DISCRETE COMPUTATION:
THEORY AND OPEN PROBLEMS

Notes for the lectures by

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Preceptorial Introduction to Computer Science for Mathematicians

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LECTURE I

1

MULTIPLICATION IN BINARY

$$\begin{array}{r} U = 101100 \\ V = \underline{111111} \\ 101100 \\ 101100 \\ \vdots \\ \hline U \times V = 101011010100 \end{array}$$

2

α = the add and shift multiplication algorithm

$T_\alpha(U,V)$ = time (number of basic operations on digits) to multiply U and V by method α.

$T_\alpha(n) = \max \{T_\alpha(U,V) \mid \ell(U) = \ell(V) = n\}$

Remark: $T_\alpha(n) = 0(n^2)$

3

Recursive algorithm for multiplication

$$U = \begin{array}{|c|c|}\hline U_1 & U_2 \\ \hline\end{array} = U_1 \cdot 2^{n/2} + U_2$$

$$V = \begin{array}{|c|c|}\hline V_1 & V_2 \\ \hline\end{array} = V_1 \cdot 2^{n/2} + V_2$$

$$U \cdot V = U_1V_1 \cdot 2^n + (U_1V_2 + U_2V_1) \cdot 2^{n/2} + U_2V_2$$

① ② ③ ④

4

ρ = recursive algorithm

$T_\rho(n) = 4T\ (n/2)$ + (time to add and shift length n numbers)

$= 4T_\rho(n/2) + 0(n)$

$= 4^kT_\rho(n/2^k) + 0(n)$

$= 0(4^{\log_2 n}) = 0(n^2)$

---

5 β = better recursive algorithm using only three half length multiplications

① $(U_1 + U_2)\cdot(V_1 + V_2)$

② U_1V_1

③ U_2V_2

$$U\cdot V = \textcircled{2}\cdot 2^n + (\textcircled{1} - \textcircled{2} - \textcircled{3})\cdot 2^{n/2} + \textcircled{3}$$

---

6 $$T_\beta(n) = 3T_\beta(n/2) + 0(n)$$

$$= 0(3^{\log_2 n})$$

$$= 0(n^{\log_2 3})$$

$$\approx 0(n^{1\cdot 6})$$

---

7 Best upper bound known for multiplication:

$$O(n \cdot \log n \cdot \log \log n)$$

by Strassen and Schönhage.

Question: What is the fastest possible way to multiply?

Need there even be one?

