INTENSITY DISCRIMINATION IN AUDITION

by

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ABSTRACT

The perception of intensity has traditionally been studied in a variety of discrimination, identification, and scaling experiments. The overall picture of the field is quite fragmented, however, in part because the different investigations have been concerned with different aspects of intensity perception. Our ultimate goal is a unified quantitative theory of intensity perception applicable to a wide variety of experiments that will enable us to understand the relationships among the various types of investigations. Our initial work in this area has led to a preliminary theory of intensity resolution which takes account of both sensory and memory limitations on performance.

This research is concerned with evaluating the preliminary theory with respect to two-interval paradigms and the relation between one- and two-interval paradigms. It is also concerned with examining certain phenomena which are evident in these paradigms, but are not accounted for by the preliminary theory. These phenomena, which we call the resolution and bias edge-effects, are characterized by the fact that performance varies over the intensity range of a given experiment.

The research consists of a set of two-interval, roving-level, discrimination experiments and a supplementary set of one-interval identification experiments. The experimental results are generally consistent with the preliminary theory, although several significant discrepancies are noted. In addition, a revision of the preliminary theory is developed which has some success in accounting for the resolution edge-effect, and several attempts are made at revising the theory to account for the bias edge-effect.

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I. INTRODUCTION

The study of the perception of intensity has revealed a wide variety of interesting phenomena and produced an abundance of detailed experimental results. Some investigations, for example, have been concerned with discrimination: the ability of observers to distinguish between two intensities that are close together. The main classical results in discrimination include Weber's law (i.e., the ability to discriminate between two intensities depends only on their ratio, regardless of their absolute level), and the fact that over a large portion of the dynamic range, two intensities can be easily distinguished if they are at least about 1 dB apart. Other investigations have been concerned with identification: the ability of observers to identify a particular intensity when it is selected at random from a large set of intensities. The classical result in identification concerns confusions made among the intensities, and is stated in terms of an upper bound on information transfer of roughly 2 - 3 bits. Still other investigations have been concerned with loudness scales: the growth of subjective loudness L with intensity I. A variety of different scales have been proposed. Each appears to have its drawbacks, however, and there have been many disagreements as to the validity of the various scales. The two most well known scales are the power law scale (i.e., $L = I^{0.3}$) obtained from ratio scaling experiments, and the logarithmic scale [i.e., $L = K \log(I)$] obtained from category scaling experiments.

Because the different investigations have been concerned with different aspects of intensity perception, the overall picture of the field has become quite fragmented. The field is strongly deficient in unifying theory. For example, it is not presently understood why observers do so well in discriminating two intensities, and yet so poorly in identifying them when they are part of a large set of intensities (the 7 $\pm$ 2 problem[1]). Similarly, although it may be presumed that loudness constitutes the primary subjective cue used by observers in identification and discrimination as well as in scaling, the relations among the results of these different types of experiments are not understood. In addition, in any one area, different paradigms, which appear to measure the same thing, often yield different results.

Our ultimate goal is to construct a unified quantitative theory of intensity perception applicable to any intensity discrimination, identification, or scaling experiment. Such a theory should enable us to understand the relationships among the various types of investigations and among the various types of paradigms.

We decided to begin our work with a study of intensity resolution (i.e., the ability of an observer to distinguish between different stimuli as evidenced by his responses). This meant we had to factor out the observer's response bias (i.e., his a priori probability of using the various responses). Although resolution was only part of the picture, we felt that it was an important part, and that it was an appropriate place to begin. In addition, we decided that it was essential to study the role of memory. We felt that memory played an important role in almost every paradigm used to study intensity perception, and that its role was quite different in different paradigms.

Initial work, based on this strategy, led to a preliminary theory of intensity resolution which takes account of both sensory and memory limitations on performance. Although preliminary, it is intended to apply to a variety of experimental situations (including identification, discrimination, and scaling paradigms), and to predict the dependence of resolution on a variety of parameters (including the number of stimuli, the intensity range covered by the stimuli, and the length of time the various stimuli must be remembered). Detailed descriptions of this preliminary theory as well as the related experimental studies, can be found in Durlach and Braida (1969); Braida and Durlach (1972); Pynn, Braida, and Durlach (1972); and Berliner and Durlach (1973). Further descriptions are also contained in later sections of this thesis.

During the past several years, a variety of experiments have been conducted to evaluate the preliminary theory and to guide its further development. Whereas the majority of this experimental work has involved one-interval paradigms, this thesis has been focused on two-interval paradigms and on the relation between performance in one- and two-interval paradigms. Certain basic aspects of the preliminary theory regarding these paradigms required testing. In addition, certain phenomena evident in these paradigms, but not accounted for by the preliminary theory, required close examination and incorporation into a revised theory.

These phenomena, characterized by the fact that performace varies over the intensity range of a given experiment, include the resolution edge-effect (i.e., the tendency of observers to exhibit better resolution near the edges of the range than in the middle of the range) and the bias edge-effect (i.e., the tendency of observers to exhibit strong and opposite response biases near the edges of the range and little response bias near the middle of the range). We considered these phenomena to be worth investigating because, while they were neither well understood nor accounted for by the preliminary theory, they were known to be large and consistent. Furthermore, both effects appeared closely related to the underlying memory process; thus an understanding of them would hopefully yield a better understanding of the memory process. This interest in the bias edge-effect indicates a departure from our initial strategy of focusing only on resolution, while factoring out and disregarding response bias. We considered this a worthwhile change, since a complete theory of intensity perception would necessarily deal with bias as well as resolution.

In this thesis, significant progress has been made in these areas. A set of experiments has been conducted to further evaluate the preliminary theory and to more closely examine the phenomena of interest. The experimental results have generally agreed with the predictions of the preliminary theory, although several significant discrepancies have been noted. Furthermore, the resolution and bias edge-effects have been examined under a variety of experimental conditions. In addition to this experimental work, theoretical work has also been done. A revision of the preliminary theory that has some success in accounting for the resolution edge-effect has been developed. In addition, several attempts have been made at revising the theory to account for the bias edge-effect. As yet, none of these has been entirely successful, although it appears that with a bit more work, one will eventually have success.

The material presented in the balance of this thesis is divided into the following chapters. In Chapter II, the basic concepts of the preliminary theory are presented. In Chapter III, the two experimental paradigms employed in this research are described and the reasons for choosing them are discussed. In Chapter IV, the details of the preliminary theory are presented. This presentation includes the assumptions and predictions of the preliminary theory, as well as a discussion of its relation to other theoretical work. In Chapter V, previous related experimental work is reviewed. In Chapter VI, the set of experiments conducted in this research is described, and the reasons for each experiment are discussed. In addition, the data analysis procedure is outlined. In Chapter VII, the experimental results of this thesis, as well as certain relevant data from other sources, are examined in three parts. Resolution averaged over intensity levels is examined first, followed by resolution as a function of level, and by criterion as a function of level. In Chapter VIII, recent theoretical work regarding the resolution and bias edge-effects is presented. Finally, in Chapter IX, the experimental and theoretical results of this thesis are summarized, and the implications for future work are discussed.

II. THE BASIC CONCEPTS OF THE PRELIMINARY THEORY OF INTENSITY RESOLUTION

During the past six years, a preliminary theory of intensity resolution has been developed primarily by N. I. Durlach and L. D. Braida, with inputs from myself and others in our research group. This theory attempts to take account of both sensory and memory limitations on resolution performance, and is applicable to certain one- and two-interval paradigms. For a quantitative description of the assumptions and implications of the theory, see Section IV; for a more complete and rigorous description of the preliminary theory, see Durlach and Braida (1969).

The preliminary theory consists of two parts: a <u>decision model</u> and an <u>internal-noise model</u>. The decision model is similar to the standard signal-detection theory model (Green and Swets, 1966). Each trial of an experiment results in a value of a random variable called the <u>decision variable</u>. Different stimuli lead to different conditional probability density functions of the decision variable. The observer bases his response on the relation of the value of the decision variable to certain <u>criteria</u> on the decision axis. The conditional response probabilities are determined by the integrals of the conditional probability density functions between the various criteria. The <u>response bias</u> is determined by the criteria and the <u>resolution</u> by the separation of the density functions.

The internal-noise model relates the resolution to the stimuli and the paradigm by describing the transformation from the stimuli to the decision variable. It is assumed that this transformation is composed of two noisy transformations: from the stimuli to the sensations, and from the sensations to the decision variable. Noise introduced in the first transformation is called sensation noise and is assumed to depend on the stimulus and the observer, but not on the experimental paradigm. It provides an upper bound on the resolution performance. Noise introduced in the second transformation is called memory noise. It accounts for limitations imposed by imperfect memory.

In considering how observers might try to reduce their memory noise, we thought of two strategies they might use. (Our thinking here was influenced both by our subjective experience as experimental observers and by the work of others, including Neisser and Pollack.)[2] These strategies correspond in our model to two modes of memory operation and two corresponding types of memory noise. In one mode, the sensory-trace mode (the model for which is essentially the same as the diffusion model proposed by Kinchla and Smyzer, 1967), the observer tries to remember a sensation itself. The memory noise in this mode comes from his memory of the sensation (the trace) wandering as time passes. In the other mode, the context-coding mode, the observer compares a sensation with the general context of sounds in the experiment, and remembers a verbal description of the comparison (called the verbal code). In the model, the verbal code is actually a numerical code and we can refer to its probability density. The memory noise in this mode comes from not remembering the context consistently, or from not making a precise comparison. The amount of noise depends on the width (i.e., the observer's perception of the range) of the context, larger widths leading to greater noise. The advantage of the sensory-trace mode is that the width of the context makes no difference. The disadvantage is that the memory noise increases as time passes following the stimulus. The advantage of the context-coding mode is that the observer can remember the code for as long as he likes. The disadvantage is that the memory noise increases as the size of the context increases.

III. EXPERIMENTAL PARADIGMS

A. The Main Experimental Paradigm:

Two-Interval Roving-Level Discrimination

In selecting an experimental procedure, the primary concern was to examine both sensory and memory factors in the perception of intensity. Thus, we wanted to force the observer to make decisions and responses based on remembered stimuli as well as current stimuli. Furthermore, we wanted to be able to vary the memory burden placed on the observer. These considerations led us to choose for the primary set of experiments in this thesis, the two-interval roving-level discrimination paradigm. In addition, a supplementary set of experiments was conducted using the one-interval identification paradigm. A discussion of the suitability of these paradigms is presented following the descriptions of the paradigms.

The format of the two-interval roving-level discrimination paradigm is shown in Figure 3.1. The signals are pulsed tones that are identical except for intensity. On each trial the observer is presented with a pair of tone pulses separated by a fixed time, T, and is required to judge which member of the pair is more intense. The term "roving-level" refers to the change in the average intensity level of the tone pairs from trial to trial. During an experimental run, the average level is allowed to vary over a fixed range, R.

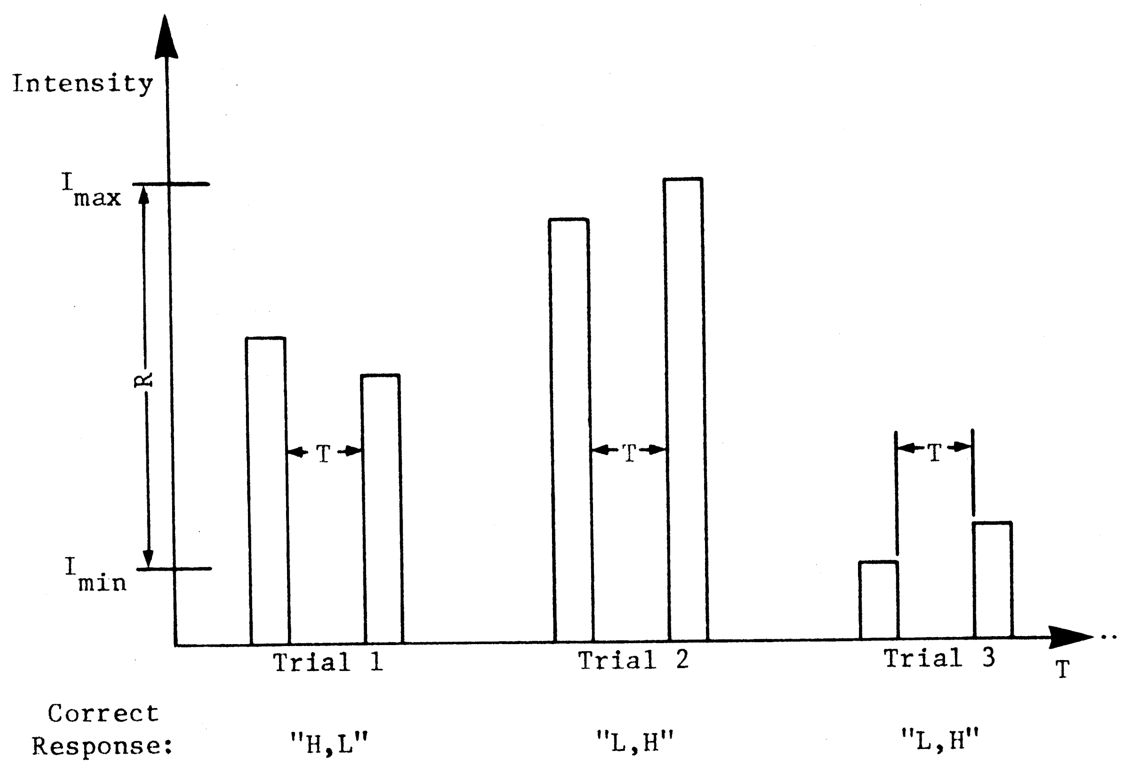
The stimulus set is composed of N discrete tone pairs. The more intense tone of each pair is denoted I_i, and the less intense tone is denoted I_i^*. Since we can have either (I, I^*) or (I^*, I) for each of

Figure 3.1

Format of the

Two-Interval Roving-Level

Discrimination Paradigm



the N levels, there are a total of 2N stimuli. On each trial the observer's response is either "H, L" (indicating that he thinks it is an $[I, I^*]$ trial) or "L, H" (indicating that he thinks it is an $[I^*, I]$ trial).[3]

The two-interval roving-level discrimination paradigm was considered to be particularly suitable for this research for a number of reasons. First, performance was clearly related to memory since the observer was forced to remember the intensity of the first tone during the interpulse period T. Second, the roving-level feature (as opposed to the fixed-level case, in which N = 1 and the observer is exposed to only two intensities) prevented the observer from establishing noise-free verbal codes of the intensities and thus diluting the effect of T. (The advantages of this feature in the study of memory have been pointed out previously by Harris and Pollack.[4] Third, by independently adjusting the parameters T and R (both of which are known to be important in determining resolution), one can not only continuously vary the overall memory load, but one can separate out different aspects of the memory problem. Fourth, the preliminary theory of intensity resolution has a number of important predictions for performance in this paradigm that had not yet been adequately tested. Fifth, and finally, the particular phenomena that constitute the primary focus of this thesis (the "edge-effects" mentioned in the Introduction) are strongly evident in this paradigm, so that the paradigm provided a useful framework for exploring these phenomena.

B. The Supplementary Experimental Paradigm:

One-Interval Identification

The format of the one-interval identification paradigm with and without a reference stimulus is shown in Figure 3.2. Again, the signals are pulsed tones that are identical except for intensity. On each trial, the observer is presented with either a fixed reference tone pulse followed by a test tone pulse, or only a test tone pulse, and is required to judge the identity of the test tone. The test tone is one member of a stimulus set composed of N discrete tones with intensities $\{I_i\}$, where $I_i < I_{i+1}$, restricted to a fixed range R. The observer's response is an integer from 1 to N, with response i corresponding to the stimulus I_i.

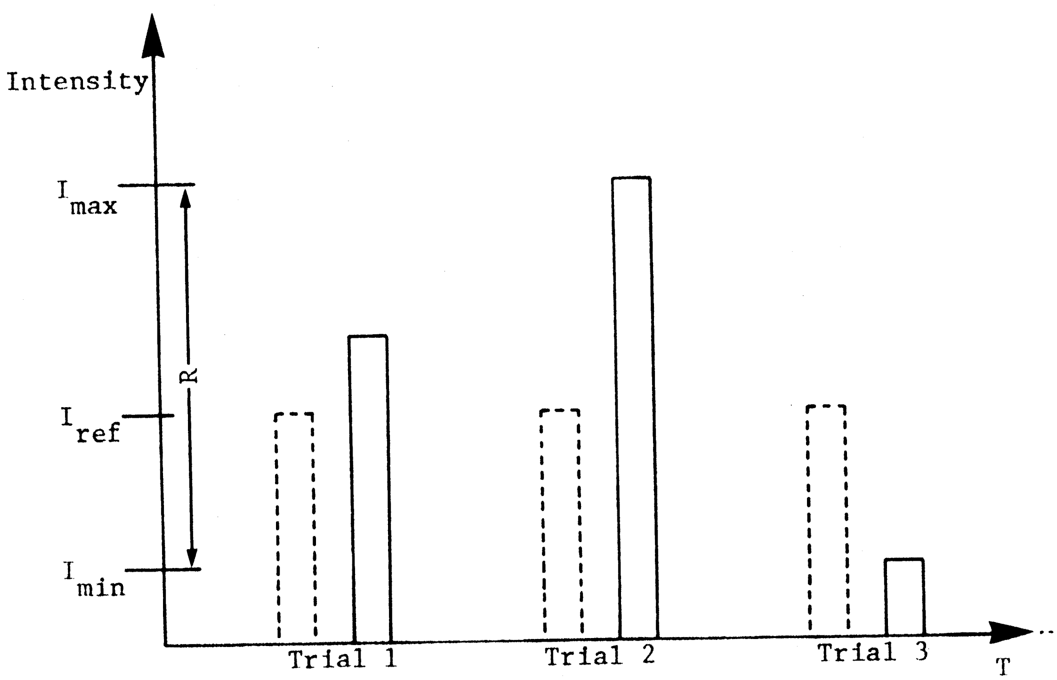
The supplementary one-interval identification experiments were conducted for several reasons. First, certain predictions of the preliminary theory concerning the relation between performance in one-interval and two-interval paradigms had not yet been adequately tested. Second, since the resolution edge-effect is strongly evident in this paradigm, it provided an additional means of examining the effect. Third, our initial interpretation of the resolution edge-effect and preliminary experimental results led us to expect that the inclusion of a reference stimulus on each trial of an identification experiment would improve resolution in the vicinity of the reference intensity. By measuring this improvement, we hoped to gain additional insight into the resolution edge-effect.

<u>Figure 3.2</u>

Format of the

Identification Paradigm

With and Without a Reference Stimulus



IV. PRESENTATION OF THE PRELIMINARY THEORY OF INTENSITY RESOLUTION [5]

A. Assumptions and Equations

1. Sensations

First, we assume there is a one-dimensional random variable Y, which we call the sensation variable. It has the property that each stimulus presentation leads to a particular value of Y. Different intensities lead to different probability density functions of Y. The conditional probability density function $p(Y|I)$ is assumed to be Gaussian with mean $\alpha(I) = K \log(I)$, and variance $= \beta^2$ (independent of I). We assume $\alpha(I) = K \log(I)$ in order to be consistent with Weber's law (i.e., so that the resolution between intensities I and CI is independent of I).

2. Trace Mode

In the trace mode, we assume that the sensation Y resulting from intensity I leads to a trace $\overline{Y}(t)$ that is a random function of t (where t denotes the time elapsed since the presentation of I). The conditional probability density function $p(\overline{Y}(T)|Y)$ at time $t = T$ is assumed to be Gaussian with mean = Y and variance = 2AT.[6] Thus the conditional probability density function $p(\overline{Y}(T)|I)$ is Gaussian with mean $= K \log(I)$, and variance $= \beta^2 + 2AT$.

3. Context-Coding Mode

In the context-coding mode, we assume that the sensation Y is compared with the context of sensations arising from previous stimuli, and the comparison leads to a numerical representation Q, the verbal code. The conditional probability density function $p(Q|Y)$ is assumed to be

Gaussian with mean = Y and variance = H^2W^2, where H is a constant and W is the width of the context. If we assume that W is given by $\alpha(I_{max})-\alpha(I_{min}) = K \log(I_{max}/I_{min})$, and if we define R, the intensity range of the experiment, by $R = \log(I_{max}/I_{min})$ and $G = HK$, then the variance of $p(Q|Y)$ is G^2R^2. Thus the conditional probability density function $p(Q|I)$ is Gaussian with mean = $K \log(I)$, and variance = $\beta^2 + G^2R^2$.

4. Decision Process in Two-Interval Paradigms

In the two-interval paradigms, the observer may operate in the trace mode or the context-coding mode, or he may combine the two modes. In the case where the modes are combined, it is assumed that they are carried out in parallel, with no mutual degradation. In the trace mode, the observer is assumed to form the decision variable X_T by subtracting the sensation arising from the second interval from the trace of the sensation arising from the first interval:

$$X_T = \overline{Y}_1(T) - Y_2 , \tag{4.1}$$

where T is the time between the two intervals (the interpulse interval). Thus the conditional probability density functions $p(X_T|I_i, I_i^*)$ and $p(X_T|I_i^*, I_i)$ are Gaussian with means = $K \log(I_i/I_i^*)$ and $K \log(I_i^*/I_i)$, and variances $\sigma_2^2 = 2(\beta^2 + AT)$.

In the context-coding mode, the observer is assumed to form the decision variable X_Q by subtracting the verbal code arising from the second interval from the verbal code arising from the first:

$$X_Q = Q_1 - Q_2 . \tag{4.2}$$

Thus the conditional probability density functions $p(X_Q|I_i, I_i^*)$ and $p(X_Q|I_i^*, I_i)$ are Gaussian with means $= K \log(I_i/I_i^*)$ and $K \log(I_i^*/I_i)$, and variances $\sigma_2^2 = 2(\beta^2 + G^2R^2)$.

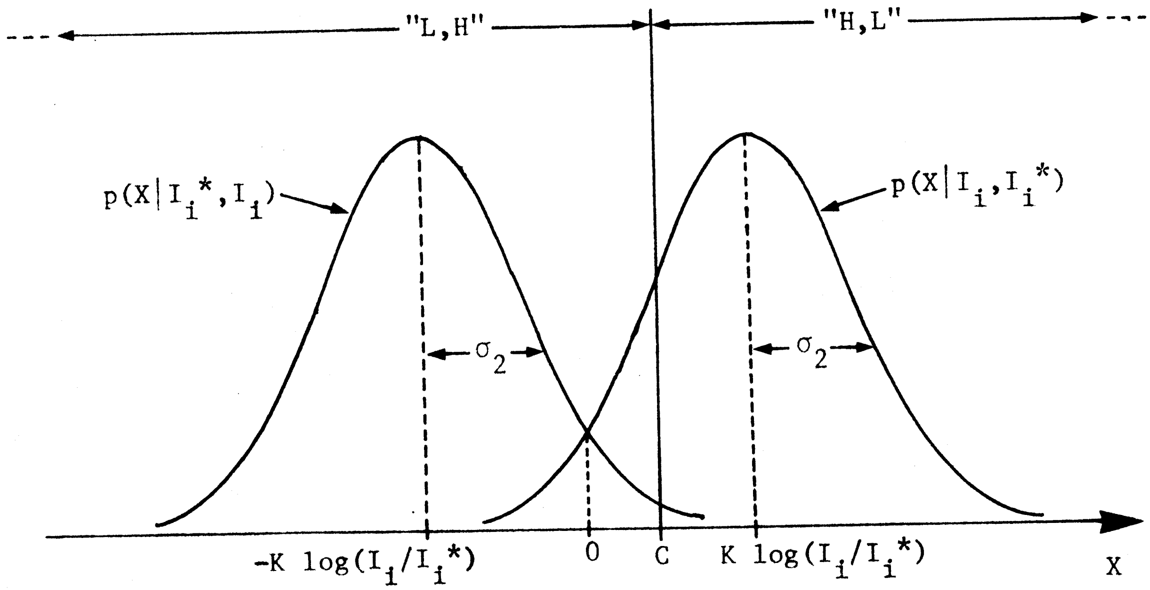
Regardless of the mode, the decision axis may be drawn as in Figure 4.1. On each trial, the decision is made by comparing the value of the decision variable X with a criterion C.[7] The observer responds "L, H" if and only if $X \leq C$, and responds "H, L" otherwise. The various conditional response probabilities are given by the integrals of the conditional probability density functions to the left and right of C. For example:

$$p(\text{"L, H"}|I_i, I_i^*) = \int_0^C p(X|I_i, I_i^*)\, dX. \tag{4.3}$$

5. Decision Process in One-Interval Paradigms

In one-interval paradigms, the observer is assumed to operate in the context-coding mode, and the decision variable X is merely the verbal code Q. Thus the conditional probability density function $p(X|I_i)$ is Gaussian with mean $= K \log(I_i)$, and variance $\sigma_1^2 = \beta^2 + G^2R^2$.

The decision axis may be drawn as in Figure 4.2. There are N conditional probability density functions $p(X|I_i)$ and M + 1 ordered criteria, $-\infty = C_0 < C_1 < \ldots < C_{M-1} < C_M = \infty$ on the decision axis. On each trial, the decision is made by comparing the value of X with the criteria; the observer responds "R_m" if and only if $C_{m-1} < X \leq C_m$. In the one-interval identification paradigm with which we are concerned in this thesis, the number of stimuli equals the number of responses (N = M) and the response R_m is merely the number m ($R_m = m$).



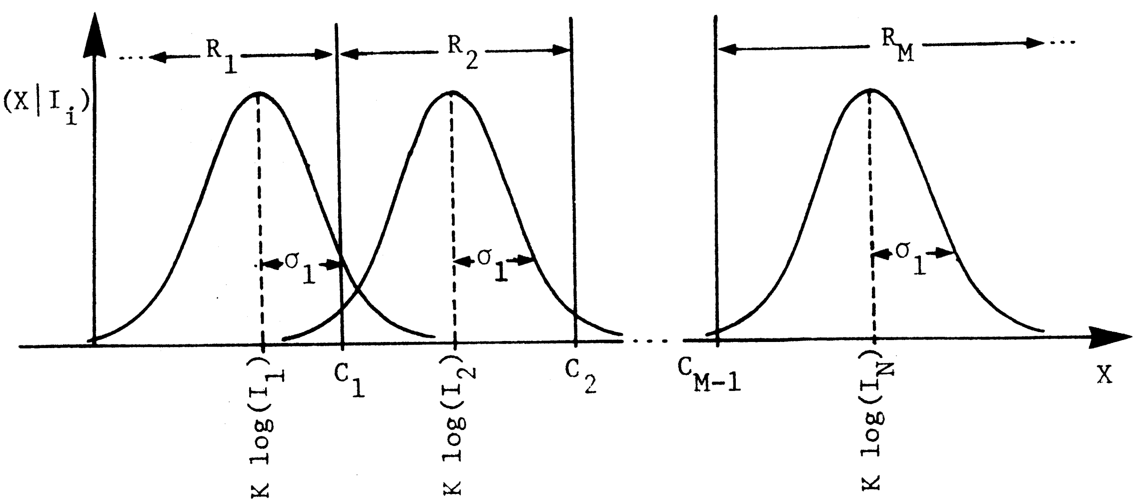
Trace Mode: $\sigma_2^2 = 2(\beta^2 + AT)$

Context-Coding Mode: $\sigma_2^2 = 2(\beta^2 + G^2R^2)$

Figure 4.1

The Decision Axis

in Two-Interval Paradigms



$$\sigma_1^2 = \beta^2 + G^2R^2$$

Figure 4.2

The Decision Axis

in One-Interval Paradigms

The various conditional response probabilities are given by the integrals of the conditional probability density functions between the appropriate criteria. Thus:

$$p(\text{"}R_m\text{"}|I_i) = \int_{C_{m-1}}^{C_m} p(X|I_i)\, dX. \tag{4.4}$$

6. Measure of Resolution

In either paradigm, the observer's resolution between two stimuli S_i and S_j is determined by the separation of the two conditional probability density functions $p(X|S_i)$ and $p(X|S_j)$. The measure of separation, and thus resolution, that we use is d', which is defined to be the difference between the means of the density functions divided by their common standard deviation σ:

$$d'(S_i;\ S_j) = \frac{E[X|S_i] - E[X|S_j]}{\sigma} \quad . \tag{4.5}$$

This measure of resolution, unlike percent correct for example, has the advantage of being independent of the criteria.

In the two-interval paradigms we can compute d' for both modes:

$$d_2'(I_i,\ I_i^*;\ I_i^*,\ I_i) = \frac{\sqrt{2}\ K\ \log(I_i/I_i^*)}{(\beta^2 + AT)^{1/2}} \text{ (trace mode)}, \tag{4.6}$$

and

$$d_2'(I_i, I_i^*; I_i^*, I_i) = \frac{\sqrt{2}\, K \log(I_i/I_i^*)}{(\beta^2 + G^2R^2)^{1/2}} \tag{4.7}$$

(context-coding mode).

It is assumed, however, that the observer combines information from both modes in an optimum fashion, by constructing the new decision variable X_c, proportional to:

$$X_{Q,T} = \ln[p(X_Q, X_T | I_i, I_i^*)/p(X_Q, X_T | I_i^*, I_i)] \tag{4.8}$$

based on the two variables $X_Q = Q_1 - Q_2$ and $X_T = \overline{Y}_1(T) - Y_2$. In this combined mode it can be shown[8] that the conditional probability density functions of the combined decision variable X_c are Gaussian with means $= K \log(I_i/I_i^*)$ and $K \log(I_i^*/I_i)$, and variances $\sigma_2^2 = 2\left(\beta^2 + \dfrac{1}{(1/AT+1/G^2R^2)}\right)$, and that the resolution is:

$$d_2'(I_i, I_i^*; I_i^*, I_i) = \frac{\sqrt{2}\, K \log(I_i/I_i^*)}{\left(\beta^2 + \dfrac{1}{(1/AT + 1/G^2R^2)}\right)^{1/2}} \tag{4.9}$$

(combined mode).

Since our primary interest concerns the influence of R and T on resolution, and since d_2' is proportional to $\log(I_i/I_i^*)$, we compute the resolution per bel δ_2', given by:

$$\delta'_2 = \frac{d'_2(I_i, I_i^*; I_i^*, I_i)}{\log(I_i/I_i^*)} = \frac{\sqrt{2}\,K}{\left(\beta^2 + \frac{1}{(1/AT + 1/G^2R^2)}\right)^{1/2}} = \frac{2K}{\sigma_2}. \tag{4.10}$$

In the one-interval paradigms, resolution is given by:

$$d'_1(I_i; I_j) = \frac{K \log(I_i/I_j)}{(\beta^2 + G^2R^2)^{1/2}} \tag{4.11}$$

and the resolution per bel δ'_1 by:

$$\delta'_1 = \frac{d'_1(I_i; I_j)}{\log(I_i/I_j)} = \frac{K}{(\beta^2 + G^2R^2)^{1/2}} = \frac{K}{\sigma_1}. \tag{4.12}$$

In addition, we may define a cumulative resolution function $d'(I_i; I_{min})$ as well as the total resolution Δ':

$$\Delta' = d'_1(I_{max}; I_{min}) = \frac{K \log(I_{max}/I_{min})}{(\beta^2 + G^2R^2)^{1/2}} = \frac{KR}{(\beta^2 + G^2R^2)^{1/2}} \tag{4.13}$$

7. Measure of Bias and Criterion

In either paradigm, the observer's response bias is determined by the placement of his criteria. The bias b' corresponding to a particular criterion C is defined as the difference between C and the point where the likelihood ratio is unity, normalized by the standard deviation of the conditional probability density functions. Thus in two-interval paradigms, the bias b' is given by:

$$b' = C/\sigma_2 \ . \tag{4.14}$$

Furthermore, since the resolution per bel δ_2' is $2K/\sigma_2$, the criterion C is determined to within a constant factor by the ratio $2b'/\delta_2'$:

$$C/K = 2b'/\delta_2' \ . \tag{4.15}$$

One interpretation of C/K is to consider it the point of subjective equality for the intensities of the first and second intervals, I_1 and I_2. For two particular intensities, i_1 and i_2, $E[X|i_1, i_2] = K \log(i_1/i_2)$. Thus when $\log(i_1/i_2) = C/K$, the conditional response probabilities $p(\text{"H, L"}|i_1, i_2)$ and $p(\text{"L, H"}|i_1, i_2)$ are equal. For example, if $C/K = 0.3$, then $\log(i_1/i_2)$ would have to be 0.3 (i.e., i_1 would have to be 3 dB greater than i_2), in order to get $p(\text{"H, L"}|i_1, i_2) = p(\text{"L, H"}|i_1, i_2) = 0.5$.

In one-interval identification paradigms, the bias b_i' corresponding to a particular criterion C_i is given by:

$$b_i' = CS_i/\sigma_1 \ , \tag{4.16}$$

where CS_i, the shift of the criterion C_i from the point of unity likelihood ratio, is defined by:

$$CS_i = C_i - (m_i + m_{i+1})/2 \ . \tag{4.17}$$

Furthermore, since the resolution per bel δ_1' is K/σ_1, the criterion shift CS_i is determined to within a constant factor by the ratio b_i'/δ_1':

$$CS_i/K = b_i'/\delta_1' \ . \tag{4.18}$$

One interpretation of CS_i/K is to consider it the shift of the point of subjective equality relative to intensities I_i and I_{i+1}. Thus a special tone of intensity I_s, where $\log(I_s) = CS_i/K + \log(I_i \cdot I_{i+1})/2$, would be identified as "I_i" (or lower) with probability = 0.5, and as "I_i" (or greater) with probability = 0.5. For example, if I_i and I_{i+1} were 60 and 70 dB SPL respectively, and if $CS_i = 0.2$, the I_s would have to be 67 dB SPL (i.e., 2 dB greater than 65 dB SPL), in order to get $p(``I_i" \text{ or lower} | I_s) = p(``I_{i+1}" \text{ or greater} | I_s) = 0.5$.

B. Implications of the Preliminary Theory

Some important implications of the preliminary theory for two-interval, roving-level, discrimination experiments are the following:

(1) The resolution per bel δ_2' is independent of all properties of the stimulus set (such as the number of levels and the intensity ratio being discriminated I_i/I_i^*) except the total intensity range R and the duration T between the two intervals.

(2) If either R or T is small (so that $G^2R^2 << \beta^2$ or $AT << \beta^2$), then $\delta_2' \simeq \sqrt{2}\, K/\beta$ independent of R and T. In this case there is no memory noise; resolution is limited only by sensation noise and is a maximum.

(3) In general, resolution declines faster with T for larger R, and it declines faster with R for larger T.

(4) The precise nature of the dependence of δ_2' on R and T, described by Eq. 4.10, depends on the values of the three parameters K/β, K/G, and $K/\sqrt{A}$. Figure 4.3 illustrates the dependence of $\log(\delta_2')$ on K/β, $R/(K/G)$, $T/(K^2/A)$. The continuous curves have been plotted under the assumption $K/\beta = 16$, and the dashed curves under the assumption $K/\beta = 8$. The numbers on the curves give the values of $R/(K/G)$.

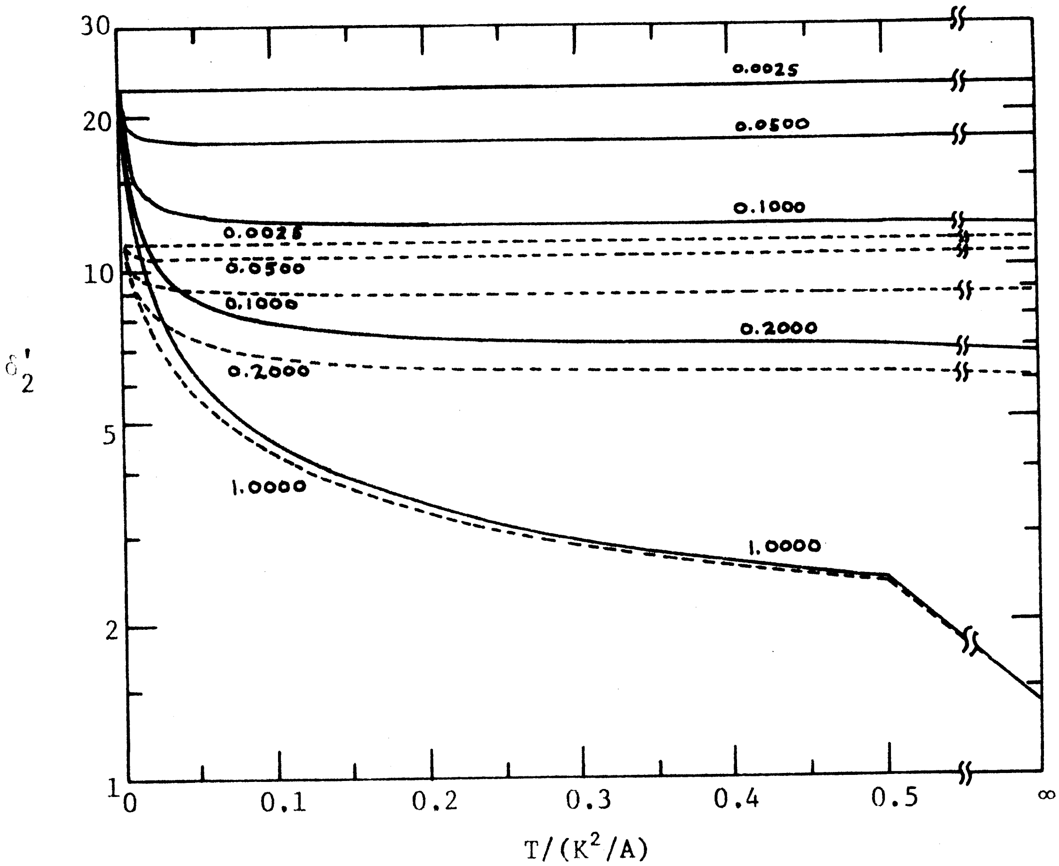


Figure 4.3 Prediction of the preliminary theory for $\log(\delta_2')$ as a function of K/β, $R/(K/G)$, and $T/(K^2/A)$. The numbers on the curves give the values of $R/(K/G)$.

——— : $K/\beta = 16$

------ : $K/\beta = 8$

Some important implications of the preliminary theory for one-interval experiments are the following:

(1) The resolution per bel δ_1' depends only on the stimulus set, not on the response set or the task. In fact, only the range R of the stimulus set affects δ_1'. Thus for a given range, δ_1' in identification, magnitude estimation, and category scaling should be the same.

(2) When the range R is large (so that $G^2R^2 \gg \beta^2$), then $\delta_1' \simeq K/GR$ is inversely proportional to R, and the total resolution $\Delta' \simeq K/G$ is independent of R. This may be considered to be a restatement of the 7 ± 2 phenomenon in identification paradigms; it is stated as an upper bound on Δ', rather than on the information transfer. Furthermore, this bound applies to a wider variety of experiments (including, for example, magnitude estimation).

(3) When the range R is small (so that $G^2R^2 \ll \beta^2$), then $\delta_1' \simeq K/\beta$ is independent of R, and $\Delta' \simeq KR/\beta$ is proportional to R. In this case there is no memory noise; resolution is limited only by sensation noise.

(4) The precise nature of the dependence of δ_1' and Δ' on R depends on the values of the two parameters K/β and K/G. Figure 4.4 illustrates the dependence of $\log(\delta_1')$ on K/β and $R/(K/G)$, while Figure 4.5 illustrates the dependence of $\Delta'/(K/G)$ on K/β and $R/(K/G)$.

Some important implications of the preliminary theory concerning the relation between resolution in two-interval roving-level discrimination and one-interval identification paradigms are the following:

(1) If the values of R used in both experiments are small (so that $G^2R^2 \ll \beta^2$ for each experiment) then performance in each experiment is

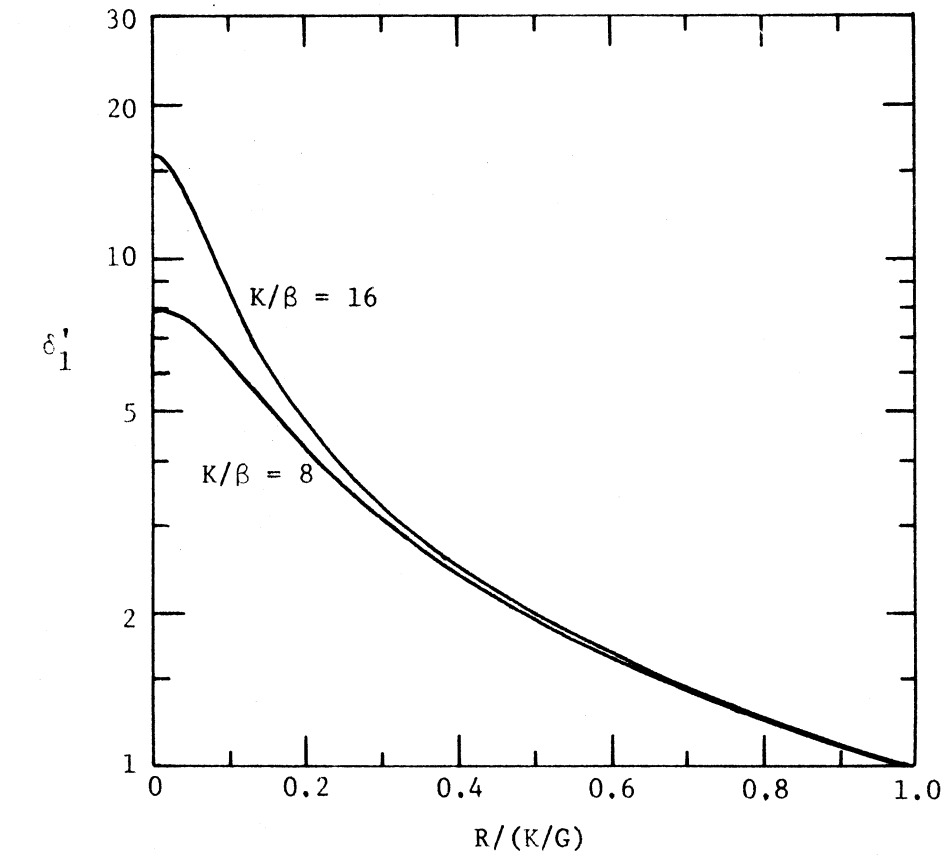


Figure 4.4 Prediction of the Preliminary Theory for $\log(\delta_1')$ as a function of K/β and $R/(K/G)$.

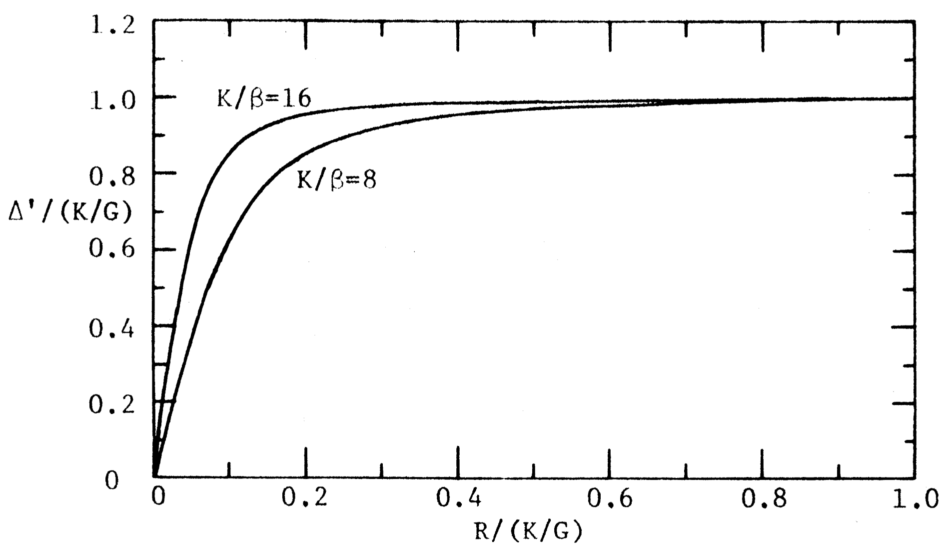


Figure 4.5 Prediction of the Preliminary Theory for $\Delta'/(K/G)$ as a function of K/β and $R/(K/G)$.

limited only by sensation noise and the resolution per bel δ_2' obtained in the discrimination experiment will equal the $\sqrt{2}$ times the δ_1' obtained in the identification experiment (a familiar result).

(2) If the same value of R is used in both experiments, then the δ_2' obtained in the discrimination experiment will be greater than or equal to the $\sqrt{2}$ times the δ_1' obtained in the identification experiment. Furthermore, δ_2' will equal $\sqrt{2}\ \delta_1'$ when T is large relative to R in the discrimination experiment (so that $AT >> G^2R^2$ and the observer is forced into the context-coding mode).

C. Relations to Other Theoretical Work

The literature contains a good deal of theoretical work that is related to ours and that has been influential in the development of the preliminary theory. Some of this work is similar to (and in some cases the same as) portions of our theory. For example, our decision model is a special case of Torgerson's "Law of Categorical Judgment"[9] and is similar to that used in signal-detection theory. In addition, the trace mode of our internal-noise model is essentially the same as the diffusion model proposed by Kinchla and Smyzer (1967). Also, although there is no other model which corresponds directly to it, the context-coding mode of our internal-noise model is related to certain ideas of Helson (1964) and of Eijkman, Thijssen, and Vendrick (1966), as well as many others.

In addition to the theoretical work that is similar to ours, there are some theories in the literature that, although different than ours, have similar goals. The three most relevant of these are the Choice Theory of Luce (1963), the Associative Strength Theory of Wickelgren (1969), and Pollack's (1956) attempt to relate performance in identification to performance in discrimination.

