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# A DISTRIBUTED LINEAR REPRESENTATION OF SURFACE RUNOFF

by

William O. Maddaus
and
Peter S. Eagleson

HYDRODYNAMICS LABORATORY
Report No. 115

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# ABSTRACT

A distributed quasi-linear model of direct catchment runoff is developed consisting of cascades of linear reservoirs connected by linear channels. By fitting to the kinematic wave, the model parameters are expressed in terms of the physical characteristics of the catchment and the impulse response function is constrained to be input-dependent.

Separate models of overland flow and streamflow are developed facilitating consideration of spatially variable inputs. Investigation into the sensitivity of the catchment to distributed inputs illustrates the failure of the kinematic wave method to provide realistic hydrograph dispersion when applied to the flood-routing problem.

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The work was performed by William O. Maddaus, Graduate Fellow and part-time Research Assistant in Civil Engineering, and essentially as given here, it constitutes his thesis presented in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering. The work was supervised by Dr. Peter S. Eagleson, Professor of Civil Engineering.

The computer work was performed at the Information Processing Center at the Massachusetts Institute of Technology.

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LIST OF SYMBOLS

$A$ = cross-sectional area of flow ($ft^2$)

$A_n$ = coefficient

$A_{smax}$ = steady-state maximum cross-sectional area ($ft^2$)

$B$ = channel width (ft)

$D$ = differential operator = $\frac{d}{dt}$ ( )

$\mathbb{F}_o$ = Froude number (dimensionless)

$H_N$ = parallel linear reservoir cascades unit impulse response function (($ft^2$/sec)/$ft^2$)

$H_N(x,t)$ = Distributed Linear Reservoir Model unit impulse response (($ft^2$/sec)/$ft^2$)

$H_n(n,t)$ = discrete representation of $H_n(x,t)$(($ft^2$/sec)/$ft^2$)

$H_N^P(n,t)$ = pulse response function (($ft^2$/sec)/$ft^2$)

$K$ = dimensionless constant of proportionality

$K_j$ = factor representing spatial variability of the storm (dimensionless)

$L$ = length of catchment or stream segment (ft)

$L_c$ = length of catchment (ft)

$L_s$ = length of stream (ft)

$N$ = number of linear elements

$N_c$ = number of linear elements in overland flow model

$N_s$ = number of linear elements in stream model

$P$ = wetted perimeter of stream channel (ft)

$Q$ = streamflow (cfs)

$Q_i$ = flow into a two-dimensional fluid system element ($ft^2$/sec)

$Q_i(x,t)$ = spatially variable input function ($ft^2$/sec)

$Q_i(n,t)$ = discrete representation of $Q_i(x,t)$($ft^2$/sec)

$Q_{ij}$ = flow into a cascade of linear reservoirs ($ft^2$/sec)

$Q_{ic}$ = average storm occurring on the catchment (in/hr)

$Q_{isj}$ = inflow to $j^{th}$ stream element ($ft^3$/sec)

$Q_{kw}$ = output predicted by kinematic wave ($ft^3$/sec)

$Q_o$ = flow out of a two-dimensional fluid system element ($ft^2$/sec)

$Q_{ocj}$ = $j^{th}$ component of the outflow from an overland flow model ($ft^2$/sec)

$Q_{oc}$ = total outflow from an overland flow model ($ft^2$/sec)

$Q_{ocr}$ = outflow from an overland flow which becomes inflow to the $j^{th}$ stream element ($ft^2$/sec)

$Q_{osj}$ = outflow from $j^{th}$ stream element ($ft^3$/sec)

$Q_{smax}$ = steady-state maximum streamflow (cfs)

$Q_{sum}$ = summation of flow out of n linear reservoir cascades ($ft^2$/sec)

R = hydraulic radius (ft)

$\mathbb{R}$ = Reynolds number (dimensionless)

$R_u$ = pulse height (in/hr)

S = storage function ($ft^2$)

$S_o$ = bottom slope (dimensionless)

$V$ = velocity (ft/sec)

$\overline{V}$ = average cross-sectional velocity (ft/sec)

$V_o$ = inflow volume ($ft^2$)

$c$ = shallow water wave speed (ft/sec)

$c_1$ = coefficient

$c_f$ = resistance coefficient (dimensionless)

$d$ = storm depth (ft)

$g$ = gravitational acceleration ($ft/sec^2$)

$h$ = catchment unit impulse response function ($(ft^2/sec)/ft^2$)

$h_n$ = linear reservoir cascade unit impulse response ($(ft^2/sec)/ft^2$)

$h(x,t)$ = spatially variable catchment impulse response function ($(ft^2/sec)/ft^2$)

$h_n(x,t)$ = $n^{th}$ component of the Distributed Linear Reservoir Model unit impulse response ($(ft^2/sec)/ft^2$)

$h_n(n,t)$ = discrete representation of $h_n(x,t)$ ($(ft^2/sec)/ft^2$)

$h_j^P(n,t)$ = $j^{th}$ component of pulse response function ($(ft^2/sec)/ft^2$)

$i(A)$ = ratio of local to average rainfall intensity

$i_e$ = rainfall excess intensity (in/hr)

$i_e^*$ = constant intensity which yields an equivalent volume of rainfall excess as the actual time-varying $i_e$ (in/hr)

$i_e^P$ = peak rainfall excess intensity (in/hr)

$i_*$ = temporally uniform rainfall excess intensity (in/hr)

$i_{mp}$ = average rainfall excess intensity contributing to peak discharge (in/hr)

$k$ = characteristic linear reservoir time constant (usually hours)

$k_c$ = overland flow linear reservoir storage constant (hours)

$k_s$ = stream linear reservoir storage constant (hours)

$k_o$ = convergence parameter (dimensionless)

$m$ = kinematic wave parameter (dimensionless)

$m_c$ = kinematic wave parameter for overland flow (dimensionless)

$m_s$ = kinematic wave parameter for streamflow (dimensionless)

$n$ = Manning's roughness coefficient (dimensionless)

$q$ = discharge per unit width ($ft^2$/sec)

$q_L$ = lateral inflow to stream due to surface runoff ($ft^2$/sec)

$q_{max}$ = steady-state maximum lateral inflow ($ft^2$/sec)

$q_p$ = maximum discharge due to $i_*$ or $i_{mp}$ ($ft^2$/sec)

$t$ = time variable (hours)

$t'$ = normalized time variable with respect to the time of concentration (dimensionless)

$t_c$ = catchment time of concentration (hours)

$t_c'$ = $m_c$ times the total wave travel time for a rainfall excess intensity $i_*$ (hours)

$t_c^*$ = catchment time of concentration corresponding to $i_e^*$(hours)

$t_\ell$ = hydrograph lag time or time from center of gravity of rainfall excess function to the peak of the catchment hydrograph (hours)

$t_{max}$ = length of impulse response function (hours)

$t_p$ = time to peak of catchment hydrograph (hours)

$t_{pc}$ = time interval of influence of rainfall on peak discharge (hours)

$t_r$ = storm duration (hours)

$t_o$ = time of beginning of rainfall influence on $q_p$ (hours)

$t_s$ = stream concentration time = $\lambda t_c$ (hours)

$t_u$ = pulse length (hours)

$x$ = horizontal coordinate distance (ft)

$x_o$ = distance measured along channel axis in direction of flow (ft)

$y$ = vertical coordinate direction (ft)

$y_o$ = average cross-sectional depth (ft)

$\alpha$ = kinematic wave parameter ($sec^{-1}$)

$\alpha_c$ = kinematic wave parameter for overland flow ($sec^{-1}$)

$\alpha_s$ = kinematic wave parameter for streamflow ($sec^{-1}$)

$\beta$ = momentum coefficient (dimensionless)

$\delta$ = Dirac delta function

$\varepsilon$ = integral square error (percent)

$\lambda$ = relative dynamic importance of streamflow to overland flow = $t_s/t_c$ (dimensionless)

$\rho$ = fluid mass density ($slugs/ft^3$)

$\sigma$ = dummy variable

$\tau$ = time constant of channel of area = A (hours)

$\tau'$ = normalized time constant with respect to the time of concentration (dimensionless)

$\tau_c$ = overland flow linear channel time delay (hours)

$\tau_s$ = stream linear channel time delay (hours)

$\tau_o$ = average boundary shear stress ($lb/ft^2$)

$\nu$ = kinematic viscosity ($ft^2/sec$)

$\Gamma$ = Gamma function

$\Delta\sigma$ = time interval between equi-spaced samples (hours)

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# Chapter 1

# INTRODUCTION

## 1-1. Statement of the Problem

The objective of this study is to develop a distributed linear model for direct catchment runoff. This is a significant extension to the common lumped linear representation in that it has the capability to handle spatial variability of the input rainfall and yet retains the simplicity of the superposition approach applied in unit-hydrograph practice.

## 1-2. Scope of the Investigation

Cascades of linear reservoirs, connected by linear channels and each having lateral input, will be used to represent the catchment. In order to relate the parameters of this model to the physical features of natural catchments, the model will be fitted to an analytical representation of nature. The kinematic wave is assumed to embody the essential features of surface runoff and will be used here for this purpose.

Separate model components will be used to represent overland flow and streamflow. A two-dimensional geometric representation of the catchment will provide the linkage between the domain of overland flow and that of streamflow. The overland flow model will receive lateral inflow in the form of an excess in rainfall over infiltration. Application of the methods developed in this work thus requires that rainfall data be transformed to a rainfall excess with a suitable infiltration model.

A significant improvement on the strictly linear approach is achieved by using an impulse response function that is input-dependent.

This distributed model will enable us to study the response of the catchment to spatially variable inputs. This will provide insight into such fundamental questions as the evaluation of errors due to lumping of the input, the importance of considering moving storms, etc.

# Chapter 2

# LINEAR RESERVOIR MODELS

## 2-1. Results from Linear Systems Theory

The physical behavior of a catchment in converting areally distributed rainfall into concentrated streamflow may be represented here by an equation of the form

$$A_n(x,Q_o,t)\frac{d^nQ_o}{dt^n} + A_{n-1}(x,Q_o,t)\frac{d^{n-1}Q_o}{dt^{n-1}} + \cdots + A_o(x,Q_o,t)Q_o = Q_i(x,t) \tag{2-1}$$

The variable coefficients determine the following characteristics of this $n^{th}$ order system:

1. The system is non-linear due to the dependence of the coefficients upon the output, $Q_o$.
2. The system is time variant, due to the time dependence of the coefficients.
3. A spatially distributed input, $Q_i$, is allowed by virtue of its dependence on the coordinate direction x.

For simplicity of analysis, the system will be reduced to a time-invariant linear system. Equation (2-1) then reduces to

$$A_n(x)\frac{d^nQ_o}{dt^n} + A_{n-1}(x)\frac{d^{n-1}Q_o}{dt^{n-1}} + \cdots + A_o(x)Q_o = Q_i(x,t) \tag{2-2}$$

For zero initial conditions, the solution of (2-2) is given by

$$Q_o(t) = \int_o^L \int_o^t Q_i(x,\sigma)h(x,t-\sigma)d\sigma dx \qquad (2\text{-}3)$$

where

L = the length of the catchment

h(x,t) = the impulse response function

h(x,t) satisfies the homogeneous equation associated with equation (2-2)

$$A_n(x)\frac{d^n h(x,t)}{dt^n} + A_{n-1}(x)\frac{d^{n-1}h(x,t)}{dt^{n-1}} + \cdots + A_o(x)h(x,t) = 0 \qquad (2\text{-}4)$$

Through equation (2-3), the kernel or impulse response function, h(x,t), uniquely characterizes the system (2-2). The problem of determining the form of h(x,t) from records of measured input and output is called "identification" and corresponds to the task of deriving the instantaneous unit hydrograph (IUH) with which hydrologists are familiar. The process to be employed here will be one of representing h(x,t) by combinations of linear reservoirs and linear channels.

## 2-2. Linear Channels and Reservoirs

Linear Reservoirs -- The process by which a catchment transforms rainfall excess into direct runoff will be separated into a storage function and a translation function. Let the storage action which causes delay, modulation and attenuation of the input rainfall excess, be represented by

$$S(t) = kQ_o(t) \tag{2-5}$$

where k is the proportionality constant or time constant relating storage $S(t)$ and outflow $Q_o(t)$ of a single linear reservoir. It is equal to the average delay time imparted by the reservoir to the inflow $Q_i(t)$. Using the continuity equation

$$Q_i(t) = Q_o(t) + \frac{d}{dt} S(t) \tag{2-6}$$

and equation (2-5), we obtain the first order linear equation

$$\frac{dQ_i}{dt} + \frac{1}{k} Q_o = \frac{1}{k} Q_i \tag{2-7}$$

For a unit step input

$$\frac{Q_i}{k} = \begin{cases} 0 & , t < 0 \\ 1 & , t \geq 0 \end{cases} \tag{2-8}$$

The total solution to (2-7) is the sum of the homogeneous and particular solutions

$$Q_o = c_1 e^{-t/k} + c_2 \qquad (2\text{-}9)$$

Noting that

$$Q_o = k \text{ as } t \to \infty$$

we require that

$$c_2 = k$$

Thus the unit step response is

$$Q_o = c_1 e^{-t/k} + k \qquad (2\text{-}10)$$

The unit impulse response for the single linear reservoir is then simply

$$h(t) = \frac{dQ_o}{dt} = -\frac{c_1}{k} e^{-t/k} \qquad (2\text{-}11)$$

This unit impulse response, as well as all others presented later, is non-zero only for positive values of the argument. For an inflow volume $V_o$, conservation of mass gives

$$\int_o^\infty h(t)dt = V_o = \int_o^\infty -\frac{c_1}{k} e^{-t/k} dt = -c_1 \qquad (2\text{-}12)$$

Thus for an instantaneous input, the response or outflow from a linear reservoir is an exponential decay from a value dependent on $V_o$

$$h(t) = \frac{V_o}{k} e^{-t/k} \qquad (2\text{-}13)$$

Successive developments in the theory and hydrologic application of this simple impulse response, or instantaneous unit hydrograph (IUH) have been carried out by Zoch (1), Nash (2) (7), Dooge (3), Singh (4) and Kulandaiswamy (5).

Linear Channels -- The case of pure translation can be handled with a linear channel, which was first introduced by Dooge (3). This concept is analogous to a channel whose area-discharge rating curve is a straight line. The linear channel, due to constant disturbance velocity at all stages, provides no change in shape of the input wave.

In a manner analogous to the linear reservoir we may write

$$A = Q_o/\overline{V} \qquad (2\text{-}14)$$

where $\overline{V}$ is the mean velocity of the channel of area A. Combining the continuity equation

$$\frac{\partial Q_o}{\partial x} + \frac{\partial A}{\partial t} = 0 \qquad (2\text{-}15)$$

with equation (2-14), the following equation is obtained

$$\frac{\partial Q_o}{\partial x} + \frac{1}{V}\frac{\partial Q_o}{\partial t} = 0 \qquad (2\text{-}16)$$

which has the solution

$$Q_o(t-\tau) = \text{constant} \qquad (2\text{-}17)$$

This of course implies only translation and $\tau$ is equal to the wave travel time. Note that the impulse response function of a linear channel is not a function of time.

The linear reservoir and linear channel placed in a series arrangement can be represented by a single block diagram as shown in Figure 2-1. Analytically, we have just a shift in the time scale from equation (2-13). That is

$$h(t) = \frac{V_o}{k} e^{-(t-\tau)/k} \quad , \; t > \tau \qquad (2\text{-}18)$$

Qualitatively this function is depicted in Figure 2-2 for a unit input ($V_o = 1$).

## 2-3. Cascades of Linear Reservoirs

The equations for a general network of linear reservoirs, as identified by March and Eagleson (6), can be derived from the configuration shown in Figure 2-3. $Q_{i1}$, $Q_{i2}$, etc. are different, time-

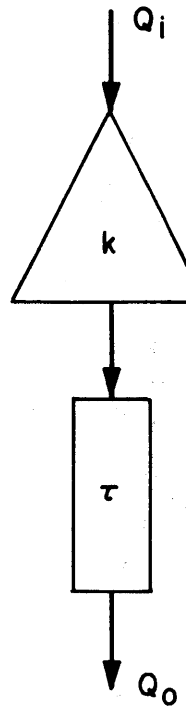


Figure 2-1 Block Diagram of Basic Linear Elements

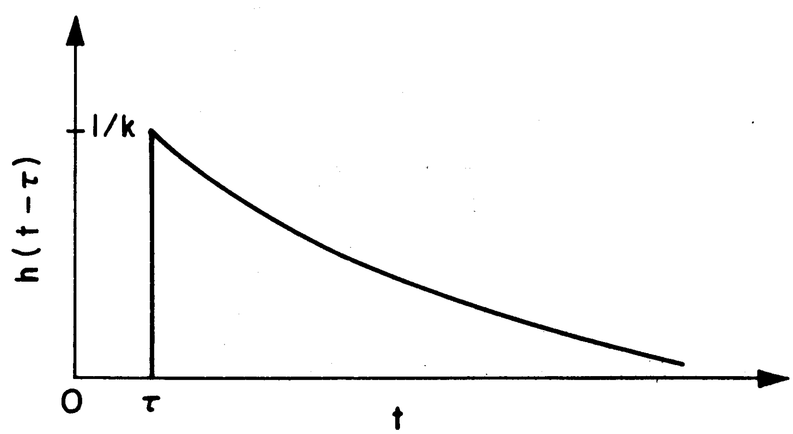


Figure 2-2 Impulse Response Function for a Single Linear Reservoir and Linear Channel Placed in Series

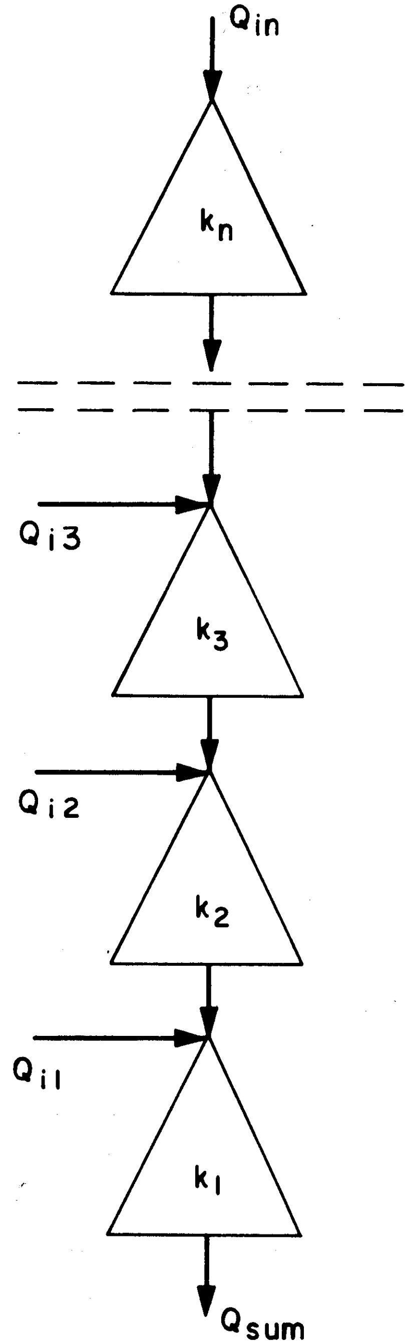


Figure 2-3 Cascade of Linear Reservoirs with Distributed Inputs

varying lateral inputs. An alternative representation, Figure 2-4, emphasizes the distributed nature of the general network by arranging these reservoirs in a parallel array of cascades of progressively higher order.

Rewriting equation (2-7) for the first cascade in terms of the differential operator ($D = \frac{d}{dt}$)

$$\frac{Q_1}{Q_{i1}} = \frac{1}{1+k_1 D} \tag{2-19}$$

and similarly for the entire system

second cascade $$\frac{Q_2}{Q_{i2}} = \frac{1}{(1+k_1 D)(1+k_2 D)} \tag{2-20}$$

$n^{th}$ cascade $$\frac{Q_n}{Q_{in}} = \frac{1}{(1+k_1 D)(1+k_2 D)\cdots(1+k_n D)} \tag{2-21}$$

Thus the process of summation yields

$$Q_{sum} = Q_1 + Q_2 + Q_3 + \cdots + Q_n \tag{2-22}$$

or

$$Q_{sum} = \frac{Q_{i1}}{(1+k_1 D)} + \frac{Q_{i2}}{(1+k_1 D)(1+k_2 D)} + \cdots + \frac{Q_{in}}{(1+k_1 D)(1+k_2 D)\cdots(1+k_n D)} \tag{2-23}$$

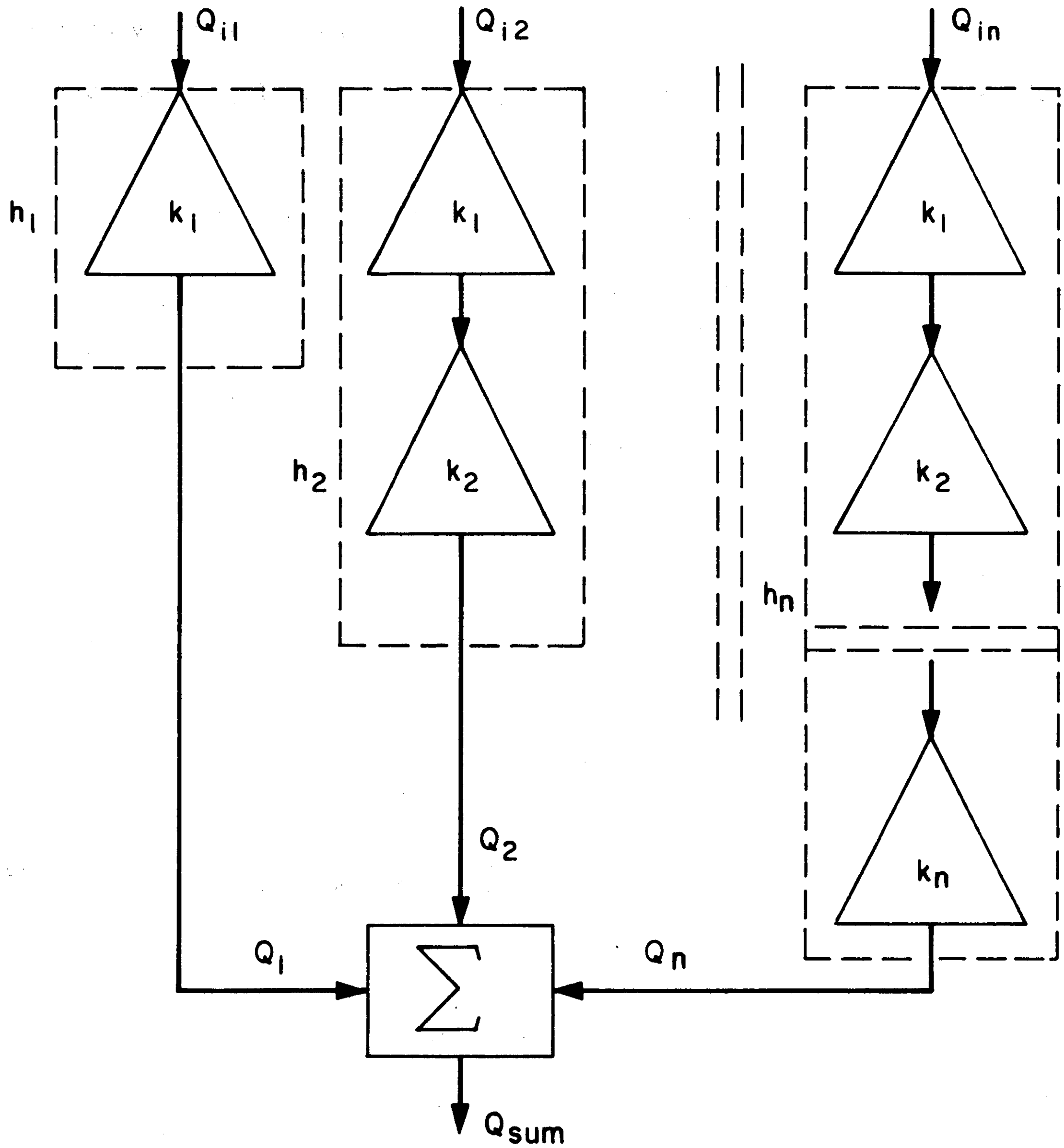


Figure 2-4 Alternative Representation of a Linear Reservoir Cascade with Distributed Inputs

Multiplying and dividing (2-23) by the denominator of the last term

$$Q_{sum} = \frac{(1+k_2D)(1+k_3D)\cdots(1+k_nD)\ Q_{i1} + \cdots + Q_{in}}{(1+k_1D)(1+k_2D)\cdots(1+k_nD)} \qquad (2\text{-}24)$$

The solution to (2-24) can be obtained by taking the Laplace transform term by term. Letting $h_j(t)$ represent the unit impulse response to the cascade having $Q_{ij}$ as input, as designated by the dashed rectangles in Figure 2-4, the output can be written as the scalar product of an $h_j$ and $Q_{ij}$, where each term in the product designates the convolution operation.

$$Q_{sum} = h_j(t)\cdot Q_{ij}(t) \qquad (2\text{-}25)$$

where

$$Q_{sum} = \left(h_1(t)+h_2(t)+\cdots+h_n(t)\right)\cdot\left(Q_{i1}(t)+Q_{i2}(t)+\cdots+Q_{in}(t)\right) \qquad (2\text{-}26)$$

or

$$Q_{sum} = h_1(t)*Q_{i1}(t) + h_2(t)*Q_{i2}(t) + \cdots + h_n(t)*Q_{in}(t) \qquad (2\text{-}27)$$

where the asterisk signifies the convolution operation.

A special simplified form of equation (2-24) results from considering the case of uniform input, $Q_{i1} = Q_{i2} = Q_{i3} = \cdots = Q_{in} = Q_i$, and equal linear elements, $k_1 = k_2 = k_3 = \cdots = k_n = k$. Equation (2-24) can then be written

$$Q_{sum} = \frac{(1+kD)^{n-1} + (1+kD)^{n-2} + \cdots + 1}{(1+kD)^n} Q_i \tag{2-28}$$

The components of the unit impulse response are thus

$$h_1(t) = \frac{e^{-t/k}}{k}$$

$$h_2(t) = \frac{e^{-t/k}\left(\frac{t}{k}\right)}{k1!}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$h_n(t) = \frac{e^{-t/k}\left(\frac{t}{k}\right)^{n-1}}{k\Gamma(n)} \tag{2-29}$$

The last of equations (2-29) is recognized as the n-element Nash model and is frequently used to represent the behavior of natural catchments. This, however, amounts to a lumping of the catchment behavior since all input passes thru the entire system.

