

CHAPTER 16 MATHEMATICS AFTER CALCULUS

16.1 Linear Algebra (page 602)

If $Ax = b$ and $AX = B$, then A times $2x + 3X$ equals $2b + 3B$. If $Ax = 0$ and $AX = 0$ then A times $2x + 3X$ equals 0 . In this case x and X are in the nullspace of A , and so is the combination $2x + 3X$. The nullspace contains all solutions to $Ax = 0$. It is a subspace, which means that when x and X are in the nullspace, so are all combinations $ax + bX$. If $x = (1, 1, 1)$ is in the nullspace then the columns add to the zero column, so they are dependent.

Another subspace is the column space of A , containing all combinations of the columns of A . The system $Ax = b$ can be solved when b is in the column space. Otherwise the best solution comes from $A^T Ax = A^T b$. Here A^T is the transpose matrix, whose rows are the columns of A . The nullspace of A^T contains all solutions to $A^T y = 0$. The column space of A^T (row space of A) is the fourth fundamental subspace. Each subspace has a basis containing as many independent vectors as possible. The number of vectors in the basis is the dimension of the subspace.

When $Ax = \lambda x$ the number λ is an eigenvalue and x is an eigenvector. The equation $dy/dt = Ay$ has the exponential solution $y = e^{At}y_0$. A 7 by 7 matrix has seven eigenvalues, whose product is the determinant D . If D is nonzero the matrix A has an inverse. Then $Ax = b$ is solved by $x = A^{-1}b$. The formula for D contains $7! = 5040$ terms, so x is better computed by elimination. On the other hand $Ax = \lambda x$ means that $A - \lambda I$ has determinant zero. The eigenvalue is computed before the eigenvector.

1 All vectors $c \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ 3 Only $x = 0$ 5 Plane of vectors with $x_1 + x_2 + x_3 = 0$

7 $x_p = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$, $A(x_p + x_0) = \begin{bmatrix} 3 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ 9 $A(x_p + x_0) = b + 0 = b$; another solution

11 $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$; $b = \begin{bmatrix} c \\ c \\ c \end{bmatrix}$

13 $CC^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 2 & 5 \end{bmatrix}$; $C^T C = \begin{bmatrix} 2 & 2 \\ 2 & 5 \end{bmatrix}$; (2 by 3) (2 by 3) is impossible

15 Any two are independent 17 C and F have independent columns

19 $\det F = 3$ 21 $F^{-1} = \frac{1}{3} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

23 $\det(F - \lambda I) = \det \begin{bmatrix} 2 - \lambda & 1 \\ 1 & 2 - \lambda \end{bmatrix} = (2 - \lambda)^2 - 1 = 3 - 4\lambda + \lambda^2 = 0$ if $\lambda = 1$ or $\lambda = 3$;

$$F \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ -1 \end{bmatrix}, F \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

25 $y = e^t \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $y = e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $y = \frac{e^t}{2} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \frac{e^{3t}}{2} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$27 \det \begin{bmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{bmatrix} = (1-\lambda)^3 - 3(1-\lambda) + 2 = \lambda^3 - 3\lambda^2 = 0 \text{ if } \lambda = 3 \text{ or } \lambda = 0 \text{ (repeated)}$$

$$29 \det \begin{bmatrix} 1-\lambda & 2 \\ 2 & 4-\lambda \end{bmatrix} = \lambda^2 - 5\lambda = 0 \text{ if } \lambda = 0 \text{ or } \lambda = 5; A \begin{bmatrix} 2 \\ -1 \end{bmatrix} = 0 \begin{bmatrix} 2 \\ -1 \end{bmatrix}, A \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 5 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$31 H = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \quad 33 \text{ F if } b \neq 0; \text{ T; T; F (} e^{\lambda t} \text{ is not a vector); T}$$

$$2 Bx = 0 \text{ if } x = c \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad 4 C^T x = 0 \text{ if } x = c \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \quad 6 Fx = 0 \text{ only if } x = 0$$

$$8 Ax = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \text{ has no solutions (parallel lines in the row picture)}$$

10 If $Ax = b$ and $Ax_P = b$ then $A(x - x_P) = 0$. The difference between any two solutions is a vector in the nullspace. (So any solution x equals $x_{\text{particular}} + x_{\text{homogeneous}}$.)

