# Overcoming the Manley-Rowe Limit for CW Terahertz Generation in $Q$-Engineered Multimodal Cavity

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**Abstract:** We present a method to overcome the Manley-Rowe limit in a $Q$-factor engineered multimodal nonlinear cavity. Cascading nonlinear processes enable continuous-wave terahertz generation with a theoretical conversion efficiency of 98.8%. 

Nonlinear frequency conversion has enabled the development of many light sources, ranging from the UV to the far-infrared wavelengths, i.e. terahertz (THz). For difference-frequency generation, the maximal energy conversion efficiency $\eta_e$ is given by the Manley-Rowe (MR) limit. In this optimal scenario, every high-energy pump photon ($\omega_0 = 2\pi \times 200\,\mathrm{THz}$) will simultaneously generate a low-energy THz photon ($\omega_T = 2\pi \times 1\,\mathrm{THz}$) and an idler photon $\omega_i$, i.e. $\omega_T = \omega_0 - \omega_i$. The maximum conversion efficiency $\eta_e$ is then given by the ratio of the THz to the pump photon energies: $\eta_e = \omega_T/\omega_0 = 0.5\%$.

The idea of recycling the idler photons to initiate subsequent cascaded nonlinear processes was proposed to overcome the MR limit [1–3]. However, competing red-shifted (THz creation) and blue-shifted (THz annihilation) processes strongly limit the possible net gain in THz conversion efficiency [2], see Fig. 1(b).

In this work, we propose a scheme for highly-efficient THz generation in an asymmetric multimodal cavity. We show that by engineering the $Q$-factors of the cavity's many modes, one can find a scenario where red-shifted cascaded nonlinear processes strongly dominate over their blue-shifted counterparts, resulting in a quasi-complete depletion of the pump energy into the THz mode. Our method enables THz conversion efficiencies as high as 98.8%. We also discuss attainable efficiencies in a realistic scenario by considering experimental parameters such as cavity dimension, nonlinear crystal, dispersion, and frequency detuning.

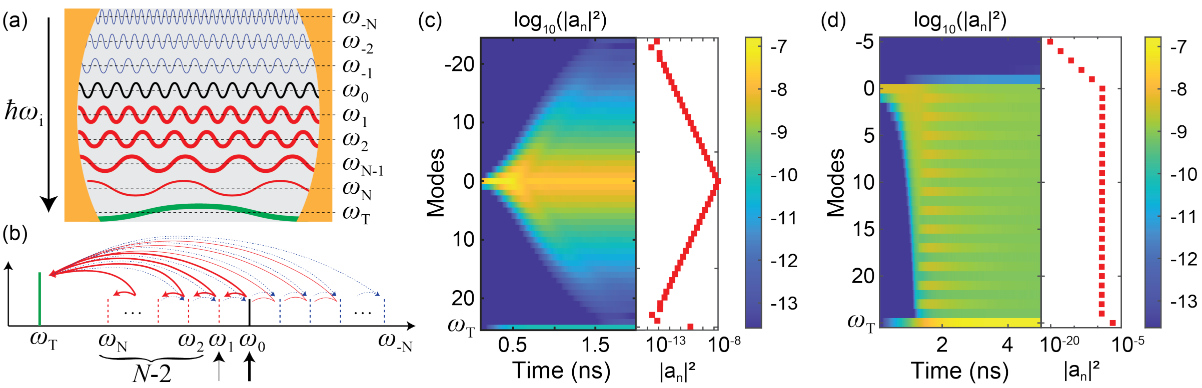


Fig. 1. (a) Highly multimodal cavity. (b) Red and blue-shifted cascaded nonlinear mode mixing. (c) Uniform $Q$ distribution: $Q_\mathrm{n} = 10^5$. Conversion efficiency $\eta_e = 0.05\%$. (d) Asymmetric $Q$ distribution: $Q_{-\mathrm{N},\ldots,-1} = Q_b = 10^2, Q_{1,\ldots,\mathrm{N}\text{-}1} = Q_r = 10^5, Q_0 = Q_\mathrm{N} = 1.3 \times 10^3$. Conversion efficiency $\eta_e = 9.7\%$.

The system under consideration is depicted in Fig.1(a). The nonlinear cavity supports $2N+1$ modes centered around the pump frequency $\omega_0$, and equally spaced by the THz frequency $\omega_T$. Figure 1(b) shows the multi-frequency mixing with red and blue-shifted cascading orders. The time-evolution of the system can be described by a set of nonlinear coupled-mode equations [4]:

$$\dot{a}_n = i(\omega_n + \delta\omega_n)a_n - \gamma_n a_n - i\omega_n\beta_n a_\mathrm{T} a_{n+1} - i\omega_n\beta_n a_\mathrm{T}^* a_{n-1} + \sqrt{2\gamma_n}s_n^+\,, \qquad (1)$$