---

8 Might have algorithms $\beta_1, \beta_2, \ldots$

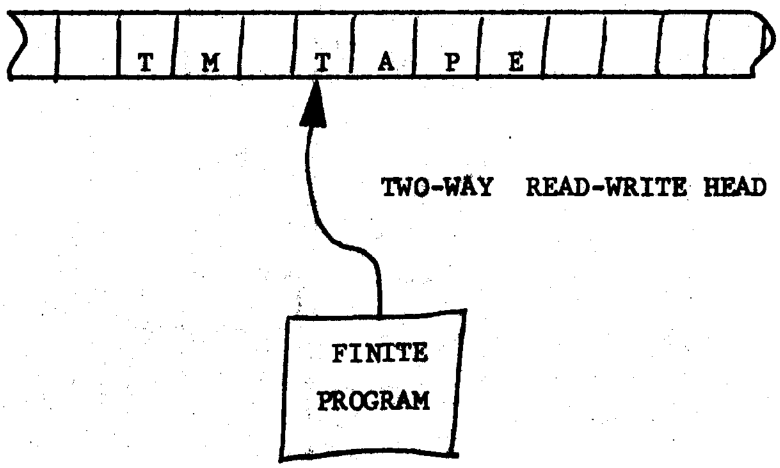
$$T_{\beta_1}(n) = n \log n$$

$$T_{\beta_2}(n) = n \sqrt{\log n}$$

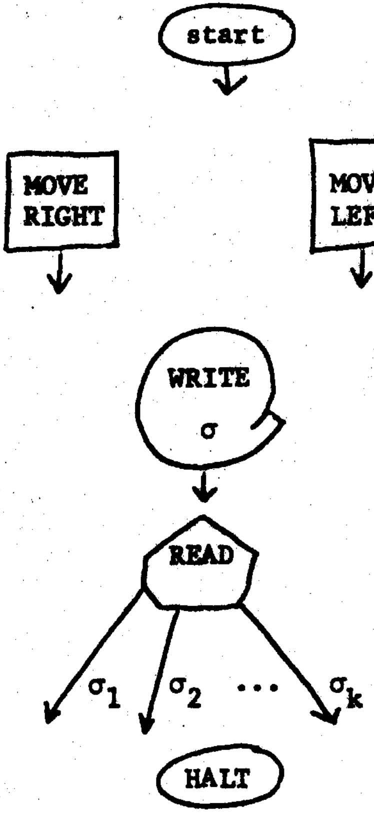
$$T_{\beta_i}(n) = n \cdot (\log n)^{\frac{1}{i}}$$

Then there is no fastest one.

Turing Machine



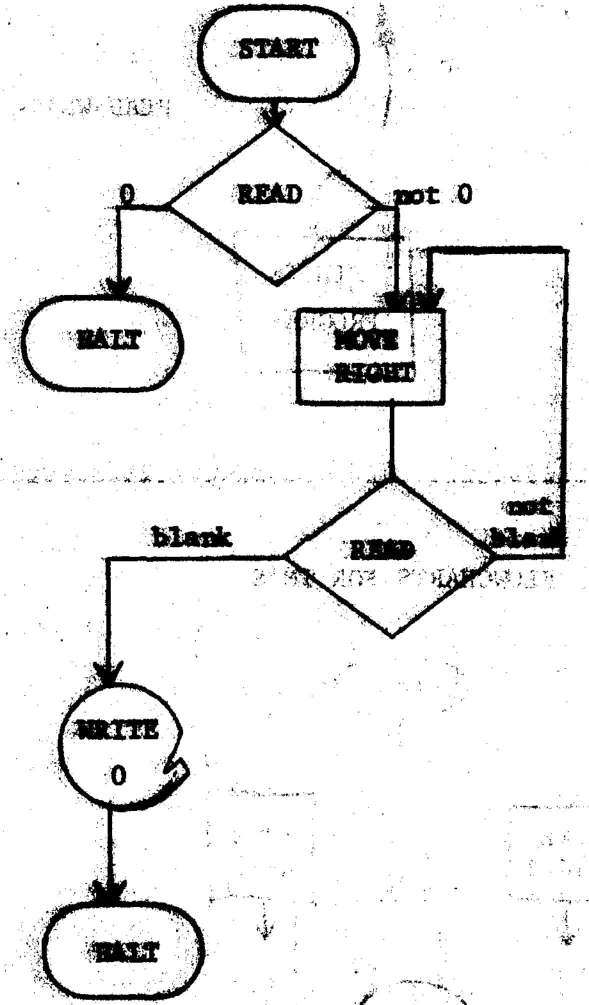
FLOWCHARTS FOR TM'S



$\sigma \in \Sigma = \{$ **tape symbols** $\}$

11 FLOWCHART FOR $F(x) = 2x$

(Input x an integer in binary notation.)



12 $\mathfrak{M}$ a T.M.,

x an input word.

$T_{\mathfrak{M}}(x)$ = number of instructions executed by $\mathfrak{M}$ on x if $\mathfrak{M}$ halts; ∞ if $\mathfrak{M}$ doesn't halt on x.

$S_{\mathfrak{M}}(x)$ = number of tape squares visited by head of $\mathfrak{M}$ with input x if $\mathfrak{M}$ halts; ∞ if $\mathfrak{M}$ does not halt.

$\varphi_{\mathfrak{M}}(x)$ = output of $\mathfrak{M}$ on x, if any; ∞ if no output.

T = <u>t</u>ime S = <u>s</u>pace φ = <u>f</u>unction

13

<u>Church's</u> <u>Thesis</u>:

The effectively (mechanically) computable functions and the Turing machine computable functions are the same.