12 The row space of $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ is the line of multiples of $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$. (So is the column space.)

The nullspace is the (perpendicular) line of multiples of $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$.

$$14 \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \text{ has no solution because the first equations give } x_1 = 1 \text{ and } x_2 = 1, \text{ which}$$

violates the third equation. So multiply by C^T : $\begin{bmatrix} 2 & 2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ gives the best least squares solution $x_1 = \frac{2}{3}, x_2 = \frac{1}{3}$.

16 The vectors $\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$ are combinations of columns of C .

18 Ax is a combination of the columns. If $Ax = 0$ happens with a nonzero x , the columns are "dependent." If $Ax = 0$ only happens when $x = 0$, the columns are "independent."

20 $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ are three independent vectors in 3D so they form a *basis*. The problem asks for three vectors with positive components not lying in a common plane.

22 $F^2 = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$ has determinant $25 - 16 = 9$ which is $3^2 = (\det F)^2$.

24 $\det(G - \lambda I) = \begin{vmatrix} 5-\lambda & 4 \\ 4 & 5-\lambda \end{vmatrix} = \lambda^2 - 10\lambda + 9$. This is $(\lambda - 9)(\lambda - 1)$ which is zero at $\lambda = 9$ and

$\lambda = 1$. Eigenvectors: $G \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 9 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $G \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. These are also the eigenvectors of F . General reasoning: If $Fx = \lambda x$ then $F^2 x = \lambda Fx = \lambda^2 x$. So x is also an eigenvector of F^2 .

26 Exponential solutions to $\frac{dy}{dt} = Gy$ are $y_1 = e^{9t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $y_2 = e^t \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. The combination $\frac{3}{2}y_1 + \frac{1}{2}y_2$

starts from $y_0 = \begin{bmatrix} 3/2 \\ 3/2 \end{bmatrix} + \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

28 An eigenvector for $Ex = 0x$, when $\lambda = 0$, is the same as a solution to $Ex = 0$ (Problem 5).

$Ex = 3x$ is solved by $x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ or any multiple $\begin{bmatrix} c \\ c \\ c \end{bmatrix}$.

30 A zero eigenvalue means there is a solution to $Ax = 0x = 0$. In this case A is not invertible and its

determinant is zero. The reasoning also goes in reverse.

- 32** If $Fx = \lambda x$ then multiplying both sides by F^{-1} and λ^{-1} gives $\lambda^{-1}x = F^{-1}x$. If F has eigenvalues $\lambda = 1$ and 3 , then F^{-1} has eigenvalues $\lambda^{-1} = 1$ and $\frac{1}{3}$ (with the same two eigenvectors x). The determinant of F is 1 times 3 and $\det F^{-1}$ is 1 times $\frac{1}{3}$.

16.2 Differential Equations (page 610)

The solution to $y' - 5y = 10$ is $y = Ae^{5t} + B$. The homogeneous part Ae^{5t} satisfies $y' - 5y = 0$. The particular solution B equals -2 . The initial condition y_0 is matched by $A = y_0 + 2$. For $y' - 5y = e^{kt}$ the right form is $y = Ae^{5t} + Be^{kt}$. For $y' - 5y = \cos t$ the form is $y = Ae^{5t} + a \cos t + b \sin t$.

The equation $y'' + 4y' + 5y = 0$ is second-order because it begins with y'' . The pure exponential solutions come from the roots of $\lambda^2 + 4\lambda + 5 = 0$, which are $r = -1$ and $s = -4$. The general solution is $y = Ae^{-t} + Be^{-4t}$. Changing $4y'$ to zero yields pure oscillation. Changing to $2y'$ yields $\lambda = -1 \pm 2i$, when the solutions become $y = Ae^{-t} \cos 2t + Be^{-t} \sin 2t$. This oscillation is underdamped. A spring with $m = 1, d = 2, k = 5$ goes directly to zero. An electrical network with $L = 1, R = 2, C = \frac{1}{5}$ also goes to zero (underdamped).