## 2-4. Distributed Linear Models

Two lines of reasoning have evolved in the search for a linear model to account for the distributed nature of catchment behavior. The first involves the class of techniques known as Time-Area Methods. Dooge (3) has introduced the concept of a routed time-area curve which recognizes the spatial variability in the input rainfall function. Each segment of the drainage basin is represented by a single linear reservoir with a separate value of k. These give the separate cascades of Figure 2-4.

Representing translation time by a linear channel in each cascade, it is shown (3) that the IUH of a given area becomes

$$h(t) = \frac{V_o}{A} \int_o^{A(t)} \frac{\delta(t - \tau)}{(1+k_1D)(1+k_2D)\cdots(1+k_nD)} \, i(A) \, dA \qquad (2\text{-}30)$$

where

$\delta$ = Dirac delta function

$\tau$ = translation time of the area element

$i(A)$ = ratio of local to average rainfall intensity

By dividing the catchment into a set of segments bounded by isochrones of constant travel time, $\tau$, to the gaging point, equation (2-30) can be converted from a surface integral to a single integral

$$h(t) = \frac{V_o}{A} \int_o^{t \le t_c} \frac{\delta(t - \tau)}{(1+k_1D)(1+k_2D)\cdots(1+k_nD)} \; i \frac{dA}{d\tau} \, d\tau \qquad (2\text{-}31)$$

where

$t_c$ = time of concentration of element

Letting $\tau' = \tau/t_c$ and $t' = t/t_c$, and defining $w(\tau') = i \frac{dA}{d\tau}$ , Dooge finally expresses the IUH as

$$h(t) = \frac{V_o}{A} \int_o^{t \le 1} \frac{\delta(t' - \tau')}{(1+k_1D)(1+k_2D)\cdots(1+k_nD)} \; w(\tau') \, d\tau' \qquad (2\text{-}32)$$

The second line of reasoning involves a special case of the general Dooge model. The simplest of these is of course the Nash model with the addition of a linear channel. This model has been studied by O'Meara (8) and is depicted in Figure 2-5. Its impulse response function is simply a lagged version of Nash equation

$$h(t) = \frac{e^{-(t-\tau)/k}}{k\Gamma(n)} \left(\frac{t-\tau}{k}\right)^{n-1} \quad , \quad t > \tau \qquad (2\text{-}33)$$

O'Meara (8) has also considered the model indicated by equation (2-28). The impulse response function is obtained by simply summing the components of equation (2-29)

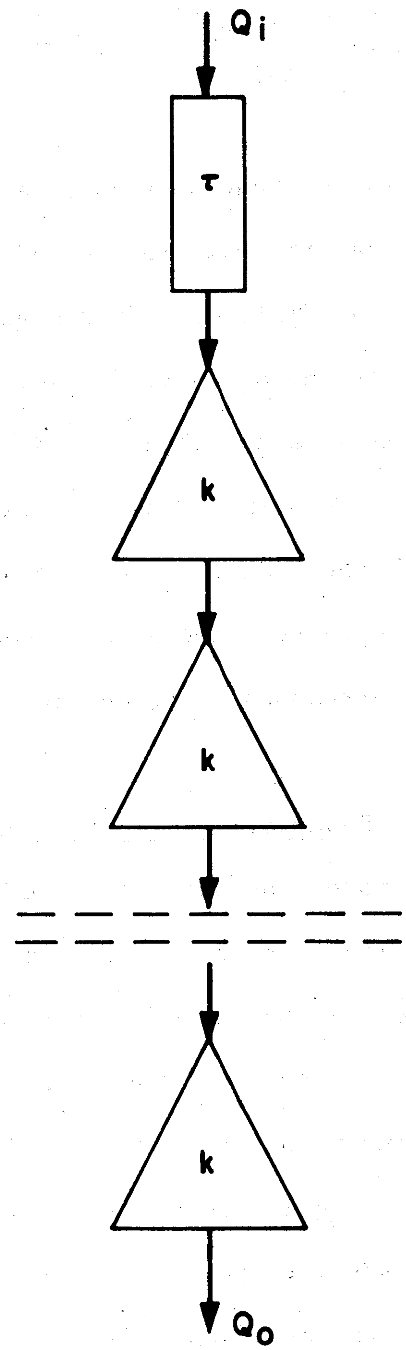


Figure 2-5 Linear Channel in Series with a Linear Reservoir Cascade

$$H_N(t) = \sum_{n=1}^{N} \frac{e^{-t/k}\,(t/k)^{n-1}}{k\Gamma(n)} \tag{2-34}$$

The system represented by equation (2-34) may be constructed of N first order linear differential equations or a single $N^{th}$ order linear differential equation. This model takes advantage of the simplicity of the Nash model but accounts for the fact that the input should be distributed areally rather than lumped into one input which is passed through the entire system. Thus each cascade of Figure 2-4 is considered to be a Nash model of the same linear elements.

This model, unlike the routed time-area method, can not handle different lateral inputs corresponding to an areally distributed input rainfall function.

## 2-5. A Distributed Linear Reservoir -- Linear Channel Model

A distributed linear model has been developed that incorporates the concepts of the three models discussed in section 2-4. The Distributed Linear Reservoir -- Linear Channel Model, hereafter called the *Distributed Linear Reservoir Model*, is composed of linear reservoirs connected by linear channels as shown in Figure 2-6. From this representation the following characteristics are evident:

1. The model recognizes the spatial variability of typical rainfall patterns by allowing different time-varying lateral inputs.
2. By not lumping the translation effect into a single linear channel (as in Figure 2-5), the concept of

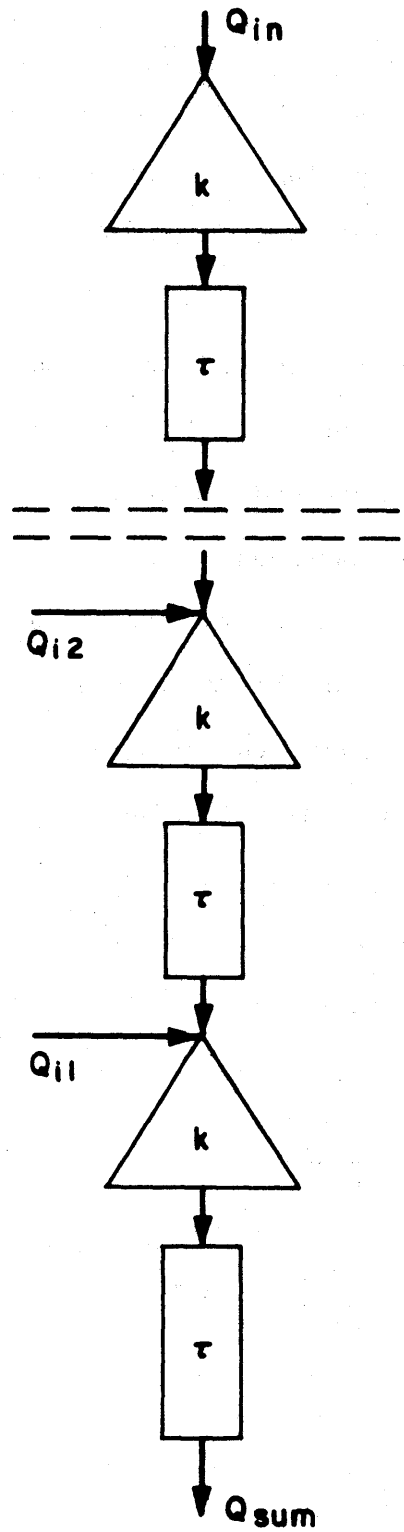


Figure 2-6 Distributed Linear Reservoir Model

different translation times from each segment of the catchment to the gaging point is retained. This is physically more realistic and is analogous to the time-area method of Dooge.

3. The storage constants, k, are equal as are the translation constants, $\tau$. This is consistent with the particular geometric model that will be chosen (Chapter 3) to represent the catchment.

In section 2-1 it was shown that a linear channel placed in series with a linear reservoir only shifted the time scale (pure translation) and was accounted for merely by redefining the time variable in the linear reservoir equation. The components of the unit impulse response function are thus dependent upon their relative location on the catchment, measured along the coordinate direction x. Equations (2-29) are thus augmented to become

$$h_1(x,t) = \frac{e^{-(t-\tau)/k}}{k} \quad , \quad t > \tau$$

$$h_2(x,t) = \frac{e^{-(t-2\tau)/k}}{k \cdot 1} \left(\frac{t-2\tau}{k}\right) \quad , \quad t > 2\tau$$

..................................

$$h_n(x,t) = \frac{e^{-(t-n\tau)/k}}{k\Gamma(n)} \left(\frac{t-n\tau}{k}\right)^{n-1} \quad , \quad t > n\tau \qquad (2\text{-}35)$$

The unit impulse response function for the Distributed Linear Reservoir Model is the summation

$$H_N(x,t) = \frac{1}{N} \sum_{n=1}^{N} \frac{e^{-(t-n\tau)/k}}{k\Gamma(n)} \left(\frac{t-n\tau}{k}\right)^{n-1} \quad , \quad t > n\tau \qquad (2\text{-}36)$$

This equation characterizes the $N^{th}$ order system described by the differential equation (2-2). The output can theoretically be obtained by the convolution indicated in equation (2-3). The IUH for equation (2-36) has the qualitative form of Figure 2-7.

The structure of the model invites a sensitivity analysis of the catchment response to areally variable input rainfall functions.

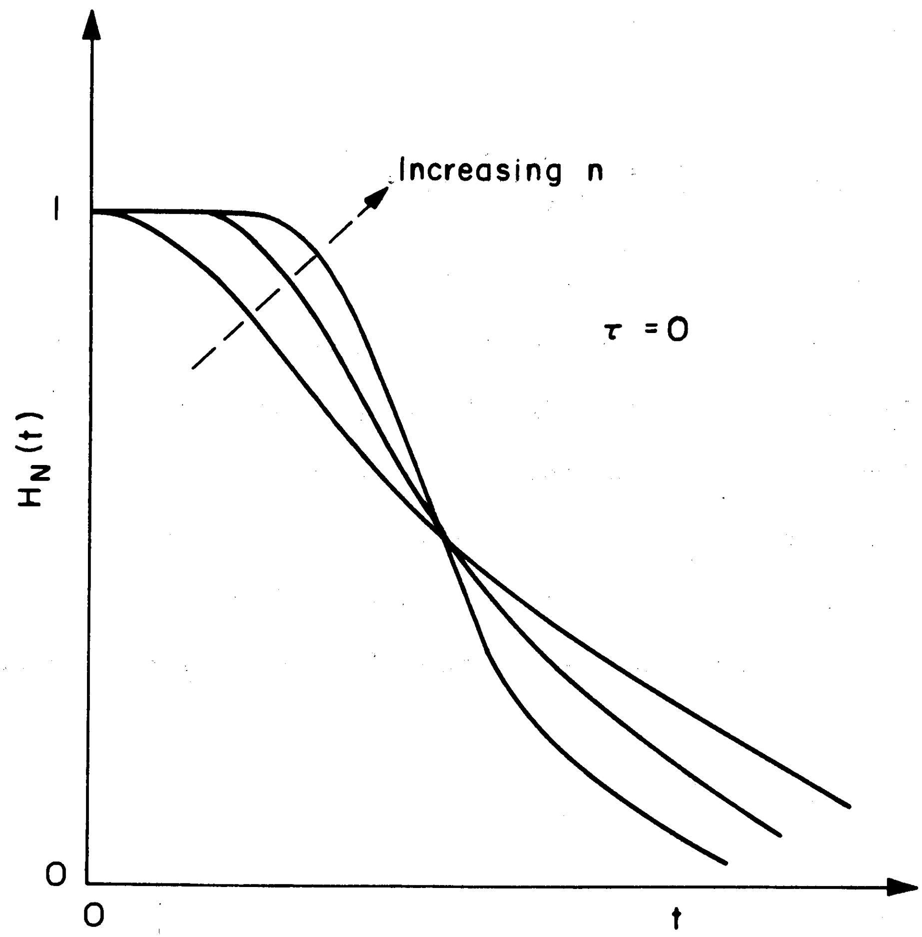


Figure 2-7 IUH for a Distributed Linear Reservoir Model

# Chapter 3

# OVERLAND FLOW AND STREAMFLOW

## 3-1. Introduction

A catchment is a very complicated physical system with stochastic inputs. However, given the physical characteristics of the system, its initial state, and the inputs, the response of the system is deterministic to a large degree. The basic element of direct catchment runoff is a surface with overland flow discharging as lateral inflow to a stream channel.

Overland flow originates from storage in surface depressions. Inflow into these depressions occurs when the rate of rainfall or snowmelt exceeds the infiltration capacity of the surface. Overland flow is generated by an excess of gravitational forces over those forces developed by surface irregularities and surface tension. This flow begins as a thin-sheet flow but is focused into small channels by surface irregularities. As these channels merge with one another, the domain of streamflow is formed. This concept of overland flow is useful in interpreting the physical meaning of simulating overland flow by means of linear reservoirs and linear channels.

## 3-2. The Equations of Motion

Direct catchment runoff can be modelled mathematically by considering conservation of mass and momentum as applied to the control volume of Figure 3-1. The resulting differential equations are quite complex but may be reduced to a tractable form by making the following primary assumptions (9):

1. The channel is of constant cross-sectional area over each reach.
2. Hydrostatic pressure distribution at each section.
3. The momentum distribution coefficient $\beta$=1.
4. Surface tension forces are negligible.
5. The transverse water surface profile at any x is horizontal.
6. The x-component of momentum flux due to lateral inflow is negligible.
7. The overpressure due to vertical inflow is negligible.

These assumptions simplify the momentum equation to

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} = g(S_o - \frac{\partial y}{\partial x}) - \frac{\tau_o}{\rho R} \tag{3-1}$$

and the continuity equation to

$$\frac{\partial (AV)}{\partial x} + \frac{\partial A}{\partial t} = q_L + Bi_e \tag{3-2}$$

where $q_L$ = lateral inflow due to surface runoff

$B$ = channel width

$i_e$ = rainfall excess intensity

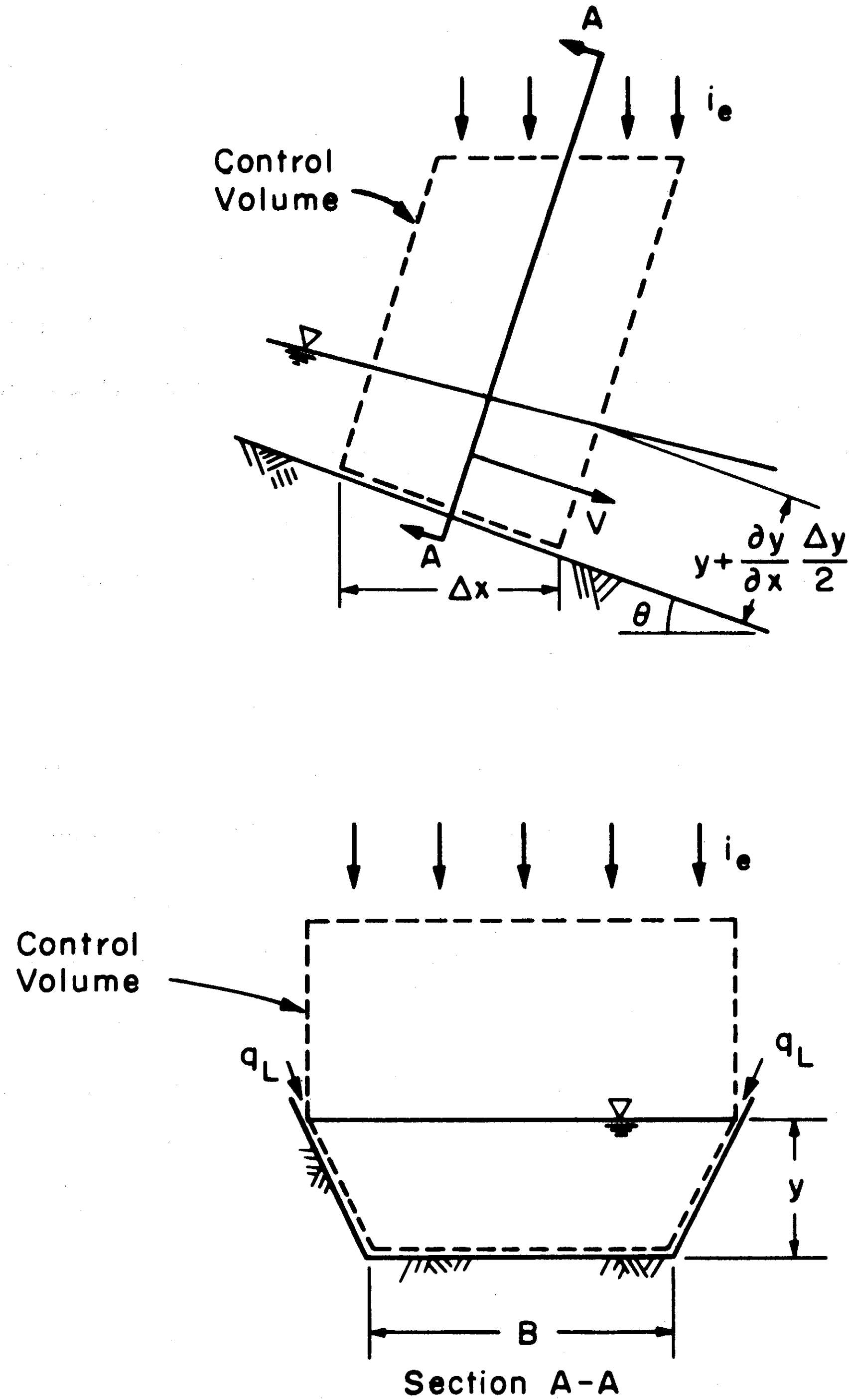


Figure 3-1 Control Volume for Flow in Small Streams

These non-linear partial differential equations have been known since the time of de St. Venant. They are applicable to one-dimensional, gradually-varied free surface flows. Analytical solutions have been restricted to special cases where suitable simplifications could be made. Numerical solutions using finite difference schemes have become feasible with the advent of the digital computer.

Under certain additional restrictions, these can be generalized to demonstrate the essential elements of the surface runoff phenomenon. A summary description of the generalization follows.

## 3-3. The Kinematic Wave Equations

The derivative terms of the momentum and continuity equation give rise to two distinct types of wave propagation. If the inertia forces are important and all inflow terms are negligible, then equations (3-1) and (3-2) describe the movement of long waves in shallow water. Flood waves in rivers are an example of these so-called dynamic waves. The other important class of problems is that in which the pressure gradient and inertia terms of equation (3-1) are small in comparison with those of gravity and friction. These are the kinematic wave conditions and are approximately satisfied by overland flow and by gradually varied flow in a prismatic channel.

The momentum equation is in a much more manageable form if the kinematic wave conditions can be assumed. However, mere assumption of this type of flow system does not preclude the actual existence of dynamic waves. Fortunately, Lighthill and Whitham (10) have shown that the dynamic component is damped (exponentially) provided that the Froude number, $\mathbb{F}_o$, satisfies

$$\mathbb{F}_o = \frac{\bar{V}}{(gy_o)^{1/2}} < 2 \qquad (3\text{-}3)$$

where $\bar{V}$ and $y_o$ are the average velocity and depth respectively at uniform flow conditions

$$q_{max} = i_e L \qquad (3\text{-}4)$$

For steady, uniform flow in a wide channel the momentum equation reduces to

$$gS_o = \frac{\tau_o}{\rho y} = \frac{c_f V^2}{2y} \qquad (3\text{-}5)$$

or

$$q = Vy = \left(\frac{2gS_o}{c_f}\right)^{1/2} y^{3/2} = \alpha_c y^{m_c} \qquad (3\text{-}6)$$

For a small depth of flow in a wide channel, the continuity equation can be simplified to the one-dimensional form

$$\frac{\partial(Vy)}{\partial x} + \frac{\partial y}{\partial t} = i_e \qquad (3\text{-}7)$$

Equations (3-6) and (3-7) are called the kinematic wave equations. By comparison of these with the "complete" equations (3-1) and (3-2), Woolhiser and Ligget (11) have demonstrated that the damping of the dynamic component will be sufficiently rapid to justify neglecting the inertia terms provided that

$$k_o = \frac{S_o L}{y_o \mathbb{F}_o^2} > 10 \qquad (3\text{-}8)$$

For $k_o$ values smaller than 10, it would be necessary to have short, channels with small slopes and high velocities. These conditions would seem more common in streamflow than in overland flow. The numerical solutions presented by both Morgali and Linsley (12) and Schaake (13), the latter of which was an urban catchment, conform very closely to the $k_o = \infty$ curve. Thus the kinematic wave equation appears to be a good approximation for most overland flow situations.

## 3-4. The Method of Characteristics

The kinematic wave equations are readily solvable by numerical methods. Woolhiser and Liggett (11) have found that solution by the method of characteristics is the most accurate and computationally efficient of the various finite difference schemes available. Assuming that $\alpha_c$ and $m_c$ remain constant with time, it can be shown (14) that equations (3-6) and (3-7) lead to the characteristic equations for two-dimensional overland flow

$$\frac{dx}{dt} = \alpha_c m_c y^{m_c - 1} = c \tag{3-9}$$

$$\frac{dq}{dx} = \frac{dy}{dt} = i_e \tag{3-10}$$

which represent the wave speed, c, and the rate of lateral inflow on the plane. The essence of the method of characteristics is to find the space-time (x-t plane) locus of the discontinuity in the partial derivatives of the important dependent variables, y(x,t) and q(x,t). The path of wave propagation, called the characteristic path, is obtained by integration of (3-9) and (3-10)

$$x - x_o = m_c \alpha_c \int_{t_o}^{t} \left( \int_{t_o}^{t} i_e(\sigma) d\sigma + y_o \right)^{m_c - 1} dt \tag{3-11}$$

Streamflow can be added to the model (15) by treating the catchment hydrograph q(x,t) as lateral inflow to the stream. The equivalent kinematic wave equation is

$$Q = VA = \left(\frac{2gS_o}{c_f P}\right)^{1/2} A^{3/2} = \alpha_s A^{m_s} \tag{3-12}$$

where P is the wetted perimeter of the stream channel. The corresponding characteristic equations for the wave speed and for the lateral inflow into the stream are

$$\frac{dx}{dt} = \alpha_s m_s A^{m_s - 1} \tag{3-13}$$

and

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q_L \tag{3-14}$$

## 3-5. A Geometric Catchment-Stream Model

To apply the foregoing analysis to natural catchments it is expedient to retain only those parameters which are essential from physical considerations. Wooding (15) has found that a simple model of the type shown in Figure 3-2 adequately preserves the main topographic features of a catchment combined with a stream. The parameters which have dominant influence upon flow in the catchment and stream are slope, roughness and flow regime. These are taken into account by the constants $\alpha_c$, $m_c$, $\alpha_s$, $m_s$ in equations (3-9) and (3-13). The component of slope in the direction of the stream is neglected on the catchment surface. As the catchment length, $L_c$, is assumed constant everywhere, it is obviously an "effective" catchment parameter which represents the average length of surface flow path tributary to the main stream. Clearly, in this model, $A = 2 L_s L_c$.

For this model a special definition for the catchment time of concentration can be derived. For temporally constant rainfall

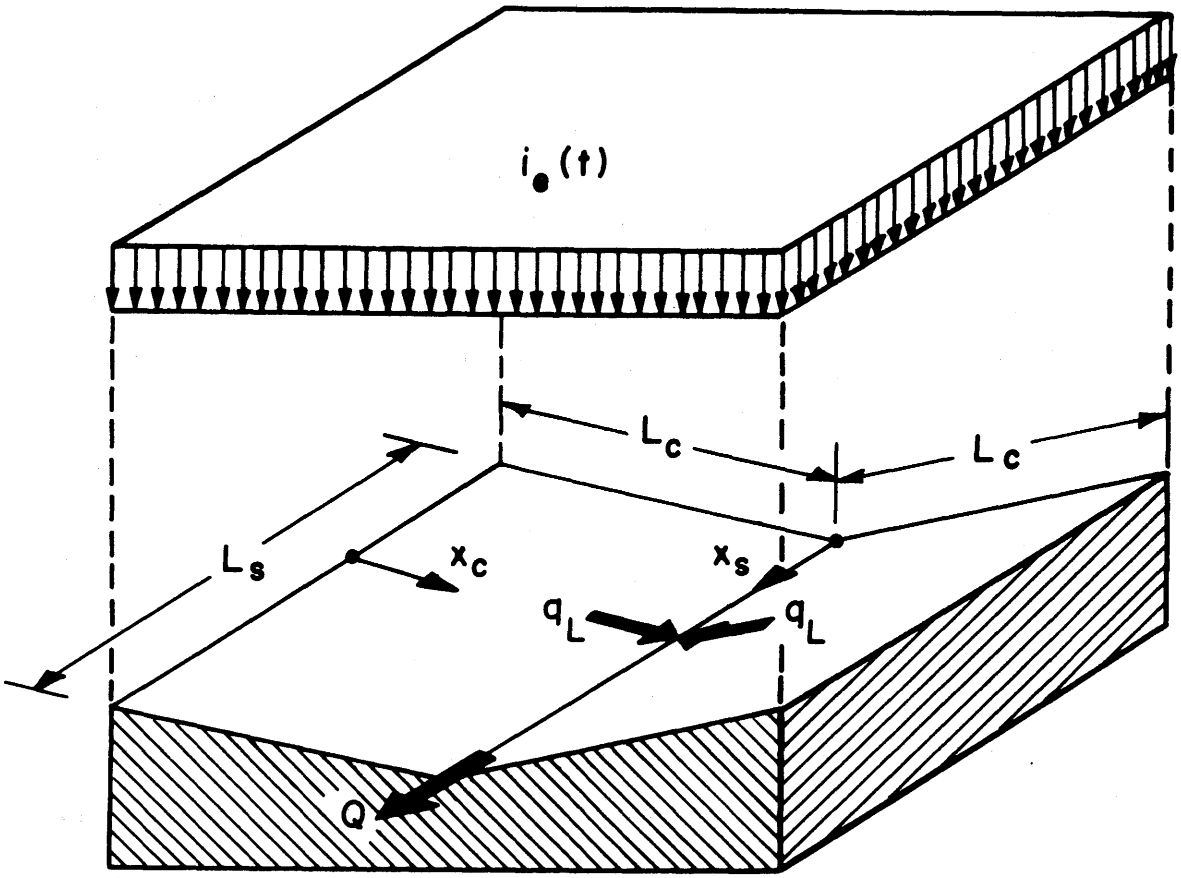


Figure 3-2 Catchment-Stream Geometric Model

excess, $i_*$, and letting $t_o = 0$ in equation (3-11), the characteristic curves are given by

$$x = \alpha_c i_*^{m_c - 1} t^{m_c} + x_o \tag{3-15}$$

For $x_o = 0$, equation (3-15) specifies the "limiting characteristic" which describes the disturbance emanating from $x = o$ where there is no inflow from upstream. For the particular case $x = L_c$, the limiting characteristic defines the maximum time during which growth of depth (and hence discharge) can occur on the catchment surface. Thus the catchment time of concentration, $t_c$, is given by

$$t_c = \left(\frac{L_c i_*^{1-m_c}}{\alpha_c}\right)^{m_c^{-1}} \tag{3-16}$$

This definition requires a storm of long duration, $t_r$, such that $t_r > t_c$. Note that if the rainfall excess stops before the characteristic reaches the end of the catchment then the right-hand side of equation (3-10) equals zero. This complicates the formulation of equation (3-11) and thus a simple expression of the time of concentration, as in equation (3-16), is no longer possible.