<u>Extended</u> <u>Church's</u> <u>Thesis:</u>

If a function is computable in time T on any reasonable computer model, then it is computable in time ≤ polynomial (T) on a Turing machine.

---

14 **Infinitely-often Speed-up Theorem:**

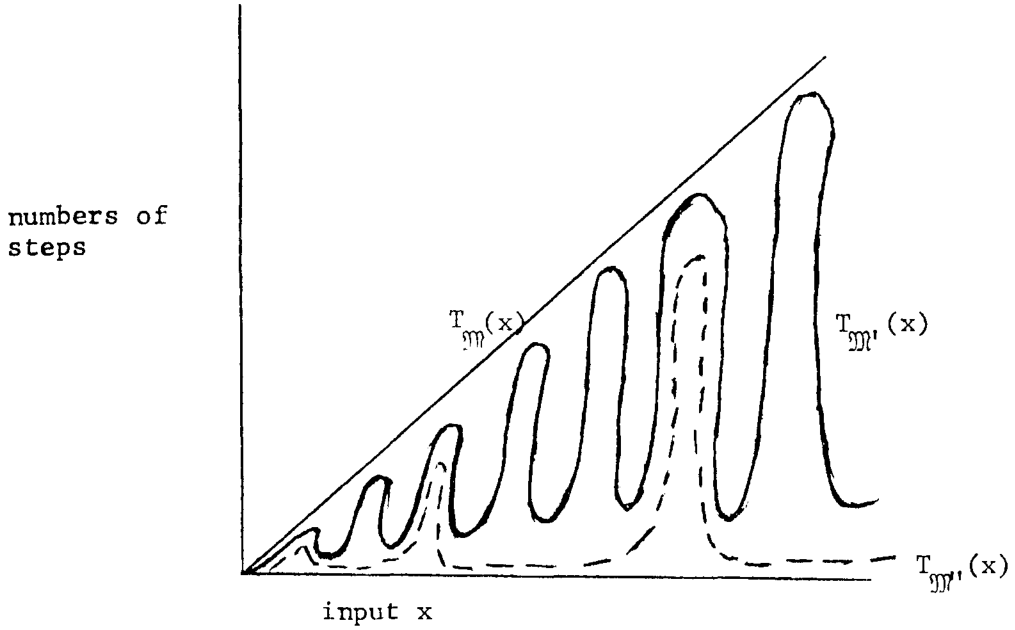
(M. BLUM). Let t: N → N be any computable function. Then there is a computable function $C_t : N \to \{0,1\}$ such that

> given any $\mathfrak{M}$ computing C_t one can construct an $\mathfrak{M}'$ also computing C_t with the property that

$T_{\mathfrak{M}}(x) > t(x)$ and $T_{\mathfrak{M}'}(x) <$ constant

for infinitely many $x \in N$.

15



$\mathfrak{M}'$ is faster than $\mathfrak{M}$ infinitely often,

$\mathfrak{M}''$ is faster than $\mathfrak{M}'$ infinitely often, etc.

16 Let $\mathfrak{M}_0, \mathfrak{M}_1, \ldots, \mathfrak{M}_i$ be an orderly list of of all Turing machines (say in order of the size of their flowcharts).

Let φ_i abbreviate $\varphi_{\mathfrak{M}_i}$,

" T_i " $T_{\mathfrak{M}_i}$.

Universal Machine Theorem:

$\varphi_i(x)$, regarded as a function of both

i and x, is computable.

16a **PADDING LEMMA**:

Given any program, one can "pad" it with instructions which it never uses. Thus, we obtain

a new program with the same behavior as the old one.

More formally,

---

17 **LEMMA:** For any T.M. $\mathbb{M}_e$ there is an infinite set PAD(e) $\subset$ N such that

(1) for any e' $\in$PAD(e)

$$\varphi_e = \varphi_{e'}, \; T_e = T_{e'}, \text{ and } S_e = S_{e'}$$

and (2) there is a T.M. which recognizes elements of PAD(e) in constant time.