Choice Theory is comparable to our decision model in that both provide a means for analyzing confusion matrices and separating response bias from stimulus confusability (i.e., resolution in our theory and stimulus similarity in Choice Theory). There are, however, two basic differences between the two theories. First, the conditional response probabilities predicted by Choice Theory are based on a set of conditional probability density functions that are not Gaussian (and that cannot be made Gaussian by a monotonic transformation). Second, there is no portion of Choice Theory that corresponds to our internal-noise model. Thus, for example, there is no attempt in Choice Theory to relate stimulus similarity to aspects of the paradigm such as the range R of the stimulus set or the interstimulus time T in two-interval paradigms.

The Associative Strength Theory of Wickelgren is comparable to our model in that both attempt to separate resolution from response bias and to relate resolution to sensory and memory limitations. There are, however, two basic differences between the two theories. First, there is no portion of Associative Strength Theory that is equivalent to the context-coding mode of our internal-noise model. Wickelgren does allow for the possibility of there being both short-term and intermediate-term memory traces (and, in fact, presents some data that support a two-trace model). However, he considers the possibility of the intermediate-term trace being a verbal code relative to the context to be a "nuisance." Second, the decision variable in Associative Strength Theory contains information only about the difference between two stimuli and nothing about the direction of the difference. This has two important implications. First, Associative Strength Theory can only handle experiments in which same-different judgments are made.[10] Thus it cannot handle, for example, identification or magnitude estimation experiments. Our theory, on the other hand, while it can handle many types of higher-lower or greater-lesser experiments, can handle only those same-different experiments which are equivalent to either same-higher or same-lower experiments. Second, it is possible to extend the Associative Strength Theory to same-different judgments about dimensionally complex stimuli such as verbal material. It would be quite difficult, on the other hand, to extend our theory to such judgments.

Pollack's (1956) study of one-interval identification and two-interval, roving-level, discrimination is comparable to the attempt made by our theory to relate the results obtained in these two paradigms. Both are concerned with the context and conclude that the range of levels used in identification and discrimination must be the same if performance in the two paradigms is to be related. There are, however, two basic differences between the two theoretical approaches. First, Pollack uses information transfer rather than resolution to evaluate performance. In our opinion, information transfer is a less useful measure than resolution to evaluate performance. Furthermore, the mathematical procedure used by Pollack to evaluate "equivalent information transmission" in the discrimination experiments does not appear to us to be valid.[11] Second, Pollack concluded that performance in identification and discrimination would be roughly equivalent if the intensity range R is the same in the two paradigms. According to our theory, this equivalence will not hold unless one of two additional conditions are satisfied. Either R must be very small (so that $G^2R^2 << \beta^2$) or the interstimulus time T in the discrimination paradigm must be large (so that $AT >> G^2R^2$).

V. PREVIOUS RELATED EXPERIMENTAL WORK

There are a number of experimental results which are related to our theoretical work. Three sets of results that are particularly relevant to the preliminary theory with respect to two-interval discrimination experiments have been reported in Berliner and Durlach (1973), Pollack's studies (1954, 1955 and 1956), and Kinchla and Smyzer (1967). Previous experimental results that are particularly relevant to the preliminary theory with respect to one-interval paradigms have been reported in Braida and Durlach (1972) and Pynn, Braida, and Durlach (1972). In addition, experiments by Rabinowitz (1970) exploring a departure from Weber's law are relevant to the preliminary theory with respect to the relation between one- and two-interval paradigms. This chapter contains a description of the relevant results of these studies, along with a tabulation of the estimates of the parameters of the model K/β, K/G, and $K/\sqrt{A}$ obtained in the studies. Some quantitative results from these studies are presented along with the experimental results of this thesis in Chapter VII.

A. Berliner and Durlach (1973)

This preliminary study consisted primarily of a series of two-interval roving-level discrimination experiments. In these tests, the intensity range R varied from 0.1 to 5.4, and the interpulse interval T varied from 0.2 to 9.0 seconds. In one test, with T = 8.7 sec, interpolated interference (consisting of a series of stimuli and tasks) was inserted in the interpulse interval. In addition, a one-interval identification experiment was conducted with R = 5.25. These experiments were preliminary inasmuch as the amount of data collected was rather small and only two observers were used.

The important results that were consistent with the preliminary theory were:

(1) Resolution did not decline significantly with R when T was small (T = 0.2 sec).

(2) In general, resolution declined faster with T for larger R, and faster with R for larger T.

(3) The resolution per bel δ'_2 obtained in the two-interval roving-level discrimination with T large (8.7 sec) and interpolated interference and with R = 5.4 was roughly equal to the $\sqrt{2}$ times the δ'_1 obtained in the one-interval identification experiment performed with virtually the same value of R.

An important result that was not predicted by the preliminary theory was:

(4) The resolution in the test that included the interpolated task (T = 8.7 sec; R = 5.4) was only slightly poorer than in a similar test without the interpolated task (T = 9.0 sec; R = 5.4). Although the theory does not predict this very surprising result, it is not necessarily inconsistent with the theory, since at present we have no model for the effects of interference.

The remaining important results that were not predicted by the preliminary theory concerned the variation of performance (both resolution and bias) over intensity within the ranges of the experiments. These results included:

(5) Resolution was better near the edges of the range than in the

middle. Under some conditions, δ' at the edge of the range was more than twice as large as δ' in the middle. We call this the <u>resolution edge-effect</u>.

(6) The observers tended to be biased in their responses. Near the bottom of the range they usually responded that the second tone was less intense than the first; near the top of the range they usually responded that the second tone was more intense. For example, while the probability of an (I_i, I_i^*) trial was always 50%, at the lower edge of the range the probability of a "H, L" response was as much as 90%, and at the upper edge as little as 10%. We call this the <u>bias edge-effect</u>.

(7) In general, resolution increased as the intensity level increased. This departure from Weber's law has been carefully studied by Rabinowitz (1970).

Items 5 and 6, the resolution and bias edge-effects, have received a close examination in this thesis. These phenomena were important factors in the selection of the experiments (see Chapter VI), they were carefully examined as part of the experimental results (see Chapter VII), and they have been the object of considerable theoretical work (see Chapter VIII).

B. Pollack (1954, 1955, and 1956)

These studies consisted of a series of fixed- and roving-level discrimination experiments. Although the discrimination results were presented in terms of percent correct (thus precluding an explicit test of our theory), they are still relevant to our model. In the fixed-level experiments, Pollack found that discrimination was relatively independent of the interstimulus interval T in the range $0 \leq T \leq 5$ seconds. In the roving-level experiments with T fixed at about 1.2 seconds, he found that discrimination was worse than in the fixed-level experiments and that the accuracy of discrimination declined as the intensity range R increased from 0 to 7.2. These results are consistent with the preliminary theory.

Some of Pollack's results indicate the same departures from the predictions of the preliminary theory as found by Berliner and Durlach. In particular, a very strong bias edge-effect was found. In fact, since resolution was not separated from response bias, the reported decline in discrimination with an increase in R is due in part to this bias. In addition, Pollack noted a departure from Weber's law similar to that found by Berliner and Durlach.

C. Kinchla and Smyzer (1967)

This study consisted of a two-interval fixed-level discrimination experiment. Resolution was measured as a function of the interpulse interval T and it was found that δ_2' decreased by about 12% as T increased from 0 to 2 seconds. Moreover, it was found that $(\delta_2')^{-2}$ increased nearly linearly with T. Although this result is consistent with the trace mode of our preliminary theory (Eq. 4.6), which is the same as the diffusion model of Kinchla and Smyzer, it is inconsistent with the overall preliminary theory. Because R was so small (R = 0.2), the observers should have been strongly in the context-coding mode, and δ_2' should have been essentially independent of T. This result is in agreement, however, with the results of this thesis for small R, and points up a significant shortcoming of the preliminary theory. This shortcoming, concerning the lack of any dependence on T in the context-coding mode, is discussed more fully in the chapters on experimental results (Chapter VII) and future work (Chapter IX).

D. Braida and Durlach (1972)

This study consisted of an extensive series of one-interval experiments, including identification, category scaling, and magnitude estimation. The major results of these experiments are listed and compared with the predictions of the preliminary theory.

(1) Resolution was not entirely independent of the paradigm. For the same value of R, the total resolution Δ' was about the same in identification and category scaling, but lower in magnitude estimation. The preliminary theory predicts that resolution is independent of the paradigm.

(2) The total resolution Δ' as a function of R agreed with the prediction of the preliminary theory (Eq. 4.13) in identification and category scaling, but not in magnitude estimation.[12]

(3) The total resolution Δ' increased by about 20% when feedback was available. The preliminary theory makes no predictions about the effects of feedback.

(4) Resolution was independent of the number of stimuli when spaced uniformly throughout a fixed intensity range, in agreement with the prediction of the preliminary theory.

(5) The total resolution Δ' depended somewhat on the pattern of stimulus spacing within a fixed intensity range, contrary to the prediction of the preliminary theory that resolution is independent of the pattern of stimulus spacing.

(6) Whereas the preliminary theory predicts that resolution is constant throughout the intensity range, it was found to vary throughout the range. Resolution was better near the edges of the range than in the middle (the resolution edge-effect). Furthermore, resolution generally increased as the intensity level increased.

E. Pynn, Braida, and Durlach (1972)

This study consisted of some small range ($0.225 \leq R \leq 0.90$), one-interval identification experiments and a two-interval fixed-level discrimination experiment. The major results are listed and compared with the predictions of the preliminary theory.

(1) The total resolution Δ' as a function of R agreed with the prediction of the preliminary theory (Eq. 4.13) over the limited range of R tested.

(2) The resolution per bel δ_2' obtained in the two-interval fixed-level discrimination experiment was approximately equal to the $\sqrt{2}$ times the δ_1' obtained in the one-interval small range ($R = 0.225$) identification experiment with the number of stimuli $N = 10$, in agreement with the prediction of the preliminary theory.

(3) The resolution per bel δ_1' obtained in identification with $N = 10$ was 26% greater than the δ_1' obtained in identification with $N = 2$, contrary to the prediction of the preliminary theory that δ_1' is independent of N.

(4) Resolution was generally constant throughout the intensity range, in agreement with the prediction of the preliminary theory, except for the largest range tested ($R = 0.90$). In this case, resolution

increased as the intensity increased.

F. Rabinowitz (1970)

The relevant portion of this study consisted of a set of two-interval fixed-level intensity discrimination experiments, and one-interval small-range (R = 0.2) identification experiments. To explore the departure from Weber's law, noted in a number of other studies, these tests were conducted at five intensity levels in the region 10 dB SPL $\leq$ I $\leq$ 72 dB SPL. The major results were:

(1) Below about 36 dB SPL, but above threshold, the resolution per bel δ' was a constant independent of I (consistent with Weber's law).

(2) Above about 36 dB SPL, δ' increased approximately linearly with I.

These results are roughly consistent with the departure from Weber's law noted in several of the preceding studies, as well as the results of McGill and Goldberg (1968), Campbell and Lasky (1967), and Viemeister (1972). In addition, Rabinowitz found that:

(3) The resolution per bel δ'_2 obtained in two-interval fixed-level discrimination was on the average about 32% greater than the $\sqrt{2}$ times the δ'_1 obtained in one-interval small-range (R = 0.2) identification, contrary to the prediction of the preliminary theory.

G. Evaluations of the Parameters of the Model

The parameters of the model K/β, K/G, and $K/\sqrt{A}$ have been estimated from the data obtained in most of the preceding studies. Some of these studies have yielded a number of estimates of these parameters. The range of values and/or a nominal value of the parameters obtained from each study are given in Table 5.1. It may be seen from the table that there is considerable variability in the estimates of these parameters between the various studies and, at times, within a particular study. In this brief summary, however, no attempt is made to analyze this variability.[13]

TABLE 5.1

Values of the Parameters of the Preliminary Theory Obtained from Previous Studies

STUDY	K/β	K/G	$K/\sqrt{A}$
Berliner & Durlach (1973)	14	11	6
Kinchla & Smyzer (1967)	8.5		(a)
Braida & Durlach (1972)	8.1 - 11.0 (9.5)	11.9 - 15.6 (13)	
Pynn, Braida & Durlach (1972)	12.7 - 14.2 (13)	8.6 (b)	
Rabinowitz (1970)	15.4		

(a) The estimate $K/\sqrt{A} \simeq 17$ may be derived from the data of Kinchla and Smyzer by assuming that the trace mode applied in this fixed-level task. According to the preliminary theory, however, the observers would be strongly in the context-coding mode in this situation, making this estimate of $K/\sqrt{A}$ a poor one.

(b) The authors considered this an unreliable estimate of K/G since the largest range tested was only R = 0.9.

VI. EXPERIMENTAL RESEARCH

A. Description of the Experiments

The main experimental work of this thesis consisted of two-interval roving-level (Expt. 1) and fixed-level (Expt. 2) discrimination experiments, performed with fixed tone pulse durations and without feedback. Additional two-interval discrimination experiments were conducted with feedback (Expts. 2 and 2A), and with variable tone pulse durations (Expts. 3 and 3A). Two supplementary sets of one-interval identification experiments were also conducted. One set (Expt. 4) was performed with different intensity ranges, and the other set (Expt. 5) was performed with different reference stimuli. A summary of all the experiments is given in Tables 6.1 and 6.2, and additional details of the experiments are presented below.

The timing of the discrimination experiments is illustrated in Fig. 6.1. In all the discrimination experiments, for each level, the a priori probabilities of the two stimuli to be discriminated $[(I_i, I_i^*)$ and $(I_i^*, I_i)]$ were equal. In the roving-level discrimination experiments, the number of overall levels, N, was always ten; the levels were always equally spaced in dB, and centered at about 63 dB SPL; the levels were always presented with equal a priori probabilities; and the increment, q, to be detected was independent of the level [i.e., $\log(I_i/I_i^*) = q$, independent of i]. For each pairwise combination of range $R = \log(I_{max}/I_{min})$ and interpulse interval T, one base value of q, $q_o(R, T)$, was chosen according to the difficulty of the experiment (as determined by preliminary tests). These values, and certain multiples of them (see

TABLE 6.1 Discrimination Experiments

(a) Expt.	T (sec)	(b) $R=\log\left[\frac{I_{max}}{I_{min}}\right]$	$q_o(R,T)$ $=\log(I_i/I_i^*)$	I_{max} (dB SPL)	T_p (sec)	Fbk	N (# of Levels)	Multiples of $q_o(R,T)$ used	(c) Observers	Total # of Trials per Observer
Expt. 1 2I RL Discrim	0.2 1.8 3.5 7.0 14.0	2.0, 3.9, 5.8 5.8 2.0, 3.9, 5.8 5.8 2.0, 3.9, 5.8	0.050, 0.050, 0.050 0.100 0.075, 0.100, 0.125 0.150 0.100, 0.150, 0.175	72, 81, 90 90 72, 81, 90 90 72, 81, 90	0.50	No	10	1,2,3,4,5	JB, KG, PN (PS), (BD)	33,000
Expt. 1A 2I FL Discrim	0.2 3.5 14.0	0.2 0.2 0.2	0.050 0.050 0.075	[36, 42, 48, 54, 60 66, 72, 78, 84, 90] 63 63	0.50	No	1	1,2,3,4,5	JB, KG, PN (PS), (BD)	11,750
Expt. 2 2I RL Discrim	0.2 3.5 14.0	2.0, 5.8 2.0, 5.8 2.0, 5.8	0.050, 0.050 0.075, 0.125 0.100, 0.175	72, 90 72, 90 72, 90	0.50	Yes	10	1, 3, 5	JB, KG, PN	18,000
Expt. 2A 2I FL Discrim	0.2 3.5 14.0	0.2 0.2 0.2	0.050 0.050 0.075	63 63 63	0.50	Yes	1	1, 3, 5	JB, KG, PN	1,800
Expt. 3 2I RL Discrim	0.0 0.2 3.5 14.0	2.9, 5.6 2.9, 5.6 2.9, 5.6 2.9, 5.6	0.050, 0.050 0.050, 0.050 0.100, 0.125 0.125, 0.175	76.5, 90 76.5, 90 76.5, 90 76.5, 90	0.20 0.50 1.25	No	10	1, 2, 3	JB, SK, JW	36,000
Expt. 3A 2I FL Discrim	0.0 0.2 3.5 14.0	0.1 0.1 0.1 0.1	0.050 0.050 0.050 0.075	63 36, 54, 72, 90 63 63	0.20 0.50 1.25	No	1	1, 2, 3	JB, SK, JW	12,600

(a) 2I RL Discrim = 2-interval, roving-level, discrimination; 2I FL Discrim = 2-interval, fixed-level, discrimination.

(b) The precise values of R employed in these experiments depend on the increment q to be detected. In the listing of R in this table, however, certain approximations have been made.

(c) Observers listed in parentheses participated in fewer than 80% of the total number of trials.

TABLE 6.2 Identification Experiments

(a) Expt.	$R=\log\left[\frac{I_{max}}{I_{min}}\right]$	I_{max} (dB SPL)	I_{ref} (dB SPL)	(b) T (sec)	T_p (sec)	Fbk	N (# of Levels)	(c) Observers	Total # of Trials per Observer
Expt. 4 1I Ident	0.3 1.8 3.6 5.4	64.5 72.0 81.0 90.0	None	—	0.50	Yes	13	JB, KG, PN (PS)	8,500
Expt. 5 1I Ident (With Ref.)	5.4	90.0	None 36.0 49.5 63.0 76.5 90.0	— 0.4 0.4 0.4 0.4 0.4	0.50	Yes	13	JB, JM BG, CB, DM (DH)	12,500

(a) 1I Ident = 1-interval, identification; (With Ref.) = Reference stimulus presented on each trial.

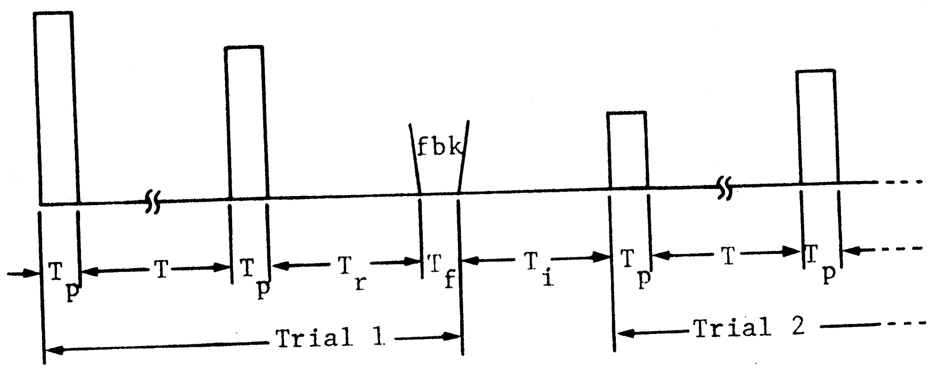
(b) T is the interval between the reference tone pulse and the test tone pulse.

(c) Observers listed in parentheses participated in fewer than 80% of the total number of trials.

Figure 6.1

Timing of the

Discrimination Experiments



T_p = Tone pulse duration = 0.20 sec - 1.25 sec

T = Interpulse interval = 0.00 sec - 14.0 sec

T_r = Response interval = 2.00 sec

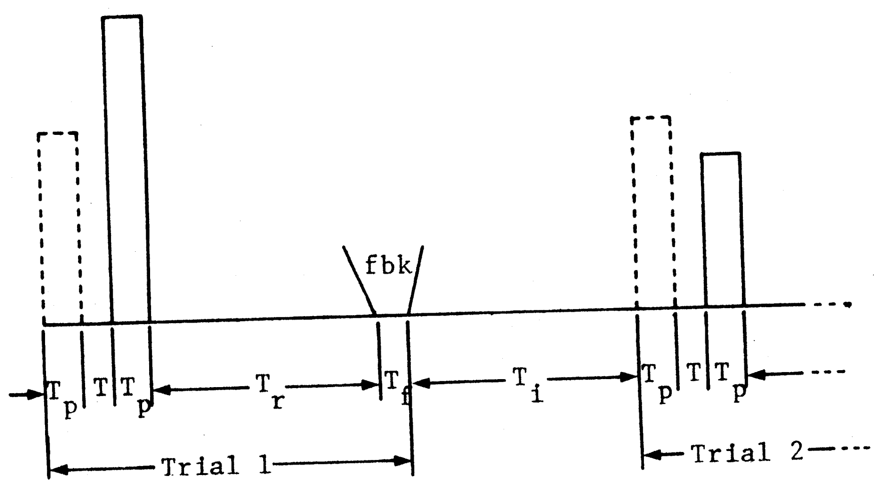
T_f = Feedback interval = 0.50 sec

T_i = Intertrial interval = 2.00 sec

Figure 6.2

Timing of the

Identification Experiments



T_p = Tone pulse duration = 0.50 sec

T = Interpulse interval = 0.40 sec

T_r = Response interval = 3.00 sec

T_f = Feedback interval = 0.40 sec

T_i = Intertrial interval = 3.00 sec

Table 6.1), were used in each of the discrimination experiments. This was done so that for each condition and observer, some values of d' would be obtained in the region between about 0.5 and 3.

The timing of the identification experiments is illustrated in Fig. 6.2. In all the identification experiments, the number of stimulus intensities to be identified, N, was always thirteen; the intensities were always equally spaced in dB, centered at 63 dB SPL, and presented with equal a priori probabilities; the tone pulse duration, T_p, was fixed at 0.50 sec; and feedback was **always** present.

In all cases, the acoustical signals were 1000-hz tone pulses with rise and decay times of 50 msec; the signals were presented to the observers in a soundproof room monaurally through TDH-39 earphones; the responses were made by pushing one of a set of buttons available to each observer on a "response box"; and the experiments were controlled on-line, and the stimuli and responses recorded by either a PDP-12 or a PDP-8 computer. In the experiments with feedback, the observers were informed (by means of a visual display) of the identity of the correct response after their own responses had been recorded.

1. The Main Experiments: Experiments 1 and 1A

Experiments 1 and 1A constituted the main experimental work of this thesis. They were two-interval roving-level (Expt. 1) and fixed-level (Expt. 1A) discrimination experiments with constant tone pulse durations T_p = 0.50 sec, and without feedback. The purpose of these experiments was to provide a rigorous test of the preliminary theory of intensity resolution in two-interval discrimination paradigms and to carefully explore the resolution and bias edge-effects. For these reasons, performance was measured for a wide variety of pairwise combinations of R and T, and an extremely large number of trials was run (about 3000 trials for each R, T combination and observer) so that for each R, T combination resolution and bias could be determined as a function of overall level. Feedback was not presented to avoid the possibility of its diluting the bias edge-effect. In addition, because earlier work in intensity resolution had indicated a departure from Weber's law,[14] the fixed-level experiment with short interpulse interval T = 0.2 sec was run at a variety of overall levels to examine this departure for each observer.

The range $R = \log(I_{max}/I_{min})$ was approximately 5.8, 3.9, or 2.0 (in Expt. 1) or approximately 0.2 (in Expt. 1A).[15] When R was 5.8, the interpulse interval T was 0.2, 1.8, 3.5, 7.0 or 14.0 sec, while when R was 0.2, 2.0 or 3.9, T was 0.2, 3.5 or 14.0 sec. In Expt. 1A, when T = 3.5 sec or 14.0 sec, the overall level was fixed at about 63 dB SPL. When T = 0.2 sec, however, ten different overall fixed levels were used. These levels ranged from 36 to 90 dB SPL in 6 dB steps. In both Expts. 1 and 1A for each R, T combination, the values of $q = \log(I_i/I_i^*)$ that were used were 1, 2, 3, 4 and 5 times $q_o(R, T)$.

Experiments 1 and 1A, which were conducted concurrently over a period of about ten months, were begun with five observers. Observer JB (myself), KG and PN completed the full experiments, while Observer PS completed about 50 percent, and BD only about 15 percent.

2. Feedback and Instructions to Eliminate The Bias Edge-Effect: Experiments 2 and 2A

Experiments 2 and 2A were two-interval, roving-level (Expt. 2) and fixed-level (Expt. 2A), discrimination experiments with fixed tone pulse durations $T_p = 0.50$ sec, and with feedback. In addition, the observers were specifically instructed to try to eliminate their bias edge-effects. The purpose of these experiments was to determine whether the bias edge-effect could be eliminated (or at least sharply reduced), and if so, what effect this would have on resolution. Since this was a supplementary experiment, performance was measured only for a subset of the R, T combinations used in Expts. 1 and 1A, although again an extremely large number of trials was run (for each R, T combination and observer about 3000 trials in Expt. 2 and 600 trials in Expt. 2A).

The range R was approximately 5.8 or 2.0 (in Expt. 2) or approximately 0.2 (in Expt. 2A).[16] The interpulse interval T was 0.2, 3.5 or 14.0 sec. In Expt. 2A, the overall level was fixed at about 63 dB SPL. In both Expts. 2 and 2A for each R, T combination, the values of q that were used were 1, 3 and 5 times $q_0(R, T)$.

Experiments 2 and 2A, which were run concurrently over a period of about three months, following the completion of Expts. 1, 1A, and 4, were conducted with the three observers who had completed these other experiments (i.e., Observers JB, KG and PN).

3. Effect of Tone Pulse Duration:

Experiments 3 and 3A

Experiments 3 and 3A were two-interval, roving-level (Expt. 3) and fixed-level (Expt. 3A), discrimination experiments with variable tone pulse durations T_p = 0.20, 0.50 or 1.25 sec, and without feedback. The purpose of these experiments was to determine the effect of tone pulse duration on resolution and bias, and to measure the resolution and bias edge-effects for very short tone pulse durations and interpulse intervals. For each value of T_p it would have been preferable to employ the same wide variety of R, T combinations (as in Expts. 1 and 1A) as well as for T = 0; however, the amount of time required was judged excessive. Therefore a more limited set of R, T combinations was used. Furthermore, because the primary purpose of these experiments was to examine average resolution (except for the condition T = 0), fewer trials were run for each R, T, T_p combination than in Expts. 1 and 1A. (About 1200 trials were run for each R, T, T_p combination where $T \neq 0$, and about 2400 trials where T = 0.) As in Expts. 1 and 1A, feedback was not presented to avoid the possibility of its diluting the bias edge-effect, and the fixed-level experiment with T = 0.2 sec was run at a variety of levels (although fewer than in Expt. 1A) to examine any departure from Weber's law for each observer.

The range R was approximately 5.6 or 2.9 (in Expt. 3) or approximately 0.1 (in Expt. 3A).[17] The interpulse interval T was 0, 0.2, 3.5 or 14.0 sec. In Expt. 3A, when T = 0, 3.5 or 14.0 sec, the overall level was fixed at about 63 dB SPL. When T = 0.2 sec, however, four different overall fixed levels were used. These levels ranged from 36 to 90 dB SPL in

18 dB steps. In both Expts. 3 and 3A, for each R, T combination, the values of q were 1, 2, and 3 times $q_o(R, T)$.

The observers for Experiments 3 and 3A were JB (myself), and two observers, JW and SK, who did not participate in any of the other experiments. Observers JB and SK completed the full experiments while Observer JW completed about 80 percent. These experiments were run concurrently over a period of about six months during the time that Expts. 1, 1A and 4 were run.

4. <u>One-Interval Identification:</u>

<u>Experiment 4</u>

Experiment 4 was a one-interval identification experiment with a variable range and no reference stimulus. The purpose of this experiment was to provide a test of the predictions of the preliminary theory with respect to the relations between one- and two-interval paradigms, to explore the resolution edge-effect in a one-interval paradigm, and to allow a comparison of this effect in one- and two-interval paradigms. For these reasons, performance was measured for approximately the same values of R used in Expts. 1 and 1A (R was 5.4, 3.6, 1.8, or 0.3); a fairly large number of trials was run (about 2100 trials for each value of R); and the experiment was run with four observers who participated in Expts. 1 and 1A. Observers JB, KG and PN completed all of Expt. 4, while Observer PS completed only about 25 percent. Expt. 4 took about two months to complete and was performed in two parts; approximately one-third of the trials were run after Expts. 1 and 1A had been half completed, while the remaining two-thirds were run after Expts. 1 and 1A had been entirely completed, but before Expts. 2 and 2A were begun.

5. One-Interval Identification with a Reference: Experiment 5

Experiment 5 was a one-interval identification experiment with a fixed range R = 5.4 centered at 63 dB SPL, and six reference stimulus conditions. The purpose of this experiment was to examine the resolution edge-effect in a one-interval paradigm, and to explore the effect of a reference stimulus on resolution. The reference stimulus, a tone pulse of the same duration as the test tone pulse (T_p = 0.50 sec) and preceding the test tone pulse by 0.8 sec, was fixed for a given run at 36.0, 49.5, 63.0, 76.5, or 90 dB SPL, or it was absent. As in Expt. 4, a fairly large number of trials was run, about 2100 trials for each reference stimulus condition. The observers for Expt. 5 were JB (myself), and five observers JM, BG, CB, DH and DM, who did not participate in any of the other experiments. All the observers completed the entire experiment except for DH who completed about 60 percent. Expt. 5 was conducted during the time that Expts. 1 and 1A were run.

B. Data Analysis Procedure

1. Discrimination Experiments

In the fixed-level discrimination experiments (Expts. 1A, 2A, and 3A), each condition (R, T, T_p, q) involved 2 stimuli [(I, I^*) and (I^*, I)] and 6 responses ("H, L" and "L, H", and 3 degrees of confidence). The data for each observer and each condition were summarized in a single

2 x 2 matrix by ignoring the confidence ratings. (The confidence ratings were ignored because the preliminary analysis indicated that, in general, each observer had chosen for each condition one favorite degree of confidence from which he seldom departed. I was reluctant to instruct the observers to use all the confidence ratings equally often for fear of disturbing their bias results.)

In the roving-level discrimination experiments (Expts. 1, 2, and 3), each condition (R, T, T_p, q) involved 20 stimuli [(I_i, I_i^*) and (I_i^*, I_i), and 10 levels $1 \leq i \leq 10$] and 6 responses ("H, L" and "L, H", and 3 degrees of confidence). The data for each observer and each condition were summarized in five 2 x 2 matrices by ignoring the confidence ratings and by pooling the data from adjacent pairs of levels. (The data from adjacent pairs of levels were pooled because it substantially reduced the lengthy time required for data analysis, and because the additional averaging helped to reduce the variability of the results.)

Estimates of resolution per bel and bias, averaged over intensity increment q, were then calculated from the matrices using a maximum likelihood estimation procedure. (For the details of this procedure, see Appendix I.) To eliminate the effects of the previously noted departure from Weber's law, the estimates of resolution per bel for each observer were normalized using the data from the fixed-level experiments which had been performed at a variety of levels. (For the details of this normalization procedure, see Appendix I.) These normalized estimates of resoltuion per bel, $\hat{\delta}_2'$, and of bias, $\hat{b}'$, were used to calculate estimates of criteria $\hat{C}/K = 2\hat{b}'/\hat{\delta}_2'$, and to calculate estimates of resolution per bel averaged over levels, $\hat{\bar{\delta}}_2'$.

Finally, the results of Expts. 3 and 3A were averaged over all the observers, and the results of Expts. 1, 1A, 2, and 2A were averaged over Observers KG and PN. The data from Observer JB (myself) were not included in this average and are presented separately because the results from Observers KG and PN were substantially different than the results from the other observers. (This difference is discussed in Chapter VII.) In addition, the data from Observers BD and PS were not included in this average, and are presented only in Appendix II, because they completed only minor portions of the experiments.[18] Tabulations of the estimates $\hat{\delta}'_2$ and $\hat{b}'$ for individual observers in all the discrimination experiments are presented in Appendix II, along with selected graphical results for individual observers.

2. Identification Experiments

In the identification experiments (Expts. 4 and 5), a single 13 stimulus by 13 response confusion matrix was obtained for each observer and experimental condition. Twelve estimates of resolution and bias were calculated. (For the details of this estimation procedure, see Appendix I.) In Expt. 4, as in the discrimination experiments, to eliminate the effects of the departure from Weber's law, the estimates of resolution for each observer were normalized using the data from the fixed-level discrimination experiments which had been performed at a variety of levels. This normalization was not done for Expt. 5, because fixed-level data had not been collected for these observers. These normalized estimates of resolution $\hat{d}'_1(I_{i+1};\ I_i)$ were used to calculate estimates of resolution per bel:

$$\hat{\delta}_1'(I_{i+1};\ I_i) = \frac{\hat{d}_1'(I_{i+1};\ I_i)}{\log(I_{i+1}/I_i)} \quad , \tag{6.1}$$

estimates of total resolution

$$\hat{\Delta}' = \sum_{i=1}^{12} \hat{d}_1'(I_{i+1};\ I_i) \ , \tag{6.2}$$

and estimates of resolution per bel averaged over levels:

$$\bar{\hat{\delta}}_1' = \hat{\Delta}'/R \ . \tag{6.3}$$

In addition, the estimates of bias $\hat{b}_i'$ were used to calculate estimates of criterion shift $C\hat{S}_i/K = \hat{b}_i'/\hat{\delta}_1'(I_{i+1};\ I_i)$.

Finally, the results of Expt. 4 were averaged over Observers KG and PN, and the results of Expt. 5 were averaged over all six observers. Tabulations of the estimates $\hat{d}_1'(I_{i+1};\ I_i)$ and $\hat{b}_i'$ for individual observers in all the identification experiments are presented in Appendix II, along with selected graphical results for individual observers.

VII. EXPERIMENTAL RESULTS

In this chapter, the experimental results of this thesis, as well as certain relevant experimental results from other sources, are presented and discussed in three phases. First, resolution averaged over intensity levels is examined with primary attention given to the pertinent predictions of the preliminary theory of intensity resolution. Next, resolution is examined as a function of level, and finally criterion is examined as a function of level.

A. Resolution Averaged Over Levels

1. Presentation of Results

a. Discrimination Experiments

Figures 7.1 - 7.3 are logarithmic[19] plots of estimated resolution per bel averaged over levels, $\overline{\hat{\delta}_2'}$, versus interpulse interval, T, with intensity range, R, as a parameter, for all the discrimination experiments (1, 1A, 2, 2A, 3, and 3A). The data averaged over Observers KG and PN from Expts. 1 and 1A are presented in Fig. 7.1a, and from Expts. 2 and 2A in Fig. 7.1b. The data for Observer JB from these experiments are presented in Fig. 7.2. The data from Expts. 3 and 3A are presented in Fig. 7.3.

Figure 7.4 is a logarithmic plot of estimated resolution per bel averaged over levels, $\overline{\hat{\delta}_2'}$, versus T, with R as a parameter, for discrimination experiments from other sources. Data are included from Berliner and Durlach (1973), Kinchla and Smyzer (1967), Pynn, Braida and Durlach (1972),

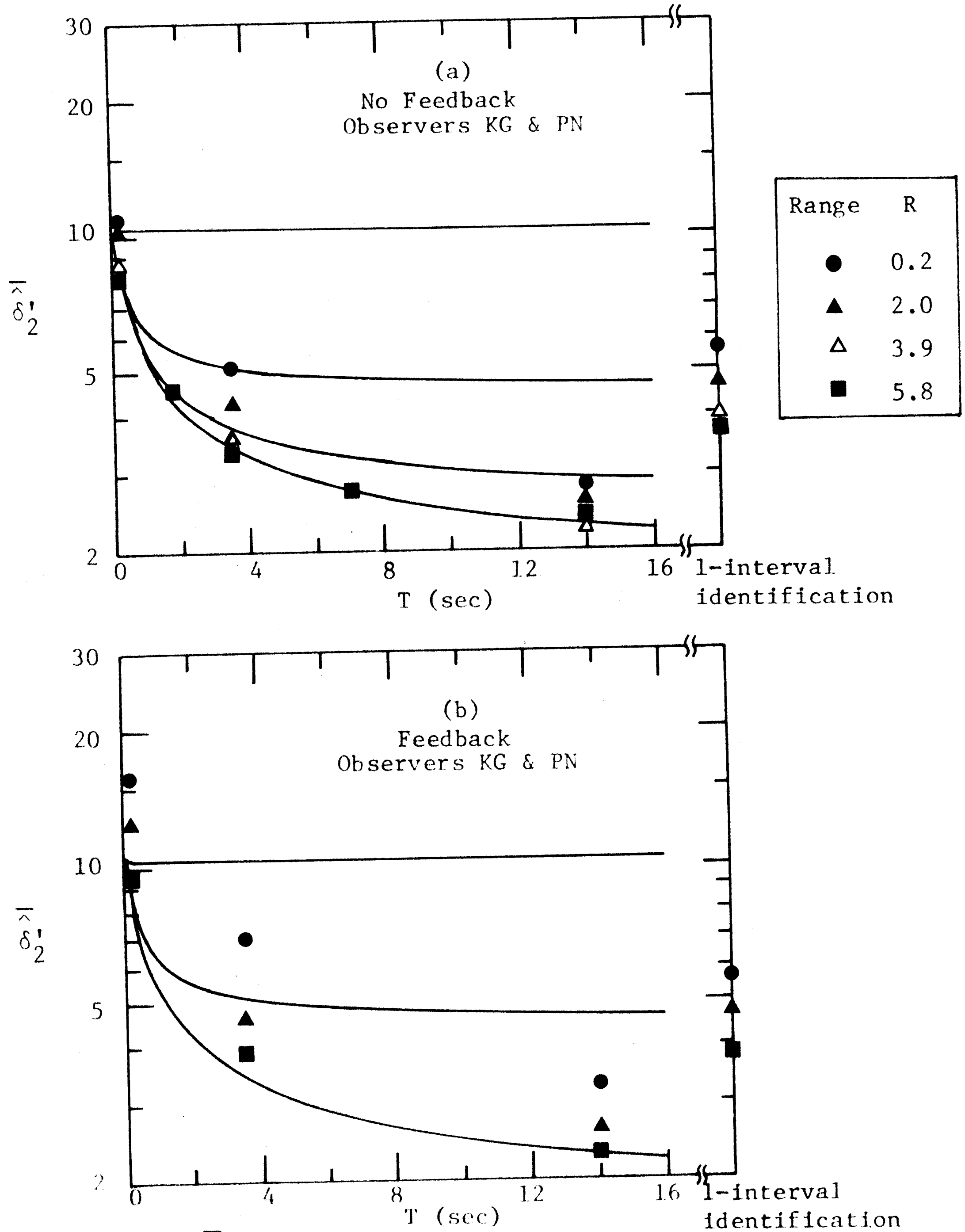


Figure 7.1 $\overline{\hat{\delta}}'_2$ versus T. Averaged over Observers KG & PN.

(a) No Feedback (Expts. 1 & 1A); (b) Feedback (Expts. 2 & 2A).

The smooth curves are derived from the preliminary theory (Eq. 4.10) under the assumption $K/\beta=7.5$, $K/G=7.0$, and $K/\sqrt{\bar{A}}= 4.2$.

At the right edges of (a) & (b) are plotted $\sqrt{2}\overline{\hat{\delta}}'_1$ from Expt. 4.

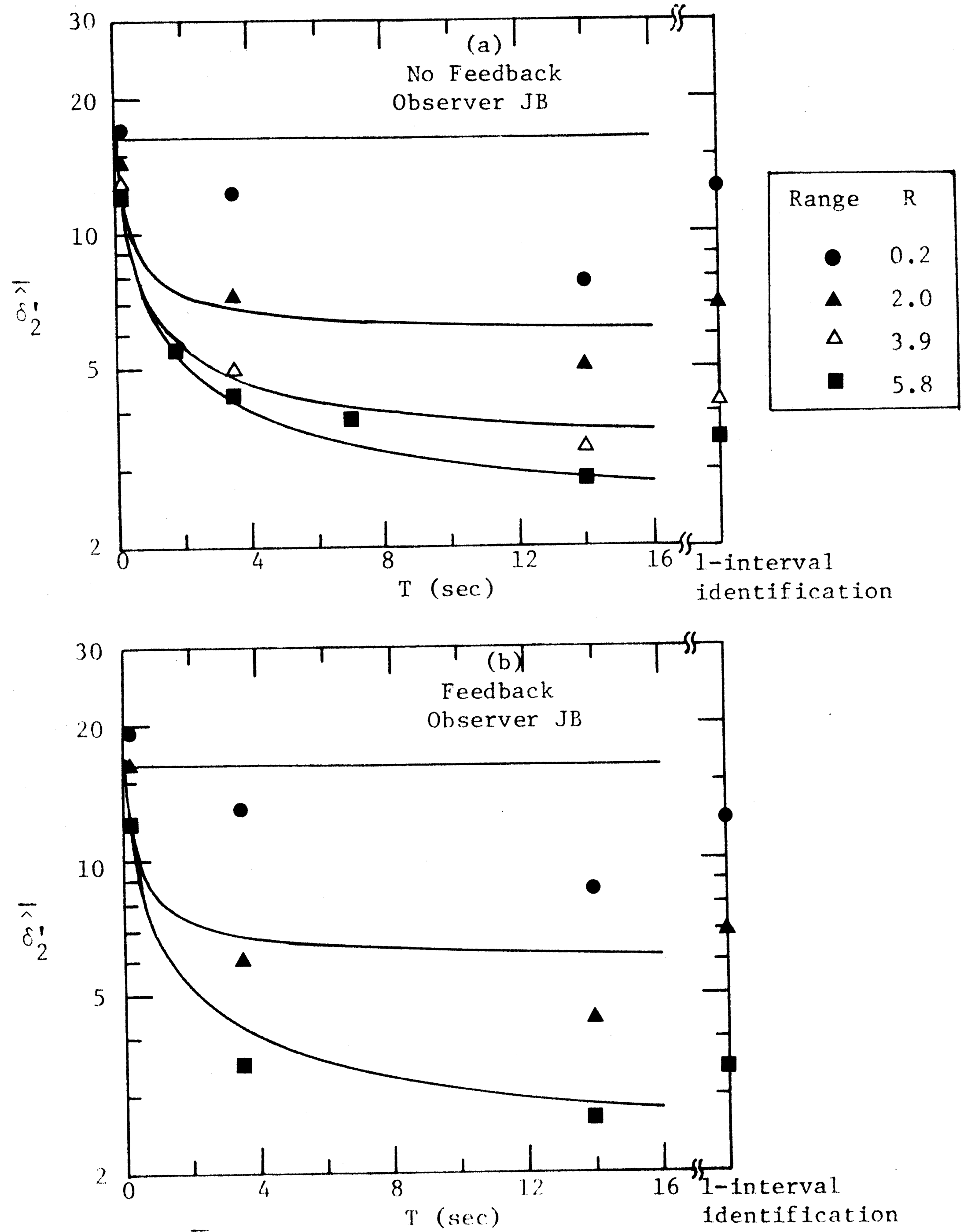
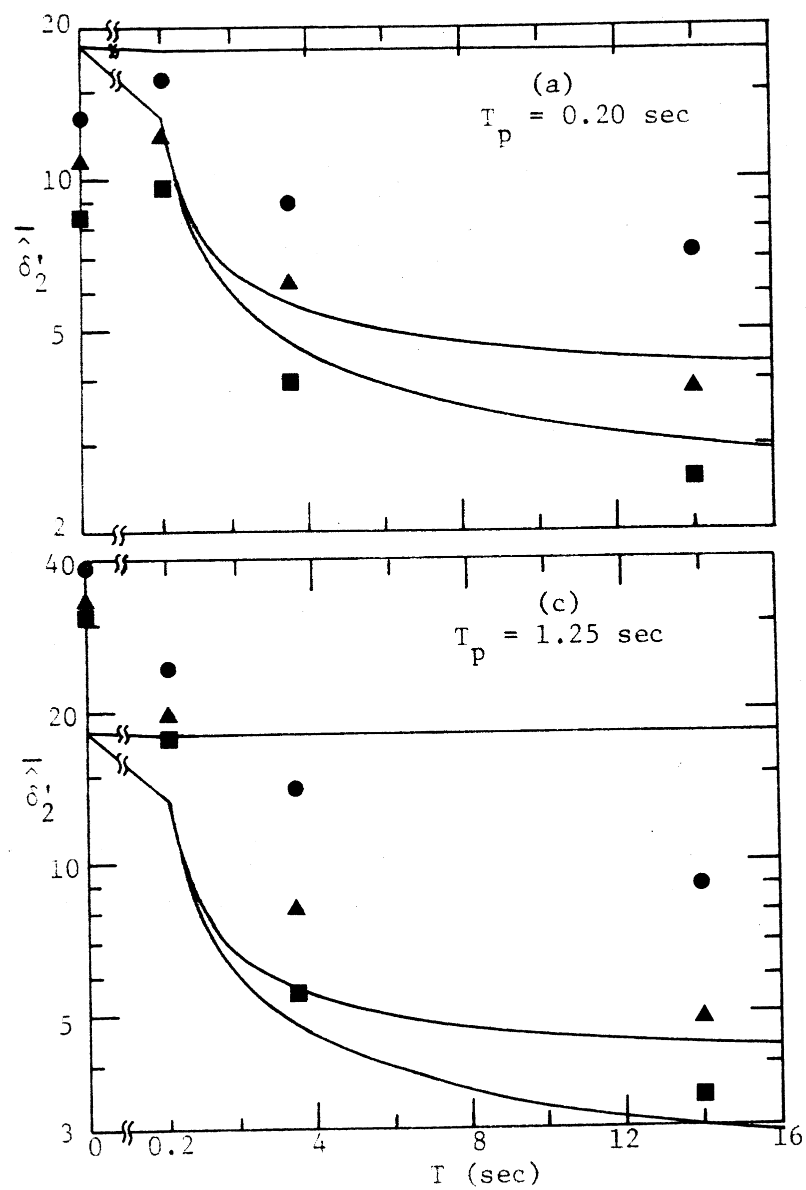


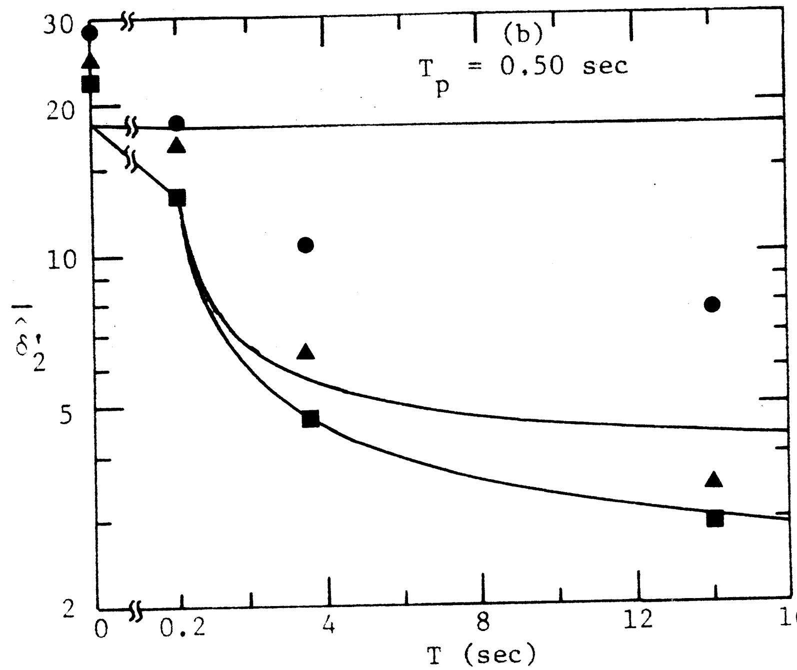
Figure 7.2 $\hat{\bar{\delta}}'_2$ versus T. Observer JB.