Maintaining the same restrictions on the problem, another useful definition is available to represent the relative dynamic importance of streamflow to overland flow.

If the lateral inflow is constant in time and space, such as would occur for $t > t_c$ for a storm of infinite duration, then the streamflow at $x_s = L_s$ will rise to a steady-state maximum, $Q_{smax}$. Letting the stream concentration time for this condition be $t_s = \lambda t_c$

$$A_{smax} = 2q_{max} t_c = \left(\frac{2q_{max} L_s}{\alpha_s}\right)^{1/m_s} = \left(\frac{Q_{smax}}{\alpha_s}\right)^{1/m_s} \quad (3\text{-}17)$$

Using equations (3-6) and (3-16), the ratio of concentration times becomes

$$\lambda = \frac{(2i_* L_c L_s/\alpha_s)^{1/m_s}}{2L_c(i_* L_c/\alpha_c)^{1/m_c}} = \frac{t_s}{t_c} \quad (3\text{-}18)$$

## 3-6. Selection of Physical Parameters

The applicability of any catchment model to an actual problem depends upon a careful choice of catchment and flow parameters. Because of the inherent geometric approximations in a two-dimensional catchment-stream model, the task of estimating average channel slopes, lengths and roughness may be difficult. A sensitivity analysis of the hydrograph to changes in these parameters has been performed by Morgali and Linsley (12).

To estimate the flow regime (i.e., laminar, turbulent, or intermediate) and the roughness coefficient, it is common practice to rely on experimental results. By definition

$$\alpha = \left(\frac{2gS_o}{c_f}\right)^{1/2} \tag{3-19}$$

where the friction coefficient, $c_f$, is a function of Reynolds number and of the relative surface roughness. For laminar flow,

$$c_f = \frac{6}{\mathbb{R}} \tag{3-20}$$

with

$$\mathbb{R} = \text{Reynolds number} = \frac{Vy}{\nu} < 500 \tag{3-21}$$

we get

$$\alpha = \frac{g\ S_o}{3\nu} \text{ and } m = 3 \tag{3-22}$$

For turbulent flow using the Manning equation

$$c_f = 0.9n^2y^{-1/3} \tag{3-23}$$

where n is the Manning roughness coefficient. Thus

$$\alpha = \frac{1.49}{n} S_o^{1/2} \text{ and } m = 5/3 \tag{3-24}$$

Overland flow is reputed to be laminar, however, the resistance coefficient, $c_f$, has been found (13) to be on the order of $10/\mathbb{R}$ , which is higher than that obtained theoretically (3-20). This is probably caused by turbulence effects due to falling raindrops on the flow surface and the lack of two-dimensional flow as assumed. For flow on natural surfaces there will be fluctuations in depth and roughness so that the flow regime may vary between laminar and turbulent. The work of Horton (18) and others indicates that overland flow in this intermediate regime is best approximated with a value of $m_c = 2$.

The streamflow component of this model is applied using $\alpha_s$ and $m_s$ as given by the Chezy or Manning formula. In storm runoff situations where the lateral inflow is negligible in comparison with streamflow (i.e., $\lambda$ very large), the kinematic wave equations are of questionable value and a more accurate procedure would be to consider the problem as one of flood routing and apply the complete equations (19).

## 3-7. The Impulse Response Function

To obtain the instantaneous hydrograph, two different approaches are available. One technique (8) involves first combining equations (3-1) and (3-2) into a single non-linear second order partial differential equation. The equation can then be linearized by referring the discharge, depth and velocity variables to their steady-state values. The solution to a Dirac delta function input can be obtained using Laplace transform techniques. Computationally it is desirable to use a reduced momentum equation (such as the kinematic solution, equation (3-5)) to derive a simplified impulse response function.

The approach to be employed here is derived from consideration of the characteristic equation (3-11). Considering a temporally constant rainfall excess, $i_*$, of short duration such that $t_r < t_c$ the catchment discharge hydrograph or pulse response can be specified (9) as

$$q = \alpha_c y^{m_c}; \quad \begin{cases} y = i_* t & , 0 < t \le t_r < t_c \\ y = i_* t_r & , t_c > t_r < t \le t_p \\ L_c = \alpha_c y^{m_c - 1}\left(y i_*^{-1} + m_c (t - t_r)\right) & , t > t_p \end{cases} \tag{3-25}$$

where the length of the peak is

$$t_p = t_r + \frac{t_c' - t_r}{m_c} \tag{3-26}$$

in which

$$t_c' = \frac{L_c}{\alpha_c d^{m_c - 1}}, \tag{3-27}$$

and

$$d = i_* t_r \tag{3-28}$$

The impulse response function can be obtained from the pulse response function by taking the limit as the pulse duration, $t_r$, goes to zero. For this limiting case, the discharge begins immediately and remains constant at the peak value, $q_p$,

$$q_p = \alpha_c d^{m_c} \tag{3-29}$$

until time $t_c'/m_c$ which is the time for the disturbance to travel the full catchment length at constant depth, d. The recession curve $(t > t_p)$ of the instantaneous hydrograph is given by

$$q = \alpha_c y^{m_c} = \alpha_c \left(\frac{L_c}{\alpha_c m_c t}\right)^{\frac{m_c}{m_c - 1}} \tag{3-30}$$

Note the similarity between this impulse response function depicted in Figure 3-3 and that obtained from the Distributed Linear Reservoir Model shown in Figure 2-7. This fact will be exploited in determining the parameters of the latter model.

## 3-8. Conclusions

The kinematic wave theory is a reasonably accurate, computationally efficient and well-documented approach to overland flow and certain streamflow problems. This is the primary reason for its use as a standard of comparison in the development of the

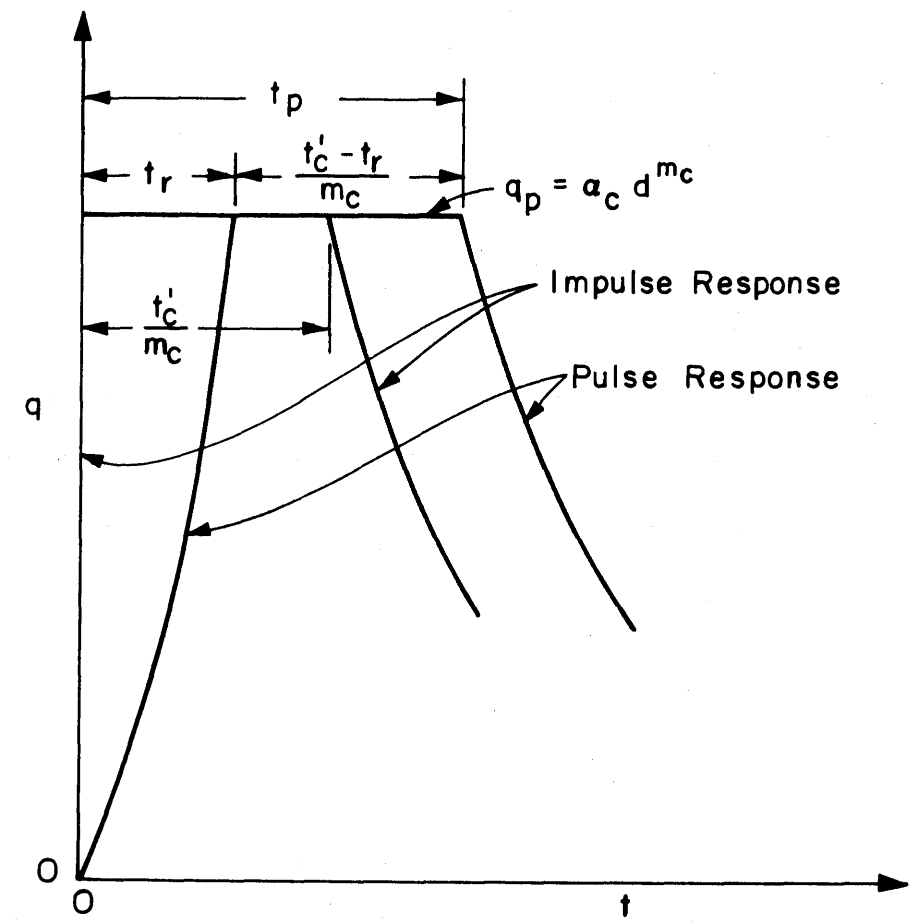


Figure 3-3 Kinematic Wave Impulse and Pulse Response Functions

Distributed Linear Reservoir Model. All concepts introduced in this chapter, including the two-dimensional catchment representation, will be incorporated into the Model. Although the Model may suffer from some of the same weaknesses of this particular application of the kinematic wave theory, it will be in no way dependent upon this approach.

# Chapter 4

# A QUASI-LINEAR REFINEMENT OF THE UNIT-HYDROGRAPH METHOD

## 4-1. Introduction

The non-linear, areally distributed nature of the rainfall-runoff process has been illustrated in the discussion of the kinematic wave presented in the previous chapter.

Traditional hydrologic methods have, for reasons of computational simplicity, adopted lumped, linear approximations of this behavior. The characteristic response function of this linear system is known as the unit hydrograph and in hydrologic terms the associated assumptions are:

1. The effects of all pertinent physical characteristics of a catchment are reflected accurately by the hydrograph of direct runoff from a storm having areally and temporally uniform rainfall excess.
2. For a given duration of rainfall excess, the duration of surface runoff is essentially constant and independent of the magnitude of the rainfall excess.
3. The discharge ordinates of direct runoff hydrographs are proportional to the total volume of direct runoff.
4. The time distribution of direct runoff from a given storm is independent of concurrent runoff from antecedent storms.

Assumptions 2 and 3 are illustrated qualitatively by the hydrographs sketched in Figure 4-1. In actual practice, Ishihara (21) has found that unit hydrographs derived from floods on the Yura River show a five-fold variation in the peak discharge and a two-fold variation in the lag time. It was observed that this non-linearity corresponded to the occurrence of overland flow during the larger storms.

The advent of digital computers has removed the need for these restrictive assumptions, since in principle, the kinematic equations (and indeed the "complete" one-dimensional equations) can now be solved for arbitrary variations of the input rainfall and catchment parameters. In practice, however, the computer time necessary for this approach is very large and we are led toward other simplifications which will produce computational efficiency without undue sacrifice in physical validity.

4-2. A Quasi-Linear Approach

The approach to be used here is to develop a model which is quasi-linear in that its response function is input-dependent and which is distributed in that different parts of the catchment have different responses.

Application of the Distributed Linear Reservoir Model requires convolving the input rainfall excess $Q_i(x,t)$ and the impulse response function $H_N(x,t)$ given in equation (2-36). The form of the convolution operation denoted in equation (2-3) can be simplified by lumping the input over the distance L/N in the direction x. The output hydrograph

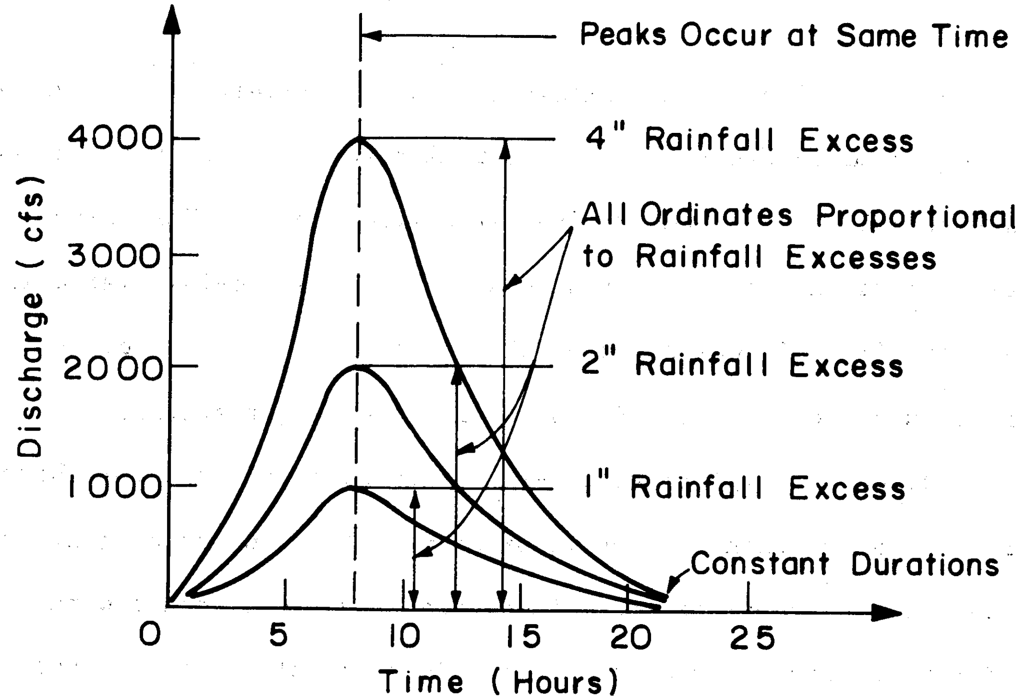


Figure 4-1 Qualitative Storm Hydrographs by the Unit-Hydrograph Method

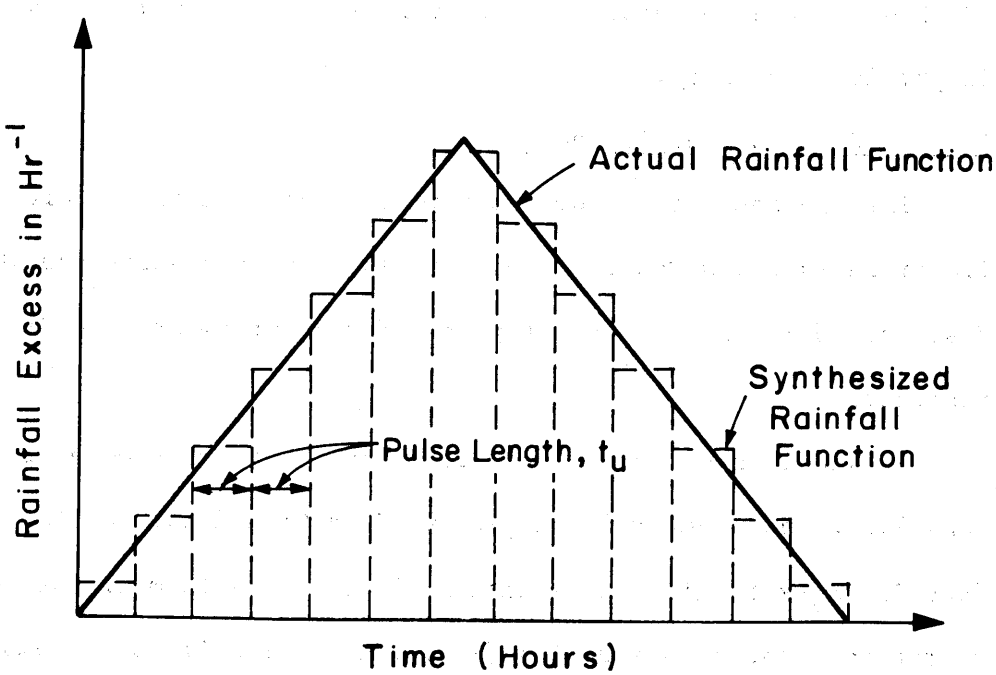


Figure 4-2 Discrete Rainfall Function

is then given by

$$Q_o(t) = \int_o^t Q_i(n,\sigma)H_N(n,t-\sigma)d\sigma \tag{4-1}$$

Due to the discontinuous nature of the input rainfall function and the form of the response function, obtaining a closed form expression for $Q_o(t)$ would rarely be feasible. Therefore, it is convenient to introduce discrete notation and to perform the integration numerically.

This implies that the input rainfall function and the impulse response function are defined at discrete intervals, spaced $\Delta\sigma$ in time. The response function is no longer instantaneous and must be defined for a unit-time. This unit-time is called the pulse length and is denoted by $t_u$.

The pulse function, $R_u(\sigma)$, is defined at discrete intervals spaced $\Delta\sigma$ in time and is of constant magnitude $R_u$ over its duration $t_u$. The components of the lumped pulse response function, $h_n^P(n,t)$, are obtained by convolving the discrete form of equation (2-35) with the pulse function

$$h_1^P(1,i) = \sum_{\sigma=1}^{i} R_u(\sigma)h_1(1,i-\sigma+1)\Delta\sigma$$

..............................

$$h_n^P(n,i) = \sum_{\sigma=1}^{i} R_u(\sigma)h_n(n,i-\sigma+1)\Delta\sigma \tag{4-2}$$

The convolution process of equation (4-2) is illustrated in Figure 4-3. The length of the output time series, $h_n^P(n,i)$, is determined as shown.

Similarly the pulse response function, $H_N^P$, in discrete form is obtained by

$$H_N^P(n,i) = \sum_{\sigma=1}^{i} R_u(\sigma) H_N(n,i-\sigma+1)\Delta\sigma \qquad (4\text{-}3)$$

To use the pulse response function, $H_N^P$, in a discrete form of equation (4-1) requires that the input rainfall function be synthesized as shown in Figure 4-2 and then normalized with respect to the pulse height

$$Q_o(i) = \sum_{\sigma=1}^{i} (Q_i(n,\sigma)/R_u) H_N^P(n,i-\sigma+1)\Delta\sigma \qquad (4\text{-}4)$$

For a given storm, the overland flow hydrographs generated by the indicated convolutions for a range of pulse heights can be compared to those obtained by a characteristics solution of the kinematic wave equation. Since the pulse height is analogous to the depth in the shallow water wave speed equation (3-9), the time to the hydrograph peak will be inversely proportional to the pulse height. This is illustrated in Figure 4-4. We will consider the optimum pulse height to be one which minimizes the following integral square error:

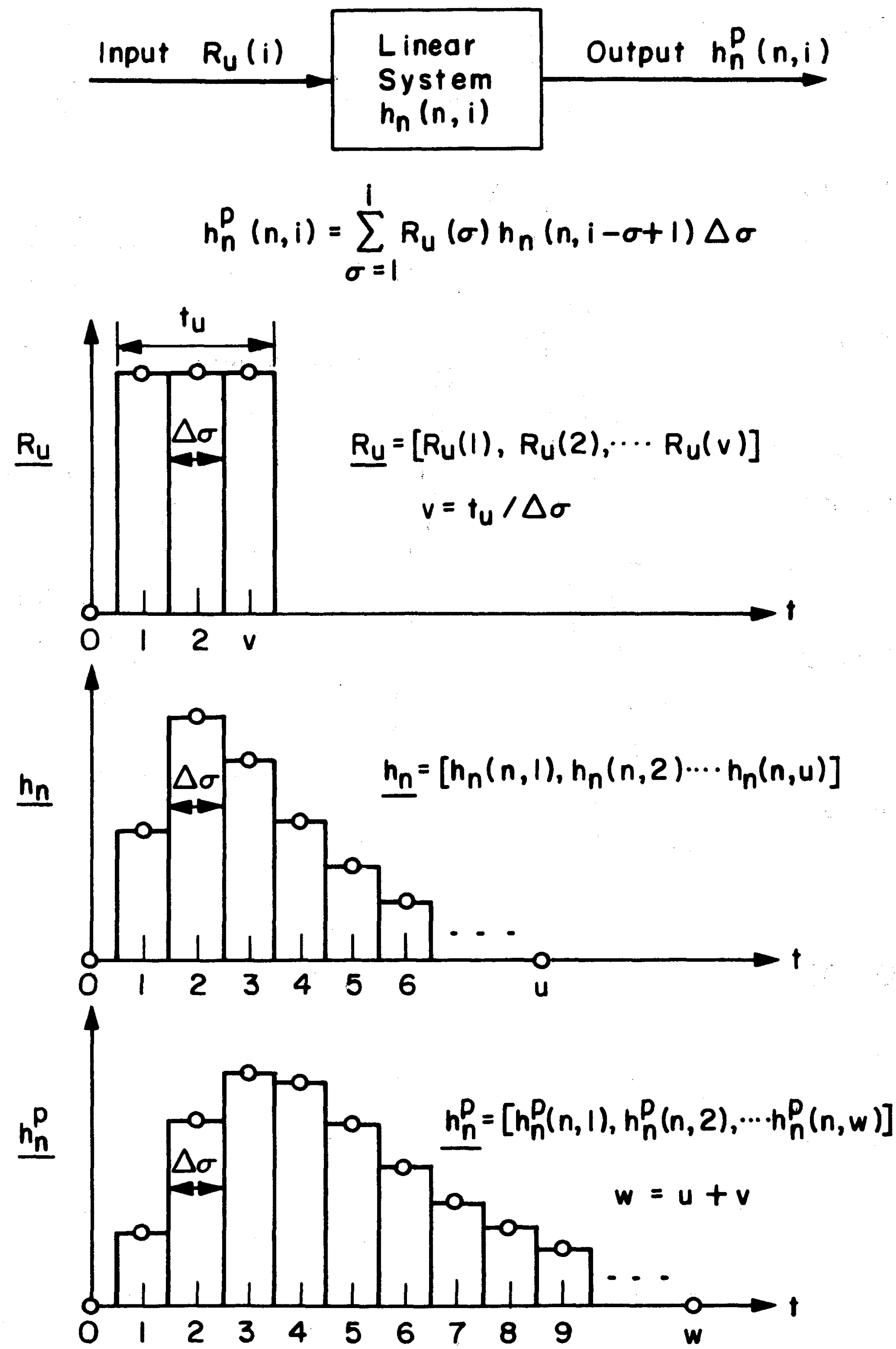


Figure 4-3 Pulse Response by Discrete Convolution

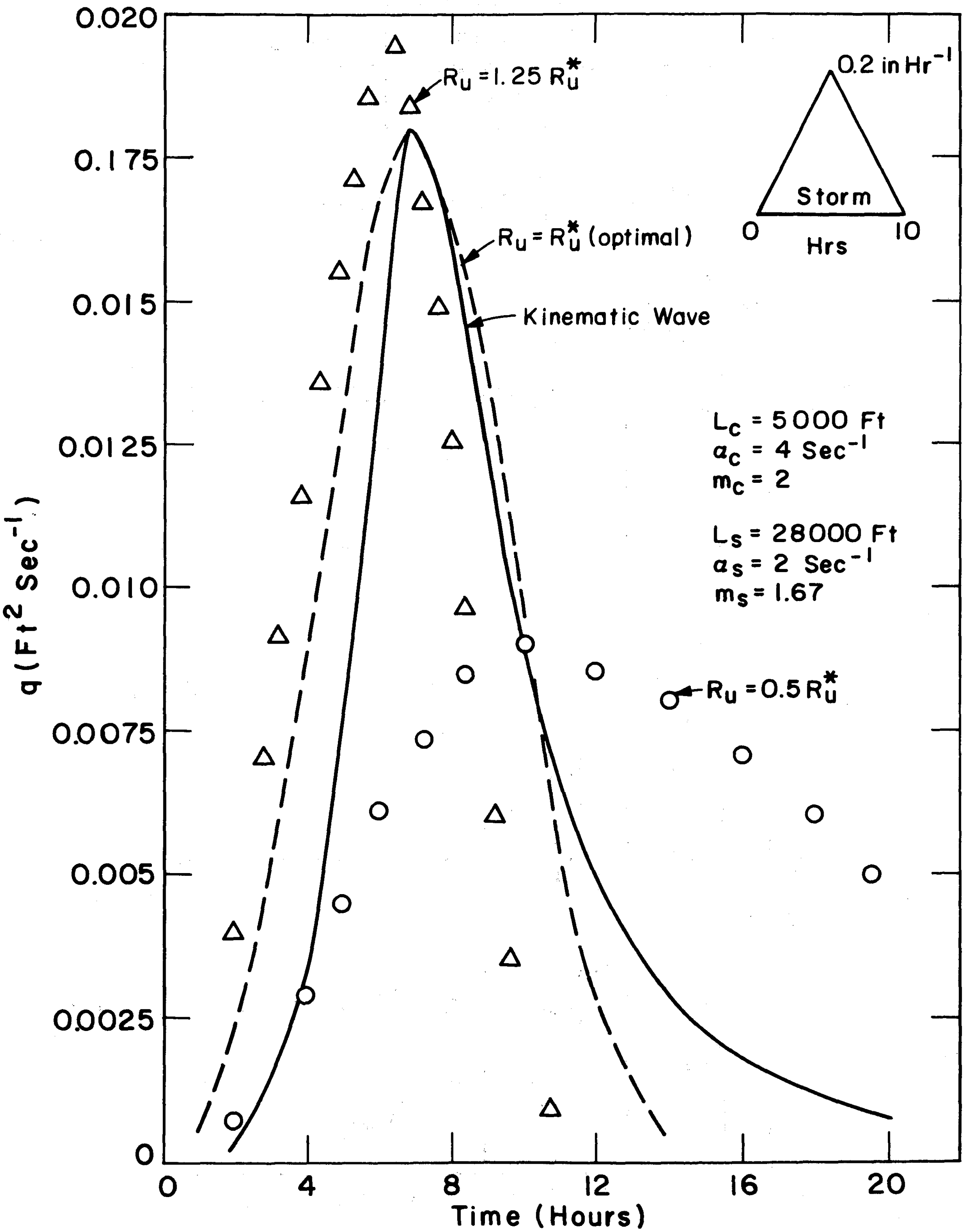


Figure 4-4 Effect of Pulse Height on the Predicted Overland Flow Hydrographs

$$\varepsilon^2 = \sum_{\ell=1}^{\infty} (Q_{kw}(\ell) - Q_o(\ell))^2 \Delta\ell \qquad (4\text{-}5)$$

where $Q_{kw}(\ell)$ is the output predicted by the kinematic wave. The derived impulse response functions of linear catchment models are actually input (storm) dependent. Thus the parameters of such a model can only be optimal if they too are input-dependent.

