1. Consumer 1 has expenditure function $e_1(p_1, p_2, u_1) = u_1\sqrt{p_1 p_2}$ and consumer 2 has utility function $u_2(x_1, x_2) = 2003 x_1^3 x_2^a$.

   (a) What are Marshallian (market) demand functions for each of the goods by each of the consumers? Denote the income of consumer 1 by m_1 and the income of consumer 2 by m_2.

   (b) For what value(s) of the parameter a will there exists an aggregate demand functions that is independent of the distribution of income?

2. Suppose that utility maximization problems and expenditure minimization problems are well defined and utility and expenditure functions satisfy all necessary "nice" properties.

   (a) Show (prove) that utility maximization implies expenditure minimization and vice versa.

   (b) List all relevant identities that are result of a.

   (c) Derive Roy's identity.

   (d) Derive Slutsky equation.

3. An economy has two kinds of consumers and two goods. Type A and type B consumers have the following utility functions

$$U^A(x_1, x_2) = 4x_1 - \frac{x_1^2}{2} + x_2,$$

$$U^B(x_1, x_2) = 2x_1 - \frac{x_1^2}{2} + x_2.$$

Consumers can only consume nonnegative quantities. The price of good 2 is 1 and all consumers have incomes of 100. There are N type A consumers and N type B consumers.

   (a) Suppose that a monopolist can produce good 1 at a constant unit cost of c per unit and cannot engage in any kind of price discrimination. Find its optimal choice of price and quantity. For what values of c will it be true that it chooses to sell to both types of consumers?

   (b) Suppose that the monopolist uses a "two-part tariff" where a consumer must pay a lump sum k in order to be able to buy as much as he likes at a price p per unit purchased. Consumers are not able to resell good 1. For $p < 4$, what is the highest amount k that a type A is willing to pay for the privilege of buying at price p? If a type A does pay the lump sum k to buy at price p, how many units will he demand? Describe the function that determines demand for good 1 by type A consumers as a function of p and k. What is the demand function for good 1 by type B consumers? Now describe the function that determines total demand for good 1 by all consumers as a function of p and k.

   (c) If the economy consisted only of N type A consumers and no type B consumers, what would be the profit-maximizing choices of p and k?

   (d) If $c < 1$, find the values of p and k that maximize the monopolist's profit subject to the constraint that both types of consumers buy from it.

   (e) For $c = 0$ calculate consumer surplus, producer surplus, and dead weight loss for both c. and d.

4. Suppose that the government can tax or subsidize a monopolist who faces inverse demand function $p(q)$ and has cost function $c(q)$ [assume both are differentiable and that $p(q)q - c(q)$ is concave in q]. What tax or subsidy per unit of output would lead the monopolist to act efficiently?