(a) No Feedback (Expts. 1 & 1A); (b) Feedback (Expts. 2 & 2A).

The smooth curves are derived from the preliminary theory (Eq. 4.10) under the assumption $K/\beta = 12$, $K/G = 9$, and $K/\sqrt{A} = 5$.

At the right edges of (a) & (b) are plotted $\sqrt{2}\hat{\bar{\delta}}'_1$ from Expt. 4.





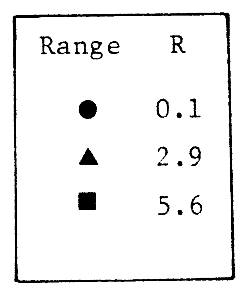


Figure 7.3 $\overline{\hat{\delta}'_2}$ versus T.
Expt. 3; Obs. JB, JW, SK.
No Feedback. (a) T_p=0.20 sec, (b) T_p=0.50 sec; (c) T_p=1.25 sec.
The smooth curves are derived from the preliminary theory under the assumption $K/\beta = 13$, $K/G = 8$, and $K/\sqrt{A} = 6$.

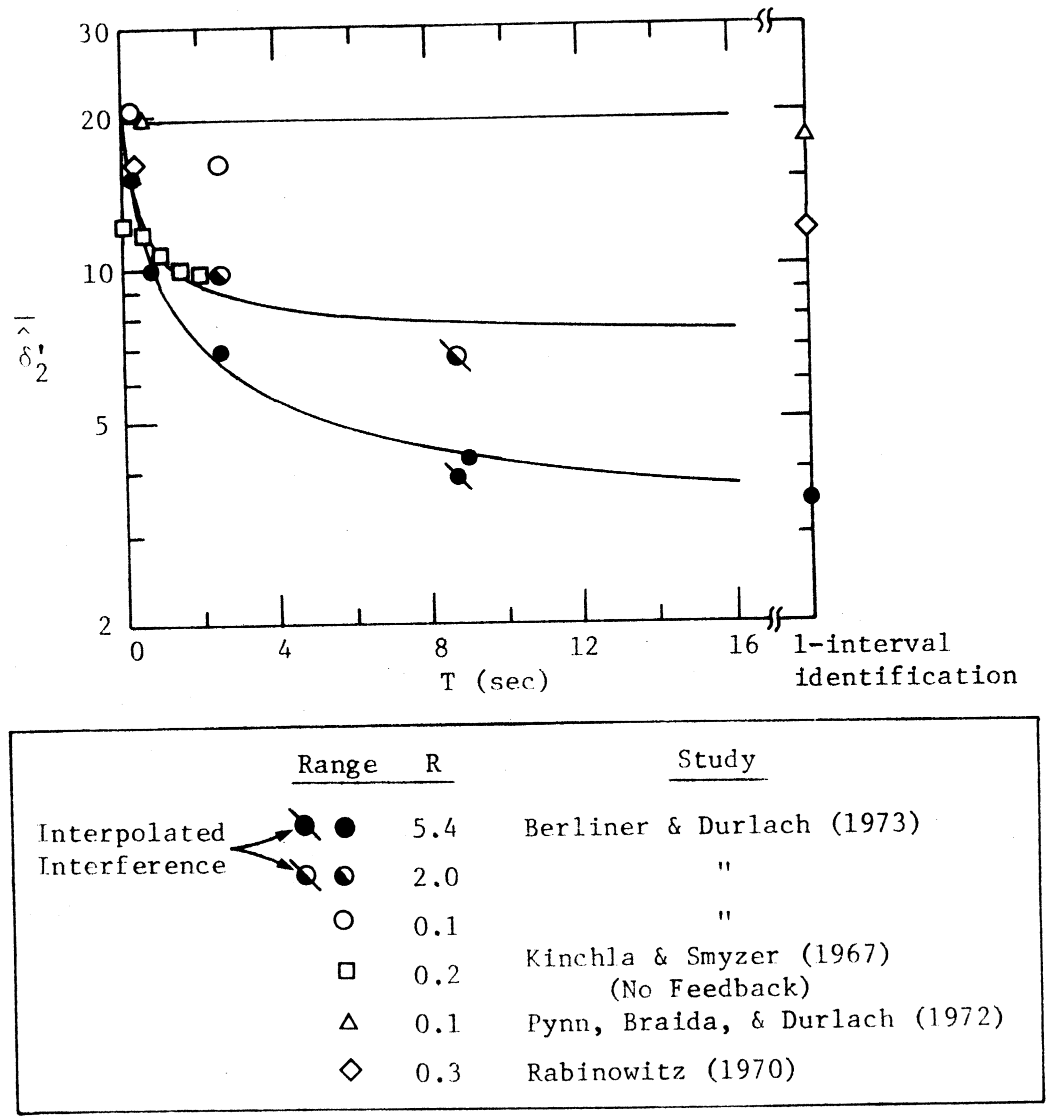


Figure 7.4 $\bar{\hat{\delta}}'_2$ versus T. Results of previous studies.

All studies were conducted with feedback except Kinchla & Smyzer.

At the right edge of the figure are plotted values of $\sqrt{2}\bar{\hat{\delta}}'_1$ obtained from 1-interval identification.

The smooth curves are derived from the preliminary theory under the assumption $K/\beta = 14$, $K/G = 11$, and $K/\sqrt{A} = 7$.

and Rabinowitz (1970). All the experiments except those of Kinchla and Smyzer were conducted with feedback.

To facilitate a comparison of the data presented in Figs. 7.1 - 7.4 with the predictions of the preliminary theory for $\overline{\delta}'_2$ (Eq. 4.10), nets of theoretical curves are drawn in each of these figures. One curve is drawn for each value of R tested. In each figure, the parameters used to derive these curves were chosen to fit the data for tone pulse duration $T_p = 0.50$ sec, and no feedback. The parameter K/B was picked to fit the point $R \simeq 0.2$ and $T = 0.2$ sec, while $K/\sqrt{A}$ and K/G were picked to fit the data for $R \simeq 5.8$. The value of $K/\sqrt{A}$ mainly influences the initial slope of the curve, while K/G mainly influences the level of the curve for large T.

b. Identification Experiments

Figures 7.5a and 7.6a are plots of estimated total resolution, $\hat{\Delta}'$, versus R, and Figs. 7.5b and 7.6b are logarithmic plots of estimated resolution per bel averaged over levels, $\overline{\hat{\delta}}'_1$, versus R from Expt. 4. The data averaged over Observers KG and PN are presented in Fig. 7.5, while the data for Observer JB are presented in Fig. 7.6.

Figures 7.7a and 7.8a are plots of $\hat{\Delta}'$ versus R, and Figs. 7.7b and 7.8b are logarithmic plots of $\overline{\hat{\delta}}'_1$ versus R from other sources. Data from Braida and Durlach (1972) are presented in Fig. 7.7, and data from Pynn, Braida and Durlach (1972) are presented in Fig. 7.8.

Figure 7.9 is a logarithmic plot of $\overline{\hat{\delta}}'_1$ versus reference intensity, I_{ref}, from Expt. 5.

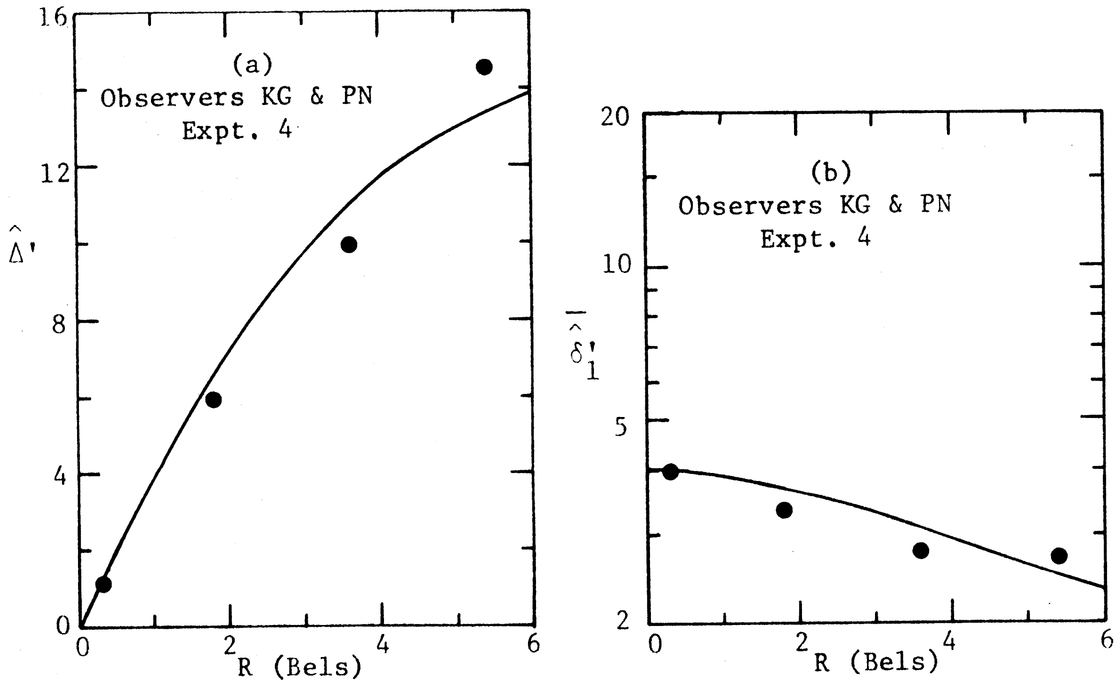


Figure 7.5 Results of Expt. 4, averaged over observers KG & PN. (a) $\hat{\Delta}'$ versus R; (b) $\bar{\hat{\delta}}'_1$ versus R. The smooth curves are derived from the preliminary theory with $K/\beta = 4$ and $K/G = 17$.

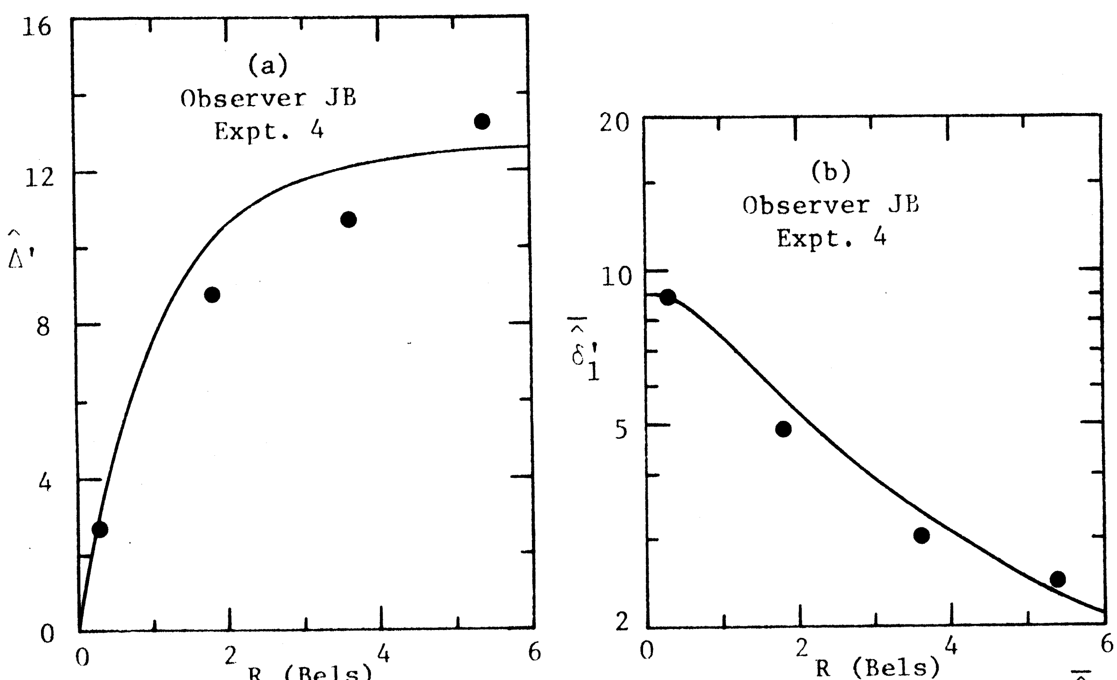


Figure 7.6 Results of Expt. 4. Observer JB. (a) $\hat{\Delta}'$ versus R; (b) $\bar{\hat{\delta}}'_1$ versus R. The smooth curves are derived from the preliminary theory with $K/\beta = 9$ and $K/G = 13$.

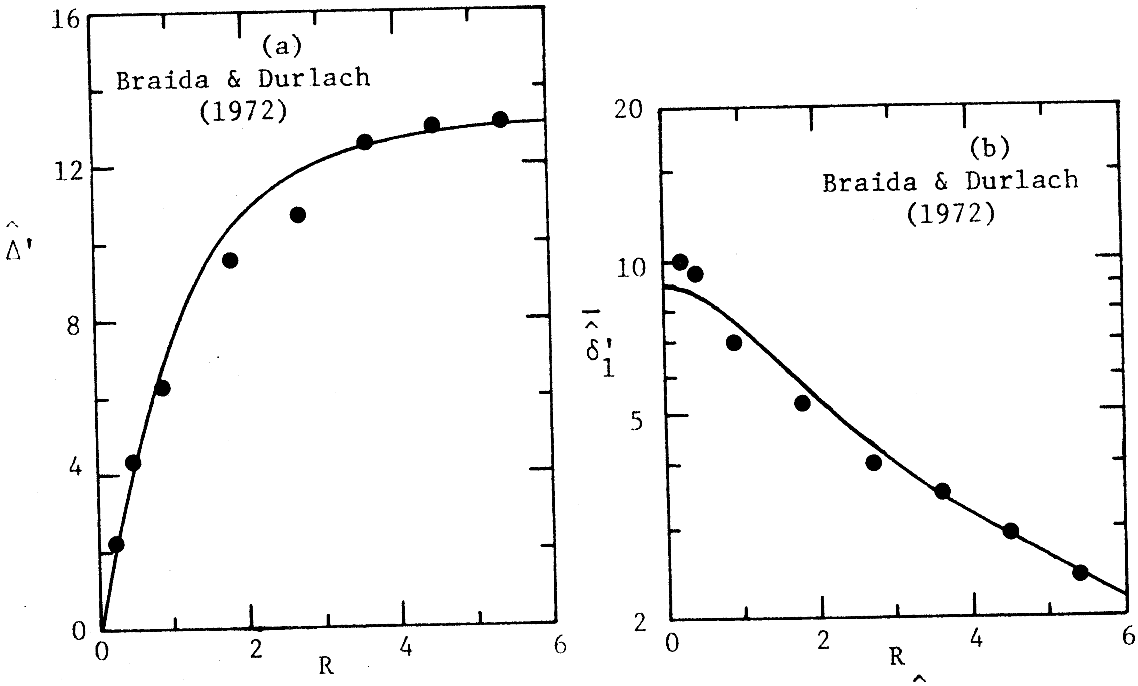


Figure 7.7 Results from Braida & Durlach (1972). (a) $\hat{\Delta}'$ versus R; (b) $\bar{\hat{\delta}}'_1$ versus R. The smooth curves are derived from the preliminary theory with $K/\beta = 9$, and $K/G = 13.5$.

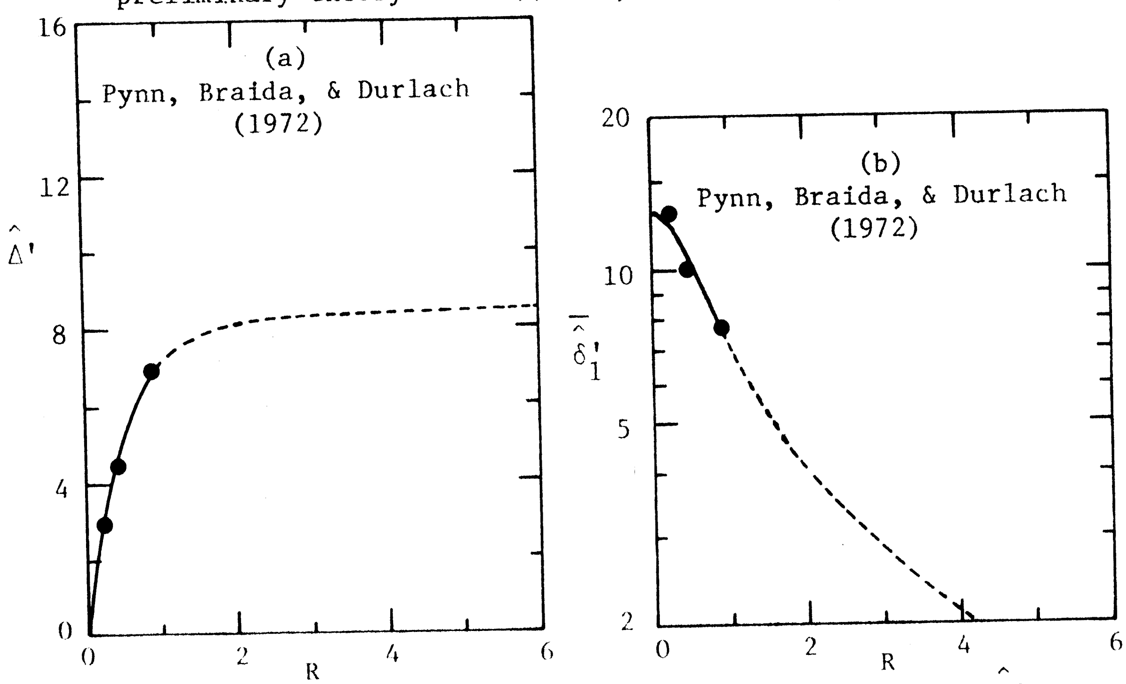


Figure 7.8 Results from Pynn, Braida, & Durlach (1972). (a) $\hat{\Delta}'$ versus R; (b) $\bar{\hat{\delta}}'_1$ versus R. The smooth curves are derived from the preliminary theory with $K/\beta = 13$ and $K/G = 8.5$.

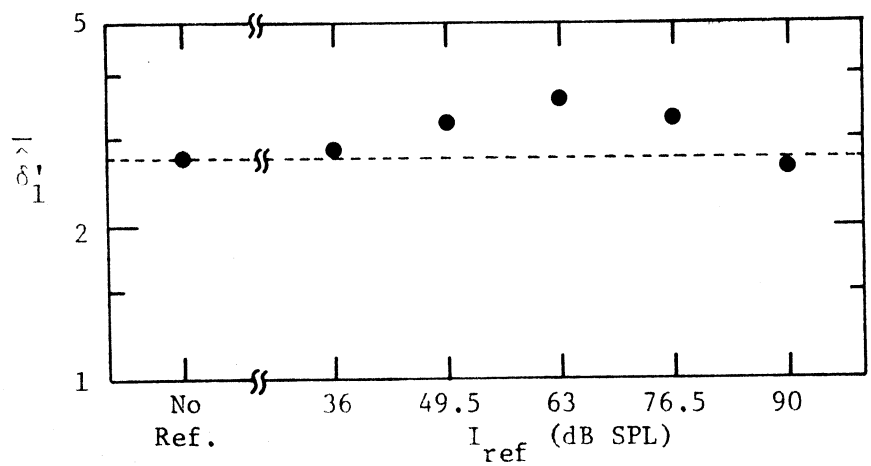


Figure 7.9 Results of Expt. 5. Averaged over all Observers $\bar{\hat{\delta}}'_1$ versus I_{ref}.

To facilitate a comparison of the data presented in Figs. 7.5 - 7.8 with the predictions of the preliminary theory for Δ' (Eq. 4.13) and $\overline{\delta}_1'$ (Eq. 4.12), theoretical curves are drawn in each of these figures. In each figure, the values of the parameters used to derive these curves were chosen to simultaneously fit the results for $\hat{\Delta}'$ and $\overline{\hat{\delta}}_1'$.

c. Relation of Discrimination and Identification

The preliminary theory has several predictions concerning the relation between the resolution per bel in the two-interval discrimination paradigm, δ_2', and the $\sqrt{2}$ times the resolution per bel in the one-interval identification paradigm, $\sqrt{2}\delta_1'$.[20]

To examine these predictions, the quantities $\sqrt{2}\overline{\hat{\delta}}_1'$ for the various ranges of Expt. 4 were plotted at the right edges of Figs. 7.1 - 7.4. The data averaged over Observers KG and PN are presented in Figs. 7.1 and 7.2, while the data for Observer JB are presented in Figs. 7.3 and 7.4.

In addition, the quantities $\sqrt{2}\overline{\hat{\delta}}_1'$ from other sources were plotted at the right edge of Fig. 7.8. Data are included from Berliner and Durlach (1973), Pynn, Braida, and Durlach (1972), and Rabinowitz (1970).

2. Discussion of Results

a. A Major Difference Between Observers KG and PN and The Other Observers

Figures 7.1a, 7.2a, and 7.3b show that for the same set of stimulus conditions (T_p = 0.50 sec and no feedback), the discrimination data of Observers KG and PN exhibit a markedly different dependence on range R and interpulse interval T than the data of Observer JB and the data averaged over the observers of Expt. 3.

For Observers KG and PN, an increase in T from 0.2 to 14.0 sec produced a decline in resolution of about 76 percent (about a factor of 4.2) regardless of the range R. For the other observers, however, the same increase in T produced a decline in resolution of about 77 percent (about a factor of 4.3) when R was large ($R \geq 5.6$), a decline of from 79 percent (about a factor of 4.8) to 64 percent (about a factor of 2.8) for intermediate values of R ($2.0 \leq R \leq 3.9$), and a decline of only about 56 percent (about a factor of 2.3) when R was small ($R \leq 0.2$).

As a result of the larger decline in resolution as a function of T for small R exhibited by Observers KG and PN, for these observers, when T = 14.0 sec an increase in R from 0.2 to 5.4 produced a decline in resolution of only about 21% (about a factor of 1.3). For the other observers, the same increase in R produced a decline in resolution of about 65% (about a factor of 2.9).

Figures 7.5 - 7.8 show that this difference between Observers KG and PN and the rest of the observers [including the observers from Braida and Durlach (1972), and from Pynn, Braida, and Durlach (1972)] is evident in the one-interval identification data as well. For KG and PN, an increase in R from 0.3 to 5.4 produced a decline in resolution per bel, $\overline{\hat{\delta}'_1}$, of only about 32 percent (about a factor of 1.5). For the other observers, however, a similar increase in R produced a decline in $\overline{\hat{\delta}'_1}$ of about 73 percent (about a factor of 3.7).

It does not appear possible to account for the differences between the data of Observers KG and PN and the rest of the observers within the context of the preliminary theory. There does, however, appear to be a plausible way to account for these results. If we assume that these two observers always used the context of the entire set of experiments (i.e., arising from the full 58 dB range) to form their verbal codes, instead of the context arising from the range employed during a particular experimental run,[21] then we would expect their resolution per bel to exhibit no dependence at all on the range R. This prediction is in the right direction, but it is somewhat too strong. We will, however, consider this same assumption concerning these observers later on in Chapter VIII while examining a revised context-coding mode that accounts for the resolution edge-effect; we will then find a better agreement between the resulting prediction and the data.

b. Dependence of Resolution on T for Small R (Discrimination)

A comparison of the data presented in Figs. 7.1 - 7.3 with the theoretical curves drawn in these figures reveals that in all the fixed-level discrimination experiments ($R \leq 0.2$), resolution declined significantly more than predicted as a function of T. Even if we ignore the data of Observers KG and PN, we find that when $R \leq 0.2$, an increase in T from 0.2 to 14.0 sec produced, on the average, a decline in resolution of about 57% (about a factor of 2.3). The preliminary theory predicts that for such small ranges, there should be virtually no decline (less than 1%) in resolution as a function of T. Similar declines in resolution with increasing T for small R are found in the data from Kinchla and Smyzer and from Berliner and Durlach (Fig. 7.4), although these data are available only for values of T between 0 and 2.5 sec.

These results indicate a significant shortcoming of the preliminary theory; some revision is clearly needed. Several reasonable approaches to revising the preliminary theory to account for these results are presented and offered as suggestions for future work in Chapter IX.

c. Dependence of Resolution on R for Small T (Discrimination)

A comparison of the data presented in Figs. 7.1 - 7.4 with the theoretical curves drawn in these figures, reveals that in the discrimination experiments (including those reported by Berliner and Durlach) with short interpulse intervals (T = 0.2 sec), the decline of resolution as a function of R is in agreement with the predictions of the preliminary theory. Restricting our attention to the experiments with $T_p = 0.50$ sec and no feedback (the condition for which the parameters of the preliminary theory were chosen), when T = 0.2 sec an increase in R from about 0.2 to 5.8 produced, on the average, a decline in resolution of about 27% (about a factor of 1.37). The preliminary theory predicts (with the parameters indicated in the figures) that for this short interpulse interval, resolution should decline, on the average, by about 25% (about a factor of 1.33).

d. Overall Dependence of Resolution on R and T (Discrimination)

Two major problems have been noted so far, a discrepancy between Observers KG and PN (who may have been employing a different strategy) and the other observers, and an excessive decline of resolution with T for small R. Except for these two problems, however, it should be pointed out that the overall dependence of resolution on R and T is in fairly good agreement with the preliminary theory. The agreement regarding the dependence of resolution on R for small T has already been discussed. In addition, Figs. 7.1 - 7.4 show that the dependence of resolution on T for large R is in good agreement with the predictions of the preliminary theory (especially with T_p = 0.50 sec and no feedback, the condition for which the parameters of the preliminary theory were chosen). Finally, the weak but general observation is made that resolution declined faster as a function of T for larger R, and declined faster as a function of R for larger T.

A summary of the values of the parameters chosen to fit the discrimination data in Figs. 7.1 - 7.4, as well as the identification data in Figs. 7.5 - 7.8, is given in Table 7.1.

e. Dependence of Resolution on R in Identification

A comparison of the data presented in Figs. 7.5 - 7.8 with the theoretical curves drawn in these figures, reveals that in the identification experiments [including those reported by Braida and Durlach (1972), and Pynn, Braida, and Durlach (1972)], the dependence of resolution on R is in good agreement with the predictions of the preliminary theory. It should be kept in mind, however, that although the identification results of Observers KG and PN are fit fairly well by the parameters $K/\beta = 4$ and and $K/G = 17$, these values are quite different from those chosen to fit their discrimination results. (The relation between the results obtained in these two paradigms is discussed further in the next section.)

TABLE 7.1

Estimates of the Parameters of the Preliminary Theory

Observer	Paradigm	Experiment	Relevant Figure	K/β	K/G	$K/\sqrt{A}$
KG & PN	2I Discrim	Expts. 1 & 1A	7.1	7.5	7	4.2
	1I Ident	Expt. 4	7.5	4	17	
JB	2I Discrim	Expts. 1 & 1A	7.2	12	9	5
	1I Ident	Expt. 4	7.6	9	13	
JB, JW, & SK	2I Discrim	Expts. 3 & 3A	7.3	13	8	6
	2I Discrim	Berliner & Durlach (1973) (a)	7.4	14	11	7
	1I Ident	Braida & Durlach (1972) (a)	7.7	9	13.5	
	1I Ident	Pynn, Braida, & Durlach (1972) (a)	7.8	13	8.5	

(a) The small differences between the values listed in this table and in Table 5.1 are due to the different estimation procedures used.

f. Relation Between One- and Two-Interval Experiments

The data presented in Figs. 7.1, 7.2, and 7.4 indicate several consistent departures from the predictions of the preliminary theory concerning the relation between 2-interval discrimination and 1-interval identification experiments. First, when the value of R used in both experiments is small (so that $G^2R^2 << \beta^2$, and performance in both experiments is limited only by sensation noise) the theory predicts that the resolution per bel δ_2' obtained in the discrimination experiment will be independent of T and will equal the $\sqrt{2}$ times the δ_1' obtained in the identification experiment. It has already been noted that the observed resolution per bel averaged over levels $\bar{\hat{\delta}}_2'$ in the discrimination experiment with small R was not constant with T. Furthermore, the $\bar{\hat{\delta}}_2'$ obtained in discrimination with both R and T small (R = 0.2 and T = 0.2 sec, so that the effect of memory noise was a minimum) was 34% greater than the $\sqrt{2}$ times the $\bar{\hat{\delta}}_1'$ obtained in identification with small R (R = 0.3) for Observer JB, and 96% greater for Observers KG and PN. For the observers of Rabinowitz the corresponding figure was 32% greater; however for the observers of Pynn, Braida, and Durlach it was only 12% greater. These differences are reflected in the higher values of K/β obtained from the discrimination data than from the identification data of the same observers. (See Table 7.1.) These results indicate that, to varying degrees, performance in the identification experiments with small R was limited by more than sensation noise alone.

Second, if the same value of R is used in both a discrimination and an identification experiment, the preliminary theory predicts that the δ_2' obtained in the discrimination experiment will be greater than or equal to the $\sqrt{2}$ times the δ'_1 obtained in the identification experiment. Furthermore, it predicts that equality will be obtained when T is large relative to R in the discrimination experiment (so that $AT >> G^2R^2$, and the observer is forced into the context-coding mode). The data presented in Figs. 7.1 and 7.2 show instead that the $\overline{\hat{\delta}'_2}$ from discrimination with T = 14.0 sec was always poorer than the $\sqrt{2}$ times the $\overline{\hat{\delta}'_1}$ from identification with the same value of R. The $\overline{\hat{\delta}'_2}$ from discrimination with T = 14.0 sec was on the average 43% (about a factor of 1.75) less than the $\sqrt{2}$ times the $\overline{\hat{\delta}'_1}$ from the identification experiments with the same value of R for Observers KG and PN, and on the average 27% (about a factor of 1.37) less for Observer JB. These differences are reflected in the lower values of K/G obtained from the discrimination data than the identification data of the same observers. (See Table 7.1.) These results indicate that resolution in discrimination with a fixed R and increasing T is not lower bounded by resolution in identification with the same R.

We now consider some results concerning resolution averaged over levels about which the preliminary theory makes no predictions.

g. Dependence of Resolution on Tone Pulse Duration T_p (Discrimination)

A careful examination of Fig. 7.3 reveals that in virtually every case, resolution per bel averaged over levels, $\overline{\hat{\delta}'_2}$, increased with increasing tone pulse duration T_p.[22] When T_p was increased from 0.20 to 0.50 sec, $\overline{\hat{\delta}'_2}$ increased on the average by about 47%, and when T_p was increased from 0.50 to 1.25 sec, $\overline{\hat{\delta}'_2}$ increased on the average by about 29%. The largest increases in resolution with increasing T_p occurred when the interpulse interval T was 0.0 sec.

With the following interesting exception, the functional form of $\bar{\hat{\delta}}'_2$ versus R and T was not much affected by T_p. When T_p was 0.50 or 1.25 sec, a decrease in T from 0.2 to 0.0 sec produced an increase in $\bar{\hat{\delta}}'_2$ (of about 61%, on the average); whereas, when T_p was only 0.20 sec, the same decrease in T produced a <u>decrease</u> in $\bar{\hat{\delta}}'_2$ (of about 13%, on the average).[23] Possibly, the combination of the short tone pulse duration and the zero interpulse interval limited the time available for the observers to assimilate the first tone pulse, thereby increasing the sensation noise associated with the first tone.

h. <u>Dependence of Resolution on Feedback (Discrimination)</u>

A comparison of Figs. 7.1a and 7.2a with Figs. 7.1b and 7.2b reveals that the presence of feedback, and instructions to eliminate the bias edge-effect, had only a small effect on resolution. Observers KG and PN experienced a slight improvement in resolution, about 16% on the average, with feedback, while Observer JB showed, on the average, no change in resolution with feedback. In general, feedback tended to be of most assistance when the range was small, and when the interpulse interval T was short.

i. <u>Dependence of Resolution on I_{ref} in Identification</u>

An examination of Fig. 7.9 reveals that the resolution per bel average over levels, $\bar{\hat{\delta}}'_1$, in identification was essentially unchanged, relative to the no reference condition, when the reference intensity coincided with the edges of the range (I_{ref} = 36 or 90 dB SPL); $\bar{\hat{\delta}}'_1$ was slightly increased when the reference was halfway between either edge and the middle of the range (I_{ref} = 49.5 or 76.5 dB SPL); and $\bar{\hat{\delta}}_1$ was moderately increased when the reference was in the middle of the range (I_{ref} = 63 dB SPL).

B. Resolution as a Function of Level

1. Discrimination Experiments

Figures 7.10 - 7.12 are logarithmic plots of estimated resolution per bel, $\hat{\delta}_2'$, versus overall intensity I with T as a parameter, for each range R, for all the roving-level discrimination experiments. The data averaged over Observers KG and PN from Expts. 1 and 2 are presented in Fig. 7.10, the data for Observer JB from these experiments are presented in Fig. 7.11, and the data from Expt. 3 are presented in Fig. 7.12.

Examination of these figures reveals that under certain experimental conditions, the resolution was better near the edges of the range than in the middle. As a measure of this phenomenon, known as the resolution edge-effect, the product of $\hat{\delta}_2'$ at each edge of the range divided by $\hat{\delta}_2'$ in the middle of the range,[24]

$$\nabla\hat{\delta}_2' = \frac{\hat{\delta}_2'(I_{max})\ \hat{\delta}_2'(I_{min})}{\left[\hat{\delta}_2'(I = 63\text{ dB SPL})\right]^2} \tag{7.1}$$

was calculated for all the roving-level discrimination experiments.

Figures 7.13 - 7.15 are logarithmic plots of $\nabla\hat{\delta}_2'$ versus T with R as a parameter. The data averaged over Observers KG and PN from Expts. 1 and 2 are presented in Fig. 7.13, the data for Observer JB from these experiments are presented in Fig. 7.14, and the data from Expt. 3 are presented in Fig. 7.15. Restricting our attention to these three figures, we can

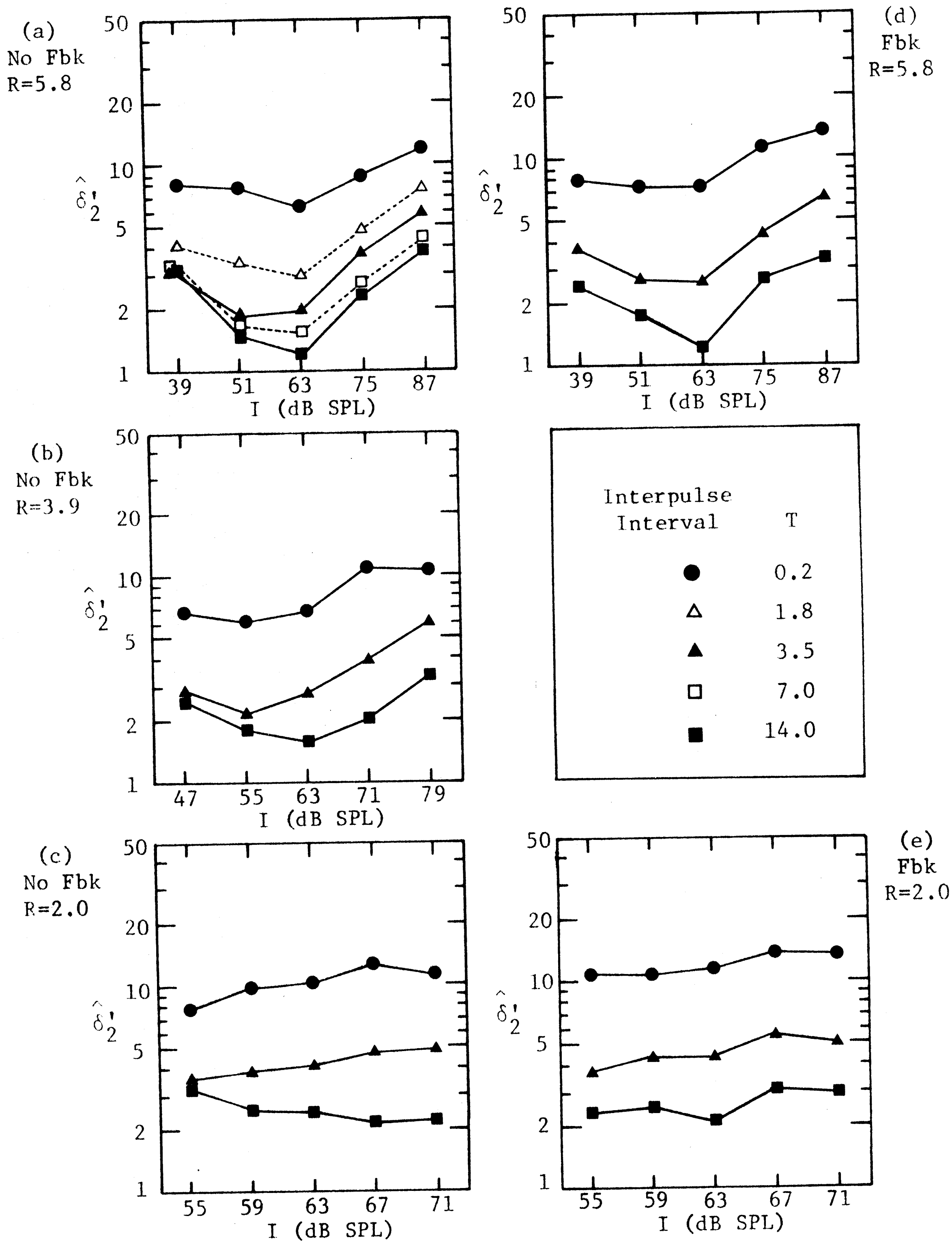
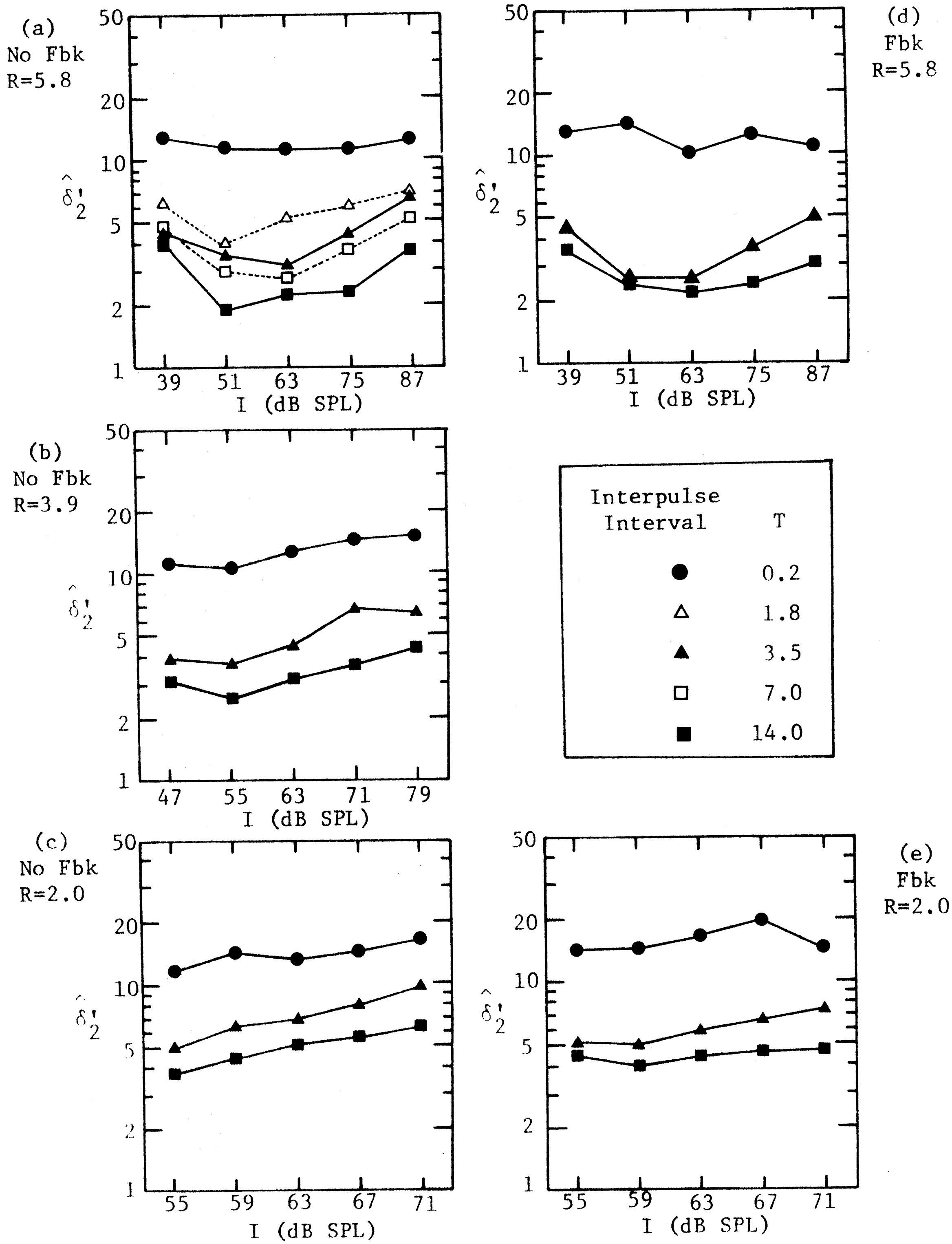


Figure 7.10 Results of Expts. 1 & 2. Averaged over Observers KG & PN. $\hat{\delta}'_2$ versus I.



<u>Figure 7.11</u> Results of Expts. 1 & 2. Observer JB.
$\hat{\delta}_2'$ versus I.

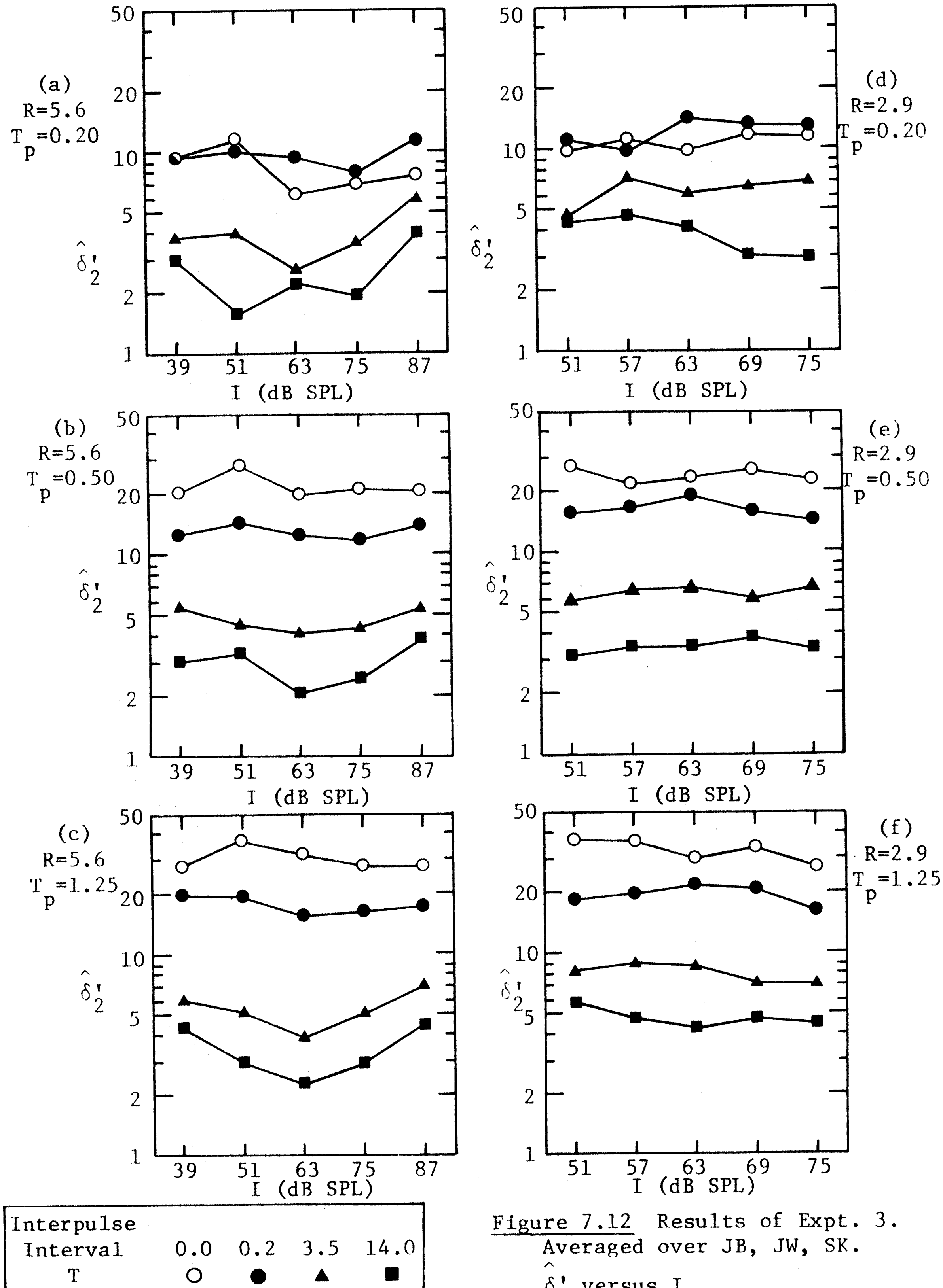


Figure 7.12 Results of Expt. 3. Averaged over JB, JW, SK. $\hat{\delta}'_2$ versus I.