## 4-3. Selection of the Optimum Pulse Height

In developing an analytical procedure to determine that pulse which best represents the lag time and the peak discharge for a given catchment and a given storm input, the mechanism which transforms rainfall excess into overland flow is of primary importance. Thorough testing of the Distributed Linear Reservoir Model has established that the optimum pulse height for a given overland flow situation is essentially independent of stream parameters, i.e., it is invariant with the fitting parameter $\lambda$ (equation 3-18).

In the manner of Ishihara (21), we can use the kinematic wave theory to define an equivalent rectangular block of rain which will produce the same peak discharge as the actual (time-varying) rainfall function.

The discharge per unit width is obtained by integrating equation (3-10) along a characteristic beginning at $x = 0$ and substituting the result in equation (3-6)

$$q(t) = \alpha_c \left( \int_{t_o}^{t} i_e(t)dt \right)^{m_c} \qquad (4\text{-}6)$$

Let $i_{mp}$ be the average rainfall intensity contributing to the peak discharge and $t_p - t_o$ as its duration, where $t_p$ = time to peak discharge. Referring to Figure 4-5, we desire to find the time $t_o$ at which this square pulse begins. Substituting $i_{mp}(t-t_o)$ for the rainfall function in equation (3-11)

$$x - x_o = L_c = m_c \alpha_c \int_{t_o}^{t_p} \left( i_{mp}(t-t_o) \right)^{m_c - 1} dt \qquad (4\text{-}7)$$

and integrating

$$L_c = \alpha_c i_{mp}^{m_c - 1} (t_p - t_o)^{m_c} \qquad (4\text{-}8)$$

We cannot solve this directly for $t_o$ since $i_{mp}$ is a function of $t_o$. However, multiplying both sides of (4-8) by $i_{mp}$ we obtain the maximum flow condition cited in equation (3-4)

$$q_p = i_{mp} L_c = \alpha_c \left( i_{mp}(t_p - t_o) \right)^{m_c} \qquad (4\text{-}9)$$

Ishihara (21) has suggested that this depth of overland flow, $i_{mp}(t_p - t_o)$, is proportional to the depth of the pulse, $R_u \cdot t_u$, used

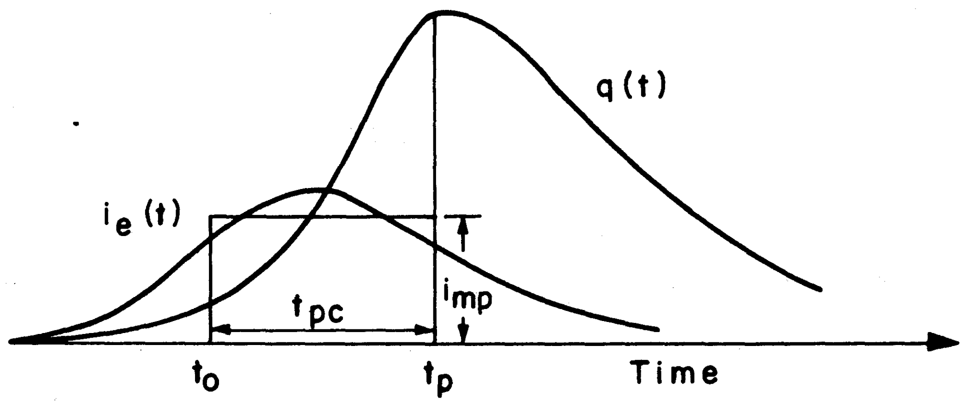


Figure 4-5 Relation Between the Rainfall Function and the Peak of the Hydrograph

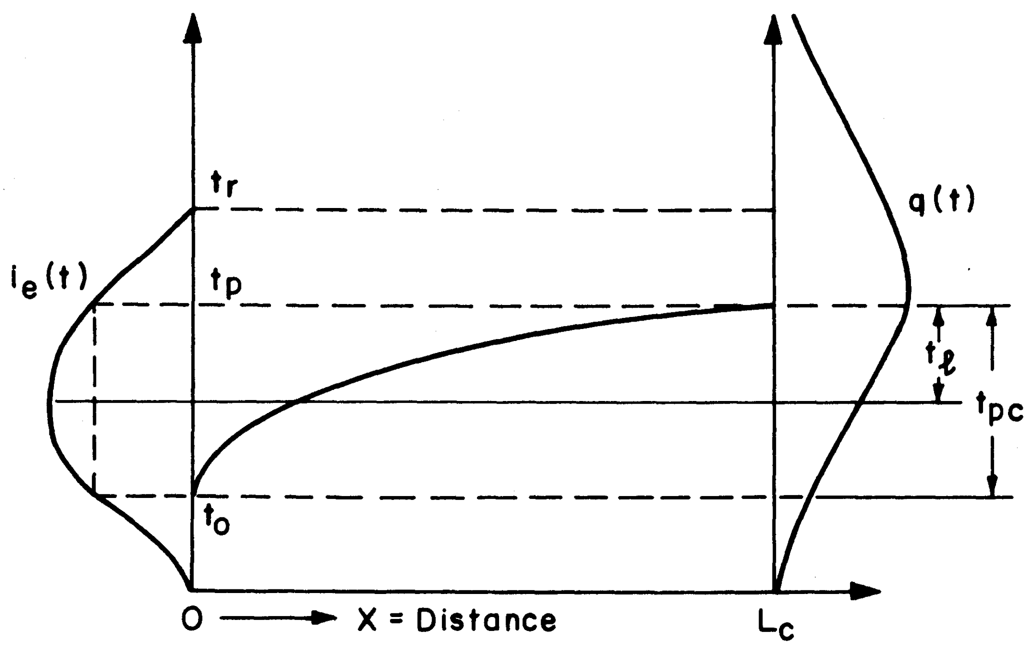


Figure 4-6 Schematic Diagram Showing the Lag Time and the Characteristic Curve

in the unit-hydrograph method. His equation for the pulse height is

$$R_u \cdot t_u = \left(\frac{1}{m_c}\right)^{1/(m_c-1)} i_{mp} t_{pc} \tag{4-10}$$

where $t_{pc} = (t_p - t_o)$. Given $i_{mp}$, $t_{pc}$ is merely the time of concentration as defined in equation (3-16). Ishihara has recommended that $t_{pc}$ can be taken as twice the lag time, $t_\ell$. These variables are depicted qualitatively in Figure 4-6.

It has been found that $i_{mp}$ can be approximated by the peak of the rainfall excess intensity, $i_e^P$, for overland flow situations of small $L_c$ and/or high values of $\alpha_c$ such that $L_c/\alpha_c < 500$. However, most overland flow situations correspond to the case of $L_c/\alpha_c > 500$ for which it has been found that $i_{mp}$ is better approximated by an average rainfall intensity defined by

$$i_e^* = \frac{1}{t_r} \int_o^{t_r} i_e(t)dt \tag{4-11}$$

Using $i_e^*$ in equation (3-16) we can define an average time of concentration, $t_c^*$

$$t_c^* = \left(\frac{L_c (i_e^*)^{1-m_c}}{\alpha_c}\right)^{m_c-1} \tag{4-12}$$

It has been found adequate to substitute $t_c^*$ for $t_{pc}$ and $i_e^*$ for $i_{mp}$ in equation (4-10) and combine these approximations and the coefficient involving $m_c$ into a constant, K

$$R_u \cdot t_u = K \cdot i_e^* \cdot t_c^* \tag{4-13}$$

The constant K was evaluated for the case of $m_c = 2$ by a linear regression analysis of the data presented in Table 1. The resulting equation for the optimum pulse height is given by

$$R_u = 1.59 \frac{i_e^* \cdot t_c^*}{t_u} \tag{4-14}$$

with a correlation coefficient of 0.96.

Table 1 compares the observed and predicted optimum pulse height for eleven catchments subjected to a variety of input conditions. It is evident that equation (4-14) is preferable to equation (4-10) for at least two reasons:

1. An a priori determination of $i_{mp}$ and $t_{pc}$ is not possible since these values must be extracted from observed data. Thus the model parameters could not be input-dependent if equation (4-10) were used.
2. Equation (4-10) was found to be inconsistent with the wide range of catchment parameters and input conditions from which equation (4-14) was derived.

Table 4-1 Prediction of the Optimum Pulse Height from Equations (4-14) and (4-10)

| $L_c/\alpha_c$ ft•sec | $i_e^P$ in/hr | $i_{mp}$ in/hr | $i_e^*$ in/hr | $t_{pc}$ hr | $t_c^*$ hr | $t_u$ hr | $R_u$ obs. | $R_u$ (4-14) | $R_u$ (4-10) |
|---|---|---|---|---|---|---|---|---|---|
| 100 | .10 | .095 | .05 | 1.2 | 2.57 | .20 | 1.0 | 1.1 | .2 |
| 100 | .20 | .19 | .10 | .8 | 1.84 | .20 | 1.4 | 1.5 | .3 |
| 100 | .35 | .34 | .175 | .6 | 1.38 | .20 | 1.8 | 1.9 | .2 |
| 100 | .50 | .49 | .25 | .5 | 1.15 | .20 | 2.1 | 2.3 | .5 |
| 100 | .70 | .67 | .35 | .4 | .99 | .20 | 2.6 | 2.8 | .5 |
| 100 | .40 | .37 | .20 | .6 | 1.29 | .10 | 4.3 | 4.1 | .8 |
| 100 | 1.0 | .93 | .5 | .4 | .82 | .10 | 5.0 | 6.5 | 1.5 |
| 100 | .15 | .13 | .075 | 1.0 | 2.22 | .30 | .8 | .9 | .2 |
| 100 | .35 | .33 | .165 | .6 | 1.41 | .30 | 1.1 | 1.2 | .3 |
| 100 | .10 | .095 | .05 | 1.0 | 2.56 | .40 | .5 | .5 | .1 |
| 100 | .25 | .235 | .125 | .6 | 1.63 | .40 | .8 | .8 | .1 |
| 150 | .20 | .19 | .10 | 1.0 | 2.25 | .20 | 1.9 | 1.8 | .4 |
| 150 | .50 | .49 | .25 | .6 | 1.42 | .20 | 3.0 | 2.8 | .6 |
| 150 | .15 | .13 | .075 | 1.2 | 2.72 | .30 | 1.1 | 1.1 | .2 |
| 150 | .33 | .30 | .165 | .7 | 1.73 | .30 | 1.7 | 1.5 | .3 |
| 200 | .10 | .09 | .05 | 1.6 | 3.65 | .20 | 1.5 | 1.4 | .3 |
| 200 | .20 | .19 | .10 | 2.4 | 2.58 | .20 | 2.5 | 2.0 | .9 |
| 200 | .40 | .37 | .20 | .8 | 1.83 | .20 | 3.4 | 2.9 | .6 |

Table 4-1 (continued)

| $L_c/\alpha_c$ ft•sec | $i_e^P$ in/hr | $i_{mp}$ in/hr | $i_e^*$ in/hr | $t_{pc}$ hr | $t_c^{**}$ hr | $t_u$ hr | $R_u$ obs. | $R_u$ (4-14) | $R_u$ (4-10) |
|---|---|---|---|---|---|---|---|---|---|
| 270 | .10 | .095 | .05 | 2.0 | 4.21 | .20 | 1.9 | 1.7 | .4 |
| 270 | .20 | .19 | .10 | 1.4 | 3.05 | .20 | 2.6 | 2.4 | .5 |
| 270 | .35 | .34 | .175 | 1.0 | 2.25 | .20 | 3.2 | 3.1 | .7 |
| 270 | .50 | .49 | .25 | .8 | 1.88 | .20 | 4.0 | 3.8 | .8 |
| 270 | .15 | .13 | .075 | 1.6 | 3.52 | .30 | 1.3 | 1.4 | .3 |
| 270 | .33 | .30 | .165 | 1.0 | 2.31 | .30 | 2.1 | 2.1 | .4 |
| 333 | .10 | .09 | .05 | 2.4 | 4.71 | .20 | 2.5 | 1.9 | .4 |
| 333 | .10 | .09 | .05 | 1.6 | 4.71 | .20 | 2.1 | 1.9 | .3 |
| 333 | .20 | .19 | .10 | 1.6 | 3.33 | .20 | 2.9 | 2.6 | .6 |
| 600 | .10 | .08 | .05 | 3.6 | 6.32 | .20 | 2.2 | 2.5 | .6 |
| 1250 | .20 | .16 | .10 | 3.6 | 6.45 | .20 | 5.0 | 5.1 | 1.2 |
| 1750 | .20 | .14 | .10 | 6.0 | 7.64 | .20 | 6.3 | 6.1 | 1.7 |
| 2500 | .10 | .05 | .05 | 10.8 | 12.9 | .20 | 4.5 | 5.1 | 1.1 |
| 2500 | .20 | .115 | .10 | 8.2 | 9.13 | .20 | 6.5 | 7.3 | 2.0 |
| 2500 | .40 | .30 | .20 | 3.8 | 6.45 | .20 | 5.5 | 5.0 | 2.3 |
| 2800 | .20 | .11 | .10 | 9.2 | 9.66 | .20 | 7.5 | 7.7 | 2.1 |
| 4700 | .10 | .07 | .05 | 16.4 | 17.64 | .40 | 3.0 | 3.5 | 1.2 |

The effect of storm duration on the error in prediction (Figure 4-7) indicates that use of an optimum pulse height in equation (4-4) is particularly important for areas such as urban catchments on which short duration storms are critical.

4-4. Selection of the Optimum Pulse Length

The convolution process that has been detailed earlier is performed over discrete time intervals with the assumption that the rainfall excess is uniform over these intervals. Such a temporal lumping of the input will cause an error in the forecast hydrograph. The pulse length should be chosen such that this error is negligible. On the other hand, in the interests of computational economy, the pulse length should not be overly small. The optimum pulse length is thus derived from theoretical and economic considerations.

Experience with the unit-hydrograph method has led to arbitrary or empirical estimates for the best unit-time or pulse length. One such determination (22) involves the lag time, $t_\ell$

$$t_u = 1/4\ t_\ell \qquad (4\text{-}15)$$

Referring to Figure 4-2, it is obvious that as the pulse length decreases the area under the synthesized rainfall function will converge to the actual depth of the storm. For the triangular shaped storms considered, the following errors as a function of pulse length or samples per storm were observed:

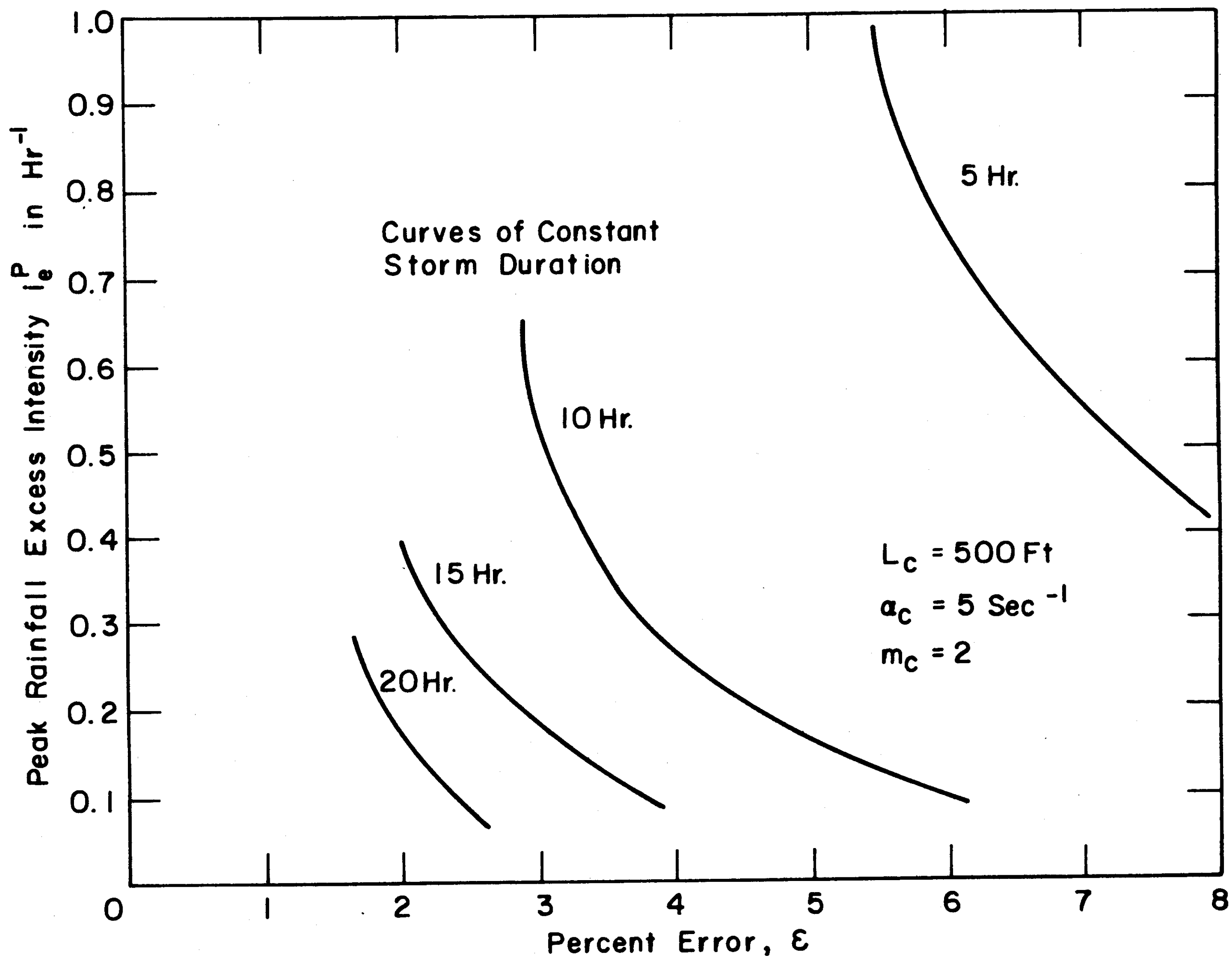


Figure 4-7 Integral Square Error Between Observed and Predicted Overland Flow Hydrographs

Table 4-2 Percent Error Due to Using a Discrete Representation of the Actual Storm

| Samples Per Storm | Actual Depth in. | Predicted Depth in. | Percent Error |
|---|---|---|---|
| 200 | 1.0 | .9991 | 0.09 |
| 100 | 1.0 | .9966 | 0.34 |
| 67 | 1.0 | .9912 | 0.88 |
| 50 | 1.0 | .9867 | 1.33 |
| 25 | 1.0 | .9450 | 5.5 |

From these results, the pulse length that is recommended for use with the Distributed Linear Reservoir Model is

$$t_u = t_r/50 \qquad (4\text{-}16)$$

Typical pulse lengths are given in Table 1. Their magnitude precludes the use of hand computation with this quasi-linear unit-hydrograph method. Note that the pulse response function $H_N^P$ must actually be defined at a finer sample spacing, $\Delta\sigma$, in order to perform the convolution required in equation (4-3).

# Chapter 5

# PARAMETER OPTIMIZATION

## 5-1. Introduction

The dynamics of how the catchment transforms rainfall excess into streamflow is represented here by a time-invariant linear system, equation (2-2). The system is characterized by its impulse response function. The conceptual model of Figure 2-6 and equation (2-36) has been proposed to represent this function. The objective of this chapter is to identify the key parameters of the model and to detail a procedure for determining their optimum values.

## 5-2. Catchment-Stream Application of the Distributed Linear Reservoir Model

The necessity for characterizing overland flow and streamflow by separate sets of equations with different values for the key parameters ($\alpha$, m, L) was detailed in Chapter 3. Without a distributed model, such as the one under consideration, these distinct catchment mechanisms must be lumped, typically by passing the input through a single cascade of linear reservoirs.

The Distributed Linear Reservoir Model is a three parameter model: the number of linear elements, N; the reservoir storage constant, k; the channel time delay, $\tau$. Thus, in recognition of catchment behavior, there should be individually parameterized models for overland flow and for streamflow.

Let the flow in a stream be simulated by a Distributed Linear Reservoir Model. The associated lateral inputs, $Q_{ij}$, are then the outputs from overland flow simulations by means of other Distributed Linear Reservoir Models. The input to the overland flow models is the rainfall excess itself.

It is convenient then to physically describe the catchment with the geometric model depicted in Figure 3-2. A more complex geometry can be envisioned for use with the Distributed Linear Reservoir Model, but this would necessarily increase the computational effort. Employment of this geometric model allows ready comparison with the characteristics solution of the kinematic wave equation.

The selected catchment-stream application of the Distributed Linear Reservoir Model is shown qualitatively for an arbitrary catchment in Figure 5-1. For simplicity, each overland flow model will have the same number of linear elements, denoted by $N_c$, each with the constants $k_c$ and $\tau_c$. The stream model parameters are similarly denoted $N_s$, $k_s$, $\tau_s$. Note that the area over which the input rainfall excess is lumped is only $(L_c/N_c)\cdot(L_s/N_s)$.

## 5-3. Catchment-Stream Simulation

It is evident from Figure 5-1 that, for the general case of an areally variable rainfall excess, each of the lateral inputs into the catchment model and thence into the stream model may be different functions of time. The equations developed in section 4-2 must therefore be generalized.

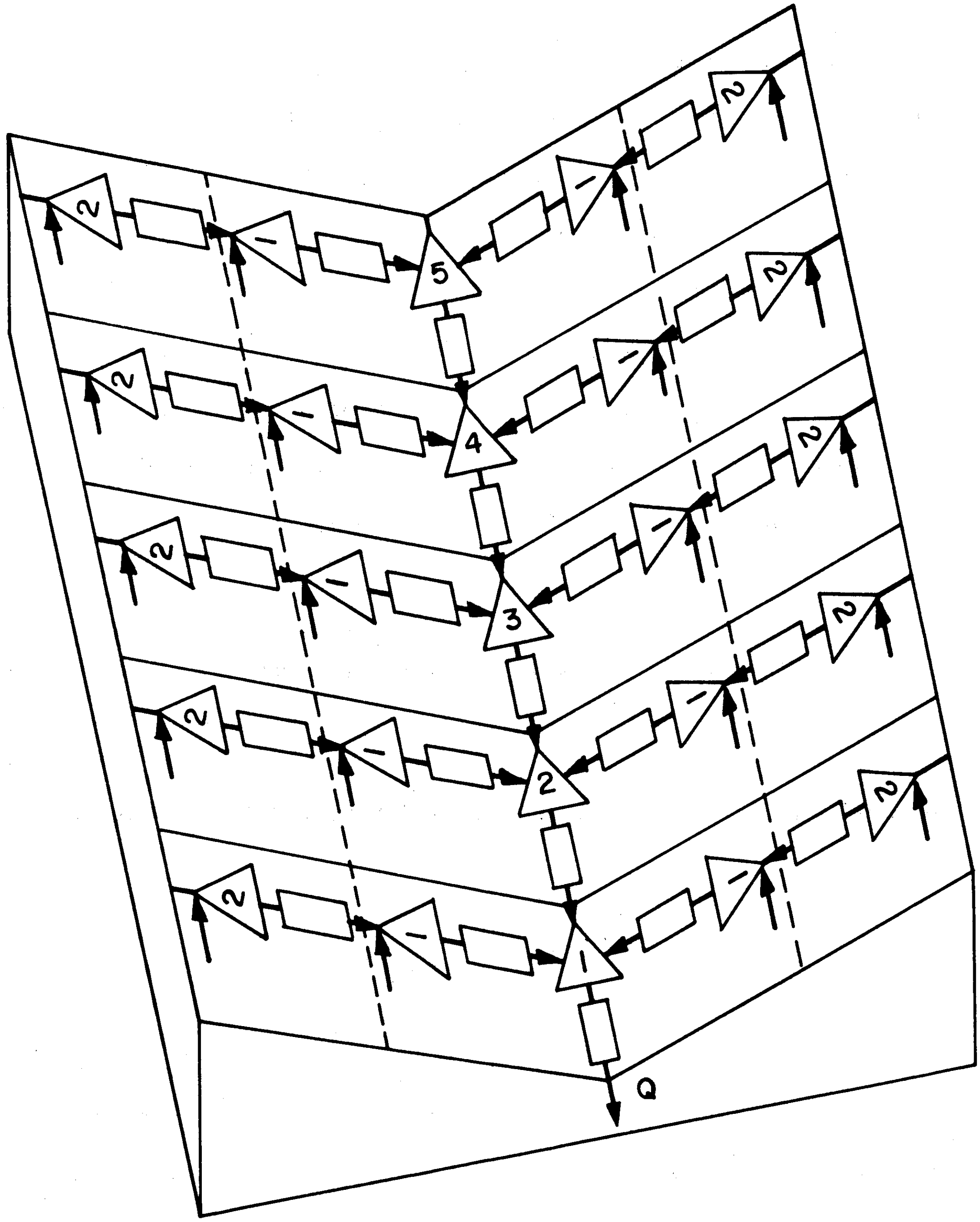


Figure 5-1 Physical Application of the Distributed Linear Reservoir Model

Catchment Simulation -- It is necessary to perform the convolution with the components, $h_j^P$ of the pulse response function, $H_N^P$. To accomplish this, an average storm, $Q_{ic}$, must be defined for each overland flow model[†]. Having determined the pulse height from equation (4-14) and the pulse length from equation (4-16), the pulse response function is then given by equation (4-3). The $j^{th}$ component of the outflow, $Q_{ocj}$, is obtained by convolution of $h_j^P$ (equation 4-2) with $K_j Q_{ic}$, which is the average storm scaled by the factor, $K_j$, representing the relative spatial variability of the actual storm. That is

$$Q_{ocj}(\ell) = \sum_{\sigma=1}^{\ell} (K_j Q_{ic}(\sigma)/R_u) h_j^P(j,\ell-\sigma+1)\Delta\sigma \qquad (5\text{-}1)$$

The total outflow from a given overland flow model, $Q_{oc}$, is thus

$$Q_{oc}(\ell) = \sum_{j=1}^{N_c} Q_{ocj}(\ell) \qquad (5\text{-}2)$$

---

[†]Depending upon the spatial variability of the storm and the sensitivity of the catchment to such variations, it may be adequate to use an average storm occurring on the entire catchment for this purpose rather than one for each chain shown in Figure 5-1.