One particular solution of $y'' + 4y = e^t$ is e^t times $\frac{1}{5}$. If the right side is $\cos t$, the form of y_P is $\frac{1}{5} \cos t$. If the right side is 1 then $y_P = \frac{1}{4}$. If the right side is $\cos 2t$ we have resonance and y_P contains an extra factor t .

- 1** $3Be^{3t} - Be^{3t} = 8e^{3t}$ gives $B = 4 : y = 4e^{3t}$ **3** $y = 3 - 2t + t^2$ **5** $Ae^t + 4e^{3t} = 7$ at $t = 0$ if $A = 3$
7 Add $y = Ae^{-t}$ because $y' + y = 0$; choose $A = -1$ so $-e^{-t} + 3 - 2t + t^2 = 2$ at $t = 0$
9 $y = \frac{e^{kt}-1}{k}; y = t$; by l'Hôpital $\lim_{k \rightarrow 0} \frac{e^{kt}-1}{k} = \lim_{k \rightarrow 0} \frac{te^{kt}}{1} = t$
11 Substitute $y = Ae^t + Bte^t + C \cos t + D \sin t$ in equation: $B = 1, C = \frac{1}{2}, D = -\frac{1}{2}$, any A
13 Particular solution $y = Ate^t + Be^t; y' = Ate^t + (A+B)e^t = c(Ate^t + Be^t) + te^t$
 gives $A = cA + 1, A + B = cB, A = \frac{1}{1-c}, B = \frac{-1}{(1-c)^2}$
15 $\lambda^2 e^{\lambda t} + 6\lambda e^{\lambda t} + 5e^{\lambda t} = 0$ gives $\lambda^2 + 6\lambda + 5 = 0, (\lambda + 5)(\lambda + 1) = 0, \lambda = -1$ or -5
 (both negative so decay); $y = Ae^{-t} + Be^{-5t}$
17 $(\lambda^2 + 2\lambda + 3)e^{\lambda t} = 0, \lambda = -1 \pm \sqrt{-2}$ has imaginary part and negative real part;
 $y = Ae^{(-1+\sqrt{2}i)t} + Be^{(-1-\sqrt{2}i)t}; y = Ce^{-t} \cos \sqrt{2}t + De^{-t} \sin \sqrt{2}t$
19 $d = 0$ no damping; $d = 1$ underdamping; $d = 2$ critical damping; $d = 3$ overdamping
21 $\lambda = -\frac{b}{2} \pm \frac{\sqrt{b^2-4c}}{2}$ is repeated when $b^2 = 4c$ and $\lambda = -\frac{b}{2}; (t\lambda^2 + 2\lambda)e^{\lambda t} + b(t\lambda + 1)e^{\lambda t} + cte^{\lambda t} = 0$
 when $\lambda^2 + b\lambda + c = 0$ and $2\lambda + b = 0$
23 $-a \cos t - b \sin t - a \sin t + b \cos t + a \cos t + b \sin t = \cos t$ if $a = 0, b = 1, y = \sin t$
25 $y = A \cos 3t + B \cos 5t; y'' + 9y = -25B \cos 5t + 9B \cos 5t = \cos 5t$ gives $B = \frac{-1}{16};$
 $y_0 = 0$ gives $A = \frac{1}{16}$
27 $y = A(\cos \omega t - \cos \omega_0 t), y'' = -A\omega^2 \cos \omega t + A\omega_0^2 \cos \omega_0 t, y'' + \omega_0^2 y = \cos \omega t$ gives $A(-\omega^2 + \omega_0^2) = 1;$
 breaks down when $\omega^2 = \omega_0^2$
29 $y = Be^{5t}; 25B + 3B = 1, B = \frac{1}{28}$ **31** $y = A + Bt = \frac{1}{2} + \frac{1}{2}t$
33 $y'' - 25y = e^{5t}; y'' + y = \sin t; y'' = 1 + t$; right side solves homogeneous equation so particular

solution needs extra factor t

35 $e^t, e^{-t}, e^{it}, e^{-it}$ **37** $y = e^{-2t} + 2te^{-2t}; y(2\pi) = (1 + 4\pi)e^{-4\pi} \approx 0$