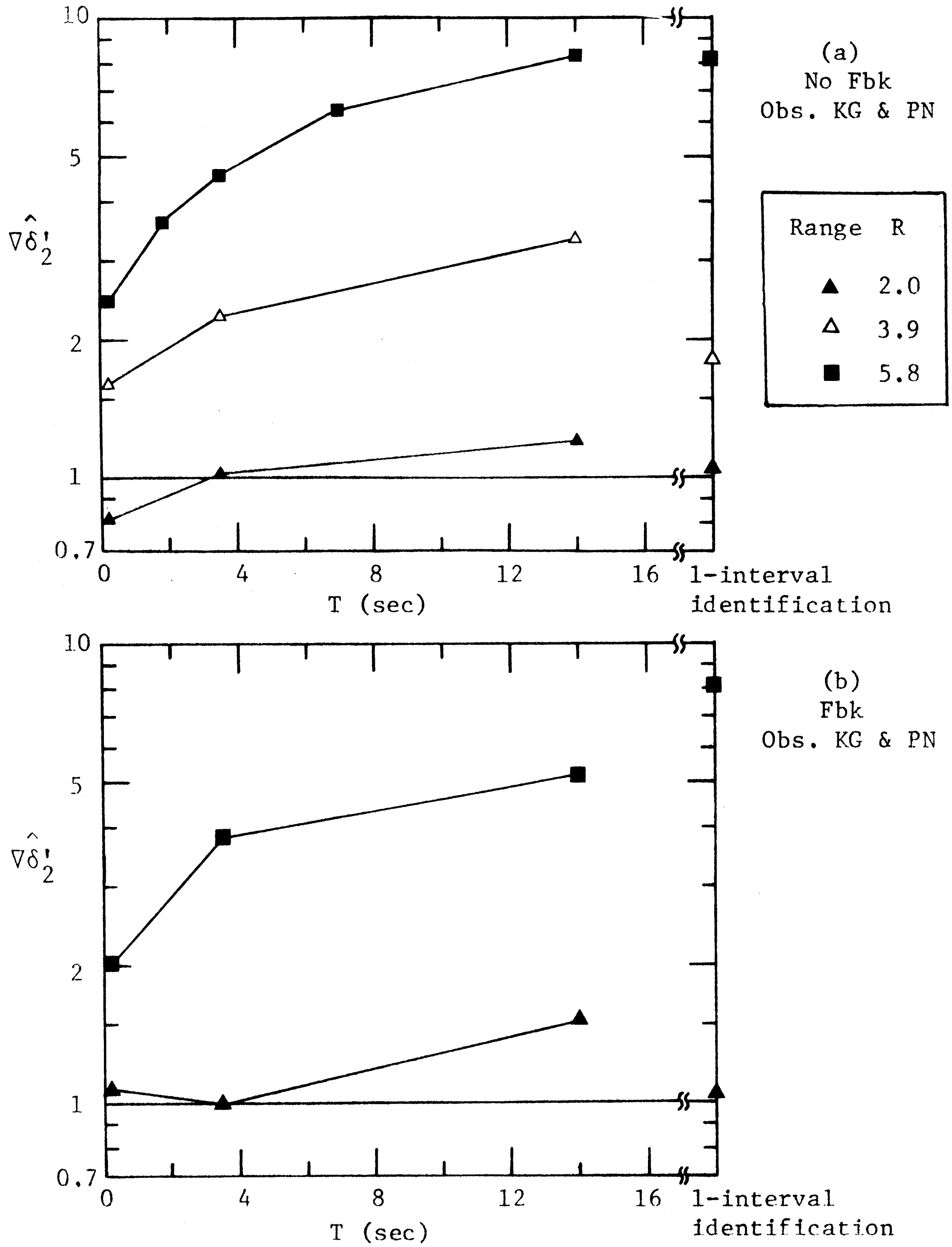


Figure 7.13 A Measure of the Resolution Edge-Effect in Expts. 1 & 2. $\nabla\hat{\delta}_2'$ versus T. Averaged over KG & PN. (a) No Feedback (Expt. 1); (b) Feedback (Expt. 2).

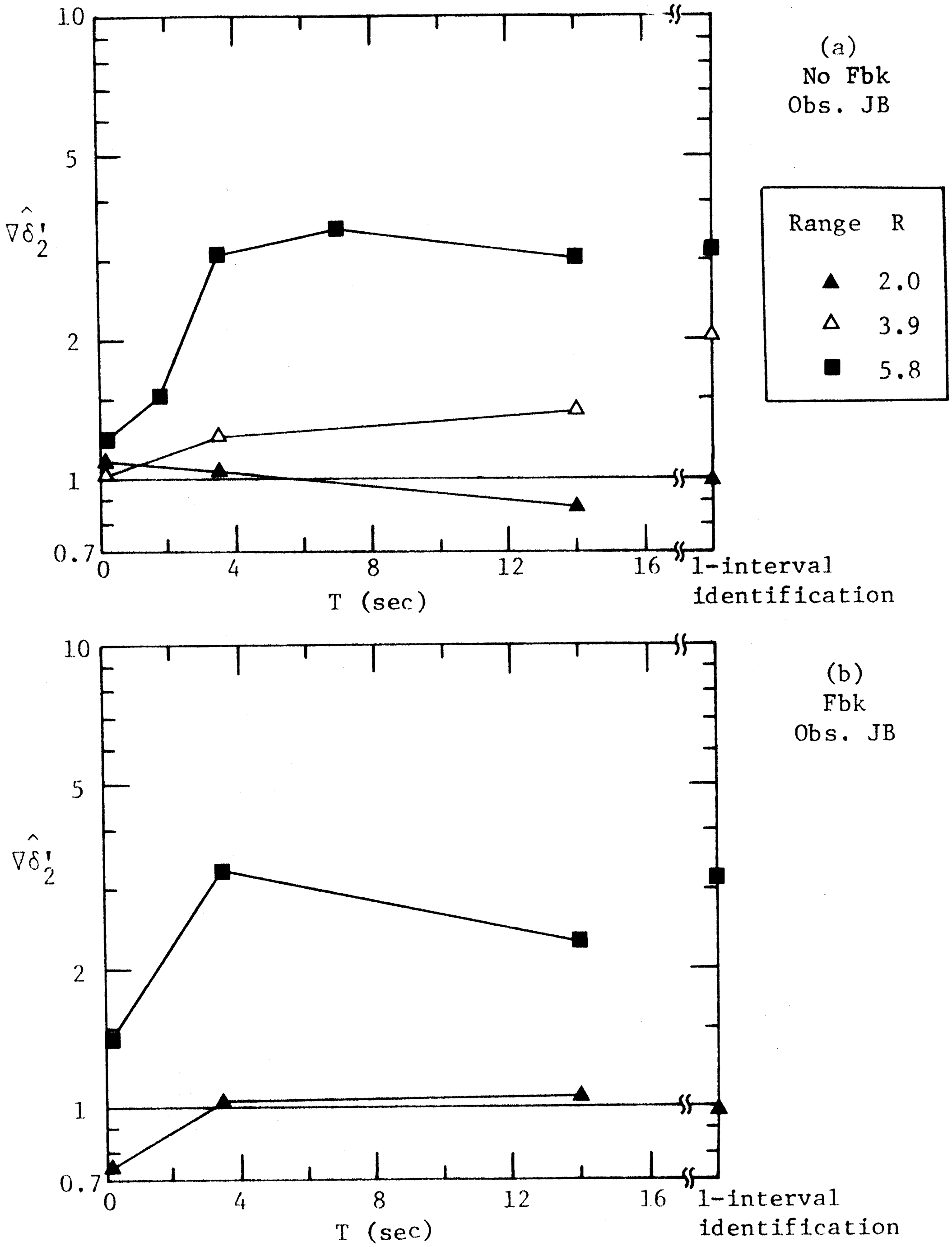


Figure 7.14 A Measure of the Resolution Edge-Effect in Expts. 1 & 2. $\hat{\nabla\delta}'_2$ versus T. Observer JB.
(a) No feedback (Expt. 1); (b) Feedback (Expt. 2).

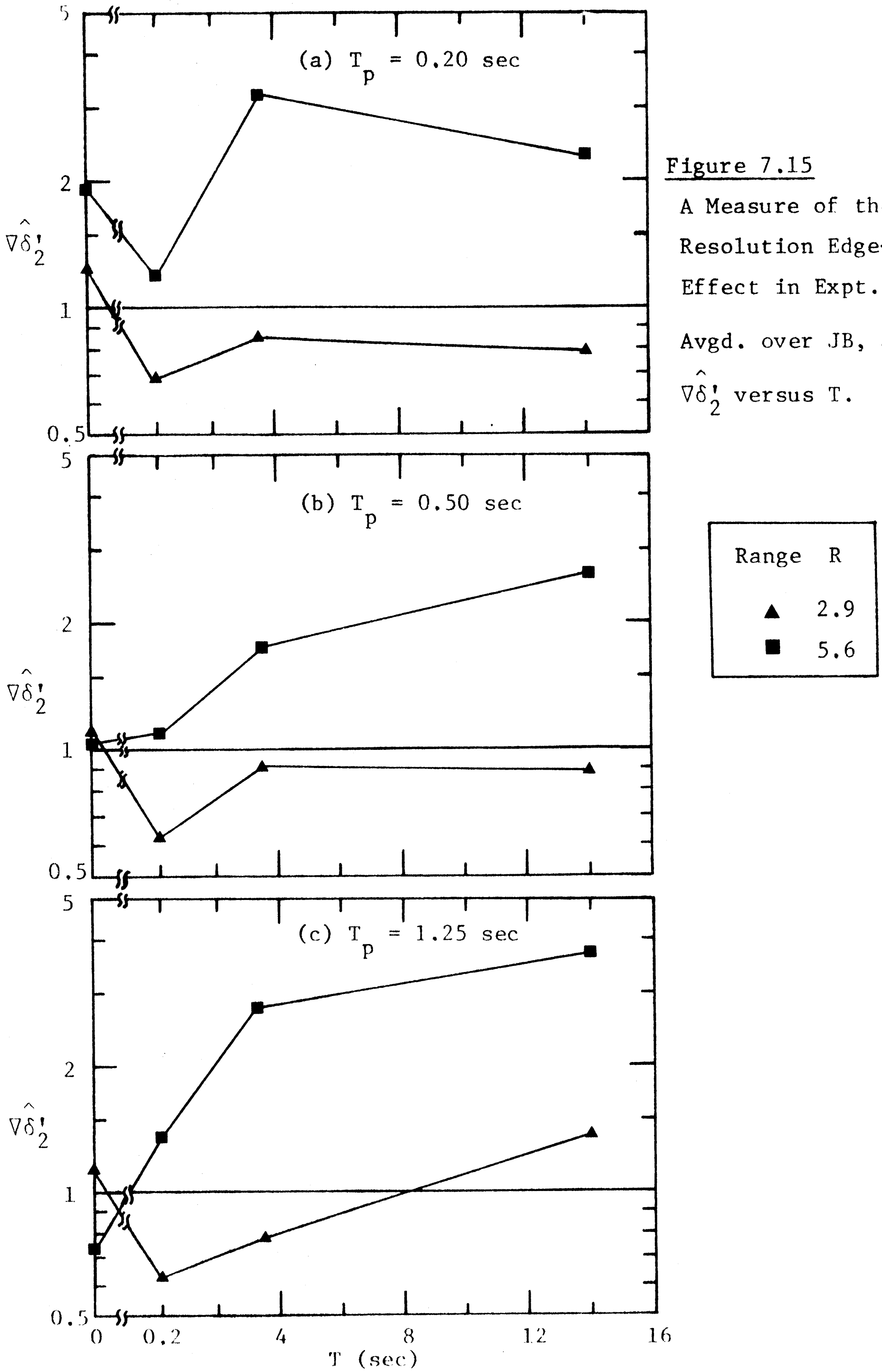


Figure 7.15

A Measure of the Resolution Edge-Effect in Expt. 3.

Avgd. over JB, JW & SK

$\nabla\hat{\delta}'_2$ versus T.

make some observations concerning the dependence of the resolution edge-effect,[25] as measured by $\nabla\hat{\delta}'_2$, on various experimental parameters.

a. Dependence of $\nabla\hat{\delta}'_2$ on R and T

Figures 7.13 - 7.15 show that the resolution edge-effect tended to be relatively small when the interpulse interval T was 0.2 sec, and generally increased with increasing T. The effect was virtually non-existent when the range R was less than 3.9, and generally increased with increasing R. These dependencies on R and T will be considered in Chapter VIII in the discussion of a new context coding model which attempts to account for the resolution edge-effect.

b. Dependence of $\nabla\hat{\delta}'_2$ on Tone Pulse Duration T_p

Examination of Fig. 7.15 reveals that, with the following interesting exception, the resolution edge-effect was generally not strongly affected by T_p. In Expt. 3 with R = 5.6, when T_p was 0.50 or 1.25 sec, a decrease in T from 0.2 to 0.0 sec produced a decrease in $\nabla\hat{\delta}'_2$ (of about 26%, on the average); whereas, when T_p was only 0.20 sec, the same decrease in T produced an increase in $\nabla\hat{\delta}'_2$ (of about 61%).

c. Dependence of $\nabla\hat{\delta}'_2$ on Feedback

A comparison of Figs. 7.13a and 7.14a with Figs. 7.13b and 7.14b reveals that feedback, and instructions to eliminate bias, had virtually no effect on the resolution edge-effect.

2. Identification Experiments Without a Reference Stimulus

Figures 7.16 and 7.17 are plots of estimated resolution, $\hat{d}'_1(I_{i+1}; I_i)$,

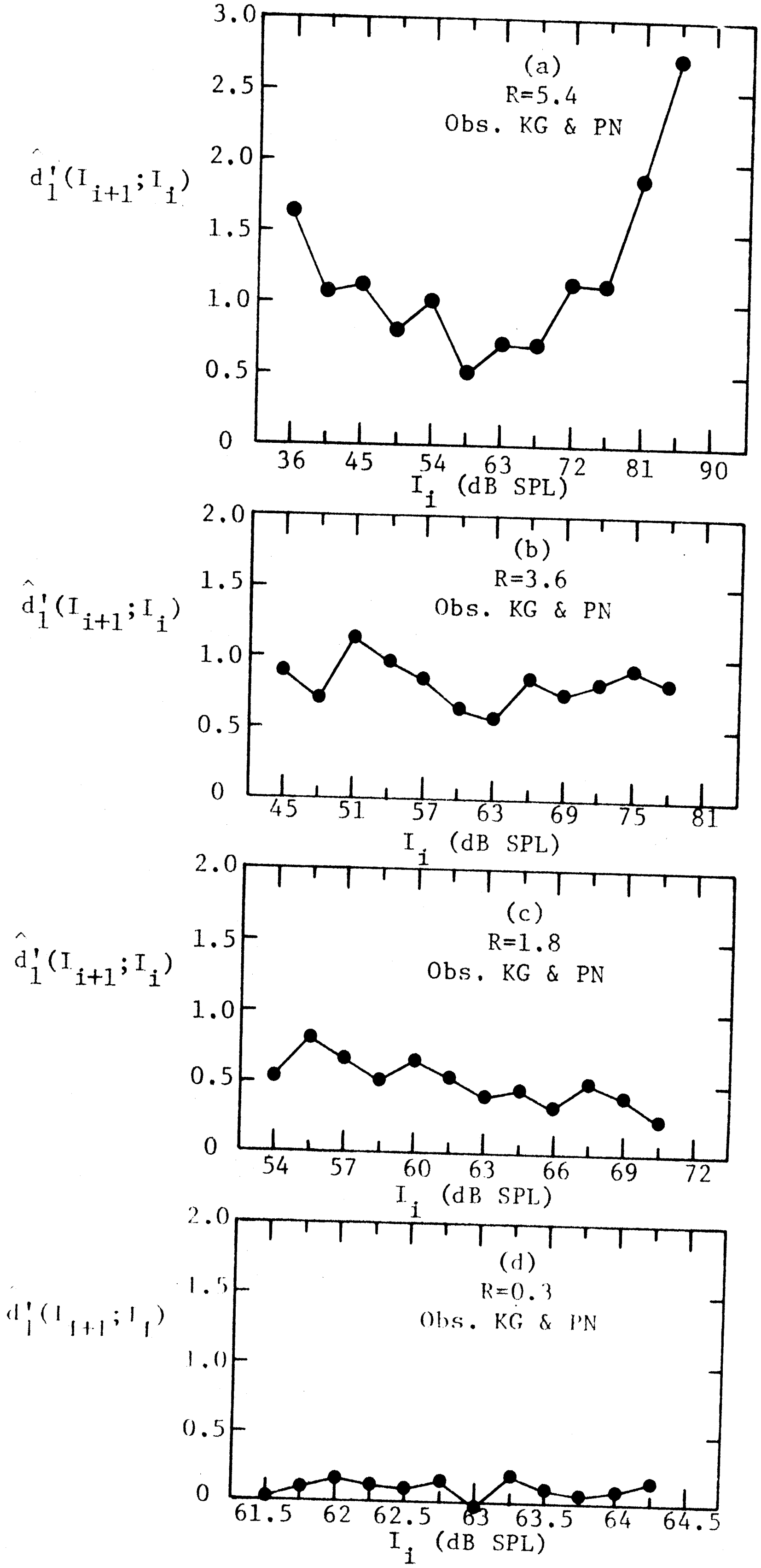


Figure 7.16

Results of Expt. 4.

Averaged over Observers KG & PN

$\hat{d}'_1(I_{i+1};I_i)$ vs. I_i.

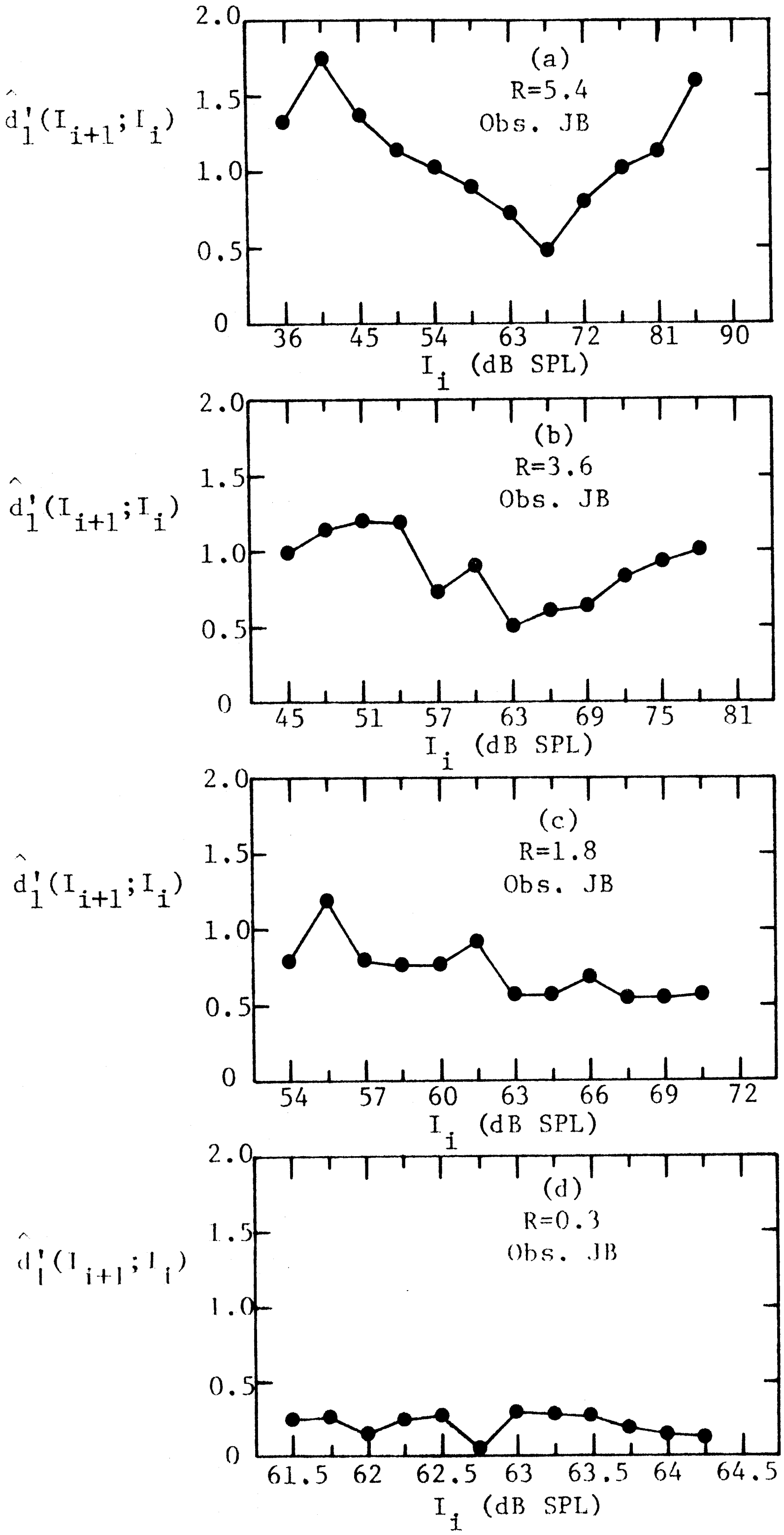


Figure 7.17

Results of Expt. 4.

Observer JB

$\hat{d}'_1(I_{i+1};I_i)$

versus I_i for each range R in Expt. 4. The data averaged over Observers KG and PN are presented in Fig. 7.16, and the data for Observer JB in Fig. 7.17.

Examination of these figures reveals that, as in the discrimination experiments, under certain experimental conditions, the resolution was better near the edges of the range than in the middle. As a measure of this resolution edge-effect, the product of $\hat{\delta}_1'$ near each edge of the range divided by $\hat{\delta}_1'$ in the middle of the range,

$$\nabla\hat{\delta}_1' = \frac{\hat{\delta}_1'(I_3;\ I_1)\ \hat{\delta}_1'(I_{13};\ I_{11})}{\left[\hat{\delta}_1'(I_8;\ I_6)\right]^2} \tag{7.2}$$

was calculated for the data of Expt. 4.

Figures 7.18 and 7.19 are logarithmic plots of $\nabla\hat{\delta}_1'$ versus R.[26] The data averaged over Observers KG and PN are presented in Fig. 7.18, and for Observer JB in Fig. 7.19.

a. Dependence of $\nabla\hat{\delta}_1'$ on R

Examination of Figs. 7.18 and 7.19 reveals that the resolution edge-effect was non-existent when the range R was less than 1.8, and that it increased with increasing R. This dependence will be considered in Chapter VIII in the discussion of a new context-coding model which attempts to account for the resolution edge-effect.

b. Relation of $\nabla\hat{\delta}'$ in Discrimination and Identification

To examine the relative magnitudes of the resolution edge-effect noted in discrimination and identification experiments, the values of $\nabla\hat{\delta}_1'$

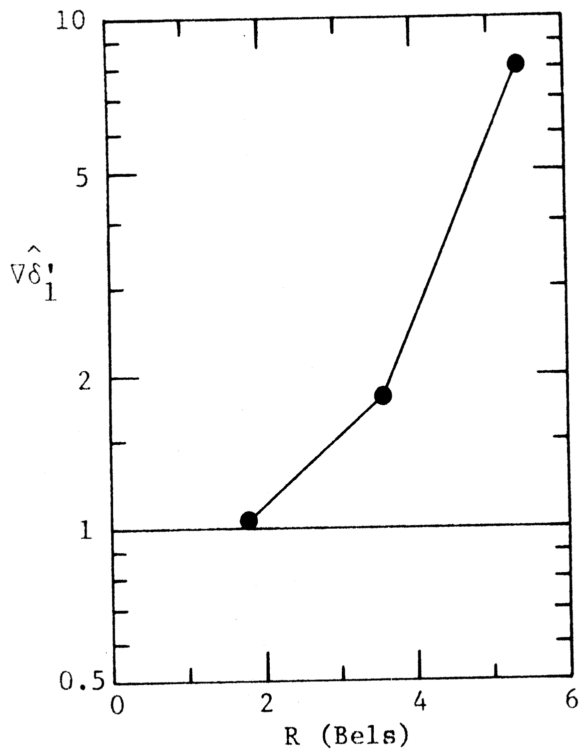


Figure 7.18

A Measure of the Resolution Edge-Effect in Expt. 4.

Averaged over KG & PN

$\nabla\hat{\delta}_1'$ versus R

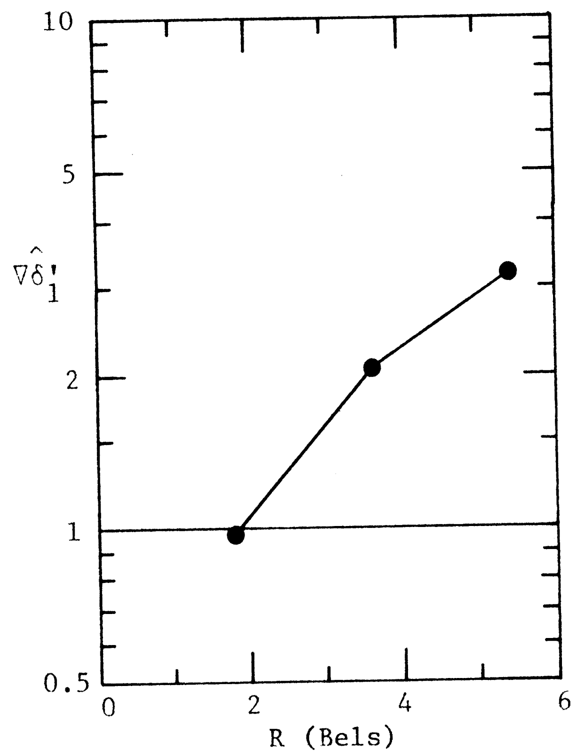


Figure 7.19

A Measure of the Resolution Edge-Effect in Expt. 4.

Observer JB

$\nabla\hat{\delta}_1'$ versus R

from Expt. 4 were plotted at the right edges of Figs. 7.13 and 7.14. These figures show a reasonable agreement between the magnitudes of the effect observed in the identification experiments and the corresponding discrimination experiments with similar values of R and large values of T.

3. Identification Experiments With a Reference Stimulus

Figures 7.20a-f are plots of estimated resolution, $\hat{d}_1'(I_{i+1};\ I_i)$, versus I_i, for each reference stimulus condition in Expt. 5. In addition to the resolution edge-effect, observable as the U-shaped trend of these curves, and the departure from Weber's law,[27] observable as the overall positive slopes of these curves, examination of these figures reveals that resolution tends to increase somewhat in the vicinity of the reference stimulus. To assess the effects of the various reference stimuli, independent of the resolution edge-effect and the departure from Weber's law, the ratio of the resolution with and without a reference,

$$\hat{d}_r'(I_{i+1};\ I_i|I_{ref}) = \frac{\hat{d}'(I_{i+1};\ I_i|I_{ref})}{\hat{d}'(I_{i+1};\ I_i|\text{No ref.})}\ , \qquad (7.3)$$

was calculated. Figures 7.21a-e are plots of this ratio versus I_i.

An examination of Figs. 7.21a-e reveals somewhat more clearly the tendency of resolution to be improved in the vicinity of the reference stimulus. This improvement did not occur when the reference intensity coincided with the edges of the range (I_{ref} = 36 or 90 dB SPL); the improvement was moderate when the reference was halfway between either edge and the middle of the range (I_{ref} = 49.5 or 76.5 dB SPL); and it was a

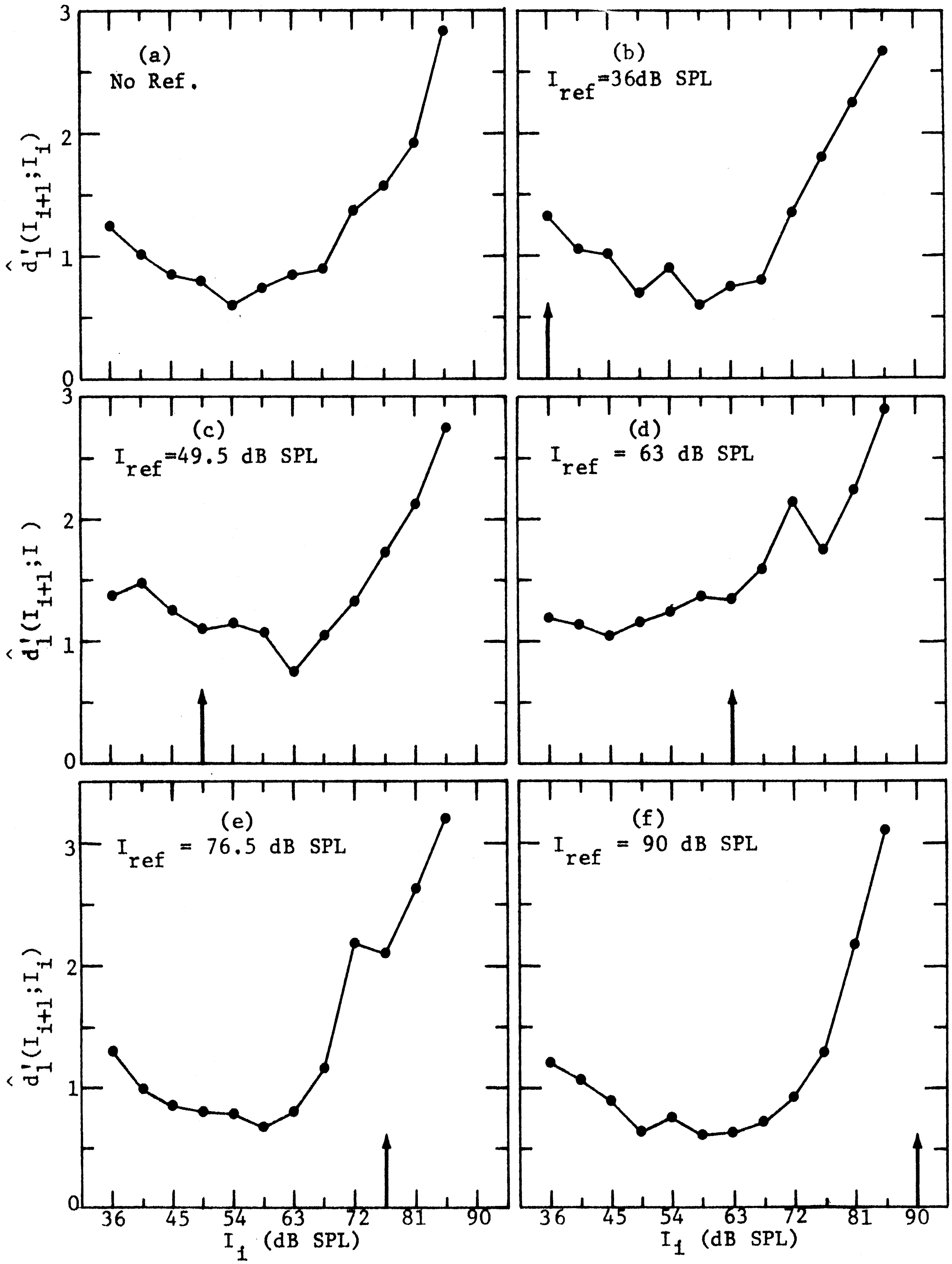


Figure 7.20 Results of Expt. 5. $\hat{d}'_1$ versus I_i with different I_{ref}. Averaged over 6 observers.

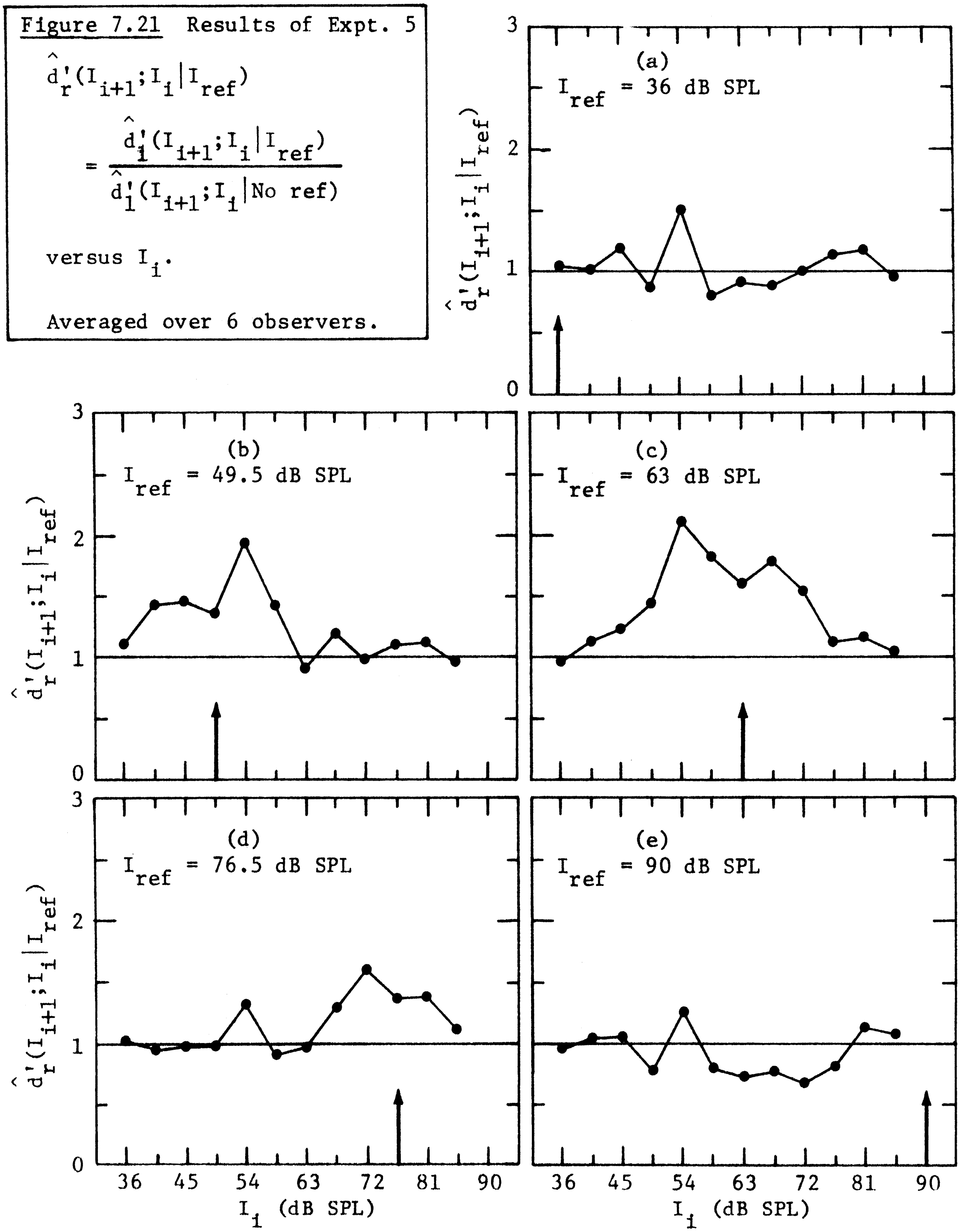
Figure 7.21 Results of Expt. 5

$\hat{d}'_r(I_{i+1};I_i|I_{ref})$

$$= \frac{\hat{d}'_i(I_{i+1};I_i|I_{ref})}{\hat{d}'_i(I_{i+1};I_i|\text{No ref})}$$

versus I_i.

Averaged over 6 observers.



maximum when the reference was in the middle of the range (I_{ref} = 63 dB SPL).

C. Criterion as a Function of Level

1. Discrimination Experiments

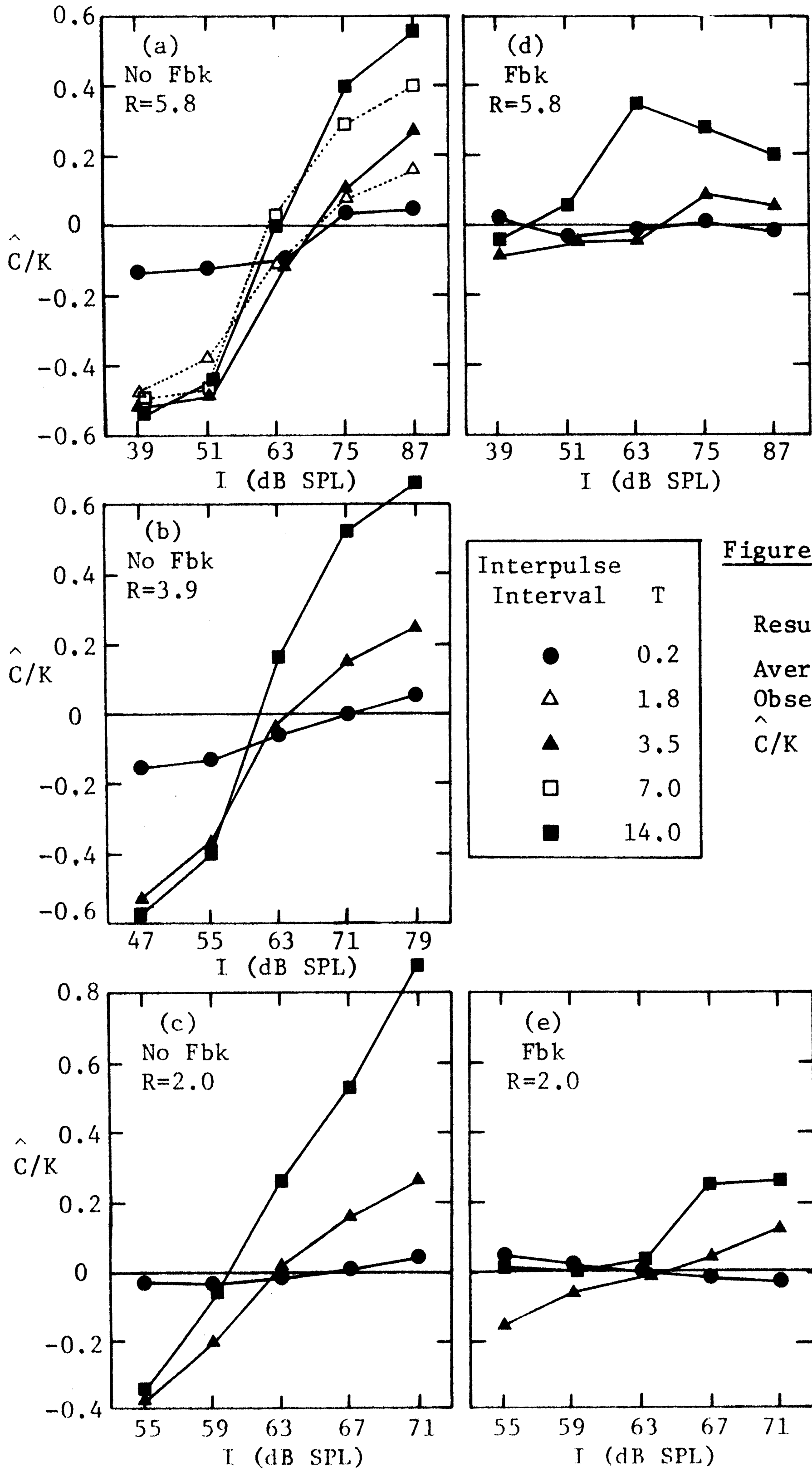
Figures 7.22 - 7.24 are plots of estimated criterion, $\hat{C}/K$, versus overall intensity I with T as a parameter, for each range R, for all the roving-level discrimination experiments. The data averaged over Observers KG and PN from Expts. 1 and 2 are presented in Fig. 7.22, the data for Observer JB from these experiments are presented in Fig. 7.23, and the data from Expt. 3 are presented in Fig. 7.24.

An examination of these figures reveals that under certain experimental conditions, the criteria were strongly negative for stimuli near the bottom of the range and were strongly positive for stimuli near the top of the range. This reflects the excessive tendencies of the observers to respond "H, L" when the stimuli were near the bottom of the range, and to respond "L, H" when the stimuli were near the top of the range. To appreciate the magnitude of the effect, recall that $\hat{C}/K$ may be considered the point of subjective equality (in bels) for the intensities of the first and second intervals.

As a measure of this phenomenon, known as the <u>bias edge-effect</u>, the difference between the criteria at the edges of the range,

$$\nabla\hat{C}/K = \hat{C}/K(I_{max}) - \hat{C}/K(I_{min}) \quad , \tag{7.4}$$

was calculated for all the roving-level discrimination experiments.



<u>Figure 7.22</u>

Results of Expts. 1 & 2.

Averaged over Observers KG & PN.
Ĉ/K versus I.

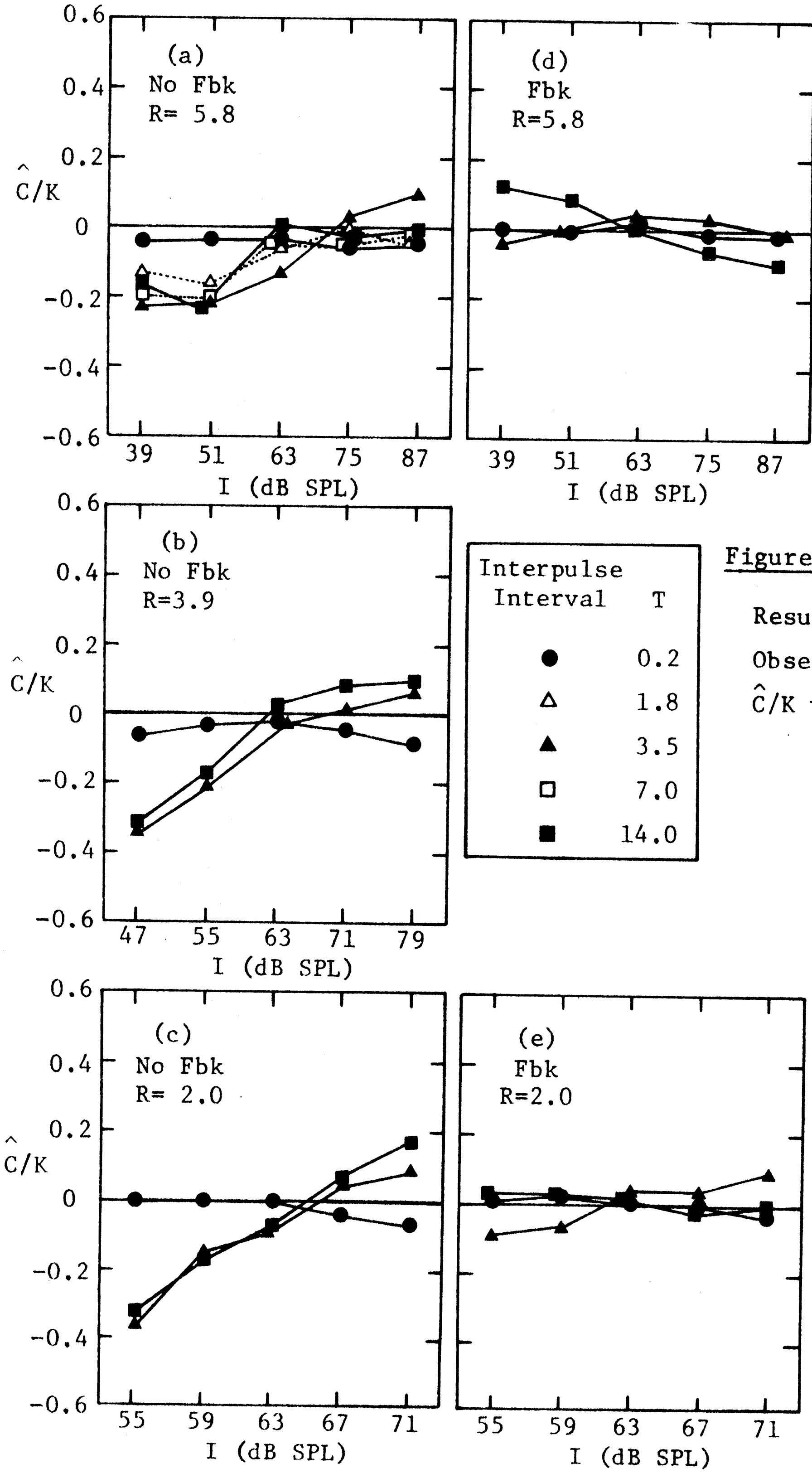


Figure 7.23

Results of Expts. 1 & 2.
Observer JB.
$\hat{C}/K$ versus I.