Streamflow Simulation -- The output from the Stream Linear Reservoir Model is given by convolving the streamflow model IUH with the overland flow hydrograph. Once again it is necessary to perform one convolution per component, $h_j$ (equation 2-35). Noting that there are actually two lateral inputs to each stream element (one from each side of the catchment), which may be different, the lateral input to the stream is thus

$$Q_{isj}(\ell) = \sum_{r=1}^{2} Q_{ocr}(\ell) \tag{5-3}$$

and the $j^{th}$ component of the outflow from the stream model is

$$Q_{osj}(\ell) = \sum_{\sigma=1}^{\ell} Q_{isj}(\sigma) h_j(j, \ell-\sigma+1) \Delta\sigma \tag{5-4}$$

the total outflow, $Q_o$ , is thus

$$Q_o(\ell) = \sum_{j=1}^{N_s} Q_{osj}(\ell) \tag{5-5}$$

For the case of distributed (unequal) inputs, it has been found convenient to let the IUH for both the overland flow and the streamflow model be generated from linear reservoir cascade responses only

(Figure 2-4, equation 2-34) and to account for the lag effect of the linear channels in the final summations of equations (5-2) and (5-5). To accomplish this, the $Q_{ocj}$ and $Q_{osj}$ are lagged by $j \cdot \tau_c$ and $j \cdot \tau_s$ respectively.

It would be well to illustrate this process by a figure. Consider the case where the spatial variability is such that the same storm can apply to each element of a given overland flow model (i.e., Figure 5-1). Thus, there is no need to perform the convolution for the overland flow model separately for each $h_j^P$ and equation (4-4) applies. Letting the asterisk denote the convolution operation, the total simulation for selected overland flow models on one half of the catchment is depicted in Figure 5-2.

## 5-4. The Method of Moments

Use of the Distributed Linear Reservoir Model requires an a priori determination of six parameters. A frequently employed technique in similar situations is the method of moments by which the system function or IUH can be found directly from the moments of the measured input and output time series.

The method of moments has definite disadvantages which in certain cases may become unacceptable.

1. The method of moments is applicable to linear systems. It thus forces the non-linearity of the catchment into the higher moments. It has been found (6) that negative parameters can arise due to the derived moments being inconsistent with the assumed model.

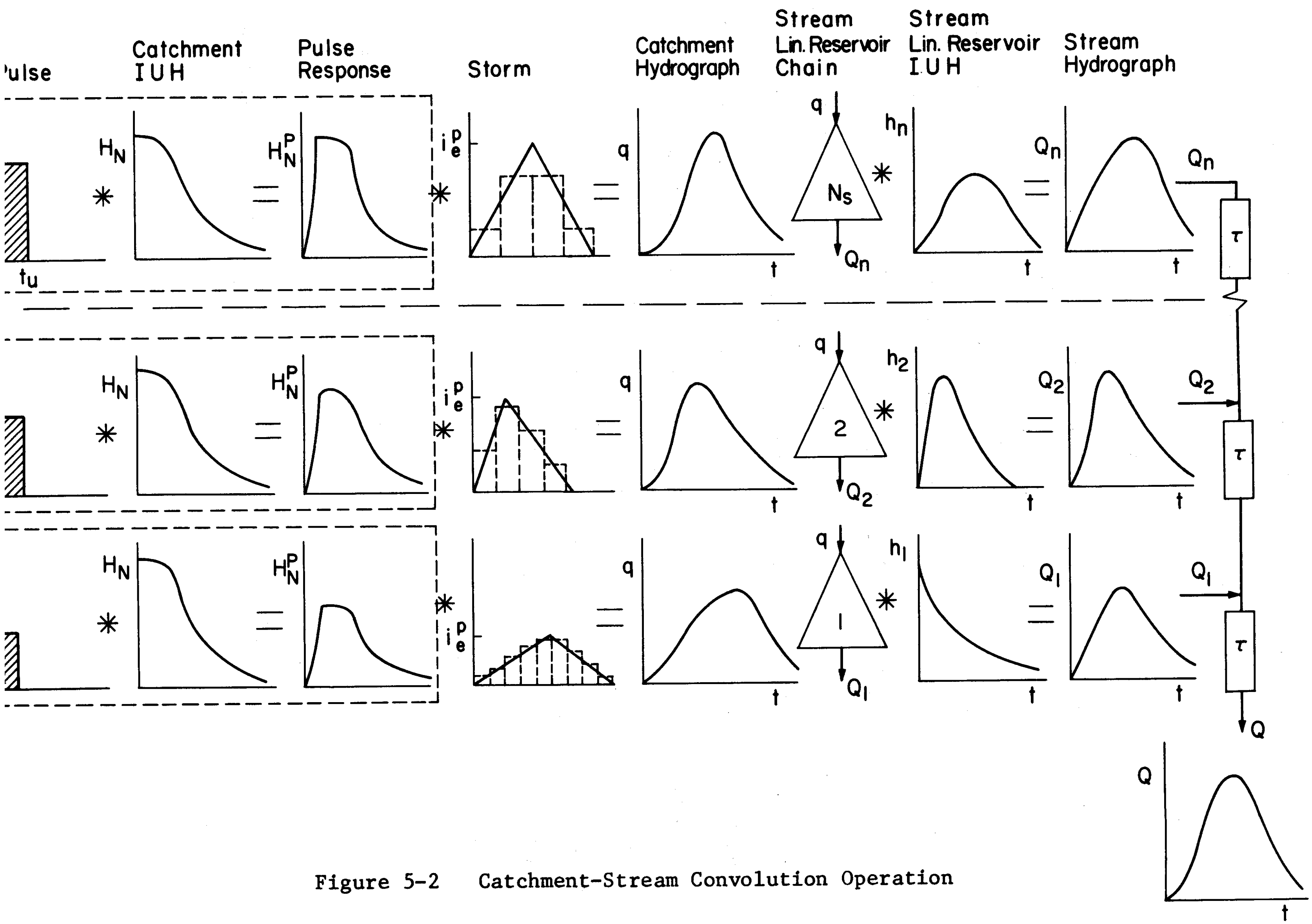


Figure 5-2 Catchment-Stream Convolution Operation

2. Most fitting methods are biased. The method of moments has been observed (2) to give more significance to the extremities of the hydrograph, consequently producing more error near the peak than at the extremities. The time of arrival and magnitude of the peak discharge are often the most important characteristics of the hydrograph.

Due to these difficulties in applying the method of moments, an entirely different and rather unique fitting method was developed.

5-5. Selection of Model Parameters

The objective is to determine an unbiased, completely general method of establishing the best model parameters. To incorporate the results of Chapter 4, the method must be input and catchment dependent.

Consider the impulse response function of the Distributed Linear Reservoir Model (Figure 2-7). The close resemblance to the impulse response function of the kinematic wave (Figure 3-3) was indicated in Chapter 3. From the definition of $H_N(n,t)$ (equation 2-36), the peak is located at $t = \tau$ and has magnitude

$$H_N(n,\tau) = \frac{1}{Nk} \qquad (5\text{-}6)$$

Convolving $H_N(n,t)$ with a pulse of depth $d = R_u t_u$ (ft.) occurring on a catchment of length $L_c$ (ft.), the peak becomes

$$H_N^P(n,\tau) = \frac{V_o}{Nk} \tag{5-7}$$

where the pulse volume, $V_o = d \cdot L_c$. This will be equated to the peak of the kinematic wave pulse response function (equation 3-29)

$$\frac{V_o}{Nk} = q_p = \alpha_c d^{m_c} \tag{5-8}$$

or

$$Nk = d^{1-m_c} L_c/\alpha_c \tag{5-9}$$

where the units (time) of $\alpha_c$ and k are the same.

The effect of the time delay, $\tau$, is to delay the origins of the components $h_j$ of $H_N(n,t)$ by $j \cdot \tau$ in time. This then produces an oscillation of the pulse response function for time $t < N\tau$. The flat portion of the function shown in Figure 2-7 thus occurs at time $t > N\tau$. The following modification to equation (5-9) was found to give excellent agreement with the kinematic wave pulse response function

$$N(k+\tau) = d^{1-m_c} L_c/\alpha_c \tag{5-10}$$

specializing this result to the catchment model

$$N_c(k_c+\tau_c) = d^{1-m_c}L_c/\alpha_c \tag{5-11}$$

Additional constraints on the value of the model parameters will adjust the particular shape of the response function as is shown qualitatively in Figure 2-7.

One approach to obtain additional equations for the model parameters would be to substitute equation (4-4) into equation (4-5), differentiate the result with respect to two of the unknown parameters and set the expressions equal to zero. Simultaneous solution of the two equations would yield the required additional conditions. However, this procedure yields unmanageable equations. Certainly a simpler method is desirable.

Through use of the pulse height and pulse length, equation (5-11) constrains the model parameters to be input-dependent. Two further relations will be presented which depend upon the physical characteristics of the catchment.

By generating hydrographs and comparing them with the kinematic wave, it became apparent that

1. The accuracy of the Distributed Linear Reservoir Model is relatively insensitive to values of N provided that k and $\tau$ are adjusted in accordance with equation (5-11).
2. The computational effort required to apply the model is inversely proportional to N.

A good fit is obtained with reasonable computational effort, for both catchment and stream models, when (as a minimum)

$$N = 2(\log_{10}L-2) + 1.0 \qquad (5\text{-}12)$$

A comparison can be made with a similar equation presented by Nash (25) as derived through application of his single cascade model to natural catchments. The extent of the agreement is shown in Figure 5-3.

Catchment Model -- The basic linear element consists of a linear reservoir and a linear channel. These components simulate the storage and the translation effect of overland flow. The form of an equation expressing the relative significance of each as a function of catchment parameters follows from consideration of the shallow water wave speed equation (3-9). For a constant unit depth

$$c = \alpha_c m_c \qquad (5\text{-}13)$$

The nature of the responses of a linear channel and a linear reservoir suggest that an equation expressing $\tau_c/k_c$ as a function of c possess the following characteristics

$$\left\{\begin{array}{ll} \text{as } \alpha_c m_c \rightarrow 0, & \text{storage predominates and } \tau_c \text{ is negligible} \\ \text{as } \alpha_c m_c \rightarrow \infty, & \text{translation predominates and } k_c \text{ is negligible} \end{array}\right\} \qquad (5\text{-}14)$$

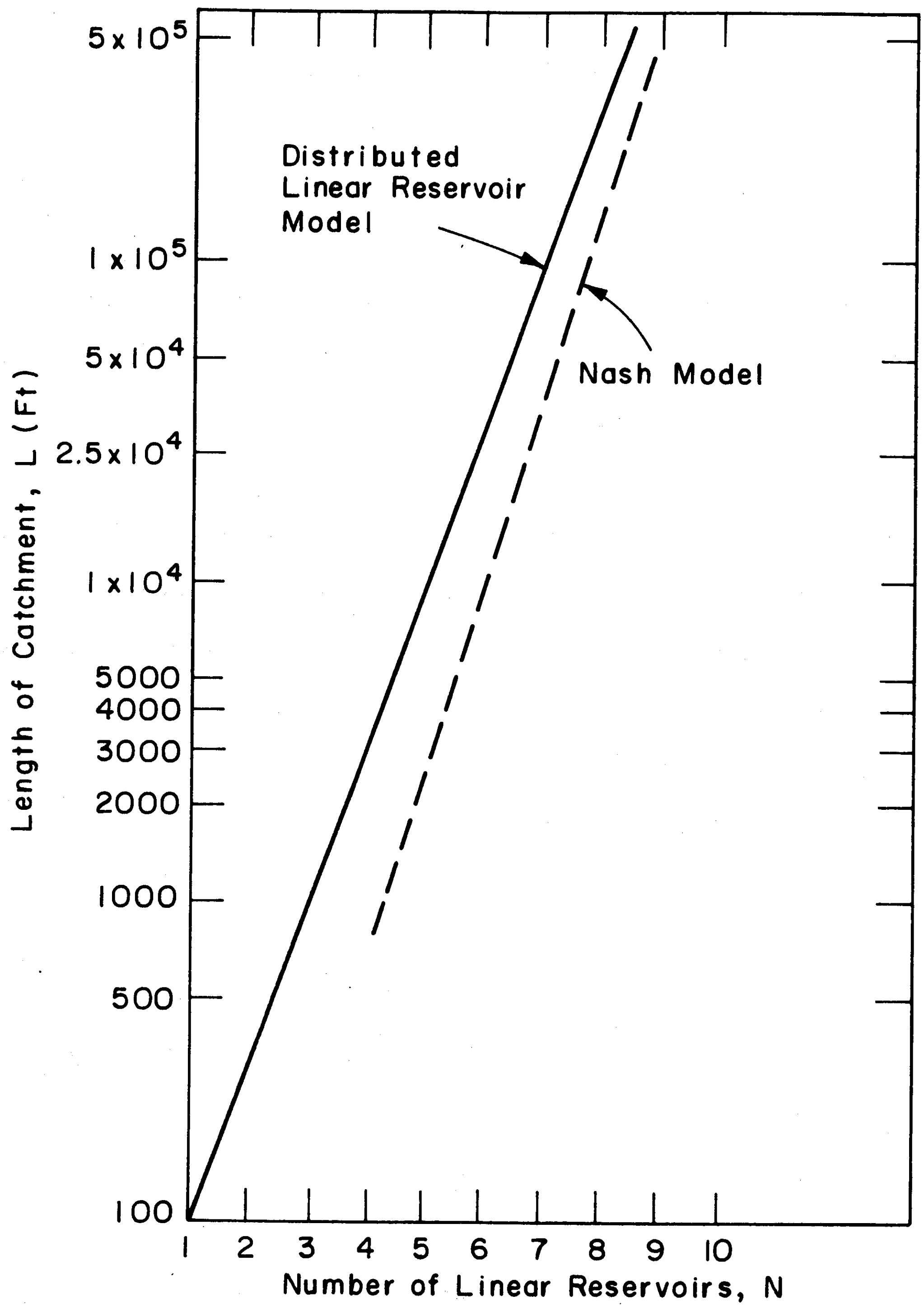


Figure 5-3 Relationship Between the Number of Linear Elements and the Length of the Catchment or Stream

Therefore, $\tau_c/k_c$ should be a monotonically increasing function of $m_c\alpha_c$. Using equations (5-11) and (5-12), the following relationship was established by minimizing the integral square error between the Distributed Linear Reservoir Model and the kinematic wave solution

$$\tau_c/k_c = .5\times10^{(m_c\alpha_c/25)} \tag{5-15}$$

This equation was derived from simulating the overland flow situations plotted in Figure 5-4. Equations (5-11), (5-12) and (5-15) completely specify $N_c$, $k_c$ and $\tau_c$ independently of the observed output.

Stream Model -- The form of the coupling between overland flow and streamflow suggests that a simple linear transformation between catchment model parameters and stream model parameters may be adequate. In section 3-5, the relative dynamic importance of streamflow with respect to overland flow was quantified by $\lambda = \frac{t_s}{t_c}$, the ratio of the time of concentrations (equation 3-18).

Since equation (5-12) also applies to $N_s$ where L becomes the length of the stream, $L_s$, only $k_s$ and $\tau_s$ remain to be specified. Letting $\lambda$ be a measure of the ratio of the total lags imposed by the respective models, we obtain

$$\frac{N_s(k_s+\tau_s)}{N_c(k_c+\tau_c)} = \lambda \tag{5-16}$$

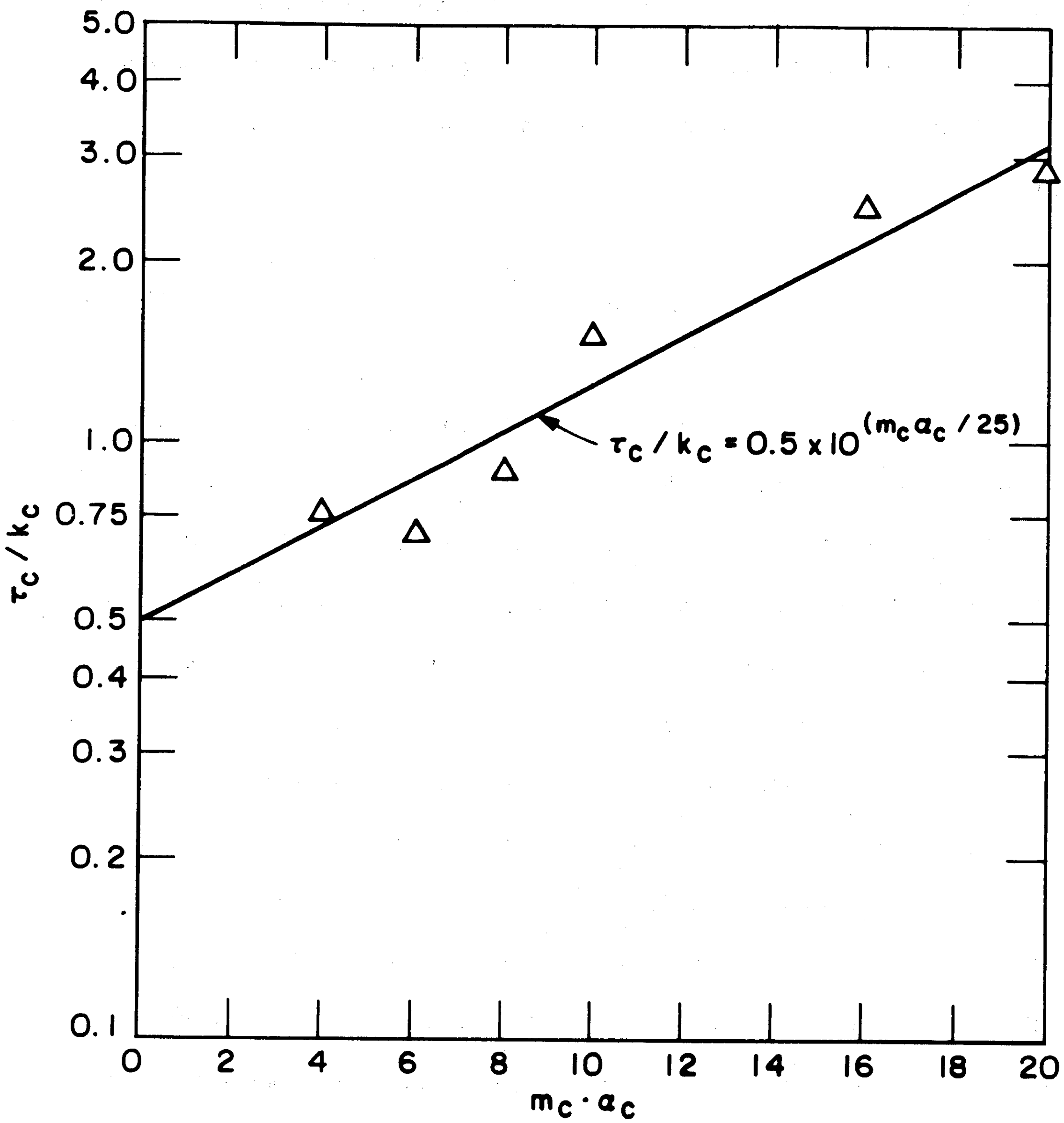


Figure 5-4 Relationship Between Model Time Constants $\tau_c/k_c$ and Physical Parameters $\alpha_c \cdot m_c$

The minimum integral square error formulation for the streamflow hydrograph established the result that $\lambda$ also applied separately to the relative time delays, $\tau$. The final equation is therefore

$$\tau_s/\tau_c = \lambda \qquad (5\text{-}17)$$

Thus, equations (5-12), (5-16) and (5-17) complete the parameter optimization for the Distributed Linear Reservoir Model.

# Chapter 6

# NUMERICAL RESULTS

## 6-1. Introduction

The Distributed Linear Reservoir Model has been developed by using the kinematic wave solution as a standard. The model can now be used to simulate the catchment response to any desired input. The input rainfall excess function specifies the pulse height and pulse length which then determine the optimal values of the model parameters from the relations of Chapter 5.

Evaluation of the performance of a mathematical model can be accomplished only after relevant measures of effectiveness have been defined. For this purpose, the following criteria will be employed:

1. Accuracy
2. Computational efficiency (speed)
3. Simplicity
4. Flexibility

## 6-2. Accuracy of the Model

Results using the Distributed Linear Reservoir Model will be compared to those obtained from a Nash model and from the kinematic wave solution.

The Nash model (equation 2-29) is a lumped-linear representation of the catchment behavior. Since all input passes through the entire system, this model is not as physically realistic as a model with distributed inputs. The Nash model was fitted to the streamflow hydrograph as predicted by the kinematic wave using the method of moments (7). When the Nash model was used to re-predict the output from which it was derived, it was found to give essentially the same integral square error as the Distributed Linear Reservoir Model but not as accurate a determination of the peak discharge and time to the peak, see Figure 6-1a. However, this is not a fair comparison. To be useful, a mathematical model must accurately represent situations other than that from which the model parameters were established. Figure 6-1b shows that the Nash model exhibits considerably more error when the peak intensity of the storm is doubled. This error is due to the non-linearity of the catchment response. The Distributed Linear Reservoir Model performs considerably better for this situation since its parameters are input-dependent. The pulse height for case (a) was 7.0 in/hr and the pulse length was 0.20 hr. Case (b) then corresponded to a pulse height of 10.0 in/hr and a pulse length of 0.20 hr.

The Distributed Linear Reservoir Model has been compared to the kinematic wave solution for a number of catchments under a variety of input conditions. The storms considered were of triangular shape with the peak intensity occurring at $t_r/2$. As was shown in Figure 4-7, the accuracy of the model is maximal for storms of low peak intensity and long duration. This effect is illustrated for catchments of similar

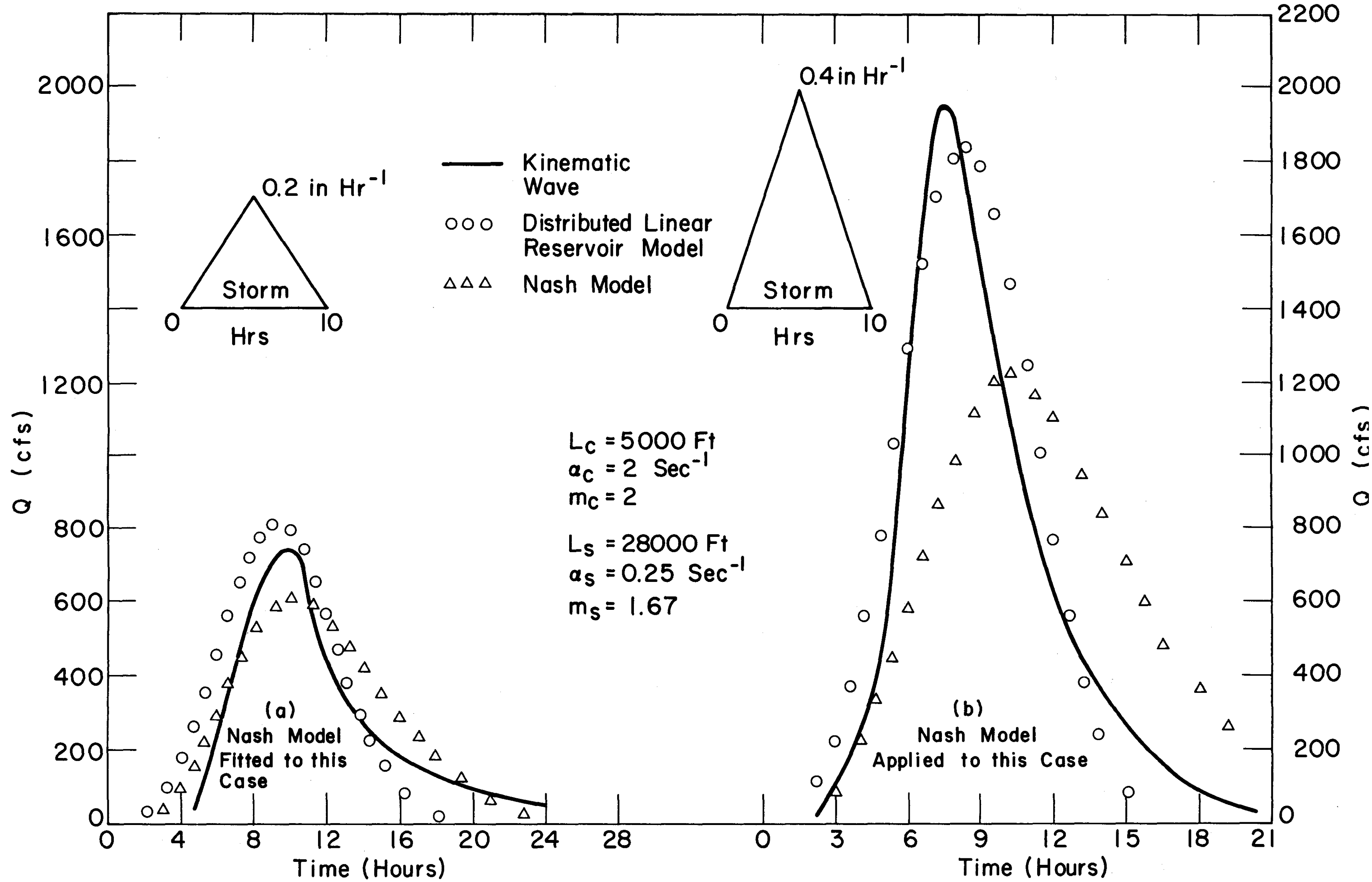


Figure 6-1 Errors Due to Linearization

lengths but different $\alpha$'s in Figures 6-2 and 6-3. The relative values of $k_c$, $\tau_c$ and $k_s$, $\tau_s$ are typical of those encountered in application of the method.

## 6-3. Computational Experience

The Distributed Linear Reservoir Model is necessarily a digital computer model. The computer program was developed for the IBM 360/65. Average running time for a spatially uniform input is 1 1/3 minutes, independent of the size of the catchment. The kinematic wave solution, with which the model was compared, was obtained by the method of characteristics. It was an early version of a model developed at M.I.T. and failed to rigorously satisfy continuity by from 5 to 15%. This disparity can be observed in the comparison figures presented in this chapter. The computational speed for the models responding to spatially uniform input was essentially the same.