39 $y = (4e^{-rt} - r^2 e^{-4t/r}) / (4 - r^2) \rightarrow 1$ as $r \rightarrow 0$ **43** $h \leq 2; h \leq 2.8$

2 $-2a \sin 2t + 2b \cos 2t + a \cos 2t + b \sin 2t = 4 \sin 2t$; collect $\sin 2t$ and $\cos 2t$ terms to find $-2a + b = 4$ and $2b + a = 0$; then $b = \frac{4}{5}, a = -\frac{8}{5}, y = -\frac{8}{5} \cos 2t + \frac{4}{5} \sin 2t$.

4 $ae^t \cos t - ae^t \sin t + be^t \sin t + be^t \cos t = 2e^t \cos t$; collect terms to find $a + b = 2, -a + b = 0$; then $a = 1, b = 1, y = e^t(\cos t + \sin t)$. This integral of $2e^t \cos t$ could be found by parts; here the correct form was assumed at the start.

6 Ae^{-t} solves the homogeneous equation $y' + y = 0$; then with $A = \frac{8}{5}$ the combination $\frac{8}{5} - \frac{8}{5} \cos 2t + \frac{4}{5} \sin 2t$ solves $y' + y = 4 \sin 2t$ with $y(0) = 0$.

8 $y = \text{constant}$ solves the homogeneous equation $y' = 0$, so a constant can be added to any particular solution of $y' = 2e^t \cos t$ (the constant A is the same as the integration constant C !).

10 Substitute homogeneous solution plus particular solution $y = Ae^t + Be^{kt}$ into the equation; then $kB - B = 1$ and also $A + B = 0$ from $y_0 = 0$; thus $B = -A = \frac{1}{k-1}$ and $y = -\frac{e^t}{k-1} + \frac{e^{kt}}{k-1}$. Apply l'Hôpital's rule (the variable is k !) to find $\lim_{k \rightarrow 1} \frac{-e^t + e^{kt}}{k-1} = \lim_{k \rightarrow 1} \frac{te^{kt}}{1} = te^t$.

12 Homogeneous solution plus particular solution = $Ae^{-t} + Be^t + Ct + D$. Substitute to find $2Be^t + C + Ct + D = e^t + t$ and $B = \frac{1}{2}, C = 1, D = -1$; then $y = Ae^{-t} + \frac{1}{2}e^t + t - 1$. (Initial value y_0 determines A .)

14 Particular solution is $B + Dt$; substitute to find $D = B + Dt + t$; then $B = -1, D = -1$; general solution $y = Ae^t - t - 1$.

16 Substitution gives $(\lambda^2 + 9)e^{\lambda t} = 0$. (a) $\lambda = 3i$ or $\lambda = -3i$; (b) Pure oscillation because the λ 's are pure imaginary (c) General solution $y = Ae^{3it} + Be^{-3it} = a \cos 3t + b \sin 3t$.

18 Substitution gives $(\lambda^2 + 6\lambda + 9)e^{\lambda t} = 0$. (a) $\lambda^2 + 6\lambda + 9 = (\lambda + 3)^2$ and the equation $(\lambda + 3)^2 = 0$ has the repeated solution $\lambda = -3$ (b) The general solution is $y = Ae^{-3t} + Bte^{-3t}$.

20 $k = 0$ gives overdamping; $k = 1$ gives critical damping; $k = 2$ gives underdamping.