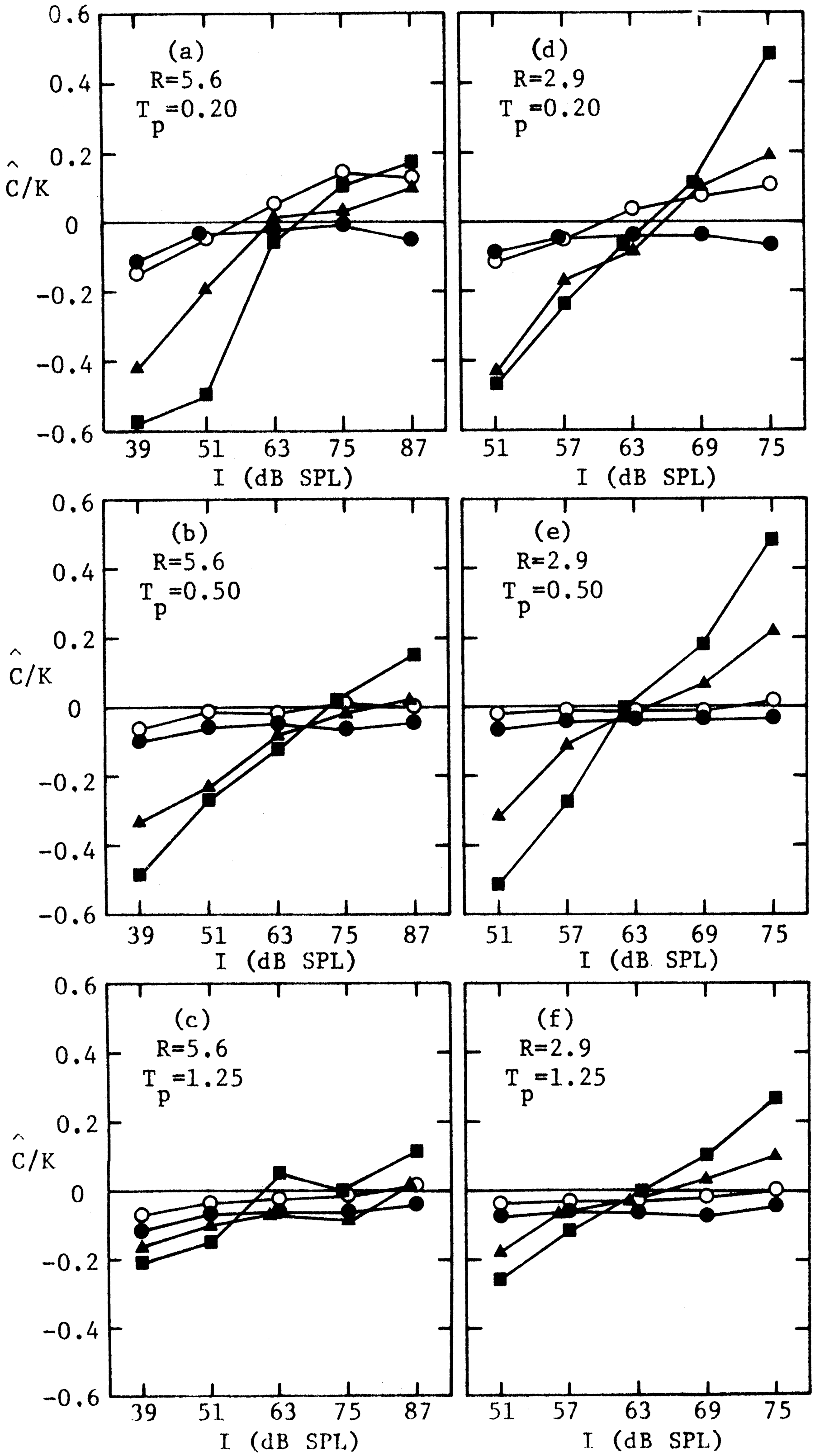
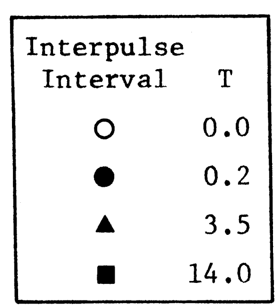


Figure 7.24

Results of Expt. 3.

Averaged over JB, JW, & SK.
$\hat{C}/K$ versus I.



Figures 7.25 - 7.27 are plots of $\nabla\hat{C}/K$ versus T with R as a parameter. The data averaged over Observers KG and PN from Expts. 1 and 2 are presented in Fig. 7.25, the data for Observer JB from these experiments are presented in Fig. 7.26, and the data from Expt. 3 are presented in Fig. 7.27. Restricting our attention to these figures, we can make some observations concerning the dependence of the bias edge-effect, as measured by $\nabla\hat{C}/K$, on various experimental parameters.

a. Dependence of $\nabla\hat{C}/K$ on R and T

Figures 7.25 - 7.27 show that the bias edge-effect was virtually non-existent when the interpulse interval T was 0.2 sec, and generally increased with increasing T. The effect was strongly evident for all values of R (i.e., in all the roving-level experiments) and tended to decrease somewhat with increasing R. These dependencies on R and T will be considered in the next chapter in the discussion of several proposed (unsuccessful) revisions of the preliminary theory which attempt to account for the bias edge-effect.

b. Dependence of $\nabla\hat{C}/K$ on Tone Pulse Duration T_p

An examination of Fig. 7.27 reveals that the bias edge-effect tended to diminish somewhat with increasing tone pulse duration T_p. With the following interesting exception, the functional form of $\nabla\hat{C}/K$ versus R and T was not much affected by T_p. In Expt. 3, when T_p was 0.50 or 1.25 sec, a decrease in T from 0.2 to 0.0 sec produced almost no change in $\nabla\hat{C}/K$; whereas, when T_p was only 0.20 sec, the same decrease in T produced a sharp increase in $\nabla\hat{C}/K$.

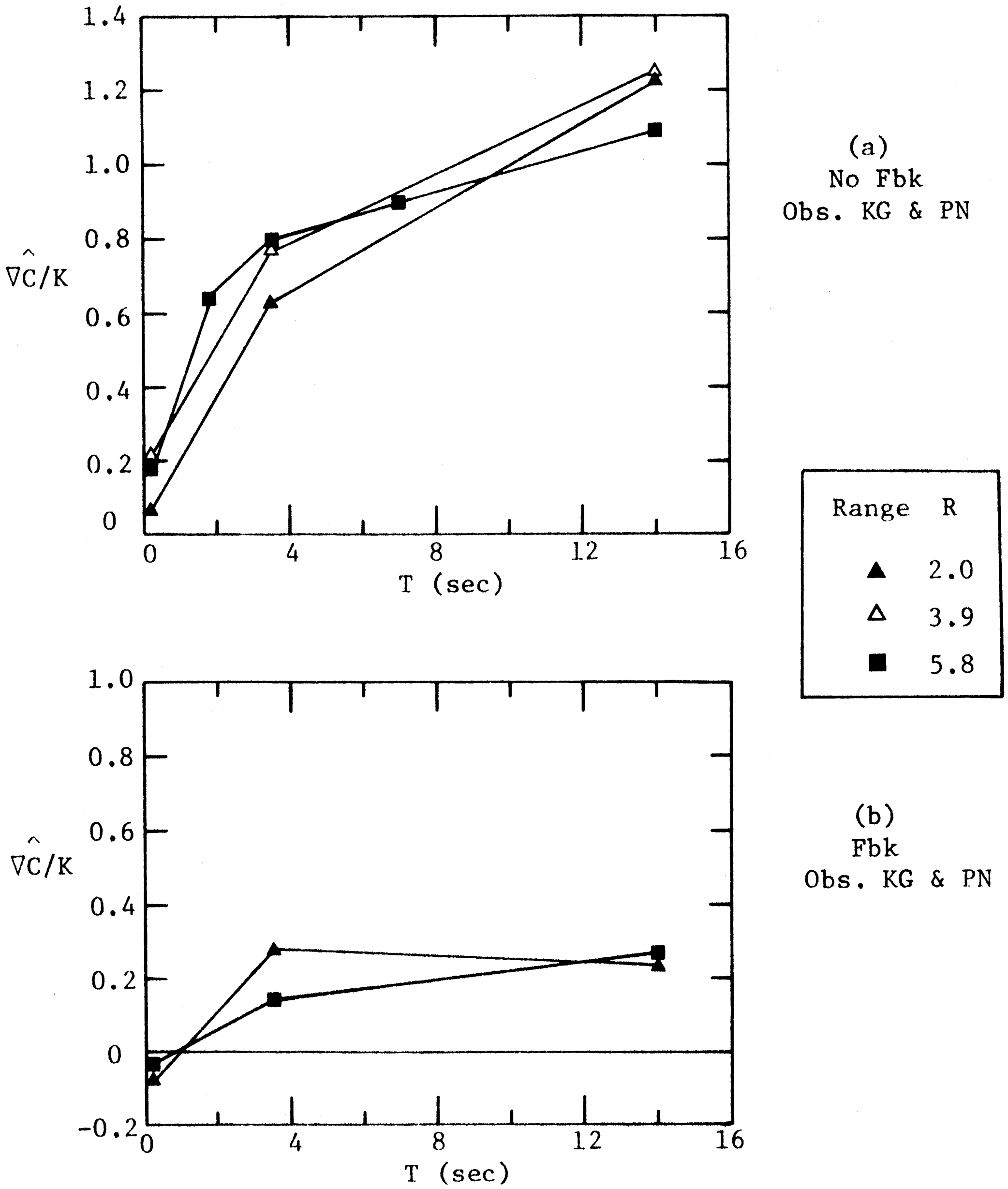


Figure 7.25 A Measure of the Bias Edge-Effect in Expts. 1 & 2. $\hat{\nabla C}/K$ versus T. Averaged over KG & PN. (a) No Feedback (Expt. 1); (b) Feedback (Expt. 2).

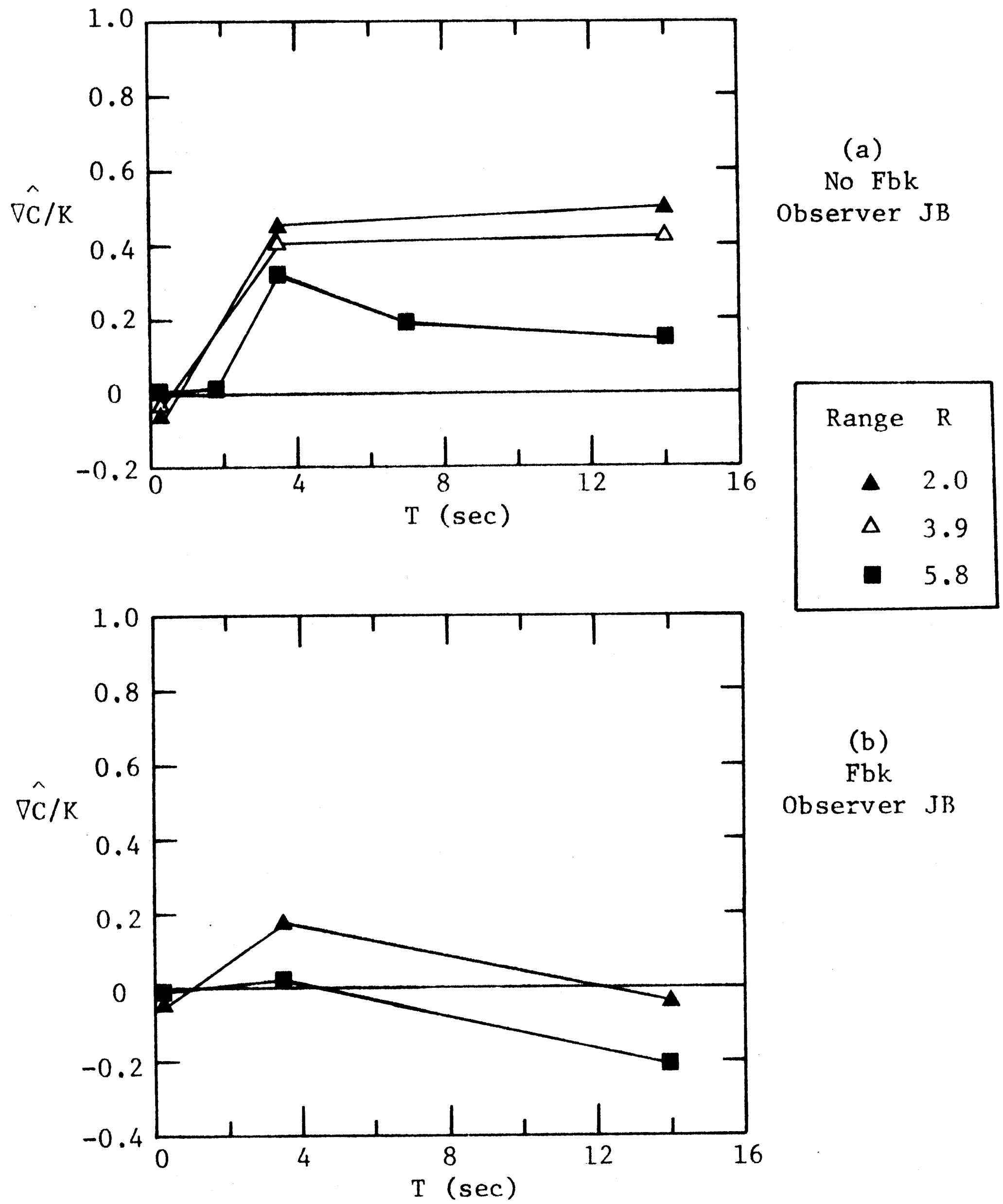


Figure 7.26 A Measure of the Bias Edge-Effect in Expts. 1 & 2. $\nabla\hat{C}/K$ versus T. Observer JB. (a) No Feedback (Expt. 1); (b) Feedback (Expt. 2).

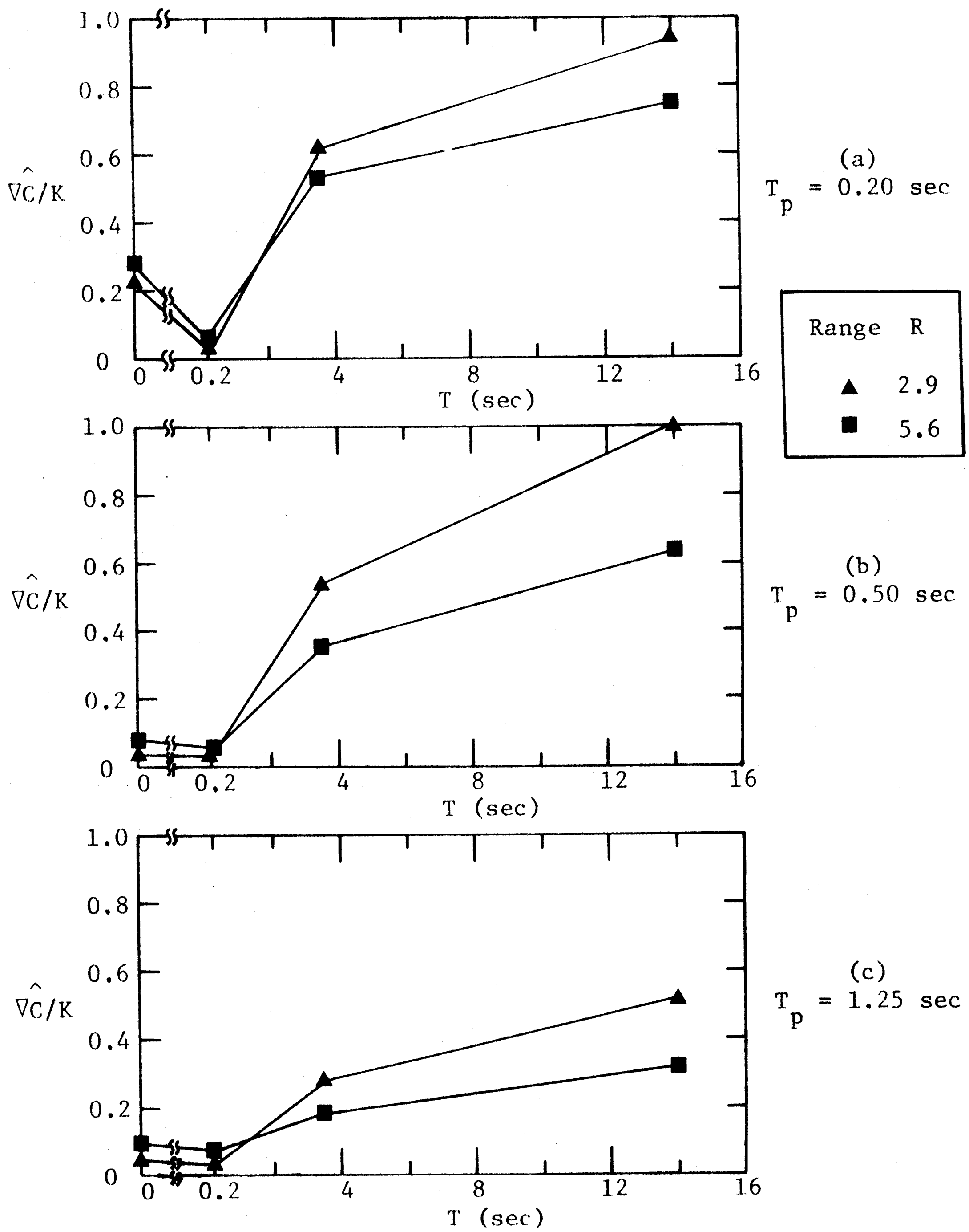


Figure 7.27 A Measure of the Bias Edge-Effect in Expt. 3. $\hat{V}C/K$ versus T. Averaged over JB, JW, & SK.

c. Dependence of $\hat{\nabla C}/K$ on Feedback

A comparison of Figs. 7.25a and 7.26a with Figs. 7.25b and 7.26b reveals that feedback, and instructions to eliminate the bias edge-effect, produced very sharp reductions in the bias edge-effect.

d. Differences in $\hat{\nabla C}/K$ Among the Observers

All the observers exhibited the bias edge-effect, although the magnitude of the effect varied greatly among the observers. See Appendix II for graphs for individual observers.

2. Identification Experiments Without a Reference Stimulus

Figures 7.28 and 7.29 are plots of estimated criterion shifts, $\hat{CS}_i/K$, versus intensity I_i, for each range R in Expt. 4. The data averaged over Observers KG and PN are presented in Fig. 7.28, and the data for Observer JB in Fig. 7.29.

From these figures, it may be seen that for the larger ranges (R = 5.4 and 3.6), the criterion shifts tended to be generally positive at the low end of the range, and negative at the high end. This represents a tendency of the actual criteria to be spread over a smaller range than unbiased criteria would be. No such trend is noticeable for the range R = 1.8. However, for the small range (R = 0.3), the criterion shifts tended to be negative at the low end of the range and positive at the high end. This represents a tendency of the actual criteria to be spread over a larger range than unbiased criteria would be. To appreciate the magnitudes of these effects, recall that $\hat{CS}_i/K$ may be considered the shift of the point of subjective equality relative to intensities I_i and I_{i+1}.

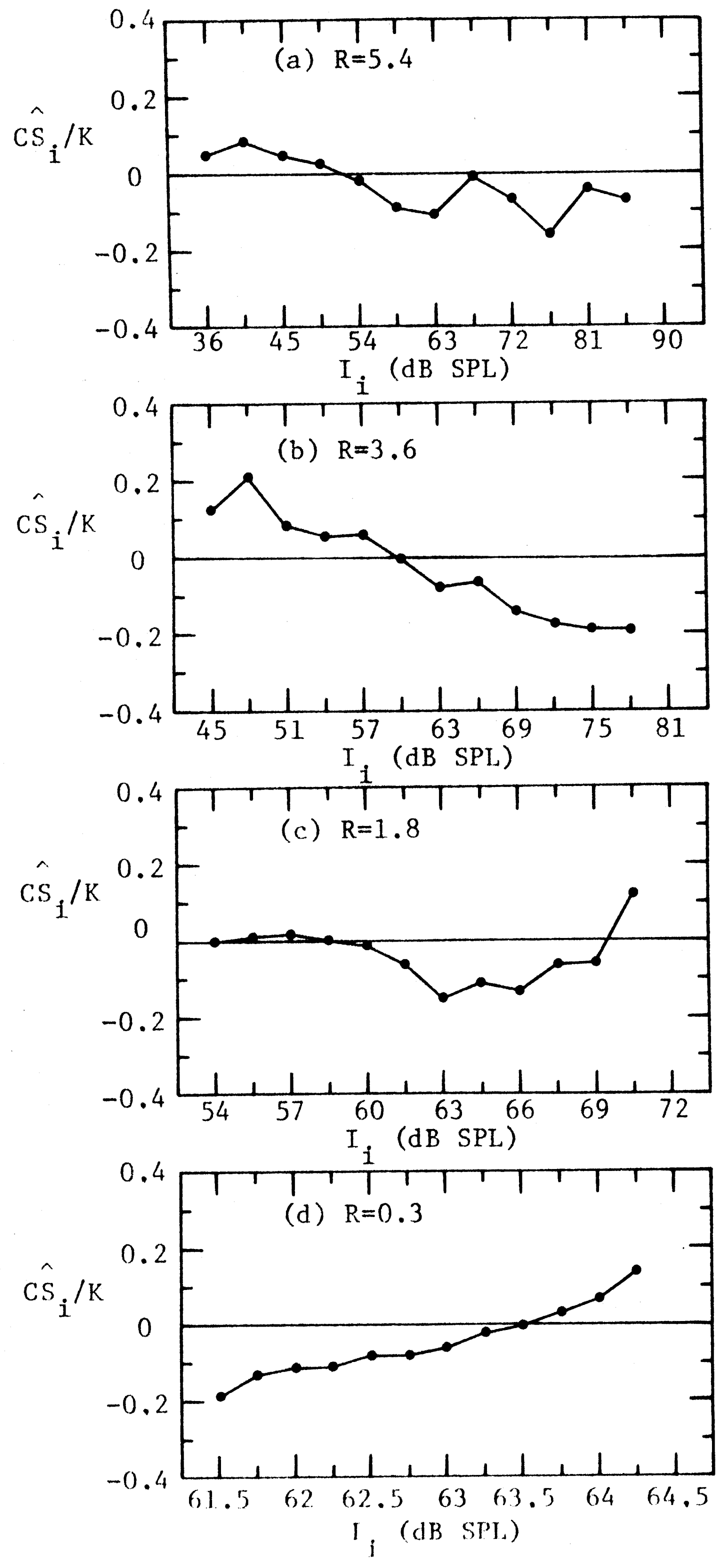
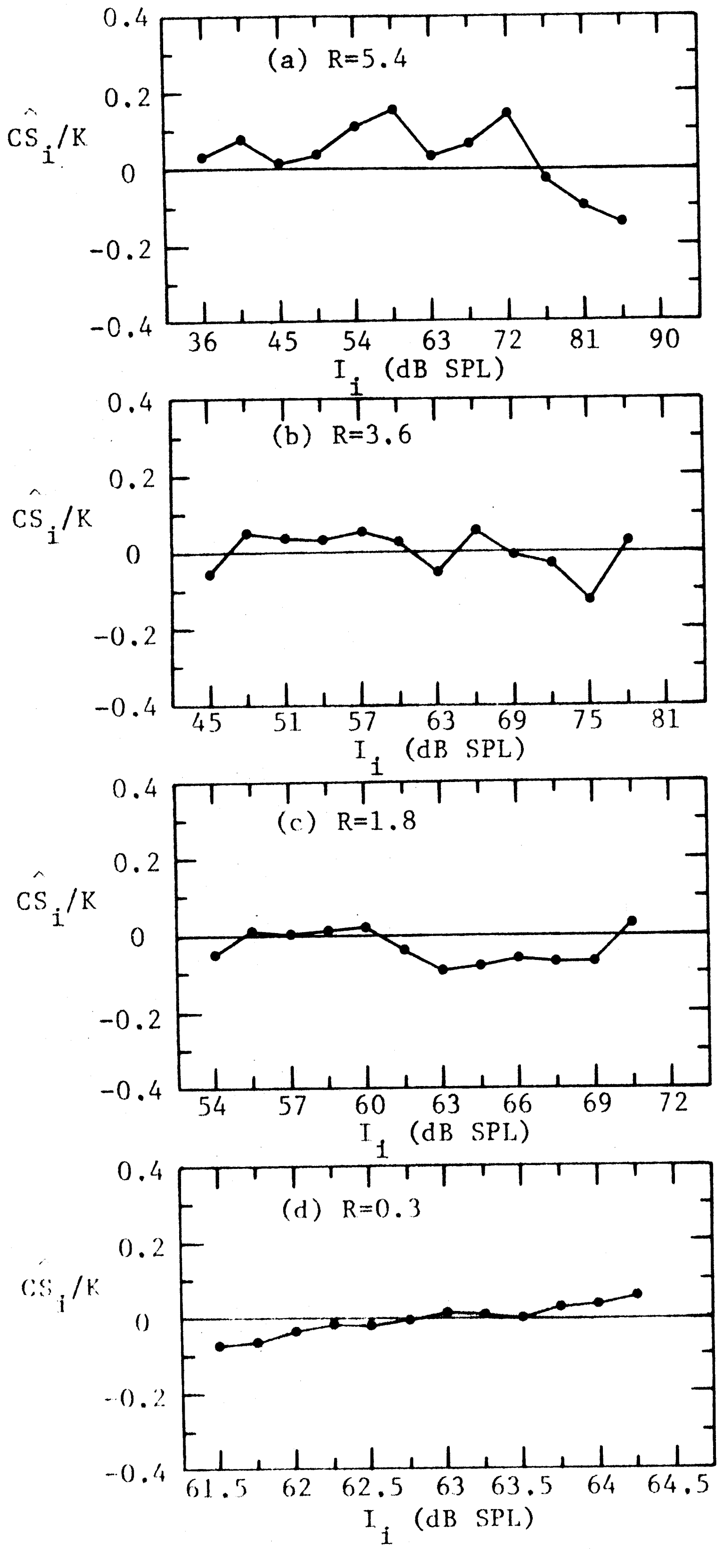


Figure 7.28

Results of Expt. 4.

Averaged over Observers KG & PN.

$\hat{CS}_i/K$ versus I_i.



<u>Figure 7.29</u>

Results of Expt. 4.

Observer JB.

$\hat{CS}_i/K$ versus I_i.

3. Identification Experiments With a Reference Stimulus

Figures 7.30a-f are plots of estimated criterion shifts, $\hat{CS}_i/K$, versus intensity I_i, for each reference stimulus condition in Expt. 5. The data plotted are the median values of $\hat{CS}_i/K$ over all six observers in this experiment. It is difficult to generalize about the results displayed in these figures, except to note that the criterion shifts tended to be far smaller in the no reference condition than in any other condition. An examination of the results for individual observers (see Appendix II) supports this view, and indicates further than when a reference stimulus was presented, large criterion shifts with large individual differences occurred.

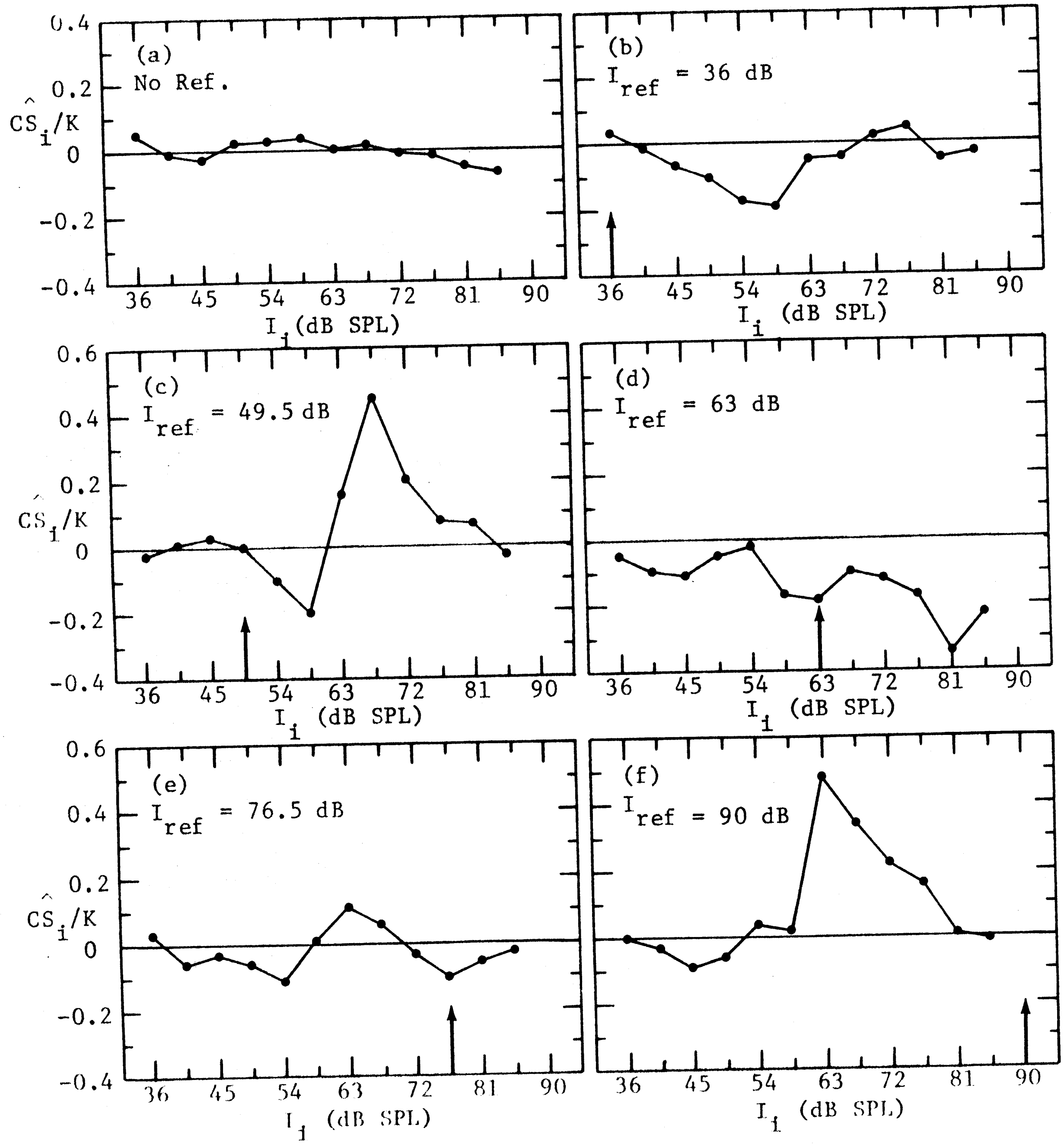


Figure 7.30 Results of Expt. 5: Identification with a Reference Stimulus

$\hat{CS}_i/K$ versus I_i. Averaged over 6 Observers.

VIII. RECENT THEORETICAL WORK

In addition to the experimental work of this thesis, the results of which have been described in Chapter VII, some theoretical work has been done. A major portion of this work entailed the construction of a perceptual-anchor model for context-coding to account for the resolution edge-effect in both one- and two-interval paradigms, and for the effect of a reference stimulus in one-interval paradigms. The other portion of the work concerned a series of attempted revisions of the preliminary theory to account for the bias edge-effect noted in two-interval paradigms.

A. Perceptual-Anchor Model for Context-Coding

One important phenomenon selected for careful study in this thesis is the resolution edge-effect; i.e., the tendency of resolution in both one- and two-interval paradigms to be better near the edges of the range of intensities covered by a given experiment than in the middle of the range. This effect has been measured in one- and two-interval paradigms under a variety of experimental conditions. A related phenomenon is the tendency of resolution in identification with a reference to be improved, over the no reference case, in the vicinity of the reference stimulus. This effect, too, has been measured.

A revision of the context-coding mode of the preliminary theory, termed the perceptual-anchor coding mode, that has some success in accounting for these phenomena is described in this section. The basic concepts of the model are presented first, followed by the detailed assumptions and equations for resolution, and a discussion of the predictions including a comparison with the data. Finally, an additional modification of the perceptual-anchor coding mode that has some success in accounting for the anomalous data of Observers KG and PN is discussed.

1. Basic Concepts

The basic strategy of the observer in the perceptual-anchor coding mode is the same as in the original context-coding mode of the preliminary theory; he compares a sensation with some aspect of his previous sensations and remembers a verbal description of the comparison, called the verbal code. However, it is now assumed that he uses the edges of the context as perceptual anchors, and that he forms verbal codes by comparing a sensation with these anchors. Furthermore, if a sensation resulting from a reference stimulus is available, it too may be used as a perceptual anchor to form an additional verbal code.[28] The noise introduced in the formation of each verbal code is assumed to depend on the distance between the sensation and the perceptual anchor, larger distances leading to more noise. The observer then combines the information from these codes in an optimal fashion to form a combined verbal code. As in the original context-coding mode, the advantage of the perceptual-anchor coding mode is that the observer can remember the code for as long as he likes[29]; the disadvantage is that the memory noise increases as the size of the context increases.

2. Assumptions and Equations

a. Formation of the Verbal Codes

In the perceptual-anchor coding mode, it is assumed that the observer compares the sensation Y resulting from intensity I with several perceptual anchors $\{\psi_a\}$ to form several numerical representations or verbal codes $\{Q_a\}$ which describe the distances from the anchors to the sensation. The perceptual anchors used are the lower edge of the context ψ_1, the upper edge of the context ψ_2, and, if a reference stimulus is available, the sensation of the reference ψ_3. These yield the verbal codes Q_1, Q_2, and Q_3 respectively.

For convenience, we define the intensities $I_{min} = I_{\psi_1}$, $I_{max} = I_{\psi_2}$, and $I_{ref} = I_{\psi_3}$. The conditional probability density functions $p(\psi_a | I_{\psi_a})$ are assumed to be Gaussian with $E(\psi_a | I_{\psi_a}) = \alpha(I_{\psi_a})$, and $Var(\psi_a | I_{\psi_a}) = \beta_\psi^2$.

It is then assumed that each verbal code Q_a is formed by counting with noisy steps from the anchor ψ_a to the sensation Y. The steps $\{S_i\}$ are assumed to be identically distributed, independent random variables with mean proportional to the width of the context, $W = \alpha(I_{\psi_2}) - \alpha(I_{\psi_1})$ $= \alpha(I_{max}) - \alpha(I_{min})$, and variance proportional to W^2. Thus, $E(S_i) = \mu W$ and $Var(S_i) = \sigma_s^2 W^2$. When $Y > \psi_a$, Q_a is the number of steps, and when $Y < \psi_a$, $-Q_a$ is the number of steps. $|Q_a|$ is thus defined by the relations:

$$\sum_{i=1}^{|Q_a|} S_i \leq |Y - \psi_a| , \tag{8.1}$$

and

$$\sum_{i=1}^{|Q_a|+1} S_i > |Y - \psi_a| . \tag{8.2}$$

In addition, we specify that μ is small so that the law of large numbers applies. It can then be shown [30] that $p(Q_a | Y, \psi_a)$ is asymptotically

Gaussian as $\mu \to 0$, with [31]

$$E[Q_a | Y, \psi_a] = (Y - \psi_a)/\mu W \quad , \tag{8.3}$$

and

$$Var[Q_a | Y, \psi_a] = |Y - \psi_a| \sigma_s^2/\mu^3 W \quad . \tag{8.4}$$

Thus the expected value of the verbal code is proportional to the difference between the anchor and the encoded sensation, and the variance of the verbal code is proportional to the absolute value of this distance.[32] The conditional probability density function $p(Q_a | I, I_{\psi_a})$, then, is Gaussian with:

$$E[Q_a | I, I_{\psi_a}] = [\alpha(I) - \alpha(I_{\psi_a})]/\mu W \quad , \tag{8.5}$$

$$Var[Q_a | I, I_{\psi_a}] = |\alpha(I) - \alpha(I_{\psi_a})| \sigma_s^2/\mu^3 W + (\beta^2 + \beta_\psi^2)/\mu^2 W^2 \quad , \tag{8.6}$$

and

$$Cov[Q_a, Q_b | I, I_{\psi_a}, I_{\psi_b}] = \beta^2/\mu^2 W^2 \quad . \tag{8.7}$$

The verbal codes $\{Q_a\}$ are then combined to form a maximum likelihood estimate of the numerical representation of the mean distance from the lower edge [33] of the context ψ_1 to the sensation Y (i.e., an estimate of $[\alpha(I) - \alpha(I_{\psi_1})]/\mu W$). Defining the combined verbal code $\bar{Q}$ to be this estimate, it can be shown [34] that the conditional probability density function $p(\bar{Q}|I)$ [35] is Gaussian with

$$E[\bar{Q}|I] = [\alpha(I) - \alpha(I_{\psi_1})]/\mu W \quad , \tag{8.8}$$

and

$$\mathrm{Var}[\overline{Q}|I] = [\beta^2 + 1/\sum_a 1/\sigma_a^2(I)]/\mu^2 W^2 \quad , \tag{8.9}$$

where

$$\sigma_a^2(I) = |\alpha(I) - \alpha(I_{\psi_a})|\sigma_s^2 W/\mu + \beta_\psi^2 \quad . \tag{8.10}$$

b. <u>One-Interval Paradigms</u>

In one-interval paradigms, the observer is assumed to make his decision in the same way as in the preliminary theory; the decision variable X is merely the combined verbal code $\overline{Q}$. The conditional probability density functions $p(X|I_i)$ and $p(X|I_j)$ have, in general, unequal variances and d' may be defined in several ways. We will, however, define d' in this situation by:[36]

$$d_1'(I_j;\, I_i) = \frac{E[X|I_j] - E[X|I_i]}{\left\{(\mathrm{Var}[X|I_j])^{1/2} + (\mathrm{Var}[X|I_i])^{1/2}\right\}\Big/2} \quad . \tag{8.11}$$

Thus:

$$d_1'(I_j;\, I_i) = \frac{\alpha(I_j) - \alpha(I_i)}{\left\{[\beta^2 + 1/\sum_a 1/\sigma_a^2(I_j)]^{1/2} + [\beta^2 + 1/\sum_a 1/\sigma_a^2(I_i)]^{1/2}\right\}\Big/2} \quad . \tag{8.12}$$

Substituting $\alpha(I) = K\log(I)$, $W = \alpha(I_{max}) - \alpha(I_{min})$, $R = \log(I_{max}/I_{min})$, and defining $G' = K\sigma_s/\mu^{1/2}$, we get:

$$d_1'(I_j;\, I_i) = \frac{K\log(I_j/I_i)}{\left\{[\beta^2 + 1/\sum_a 1/\sigma_a^2(I_j)]^{1/2} + [\beta^2 + 1/\sum_a 1/\sigma_a^2(I_i)]^{1/2}\right\}\Big/2} \quad , \tag{8.13}$$

where the variance of the coding noise $\sigma_a^2(I_i)$ is given by:

$$\sigma_a^2(I_i) = |\log(I_i/I_{\psi_a})|RG'^2 + \beta_\psi^2 \quad . \tag{8.14}$$

Restricting our attention to pairs of adjacent intensities (so that $j = i+1$), we make the approximation $\sigma_a^2(I_i) \simeq \sigma_a^2(I_{i+1}) \simeq \sigma_a^2(I_i')$, where I_i' is defined by $\alpha(I_i') = [\alpha(I_i) + \alpha(I_{i+1})]/2$. The equation for d' then becomes:

$$d'(I_{i+1};\ I_i) \simeq \frac{K \log(I_{i+1}/I_i)}{\left[\beta^2 + 1/\sum_a 1/\sigma_a^2(I_i')\right]^{1/2}} \quad , \tag{8.15}$$

and the resolution per bel is given by:

$$\delta_1'(I_i') \simeq \frac{K}{\left[\beta^2 + 1/\sum_a 1/\sigma_a^2(I_i')\right]^{1/2}} \quad . \tag{8.16}$$

In fitting the model to the data, it was found that in cases where a reference is available, a better fit is obtained if the observer is restricted to using only two anchors, the two anchors surrounding the sensation to be coded. (Thus if $I > I_{ref}$, ψ_3 is substituted for ψ_1, and if $I < I_{ref}$, ψ_3 is substituted for ψ_2.) The summation $\sum_a$ in the preceding equations, then, indicates a summation over only the two appropriate anchors.

c. Two-Interval Paradigms

In two-interval paradigms, the observer is assumed to use the trace mode and the perceptual-anchor coding mode to form the respective decision variables X_T and X_Q, and to then combine the information from them in an optimal fashion. X_T is formed by subtracting the sensation arising from the second interval from the trace of the sensation arising from the first interval:

$$X_T = \overline{Y}_1(T) - Y_2 , \tag{8.17}$$

and X_Q is formed by subtracting the combined verbal code arising from the second interval from the combined verbal code arising from the first interval:

$$X_Q = \overline{Q}_1 - \overline{Q}_2 . \tag{8.18}$$

The conditional probability density functions $p(X_T|I_i, I_i^*)$ and $p(X_T|I_i^*, I_i)$ are Gaussian with:

$$E[X_T|I_i, I_i^*] = \alpha(I_i) - \alpha(I_i^*) , \tag{8.19}$$

$$E[X_T|I_i^*, I_i] = \alpha(I_i^*) - \alpha(I_i) , \tag{8.20}$$

and

$$Var[X_T|I_i, I_i^*] = Var[X_T|I_i^*, I_i] = 2(\beta^2 + AT) . \tag{8.21}$$

The conditional probability density functions $p(X_Q|I_i, I_i^*)$ and $p(X_Q|I_i^*, I_i)$ are Gaussian with:

$$E[X_Q|I_i, I_i^*] = [\alpha(I_i) - \alpha(I_i^*)]/\mu W , \tag{8.22}$$

$$E[X_Q|I_i^*, I_i] = [\alpha(I_i^*) - \alpha(I_i)]/\mu W , \tag{8.23}$$

and

$$\mathrm{Var}[X_Q | I_i, I_i^*] = \mathrm{Var}[X_Q | I_i^*, I_i]$$

$$= [2\beta^2 + 1/\sum_a 1/\sigma_a^2(I_i) + 1/\sum_a 1/\sigma_a^2(I_i^*)]/\mu^2 W^2 \quad . \tag{8.24}$$

Furthermore,

$$\mathrm{Cov}[X_Q, X_T | I_i, I_i^*] = \mathrm{Cov}[X_Q, X_T | I_i^*, I_i] = 2\beta^2/\mu W \quad . \tag{8.25}$$

The observer is assumed to combine the information from X_T and X_Q in an optimum fashion, by constructing a new decision variable, X_c, proportional to:

$$X_{Q,T} = \ln[p(X_Q, X_T | I_i, I_i^*)/p(X_Q, X_T | I_i^*, I_i)] \quad . \tag{8.26}$$

It can then be shown [37] that the conditional probability density functions $p(X_c | I_i, I_i^*)$ and $p(X_c | I_i^*, I_i)$ are Gaussian with:

$$E[X_c | I_i, I_i^*] = \alpha(I_i) - \alpha(I_i^*) \quad , \tag{8.27}$$

$$E[X_c | I_i^*, I_i] = \alpha(I_i^*) - \alpha(I_i) \quad , \tag{8.28}$$

and

$$\mathrm{Var}[X_c] = \mathrm{Var}[X_c | I_i, I_i^*] = \mathrm{Var}[X_c | I_i^*, I_i]$$

$$= 2\beta^2 + \cfrac{1}{\cfrac{1}{2AT} + \cfrac{1}{1/\sum_a 1/\sigma_a^2(I_i) + 1/\sum_a 1/\sigma_a^2(I_i^*)}} \quad . \tag{8.29}$$

Before writing the equation for d', however, we make the approximation $\sigma_a^2(I_i) \simeq \sigma_a^2(I_i^*) \simeq \sigma_a^2(I_i')$, where I_i', defined by $\alpha(I_i') = [\alpha(I_i) + \alpha(I_i^*)]/2$, represents the overall intensity of the i^{th} level. The equation for the variance then becomes:

$$\mathrm{Var}[X_c] \simeq 2\beta^2 + \frac{2}{1/AT + \sum\limits_a 1/\sigma_a^2(I_i')} , \tag{8.30}$$

and d_2' is given by:

$$d_2'(I_i, I_i^*; I_i, I_i^*) \simeq \frac{\sqrt{2}[\alpha(I_i) - \alpha(I_i^*)]}{\left[\beta^2 + \dfrac{1}{1/AT + \sum\limits_a 1/\sigma_a^2(I_i')}\right]^{1/2}} . \tag{8.31}$$

Substituting $\alpha(I) = K \log(I)$, $W = \alpha(I_{max}) - \alpha(I_{min})$, $R = \log(I_{max}/I_{min})$ and $G' = K\sigma_s/\mu^{1/2}$, we get:

$$d'(I_i, I_i^*; I_i^*, I_i) \simeq \frac{\sqrt{2}\ K \log(I_i/I_i^*)}{\left[\beta^2 + \dfrac{1}{1/AT + \sum\limits_a 1/\sigma_a^2(I_i')}\right]^{1/2}} , \tag{8.32}$$

and

$$\delta_2'(I_i') \simeq \frac{\sqrt{2}\ K}{\left[\beta^2 + \dfrac{1}{1/AT + \sum\limits_a 1/\sigma_a^2(I_i')}\right]^{1/2}} , \tag{8.33}$$

where

$$\sigma_a^2(I_i') = \left| \log(I_i'/I_{\psi_a}) \right| G'^2 R + \beta_\psi^2 \quad . \tag{8.34}$$

3. Discussion of The Perceptual-Anchor Coding Model

In this section, a preliminary discussion of the implications of the revised theory incorporating the perceptual-anchor coding model is presented in two parts. The first part deals with the predictions for resolution averaged over intensity levels, and the second part with resolution as a function of intensity level.