The computational effort for the Distributed Linear Reservoir Model is directly proportional to the time step used in the computations. In Chapter 4 it was noted that the time step, $\Delta\sigma$, must be less than the pulse length, $t_u$, for an accurate discrete representation of the pulse function $R_u(\sigma)$. Accuracy here is measured in terms of continuity. The time step was considered adequate if the integrated area under the curve was within 95% of the theoretical value. The components of the impulse response function, $h_j$, were found to be very sensitive to the time step used. Obviously, as the linear reservoir constant k decreases, the time step must also decrease. In general, these

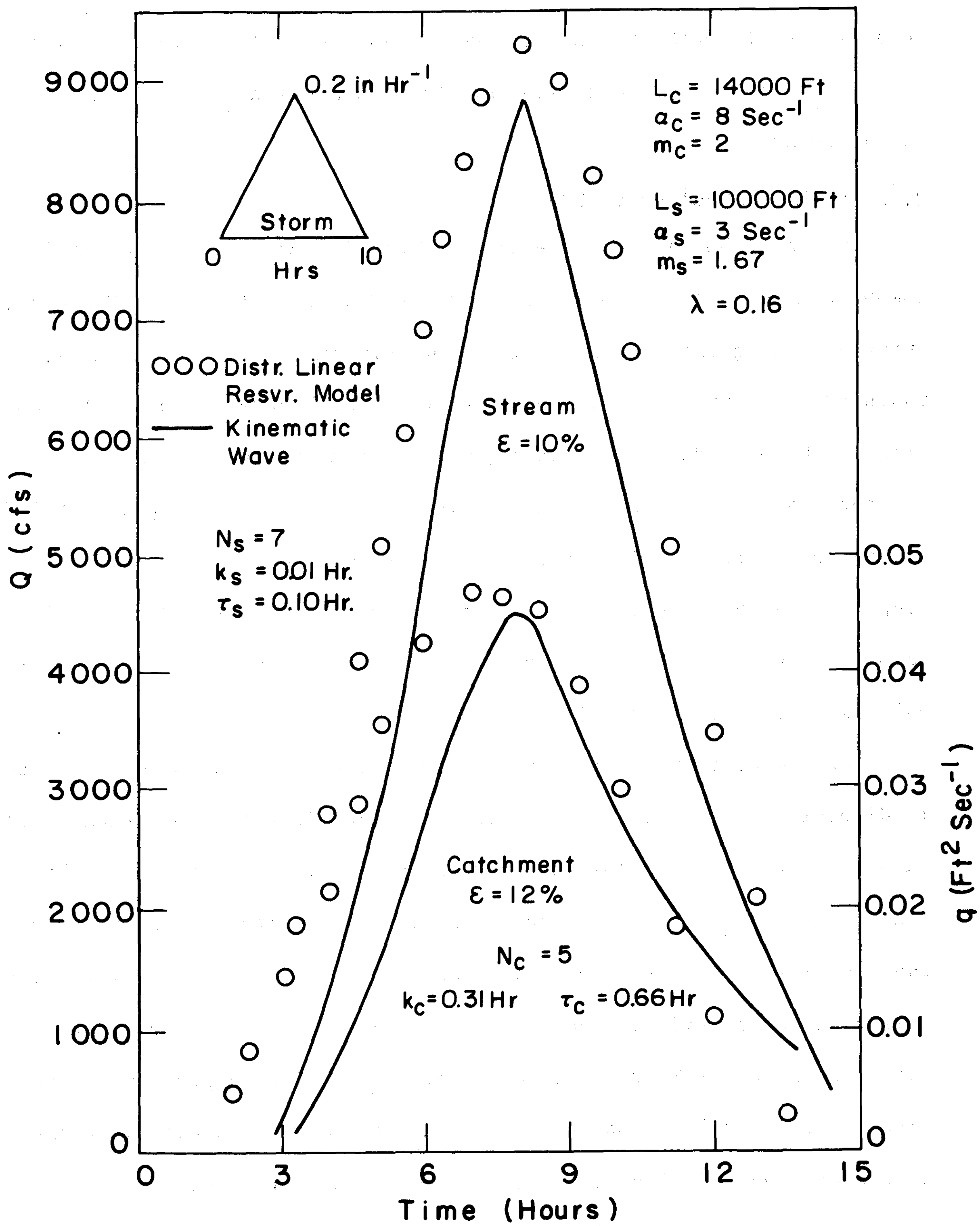


Figure 6-2 Typical Agreement Between the Kinematic Wave Solution and the Distributed Linear Reservoir Model

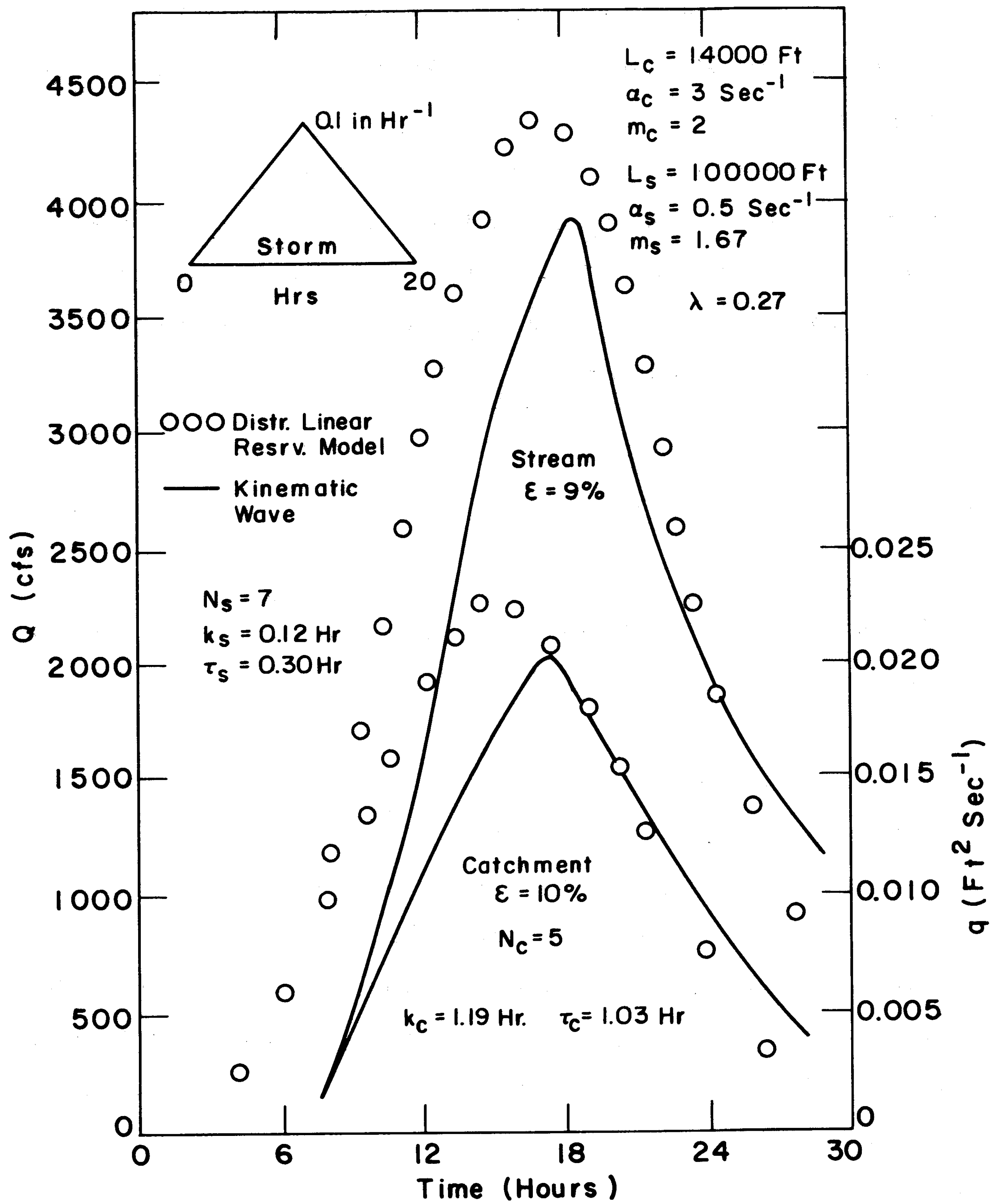


Figure 6-3 Typical Agreement Between the Kinematic Wave Solution and the Distributed Linear Reservoir Model

functions were generated at a time interval of $\Delta\sigma = 0.04$ hr. This provided a good definition of the time lag $\tau$ since each $h_j$ is actually lagged $j \cdot \tau/\Delta\sigma$ units.

Because convolution is a time consuming process, we wish to truncate the components, $h_j$, of the impulse response function after they have decayed to a negligible level. An estimate of the length of the impulse response function, $t_{max}$ (hours), was made from the following empirically-determined relation

$$t_{max} = 8.35 \left(\log_{10}(N(k+\tau))\right) + 6.0 \qquad (6\text{-}1)$$

The functions were truncated at an earlier time if their value was less than 0.001.

Adequate definition of the catchment and stream hydrographs was accomplished with a time step of $t_r/50$. Linear interpolation was used where necessary for convolution purposes.

## 6-4. Application to Natural Catchments

The Distributed Linear Reservoir Model is extremely easy to use. Only the catchment geometry and storm characteristics need be specified. However, it is not readily apparent how one applies the geometric model of Figure 3-2 to a field situation. The following two stage procedure is recommended.

Stage 1

1. Let $L_s$ = the length of the main stream.

2. Determine $L_c$ such that $2 x L_c x L_s$ = the catchment area (for continuity). The ratio of $L_c/L_s$ is commonly 1/6 (9).

3. Let $\alpha_s$ be determined from the average slope and roughness of the main stream.

4. Let $\alpha_c$ be determined from the tributary catchment slope and roughness.

5. Use $m_c$ = 2, see reference (18).

6. Use $m_s$ = 1.67 or 1.5 depending on whether the Manning or the Chezy equation is preferred.

If storm and streamflow data are available, continue to the next stage, otherwise, terminate.

Stage 2

1. Following the approach used by Wooding for application of the kinematic wave model (16), the Distributed Linear Reservoir Model can be fitted to the observed hydrographs by iterating on the parameter $\lambda$ (equation 3-18).

Application of mathematical models to natural catchments is an art which must be developed with experience. It was not attempted in the development of the Distributed Linear Reservoir Model since many catchments were required to establish the equations of Chapter 5 for the optimal model parameters.

## 6-5. Catchment Response to Distributed Inputs

Development of a distributed model opens the door to entirely new areas of fruitful research. Presently, little is known about the sensitivity of catchment behavior to spatially distributed inputs. Such knowledge is indeed crucial for at least two reasons:

1. When is a simple two-dimensional geometric model of the catchment adequate?

2. How much research effort should be expended on the understanding of the internal mechanics of storm movements?

Insight into these considerations can be gained from a few examples. The response of the catchment to a localized input is certainly dependent upon the location of that input relative to the observer (refer to Figure 5-1). To illustrate this, the results of routing the same storm through overland flow models tributary to different stream elements is shown in Figure 6-4. Applying the storm to the most distant portion of the catchment (overland flow model tributary to stream element No. 8) produces a significant time lag compared to that when the storm is applied near the catchment mouth (overland flow model tributary to stream element No. 2). Figure 6-3 corresponds to the same catchment subjected to a similar spatially uniform storm. In comparison with Figure 6-4, it can be observed that the contribution to the peak discharge comes primarily from the overland flow models near the catchment mouth.

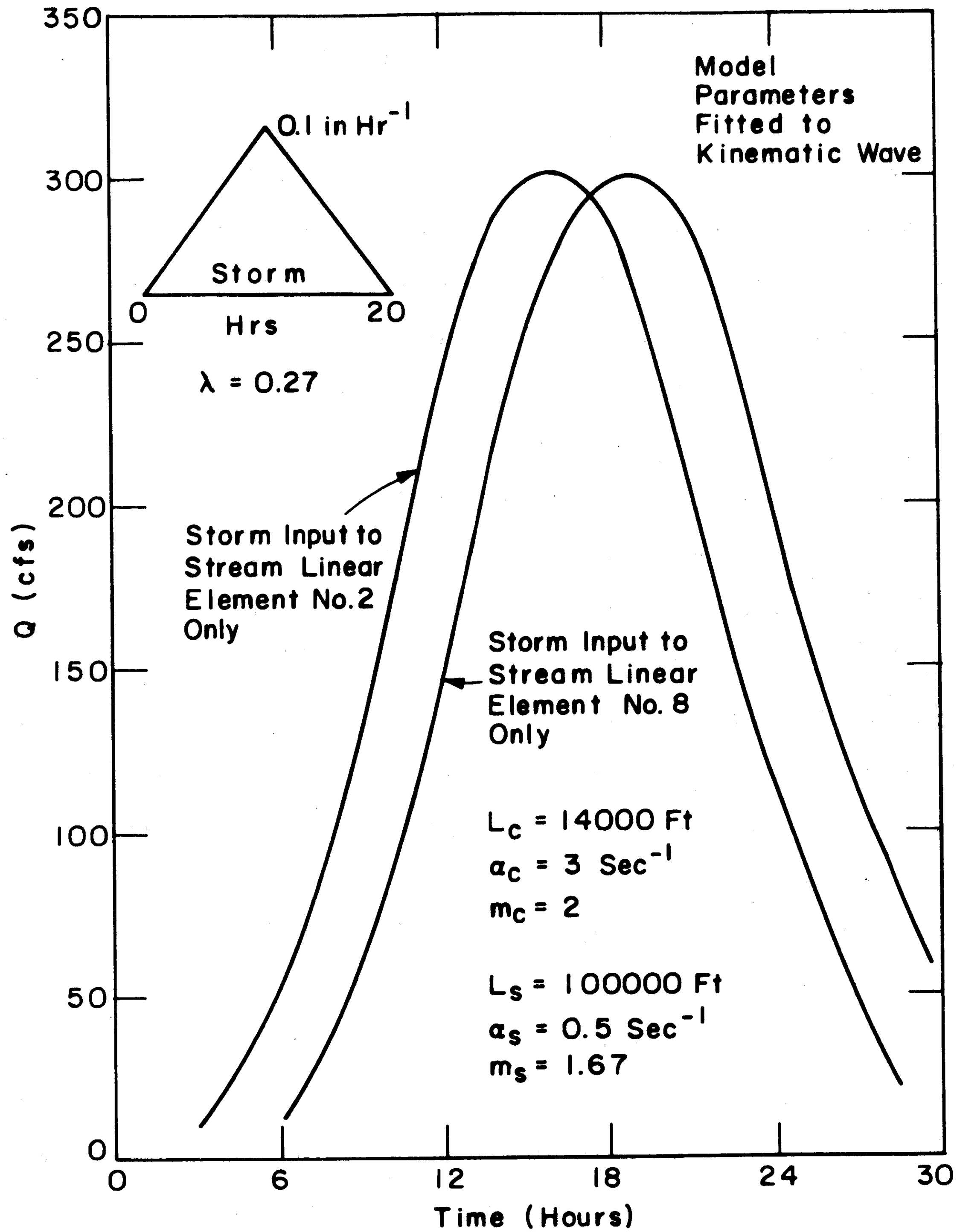


Figure 6-4 Comparison of the Response from Equal Lateral Inputs to Two Different Overland Flow Models

It is important to note in Figure 6-4 that there is virtually no difference in the hydrograph shape for the two cases considered even though the travel distance in the stream varied greatly. This is intuitively incorrect, however, this example is actually a flood-routing problem which certainly violates the assumptions implicit in the kinematic approach to which the model was fitted.

An investigation into the sensitivity of the response of overland flow to localized inputs can be performed in an analogous manner. For the same overland flow models considered in Figure 6-4, an identical storm was input into a single linear element in each model. The result of this operation is given in Figure 6-5. The response of the most distant linear catchment element is considerably lagged and dispersed from that obtained by applying the storm to a low-order element near the catchment mouth. Comparison with Figure 6-4 indicates that all peak attenuation and much of the lag is derived from passing the input through the overland flow model. In this example, the effect of the streamflow model is then simply to increase this lag.

The following example provides an indication of the errors due to spatially lumping the input function. Actual rainfall is likely to vary randomly over the catchment. To simulate this effect, a random number (between 0 and 1) was selected to scale the input into each catchment element. The factors were then averaged for each overland flow model (see Figure 5-1) corresponding to equal lateral inputs. These two cases can then be compared to lumping the input over the entire

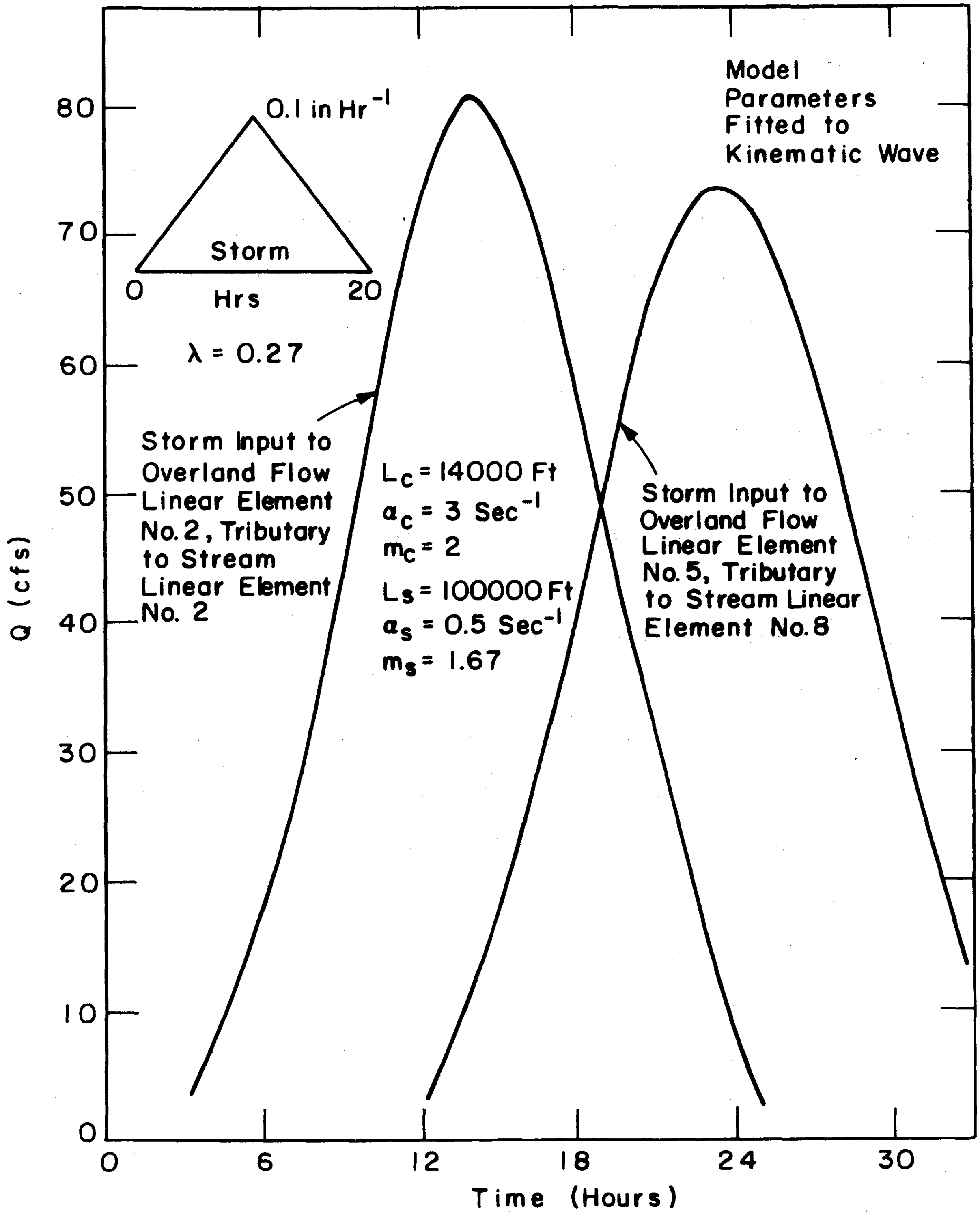


Figure 6-5 Comparison of the Response from a Single Lateral Input to Two Different Overland Flow Models

catchment by using a uniform storm having the average intensity of the above random case. The same pulse height was used for all three cases. The results from these lumping operations are shown in Figure 6-6.

For the totally random case, the catchment has smoothed the spatial variability in the input through the convolution and summation processes involved. Progressively more lumping, first per chain, then over the entire catchment has the effect of shifting the peak earlier in time and to a higher value. Thus the random variation in storm intensity has produced a dispersion effect on the streamflow hydrograph.

The totally random case corresponded to seventy catchment linear elements for this example. The computational effort required was only 50% greater than the typical spatially uniform input case. When the input was averaged to produce equal lateral input to each overland flow model (fourteen for this example) the computational effort was reduced to the level of spatially uniform input. Thus this model can efficiently utilize the greatly increased spatial resolution of storm intensity that the latest rainfall measurement techniques (24) promise to provide.

The final consideration is the importance of moving storms. The Distributed Linear Reservoir Model is capable of handling these situations merely by lagging the storm in proportion to its speed. The example considered was that of a storm moving parallel to the stream axis at a constant rate of speed. The same random scaling factors were used for each linear element as in Figure 6-6. The results

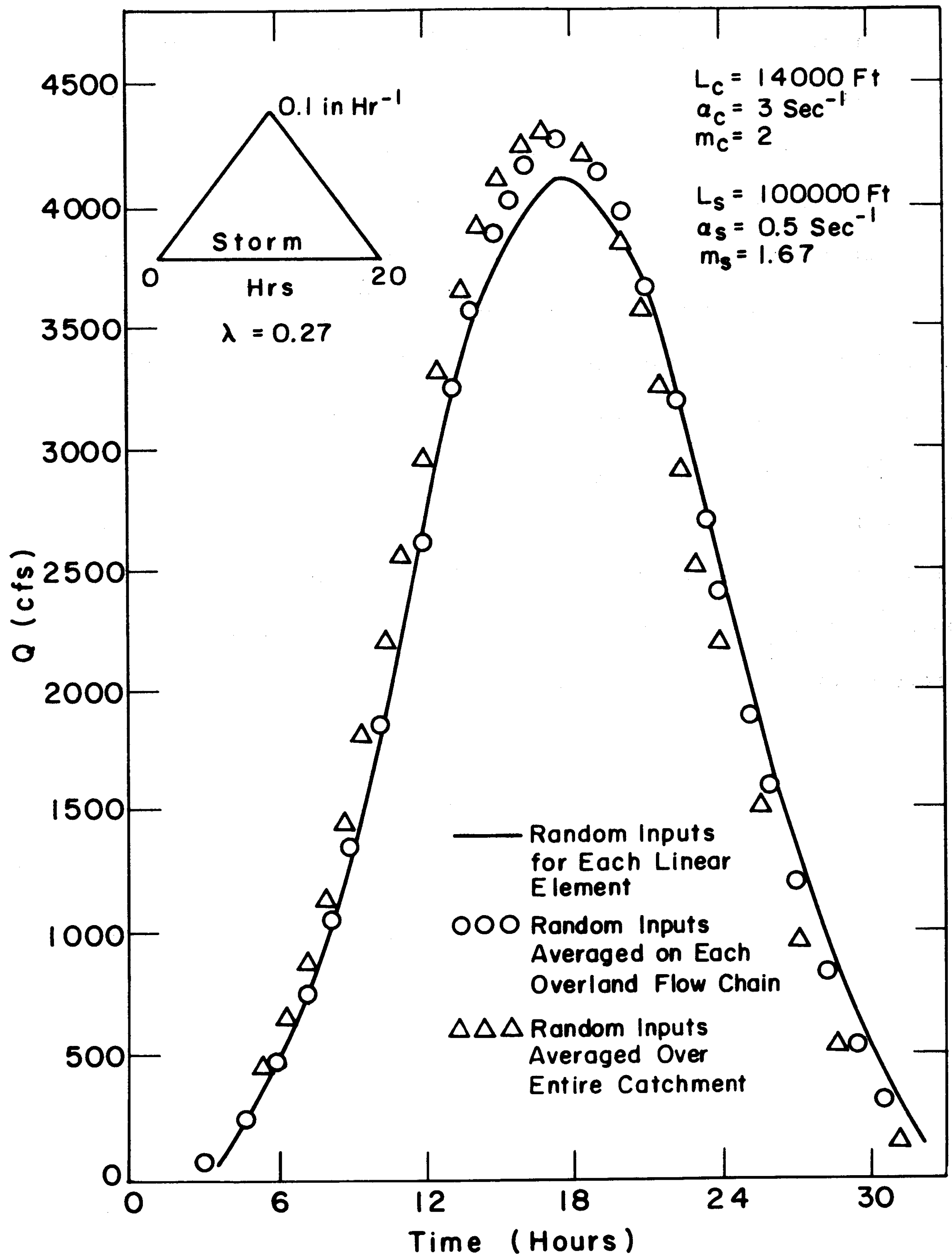


Figure 6-6 Effect of Lumping the Input from a Spatially Varying Rainfall Function

shown in Figure 6-7 indicate a time lag and peak attenuation associated with a moving storm.

It is interesting to note that a storm moving upslope produces more dispersion than a storm moving in the opposite direction. This is to be expected. As the storm moves upstream, the peaks of the lateral inflow become separated more and more from the peak of the streamflow already produced in lower reaches. As the storm moves downstream, it tends to become synchronous with the flood wave in the stream.

The apparent lack of the catchment to recognize spatial variability in the input rainfall excess is due in part to the considerations of Figure 6-4. It is believed that fitting the Distributed Linear Reservoir Model to a more exact streamflow model would accentuate the sensitivity of the catchment to distributed inputs.