(Think of PAD(e) being binary numbers of the form

e	#	irrelevant

)

---

18 **Proof of L.O. Speed-up Thm:**

Let

$$C_t(x) \overset{df}{=} \begin{cases} 1 \dot{-} \varphi_x(x) & \text{if } T_x(x) \le t(x), \\ 0 & \text{otherwise.} \end{cases}$$

$$1 \dot{-} y \overset{df.}{=} \begin{cases} 0 & \text{if } y \ge 1, \\ 1 & \text{if } y = 0. \end{cases}$$

(1) C_t is computable (implicit in the Universal Machine Thm.)

(2) If $\varphi_{e'} = C_t$, then $T_{e'}(e') > t(e')$ and $C_t(e') = 0$ (by def. of C_t).

(3) Say $\varphi_e = C_t$. Then for any $e' \in PAD(e)$,

$$t(e') < T_{e'}(e') = T_e(e')$$

and $C_t(e') = 0$.

So speed-up $\mathfrak{M}_e$ by always testing if the input is in PAD(e), and if so immediately print output 0.

19 **Def.** $\mathrm{Time}(t) = \{\, \varphi_i : N \to N \;\mid\; T_i(x) \le t(x)$

almost everywhere$\}$

$\mathrm{Space}(t) = \;\cdots\; \qquad S_i \;\cdots$

Corollary. $C_t \notin \mathrm{Time}(t)$ for any computable t.

Remark: Time to compute C_t depends on time to compute t.

Convention: $n = \mathrm{length}(x) = \ell(x) \approx \log_2 x$. Thus,

$\mathrm{Time}\ (2^n) = \mathrm{Time}\ (2^{\ell(x)}) = \mathrm{Time}\ (x)$,

$\mathrm{Time}\ (2^{2n}) = \mathrm{Time}\ (x^2)$, etc.

20 **Def.** A computable $t: N \to N$ is time-honest iff $t \in \mathrm{Time}(t^3)$ and $t(x) \geq \ell(x)$.

Cor. (Compression Theorem, Hartmanis-Stearns) For any time-honest t, $C_t \in \mathrm{Time}(t^4) - \mathrm{Time}(t)$.

Remark: Lots of time-honest fcns.

$$\lceil \log_2 x \rceil,\ x,\ 2^x,\ \left.2^{2^{\cdot^{\cdot^{\cdot^{2}}}}}\right\} x$$

closed under +, •, exp, composition.

Z1 Is more time better than less?

Is

$$\mathrm{Time}(t^4) - \mathrm{Time}(t) \neq \emptyset$$

for all computable t?

NOT NECESSARILY!

Z2

Gap Theorem: (Trachtenbrot, Borodin) For any computable g, there exist arbitrarily large computable t such that

$$\mathrm{Time}(t) = \mathrm{Time}(g \circ t)$$

Z2a Proof of Gap Theorem:

Given g, define

t(x) = the least z such that

$(\forall i \leq x)[\, T_i(x) < z$ or $T_i(x) > g(z)\,]$.

---

23 **Honesty Theorem** (McCreight, Meyer)

For every computable t, there is a time-honest t' such that

$$\mathrm{Time}(t) = \mathrm{Time}(t')$$

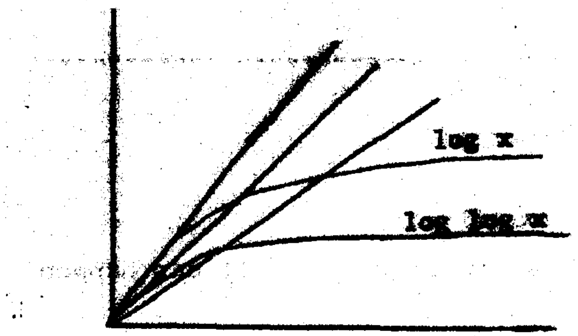
24

Summary:

For arbitrarily large t, t' it can happen that

$$\mathrm{Time}(2^{2^{t}}) \underset{\substack{\nearrow \\ \text{GAP}}}{=} \mathrm{Time}(t) \underset{\substack{\uparrow \\ \text{HONESTY}}}{=} \mathrm{Time}(t') \underset{\substack{\nwarrow \\ \text{COMPRESSION}}}{\subsetneqq} \mathrm{Time}((t')^{4})$$

MNEMONIC PICTURE



Lines(f) = radial lines almost everywhere under f

Compression: For any line $L \neq 0$,

$$\text{Lines}(2L) \supsetneq \text{Lines}(L)$$

Honesty: For any function f, there is a line L,

Lines(f) = Lines(L).