The discussion is of a preliminary nature primarily because the work on the revised theory is not yet completed. Some of the assumptions of the model may require modification. For example, it might be more appropriate to assume that $E[\psi_1|I_{min}] = \alpha(I_{min}) - \beta_\psi$ and $E[\psi_2|I_{max} = \alpha(I_{max}) + \beta_\psi$ [rather than merely $\alpha(I_{min})$ and $\alpha(I_{max})$ respectively], since these are the points where the "sensation gradient" $\left|\frac{\partial p(Y)}{\partial Y}\right|$ is a maximum. In addition, in the analysis performed to date, the simplifying assumption has been made that the variance of the anchors equals the sensation variance, $\beta_\psi^2 = \beta^2$, thereby reducing the number of free parameters in the revised theory to three ($K/\beta = K/\beta_\psi$, K/G', and $K/\sqrt{A}$), the same number as in the preliminary theory. It was found that making this assumption did not prohibit our obtaining a reasonable fit to the data, and that the fit is not very sensitive to the choice of K/β_ψ anyway.

a. Resolution Averaged over Levels

Although the similarity of the predictions of the preliminary and revised theories for resolution averaged over levels may not be readily apparent from a comparison of equations 8.16 and 8.33 with equations 4.12 and 4.10, the following considerations and graphical comparisons should make it clear.

The major difficulty in the comparison is that the range R enters the expression for the variance of the coding noise, σ_a^2, (Eqs. 8.14 and 8.34) in the revised theory in two ways. It is included explicitly as well as implicitly, since the average (over levels) value of $|\log(I_i'/I_{\psi_a})|$ is proportional to R. Ignoring the variance of the anchors β_ψ^2 (which is significant mainly for small R), the variance of the coding noise, $1/\sum_a 1/\sigma_a^2(I_i')$, in the middle of the range would be $G'^2R^2/4$, while at the edges it would be zero. Making the approximation that the variance of the coding noise varies linearly between the middle of the range and the edges, its average value would be $G'^2R^2/8$. Comparing this to the variance of the coding noise from the preliminary theory G^2R^2, it is seen that they are both proportional to R^2, and that numerical agreement would be obtained when $G' = \sqrt{8}\,G$.

Another, minor, difficulty arises at small values of R. Whereas in the preliminary theory the coding noise vanishes for small R, in the revised theory the coding noise is limited by the variance of the anchors and is never less than $\beta_\psi^2/2$. Consequently, if it is assumed that $\beta_\psi^2 = \beta^2$, the expressions for resolution in the preliminary and revised theories will differ by as much as a factor of $\sqrt{1.5} \simeq 1.22$, with the maximum difference occurring when R is near zero and the observer is strongly in the context-coding mode.

Figure 8.1 provides a graphical comparison of the predictions of the preliminary and revised theories for resolution averaged over levels, $\overline{\delta}_1'$, in one-interval paradigms with no reference stimulus. The dashed curves are logarithmic plots of $\overline{\delta}_1'$ versus $R/(K/G)$ from the preliminary theory with $K/\beta = 8$ and 16, while the solid curves are logarithmic plots of $\overline{\delta}_1'$ versus $R/(\sqrt{8}\ K/G')$ from the revised theory with $K/\beta = K/\beta_\psi = 8$ and 16.[38]

From Fig. 8.1, it may be seen that (with $G' = \sqrt{8}\ G$) for large values of R, the preliminary and revised theories are in good agreement; while for small values of R, they differ by as much as 18% (about a factor of 1.22), as expected. Note that agreement would be obtained for all values of R if $K/\beta = K/\beta_\psi$ in the revised theory were chosen to be $\sqrt{1.5}\ K/\beta$ in the preliminary theory.

Figure 8.2 illustrates the predictions of the revised theory for the relative values of $\overline{\delta}_1'$ with and without a reference stimulus. (The preliminary theory makes no predictions concerning the effect of a reference stimulus.) The solid curve in Fig. 8.2 indicates the values of the ratio $\overline{\delta}_1'(I_{ref})/\overline{\delta}_1'(\text{No ref.})$, derived from the revised theory under the assumption $K/\beta = K/\beta_\psi = 11$ and $\sqrt{8}\ K/G' = 15$. (Since the observers of Expt. 5 were tested at only one value of R, the parameters could not be accurately estimated. The value of K/G' was picked to match roughly the Δ'(no ref) from Expt. 5, while $K/\beta = K/\beta_\psi$ was picked, rather arbitrarily, to match roughly $\sqrt{1.5}$ times the value of K/β obtained from the results of Observer JB from Expt. 4. However, since the quantity plotted in Fig. 8.2 is a

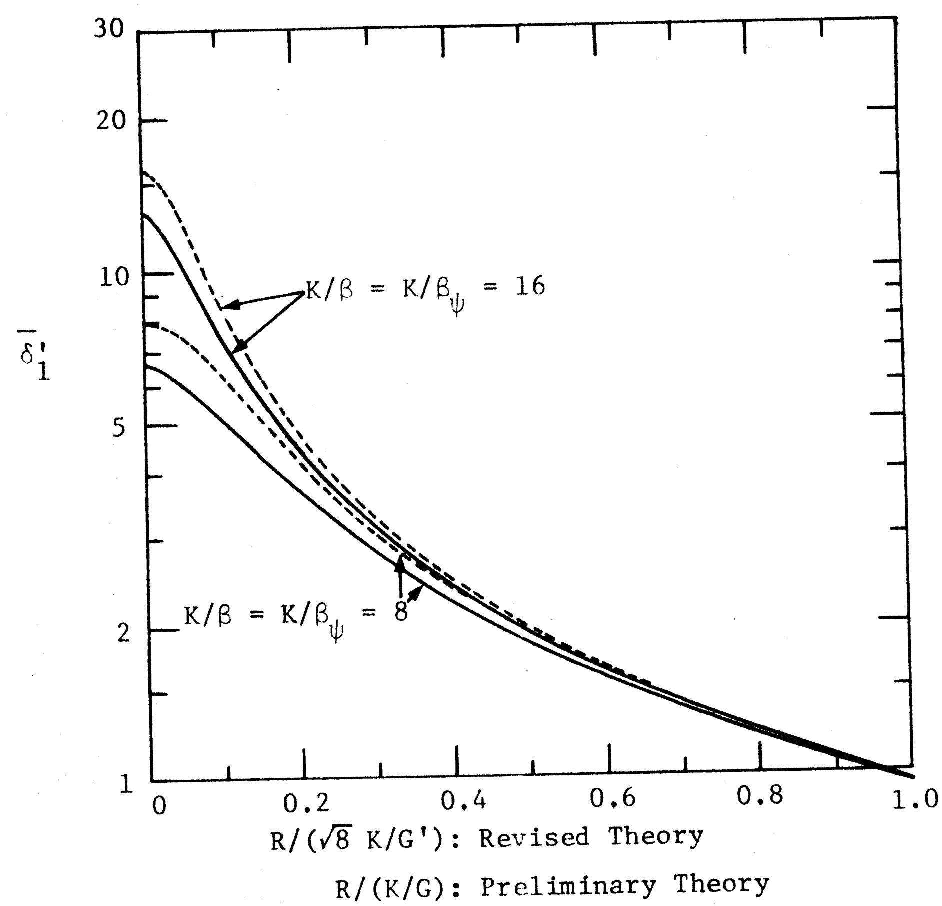
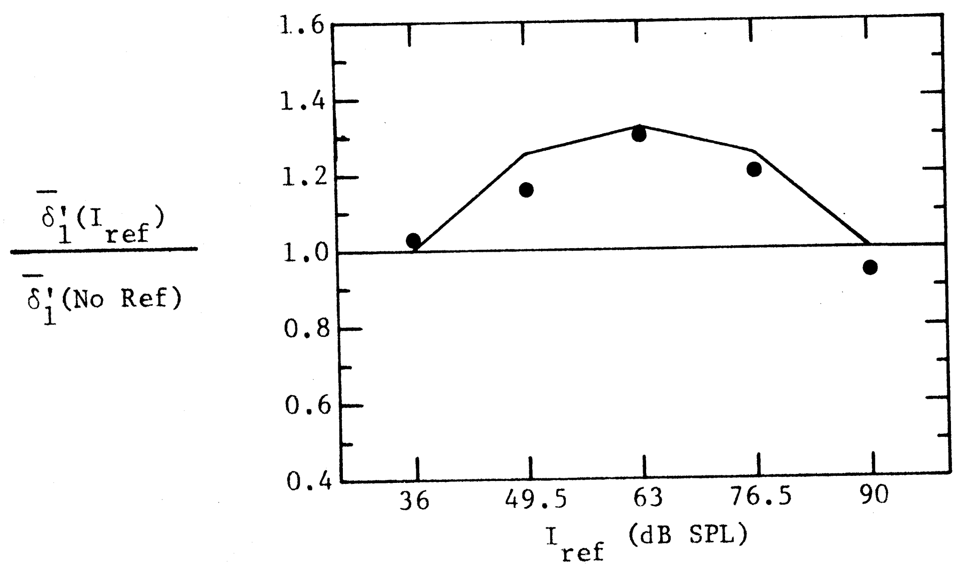


Figure 8.1 Predictions of the Preliminary and Revised Theories for Resolution Averaged over Levels in One-Interval Paradigms.

$\log(\overline{\delta}_1')$ as a function of K/β and $R/(K/G)$ or $R/(\sqrt{8}\ K/G')$.

———: Revised Theory

-------: Preliminary Theory



<u>Figure 8.2</u> $\frac{\bar{\delta}_1'(I_{ref})}{\bar{\delta}_1'(\text{No Ref})}$ versus I_{ref}.

The data points are from Expt. 5, averaged over 6 observers.

The solid curve was derived from the revised theory under the assumption $K/\beta = K/\beta_\psi = 11$ and $\sqrt{8}\ K/G' = 15$.

ratio of $\overline{\delta}_1^{\prime}$s, it is relatively insensitive to the values of the parameters anyway.) The filled circles in Fig. 8.2 are the values of the ratio $\overline{\hat{\delta}}_1^{\prime}(I_{ref})/\overline{\hat{\delta}}^{\prime}(\text{No ref.})$ obtained from the observers of Expt. 5. Comparison of these points with the theoretical curve reveals a fairly good agreement between the predicted and actual results.

Figure 8.3 provides a graphical comparison of the predictions of the preliminary and revised theories for resolution averaged over levels in two-interval paradigms, $\overline{\delta}_2^{\prime}$. The dashed curves are logarithmic plots of $\overline{\delta}_2^{\prime}$ versus $T/(K^2/A)$ with $R/(K/G)$ as a parameter from the preliminary theory with K/β fixed at 12, while the solid curves are logarithmic plots of $\overline{\delta}_2^{\prime}$ versus $T/(K^2/A)$ with $R/(\sqrt{8}\ K/G^{\prime})$ as a parameter from the revised theory with $K/\beta = K/\beta_{\psi}$ fixed at 12.[39]

From Fig. 8.3, it may be seen that as in one-interval paradigms (with $G^{\prime} = \sqrt{8}\ G$), for large values of R, the preliminary and revised theories are in good agreement; while for small values of R, they again differ by as much as 18% (about a factor of 1.22), as expected. This difference at small values of R, however, diminishes with decreasing T because the trace mode (which is unchanged in the revised theory) is dominant for small values of T.

Consequently, if our attention is restricted to the case where R is small, the revised theory represents a small improvement over the preliminary theory with regard to $\overline{\delta}_2^{\prime}$ as a function of T, and with regard to the relation between $\overline{\delta}_2^{\prime}$ and $\overline{\delta}_1^{\prime}$. Whereas the preliminary theory predicts virtually no decline in $\overline{\delta}_2^{\prime}$ with increasing T, the revised theory predicts a maximum decline of about 18%. Furthermore, whereas the preliminary

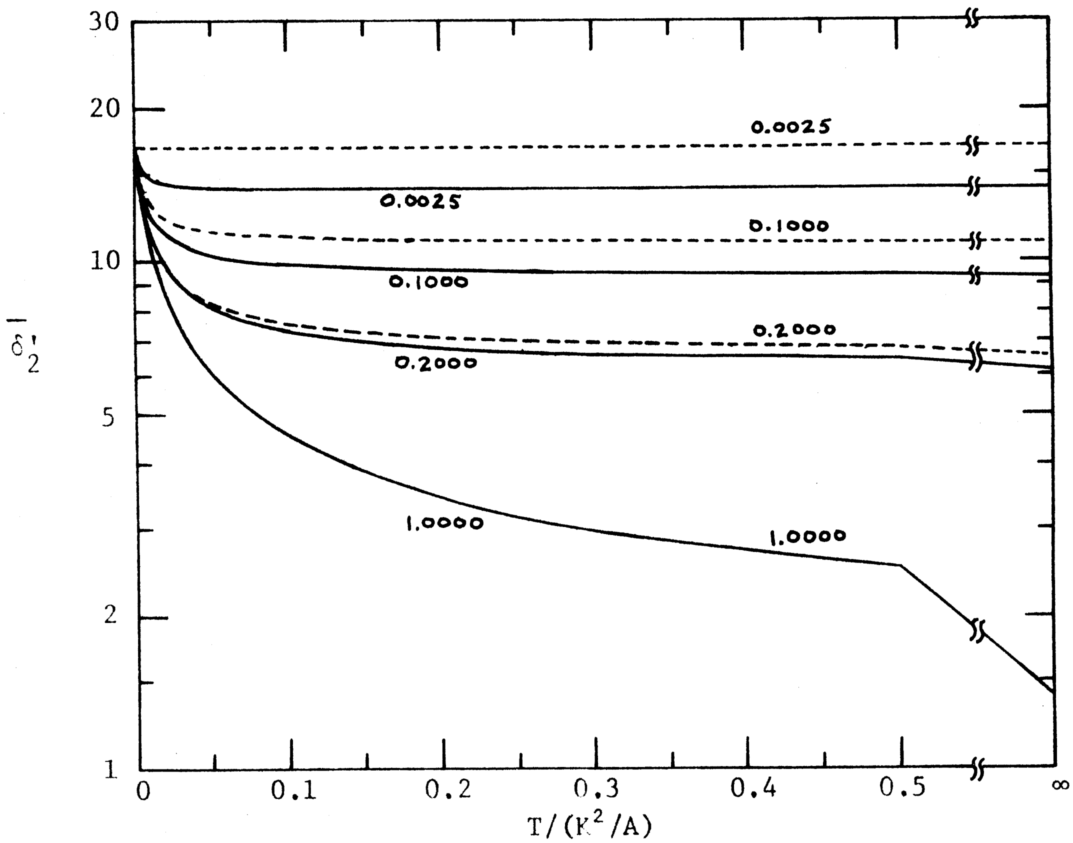


Figure 8.3 Predictions of the Preliminary and Revised Theories for Resolution Averaged over Levels in Two-Interval Paradigms.

$\log(\bar{\delta}'_2)$ versus $T/(K^2/A)$, with $K/\beta = K/\beta_\psi = 12$.

———: Revised Theory. The numbers on the curves give the values of $R/(\sqrt{8}\ K/G')$.

-------: Preliminary Theory. The numbers on the curves give the values of $R/(K/G)$.

theory predicts that $\overline{\delta}'_2$ obtained with small T is equal to $\sqrt{2}\,\overline{\delta}'_1$, the revised theory predicts that $\overline{\delta}'_2$ obtained with small T is about 22% greater than $\sqrt{2}\,\delta'_1$. Considering the data collected at small values of R, (and ignoring the data from Observers KG and PN) it was noted in Chapter VII (see Figs. 7.2 - 7.4) that $\overline{\hat{\delta}}'_2$ declined by about 57% as T was increased from 0.2 to 14.0 sec, and that $\overline{\hat{\delta}}'_2$ obtained with small T was from 12% to 34% greater than $\sqrt{2}\,\overline{\hat{\delta}}'_1$.

b. Resolution as a Function of Level

The revised theory, incorporating the perceptual-anchor coding model, has the following three implications concerning resolution as a function of level:

1) In one-interval paradigms with no reference stimulus, resolution is predicted to be a maximum at the edges of the range, and to diminish towards the middle of the range (a resolution edge-effect).

2) In one-interval paradigms with a reference stimulus, resolution is predicted to be improved (over the no reference case) in the vicinity of the reference.

3) In two-interval paradigms, a resolution edge-effect is predicted for conditions where the observer makes significant use of the coding mode.

These three predictions are qualitatively consistent with the experimental results reported in the preceding chapter; a quantitative examination of the predictions and the experimental results is presented below.

Figures 8.4 and 8.5 provide comparisons of the predictions of the revised theory for resolution as a function of level in one-interval

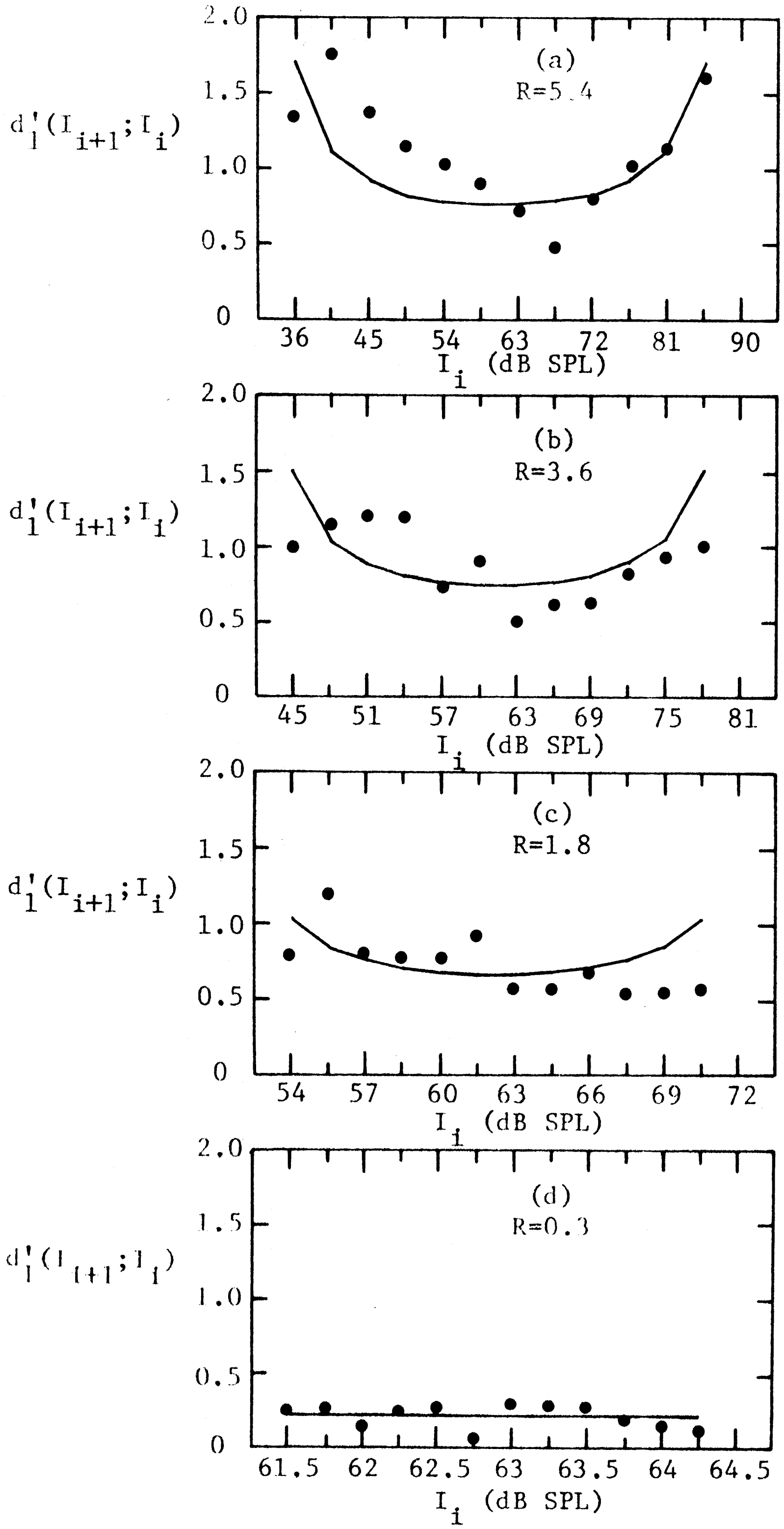


Figure 8.4

Comparison of the Revised Theory with the Results of Expt. 4.

$d_1'(I_{i+1};I_i)$ vs. I_i.

Data points: Obs. JB

Solid curves: Revised Theory with $K/\beta = K/\beta_\psi = 11$, and $\sqrt{8}\ K/G' = 13$.

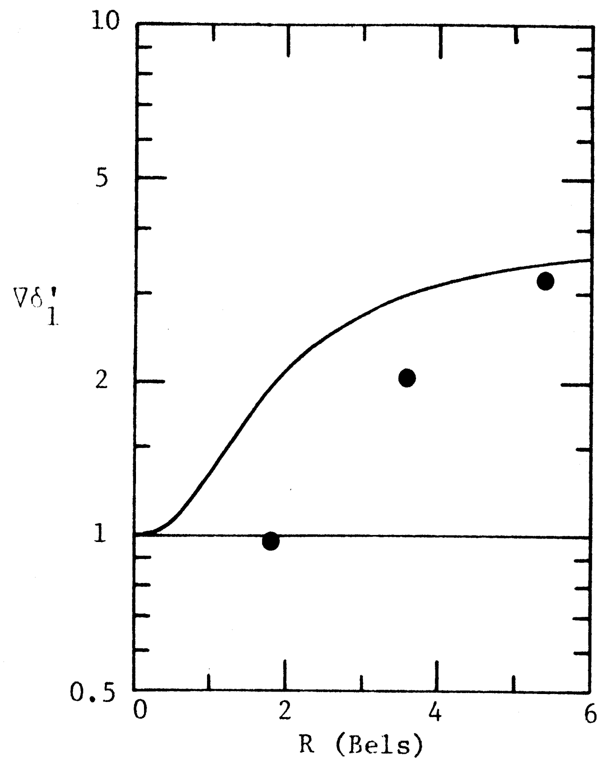


Figure 8.5 Comparison of the Revised Theory with the results of Expt. 4.

$\nabla\delta_1'$ versus R.

Data Points: Observer JB

Solid Curve: Revised Theory with

$K/\beta = K/\beta_\psi = 11$, and $\sqrt{8}\ K/G' = 13$.

paradigms with the data of Observer JB from Expt. 4. (The data averaged over Observers KG and PN are examined separately, in a later section.) In Figs. 8.4a-d are plotted the actual values of $\hat{d}_1'(I_{i+1}; I_i)$ (the filled circles) versus I_i for each range R from Expt. 4, as well as theoretical values of $d_1'(I_{i+1}; I_i)$ (the continuous curves) from the revised theory. In Fig. 8.5 are plotted the actual values of the quantity used as a measure of the resolution edge-effect in one-interval paradigms, $\nabla\hat{\delta}_1'$, (the filled circles) versus R, as well as a theoretical curve of $\nabla\delta_1'$ from the revised theory. In both Figs. 8.4 and 8.5, the theoretical values of $d_1'(I_{i+1}; I_i)$ and $\nabla\delta_1'$ were derived from the revised theory under the assumption $K/\beta = K/\beta_\psi = 11 \simeq 9\sqrt{1.5}$ and $\sqrt{8}\ K/G' = 13.$[40] These parameters were selected to give the same predictions as the preliminary theory for resolution averaged over levels, with parameters $K/\beta = 9$ and $K/G = 13$. See Fig. 7.6

Apart from the apparent noisiness of the data, Figs. 8.4 and 8.5 indicate that the dependence of resolution on level in a one-interval paradigm predicted by the revised theory is in rough agreement with the data. A large resolution edge-effect is both predicted and observed in the data for R = 5.4; no effect is either predicted or observed for R = 0.3; however, the predicted effect is too large for intermediate values of R.

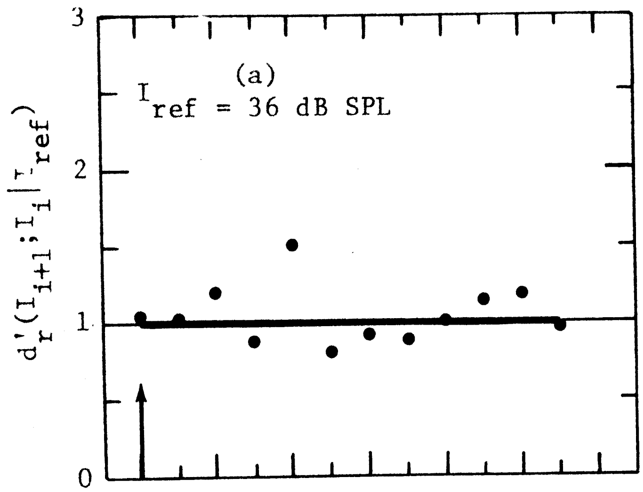
Figure 8.6 provides a comparison of the data from Expt. 5 with the predictions of the revised theory for the relative values of $d_1'(I_{i+1}; I_i)$ with and without a reference stimulus. In Figs. 8.6a-e, are plotted the actual values of the quantity used to assess the effect of the different reference stimuli relative to the no reference condition, $d_r'(I_{i+1}; I_i|I_{ref})$ (the filled circles) versus I_i, for five different reference stimulus

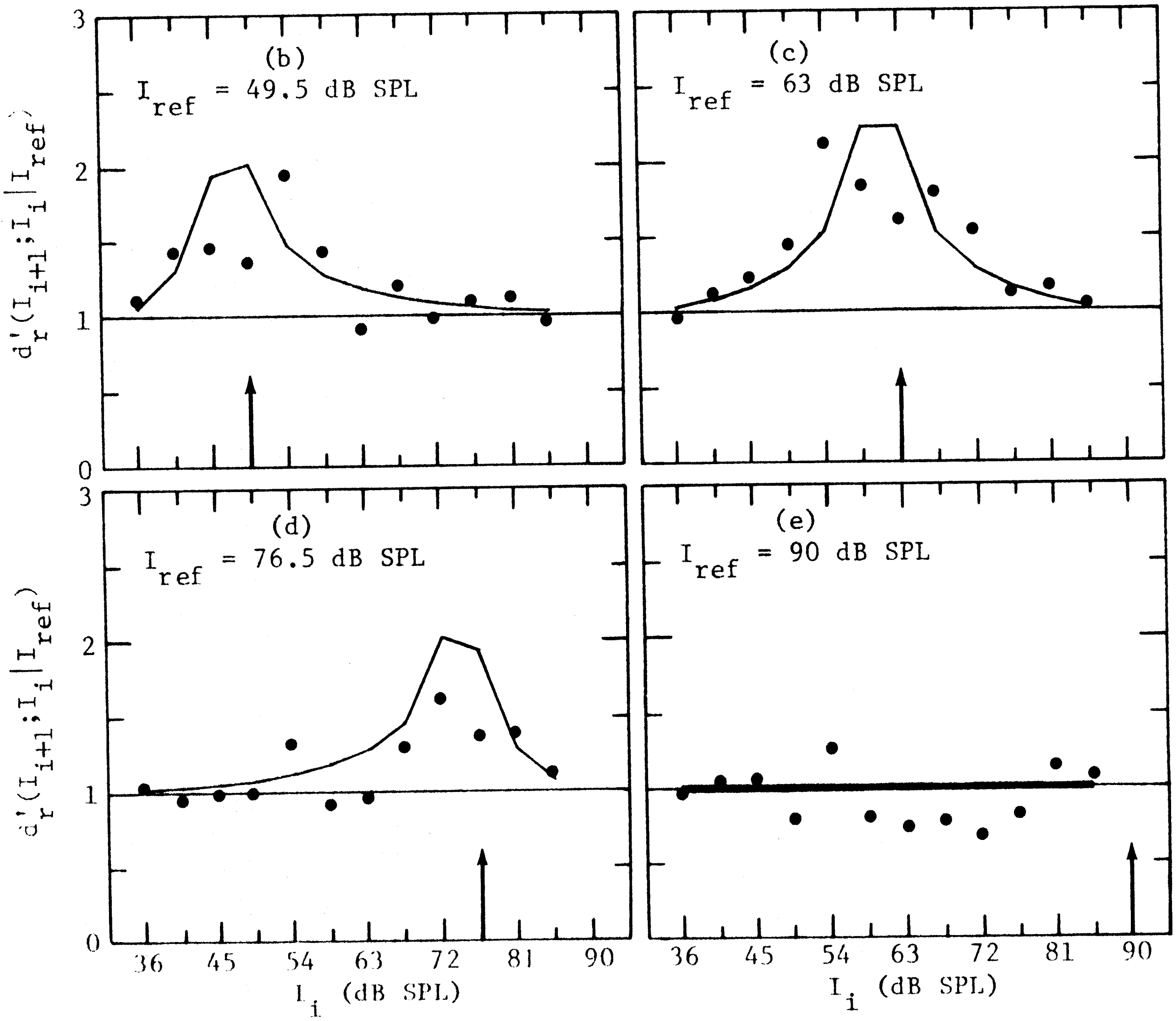
Figure 8.6 Comparison of the Revised Theory with the Results of Expt. 5.

$d'_r(I_{i+1};I_i|I_{ref})$ versus I_i .

Data points: Expt. 5, averaged over 6 observers.

Solid curves: Revised Theory with $K/\beta = K/\beta_\psi = 11$, and $\sqrt{8}\ K/G' = 15$.





intensities; as well as the theoretical values of $d_r'(I_{i+1}; I_i | I_{ref})$ (the solid curve), derived from the revised theory under the assumption $K/\beta = K/\beta_\psi = 11$ and $\sqrt{8}\ K/G' = 15$ [41] (the same as in Fig. 8.2).

Again, apart from the apparent noisiness of the data, Fig. 8.6 shows that the revised theory provides a fairly good fit to the data. No improvements in resolution over the no reference case are either predicted or observed in the data when the reference is at either edge of the range; improvements in resolution near the reference stimulus are both predicted and observed when the reference is away from the edges, although the predicted improvements are slightly too large.

Figures 8.7 and 8.8 provide comparisons of the predictions of the revised theory for resolution as a function of level in two-interval paradigms with some of the data of Observer JB from Expt. 1, as well as data from Expt. 3. (The data averaged over Observers KG and PN from Expt. 1 are examined separately, in a later section.) In Figs. 8.7a-d are plotted the actual values of $\hat{\delta}_2'$ (the filled symbols) versus overall intensity I, with T as a parameter, for several ranges R from Expts. 1 and 3; as well as theoretical values of δ_2' (the continuous curves) from the revised theory. In Figs. 8.8a-b are plotted the actual values of the quantity used as a measure of the resolution edge-effect in two-interval paradigms, $V\hat{\delta}_2'$, (the triangles and squares) versus T, with R as a parameter; as well as theoretical values of $V\delta_2'$ (the continuous curves) from the revised theory. The data of Observer JB from Expt. 1 are presented in Figs. 8.7a-b and 8.8a, and the data averaged over the observers of Expt. 3 are presented in Figs. 8.7c-d and 8.8b.

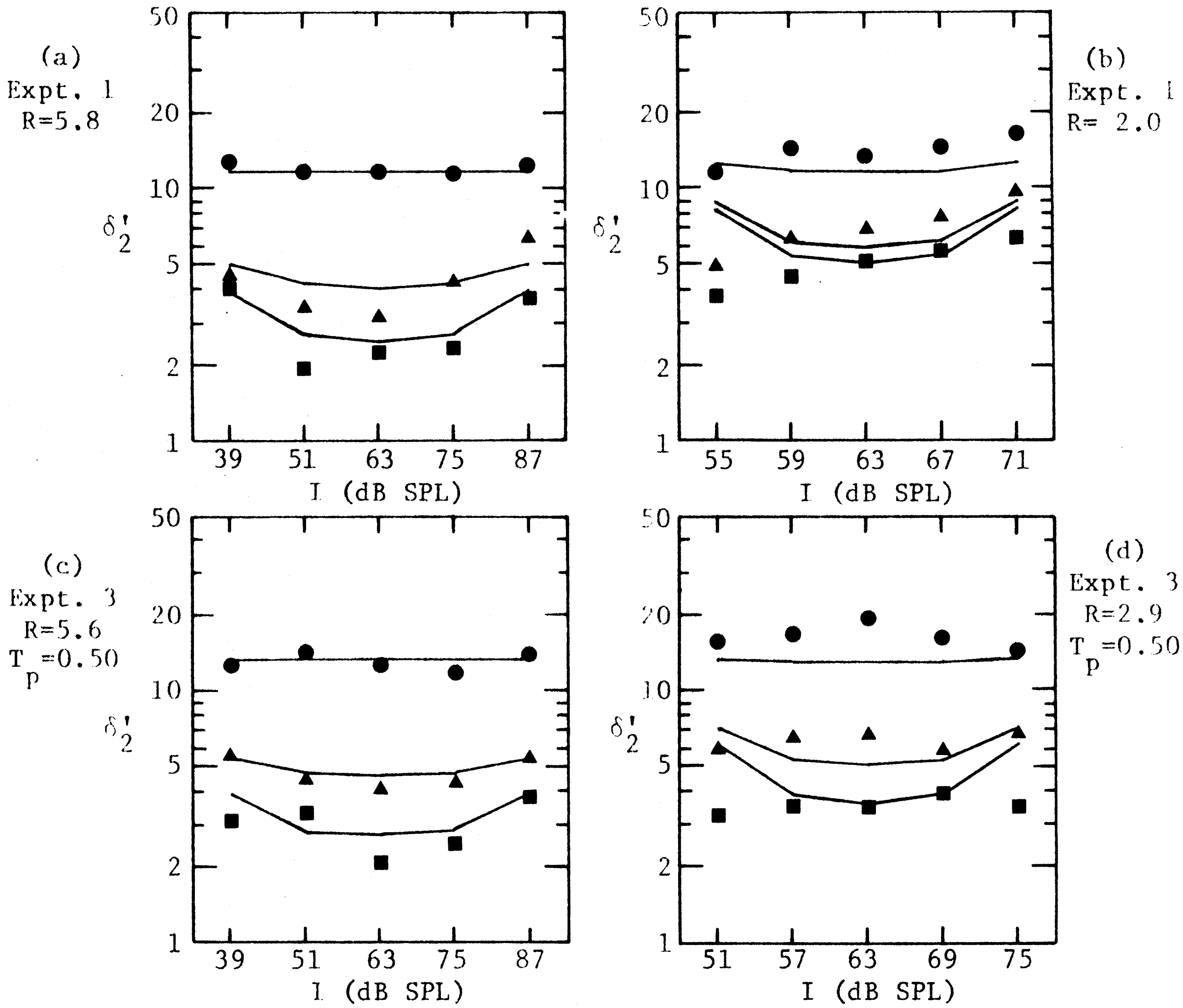


Figure 8.7 Comparison of the Revised Theory with the Results of Expts. 1 & 3.

Interpulse Interval	T
●	0.2
▲	3.5
■	14.0

δ_2' versus I.

(a) & (b) Data points from Expt.1 - Obs. JB.
Solid curves: Revised Theory with
$K/\beta = K/\beta_\psi = 12$, $\sqrt{8}\ K/G' = 9$ & $K/\sqrt{A} = 5$.

(c) & (d) Data points from Expt. 3 - Averaged over JB, JW, & SK.
Solid curves: Revised Theory with
$K/\beta = K/\beta_\psi = 13$, $\sqrt{8}\ K/G' = 8$ & $K/\sqrt{A} = 6$.

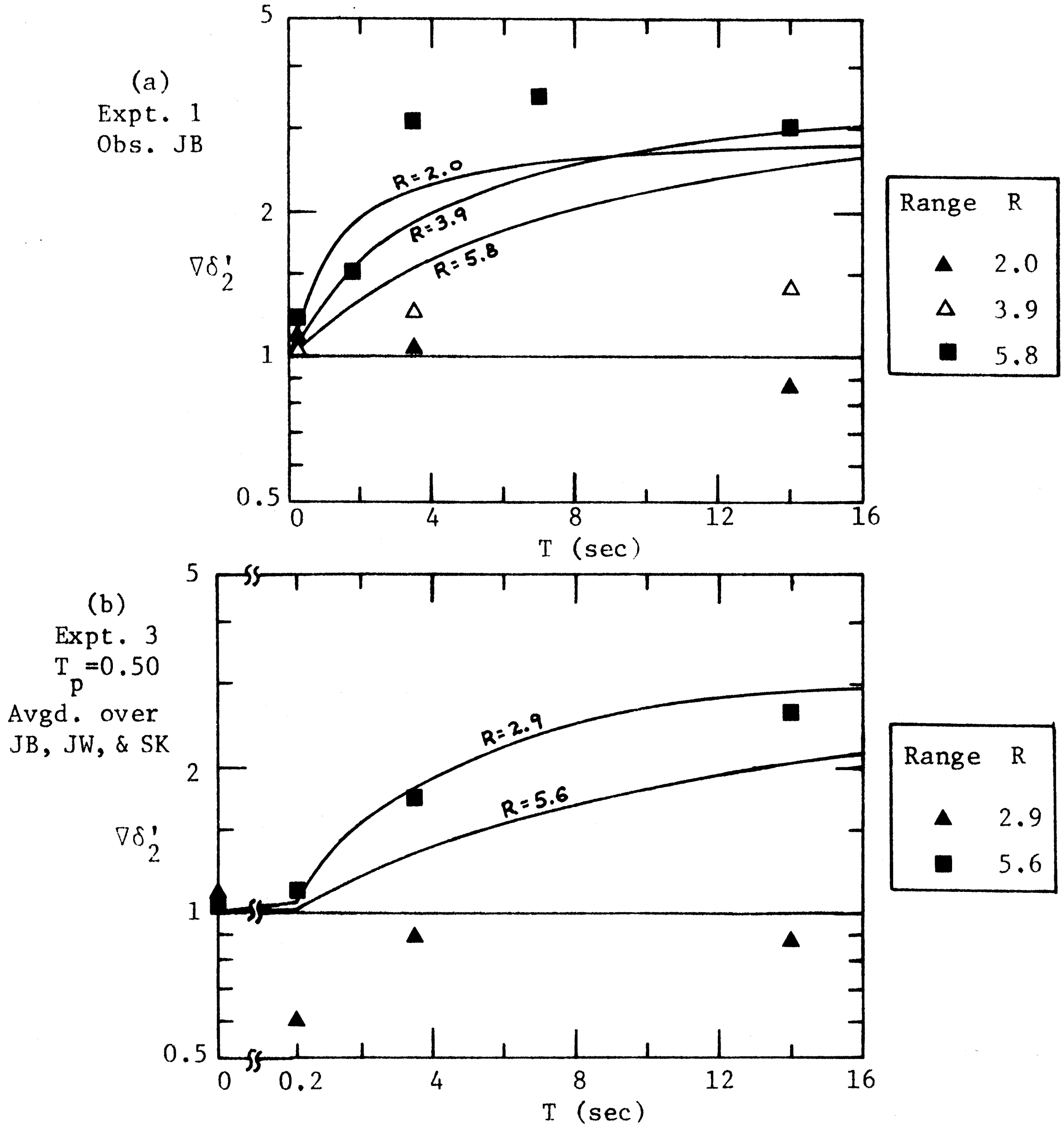


Figure 8.8 Comparison of the revised theory with the results of Expts. 1 & 3.

$\nabla\delta_2'$ versus T.

(a) Data points: Experiment 1 - Observer JB.
Solid curves: Revised theory with
$K/\beta = K/\beta_\psi = 12$, $\sqrt{8}\ K/G' = 9$, & $K/\sqrt{A} = 5$.

(b) Data points: Experiment 3 - Averaged over JB, JW, & SK.
Solid curves: Revised theory with
$K/\beta = K/\beta_\psi = 13$, $\sqrt{8}\ K/G' = 8$, & $K/\sqrt{A} = 6$.

In these figures, the theoretical values of δ_2' and $\nabla\delta_2'$ were derived from the revised theory under the assumption $K/\beta = K/\beta_\psi = 12$, $\sqrt{8}\ K/G' = 9$ and $K/\sqrt{A} = 5$ for Observer JB, and under the assumption $K/\beta = K/\beta_\psi = 13$, $\sqrt{8}\ K/G' = 8$, and $K/\sqrt{A} = 6$ for the observers of Expt. 3.[42] These parameters were selected to fit the data from these observers for resolution averaged over levels, $\hat{\delta}_2'$; and thus are equal to the values chosen in Chapter VII (see Figs. 7.2 and 7.3), since the preliminary and revised theories give virtually the same predictions for $\bar{\delta}_2'$.

Figures 8.7 and 8.8 indicate that the dependence of resolution on level in a two-interval paradigm predicted by the revised theory is in only fair agreement with the data. The effect is generally larger than predicted for large values of R, and considerably smaller than predicted for smaller values of R. Furthermore, the prediction of the theory that for fixed values of T, the effect grows with decreasing R (at least for $2.0 \leq R \leq 5.8$ and $T < 8$ sec) is clearly in error. This prediction results from the fact that for fixed values of T, as R is decreased, the observer's dependence on the perceptual-anchor coding mode increases.

In summary, Figures 8.1 - 8.8 indicate that the revised theory provides at least a first-order fit to the data for resolution as a function of level, while generally preserving the predictions of the preliminary theory for resolution averaged over levels. Considering that the revised theory has the same number of free parameters as the preliminary theory, and that in each case these parameters were picked to fit the data for resolution averaged over levels, it is clear that the revised theory represents an improvement over the preliminary theory.

There are, however, a number of aspects of the revised theory which require further work. The first concerns the evaluation of the revised theory, in order to guide its further development. An important task here is the estimation of the relative statistical significance of the various disagreements noted between the data and the revised theory. Second, if it is assumed that on each trial the observer uses the previous trial as an anchor, the revised theory would have predictions for the influence of the previous trial, as well as for the effect of feedback.[43] Third, since in one-interval paradigms the conditional probability density functions of the decision variable have, in general, different variances, the receiver operating characteristics, the ROC's, should be linear and of nonunity slope, when plotted on normal-normal paper. A preliminary examination of ROC's did not support this prediction, although the problem is a delicate one.[44] Fourth, our present procedure for estimating the resolution and criteria from the confusion matrices is based on a decision model in which it is assumed that the densities have equal variances. This model is in contradiction with the revised theory and, although the differences would probably be rather small, the data should be reprocessed using a procedure that is consistent with the revised theory.

4. Modification to Account for The Data of Observers KG and PN

An important experimental result noted in Chapter VII is that the resolution per bel obtained from Observers KG and PN in both one- and two-interval paradigms exhibited a weaker dependence on the range, R, than the resolution per bel obtained from the other observers or predicted by the preliminary theory. The assumption that these two observers always used the context of the entire experiment (i.e., arising from the full 58 dB range) to form their verbal codes, instead of the context arising from the range employed during a particular experimental run, was considered in Chapter VII. This assumption, however, implied that the resolution per bel would be entirely independent of R, a change in the right direction, but too far. Now, a similar modification of the perceptual-anchor coding mode is considered; it is assumed that the perceptual anchors are determined by the context of the entire experiment, but that the statistics of the steps $\{S_i\}$ (i.e., $E[S_i]$ and $Var[S_i]$) are determined by the context of a particular experimental run. [45]

Figures 8.9 and 8.10 provide comparisons of the predictions of this modified revised theory for resolution averaged over levels in both one- and two-interval paradigms, with data averaged over Observers KG and PN. In Figs. 8.9a and b, respectively, are plotted the actual values of $\hat{\Delta}'$ and $\overline{\hat{\delta}}'_1$ obtained in Expt. 4 (the filled circles), as well as theoretical curves of Δ' and $\overline{\delta}'_1$ from the modified revised theory (the continuous curves). The values of the parameters used to derive these curves ($K/\beta = K/\beta_\psi = 5.5$ and $\sqrt{8}\ K/G' = 20$) were chosen to simultaneously fit the results for $\hat{\Delta}'$ and $\overline{\hat{\delta}}'_1$. In Fig. 8.10 are plotted the actual values of $\overline{\hat{\delta}}'_2$ obtained in Expts. 1 and 1A (the circles, triangles and squares), as well as theoretical curves of $\overline{\delta}'_2$ from the modified revised theory (the continuous curves). The values of the parameters used to derive these curves ($K/\beta = K/\beta_\psi = 7.5$, $\sqrt{8}\ K/G' = 7$, and $K/\sqrt{A} = 4.2$) are the same values used to fit the preliminary theory to the data. See Fig. 7.1.