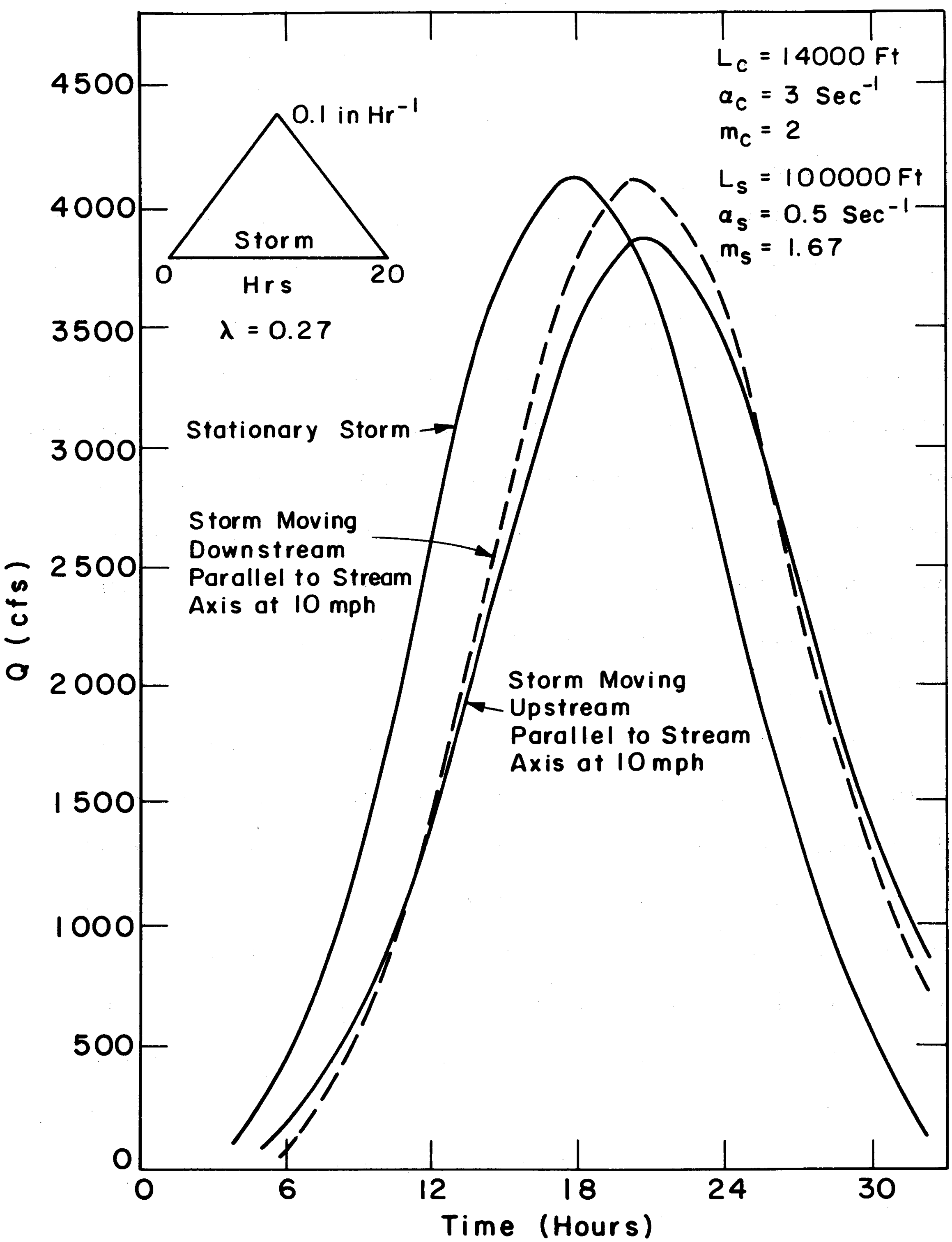


Figure 6-7 Effect of a Moving Spatially Varying Rainfall Function

# Chapter 7

# CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

## 7-1. Conclusions

A distributed quasi-linear model of direct catchment runoff has been developed. This model, called the Distributed Linear Reservoir Model, consists of cascades of linear reservoirs connected by linear channels. The model parameters were fitted to the kinematic wave formulation of Chapter 3. By determining the optimum pulse height and pulse length as a function of input and catchment parameters, the traditional unit-hydrograph method has been significantly improved. Incorporating these results into the model parameters, the impulse response function is therefore constrained to be input as well as catchment dependent.

Separate models of overland flow and streamflow allow simulation of the catchment response to spatially variable inputs. In Chapter 6 the sensitivity of the catchment to distributed inputs was investigated. The kinematic wave method has been found incapable of providing realistic hydrograph dispersion when applied to streamflow or flood-routing problems. Part of the lack of sensitivity of the catchment to a randomly varying storm pattern can be traced to the negligible streamflow dispersion exhibited by the model. Lumping of the input has been shown to slightly reduce the dispersion provided by random variations or by moving storm patterns.

Use of a linear or quasi-linear model of catchment behavior in an age when more exact solutions exist needs justification. What is involved is the trade-off between errors due to linearization and a greater computational efficiency through use of a linear model. For a spatially uniform rainfall function, the two models studied were of identical computational efficiency. However, when considering spatially varying inputs, the linear approach offers an order of magnitude better computational efficiency.

## 7-2. Suggestions for Future Work

1. A more exact streamflow solution than is provided by the kinematic wave of Chapter 3 should be employed in the parameter optimization. Then the Distributed Linear Reservoir Model would give, hopefully, better accuracy than the equally efficient kinematic wave method when compared to natural catchment data. A further refinement would be to use actual streamflow data in the parameter optimization process.
2. Development of an infiltration and groundwater model, parallel to the direct runoff model depicted in Figure 5-1, would be a significant extension of this approach. If a separate model analogous to the kinematic wave solution for direct runoff could be used as a standard, then this work could proceed independently of the quasi-linear direct runoff model already developed. Otherwise, it is recommended that suggestion No. 1 be pursued first and then development of

a parallel groundwater system could be done by comparison with a total runoff model. Accomplishing this task would increase the advantages of the linear approach to catchment behavior.

3. Upon completion of suggestion No.1 (and preferably suggestion No. 2 also) a more intensive study of the catchment sensitivity to distributed inputs could be performed. If necessary, a more elaborate physical model of the catchment (incorporating many of the simple two-dimensional models and/or fractions thereof, all with appropriate connectivity) could be utilized to represent highly-complex natural catchments.

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## Appendix

Contained herein is the computer program for the Distributed Linear Reservoir Model. The program was written in FORTRAN IV for the IBM 360/65. The options of the program allow either a spatially uniform or a spatially non-uniform rainfall excess. An average storm on the catchment is synthesized by inputing the essential storm characteristics as specified in the program listing. Alternatively, actual storm data, sampled at any time interval, may be input. Spatial non-uniformity is obtained by specifying the factors which multiply the input into each catchment linear element. The input variables and required format are described in the program listing.

The functions of the subprograms are briefly:

MAIN - Handles input of data, transfers to MODEL, computes pulse response for uniform rainfall excess

MODEL - Computes essential catchment response to storm and determines pulse height, pulse length and model parameters

STORM - Synthesizes a storm from input storm shape, length, volume, etc. or interpolates storm data on an equal time step if this option is used

DISTRB - Calculates the responses of each overland flow model to a distributed input if the storm is spatially non-uniform or is moving

STREAM - Takes the overland flow model hydrographs as input to the stream model and routes them down the channel

CONVOL - Generalized convolution subprogram

XKINIT - Determines catchment and stream model N, k, $\tau$

HYDGEN - Generates N-element linear reservoir cascade responses

HYDSUM - Combines the cascade responses, incorporating the appropriate time delays

INTRPL - Performs straight-line interpolation

SIMSON - Integration by Simpson's Rule

ERROR - Computes integral square error and correlation coefficient between observed and predicted time series

