.E-0 ;
FTIL(12,11)=A_/TSSD ; /*2*/
DE_=(1.E-0-ALFA)*1.E-0/RW ; /*2*/
FT_=(SDC*-DE_)/(TAE*E) ; /*3*/
CGT_=EGT*SCGT*FT_ ; /*3*/
GT_=CGT_ ; /*3*/
CGS_=EGS*FT_*SCGS ; /*3*/
Y_=1.E-0*FCU ; /*2*/
T_=Y_*NTR*SGYT ; /*2*/
CDY_=Y_-(T_-GT_) ; /*3*/
TMS_=EM*(SCMS*FT_) ; /*3*/
G_=CGS_ ; /*3*/
FS_=G_ ; /*3*/
A_=FS_ ; /*3*/
RCM_=M*FT_*SCMG+(STM*TMS_)/TAM ; /*3*/
FTIL(1,12)=CDY_/TSY ; /*3*/
FTIL(2,12)=CDY_ ; /*3*/
FTIL(4,12)=DE_/TAE ; /*2*/
FTIL(7,12)=A_/TSLD ; /*3*/
FTIL(9,12)=RCM_ ; /*3*/
FTIL(11,12)=Y_-FS_ ; /*3*/
FTIL(12,12)=(A_-1.E-0)/TSSD ; /*3*/
FTIL(13,12)=Y_/TSAY ; /*2*/
R_=LR*EXP((-1.E-0/LEM)*LOG((EY*P)/(M*EYVM)))*EXP_((-YEM/LEM)*LOG(
  AY/EY),(-YEM/LEM)*LOG_(AY/EY,1.E-0/EY)) ; /*3*/
DK_=-((ALFA*LED*_R_)/(((1.E-0/ALK)+R)*((1.E-0/ALK)+R))) ; /*3*/
IVST_=DK_/TAK ; /*3*/
FS_=IVST_ ; /*3*/
A_=FS_ ; /*3*/
FTIL(3,13)=IVST_ ; /*3*/
FTIL(6,13)=IVST_ ; /*3*/
FTIL(7,13)=A_/TSLD ; /*3*/
FTIL(11,13)=-FS_ ; /*3*/
FTIL(12,13)=A_/TSSD ; /*3*/
FTIL(13,13)=-1.E-0/TSAY ; /*2*/
FTIL(14,14)=0.E-0 ;
CNFTIL = 0 ;
A_=ANSE ; /*2*/
LED_=A_/TSLD ; /*2*/
SED_=A_/TSSD ; /*2*/
TEMPB(2,1)=SED_ ;
TEMPB(10,1)=LED_ ;
PTY_=YNSE ; /*2*/
Y_=PTY_*(1.E-0-FCU) ; /*2*/
T_=Y_*NTR*SGYT ; /*2*/

```

```

CDY_=Y_-T_ ; /*2*/
PY_=CDY_/TSY ; /*2*/
OVCDY_=CDY_ ; /*2*/
IV_=Y_ ; /*2*/
AY_=Y_/TSAY ; /*2*/
TEMPB(1,2)=PY_ ;
TEMPB(7,2)=IV_ ;
TEMPB(8,2)=AY_ ;
TEMPD(1,2)=OVCDY_ ;
RETURN ;
END ;
IC ; PROCEDURE ;
U=NRU ;
LU=U ;
SED=EY ;
RW=((1.E-0-ALFA)*EY)/EE ;
DE=((1.E-0-ALFA)*SED)/RW ;
E=EE ;
PT=LU-NRU+(SDC*(E-DE))/(TAE*E) ;
CGT=EGT*SCGT*PT ;
GT=EGT+CGT ;
NTR=(EGS+EGT)/EY ;
EK=(ALFA*EY)/((1.E-0/ALK)+LR) ;
K=EK ;
PTY=EY*EXP((1.E-0-ALFA)*LOG(E/EE))*EXP(ALFA*LOG(K/EK))+NOISE(YNSE)
;
Y=PTY*(1.E-0-FCU)+SED*FCU ;
T=(EGS+EGT)*(1.E-0-SGYT)+Y*NTR*SGYT ;
PY=EY-(T-GT) ;
CDY=Y-(T-GT) ;
OVCDY=CDY ;
LED=EY ;
P=EP ;
EM=(EY*EP)/EYVM ;
M=EM ;
AY=EY ;
R=LR*EXP((-1.E-0/IEM)*LOG((EY*P)/(M*EYVM)))*EXP((-YEM/IEM)*LOG(AY/
EY)) ;
DK=(ALFA*LED)/((1.E-0/ALK)+R) ;
KD=K/ALK ;
IVST=KD+(DK-K)/TAK ;
OVI=IVST ;
IV=EY*NIC ;
OVIIV=IV ;
OVE=E ;
AFC=(PY-KD)/PY ;
RETURN ;
END ;
CHANGE ; PROCEDURE ;
PUT SKIP(3) LIST('ANY CHANGES? CONSTANTS OR OUTPUT. D(?)=1 FOR TASK');
PUT SKIP LIST('1>EVEP, 2>EVEL, 3>FR, 4>GC, 5>FREP, 6>FREC. END W ;.');

```