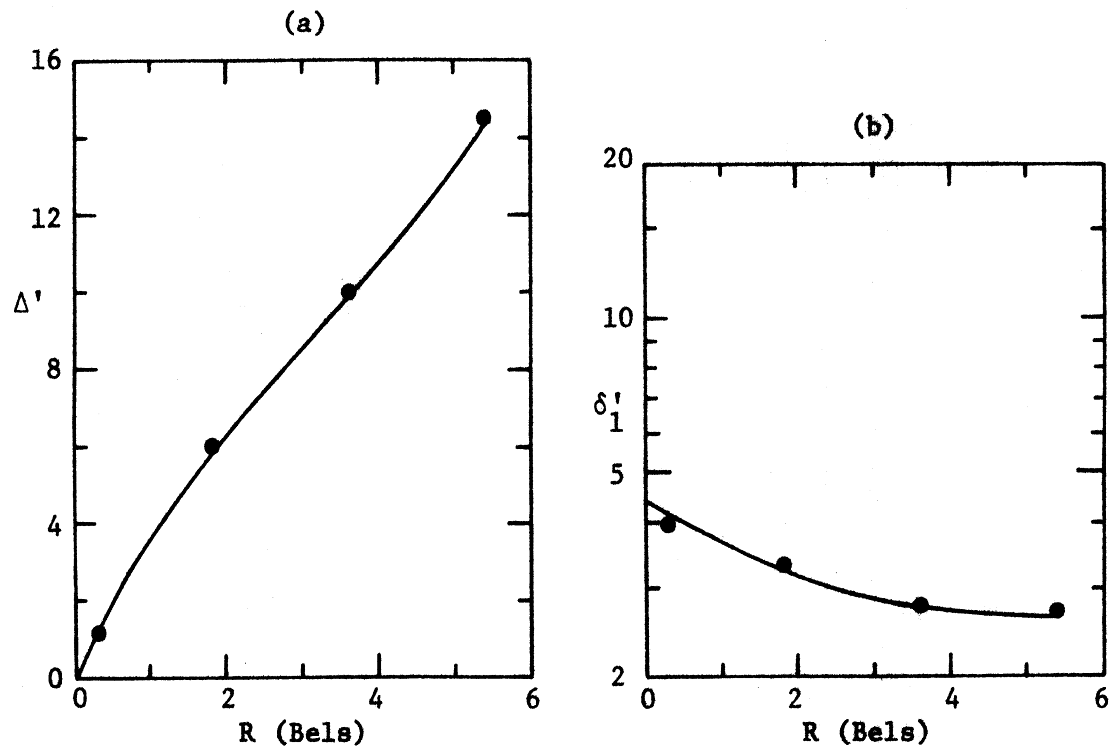


Figure 8.9 Comparison of the Modified Revised Theory with data averaged over KG & PN from Expt. 4.

(a) Δ' versus R.

(b) δ_1' versus R.

The solid curves are derived from the modified revised theory with $K/\beta = K/\beta_\psi = 5.5$ & $\sqrt{8}\ K/G' = 20$.

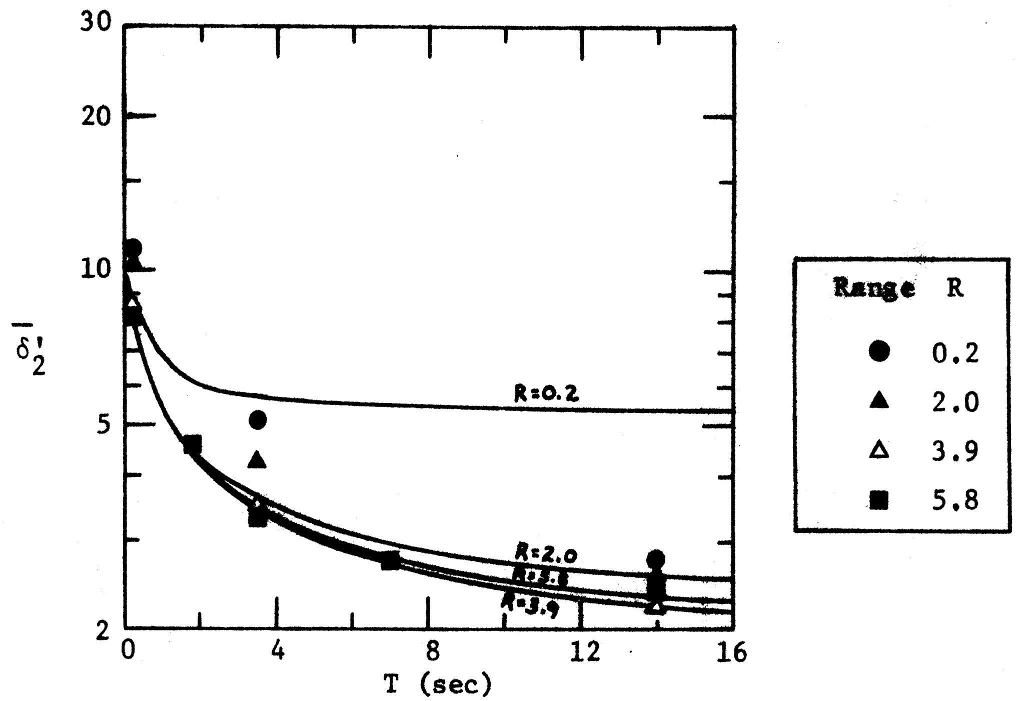


Figure 8.10 Comparison of the Modified Revised Theory with data averaged over KG & PN from Expts. 1 & 1A.

$\overline{\delta'_2}$ versus T.

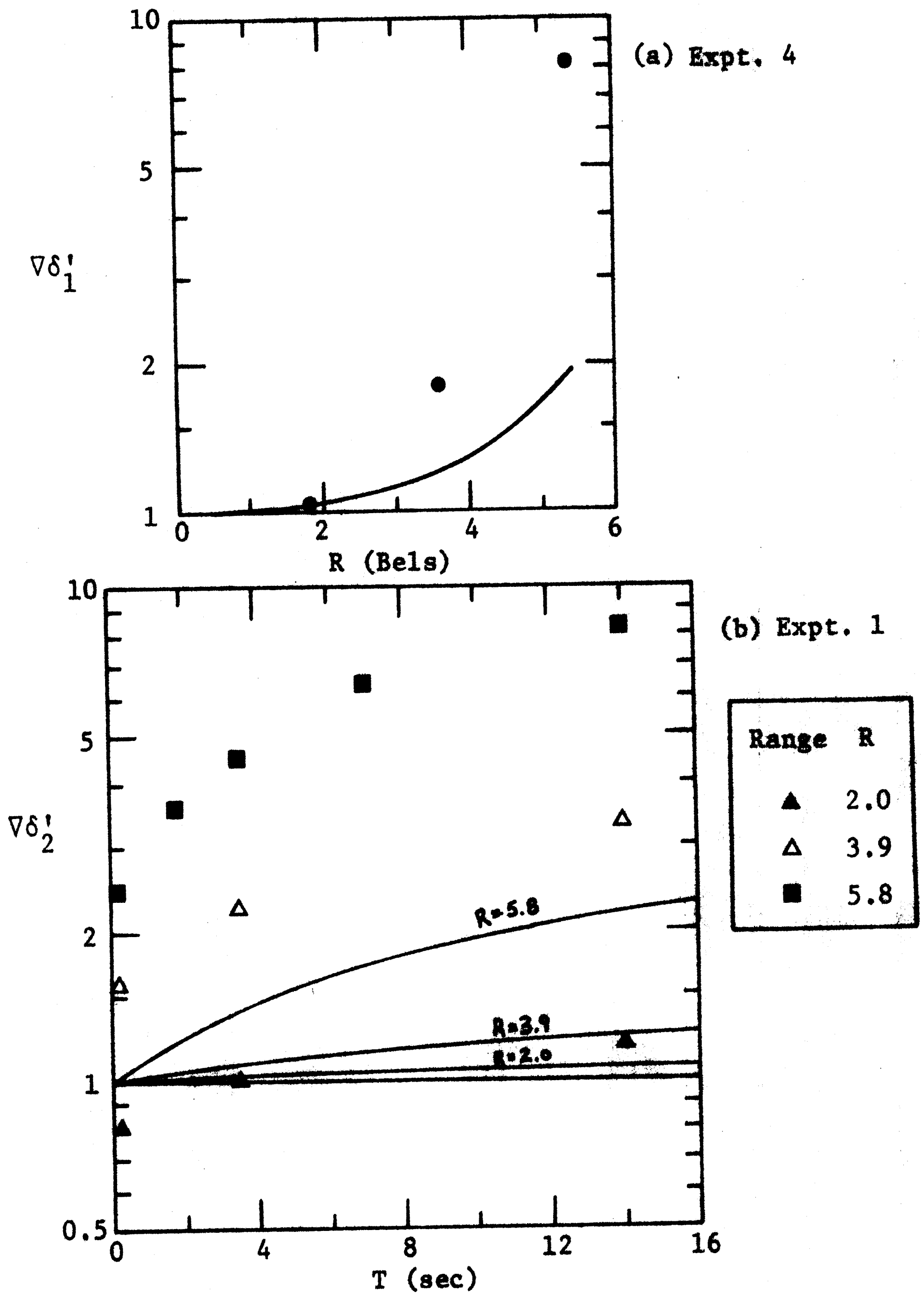
The solid curves are derived from the modified revised theory with

$K/\beta = K/\beta_\psi = 7.5$, $\sqrt{8}\ K/G' = 7$, & $K/\sqrt{A} = 4.2$.

An examination of Figs. 8.9 and 8.10, and a comparison with Figs. 7.5 and 7.1, reveals that the modified revised theory provides a better fit than the preliminary theory to the data of Observers KG and PN for resolution averaged over levels. The major remaining area of disagreement between the theory and the data concerns the dependence of $\overline{\hat{\delta}}'_2$ on T for small R; when R = 0.2 and T = 14.0 sec, the value of $\overline{\hat{\delta}}'_2$ is about 49% less than predicted. This disagreement, however, is now quite similar to the corresponding disagreement of about 57%, noted previously in Chapter VII, between the preliminary theory and the data of the other observers. (See Figs. 7.2 and 7.3.)

Figure 8.11 provides comparisons of the predictions of the modified revised theory for the quantities used as measures of the resolution edge-effect in one- and two-interval paradigms, $\nabla\delta'_1$ and $\nabla\delta'_2$, with data averaged over Observers KG and PN. In Fig. 8.11a are plotted the actual values of $\nabla\hat{\delta}'_1$ obtained in Expt. 4 (the filled circles), as well as the theoretical curve of $\nabla\delta'_1$ from the modified revised theory (the continuous curve) derived under the assumption $K/\beta = K/\beta_\psi = 5.5$ and $\sqrt{8}\ K/G' = 20$. These are the same values used to fit the data for $\hat{\Delta}'$ and $\overline{\hat{\delta}}'_1$ in Fig. 8.9. In Fig. 8.11b are plotted the actual values of $\nabla\hat{\delta}'_2$ obtained in Expt. 1 (the triangles and squares), as well as theoretical curves of $\nabla\delta'_2$ from the modified revised theory (the continuous curve) derived under the assumption $K/\beta = K/\beta_\psi = 7.5$, $\sqrt{8}\ K/G' = 7$, and $K/\sqrt{A} = 4.2$. These are the same values used to fit the data for $\overline{\hat{\delta}}'_2$ in Fig. 8.10.

An examination of Fig. 8.11a, and a comparison with Fig. 8.5, reveals that whereas the predicted values of $\nabla\delta'_1$ from the revised theory



<u>Figure 8.11</u> Comparison of the Modified Revised Theory with data averaged over KG & PN from Expts. 1 & 4.

(a) $\nabla\delta_1'$ versus R. Data points: Expt. 4
Solid curve: Modified revised theory with
$K/\beta = K/\beta_\psi = 5.5$ & $\sqrt{8}\ K/G' = 20$.

(b) $\nabla\delta_2'$ versus T. Data points: Expt. 1
Solid curve: Modified revised theory with
$K/\beta = K/\beta_\psi = 7.5$, $\sqrt{8}\ K/G' = 7$, & $K/\sqrt{A} = 4.2$.

are generally too large, the predicted values of $\nabla\delta_1'$ from the modified revised theory are generally too small. An examination of Fig. 8.11b, and a comparison with Fig. 8.8, reveals that whereas the predicted values of $\nabla\delta_2'$ from the revised theory are generally too small for $R \simeq 5.8$, and generally too large for smaller values of R; the predicted values of $\nabla\delta_2'$ from the modified revised theory are generally too small for $R \geq 3.9$, and about right for $R = 2.0$. Furthermore, whereas the revised theory predicted incorrectly that for fixed T, $\nabla\delta_2'$ grows with decreasing R; the modified revised theory correctly predicts that $\nabla\delta_2'$ grows with increasing R.

Finally, it is noted that the dependence of resolution on range, R, exhibited by Observers KG and PN is similar to the dependence exhibited by observers in a magnitude estimation experiment reported by Braida and Durlach (1972). The authors interpreted their results by assuming that the context used by these observers arose from a combination of the context of a particular run and a fixed large "natural" context, corresponding to a range $R_o > 7$, independent of the experiment.

Figure 8.12 provides a comparison of the predictions of the modified revised theory with the magnitude estimation data reported by Braida and Durlach. Actual values of $\hat{\Delta}'$ and $\overline{\hat{\delta}_1'}$ are plotted along with values of Δ' and $\overline{\delta_1'}$ derived from the theory, under the assumption $K/\beta = K/\beta_\psi = 4$, $\sqrt{8}\ K/G' = 15.5$, and $R_o = 7$. Fig. 8.12 shows that the modified revised theory provides an excellent fit to the data.

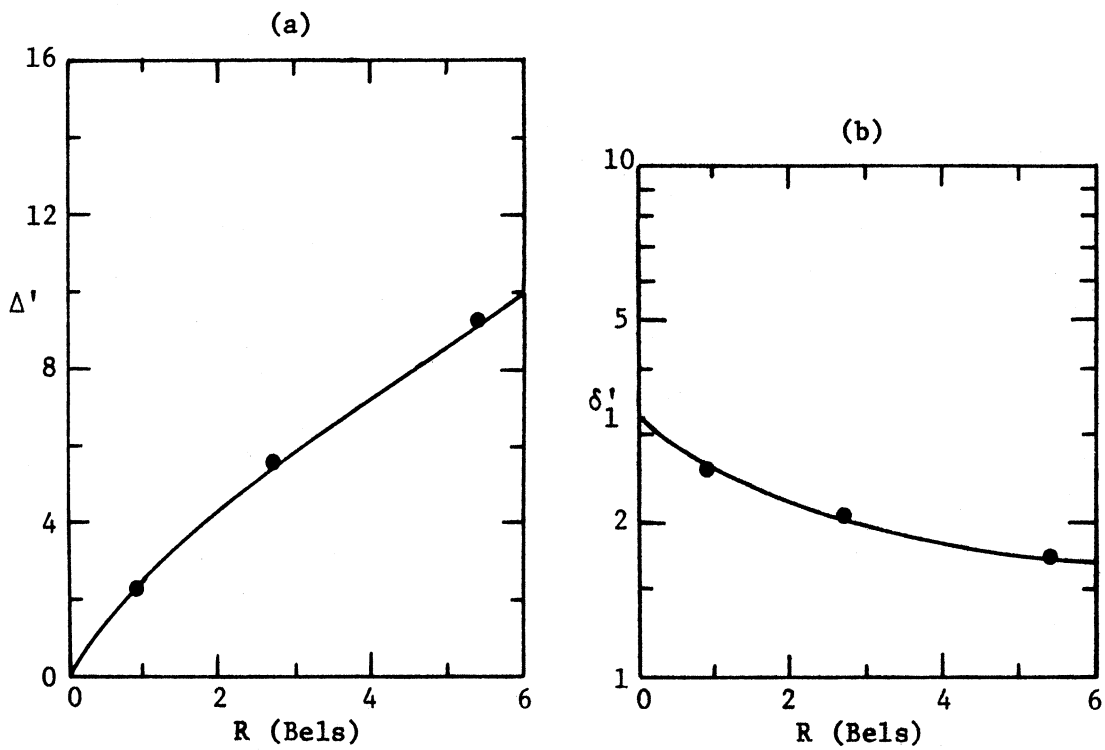


Figure 8.12 Comparison of the Modified Revised Theory with Magnitude Estimation data reported by Braida & Durlach (1972).

(a) Δ' versus R.

(b) δ_1' versus R.

The solid curves are derived from the modified revised theory with $K/\beta = K/\beta_\psi = 4$, $\sqrt{8}\ K/G' = 15.5$, & $R_o = 7$.

B. Attempts at a Model for Bias

In a departure from the initial strategy adopted by our research group, a significant effort has been made in this research to examine response bias. In particular, an important phenomenon selected for careful study in this thesis is the bias edge-effect; i.e., the tendency of observers to exhibit strong and opposite response biases near the edges of the range, and little response bias near the middle of the range.

In this section, several attempts at accounting for the bias edge-effect are presented. These include the original approach to the bias edge-effect (variable criterion), a trace-drift (substitution-errors) model, and an unequal-weights (strategy) model. As yet, none of these has been entirely successful, although it appears that the trace-drift model may eventually have success once it is combined with a context-coding mode.

1. Original Approach to the Bias Edge-Effect

The original approach to the bias edge-effect is a straightforward extension of the preliminary theory. In order to account for the bias edge-effect, the criterion, C, is allowed to depend on the intensity level, and is written C_i. This approach was prompted by the traditional detection-theory picture. In this picture, the criterion which determines the bias is under the control of the observer (and may be influenced by such things as a priori stimulus probabilities and payoffs), and is independent of the conditional probability density functions.

This approach to the bias edge-effect has three shortcomings. First, it leads to an inconsistency. Since the criteria are assumed to vary with the intensity level, the observer must identify the level on each trial to pick C_i. However, his ability to identify the levels (which is essentially equivalent to his ability to identify the intensities in a one-interval identification experiment) is limited, so that he will make errors. Thus, for any stimulus pair, he will not always use the same criterion. This phenomenon, which would tend to reduce resolution, is ignored in this approach.

Second, whereas the criteria are normally thought to be at least partly under the conscious control of the observers, the bias edge-effect appears not to be a result of conscious control by the observers. For example, although naive observers are unaware of the bias edge-effect (and remain so unless given some kind of feedback), it is very large and very resistant to attempts at compensation. Furthermore, the effect tends to sharply reduce the percent correct, which the observer ordinarily tries to maximize. These facts both imply that the bias edge-effect is not a result of conscious control by the ovserver.

Third, the approach is not a model for bias. No predictions are made concerning the bias or criteria (the dependence of C_i on i is unspecified); the criteria are merely allowed to vary arbitrarily over the range of levels. The bias edge-effect, however, is large and consistent; bias does not vary arbitrarily over the range of levels. Thus, we would prefer to construct a model that does make predictions concerning bias as a function of level. Two attempts at such a model are presented in the remainder of this section.

2. Trace-Drift (Substitution-Errors) Model

a. Basic Concepts

The trace-drift model is a modification of the trace mode of the preliminary theory. In this model, it is assumed that as a result of substitution errors (i.e., errors in which the trace of a previous sensation is inadvertently substituted for a current trace),[46] the mean of the trace drifts towards the middle of the range and the variance of the trace grows with time. Regardless of the details of the model, it has the following implications:

1) The growth rate of the variance of the trace and the drift rate of the mean of the trace will be quantitatively related.

2) The bias edge-effect is predicted only in the (revised) trace mode, not in the context-coding mode.

3) Consequently, the effect is predicted to be small in experiments with short interpulse intervals T (time is needed for the trace to drift),[47] as well as with very long T (so that the observer is in the context-coding mode).

A variety of trace-drift models have been considered. Because of computational difficulties, however, quantitative predictions have been obtained for only one class of these models. In this class, the trace is assumed to undergo a random walk in which the probabilities of stepping in either direction or the step sizes (or both) are biased towards the middle of the range. The notion of finite substitution errors is not carried much further; its influence on the model is mostly motivational. The properties of the random walk process are examined in the limit as the step size approaches zero and the number of steps per unit time approaches infinity.

Quantitative predictions for resolution and bias from the trace-drift model are derived in the next section. Such predictions have not yet been obtained for a complete revised theory in which the trace-drift model is combined with a context-coding model, although qualitative remarks are made concerning the implications of such a combined-mode theory, during the discussion of the trace-drift model.

b. Assumptions and Equations

A number of these trace-drift models have been investigated. The model presented in this section appears to come the closest of any investigated so far to accounting for the bias edge-effect results.

In any of these models, it is assumed that the sensation Y resulting from intensity I is the same as in the preliminary theory, and leads to a trace $\bar{Y}(t)$ that is a random function of time, with $\bar{Y}(0) = Y$. This random function is assumed to result from a continuous Markov random-walk process in which both the probabilities of stepping in either direction and the step sizes depend on $\Phi(t)$, the current value of the trace relative to the context. Specifically, $\Phi(t)$ is defined as the difference between the value of the trace at time t, $\bar{Y}(t)$, and the middle of the context, $m = [\alpha(I_{max}) + \alpha(I_{min})]/2$:

$$\Phi(t) = \bar{Y}(t) - m \,. \tag{8.35}$$

In the present model, it is assumed that both the probabilities and the step sizes are biased towards the middle of the range, although the predictions would be essentially unchanged if either one alone were biased. Specifically, it is assumed that at time t, the next step, $\Delta\Phi(t) = \Phi(t + \Delta t) - \Phi(t)$, is given by either

$$\Delta\Phi(t) = s[1 - 2s\Phi(t)/J^2W^2] \tag{8.36}$$

with a probability of $[1/2 - 2s\Phi(t)/J^2W^2]$, or

$$\Delta\Phi(t) = -s[1 + 2s\Phi(t)/J^2W^2] \tag{8.37}$$

with a probability of $[1/2 + 2s\Phi(t)/J^2W^2]$, where J and s are constants, and $W = \alpha(I_{max}) - \alpha(I_{min})$ is the width of the context.

The salient feature of the model is the bias term, $\pm 2s\Phi(t)/J^2W^2$. Two properties of this term should be noted. First, when the trace is in the middle of the range (so that $\Phi(t) = 0$), the bias term is zero; and when the trace is at either end of the range (so that $\Phi(t) = \pm W/2$), the bias term is $\mp s/J^2W$, inversely proportional to W. Second, as $J \to \infty$ (so that the bias term is always zero), the model is identical to the trace mode of the preliminary theory.

Next, we let s and Δt approach zero such that $s^2/\Delta t = 2A$ [48], and calculate the infinitesimal mean, $E'(\phi, t)$ and variance, $V'(\phi, t)$, of the process:

$$E'(\phi, t) = \lim_{s,\Delta t \to 0} \frac{E[\Delta\Phi(t) | \Phi(t) = \phi]}{\Delta t}$$

$$= \lim_{s,\Delta t \to 0} \{[s(1 - 2s\phi/J^2W^2)(1/2 - 2s\phi/J^2W^2)$$

$$- s(1 + 2s\phi/J^2W^2)(1/2 + 2s\phi/J^2W^2)]/\Delta t\}$$

$$= - 12A\phi/J^2W^2 \ , \qquad (8.38)$$

and (ignoring all terms of higher order than s^2)

$$V'(\phi, t) = \lim_{s,\Delta t \to 0} \frac{Var[\Delta\Phi(t) | \Phi(t) = \phi]}{\Delta t}$$

$$= \lim_{s,\Delta t \to 0} s^2/\Delta t$$

$$= 2A \ . \qquad (8.39)$$

The forward Kolmogorov equation,

$$\frac{1}{2}\frac{\partial^2}{\partial\phi^2} \{V'(\phi, t)p\} - \frac{\partial}{\partial\phi}\{E'(\phi, t)p\} = \frac{\partial p}{\partial t} \ , \qquad (8.40)$$

[where $p = p(\Phi(t); t)$, the probability density function of $\Phi(t)$ at time t] is then:

$$A \frac{\partial^2 p}{\partial\phi^2} + (12A/J^2W^2) \frac{\partial(\phi p)}{\partial\phi} = \frac{\partial p}{\partial t} \ . \qquad (8.41)$$

The solution to this equation, with the initial condition $\Phi(t = 0) = \phi_o$, is the conditional probability density function $p(\Phi(t);\ t|\phi_o)$ that is Gaussian with mean:[49]

$$E[\Phi(t)|\phi_o] = \phi_o \exp(-\ 12\ At/J^2W^2)\ , \tag{8.42}$$

and variance:

$$Var[\Phi(t)|\phi_o] = \frac{J^2W^2}{12}\ [1 - \exp(-\ 24\ At/J^2W^2)]\ . \tag{8.43}$$

Since $\Phi(t) = \overline{Y}(t) - m$, and $\overline{Y}(0) = Y$, the conditional probability density function $p(\overline{Y}(t)|Y)$ is Gaussian with mean:

$$E[\overline{Y}(t)|Y] = (Y - m)\ \exp(-\ 12\ At/J^2W^2) + m\ , \tag{8.44}$$

and variance:

$$Var[\overline{Y}(t)|Y] = \frac{J^2W^2}{12}\ [1 - \exp(-\ 24\ At/J^2W^2)]\ . \tag{8.45}$$

Furthermore, since the assumptions concerning the formation of the sensation are the same as in the preliminary theory, the conditional probability density function $p(\overline{Y}(t)|Y)$ is Gaussian with mean:

$$E[\overline{Y}(t)|I] = \alpha(I)\exp(-\ 12\ At/J^2W^2) + m[1 - \exp(-\ 12\ At/J^2W^2)]\ , \tag{8.46}$$

and variance:

$$\mathrm{Var}[\bar{Y}(t)|I] = \beta^2 \exp(-\ 24\ At/J^2W^2) + \frac{J^2W^2}{12}\ [1 - \exp(-\ 24\ At/J^2W^2)]\ . \tag{8.47}$$

Thus the mean of $\bar{Y}(t)$ is initially $\alpha(I)$ and approaches m exponentially, while the variance of $\bar{Y}(t)$ is initially β^2 and approaches $J^2W^2/12$ exponentially.

At this point, a judgment can be made about the approximate value of the parameter J. The trace-drift model was motivated by the notion that the drift is a result of substitution errors. If this were strictly so, the steady-state distribution of $\bar{Y}(t)$ would be uniform between $m - W/2$ and $m + W/2$; consequently, its steady-state variance would be $W^2/12$. Although in the trace-drift model the distribution of $\bar{Y}(t)$ is Gaussian rather than uniform, it is assumed that its steady-state variance is $W^2/12$, and hence that $J^2 = 1$. This value of J will be of importance later, when discussing the implications of the trace-drift model.

In the trace-drift model, the observer is assumed to form the decision variable, X_T, as in the normal trace mode by subtracting the sensation arising from the second interval from the trace arising from the first interval:

$$X_T = \bar{Y}_1(T) - Y_2\ . \tag{8.48}$$

Thus the conditional probability density functions $p(X_T|I_i,\ I_i^*)$ and

$p(X_T | I_i^*, I_i)$ are Gaussian, with

$$E[X_T | I_i, I_i^*] = \alpha(I_i)\exp(-12AT/J^2W^2) + m[1-\exp(-12AT/J^2W^2)] - \alpha(I_i^*) \quad , \tag{8.49}$$

$$E[X_T | I_i^*, I_i] = \alpha(I_i^*)\exp(-12AT/J^2W^2) + m[1-\exp(-12AT/J^2W^2)] - \alpha(I_i) \quad , \tag{8.50}$$

and

$$\mathrm{Var}[X_T] = \beta^2[1+\exp(-24AT/J^2W^2)] + \frac{J^2W^2}{12}[1-\exp(-24AT/J^2W^2)] \ . \tag{8.51}$$

Finally, it is assumed that the observer makes his decision by comparing X_T with a fixed criterion C', which is independent of level. The resolution, d_2', is then given by:

$$d_2'(I_i, I_i^*; I_i^*, I_i) = \frac{[\alpha(I_i)-\alpha(I_i^*)][1+\exp(-12AT/J^2W^2)]}{\left\{\beta^2[1+\exp(-24AT/J^2W^2)] + \frac{J^2W^2}{12}[1-\exp(-24AT/J^2W^2)]\right\}^{1/2}} \ . \tag{8.52}$$

Substituting $\alpha(I) = K \log(I)$, $W = \alpha(I_{max}) - \alpha(I_{min})$, and $R = \log(I_{max}/I_{min})$, d_2' becomes:

$d_2'(I_i, I_i^*; I_i^*, I_i)$

$$= \frac{K \log(I_i/I_i^*)[1+\exp(-12AT/J^2K^2R^2)]}{\left\{\beta^2[1+\exp(-24AT/J^2K^2R^2)] + \frac{J^2K^2R^2}{12}[1-\exp(-24AT/J^2K^2R^2)]\right\}^{1/2}} , \tag{8.53}$$

and the resolution per bel δ_2' is: [50]

$$\delta_2' = \frac{K[1+\exp(-12AT/J^2K^2R^2)]}{\left\{\beta^2[1+\exp(-24AT/J^2K^2R^2)] + \frac{J^2K^2R^2}{12}[1-\exp(-24AT/J^2K^2R^2)]\right\}^{1/2}} . \tag{8.54}$$

Since the two conditional probability density functions $p(X_T|I_i, I_i^*)$ and $p(X_T|I_i^*, I_i)$ are no longer located symmetrically about the point $X_T = 0$ (as they were in the preliminary theory), the bias, b', is defined by:

$$b'(I_i,I_i^*; I_i^*,I_i) = \frac{C'-(E[X_T|I_i, I_i^*]+E[X_T|I_i^*, I_i])/2}{(\mathrm{Var}[X_T])^{1/2}} , \tag{8.55}$$

which is:

$$b'(I_i') = \frac{C' + [\alpha(I_i')-m][1-\exp(-12AT/J^2W^2)]}{\left\{\beta^2[1+\exp(-24AT/J^2W^2)] + \frac{J^2W^2}{12}[1-\exp(-24AT/J^2W^2)]\right\}^{1/2}} ,$$

(8.56)

where I_i', defined by $\alpha(I_i') = [\alpha(I_i) + \alpha(I_i^*)]/2$, represents the overall intensity of the i^{th} level. Substituting $\alpha(I) = K \log(I)$, $W = \alpha(I_{max}) - \alpha(I_{min})$, $m = [\alpha(I_{max}) + \alpha(I_{min})]/2$, and $R = \log(I_{max}/I_{min})$, $b'(I_i')$ becomes:

$$b'(I_i') = \frac{C' + K \log[I_i'/(I_{max}I_{min})^{1/2}][1-\exp(-12AT/J^2K^2R^2)]}{\left\{\beta^2[1+\exp(-24AT/J^2K^2R^2)] + \frac{J^2K^2R^2}{12}[1-\exp(-24AT/J^2K^2R^2)]\right\}^{1/2}} .$$

(8.57)

The measure of criterion $C/K = 2b'/\delta_2'$, derived from the equations of the preliminary theory, is now a measure of apparent criterion, although it may still be interpreted as the point of subjective equality for the intensities of the first and second intervals. It is given by:

$$C(I_i')/K = \frac{2C'}{K[1+\exp(-12AT/J^2K^2R^2)]} + \log[I_i'^2/(I_{max}I_{min})]\tanh(6AT/J^2K^2R^2) .$$

(8.58)

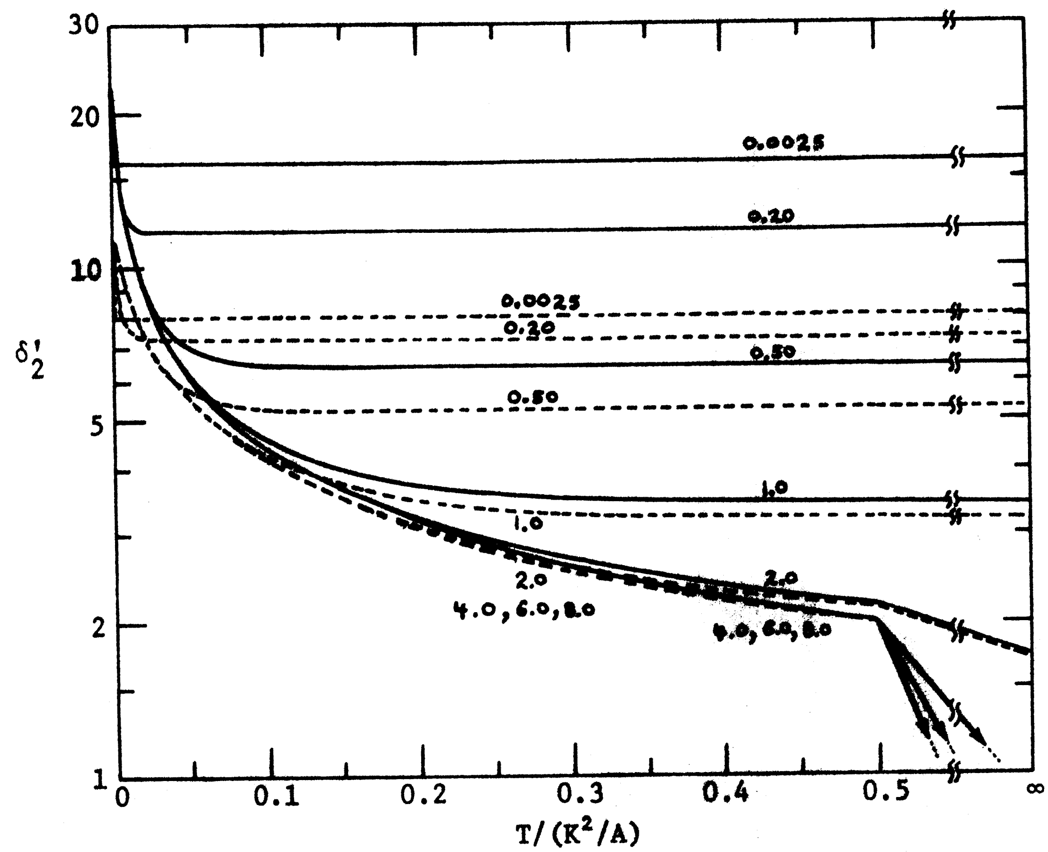
The measure of the bias edge-effect, $\nabla C/K = C(I_{max})/K - C(I_{min})/K$, is then:

$$\nabla C/K = 2R \tanh(6AT/J^2K^2R^2). \tag{8.59}$$

c. Implications of the Trace-Drift Model

First, the implications of the trace-drift model for resolution δ_2' are considered, keeping in mind that it has not yet been incorporated into a complete model by combining it with a context-coding model. The dependence of δ_2' on R and T, described by Eq. 8.54, depends on the values of the three parameters K/β, J, and $K/\sqrt{A}$. Figure 8.13 illustrates the dependence of $\log(\delta_2')$ on K/β, RJ and $T/(K^2/A)$. The solid curves have been plotted under the assumption $K/\beta = 16$, and the dashed curves under the assumption $K/\beta = 8$. The numbers on the curves give the values of RJ.

In order to interpret meaningfully the theoretical curves drawn in Fig. 8.13, several comments are made concerning the three parameters. First, consider the results of the discrimination experiments (listed in Table 7.1) that $7.5 \leq K/\beta \leq 14$, and that $4.2 \leq K/\sqrt{A} \leq 7$; and recall that the parameters K/β and $K/\sqrt{A}$ have been defined in exactly the same manner in the trace-drift model as in the preliminary theory. It can then be concluded that the range of K/β included in Fig. 7.13 encompasses typical values of K/β; and that the finite upper limit of $T/(K^2/A) = 0.5$ included in Fig. 7.13 corresponds to a value of T of from 8.8 to 24.5 sec, so that the range of $T/(K^2/A)$ encompasses all or nearly all (depending on the particular value of $K/\sqrt{A}$) the values of T actually employed in the experiments. Second, since it has been assumed that J = 1, the values of RJ on



<u>Figure 8.13</u> Prediction of the trace-drift model for $\log(\delta_2')$ as a function of K/β, RJ, and $T/(K^2/A)$. The numbers on the curves give the values of RJ.

———: $K/\beta = 16$

------: $K/\beta = 8$

the curves in Fig. 8.13 may be interpreted directly as values of range, R.

With these comments in mind, it may be seen from Fig. 8.13 that although δ_2' is generally strongly dependent on R, for the values of R and T employed in the roving-level discrimination experiments ($2.0 \leq R \leq 5.8$, and $0 \leq T \leq 14.0$ sec), δ_2' is nearly independent of R. This point may be clarified by referring to Eq. 8.54 and noting that for most of these values of R and T, $12AT/J^2K^2R^2$ is much less than 1. Thus $\exp(-12AT/J^2K^2R^2)$ is approximately equal to $1 - 12AT/J^2K^2R^2$, and Eq. 8.54 becomes approximately:

$$\delta_2' \simeq \frac{2K}{\left(2\beta^2 + 2AT\right)^{1/2}} \quad . \tag{8.60}$$

Comparing this with Eq. 4.6, we conclude that for the values of R and T employed in the roving-level experiments, the predictions for δ_2' of the trace-drift model are nearly identical with the predictions of the trace mode of the preliminary theory. Consequently, although the details have not yet been worked out, it appears likely that when the trace-drift model is combined with a context-coding model, the resulting predictions for δ_2' will be nearly identical with the predictions of the preliminary theory.

Next, the implications of the trace-drift model for bias, in particular the bias edge-effect, are considered, again keeping in mind that it has not yet been combined with a context-coding model. The dependence of $\nabla C/K$ on R and T, described by Eq. 8.59, depends on the two parameters J and $K/\sqrt{A}$. Figure 8.14 illustrates the dependence of $(\nabla C/K)J$ on RJ and $T/(K^2/A)$. The numbers on the curves give the values of RJ.

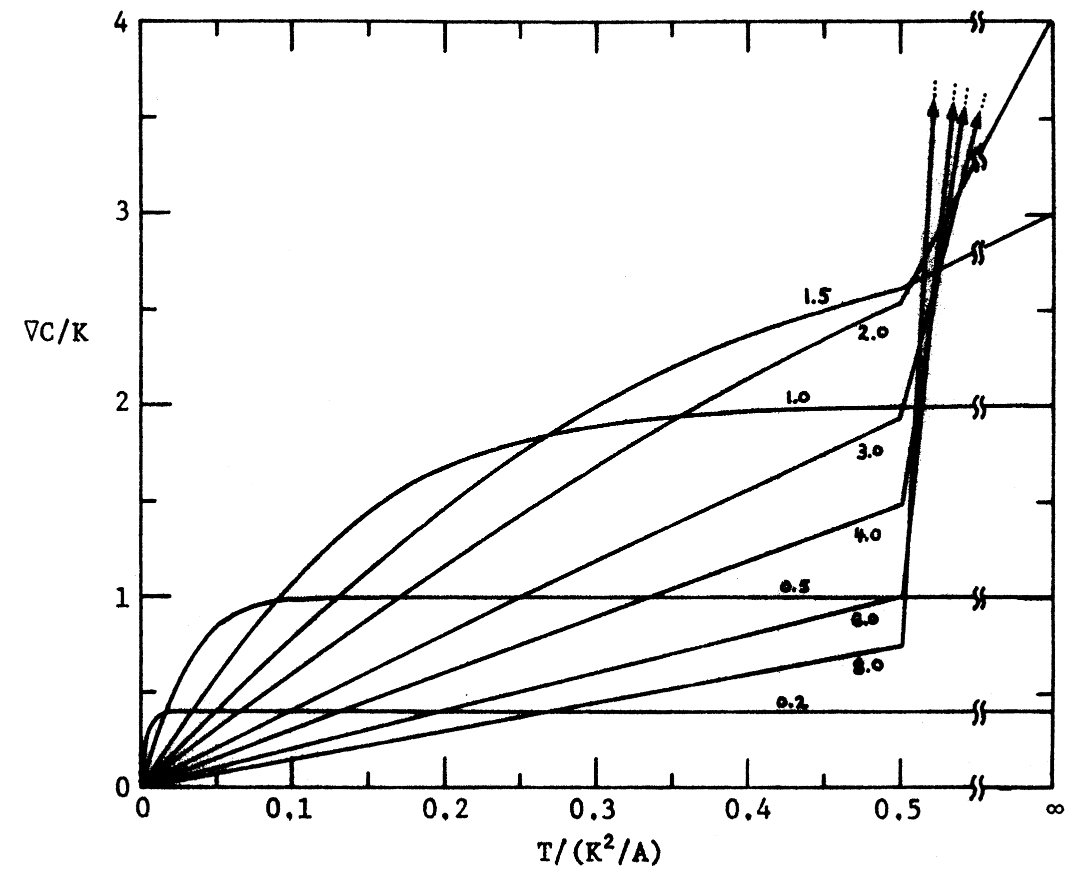


Figure 8.14 Prediction of the trace-drift model for $\nabla C/K$ as a function of RJ, and $T/(K^2/A)$. The numbers on the curves give the values of RJ.

Recalling the comments made previously concerning the values of the parameters, we interpret the curves in Fig. 8.14 to indicate that for the values of R and T employed in the roving-level discrimination experiments ($2.0 \leq R \leq 5.8$, and $0 \leq T \leq 14.0$ sec), $\nabla C/K$ is proportional to T and inversely proportional to R. This point may be clarified by referring to Eq. 8.59 and again noting that for most of these values of R and T, $6AT/J^2K^2R^2$ is much less than 1. Thus $\tanh(6AT/J^2K^2R^2)$ is approximately given by $6AT/J^2K^2R^2$, and Eq. 8.59 becomes approximately:

$$\nabla C/K \simeq 12AT/J^2K^2R \quad . \tag{8.61}$$

Before comparing the predictions of the model for $\nabla C/K$ with the results of Expts. 1 and 3, we consider qualitatively the effect of combining the trace-drift model with a context-coding model. In such a combined-mode model, the observer's relative dependence on the trace-mode would generally diminish with decreasing R and with increasing T. Thus, $\nabla C/K$ should generally be somewhat less than indicated in Fig. 8.14, $\nabla C/K$ should be concave down as a function of T for fixed R, and $\nabla C/K$ should increase less than shown in Fig. 8.14 with decreasing R for fixed T.

An examination of Figs. 7.25a, 7.25b, and 7.27, indicates that these predictions are not inconsistent with the results of Expts. 1 and 3. A quantitative comparison of the data and the theory, however, must await the construction of a combined-modes model. This work is currently underway and should be completed in the near future, although not in time for inclusion in this thesis.

3. Unequal-Weights (Strategy) Model

Until recently, I had believed that the unequal-weights model had the best chance to account for the bias edge-effect. Now, however, it seems that the trace-drift model will ultimately prove successful, although the unequal-weights model may eventually have success, as well.

a. Basic Concepts

The unequal-weights (strategy) model is a modification of the internal-noise model of the preliminary theory. In this model, it is assumed that in two-interval paradigms, because the information concerning the intensity of the first interval is corrupted by memory noise, the observer gives it less weight in forming his decision variable, than the information concerning the intensity of the second interval. The relative weights are determined by the relative amounts of noise corresponding to each interval, as well as by the range of the stimuli R, since as R increases, the observer must pay an increasing penalty (in terms of percent correct) for ignoring the first interval.

The model is still under development; regardless of its eventual details, however, it will have the following implications. First, the predictions for resolution and for the bias edge-effect will be quantitatively related. Second, the bias edge-effect is predicted to occur in both the trace and context-coding modes.

b. Current Formulation of the Unequal-Weights Model

Because the unequal-weights model is still under development, it is not yet possible to give a complete picture of it. Its current, incomplete, formulation is described in this section.

In the unequal-weights model, it is assumed that the trace and context modes operate in basically the same way as in the preliminary theory, up to the formation of the decision variable. Thus, after time T, the observer has available to him the trace of the first sensation $\overline{Y}_1(T)$, and the verbal code of the first sensation Q_1. To simplify the later description of the model, the following change is made in the context-coding mode. Instead of encoding the second sensation to form Q_2 for a comparison with Q_1, as in the preliminary theory, the observer is now assumed to decode Q_1, producing Y_{Q1}, the decoded verbal code of the first sensation. It is assumed that so doing entails the addition of zero-mean Gaussian decoding noise of variance G^2R^2 to Q_1. Thus the conditional probability density functions $p(\overline{Y}_1(T)|I_1)$ and $p(Y_{Q1}|I_1)$ are jointly Gaussian, with:

$$E[\overline{Y}_1(T)|I_1] = E[Y_{Q1}|I_1] = \alpha(I_1) \ , \tag{8.62}$$

$$\mathrm{Var}[\overline{Y}_1(T)|I_1] = \beta^2 + 2AT \ , \tag{8.63}$$

$$\mathrm{Var}[Y_{Q1}(T)|I_1] = \beta^2 + 2G^2R^2 \ , \tag{8.64}$$

and

$$\mathrm{Cov}[\overline{Y}_1(T),\ Y_{Q1}(T)|I_1] = \beta^2 \ . \tag{8.65}$$

The observer is then assumed to optimally combine $\overline{Y}_1(T)$ and Y_{Q1} to form $Y_{Q,T}$, a maximum likelihood estimate of $\alpha(I_1)$. The conditional probability density function $p(Y_{Q,T}|I_1)$ is Gaussian with: [51]

$$E[Y_{Q,T}|I_1] = \alpha(I_1) \ , \tag{8.66}$$

and

$$\mathrm{Var}[Y_{Q,T}|I_1] = \beta^2 + \frac{2}{1/AT + 1/G^2R^2} \ . \tag{8.67}$$

The changes from the preliminary theory up to this point are of no real consequence; they were made primarily for convenience. If the observer were to form the combined decision variable, X_c, by subtracting Y_2 from $Y_{Q,T}$, the model would be identical to the preliminary theory. However, unlike the preliminary theory, in the unequal-weights model, the observer is assumed to be free to form his decision variable, X_c, by computing any linear combination of $Y_{Q,T}$ and Y_2. Since a multiplicative transformation of the decision variable would have no effect on performance, X_c is defined:

$$X_c = VY_{Q,T} - Y_2 \ , \tag{8.68}$$

where V is the weighting factor of $Y_{Q,T}$, and in general $0 \leq V \leq 1$.

