14.04 Midterm Exam 3 Prof. Sergei Izmalkov
Wed, Dec 3

1. Consider an industry with 3 firms, each having marginal costs equal to 0. The inverse demand curve facing this industry is
$$P(Q) = 60 - Q,$$
where $Q = q_1 + q_2 + q_3$ is total output.

   (a) If each firm behaves as a Cournot competitor, what is firm 1's best response function — optimal choice given other firms outputs?

   (b) Calculate the Cournot equilibrium.

   (c) Firms 2 and 3 decided to merge and form a single firm with marginal costs still equal to 0. Calculate new industry equilibrium. Is firm 1 is worse of or better of as a result? Was it a good idea for firms 2 and 3 to merge? Would it be a good idea for all three firms to organize the cartel?

   (d) Suppose firm 1 can commit to a certain level of output in advance. If the choice of firm 1 is q_1, what would be the optimal choices of firms 2 and 3? (Hint: After observing q_1 firms 2 and 3 would engage in (Cournot) duopolistic competition. What is the optimal level of q_1? Calculate profits of firm 1, compare with (b).

2. Consider an economy with 3 firms and 2 consumers. Each consumer owns 10 units of *Land*. Firm 1 produces *Food* and *Wood* using technology $(-L, F, W) = (-1, 1, 2)$. Firm 2 produces only *Food* with technology $(-L, F) = (-2, 1)$ and firm 3 produces only *Wood* with technology $(-L, W) = (-1, 1)$. Firms 2 and 3 are owned by consumer 1, firm 1 is owned by consumer 2. Consumers have identical utilities $u(w, f) = \sqrt{wf}$.

   Calculate Walrasian equilibrium.

3. There is one consumer and one firm. The firm may have a high quality indivisible product (with probability q) or a low quality product (with probability $1 - q$). The firm knows the value of the product, while the consumer cannot observe it prior to the sale. The consumer's utility from a product of given quality is $v_i - p$, where $v_h = 8$, $v_l = 4$, and p is the price paid. Costs of production are $c_h = 3$, $c_l = 1$.

   (a) Under what conditions on q consumers will be willing to buy the product at a prespecified price p? What qualities of the product would be sold? (Hint: Analyze it case by case. E.g., if both qualities are sold, what is the expected utility of the consumer? Would she by? Would both types of the firm sell?)

   (b) Suppose a firm can spend some money A on advertisement of its product, and A is observable by the consumers. Present a separating equilibrium, where the high quality firm advertises, $A_h^* > 0$, and sells the product at price $p = 8$, while the low quality firm does not advertise, $A_l^* = 0$ and charges $p = 4$. (It would suffice if you check incentive compatibility and individual rationality constraints for only two choices of A, A_h^* and A_l^*.)