```
GET DATA (DT,TSSD,ALFA,EGS,SIC,TAM,CMPPER,SFC,TSU,TAI,SCMS,LENGTH,  
FCU,IEM,TAK,EGT,LR,SCMG,SCGT,SCGS,PRTPER,EP,TSY,TSLD,NRU,YEM,SAUPER  
SPRPER,TAE,ALK,STM,TSAY,PLTPER,NIC,SGYT,EY,EE,EYVM,D) ;  
PUT LIST ('CONTINUING...');  
RETURN ;  
END ;  
END MODEL ;
```

APPENDIX F

CREATING PL/1 PROGRAM FROM DYNASTAT OUTPUT

BELOW IS AN EXEC FILE OF INSTRUCTIONS FOR CONVERTING DYNASTAT OUTPUT INTO THE PROGRAM SEEN IN APPENDIX E. TWO BLOCKS OF INSTRUCTIONS, INSERTA AND INSERTB, ARE IDENTIFIED IN APPENDIX E.

```

ERASE &1 FLI
DS &1
&BEGSTACK
V OFF
C/LOGN/LOG/* *
V ON
TOP
/DECLARE/
T 7
/CNFTIL =/
/QTIL/
DEL 3
U
I TEMPB(2,1)=SED_ ;
I TEMPB(10,1)=LED_ ;
/QTIL/
DEL 10
U
I TEMPB(1,2)=PY_ ;
I TEMPB(7,2)=IV_ ;
I TEMPB(8,2)=AY_ ;
I TEMPD(1,2)=OVCDY_ ;
TOP
/DIM ;/
DEL 5
/DT=/
DEL
/COVX=/
DEL 3
U
I PMX(1)=TAE ;
I PMX(2)=TAI ;
I PMX(3)=EYVM ;
I PMX(4)=LR ;
I PMX(5)=EGS ;
I PMX(6)=NRU ;
I PMX(7)=EGT ;
I PMX(8)=TAK ;

```

```

I PMX(9)=IEM ;
I PMX(10)=ALK ;
I PMX(11)=ALFA ;
I PMX(12)=YEM ;
I PMX(13)=TSY ;
I PMX(14)=TSSD ;
I PMX(15)=NIC ;
I PMX(16)=SPC ;
I PMX(17)=TSLD ;
I PMX(18)=TSAY ;
I PMX(19)=FCU ;
I PMX(20)=SGYT ;
I PMX(21)=SCGS ;
I PMX(22)=SCGT ;
I PMX(23)=SCMG ;
I PMX(24)=SCMS ;
/XHATP/
DEL 14
TOP
/+NOISE(/
C/+NOISE(YNSE)//
/+NOISE(/
C/+NOISE(ANSE)//
/TIME=/
DEL 3
/CNQTIL =/
DEL 15
/TM=/
DEL
/TIME=/
DEL 2
/NALPHA :/
DEL 2
TOP
N2
DEL2
U
GET INSERTA PLI
N5
GET INSERTB PLI
/ENTRY/
C/ENTRY/PROCEDURE/
/RETURN/
I END;
/ENTRY/
C/ENTRY/PROCEDURE/
/RETURN/
I END ;
/ENTRY/
C/ENTRY/PROCEDURE/
/RETURN/

```



```

I END;
/ENTRY/
C/ENTRY/PROCEDURE/
/RETURN/
I END ;
/ENTRY/
C/ENTRY/PROCEDURE/
/GET DATA/
U
I PUT SKIP(3) LIST('ANY CHANGES? CONSTANTS OR OUTPUT, I(?)=1 FOR 1
I PUT SKIP LIST('1>EVEP, 2>EVEL, 3>FR, 4>GC, 5>FREP, 6>FREC. END U
N3
C//,D)//
I PUT LIST ('CONTINUING...');
/RETURN/
I END ;
TOP
N7
V OFF
C/DXZ/14/ 23 G
TOP
N7 .
C/DU1/3/ 23 G
TOP
N7
C/DX/10/ 23 G
TOP
N7
C/DZ/4/ 23 G
TOP
N7
C/DU/2/ 23 G
TOP
N7
C/DF/24/ 23 G
TOP
N7
C/DF/20/ 23 G
V ON
TOP
/PD24(/
C/PD24/PIDP/
FILE
&END
EDIT &1 FLI

```