where $a_n$ is the amplitude of the mode with index $n \in [-N,N]$, with frequency $\omega_n$, detuned by $\delta\omega_n$, decay rate $\gamma_n = \omega_n/2Q_n$ and effective nonlinear coupling coefficient $\beta_n$. The mode $n$ is pumped with input power $s_n^+$. Only modes 0 and 1 are pumped with $s_0^+ \gg s_1^+$ (arrows in Fig. 1(b)). The mode amplitudes are normalized such that $|a_n|^2$ is the energy in mode $n$. Additionally, the THz mode amplitude is given by $\dot{a}_\mathrm{T} = i\omega_\mathrm{T}a_\mathrm{T} - \gamma_T a_\mathrm{T} - i\omega_\mathrm{T}\beta_\mathrm{T}\sum_{n=-N}^{N-1} a_n a_{n+1}^*$. We first consider an ideal situation where we neglect frequency detuning $\delta\omega_n$. It is known that by engineering the phase-matching between blue and red-shifted modes, one can favor red-shifted cascaded processes [2]. We make a similar observation via $Q$-engineering the multimodal cavity, by considering low-$Q$ blue-shifted modes $Q_b$ and high-$Q$ red-shifted modes $Q_r$ (except the modes with indices 0 and $N$, which are assumed to have a lower $Q$). Figure 1(c) and (d) show the time-evolution (left panel) and steady-state (right panel) of the mode energies $|a_n|^2$ for a uniform and asymmetric system, respectively. While a uniform $Q$-factor distribution results in a symmetrical energy decay around the pump mode ($\eta_e = 0.05\%$), the asymmetric $Q$ distribution favors red-shifted processes, thus resulting in a significant conversion efficiency enhancement ($\eta_e = 9.7\%$).

With a $Q$-engineered multimodal cavity of $N = 199$ red-shifted modes ($\omega_0 = 2\pi \times 200\,\text{THz}, \omega_T = 2\pi \times 1\,\text{THz}$), a theoretical THz conversion efficiency as high as 98.8% is achieved. Figure 2(a) shows how most of the energy flows through the red-shifted modes, contributing to a THz photon generation at each step. At steady state the photon efficiency shows that a single pump photon produced 197 $\omega_T$ photons ($\eta_{ph} = 19{,}700\%$) and 1 $\omega_N$ photon, see Fig. 2(a). This is a quasi-full depletion of the pump energy into the THz mode and almost a 200-fold enhancement compared to the conventional MR limit when considering a single difference frequency generation process.

We then consider an experimentally realistic scenario with $N = 24$ red-shifted modes, corresponding to an optical bandwidth of 166 nm for pump wavelength $\lambda_0 = 1.55\,\mu\text{m}$ and THz frequency $\omega_T = 2\pi \times 1\,\text{THz}$. We assume a bow-tie free-space optical cavity with length 0.6 m and a 10 µm $LiNbO_3$ crystal. Figure 2(b) shows the time evolution of the photon efficiency. We predict a photon efficiency of $\eta_{ph} = 2160\%$ at a pump power $P_0 = 100\,\text{W}$, corresponding to an energy conversion efficiency of $\eta_e = 11.2\%$. This is more than 20 times the MR limit.

Interestingly, the system under study is equivalent to a resonator chain coupled along a synthetic dimension in the tight-binding approximation. Modes 0 to $N-1$ are coupled along the synthetic dimension corresponding to the mode frequency $\omega$. In real space, such tight-binding configurations are known to be efficient waveguiding systems [5]. Assuming fast decay of the THz signal compared to the red-shifted IR modes, only nearest-neighbor modes can couple along this synthetic dimension through an effective nonlinear coefficient $\kappa = -i\omega_n\beta_n a_\text{T} \ll Q_r$. In our scheme, the pumping and $Q$ distribution are chosen to favor the excitation of a forward propagating mode along the mode index direction $N$ (towards decreasing values in synthetic frequency space $\omega$). Importantly, the mode $N$ at the end of the chain is chosen to minimize reflection at the boundary of the synthetic dimension with $\gamma_N \sim \kappa$ (since a backward propagating wave would result in THz annihilation).

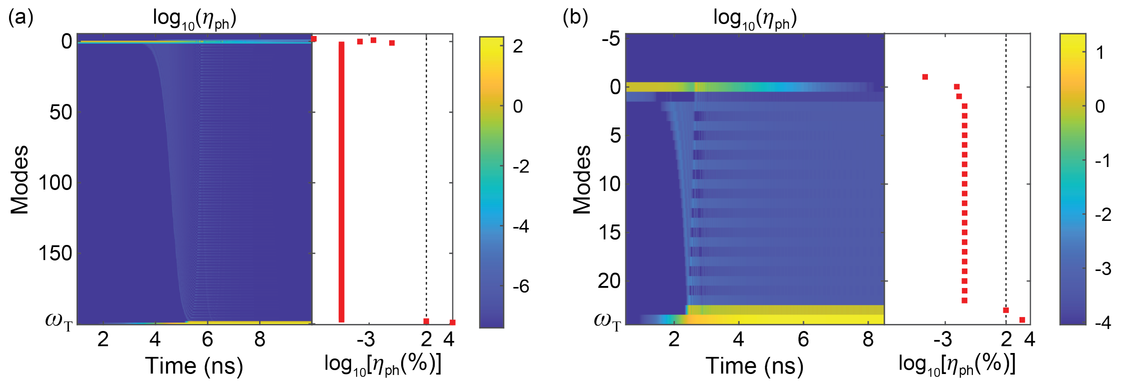


Fig. 2. (a) Ideal scenario with $N = 199$: $Q_{-\text{N},\dots,-1} = 1, Q_{1,\dots,\text{N-1}} = 10^{10}, Q_0 = Q_\text{N} = 4 \times 10^2$. (b) Realistic scenario with $N = 24$: $Q_{-\text{N},\dots,-1} = 30, Q_{1,\dots,\text{N-1}} = 10^7, Q_0 = Q_\text{N} = 5 \times 10^3$. Efficiency $\eta_e$ for (a) and (b) is respectively 98.8% and 11.2%.

In conclusion, we propose a novel scheme for highly-efficient THz generation in $Q$-engineered multimodal cavities. By engineering the $Q$-factor distribution, one can favor nonlinear processes involving red-shifted modes and achieve almost complete depletion of the pump infrared modes into a THz mode. Our findings open the way to highly-efficient continuous-wave THz generation with realistic experimental parameters.

## References


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