Gap: Lines(t) = Lines (2^t) = {zero line}

for t = loglog.

26 *Def.* Let f be a computable function. A sequence $t_1, t_2, \ldots$ of functions is a (space) *complexity sequence for f* iff

(1) If $\varphi_e = f$, then $S_e \geq t_i$ almost every where for some i,

and (2) For every i, there is a $\varphi_e = f$ such that

$$t_i \geq S_e \text{ almost everywhere.}$$

---

27 *Def.* A sequence of functions $p_1, p_2, \ldots$ is an *r.e. complexity sequence* (for space) iff

(1) $p_{i+1} \leq \left\lceil \frac{1}{2} \cdot p_i \right\rceil$ for all i,

(2) for each i there is a j such that,

$$p_i = S_j$$

and (3) $p_i(x)$ is a computable function of i and x.

---

28 Theorem (Meyer, Schnorr) Every computable function has an r.e. complexity sequence.

Every r.e. complexity sequence is a complexity sequence for some 0-1 valued computable function.

29

Example:

$$\text{Let } t_i(x) = \left.2^{2^{\cdot^{\cdot^{\cdot^{2}}}}}\right\} x \dot{-} i$$

So $t_{i+1} = \log_2 t_i$ almost everywhere.

Cor. Almost everywhere Speed-up

(Blum) There is a 0-1 valued computable function, c, such that for any T.M. computing c there is another T.M. computing c which uses exponentially less space at almost all arguments.

LECTURE II

1

Σ = finite set called the <u>alphabet</u> or
<u>vocabulary</u>,

an element $\sigma \in \Sigma$ is called a <u>letter</u>.

Σ^* = set of all finite sequence of
letters,

an element $x \in \Sigma^*$ is called a <u>word</u>.

2

Binary operation <u>concatenation</u>, written " • "
on Σ^*:

$$x \cdot y = xy = \text{word } x \text{ followed by word } y.$$

<u>Example</u>: $001 \cdot 01 = 00101$

$\ell(x)$ = length (number of occurrences of letter) of the word x.

$\ell(001) = 3$

$\ell(x \cdot y) = \ell(x) + \ell(y).$

---

3 $\underline{\lambda} \in \Sigma^*$ acts as an identity element under concatenation.

$$\lambda \cdot x = x \cdot \lambda = x \quad \text{for all } x \in \Sigma^*,$$

$$\ell(\lambda) = 0.$$

Remark 1: $<\Sigma^*, \cdot>$ is the free monoid generated by Σ with identity λ.

Remark 2: Remark 1 is irrelevant.

Remark 3: λ is introduced as a technical convenience and could be eliminated in what follows at the expense of some minor awkwardness.

4

A set $L \subset \Sigma^*$ is called a language. Extending concatenation to languages in the usual way:

$L \cdot M$, also written $LM \overset{\text{def.}}{=}$

$$\{x \cdot y \mid x \in L \text{ and } y \in M\}$$

Example: $\{0\} \cdot \{0,1\} = \{00,01\}$

$$\{0,00\} \cdot \{1,01\} = \{01,001,0001\}$$

$\{0,1,\lambda\} \cdot \{0,1,\lambda\} \cdot \{0,1,\lambda\}$ = all binary words of length ≤ 3 (including λ).

5

For $x \in \Sigma^*$, $n \in N$,

$$x^n = \underbrace{x \cdot x \cdots x}_{n}$$

$$x^0 = \lambda$$

Example: $(01)^3 = 010101$

Similarly for $A \subset \Sigma^*$

$$A^n = \underbrace{A \cdot A \cdots A}_{n}$$

$$A^0 = \{\lambda\}$$

6

Example:

$\{0,1\}^4$ = all binary words of length <u>exactly</u> 4.