22 Substitute $(te^{-t})'' + 3(te^{-t})' + 2(te^{-t}) = (t-2)e^{-t} + 3(1-t)e^{-t} + 2te^{-t} = e^{-t} \neq \text{zero}$. So te^{-t} is not a homogeneous solution. (It would be if the coefficients in the equation changed from 1,3,2 to 1,2,1; then $\lambda^2 + 2\lambda + 1$ has $\lambda = -1$ as a double root.)

24 Substitute y to find $-\omega^2 a \cos \omega t - \omega^2 b \sin \omega t - \omega a \sin \omega t + \omega b \cos \omega t + a \cos \omega t + b \sin \omega t = \sin \omega t$. Then $(1 - \omega^2)a + \omega b = 0$ and $-\omega a + (1 - \omega^2)b = 1$, which gives $a = \frac{-\omega}{\omega^2 + (1 - \omega^2)^2}$ and $b = \frac{1 - \omega^2}{\omega^2 + (1 - \omega^2)^2}$.

26 The graph of $\cos 5t - \cos 3t$ will look like the figure following the exercises in Section 7.2, which shows $\sin 10x \sin x = \frac{1}{2}(\cos 11x - \cos 9x)$.

28 Substitute to find $(i\omega L + R + \frac{1}{i\omega C})Ae^{i\omega t} = Ve^{i\omega t}$. Then $A = \frac{V}{i\omega L + R + (i\omega C)^{-1}}$.

30 Substitute $y_{\text{particular}} = A \sin t + B \cos t$ to find $-A \sin t - B \cos t + 3A \sin t + 3B \cos t = \sin t$. Then $2A = 1$ and $B = 0$ give $y = \frac{1}{2} \sin t$. (Note: the homogeneous solution involves $\cos \sqrt{3}t$ and $\sin \sqrt{3}t$.)

32 Substitute $y_{\text{particular}} = Ae^t \cos t + Be^t \sin t$ to find $-2Ae^t \sin t + 2Be^t \cos t + 2Ae^t \cos t + 2Be^t \sin t = e^t \cos t$. Then $-2A + 2B = 0$ and $2B + 2A = 1$ and $A = B = \frac{1}{4}$.

34 Equation (2) used l'Hôpital's Rule for the limit of $y = \frac{e^{kt} - e^{ct}}{k - c}$ as k approached c . This exercise uses the series $e^{kt} = 1 + kt + \frac{1}{2}k^2 t^2 + \dots$ and $e^{ct} = 1 + ct + \frac{1}{2}c^2 t^2 + \dots$ to find $\frac{e^{kt} - e^{ct}}{k - c} = \frac{kt - ct + \frac{1}{2}k^2 t^2 - \frac{1}{2}c^2 t^2 + \dots}{k - c} = t + \frac{1}{2}(k + c)t^2 + \dots$. As k approaches c this is $t + ct^2 + \dots = t(1 + ct + \dots) = te^{ct}$.

36 Substitute $y_{\text{particular}} = Ae^t$ to find $Ae^t + Ae^t = e^t$. Then $A = \frac{1}{2}$ and $y = \frac{1}{2}e^t$.

38 $y_0 = 1$ gives $A + B = 1; y'_0 = 0$ gives $-A - 4B = 0$; the solution has $A = \frac{4}{3}, B = -\frac{1}{3}$, and $y = \frac{4}{3}e^{-t} - \frac{1}{3}e^{-4t}$. At $t = 2\pi$ the value is $\frac{4}{3}e^{-2\pi} - \frac{1}{3}e^{-8\pi} \approx \frac{4}{3}e^{-2\pi}$.

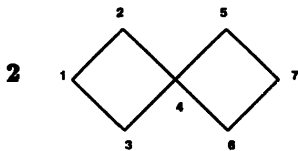
- 40 In exponential solutions Ae^{kt} all derivatives are proportional to the function. For $y'' = 6y^2$ substitute $y = x^n$ to find $n(n-1)x^{n-2} = 6x^{2n}$. Comparing exponents gives $n-2 = 2n$ or $n = -2$. Then $n(n-1) = 6$ and a solution is $y = x^{-2}$. (If the 6 were changed, this solution would become Ax^{-2} .)
- 44 Runge-Kutta is stable with $h = .02$ but 100 steps with $h = .03$ lead to $y_{100} \approx 670,000,000,000$.