PLOT - Generalized printer-plotter

```
C
C     DISTRIBUTED LINEAR RESERVOIR--LINEAR CHANNEL MODEL
C
C     PROGRAM WRITTEN IN FORTRAN IV, G LEVEL
C     STORAGE REQUIREMENTS ARE APPROXIMATELY 250K
C     AVERAGE RUNNING TIME IS 1-1/3 MIN. ON THE IBM 360/65
C
C
C     INPUT DATA
C        ELC=LENGTH OF CATCHMENT (FEET)
C        ELS=LENGTH OF STREAM (FEET)
C        EMC,ALFC(1/SEC),EMS,ALFS(1/SEC) ARE THE PHYSICAL
C        CATCHMENT PARAMETERS FOR THE TWO-DIMENSIONAL MODEL
C        TDUR=STORM DURATION (HOURS)
C        TPEAK=TIME TO PEAK INTENSITY (HOURS)
C        DEP=TOTAL STORM VOLUME (INCHES)
C        XIEP=PEAK INTENSITY (IN/HR)
C
C     IOPTD=0 IMPLIES SPATIALLY UNIFORM RAINFALL EXCESS
C     IOPTD=1 IMPLIES THE DISTRIBUTED MODEL IS TO BE USED
C     IOPTD=2 IMPLIES A MOVING STORM WITH THE DISTR. MODEL
C
C     IOPT=1 IMPLIES THE STORM IS OF TRIANGULAR SHAPE
C     IOPT=2 IMPLIES A RECTANGULAR BLOCK
C     IOPT=3 IMPLIES STORM DATA IS INPUT BY ARRAY
C
      DIMENSION RAIN(50),GP(700),NHL(10),A(50,2)
      COMMON/CATPAR/ELC,EMC,ALFC,ELS,EMS,ALFS
      COMMON/CATCAR/RUOPT,DUR,XLAMDA,CUTOFF
      COMMON/CATSTR/TDUR,TPEAK,DEP,XIEP
      COMMON/MODPAR/XNC,NC,XKC,TDC,XNS,NS,XKS,TDS
      COMMON/MODTIM/TMAX,DTS,DTQ,DTH,IOPTD,IOPT
      COMMON/MODLIM/LIMIT,LIMGO,LIMGOS,LIMGS,NL
      COMMON/CATCH/CAT(10,500),CUH(10,500)
      COMMON/MODATA/UH(10000),CATCH(10,500),HSUM(1500)
      COMMON/TIMFAC/FAC(2,10,10),TIMES(2,10,10)
    1 READ(5,118) ELC,EMC,ALFC,ELS,EMS,ALFS,IOPT,IOPTD
  118 FORMAT(6F10.3,2I1)
      READ(5,100) TDUR,TPEAK,DEP,XIEP
  100 FORMAT(4F10.3)
      WRITE(6,101)
      WRITE(6,102)
      WRITE(6,103) ELC,ELS,ALFC,ALFS,EMC,EMS
      WRITE(6,104)
      WRITE(6,105) DEP,TDUR,XIEP,TPEAK
  101 FORMAT(1H1,5X'DISTRIBUTED LINEAR RESERVOIR--LINEAR'
     1' CHANNEL MODEL'///)
  102 FORMAT(10X,'TWO-DIMENSIONAL CATCHMENT PARAMETERS')
  103 FORMAT (//11X'LC ='F10.1,5X'LS ='F10.1/11X'ALFC ='F8.3,
     15X,'ALFS ='F8.3/11X'MC ='F10.3,5X'MS ='F10.3/)
```

```
 104 FORMAT(//,15X'STORM CHARACTERISTICS')
 105 FORMAT(//10X'STORM DEPTH ='F6.2,1X'IN.',10X,
    1'STORM DURATION ='F6.2,1X'HR.',/10X'PEAK INTENSITY ='
    2F6.2,'IN/HR',6X,'TIME TO PEAK ='F6.2,1X,'HR.')
     NC=2.0*(ALOG10(ELC)-2.0)+1.5
     IF(NC .LE. 10) GO TO 5
     NC=10
   5 XNC=NC
     NS=2.0*(ALOG10(ELS)-2.0)+1.5
     XNS=NS
C
C     THE SUBPROGRAM 'ERASE' IS A SPECIAL SUBPROGRAM
C     WHICH ZEROES OUT ONE-DIMENSIONAL ARRAYS
C     IF NOT AVAILABLE IT SHOULD BE REPLACED BY
C     APPROPRIATE DO LOOPS TO PERFORM THIS TASK
C
     CALL ERASE(RAIN,50)
     IF(IOPTD .GE. 1) GO TO 10
     FACAVG=1.0
     CALL MODEL(FACAVG,RAIN)
     GO TO 9
  10 WRITE(6,130)
 130 FORMAT(//,5X'STORM FACTORS FOR LINEAR ELEMENTS '
    1'IN THE DISTRIBUTED CATCHMENT MODEL'//)
C    EACH OF THE 2*NS DATA CARDS CONTAINS NC STORM FACTORS
C    LEFT SIDE OF TWO-DIMENSIONAL MODEL FACING UPSTREAM
C    PRECEDES THE RIGHT SIDE IN INPUT DATA STRUCTURE
     DO 23 IS=1,2
     READ(5,120) ((FAC(IS,J,I),I=1,NC),J=1,NS)
     WRITE(6,120)((FAC(IS,J,I),I=1,NC),J=1,NS)
 120 FORMAT(5F10.2)
  23 CONTINUE
     NLRNO=0
     FACSUM=0.0
     DO 25 IS=1,2
     DO 25 J=1,NS
     DO 25 I=1,NC
     FACTOR=FAC(IS,J,I)
     IF(FACTOR .LT. 0.01) GO TO 25
     FACSUM=FACSUM+FACTOR
     NLRNO=NLRNO+1
  25 CONTINUE
     FACAVG=FACSUM/NLRNO
     WRITE(6,121) FACAVG
 121 FORMAT(/10X'STORM LINEAR SCALE FACTOR AVERAGE ='F6.3)
     IF(IOPTD .GE. 2) GO TO 14
     DO 26 IS=1,2
     DO 26 J=1,NS
     DO 26 I=1,NC
  26 TIMES(IS,J,I)=0.0
```

```
      CALL MODEL(FACAVG,RAIN)
      GO TO 40
   14 WRITE(6,126)
  126 FORMAT (//,5X'STORM TIME LAGS FOR LINEAR ELEMENTS '
     1'IN THE DISTRIBUTED CATCHMENT MODEL'//)
C     MOVING STORM DATA READ IN
C     EACH OF THE 2*NS DATA CARDS CONTAINS NC TIME DELAYS
C     DATA STRUCTURE SIMILAR TO STORM FACTOR DATA
      DO 24 IS=1,2
      READ(5,120) ((TIMES(IS,J,I),I=1,NC),J=1,NS)
      WRITE(6,120)((TIMES(IS,J,I),I=1,NC),J=1,NS)
   24 CONTINUE
      CALL MODEL(FACAVG,RAIN)
      GO TO 40
    9 IT=LIMGO/50
      DT=IT*DTQ
      T=0.0
      A(1,1)=0.0
      A(1,2)=0.0
      DO 12 I=2,50
      II=IT*(I-1)+1
      T=T+DT
   12 A(I,1)=T
      CALL HYDGEN(XNC,NC,XKC,DTH,DTH,NHL,1)
      CALL HYDSUM(XNC,NC,TDC,DTH,DTQ,LIMIT,LIMGO)
      CALL SIMSON(DTQ,LIMGO,HSUM,SUM)
      CALL CONVOL(DTQ,DTQ,DTQ,NL,LIMGO,LIMG,RAIN,HSUM,GP)
C     CONVERSION TO CFS UNITS
      DO 21 I=1,LIMGO
   21 GP(I)=GP(I)*(ELC/43200.0)
      DO 18 I=2,50
      II=IT*(I-1)+1
      A(I,2)=GP(II)
   18 CONTINUE
      CALL SIMSON(DTQ,LIMGO,GP,SUM)
      WRITE(6,150)
  150 FORMAT(1H1,60X,'OPTIMUM PULSE RESPONSE',//)
   30 CALL PLOT(1,A,50,2,0,0,50,2)
   40 CALL STORM(RAIN,GP)
      GO TO 1
      END
```

```
      SUBROUTINE MODEL(FACAVG,RAIN)
C     THIS SUBROUTINE CALCULATES THE ESSENTIAL CATCHMENT
C     CHARACTERISTICS AND DETERMINES THE OPTIMUM
C     PULSE HEIGHT, PULSE LENGTH, TIME STEPS ETC.
      DIMENSION RAIN(50)
      COMMON/CATPAR/ELC,EMC,ALFC,ELS,EMS,ALFS
      COMMON/CATCAR/RUOPT,DUR,XLAMDA,CUTOFF
      COMMON/CATSTR/TDUR,TPEAK,DEP,XIEP
      COMMON/MODPAR/XNC,NC,XKC,TDC,XNS,NS,XKS,TDS
      COMMON/MODTIM/TMAX,DTS,DTQ,DTH,IOPTD,IOPT
      COMMON/MODLIM/LIMIT,LIMGO,LIMGOS,LIMGS,NL
      COMMON/MODATA/UH(10000),CATCH(10,500),HSUM(1500)
      DEP=DEP*FACAVG
      XIAV=DEP/TDUR
      TC=(ELC/(ALFC*3600.0*(XIAV/12.0)))**(1.0/EMC)
      XLAMDA=(2.*XIAV*ELC*ELS/ALFS)**(1./EMS)/(2.*ELC*
     1(XIAV*ELC/ALFC)**(1./EMC))
      DUR=TDUR/50.0
      IDT=DUR/0.050
      DUR=IDT*0.050
C     DEFINITON OF CATCHMENT AND STREAM HYDROGRAPH TIME STEP
      DTS=TDUR/50.0
C     DEFINITON OF PULSE RESPONSE TIME STEP
      DTQ=DTS/5.0
C     DECREASING THE TIME STEPS PROVIDES BETER CONTINUITY
C     BUT REQUIRES PROPORTIONATELY MORE COMPUTATION TIME
      IF(DUR .GT. 0.0550) GO TO 4
      DUR=0.050
      DTQ=0.010
    4 RUOPT=1.59*TC*XIAV/DUR
      IR=RUOPT/0.50+0.25
      RUOPT=IR*0.50
      VOLR=RUOPT*DUR
      VOLUME=VOLR*(ELC/43200.0)
      QPEAK=ALFC*((VOLR/12.0)**EMC)
      CUTOFF=QPEAK*0.010*DEP/(TDUR*RUOPT)
      VOLTOT=DEP*ELC*ELS*2.0/43200.0
      VOLLRB=VOLTOT/(XNC*XNS*2.0)
      WRITE(6,133) TC
  133 FORMAT(//5X'CATCHMENT TIME OF CONCENTRATION ='F6.2,
     12X'HOURS')
      WRITE(6,134) RUOPT
  134 FORMAT (/5X,'OPTIMUM PULSE HEIGHT ='F6.2,2X'IN/HR')
      WRITE(6,136) DUR
  136 FORMAT (/5X'OPTIMUM PULSE LENGTH ='F6.3,2X'HOURS')
C     THE FOLLOWING VOLUMES PROVIDE A CONTINUITY
C     CHECK ON THE MODEL
C     LACK OF CONTINUITY WILL BE DUE TO TOO LARGE
C     A TIME STEP USED IN THE COMPUTATIONS
      WRITE(6,135) VOLLRB
```

```
135 FORMAT(/10X'VOLUME OF DIRECT RUNOFF PER LINEAR '
   1'ELEMENT ='E10.4,2X'FEET')
    WRITE(6,137) VOLTOT
137 FORMAT (/10X'TOTAL VOLUME OF DIRECT RUNOFF FOR STORM'
   1' ON THE ENTIRE CATCHMENT ='E10.4,2X'FEET')
    WRITE(6,105)
105 FORMAT(//,10X'OVERLAND FLOW MODEL PARAMETERS')
    CALL XKINIT(XNC,XKC,TDC,QPEAK,VOLUME,1)
    XKT=XKC+TDC
    ITMAX=8.35*(ALOG10(XNC*XKT))+6.1
    TMAX=ITMAX
    IF(TMAX .GT. 2.0) GO TO 12
    TMAX=2.0
 12 LIMGO=TMAX/DTQ+0.5
    IF(LIMGO .LE. 600) GO TO 18
    DTQ=DTQ*2.0
    GO TO 12
 18 DTH=DTQ
    XLIM=TMAX/DTH+0.5
    LIMIT=XLIM
    TDS=XLAMDA*TDC
    ITDC=TDC/DTQ+0.50
    TDC=ITDC*DTQ
    LAG=TDC/DTH+0.5
    L=LIMIT+LAG*NC
    NUHL=NC*L
    CALL ERASE(UH,NUHL)
    WRITE(6,138) DTS,DTQ
138 FORMAT(//10X'TIME STEPS USED IN COMPUTATIONS'/11X,
   1'DTS ='F5.3,5X'DTQ ='F5.3)
C     PULSE RESPONSE TIME STEP MUST BE SMALL ENOUGH TO
C     PROVIDE A GOOD DISCRETE DEFINITION OF THE PULSE
C     AT LEAST 5 SAMPLES (NL>5) ARE NORMALLY REQUIRED
    NL=DUR/DTQ+1.1
    FACR=2.0
  7 RAIN(1)=RUOPT/FACR*DTQ
    RAIN(NL)=RUOPT/FACR*DTQ
    NL1=NL-1
    DO 10 I=2,NL1
 10 RAIN(I)=RUOPT*DTQ
    RETURN
    END
```

```
      SUBROUTINE STORM(RAIN,GP)
C     THIS SUBROUTINE GENERATES A STORM FOR THE CATCHMENT
      DIMENSION GP(1),RAIN(50),STR(500),GPS(700),
     1TIME(200),FACSTR(10),D(50,2)
      COMMON/CATPAR/ELC,EMC,ALFC,ELS,EMS,ALFS
      COMMON/CATCAR/RUOPT,DUR,XLAMDA,CUTCFF
      COMMON/CATSTR/TDUR,TPEAK,DEP,XIEP
      COMMON/MODPAR/XNC,NC,XKC,TDC,XNS,NS,XKS,TDS
      COMMON/MODTIM/TMAX,DTS,DTQ,DTH,IOPTD,IOPT
      COMMON/MODLIM/LIMIT,LIMGO,LIMGOS,LIMGS,NL
      WRITE(6,101)
  101 FORMAT(1H1,10X'SYNTHESIZED CATCHMENT STORM'//)
      XLIM=TDUR/DUR+1.5
      LIM=XLIM
      XLIMP=TPEAK/DUR+0.5
      LIMP=XLIMP
      LIMGOS=TDUR/DTS+0.5
      CALL ERASE(STR,LIM,GPS,600)
      GO TO (14,13,12),IOPT
C     INPUT STORM DATA BY ARRAY
C     LIMSTR=NUMBER OF INPUT DATA POINTS
   12 READ(5,117) LIMSTR
      READ(5,118) (TIME(I),GPS(I),I=1,LIMSTR)
  117 FORMAT(I10)
  118 FORMAT(2F10.4)
C     INPUT STORM DATA CONVERTED TO AN EQUAL TIME STEP
      CALL INTRPL(STRLIM,LIM,DUR,DUR,TIME,GPS,STR,2)
      GO TO 17
   13 HST=DEP/TDUR
      DHS=HST/RUOPT
      DO 16 I=1,LIM
   16 STR(I)=DHS
      WRITE(6,105) (STR(I),I=1,LIM)
  105 FORMAT(10F10.5)
      GO TO 20
   14 HST=XIEP
      HSM=HST
      IF(LIMP .LT. 2) GO TO 21
      DHST=HST/LIMP
      HST=0.0
      DO 10 I=2,LIMP
      HST=HST+DHST
   10 STR(I)=HST
      LIMP=LIMP+1
      STR(LIMP)=HST+DHST/2.0
      HST=HST+DHST
   21 DHST=HSM/(LIM-LIMP)
      L=LIMP+1
      DO 15 I=L,LIM
      HST=HST-DHST
```

```
   15 STR(I)=HST
      LIM=LIM-1
      CALL SIMSON(DUR,LIM,STR,SUM)
      WRITE(6,125) (I,STR(I),I=1,LIM)
  125 FORMAT(5(I5,F10.5))
   17 DO 18 J=1,LIM
   18 STR(J)=STR(J)/RUDPT
      IF(IOPTD .LT. 1) GO TO 20
      CALL DISTRB(RAIN,STR)
      GO TO 40
   20 XNM=TDUR/DUR+0.5
      NM=XNM
      CALL CONVOL(DUR,DTQ,DTS,NM,LIMGC,LGPS,STR,GP,GPS)
      IF(LGPS .GE. LIMGOS) GO TO 25
      DO 26 I=LGPS,LIMGOS
   26 GPS(I)=0.0
   25 WRITE(6,145)
  145 FORMAT(1H1,15X'PREDICTED CATCHMENT HYDROGRAPH'//)
      LIMGOS=LGPS
C     CONTINUITY CHECK
      CALL SIMSON(DTS,LIMGOS,GPS,SUM)
   27 IT=LIMGOS/50+0.95
      T=0.0
      DT=IT*DTS
      D(1,1)=0.0
      D(1,2)=0.0
      DO 30 I=2,50
      II=IT*(I-1)+1
      T=T+DT
      D(I,1)=T
      D(I,2)=GPS(II)
   30 WRITE(6,140) T,GPS(II)
  140 FORMAT(15X,F8.2,1F10.5)
      WRITE(6,150)
  150 FORMAT(1H1,60X,'CATCHMENT HYDROGRAPH',//)
      CALL PLOT(1,D,50,2,0,0,50,2)
      CALL STREAM(GPS,FACSTR)
   40 RETURN
      END
```

```
      SUBROUTINE DISTRB (RAIN,STR)
C     THIS SUBROUTINE HANDLES UNEQUAL LATERAL
C     INPUTS INTO THE OVERLAND FLOW MODEL
      DIMENSION NHL(10),UHLP(600),UPH(1500),STRC(400),
     1STR(1),GPS(700),FACSTR(10),RAIN(1),GPD(700)
      COMMON/CATPAR/ELC,EMC,ALFC,ELS,EMS,ALFS
      COMMON/CATCAR/RUNPT,DUR,XLAMDA,CUTOFF
      COMMON/CATSTR/TDUR,TPEAK,DEP,XIEP
      COMMON/MODPAR/XNC,NC,XKC,TDC,XNS,NS,XKS,TDS
      COMMON/MODTIM/TMAX,DTS,DTQ,DTH,IOPTD,IOPT
      COMMON/MODLIM/LIMIT,LIMGO,LIMGOS,LIMGS,NL
      COMMON/CATCH/CAT(10,500),CUH(10,500)
      COMMON/MODATA/UH(10000),CATCH(10,500),HSUM(1500)
      COMMON/TIMFAC/FAC(2,10,10),TIMES(2,10,10)
      WRITE(6,105)
  105 FORMAT(///10X,'DISTRIBUTED CATCHMENT MODEL'//)
      JSAME=0
      LAG=TDC/DTH+1.5
      L=LIMIT+NC*LAG+10
      NLIM=L*NC
      CALL ERASE(UHLR,L,UH,NLIM)
      XMNS=TDUR/DUR+0.5
      NMS=XMNS
      NL=DUR/DTQ+1.1
      DO 5 J=1,NS
    5 FACSTR(J)=1.0
      DO 10 J=1,NS
      DO 10 JJ=1,500
      CATCH(J,JJ)=0.0
      CUH(J,JJ)=0.0
   10 CAT(J,JJ)=0.0
      DO 50 IS=1,2
      DO 40 J=1,NS
      IF(IOPTD .EQ. 2) GO TO 15
      IZERO=0
      ISAME=0
      FACT=1.0
      DO 17 I=1,NC
      FACTOR=FAC(IS,J,I)
      IF(FACTOR .GT. 0.010) GO TO 16
      IZERO=IZERO+1
      GO TO 17
   16 IF((ABS(FACTOR-FACT)) .GT. 0.01) GO TO 17
      ISAME=ISAME+1
   17 FACT=FACTOR
      WRITE(6,100) J,IZERO,ISAME
      IF(IZERO .LT. NC) GO TO 14
      DO 19 JJ=1,500
   19 CAT(J,JJ)=0.0
      IF(IS .GT. 1) GO TO 40
```

```
      FACSTR(J)=0.0
      GO TO 40
   14 IF(ISAME .LT. (NC-1)) GO TO 15
      WRITE(6,110) J
  110 FORMAT(//10X,'EQUAL LATERAL INPUTS FOR CHAIN NO.'I2)
      IF(JSAME .EQ. 1) GO TO 18
C     ROUTINE FOR EQUAL LATERAL INPUTS
C     TO A GIVEN OVERLAND FLOW MODEL
      CALL HYDGEN(XNC,NHC,XKC,DTH,DTH,NHL,1)
      CALL HYDSUM(XNC,NHC,TDC,DTH,DTQ,LIMIT,LIMGO)
      CALL SIMSON(DTQ,LIMGO,HSUM,SUM)
      CALL CONVOL(DTQ,DTQ,DTQ,NL,LIMGO,LIMG,RAIN,HSUM,UPH)
      CALL SIMSON(DTQ,LIMG,UPH,SUM)
      CALL CONVOL(DUR,DTQ,DTS,NMS,LIMG,LIMGS,STR,UPH,GPD)
      CALL SIMSON(DTS,LIMGS,GPD,SUM)
   18 FACTOR=FAC(IS,J,1)
      CFS=FACTOR*(ELC/43200.0)
      WRITE(6,100)  JSAME,NL,LIMGS,CFS,FACTOR
  100 FORMAT(3I10,4F10.2)
      JSAME=1
      DO 27 JJ=1,LIMGS
   27 CAT(J,JJ)=GPD(JJ)*CFS
      GO TO 40
   15 WRITE(6,115) J
  115 FORMAT(//10X'DISTRIBUTED LATERAL INPUTS FOR '
     1'OVERLAND FLOW MODEL NO.'I3,/)
C     ROUTINE FOR UNEQUAL LATERAL INPUTS
C     TO A GIVEN OVERLAND FLOW MODEL
      DO 35 I=1,NC
      IL=(I-1)*LIMGS
      FACTOR=FAC(IS,J,I)
      IF(FACTOR .GT. 0.010) GO TO 21
      DO 20 IC=1,LIMGS
   20 CUH(I,IC)=0.0
      GO TO 35
   21 XNH=I
      CALL HYDGEN(XNH,NHC,XKC,DTH,DTH,NHL,2)
      LIMUH=NHL(I)
      DO 22 JJ=1,LIMUH
   22 UHLR(JJ)=UH(JJ)
      CALL SIMSON(DTH,LIMUH,UHLR,SUM)
      CALL CONVOL(DTQ,DTH,DTQ,NL,LIMUH,LIMG,RAIN,UHLR,UPH)
      CALL SIMSON(DTQ,LIMG,UPH,SUM)
      LIMS=NMS
      TOA=TIMES(IS,J,I)
      CALL MOVFAC(TOA,DTS,LIMS,FACTOR,STR,STRC)
      CALL CONVOL(DUR,DTQ,DTS,LIMS,LIMG,LIMGS,STRC,UPH,GPS)
      CALL SIMSON(DTS,LIMGS,GPS,SUM)
      CFS=(ELC/(XNC*43200.0))
  125 FORMAT(10X,10F10.3)
```

```
      DO 24 JJ=1,LIMGS
   24 CUH(I,JJ)=GPS(JJ)*CFS
   35 CONTINUE
      IL=NC*LIMGS
      CALL ERASE(UH,IL)
      DO 36 I=1,NC
      IL=(I-1)*LIMGS
      DO 36 JJ=1,LIMGS
   36 UH(IL+JJ)=CUH(I,JJ)
      CALL HYDSUM(XNC,NC,TPC,DTS,DTS,LIMGS,LIMGS)
      DO 38 JJ=1,LIMGS
   38 CAT(J,JJ)=HSUM(JJ)*XNC
   40 CONTINUE
      IF(IS .GT. 1) GO TO 46
      DO 45 J=1,NS
      DO 45 JS=1,LIMGS
   45 CATCH(J,JS)=CAT(J,JS)
      GO TO 50
   46 DO 48 J=1,NS
      DO 48 JS=1,LIMGS
   48 CATCH(J,JS)=CATCH(J,JS)+CAT(J,JS)
   50 CONTINUE
      CALL STREAM(GPS,FACSTR)
      RETURN
      END
```

```
      SUBROUTINE MOVFAC(TOA,DTS,LIMIT,FACTOR,STR,STRC)
C     THIS SUBROUTINE MULTIPLIES THE STORM INPUT BY THE
C     LINEAR ELEMENT FACTOR AND LAGS THE BEGINNING OF THE
C     STORM ACCORDING TO THE STORM TIME DELAYS
      DIMENSION STR(1),STRC(400)
      WRITE(6,100) TOA,FACTOR
  100 FORMAT(//10X,'SHIFT TIME ORIGIN BY'F10.2,2X'HOURS',
     1/10X'MULTIPLY LATERAL INPUT BY AREAL FACTOR 'F10.2,/)
      NMOVE=TOA/DTS
      NLIMIT=LIMIT+NMOVE
      DO 10 I=1,LIMIT
      J=NLIMIT-I+1
      STRC(J)=STR(J-NMOVE)*FACTOR
   10 CONTINUE
      IF(NMOVE .EQ. 0) GO TO 18
      DO 15 I=1,NMOVE
   15 STRC(I)=0.0
   18 LIMIT=NLIMIT
      RETURN
      END
```

```
      SUBROUTINE STREAM(GPS,FACSTR)
C     THIS SUBROUTINE ROUTS THE OVERLAND FLOW HYDROGRAPH
C     THROUGH THE STREAM MODEL
      DIMENSION GPS(700),UHGP(500),STRGPS(500),D(50,2),
     2NHL(10),UHGP1(4000),TIME(700),UHLR(600),FACSTR(10)
      COMMON/CATPAR/ELC,EMC,ALFC,ELS,EMS,ALFS
      COMMON/CATCAR/RUOPT,DUR,XLAMDA,CUTOFF
      COMMON/CATSTR/TDUR,TPEAK,DEP,XIEP
      COMMON/MODPAR/XNC,NC,XKC,TDC,XNS,NS,XKS,TDS
      COMMON/MODTIM/TMAX,DTS,DTQ,DTH,IOPTD,IOPT
      COMMON/MODLIM/LIMIT,LIMGO,LIMGOS,LIMGS,NL
      COMMON/CATCH/CAT(10,500),CUH(10,500)
      COMMON/MODATA/UH(10000),CATCH(10,500),HSUM(1500)
      WRITE(6,110) XLAMDA
  110 FORMAT(1H1,///15X,'CATCHMENT-STREAM MODEL'//,
     120X,'LAMDA ='F10.4,//)
      XLR=ELS/XNS
      T=0.0
      DT=DTS
      DO 6 I=1,600
      T=T+DT
    6 TIME(I)=T
      T=0.0
      DO 7 J=1,10
      DO 7 JJ=1,500
    7 CUH(J,JJ)=0.0
      DO 8 JJ=1,600
    8 STRGPS(JJ)=0.0
      ITDS=TDS/DTS+0.25
      TDS=ITDS*DTS
      WRITE(6,105)
  105 FORMAT(//,15X'STREAM MODEL PARAMETERS')
    2 CALL XKINIT(XNS,XKS,TDS,QPEAK,VOL,2)
      IF(IOPTD .GE. 1) GO TO 49
   47 LIMGS=LIMGOS
      DO 45 J=1,NS
   45 FACSTR(J)=1.0
      DO 46 J=1,NS
      DO 46 I=1,LIMGOS
   46 CATCH(J,I)=GPS(I)*2.0
      GO TO 50
   49 WRITE(6,140)
  140 FORMAT(//10X,'DISTRIBUTED STREAM MODEL'//)
   50 DO 70 J=1,NS
      JL=(J-1)*LIMGS
      XN=J
      DT1=DTS
      LGS=LIMGS
      CALL ERASE(GPS,600)
      IF(FACSTR(J) .GT. 0.10) GO TO 52
```

```
      DO 51 JJ=1,LIMGS
   51 CUH(J,JJ)=0.0
      GO TO 70
   52 WRITE(6,141) J
  141 FORMAT(//10X'LINEAR RESERVOIR CHAIN NO.'I3,2X'IUH'/)
      IF(J .GT. 3) GO TO 60
   56 LIMIT=TMAX/DT1
      CALL  HYDGEN(XN,NHS,XKS,DT1,DT1,NHL,2)
      LIMUH=NHL(J)
      DO 53 I=1,LIMUH
   53 UHLR(I)=UH(I)*DT1
      WRITE(6,166) (UH(I),I=1,LIMUH)
  166 FORMAT(5F10.4)
      CALL SIMSON(DT1,LIMUH,UH,SUM)
      IF(SUM .GT. 0.90) GO TO 58
      DT2=DT1/2.0
      IF(DT2 .LT. 0.0095) GO TO 58
      DT1=DT2
      GO TO 56
   58 DO 57 I=1,LIMGS
      UHGP(I)=CATCH(J,I)
   57 CONTINUE
      LGS=LIMGS*(DTS/DT1)
      CALL  INTRPL(LIMGS,LGS,DT1,DTS,TIME,UHGP,UHGP1,2)
      CALL  CONVOL(DT1,DT1,DTS,LGS,LIMUH,LIMGOS,UHGP1,
     1UHLR,GPS)
      CALL  SIMSON(DTS,LIMGOS,GPS,SUM)
      DO 55 IS=1,LIMGOS
   55 CUH(J,IS)=GPS(IS)*XLR
      GO TO 70
   60 LIMIT=TMAX/DTS
      CALL  HYDGEN (XN,NHS,XKS,DTS,DTS,NHL,2)
      LIMUH=NHL(J)
      DO 62 I=1,LIMUH
   62 UHLR(I)=UH(I)*DTS
      WRITE(6,166) (UH(I),I=1,LIMUH)
      CALL  SIMSON(DTS,LIMUH,UH,SUM)
C     ITERATE UNTIL ADEQUATE CONTINUITY IS ACHIEVED
C     FOR THE LINEAR RESERVOIR RESPONSE
      IF(SUM .GT. 0.90 .AND. SUM .LT. 1.10) GO TO 64
      DT1=DTS/10.0
      GO TO 56
   64 DO 67 I=1,LIMGS
   63 UHGP(I)=CATCH(J,I)
   67 CONTINUE
      CALL  CONVOL(DTS,DTS,DTS,LIMGS,LIMUH,LIMGOS,UHGP,
     1UHLR,GPS)
      CALL  SIMSON(DTS,LIMGOS,GPS,SUM)
      DO 65 IS=1,LIMGOS
   65 CUH(J,IS)=GPS(IS)*XLR
```

```
 70 CONTINUE
    JL=NS*LIMGOS+300
    CALL ERASE(UH,JL)
    DO 75 J=1,NS
    JL=(J-1)*LIMGOS
    DO 75 JJ=1,LIMGOS
 75 UH(JL+JJ)=CUH(J,JJ)
    CALL HYDSUM(XNS,NS,TDS,DTS,DTS,LIMGOS,LIMGOS)
    DO 76 I=1,LIMGOS
 76 STRGPS(I)=HSUM(I)*XNS
    WRITE(6,135)
135 FORMAT(1H1,15X'PREDICTED STREAMFLOW HYDROGRAPH'//)
C   CONTINUITY CHECK
    CALL SIMSON(DTS,LIMGOS,STRGPS,SUM)
 80 IL=LIMGOS/50+1.05
    DT=IL*DTS
    D(1,1)=0.0
    D(1,2)=0.0
    DO 86 I=2,50
    II=IL*(I-1)+1
    T=T+DT
    D(I,1)=T
    D(I,2)=STRGPS(II)
    WRITE(6,125) T,STRGPS(II)
125 FORMAT(15X,F8.2,1F10.3)
 86 CONTINUE
    IF(IOPTD .LT. 1) GO TO 33
    WRITE(6,160)
160 FORMAT (1H1,50X,'DISTRIBUTED CATCHMENT-STREAM '
   1'HYDROGRAPH'//)
    GO TO 35
 33 WRITE(6,150)
150 FORMAT(1H1,60X,'CATCHMENT-STREAM HYDROGRAPH',//)
 35 CALL PLOT(2,D,50,2,0,0,50,2)
 40 RETURN
    END
```

```
      SUBROUTINE CONVOL(DOR,DTQ,DTS,NM,LIMH,LIMG,RAIN,UHP,GSS)
C     THIS SUBROUTINE PERFORMS THE CONVOLUTION
      DIMENSION RAIN(1),UHP(1),G(4000),GSS(700),TIME(1500)
      COMMON/CATCAR/RUOPT,DUR,XLAMDA,CUTOFF
      NB=1
      T=0.0
      XLAG=DOR/DTQ+0.25
      LAG=XLAG
      LM=LIMH+LAG*NM
      CALL ERASE(G,4000,GSS,700)
      LIM=LIMH
      DO 25 N=1,NM
      K=LAG*(N-1)
      DO 18 I=NB,LIM
      J=I-K
      IF(UHP(J) .LT. CUTOFF) GO TO 18
   17 G(I)=G(I)+UHP(J)*RAIN(N)
   18 CONTINUE
   19 NB=NB+LAG
      LIM=LIM+LAG
   25 CONTINUE
C     CONVERSION TO TIME STEP=DTS
      XLG=DTS/DTQ
      LG=XLG
      DLG=XLG-LG
      IF(DLG .GE. 0.10) GO TO 40
      IF(XLG .GT. 1.05) GO TO 28
      DO 26 I=1,LIM
   26 GSS(I)=G(I)
      LIMG=LM
      GO TO 50
   28 LIMG=LIM/LG
      GSS(1)=0.0
      DO 30 I=2,LIMG
      II=LG*(I-1)+1
   30 GSS(I)=G(II)
      GO TO 50
   40 DO 45 I=1,LIM
      T=T+DTQ
   45 TIME(I)=T
      LIMG=LIM*DTQ/DTS
      CALL INTPPL(LIM,LIMG,DTS,DTQ,TIME,G,GSS,2)
   50 RETURN
      END
```

```
      SUBROUTINE XKINIT (XNH,XK,TD,QPEAK,VOL,JOPT)
C     THIS SUBROUTINE SETS THE VALUE OF THE MODEL
C     PARAMETERS K AND TD FOR THE APPPOPRIATE N
      DIMENSION XCAT(2)
      COMMON/CATPAR/ELC,EMC,ALFC,ELS,EMS,ALFS
      COMMON/CATCAR/RUOPT,DUR,XLAMDA,CUTOFF
      GO TO (1,5),JOPT
C     DETERMINATION OF CATCHMENT MODEL PARAMETERS
C     THE PEAK DISCHARGE OF THE KINEMATIC WAVE PULSE
C     RESPONSE IS EQUATED TO THE PEAK OF THE MODEL
C     PULSE RESPONSE TO DETERMINE (XK+TD)=TDKSUM
    1 TDKSUM=1.0/(XNH*QPEAK/VOL)
      XCAT(1)=XNH
      XCAT(2)=TDKSUM
C     RATIO OF TD/XK=F(ALFC,EMC)
      TDXKR=0.50*10.0**(EMC*ALFC/25.0)
      XK=TDKSUM/(1.0+TDXKR)
      TD=TDKSUM-XK
    2 IF(XK .GT. 0.020) GO TO 3
      XK=TDKSUM
      TD=0.0
    3 WRITE(6,100) XNH,XK,TD
  100 FORMAT    (//5X,'LINEAR RESERVOIR--LINEAR CHANNEL '
     1'MODEL OF'F5.2,1X,'CHAINS',//10X'K ='F7.4,1X'HOURS'
     24X'TIME DELAY ='F7.4,1X'HOURS'/)
   10 RETURN
C     DETERMINATION OF STREAM MODEL PARAMETERS
    5 TDKSUM=XLAMDA*XCAT(1)*XCAT(2)/XNH
      XK=TDKSUM-TD
      GO TO 2
      END
```

```
      SUBROUTINE HYDGEN(XNH,NH,XK,DTH,DTLR1,NHL,IOPT)
C     THIS SUBROUTINE GENERATES THE LINEAR RESERVOIR RESPONSES
C     EACH CHAIN IS A NASH MODEL OF N RESERVOIRS
      DIMENSION NHL(10)
      DOUBLE PRECISION XKD,XKN,XNUM,XDEM
      COMMON/MODLIM/LIMIT,LIMGO,LIMGOS,LIMGS,NL
      COMMON/MODATA/UH(10000),CATCH(10,500),HSUM(1500)
      WRITE(6,105)
  105 FORMAT(/10X,'GENERATED L.R. CHAINS FROM HYDGEN'/)
      XKD=XK
      IF(IOPT .EQ. 1) GO TO 1
      N=XNH
      XN=XNH
      DT=DTH
      NM=0
      FAC=1.0
      GO TO 11
    1 XN=0.0
      N=0
      FAC=1.00
    5 XN=XN+1.00
      IF(DTH .LT. 0.010 .OR. XN .GT. 1.5) GO TO 7
      DT=DTLR1
      GO TO 8
    7 DT=DTH
    8 DXN=XNH/XN
      IF(DXN .GT. 0.99) GO TO 10
      FAC=1.00-(XN-XNH)
      XN=XNH
   10 N=N+1
      NM=(N-1)*LIMIT
   11 T=0.0
      UH(NM+1)=0.0
      HMAX=0.0
      Z=XN
      GN=GAMMA(Z)
      XKN=XKD**XN
      XDEM=XKN*GN
      DO 18 J=2,LIMIT
      T=T+DT
      LJ=J
      XNUM=(T**(XN-1.0))*DEXP(-T/XKD)
      H=XNUM/XDEM
      UH(NM+J)=H*FAC
      IF(H .GT. HMAX) GO TO 17
      IF(H .LT. 1.0E-03) GO TO 19
      GO TO 18
   17 HMAX=H
   18 CONTINUE
   19 IF(LJ .GE. LIMIT) GO TO 25
```

```
      DO 20 J=LJ,LIMIT
   20 UH(NM+J)=0.0
   25 DX=XNH-XN
      NHL(N)=LJ
      WRITE(6,100) N,T,DT,LIMIT,LJ
  100 FORMAT(I10,2F10.4,2I10/)
      IF(DX-0.01)50,50,5
   50 NH=N
      RETURN
      END
```

```
      SUBROUTINE HYDSUM(XNH,NM,TD,DTH,DTQ,LIMIT,LIMGO)
C     THIS SUBROUTINE COMBINES THE COMPONENT RESPONSES
C     AND INCORPORATES THE TIME DELAY TD
      DIMENSION HS(3000),TH(3000)
      COMMON/MODATA/UH(10000),CATCH(10,500),HSUM(1500)
      XLAG=TD/DTH+0.5
      LAG=XLAG
      DLAG=XLAG-LAG
      IF(DLAG .LT. 0.50) GO TO 5
      LAG=LAG+1
    5 L=LIMIT+LAG*NM+100
      CALL ERASE(HS,L,TH,L)
      NB=1
      LIM=LIMIT
      DO 25 N=1,NM
      NH=(N-1)*LIMIT
      K=LAG*(N-1)
      DO 18 I=NB,LIM
      J=I-K
   17 HS(I)=HS(I)+UH(NH+J)
   18 CONTINUE
      NB=NB+LAG
      LIM=LIM+LAG
   25 CONTINUE
      T=-DTH
      DO 30 I=1,LIM
      T=T+DTH
      TH(I)=T
      HS(I)=HS(I)/XNH
   30 CONTINUE
      LG=IABS(LIMIT-LIMGO)
      IF(LG .GT. 10) GO TO 40
      DO 35 I=1,LIMIT
   35 HSUM(I)=HS(I)
      GO TO 45
   40 CALL INTRPL(LIMGO,LIMGO,DTQ,DTH,TH,HS,HSUM,1)
   45 RETURN
      END
```

```
      SUBROUTINE INTRPL(L,LIM,DTX,DTH,TH,X,Y,JOPT)
      DIMENSION TH(1),X(1),Y(4000)
      IF(JOPT .GT. 1) GO TO 8
C     THIS BRANCH DETERMINES THE VALUE OF Y CORRESPONDING TO
C     A PARTICULAR X AT TIME T
      J=0
      K=1
      T=0.0
    1 IF(T .GE. TH(K)) GO TO 5
      T=T+DTX
      GO TO 1
    5 IF(T .GE. TH(K) .AND. T .LE. TH(K+1)) GO TO 7
      K=K+1
      GO TO 1
    7 DTD=(T-TH(K))/DTH
      J=J+1
      Y(J)=(X(K+1)-X(K))*DTD+X(K)
      T=T+DTX
      IF(J .GE. L) GO TO 25
      GO TO 1
C     THIS BRANCH PERFORMS CONTINUOUS STRAIGHT LINE
C     INTERPOLATION BETWEEN TWO VALUES OF X
    8 TH(1)=0.0
      T=0.0
      J=0
      Y(1)=0.0
    9 T=T+DTX
      J=J+1
      DO 10 I=1,L
      IF(T .LE. TH(I+1)) GO TO 15
   10 CONTINUE
      Y(J)=0.0
      GO TO 20
   15 PDX=(T-TH(I))/(TH(I+1)-TH(I))
      Y(J)=X(I)+(X(I+1)-X(I))*PDX
   20 IF(J .GE. LIM) GO TO 25
      GO TO 9
   25 RETURN
      END
```

```
      SUBROUTINE SIMSON (DT,N,Y,SUM)
C     THIS SUBROUTINE CARRIES OUT THE INTEGRATION
C     ACCORDING TO SIMPSON'S RULE
C     Y IS THE FUNCTION TO BE INTEGRATED FROM A TO B
C     APLICATION HERE IS FOR CONTINUITY CHECKS TO
C     DETERMINE THE PROPER TIME STEPS FOR THE COMPUTATIONS
      DIMENSION Y(1)
      COMMON/CATCAR/TCC,TCS,DUR,TRAIN,XI,TDUR,XLAMDA,CUTOFF
      H=DT
      SUM=0.0
      NL=(N/2)-2
      DO 18 I=1,NL
      J=2*I-1
      SUM=SUM+Y(J)+4.*Y(J+1)+Y(J+2)
   18 CONTINUE
   19 SUM=SUM*H/3.0
   20 WRITE(6,100) SUM
  100 FORMAT (/15X,'INTEGRATED AREA UNCER CURVE ='E10.4)
      RETURN
      END
```

```
      SUBROUTINE ERROR (G,GP,N,DT)
C     THIS SUBROUTINE COMPUTES THE INTEGRABLE SQUARE ERROR
C     AND THE CORRELATION COEFFICIENT BETWEEN THE TIMES SERIES
C     G (ASSUMED TO BE THE NORM) AND GP
C     APPLICATION HERE IS FOR THE FITTING PROCESS
      DIMENSION G(1),GP(1)
      SUMA=0.0
      SUMB=0.0
      SUMC=0.0
      SUMD=0.0
      SUME=0.0
      N2=N/2+0.5
      DO 18 I=1,N
      SUMA=SUMA+G(I)**2
      SUMB=SUMB+G(I)*DT
      SUMC=SUMC+((GP(I)-G(I))**2)*DT
      SUMD=SUMD+GP(I)**2
   18 SUME=SUME+G(I)*GP(I)
   19 SUMX=ABS((2.0*SUME-SUMD)/SUMA)
      ERRORS=SQRT(SUMC)*100./SUMB
      CORR=SQRT(SUMX)
      WRITE(6,100) ERRORS,CORR
  100 FORMAT(///,10X'INTEGRAL SQUARE ERROR ='F8.3,/,10X,
     1'CORRELATICN COEFFICIENT ='F8.4/)
      RETURN
      END
```

```
      SUBROUTINE PLOT(NO,B,N,M,NL,NS,KX,JX)
C     THIS SUBROUTINE IS A GENERAL PRINTER-PLOTTER
      DIMENSION OUT(101),YPR(11),IANG(9),A(200),B(KX,JX)
      INTEGER IDUM/'1'/,IANG/'.','*','+','-','A','B',
     1'C','D','E'/
      INTEGER OUT
      I=1
      DO 39 J=1,M
      DO 39 K=1,N
      A(I)=B(K,J)
      I=I+1
 39   CONTINUE
    2 FORMAT(1H ,F11.3,5X,101A1)
    3 FORMAT(1H )
    7 FORMAT(/,16X,'.         .         .         .'
     1'         .         .         .         .'
     2'         .         .         .')
    8 FORMAT(1H0,9X,11F10.3,//60X'DISCHARGE-CFS'///)
      NLL=NL
      IF(NS) 16, 16, 10
   10 DO 15 I=1,N
      DO 14 J=I,N
      IF(A(I)-A(J)) 14, 14, 11
   11 L=I-N
      LL=J-N
      DO 12 K=1,M
      L=L+N
      LL=LL+N
      F=A(L)
      A(L)=A(LL)
   12 A(LL)=F
   14 CONTINUE
   15 CONTINUE
   16 IF(NLL) 20, 18, 20
   18 NLL=50
   20 BLANK=0
      FL=FLOAT(NLL-1)
      IF(FL .LT. 0.010) GO TO 95
      XSCAL=(A(N)-A(1))/FL
      YMIN=1.0E75
      YMAX=-1.0E75
      M1=N+1
      M2=M*N
      DO 40 J=M1,M2
      IF (A(J) .GT. YMAX) YMAX=A(J)
      IF (A(J) .GT. YMAX) YMAX=A(J)
      IF (A(J) .LT. YMIN) YMIN=A(J)
   40 CONTINUE
      YSCAL=(YMAX-YMIN)/100.0
      XB=A(1)
```

```
      L=1
      MY=M-1
      DO 80 I=1,NLL
      F=I-1
      XPR=XB+F*XSCAL
   50 DO 55 IX=1,101
   55 OUT(IX)=BLANK
      DO 60 J=1,MY
      LL=L+J*N
      JP=((A(LL)-YMIN)/YSCAL)+1.0
      OUT(JP)=IANG(J)
   60 CONTINUE
      WRITE(6,2) XPR,(OUT(IZ),IZ=1,101)
      L=L+1
      GO TO 80
   70 WRITE(6,3)
   80 CONTINUE
      WRITE(6,7)
      YPR(1)=YMIN
      DO 90 KN=1,9
   90 YPR(KN+1)=YPR(KN)+YSCAL*10.0
      YPR(11)=YMAX
      WRITE(6,8)(YPR(IP),IP=1,11)
   95 RETURN
      END
```

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