$\{0,1,\lambda\}^4$ = all binary words of length ≤ 4.

$(((\{0,1\}^2)^2)^2)^2 = \{0,1\}^{2^4}$ = all binary words of length 16.

---

7 Important example:

$$(01)^{n} = \underbrace{0101\cdots01}_{2n} =$$

$$= \{0,1\}^{2n} - (1\cdot\{0,1,\lambda\}^{2n} \cup$$
$$\{0,1,\lambda\}^{2n}\cdot 0 \cup$$
$$\{0,1,\lambda\}^{2n}\cdot\{00,11\}\cdot\{0,1,\lambda\}^{2n})$$

= all binary words of length 2n which do not

(1) start wrong

or (2) end wrong

or (3) move wrong (contain a forbidden local pattern)

8

<u>Problem</u>: Given two <u>expressions</u> involving letters in Σ,λ, and operations

" $\cdot$ "	concatenation
" $\cup$ "	union
" 2 "	squaring
" $\cap$ "	intersection
" - "	set difference

is there a way to tell if they describe the same language?

---

9 YES!

BUT NO GOOD WAY!!

10

Lemma. An expression containing n operation symbols describes a subset of

$$(\Sigma \cup \lambda)^{2^n}$$

Proof. By induction on n:

If $n=0$, the expression must consist of a single letter or λ.

11

If E is an expression containing n+1 operations, then E is of the form

$$E_1 \cdot E_2$$
$$E_1 \cup E_2$$
$$(E_1)^2$$
$$E_1 \cap E_2$$
$$E_1 - E_2$$

where E_1, E_2 are expressions containing $\leq$ n operation symbols. Proof follows immediately.

12

For any expression E, let

$\mathcal{L}(E) \subset \Sigma^*$ be the language described by E.

<u>Remark</u>: Formally, $E_1 = E_2$ means that E_1 and E_2 are identical expressions. E_1 and E_2 are <u>equivalent</u> (written $E_1 \equiv E_2$) iff $\mathcal{L}(E_1) = \mathcal{L}(E_2)$.

13

$E_1 \equiv E_2$ iff $(E_1 - E_2) \cup (E_2 - E_1) = \emptyset$

Hence sufficient to test whether an expression describes the empty set.

14

To test if $\mathcal{L}(E) = \emptyset$, convert E to a list of the words in $\mathcal{L}(E)$ beginning at the "innermost" subexpressions of E and working out.

See if the list is empty when you finish.

<u>Difficulty</u>: The list for

$$(\cdots(((0 \cup 1)^{2}\underbrace{)^{2})\cdots)}_{n}{}^{2}$$

contains

---

15 $2^n \cdot 2^{2^n}$ bits

16

Theorem 1. There is (for any finite Σ) a constant $k > 0$ and a Turing machine $\mathfrak{M}$ such that

(1) $\mathfrak{M}$ accepts an input w iff w is a well-formed expression and $\mathcal{L}(w) = \phi$.

(2) $T_{\mathfrak{M}}(n) \overset{\text{df.}}{=} \max\{T_{\mathfrak{M}}(x) \mid \ell(x) = n\} \leq$

$$\leq 2^{2^{kn}}$$

17

Theorem 2. There is a finite Σ and a constant $k > 1$ such that

if $\mathfrak{M}$ is *any* T.M. accepting precisely the expressions over Σ describing the empty set, then

$$T_{\mathfrak{M}}(n) > 2^{\sqrt[k]{n}}$$

for infinitely many n.

(That is, $\{E \mid \mathcal{L}(E) = \phi\} \notin \text{Time}(2^{\sqrt[k]{n}})$)

---

18 To prove Theorem 2:

(i) Define a relation on languages

$$L_1 \prec L_2$$

with intuitive meaning that L_1 is easy to decide given L_2.

(ii) Show that for <u>any</u> $L \in \text{Time}(2^n)$

$$L \prec \{E \mid \mathcal{L}(E) = \emptyset\}.$$

(iii) Deduce from the Compression Theorem that there is an $L \in \text{Time}(2^n)$ which is hard to decide.