16.3 Discrete Mathematics (page 615)

A graph is a set V of nodes or vertices and a set E of edges. With 6 nodes, a complete graph has 15 edges. A spanning tree has only 5 edges. A tree is defined as a graph with no loops, and it is spanning if it contains all nodes. It has one path between each pair of nodes.

To find a path from node i to node j , two search methods are depth first search and breadth first search. As nodes are reached, DFS looks out from the latest node for a new one. BFS looks out from the earliest node. DFS must be prepared to backtrack to earlier nodes. In case of fire, BFS locates all doors from the room you are in before it chooses one.

- 1 Two then two then last one; go around hexagon 3 Six (each deletes one edge)
- 5 Connected: there is a path between any two nodes; connecting each new node requires an edge
- 13 Edge lengths 1,2,4
- 15 No; 1,3,4 on left connect only to 2,3 on right; 1,3 on right connect only to 2 on left 17 4
- 19 Yes 21 F (may loop); T 25 16



Breadth first search from node 4: 2, 3, 5, 6, 1, 7.
 Depth first search from node 4: 2, 1, 3, (backtrack to 4), 5, 7, 6

- 4 A spanning tree omits one edge from each square. There are 4×4 choices of edges to omit: 16 spanning trees.
- 6 A loop is a sequence of edges from x_0 to x_1 , x_1 to x_2 , ..., x_k to x_0 : the loop (or circuit) is a closed path on the graph. A loop involving k nodes uses k edges. That leaves only $8 - k$ edges to connect the loop to the remaining $9 - k$ nodes, which is impossible.
- 8 Breadth first search finds all neighbors, then all neighbors of neighbors, and so on. The farthest state is found last.
- 10 Please send answer!
- 12 A matching is a set of edges, no two of which touch the same node.
- 14 The shortest path tree in network A contains the edges of length 4, 5, 1.
- 16 The loop in network B contains the edges of length 1, 4, 2, 3. Minimum spanning tree by Method 1, starting top left: 7, 2, 3, 1, 5, 6, 8. Minimum spanning tree by Method 2 (increasing lengths, avoiding loops): 1, 2, 3, (avoid 4), 5, 6, 7, 8.
- 18 If one node has 4 edges going out, then every other node has one of these edges coming in: a node with 0 edges cannot exist.

- 20** Suppose a spanning tree contains edge (8) but not (6). Then it is shorter if we substitute edge (6) for edge (8). Also *it is still a spanning tree*: It contains the correct number of edges and no loops. Edge (6) cannot be in a loop since edge (8) was removed; and those are the only edges into their common node.
- 22** (a) Starting from node s , a tree that connects each node to the next is perfect for depth first search. There are no backtracks. (b) For breadth first search the perfect tree connects node s directly to each other node. On that tree, depth first search would have to backtrack after each new node.
- 24** The code intends to find shortest distances d_{ij} between every pair of nodes (like a road atlas).
Correction: *The loop on k should be the outer loop*. Then $k = 1$ allows direct edges i to j and paths from i to 1 to j . Next $k = 2$ also allows paths from i to 2 to j and from i to 2 to 1 to j and from i to 1 to 2 to j . Eventually all intermediate nodes and all paths are allowed.
- 26** Maximum spanning tree: Method 1 adds the longest edge that goes out from the current tree (start at any node). Method 2 adds edges in decreasing order, longest first, rejecting any edge that closes a loop.

MIT OpenCourseWare
<http://ocw.mit.edu>

Resource: Calculus Online Textbook
Gilbert Strang

The following may not correspond to a particular course on MIT OpenCourseWare, but has been provided by the author as an individual learning resource.

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.