(iv) Conclude that $\{E \mid \mathcal{L}(E) = \emptyset\}$ is hard to decide.

19

Def. For $L_1 \subset \Sigma_1^*$, $L_2 \subset \Sigma_2^*$ we say $L_1 \prec L_2$ (L_1 is polynomial time reducible to L_2) iff there exists a function $f: \Sigma_1^* \to \Sigma_2^*$

(1) f is computable in time bounded by a polynomial in the length of its argument ($f \in \text{Time}(\, p \circ \ell\,)$ where $p: N \to N$ is a polynomial and $\ell: \Sigma_1^* \to N$ is the length function).

(2) $x \in L_1 \Leftrightarrow f(x) \in L_2$ for all $x \in \Sigma_1^*$.

---

20 **Lemma.** Let $t: N \rightarrow N$ be nondecreasing, and $t(n) \geq n$.

If $L_1 < L_2$ and $L_2 \in \text{Time}(t(n))$, then $L_1 \in \text{Time}(t(p(n)))$ for some polynomial $p: N \rightarrow N$

Contrapositive. If $L_1 \notin \text{Time}(\ 2^{n/4}\ )$ and $L_1 < L_2$, then $\exists\ k > 1$ such that $L_2 \notin \text{Time}(\ 2^{\sqrt[k]{n}}\ )$

---

21 Because $2^{\ell(x)/4}$ is time-honest, the compression theorem implies $\exists\ L_1$ such that

$$L_1 \in \text{Time}(2^n) - \text{Time}(2^{n/4}).$$

Thm. 2 follows immediately from the preceding contrapositive if we show

---

22 Main Construction for Theorem 2.

Lemma. For any $L \in \text{Time}(2^n)$, there is an alphabet Σ such that

$$L < \{E \mid E \text{ is an expression over } \Sigma \text{ and } \mathcal{L}(E) = \emptyset\}.$$

---

23 Choose any $L \in \mathrm{Time}(2^n)$

Say $L \subset \Delta^*$ for some alphabet Δ.

Let M be a Turing machine which

(i) halts on any input of length n in $\leq 2^n$ steps, and

(ii) halts scanning a symbol " 1 " on its tape iff the input is in L.

24 Let Q be the states (boxes in the flowchart) of $\mathfrak{M}$,

let W be the tape symbols of $\mathfrak{M}$ including $b \in W$ for the blank tape symbol, let # be still another symbol.

$$\Sigma \overset{\text{def.}}{=} Q \cup W \cup \{\#\}.$$

25

For $x \in \Delta^*$, $\ell(x)=n$,

Comp$(x) \in \Sigma^*$ is to be:

$$\# \, b^{2^n} \cdot \boxed{\text{start}} \cdot x \, b^{2^n} \#\text{(tape after one step)}\#\cdots$$

$$\cdots \# \text{ (tape after k steps) } \# \text{ (tape after k+1 steps)} \#\cdots$$

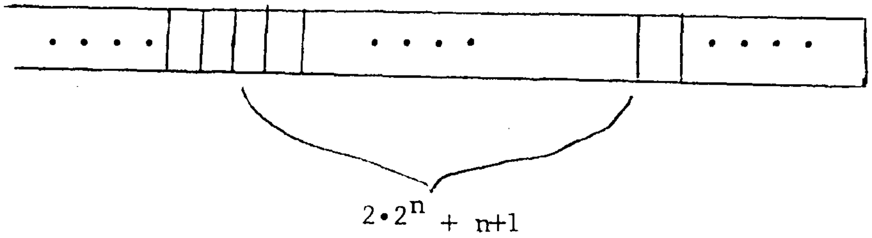
$$\cdots \# \text{ tape } \boxed{\text{halt}} \text{ symbols } \#$$

Exactly $2 \cdot 2^n + n+1$ symbols between successive #'s.

$$\ell(\text{Comp}(x)) \leq 2^{3(n+1)} \overset{\text{df.}}{=} N$$

---

26 Comp(x) has the property that any four consecutive letters determine the letter $2\cdot 2^n + n$ to their right:



Let $F = \{(\sigma_1,\sigma_2,\sigma_3,\sigma_4,\sigma_5) \mid \sigma_5$ is not the letter determined by $\sigma_1\sigma_2\sigma_3\sigma_4\}$. This follows from the fact that at any step the next move of $\mathfrak{M}$ is determined by the state and the tape symbol being scanned.

27

$$\text{Comp}(x) = (\text{starts right}) \cap (\text{ends right}) \cap ((\Sigma \cup \lambda)^N - (\text{moves wrong}))$$

---

starts right: $\#b^{2^n} \cdot \boxed{\text{start}} \cdot x \cdot b^{2^n} \cdot \# \cdot (\Sigma \cup \lambda)^N$

ends right: $(\Sigma \cup \lambda)^N \cdot \boxed{\text{halt}} \cdot (\Sigma \cup \lambda)^N \cdot \#$

---

28 moves wrong:

$$(\Sigma \cup \lambda)^N \cdot \left(\bigcup_F (\sigma_1\sigma_2\sigma_3\sigma_4 \; \Sigma^{2\cdot 2^n+n-1} \cdot \sigma_5)\right) \cdot (\Sigma \cup \lambda)^N$$

---

29 Let $\text{Rejects}(x) =$

$$(\Sigma \cup \lambda)^N \cdot \boxed{\text{halt}} \cdot (\Sigma - \{1\}) \cdot (\Sigma \cup \lambda)^N.$$

Then

$x \in L \Leftrightarrow \mathfrak{M}$ halts reading a 1

$\Leftrightarrow \text{Comp}(x) \cap \text{Rejects}(x) = \emptyset.$

But expressions for Comp(x) and Rejects(x) can be constructed in polynomial time in $\ell(x)$, so $L < \{E \text{ over } \Sigma \mid \mathcal{L}(E) = \emptyset\}$.

Q.E.D.

30

Remarks: (1) Thm. 2 holds for expressions using only " $\cdot$ ", " $\cup$ ", " 2" and letters 0,1 .

(2) If we allow "$\{0,1\}^*$" to be used in expressions Stockmeyer has shown that

$$\{E \text{ with } \{0,1\}^* \mid \mathcal{L}(E) = \emptyset\} \in \text{Time } \left.2^{2^{\cdot^{\cdot^{\cdot^{2}}}}}\right\} n$$

$$\text{but } \notin \text{Time } \left.2^{2^{\cdot^{\cdot^{\cdot^{2^n}}}}}\right\} \epsilon \cdot \log_2 n$$

for some fixed $\epsilon > 0$.

(3) If we allow only " $\cup$ ", " $\cdot$ " , the equivalence problem is complete in $\mathcal{NP}$ (discussed in Karp's lecture).

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31 <u>Remark:</u> Most decidable theories studied in mathematical logic require exponential time or worse. (An important exception being the propositional calculus, for which lower bounds larger than a polynomial are unknown.)

32

<u>Open problems</u>:

(1) Can the satisfiable formulas of the propositional calculus be recognized in polynomial time? (This is the $\mathcal{P} = \mathcal{NP}$ question of Cook and Karp).

(2) Can a multi-tape Turing machine multiply integers (in binary notation) in linear time?

---

33 (3) What is the relation between time and space?

Known: $S_{\mathfrak{M}}(n) \leq T_{\mathfrak{M}}(n) \leq c^{S_{\mathfrak{M}}(n)}$

($c > 1$ depends on $\mathfrak{M}$)

Open: If $L \in \text{Time}(2^n)$ is $L \in \text{Space}(n)$?

(4) Is Space(n) = Nondeterministic Space(n)?
(The LBA problem of Myhill)

---

(5) Are linear time 3 tape T.M.'s more powerful than linear time 2-tape T.M.'s?

(6) Can the primes (represented in binary) be recognized in linear time?

Can the context-free languages?

(7) Can two nxn matrices be multiplied in proportional to $n^{2.8}$ arithmetic operations?

($n^{2.9}$ is known to be possible.)

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DISCRETE COMPUTATION: THEORY AND OPEN PROBLEMS

Albert R. Meyer

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