For a particular choice of V, the conditional probability density functions $p(X_c|I_i, I_i^*)$ and $p(X_c|I_i^*, I_i)$ are Gaussian, with:

$$E[X_c|I_i, I_i^*, V] = V\alpha(I_i) - \alpha(I_i^*) \ , \tag{8.69}$$

$$E[X_c|I_i^*, I_i, V] = V\alpha(I_i^*) - \alpha(I_i) \ , \tag{8.70}$$

and

$$\mathrm{Var}[X_c|V] = \beta^2(1 + V^2) + \frac{2V^2}{1/AT + 1/G^2R^2} \quad . \tag{8.71}$$

Assuming the observer makes his decision by comparing X_c with a fixed criterion, C', which is independent of level, and substituting $\alpha(I) = K \log(I)$, the resolution is given by:

$$d_2'(I_i, I_i^*; I_i^*, I_i|V) = \frac{(1+V)K \log(I_i/I_i^*)}{\left[\beta^2(1+V^2) + \dfrac{2V^2}{1/AT + G^2R^2}\right]^{1/2}} \quad , \tag{8.72}$$

and the resolution per bel by:

$$\delta_2'(V) = \frac{(1+V)K}{\left[\beta^2(1+V^2) + \dfrac{2V^2}{1/AT + 1/G^2R^2}\right]^{1/2}} \quad . \tag{8.73}$$

The bias in this situation is defined by:

$$b'(I_i, I_i^*; I_i^*, I_i|V) = \frac{C' - (E[X_c|I_i, I_i^*, V] + E[X_c|I_i^*, I_i, V])/2}{(\mathrm{Var}[X_c|V])^{1/2}} \quad , \tag{8.74}$$

which is:

$$b'(I_i'|V) = \frac{C' + (1-V)\alpha(I_i')}{\left[\beta^2(1+V^2) + \dfrac{2V^2}{1/AT + 1/G^2R^2}\right]^{1/2}} \quad , \tag{8.75}$$

where I_i', defined by $\alpha(I_i') = [\alpha(I_i) + \alpha(I_i^*)]/2$, represents the overall intensity of the i^{th} level. Substituting $\alpha(I) = K \log(I)$, $b'(I_i'|V)$ becomes:

$$b'(I_i'|V) = \frac{C' + (1-V)\ K \log(I_i')}{\left[\beta^2(1+V^2) + \dfrac{2V^2}{1/AT + 1/G^2R^2}\right]^{1/2}} \quad . \tag{8.76}$$

The measure of criterion $C/K = 2b'/\delta_2'$ is given by:

$$C(I_i'|V)/K = \frac{2C' + 2(1-V)\ K \log(I_i')}{(1+V)K} \quad , \tag{8.77}$$

and the measure of the bias edge-effect, $\nabla C/K = C(I_{max})/K - C(I_{min})/K$, is then:

$$\nabla C(V)/K = \frac{2R(1-V)}{(1+V)} \quad . \tag{8.78}$$

c. Implications of the Unequal-Weights Model

At this point in the development of the model, V is still a free parameter. It is hoped that eventually a strategy, probably stated in terms of maximizing the percent correct, will be defined which, in turn, will define V in any situation. For the present, however, a few specific paradigms will be examined, and the implications of the model briefly explored.

Consider first an unsymmetric fixed-level discrimination paradigm. The first tone is always of intensity I, and the second is either I^+ or I^-, where $\log(I^+/I) = \log(I/I^-) = q$. If the observer operates in the normal manner, described by the preliminary theory, by computing $X_c = Y_{Q,T} - Y_2$ (i.e., $V = 1$), his resolution will be:

$$d'(I, I^-; I, I^+|V = 1) = \sqrt{2}\, q\, K/\beta \quad . \tag{8.79}$$

In the unequal-weights model, however, he is allowed to ignore $Y_{Q,T}$ and use merely $X_c = -Y_2$ as his decision variable (i.e., $V = 0$). Consequently, his resolution will be

$$d'(I, I^-; I, I^+|V = 0) = 2\, q\, K/\beta \quad , \tag{8.80}$$

an increase by a factor of $\sqrt{2}$. This improvement occurs because the first interval contributes no information, only sensation noise, to the decision variable X_c.

In a symmetric fixed-level paradigm, on the other hand, where the stimuli are either (I, I^*) or (I^*, I), and $\log(I/I^*) = q$, the observer would do best by computing $X_c = Y_{Q,T} - Y_2$ (i.e., $V = 1$). Consequently, his resolution would be:

$$d'(I, I^*; I^*, I|V = 1) = \sqrt{2}\, q\, K/\beta \quad . \tag{8.81}$$

Thus a comparison of resolution in the unsymmetric and symmetric fixed-level paradigms would provide a test of the unequal-weights model.

The preceding examples were fixed-level paradigms, in which no bias edge-effect is observable, regardless of the value of the weighting factor V. Consider next a symmetric roving-level discrimination paradigm, with an intensity range R, and an interpulse interval T. In the case where either $G^2R^2 << \beta^2$ or $AT << \beta^2$, $\delta_2'(V)$ becomes (see Eq. 8.73):

$$\delta_2'(V) = \frac{(1 + V)K}{\left[\beta^2(1 + V^2)\right]^{1/2}} \quad . \tag{8.82}$$

Thus the observer could maximize δ_2' by setting V = 1, so that $\delta_2' = \sqrt{2}\ K/\beta$. Furthermore, since V = 1, $\nabla C/K = 0$ (see Eq. 8.78), and there is no bias edge-effect so that his percent correct would also be maximized.

The case where neither $G^2R^2 << \beta^2$ nor $AT << \beta^2$ is considerably more complicated. If the observer chooses V = 0, thereby ignoring the first interval, δ_2' would be given by K/β (see Eq. 8.73). Therefore, if he were free to choose V to maximize δ_2', and ignored the resultant bias, δ_2' would never diminish by more than a factor of $\sqrt{2}$, relative to the case where there is no memory noise. Setting V = 0, however, maximizes the bias edge-effect (see Eq. 8.78):

$$\nabla C(V{=}0)/K = 2R \quad . \tag{8.83}$$

Thus maximizing δ_2' by adjusting V may introduce a large bias edge-effect, and consequently severely limit the overall percent correct.

Unfortunately, further work on this model must await the formulation of a specific rule for selecting the weighting factor, V. At this point, it seems plausible that the model, once completed, will account for the bias edge-effect.

IX. SUMMARY OF RESULTS AND IMPLICATIONS FOR FUTURE WORK

A. Summary of Experimental Results

The experimental results are discussed in three parts: resolution per bel averaged over levels, resolution per bel as a function of level, and criterion as a function of level.

1. Resolution per Bel Averaged over Levels: $\overline{\hat{\delta}}'_1$ and $\overline{\hat{\delta}}'_2$

With the following two exceptions, the overall dependence of $\overline{\hat{\delta}}'_1$ on range, R, and of $\overline{\hat{\delta}}'_2$ on R and interpulse interval, T, were in fairly good agreement with the predictions of the preliminary theory. For Observers KG and PN, $\overline{\hat{\delta}}'_1$ and $\overline{\hat{\delta}}'_2$ exhibited a weaker dependence on R than for the other observers. For all the observers in the fixed-level discrimination experiments, $\overline{\hat{\delta}}'_2$ declined significantly more than predicted as a function of T.

Concerning the relation of resolution in one- and two-interval paradigms, it was found that $\overline{\hat{\delta}}'_1$ in identification with small R was poorer than it should have been, considering the results for $\overline{\hat{\delta}}'_2$ in discrimination with small T and R. In addition, it was found that $\overline{\hat{\delta}}'_2$ in discrimination with T = 14.0 sec and various values of R was poorer than it should have been, considering the results for $\overline{\hat{\delta}}'_1$ in identification with comparable values of R.

Some results about which the preliminary theory makes no predictions are the following. The effect of increasing tone pulse duration, T_p, in discrimination was to increase $\overline{\hat{\delta}}'_2$. In addition, despite the tendency of $\overline{\hat{\delta}}'_2$ to increase with decreasing T, when T_p was short (0.2 sec) $\overline{\hat{\delta}}'_2$ decreased when T was decreased from 0.2 to 0.0 sec. The presence of feedback and instructions to eliminate the bias edge-effect produced a slight improvement in $\overline{\hat{\delta}'_2}$. Finally, $\overline{\hat{\delta}'_1}$ in identification with a reference was essentially unchanged, relative to the no reference condition, when the reference intensity I_{ref} coincided with the edges of the range, and was most improved when I_{ref} was in the middle of the range.

2. Resolution per Bel as a Function of Level

The primary results with regard to resolution as a function of level concern the resolution edge-effect; i.e., the tendency of resolution in both one- and two-interval paradigms, under certain experimental conditions, to be better near the edges of the range than in the middle of the range. The quantities used as measures of this phenomenon are $\nabla\hat{\delta}'_1$ and $\nabla\hat{\delta}'_2$ in one- and two-interval paradigms, respectively. It was found that $\nabla\hat{\delta}'_1$ increased with increasing R, that $\nabla\hat{\delta}'_2$ increased with increasing R and T, and that $\nabla\hat{\delta}'_1$ in identification with various values of R was roughly equal to $\nabla\hat{\delta}'_2$ obtained in discrimination with similar values of R and large values of T. The tone pulse duration, T_p, generally did not affect $\nabla\hat{\delta}'_2$. However, in one instance when T_p was short (0.2 sec), the tendency of $\nabla\hat{\delta}'_2$ to increase with increasing T was reversed; in this case, $\nabla\hat{\delta}'_2$ decreased when T was increased from 0.0 to 0.2 sec. The presence of feedback, and instructions to eliminate the bias edge-effect had no effect on $\nabla\hat{\delta}'_2$.

The remaining, related, result with regard to resolution as a function of level concerns the effect of a reference stimulus in identification. It was found that resolution was improved, relative to the no reference condition, in the vicinity of the reference stimulus.

3. Criterion as a Function of Level

The primary results with regard to criterion as a function of level concern the bias edge-effect; i.e., the tendency of observers in two-interval paradigms to exhibit strong and opposite response biases near the edges of the range and little response bias near the middle of the range. The quantity used as a measure of this effect is $\nabla\hat{C}/K$. It was found that $\nabla\hat{C}/K$ generally increased with increasing T, and decreased somewhat with increasing R and with increasing T_p. However, in one instance when T_p was short (0.2 sec), the tendency of $\nabla\hat{C}/K$ to increase with increasing T was reversed; in this case, $\nabla\hat{C}/K$ decreased markedly as T was increased from 0.0 to 0.2 sec. The presence of feedback, and instructions to eliminate the bias edge-effect sharply reduced $\nabla\hat{C}/K$.

Additional results with regard to criterion as a function of level concern identification paradigms. It was found that in experiments with large ranges, the criteria were spread over a smaller range than unbiased criteria would be, and that in experiments with a small range, the criteria were spread over a larger range than unbiased criteria would be. Finally, it was found that the presence of a reference stimulus in identification produced large, irregular criterion shifts.

B. Summary of Theoretical Results

Theoretical work has been done in two major areas. One portion of this work involved the construction of a perceptual-anchor model for context coding to account for the resolution edge-effect and for the effect of a reference stimulus in one-interval paradigms. The other portion of this work concerned a series of attempted revisions of the preliminary theory to account for the bias edge-effect in two-interval paradigms.

1. Perceptual-Anchor Model

A modification of the context-coding mode of the preliminary theory, termed the perceptual-anchor model for context-coding, has been developed and incorporated into a revised theory of intensity resolution. In this model, the observer is assumed to use the edges of the range and the reference stimulus (when present) as perceptual anchors, and to estimate the intensity of the stimulus to be judged by comparing it with these anchors. Although it was found that, in some cases, the predictions of this revised theory for the resolution edge-effect were somewhat too large, the theory provided at least a first-order fit to the data for resolution as a function of level, while generally preserving the predictions of the preliminary theory for resolution averaged over levels. In addition, it was found that the rather weak dependence of resolution per bel on intensity range, R, exhibited by Observers KG and PN could be accounted for by a relatively simple modification of the revised theory; it is assumed that the perceptual anchors used by these two observers were determined by the context of the entire set of experiments, rather than by the context of a particular experimental run.

2. Models for Bias

It has been concluded that the original approach to the bias edge-effect was unsatisfactory, and two, new models have been attempted. One of these, the trace-drift model, is a modification of the trace mode of the preliminary theory. In this model, it is assumed that the mean of the trace, instead of remaining stationary, tends to drift towards the middle of the range. It appears that the trace-drift model has a good chance of accounting for the bias edge-effect once it is combined with a context-coding mode.

The other model, the unequal-weights model, involves a change in the formation of the decision variable. It is assumed that in two-interval paradigms, because the information from the first interval is corrupted by memory noise, the observer gives it less weight than the information from the second interval. This model is still under development.

C. Implications for Future Work

The research reported in this thesis has a number of direct implications for future experimental and theoretical work. Some of the theoretical work undertaken in this thesis should be pursued further. One such area concerns the expansion of the perceptual-anchor model for context-coding to account for the influence of the preceding trial and feedback in one-interval paradigms. Another area concerns the models for bias. The trace-drift model must be incorporated into a more complete model by combining it with a context-coding model. In addition, because I find its basic idea intuitively appealing, I would favor the continued development of the unequal-weights model.

Another important area involves accounting for the substantial decline of resolution as a function of T in the fixed-level discrimination experiments. Two approaches to this problem have been considered, although as yet, little work has been done. One possibility would be to modify the context-coding mode (or the perceptual-anchor coding mode) to allow for the context (or the anchors) to vary as a function of time following the encoding of the first stimulus. Another possibility is that the observers failed to optimally combine the two modes; they may have given less than optimum weight to the context-coding mode, and more than optimum weight to the "more natural" trace mode.

A possibly related area involves accounting for the discrepancies noted between the results from one- and two-interval paradigms. In this regard, it will be important to separate the various sources of noise in the context-coding mode which apply differently to each paradigm. For example, variations of the context from trial to trial are significant in one-interval paradigms, but irrelevant in two-interval paradigms. In addition, the notion of observers giving too much weight to the trace mode in two-interval paradigms may help account for some of these discrepancies.

Another area requiring theoretical work concerns the effect of the tone pulse duration, T_p. First, the data must be examined (and perhaps more collected) to assess the relative influence of T_p on sensation noise and memory noise.[52] In addition, the anomalies which occurred when both T_p and T were small (T_p = 0.2 sec, T = 0.0 sec) regarding resolution averaged over levels, as well as the resolution and bias edge-effects, must be accounted for.

Finally, there is a need for some improvements in data analysis techniques. One area for work, begun by Lippmann (1973), is the increased use of statistical tests to help reveal the relative strengths and weaknesses of the theory, as a guide to its further development. In addition, the procedure for estimating the resolution and criteria in one-interval paradigms must be revised to be consistent with the perceptual-anchor coding model.

Regarding the implications for future experimental work, one current requirement is for a series of two-interval roving-level discrimination experiments performed with relatively small ranges, $0 < R < 2.0$. The current experimental results indicate that the bias edge-effect tends to increase slightly as R is decreased from 5.4 to 2.0. It is expected that the effect will eventually be diminished as R is decreased further and that this is likely to be a sensitive region for the evaluation of the models for bias, once they are completed. In addition, the unequal-weights model has a prediction, which should be tested, for the relation between resolution in the symmetric and unsymmetric two-interval, fixed-level, discrimination paradigms.

Another requirement is the general need for additional data involving more trials and more observers. Such data would help reduce the apparent experimental scatter which hampered, in some cases, the evaluation of the perceptual-anchor coding model. More data are also needed to accurately determine the shapes of the ROC's in one-interval paradigms, about which this model has important predictions. In addition, experiments employing additional observers would serve to indicate, for example, whether Observers KG and PN are as unusual as is currently believed.

Finally, it would be appropriate to broaden the scope of the research to include dimensions other than intensity, as well as sense modalities other than hearing.

APPENDIX I. DETAILS OF DATA ANALYSIS

The procedure used to calculate the maximum likelihood estimates of resolution per bel, δ_2', and bias, b', in the discrimination experiments is described in part A of this appendix. The procedure used to calculate the estimates of resolution, $d_1'(I_{i+1}; I_i)$, and bias, b_i', in the identification experiments is described in part B. The Weber's law normalization procedure is described in part C.

A. Maximum Likelihood Estimation of δ_2' and b' in Discrimination

Each maximum likelihood estimate of δ_2' and b', denoted $\hat{\delta}'$ and $\hat{b}'$, is based on a data set consisting of either three or five 2 x 2 confusion matrices. (The number of matrices depends on how many intensity increments, q, had been used.) To simplify the description of the estimation procedure, some notation is now established. The different values of q are denoted $\{q_j\}$; the two stimulus conditions are denoted S = 0 or 1, corresponding to (I, I^*) and (I^*, I), respectively; and the two response categories are denoted R = 0 or 1, corresponding to "H, L" and "L, H", respectively. The entire data set is denoted $\underline{O}$, and a particular element of the data set is specified by $O(q_j, S, R)$.

The Gaussian probability density function and its integral are represented by G(X) and N(Z), where

$$G(X) = \frac{\exp(-X^2/2)}{\sqrt{2\pi}} \quad , \qquad (A1.1)$$

and

$$N(Z) = \int_{-\infty}^{Z} G(X)\, dX \quad . \tag{A1.2}$$

In addition, the inverse function of N(Z), $N^{-1}(Z)$ is defined such that for all N(Z) in the region $0 < N(Z) < 1$,

$$N^{-1}[N(Z)] = Z \ . \tag{A1.3}$$

The object of the estimation procedure is to determine the (δ_2', b') pair which maximizes the conditional probability or likelihood, $P(\underline{O}|\delta_2', b') = L$. To do this, L must be expressed in terms of δ_2' and b'. If we assume that the trials of an experiment are independent, then L is given by:

$$L = \prod_{q_j,S} BC(q_j,S)\ P(q_j,S,0)^{O(q_j,S,0)}\ P(q_j,S,1)^{O(q_j,S,1)} \quad , \tag{A1.4}$$

where $P(q_j,S,R)$ is the probability of response R when the intensity increment is q_j and the stimulus is S, and BC(q,S) is the binomial coefficient:

$$BC(q,S) = \frac{[O(q_j,S,0) + O(q_j,S,1)]!}{[O(q_j,S,0)]!\ [O(q_j,S,1)]!} \quad . \tag{A1.5}$$

Since the natural log function is monotonically increasing, the (δ_2', b') pair which maximizes L will also maximize ln(L), given by:

$$\ln(L) = \sum_{q_j,S} BC(q_j,S) + \sum_{q_j,S,R} O(q_j,S,R)\ \ln[P(q_j,S,R)] \ . \tag{A1.6}$$

Necessary conditions for this maximization are specified by the two likelihood equations, obtained by setting the derivatives of ln(L) equal to zero:

$$\frac{\partial \ln(L)}{\partial \delta_2'} = \sum_{Q_j,S,R} \frac{O(q_j,S,R)}{P(q_j,S,R)} \frac{\partial P(q_j,S,R)}{\partial \delta_2'} = 0 \quad , \tag{A1.7a}$$

and

$$\frac{\partial \ln(L)}{\partial b'} = \sum_{Q_j,S,R} \frac{O(q_j,S,R)}{P(q_j,S,R)} \frac{\partial P(q_j,S,R)}{\partial b'} = 0 \ . \tag{A1.7b}$$

To proceed further, $P(q_j,S,R)$ and its derivatives must be expressed in terms of $\delta_2' = d_2'/q_j$ and b'. Figure A1.1 represents the decision axis in two-interval paradigms, transformed by $X' = X/\sigma_2$(see Fig. 4.1) so that the decision variable has unit variance. From Fig. A1.1, we observe that:

$$P(q_j,1,1) = N(b' + d_2'/2) \tag{A1.8}$$

and

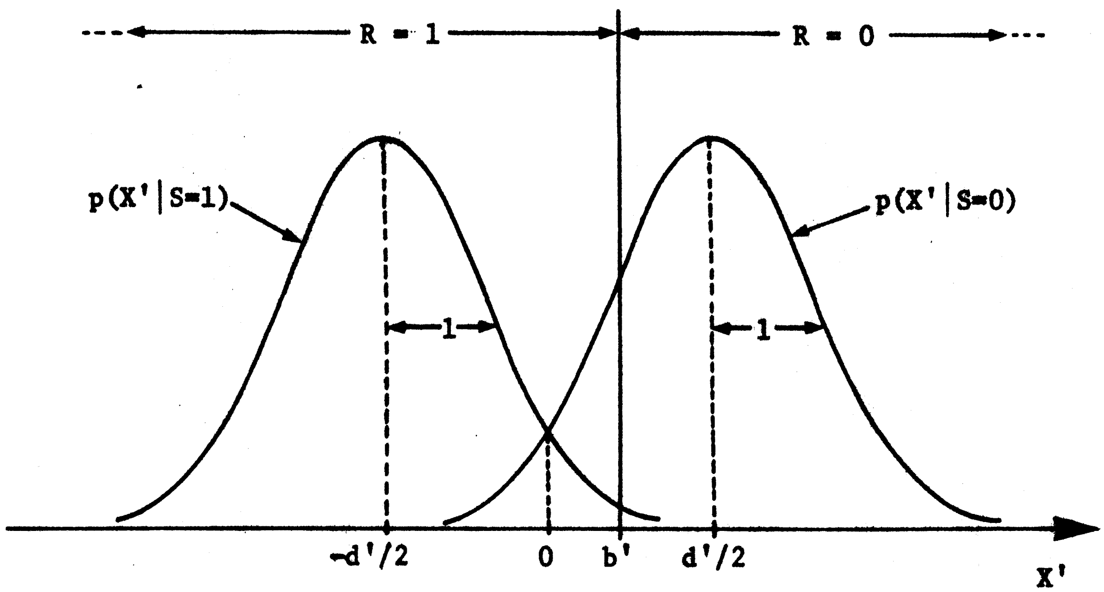
$$P(q_j,0,1) = N(b' - d_2'/2) \ . \tag{A1.9}$$

Substituting $d_2' = \delta_2'\, q_j$, and generalizing for S = 0 or 1, we obtain:

$$P(q_j,S,1) = N[b' + (-1^{S+1})\ \delta_2'\ q_j/2] \ . \tag{A1.10}$$

In addition, from Fig. A1.1, we observe that:

$$P(q_j,S,0) = 1 - P(q_j,S,1) \ . \tag{A1.11}$$



<u>Figure A1.1</u>

The Decision Axis
in Two-Interval Paradigms
Transformed to have Unit Variance

The derivatives of $P(q_j,S,R)$ are computed by first noting that:

$$\frac{\partial\ N(X)}{\partial\ X} = G(X)\ . \tag{A1.12}$$

Thus:

$$\frac{\partial\ P(q_j,S,1)}{\partial\ \delta_2'} = [(-1^{S+1})\ q_j/2]\ G[b' + (-1^{S+1})\ \delta_2'\ q_j/2]\ , \tag{A1.13}$$

and

$$\frac{\partial\ P(q_j,S,0)}{\partial\ \delta_2'} = \frac{-\partial\ P(q_j,S,1)}{\partial\ \delta_2'}\ . \tag{A1.14}$$

Generalizing for R = 0 or 1, we obtain:

$$\frac{\partial\ P(q_j,S,R)}{\partial\ \delta_2'} = [(-1^{S+R})\ q_j/2]\ G[b' + (-1^{S+1})\ \delta_2'\ q_j/2]\ . \tag{A1.15}$$

Similarly, we obtain:

$$\frac{\partial\ P(q_j,S,R)}{\partial\ b'} = (-1^{R+1})\ G[b' + (-1^{S+1})\ \delta_2'\ q_j/2]\ . \tag{A1.16}$$

Substituting these derivatives into the likelihood equations, and defining the auxiliary function $T(q_j,S,R)$:

$$T(q_j,S,R) = \frac{G[b' + (-1^{S+1})\ \delta_2'\ q_j/2]}{P(q_j,S,R)}\ , \tag{A1.17}$$

the likelihood equations become:

$$\frac{\partial \ln(L)}{\partial \delta_2'} = \sum_{q_j,S,R} (-1^{S+R}) \frac{q_j}{2} O(q_j,S,R)\ T(q_j,S,R) = 0 \ , \qquad (A1.18)$$

and

$$\frac{\partial \ln(L)}{\partial b'} = \sum_{q_j,S,R} (-1^{R+1}) O(q_j,S,R)\ T(q_j,S,R) = 0 \ . \qquad (A1.19)$$

The solution to these two non-linear likelihood equations may be obtained by using the Newton-Raphson iteration method, an extension of Newton's method to matrix equations. Specifically, the unknowns are defined as the vector $\underline{\Gamma}$:

$$\underline{\Gamma} = \begin{bmatrix} \delta_2' \\ \\ b' \end{bmatrix} \ . \qquad (A1.20)$$

Beginning with a vector of initial estimates, $\underline{\Gamma}_0$, successive improved estimates, $\underline{\Gamma}_i$, are obtained by

$$\underline{\Gamma}_i = \underline{\Gamma}_{i-1} - \underline{D}^{-1} \cdot \underline{B} \ , \qquad (A1.21)$$

where $\underline{B}$ is the vector of first derivatives:

$$\underline{B} = \begin{bmatrix} \dfrac{\partial \ln(L)}{\partial \delta_2'} \\ \\ \dfrac{\partial \ln(L)}{\partial b'} \end{bmatrix} \ , \qquad (A1.22)$$

and $\underline{D}$ is the matrix of second derivatives:

$$\underline{D} = \begin{bmatrix} \dfrac{\partial^2 \ln(L)}{\partial \delta_2'^2} & \dfrac{\partial^2 \ln(L)}{\partial \delta_2' \, \partial b'} \\ \\ \dfrac{\partial^2 \ln(L)}{\partial b' \, \partial \delta_2'} & \dfrac{\partial^2 \ln(L)}{\partial b'^2} \end{bmatrix} . \tag{A1.23}$$

To compute the second derivatives in $\underline{D}$, the derivatives of $T(q_j,S,R)$ must be obtained. Noting that:

$$\frac{\partial G(X)}{\partial X} = - XG(X) , \tag{A1.24}$$

and making use of the derivatives of $P(q_j,S,R)$ given in Eqs. A1.15 and A1.16, we find

$$\frac{\partial T(q_j,S,R)}{\partial \delta_2'} = (-1^{S+1}) \frac{q_j}{2} T(q_j,S,R) \; H(q_j,S,R) \quad , \tag{A1.25}$$

and

$$\frac{\partial T(q_j,S,R)}{\partial b'} = T(q_j,S,R) \; H(q_j,S,R) , \tag{A1.26}$$

where another auxiliary function $H(q_j,S,R)$ is defined:

$$H(q_j,S,R) = - b' + (-1^S) \; \delta_2' \; q_j/2 + (-1^R) \; T(q_j,S,R) \quad . \tag{A1.27}$$

Now, referring to the first derivatives of ln(L) given in Eqs. A1.18 and A1.19, and the derivatives of $T(q_j,S,R)$ just computed, the second derivatives of ln(L) are found:

$$\frac{\partial^2 \ln(L)}{\partial \delta_2'^2} = \sum_{q_j,S,R} (-1^{R+1}) \frac{q_j^2}{4} O(q_j,S,R)\ T(q_j,S,R)\ H(q_j,S,R)\ , \tag{A1.28}$$

$$\frac{\partial^2 \ln(L)}{\partial \delta_2' \partial b'} = \frac{\partial^2 \ln(L)}{\partial b' \partial \delta_2'}$$

$$= \sum_{q_j,S,R} (-1^{S+R}) \frac{q_j}{2} O(q_j,S,R)\ T(q_j,S,R)\ H(q_j,S,R)\ , \tag{A1.29}$$

and

$$\frac{\partial^2 \ln(L)}{\partial b'^2} = \sum_{q_j,S,R} (-1^{R+1})\ O(q_j,S,R)\ T(q_j,S,R)\ H(q_j,S,R)\ . \tag{A1.30}$$

Now, expressions for all the elements of $\underline{B}$ and $\underline{D}$ have been obtained, and since $\underline{D}$ is only a 2 x 2 matrix, D^{-1} is easily found. Thus, once an initial pair of estimates:

$$\underline{\Gamma}_0 = \begin{bmatrix} \delta_0' \\ \\ b_0' \end{bmatrix}\ , \tag{A1.31}$$

is chosen, equation A1.21 may be used to generate successive estimates.

The initial estimates used in analyzing the data in this thesis were obtained by the following simple averaging procedure. First, the fractions

$$F(q_j,S,1) = \frac{O(q_j,S,1)}{O(q_j,S,0) + O(q_j,S,1)} \tag{A1.32}$$

were calculated as estimates of $P(q_j,S,1)$. Then for each value of q, initial estimates $\delta_0'(q)$ and $b_0'(q)$ were obtained according to: (see Fig. A1.1)

$$\delta_0'(q_j) = d_0'(q_j)/q_j = \frac{N^{-1}[F(q_j,1,1)] - N^{-1}[F(q_j,0,1)]}{q}, \tag{A1.33}$$

and

$$b_0'(q_j) = \frac{N^{-1}[F(q_j,1,1)] + N^{-1}[F(q_j,0,1)]}{2} . \tag{A1.34}$$

Finally, the initial estimates δ_0' and b_0' were found by averaging over q_j:

$$\delta_0' = \underset{q_j}{\mathrm{AV}}\ \delta_0'(q_j) \tag{A1.35}$$

and

$$b_0' = \underset{q_j}{\mathrm{AV}}\ b_0'(q_j) . \tag{A1.36}$$

It was found that when these initial estimates were used, and Eq. A1.21 was used to generate successive estimates, the solution to the likelihood equations (i.e., the maximum likelihood estimates) converged after no more than four or five iterations.

B. Estimation of Resolution and Bias in Identification

The estimates of resolution and bias for the identification experiments are based on a single 13 x 13 confusion matrix. The thirteen stimuli are denoted $\{S_i\}$, the thirteen responses are denoted $\{R_j\}$, the confusion matrix is denoted $\underline{O}$, and a particular element of the matrix is denoted $O_{i,j}$.

In this estimation procedure, the fractions

$$F_{i,j} = \frac{O_{i,j}}{\sum_{j=1}^{13} O_{i,j}} \tag{A1.37}$$

are computed first. Then a cumulative fraction matrix is computed with the elements:

$$CF_{i,j} = \sum_{k=1}^{j} F_{i,k} \quad , \tag{A1.38}$$

and a Z-score matrix is computed with the elements:

$$Z_{i,j} = N^{-1}(CF_{i,j}) \quad . \tag{A1.39}$$

Since the $CF_{i,13}$ are equal to one, the $Z_{i,13}$ need not be computed, and the dimensions of the Z-score matrix are 13 x 12.

The estimates of d_i' and b_j' are then calculated according to the following equations:

$$\hat{d}_i'(I_{i+1};\, I_i) = \hat{d}_i' = \underset{j}{AV}\,(Z_{i,j} - Z_{i+1,j}) \qquad i = 1, 2, \ldots, 12 \tag{A1.40}$$

and (defining $\hat{d}_0' = 0$),

$$\hat{b}_j' = \underset{i}{AV}\left(Z_{i,j} + \sum_{k=0}^{i-1} \hat{d}_k' - \sum_{k=0}^{j-1} \hat{d}_k' - d_j'/2\right). \tag{A1.41}$$

$$j = 1, 2, \ldots, 12$$

The average in these equations is over all $Z_{i,j}$ resulting from $CF_{i,j}$ between 0.05 and 0.95. In the few cases where no such ($CF_{i,j}$; $CF_{i+1,j}$) pair existed, the region of permissible $CF_{i,j}$ was increased enough to include one ($CF_{i,j}$; $CF_{i+1,j}$) pair.

C. Weber's Law Normalization

In this thesis, as in many previous studies of intensity resolution, a departure from Weber's law has been noted; resolution generally increased with increasing intensity level. This tendency has been examined previously in some detail [e.g., Rabinowitz (1970) and Viemeister (1972)], and was not of much interest in this thesis. In fact, a different type of variation in resolution as a function of level, the resolution edge-effect, was of major interest. Consequently, we sought to factor out from the results of this thesis the variation in resolution due to the deviation from Weber's law.

We chose to characterize the deviation from Weber's law by assuming that in two-interval paradigms:

$$\delta_2'(I) = \frac{2Kg(I)}{\sigma_2} \quad , \tag{A1.42}$$

and in one-interval paradigms:

$$\delta_1'(I) = \frac{Kg(I)}{\sigma_1} \quad , \tag{A1.43}$$

where $\delta'(I)$ is the resolution per bel at level I, and g(I) represents the deviation from Weber's law. If, as assumed in the preliminary theory, Weber's law were valid, then g(I) would be identically unity, and Eqs. A1.42 and A1.43 would be merely restatements of Eqs. 4.10 and 4.12.

This characterization was considered appropriate (rather than making either the sensation noise, or the memory noise, alone functions of I) because the deviation in Weber's law has been noted in experiments in which the memory load is large (e.g., one-interval, large-range, identification), as well as in experiments in which there is virtually no memory load (e.g., two-interval, fixed-level, discrimination with very short T).

Because we do not yet have a satisfactory technique for separating g(I) from the resolution edge-effect in a given experiment, we estimated g(I) from the fixed-level discrimination experiments, in which there could be no resolution edge-effect. In fact, these experiments were conducted at a variety of levels primarily for this purpose. Specifically, for each observer, g(I) was estimated according to:

$$\hat{g}(I) = \hat{\delta}_{f1}'(I)/\hat{\delta}_{f1}'(I = 63 \text{ dB SPL}) \quad , \tag{A1.44}$$

where $\hat{\delta}_{f1}'(I)$ is the resolution per bel obtained at level I in the fixed-level discrimination experiment with T = 0.20 sec, T_p = 0.50 sec, and no feedback. Values of $\hat{g}(I)$ for intermediate values of I were obtained by linearly interpolating $\hat{g}(I)$ between the values of I actually tested.

Finally, normalized estimates of resolution per bel, $\hat{\delta}'_{norm}(I)$, for both the one- and two-interval paradigms, were calculated from the original estimates, $\hat{\delta}'_{orig}(I)$, according to:

$$\hat{\delta}'_{norm}(I) = \hat{\delta}'_{orig}(I)/\hat{g}(I) \quad . \tag{A1.45}$$

APPENDIX II. TABLES OF RESULTS AND SELECTED GRAPHS FOR INDIVIDUAL OBSERVERS

A. Tables of Results

Estimates of resolution per bel, $\hat{\delta}'_2$, and bias, $\hat{b}'$, for individual observers in the discrimination experiments are listed in Tables A2.1-A2.11. Tables A2.1-A2.5 contain results from Expts. 1 and 1A, Tables A2.6-A2.8 contain results from Expts. 2 and 2A, and Tables A2.9-A2.11 contain results from Expts. 3 and 3A. Estimates of resolution, $\hat{d}'_1(I_{i+1};\ I_i)$, and bias, $\hat{b}'_i$, for individual observers in the identification experiments are listed in Tables A2.12-A2.21. Tables A2.12-A2.15 contain results from Expt. 4, and Tables A2.16-A2.21 contain results from Expt. 5. In all cases, the values of $\hat{\delta}'_2$ and $\hat{d}'_1(I_{i+1};\ I_i)$ listed in the tables have not undergone the Weber's law normalization procedure.

B. Selected Graphs

Graphs which summarize the results of individual observers in the discrimination experiments are presented in Figs. A2.1-A2.7. Figures A2.1-A2.4 contain results from Expts. 1, 1A, 2, and 2A, and Figs. A2.5-A2.7 contain results from Expts. 3 and 3A. (Note that graphs of the results of Observer JB from Expts. 1, 1A, 2, 2A, and 4 are not included in this appendix, since they have been presented in Chapter VII.) These figures contain graphs of resolution per bel averaged over levels, $\bar{\hat{\delta}}'_2$, as well as measures of the resolution and bias edge-effects, $\nabla\hat{\delta}'_2$ and $\nabla\hat{C}/K$ respectively.

Graphs presenting certain results of individual observers in the identification experiments are presented in Figs. A2.8-A2.10. Figures A2.8-A2.9 contain graphs of total resolution, Δ', and resolution per bel averaged over levels, $\overline{\hat{\delta}'_1}$, as well as graphs of the measure of the resolution edge-effect, $\nabla\hat{\delta}'_1$, from Expt. 4. Figure A2.10 is a graph of the criterion shifts, $\hat{CS}_i/K$, for all six observers from Expt. 5.

TABLE A2.1

$\hat{\delta}'_2$ and $\hat{b}'$ for Observer JB in Expts. 1 & 1A

R (Bels)	T (sec)		I (dB SPL)				
			39	51	63	75	87
5.8	0.2	δ'	8.712	9.412	11.55	15.13	19.95
		b'	-0.254	-0.200	-0.180	-0.268	-0.292
	1.8	δ'	4.181	3.129	5.129	7.887	10.30
		b'	-0.411	-0.306	-0.135	-0.002	-0.121
	3.5	δ'	3.007	2.749	3.070	5.718	10.27
		b'	-0.513	-0.379	-0.204	0.075	0.315
	7.0	δ'	3.248	2.367	2.694	4.889	8.400
		b'	-0.489	-0.291	-0.067	-0.087	-0.046
	14.0	δ'	2.842	1.522	2.258	3.078	5.802
		b'	-0.338	-0.214	-0.011	-0.025	-0.023

R (Bels)	T (sec)		I (dB SPL)				
			47	55	63	71	79
3.9	0.2	δ'	8.644	9.327	12.90	18.08	21.33
		b'	-0.323	-0.172	-0.159	-0.292	-0.622
	3.5	δ'	2.883	3.149	4.420	8.149	9.085
		b'	-0.632	-0.377	-0.049	0.023	0.218
	14.0	δ'	2.282	2.161	3.067	4.364	6.249
		b'	-0.481	-0.214	0.038	0.159	0.210

R (Bels)	T (sec)		I (dB SPL)				
			55	59	63	67	71
2.0	0.2	δ'	10.12	13.61	13.32	16.17	20.47
		b'	0.001	0.035	-0.002	-0.286	-0.578
	3.5	δ'	4.320	5.847	6.870	8.779	12.04
		b'	-0.893	-0.486	-0.298	0.190	0.443
	14.0	δ'	3.264	4.152	5.191	6.263	7.615
		b'	-0.605	-0.366	-0.186	0.200	0.530

R (Bels)	T (sec)		I (dB SPL)				
			39	51	63	75	87
0.2	0.2	δ'	11.49	13.61	16.84	22.43	26.81
		b'	-0.222	-0.238	-0.221	-0.220	-0.100
	3.5	δ'			12.22		
		b'			-0.061		
	14.0	δ'			7.740		
		b'			-0.045		

TABLE A2.2

$\hat{\delta}_2'$ and $\hat{b}'$ for Observer KG in Expts. 1 & 1A

R (Bels)	T (sec)		I (dB SPL) 39	51	63	75	87
5.8	0.2	δ'	4.753	4.517	5.979	6.602	7.892
		b'	-0.100	-0.154	-0.144	0.090	-0.115
	1.8	δ'	2.374	2.130	2.636	3.846	5.332
		b'	-0.607	-0.455	-0.175	-0.056	0.184
	3.5	δ'	1.525	0.988	1.812	3.092	4.709
		b'	-0.536	-0.361	-0.192	-0.003	0.444
	7.0	δ'	1.764	0.853	1.352	2.408	3.439
		b'	-0.512	-0.321	-0.005	0.122	0.578
	14.0	δ'	1.941	0.676	1.068	2.039	2.849
		b'	-0.577	-0.210	-0.113	0.144	0.719

R (Bels)	T (sec)		I (dB SPL) 47	55	63	71	79
3.9	0.2	δ'	4.867	3.927	6.137	9.555	9.085
		b'	0.044	-0.199	-0.179	-0.205	-0.048
	3.5	δ'	1.664	1.532	2.453	3.872	4.867
		b'	-0.487	-0.198	-0.073	0.015	0.356
	14.0	δ'	1.776	1.238	1.410	2.049	3.061
		b'	-0.532	-0.275	0.117	0.225	0.790

R (Bels)	T (sec)		I (dB SPL) 55	59	63	67	71
2.0	0.2	δ'	5.609	7.192	9.250	12.48	10.06
		b'	0.152	-0.101	-0.153	-0.237	-0.244
	3.5	δ'	2.120	2.696	3.981	4.359	3.695
		b'	-0.440	-0.336	-0.181	0.078	0.296
	14.0	δ'	2.082	1.775	2.159	1.662	2.135
		b'	-0.438	-0.110	0.133	0.329	0.629

R (Bels)	T (sec)		I (dB SPL) 39	51	63	75	87
0.2	0.2	δ'	5.964	6.763	10.55	9.802	8.720
		b'	-0.179	-0.127	-0.012	0.086	0.337
	3.5	δ'			4.845		
		b'			-0.135		
	14.0	δ'			2.966		
		b'			0.151		

TABLE A2.3

$\hat{\delta}'_2$ and $\hat{b}'$ for Observer PN in Expts. 1 & 1A

R (Bels)	T (sec)		I (dB SPL) 39	51	63	75	87
5.8	0.2	δ'	4.165	5.167	6.649	8.502	10.37
		b'	-0.909	-0.824	-0.487	0.307	0.975
	1.8	δ'	2.147	2.068	3.252	4.497	6.072
		b'	-1.354	-0.800	-0.122	0.571	1.177
	3.5	δ'	1.889	1.333	2.188	3.272	4.127
		b'	-1.151	-0.542	0.008	0.457	1.171
	7.0	δ'	2.036	1.314	1.776	2.113	3.243
		b'	-1.294	-0.489	0.066	0.653	1.212
	14.0	δ'	1.491	1.282	1.377	1.946	3.101
		b'	-1.011	-0.502	0.141	0.835	1.596

R (Bels)	T (sec)		I (dB SPL) 47	55	63	71	79
3.9	0.2	δ'	3.302	5.300	7.466	10.70	8.784
		b'	-0.907	-0.693	-0.259	0.217	0.697
	3.5	δ'	1.726	1.713	3.074	3.300	4.885
		b'	-1.067	-0.623	-0.014	0.563	1.182
	14.0	δ'	1.277	1.492	1.804	1.684	2.417
		b'	-0.906	-0.436	0.150	0.818	1.381

R (Bels)	T (sec)		I (dB SPL) 55	59	63	67	71
2.0	0.2	δ'	6.088	9.745	11.97	12.21	10.59
		b'	-0.408	-0.266	-0.029	0.259	0.731
	3.5	δ'	3.310	4.261	4.348	4.645	5.411
		b'	-0.954	-0.467	0.287	0.726	1.177
	14.0	δ'	2.804	2.804	2.840	2.584	1.938
		b'	-0.738	0.015	0.558	0.934	1.373

R (Bels)	T (sec)		I (dB SPL) 39	51	63	75	87
0.2	0.2	δ'	6.073	6.825	11.42	8.881	7.940
		b'	-0.178	-0.094	0.151	0.298	0.637
	3.5	δ'			5.469		
		b'			0.184		
	14.0	δ'			2.538		
		b'			0.487		

TABLE A2.4

$\hat{\delta}'_2$ and $\hat{b}'$ for Observer PS in Expts. 1 & 1A

R (Bels)	T (sec)		I (dB SPL) 39	51	63	75	87
5.8	0.2	δ'	2.674	6.461	7.185	8.663	12.36
		b'	-1.355	-0.585	0.303	0.403	0.457
	1.8	δ'	1.865	2.223	4.307	3.174	7.874
		b'	-1.740	-0.897	0.036	0.488	0.634
	3.5	δ'	1.955	0.960	2.498	1.867	4.432
		b'	-1.641	-0.762	0.055	0.664	0.711
	7.0	δ'	0.825	3.510	2.790	1.985	3.091
		b'	-1.475	-0.813	-0.112	0.448	0.814
	14.0	δ'	1.788	1.756	1.360	0.668	4.055
		b'	-1.824	-0.846	0.001	0.515	0.988
R (Bels)	**T (sec)**		**I (dB SPL) 47**	**55**	**63**	**71**	**79**
3.9	0.2	δ'	5.787	6.660	7.713	12.60	14.23
		b'	-1.271	-0.557	0.176	0.321	0.425
	3.5	δ'	1.190	2.394	2.557	4.924	4.319
		b'	-1.369	-0.506	0.040	0.336	0.910
	14.0	δ'	1.105	1.039	1.724	1.972	2.967
		b'	-1.444	-0.772	-0.039	0.307	0.941
R (Bels)	**T (sec)**		**I (dB SPL) 55**	**59**	**63**	**67**	**71**
2.0	0.2	δ'	5.237	7.300	8.871	10.72	13.30
		b'	-0.778	-0.484	-0.101	0.169	0.514
	3.5	δ'	1.806	3.469	4.096	5.754	5.594
		b'	-0.859	-0.621	-0.320	0.269	0.780
	14.0	δ'	0.841	1.591	3.556	3.599	4.600
		b'	-1.175	-0.797	-0.335	0.390	0.991
R (Bels)	**T (sec)**		**I (dB SPL) 39**	**51**	**63**	**75**	**87**
0.2	0.2	δ'	4.802	5.746	11.46	12.24	13.91
		b'	-0.341	-0.229	-0.204	-0.126	-0.081
	3.5	δ'			5.395		
		b'			-0.064		
	14.0	δ'			6.573		
		b'			-0.098		

TABLE A2.5

$\hat{\delta}_2'$ and $\hat{b}'$ for Observer BD in Expts. 1 & 1A

R (Bels)	T (sec)		I (dB SPL) 39	51	63	75	87
5.8	0.2	δ'	5.635	6.638	12.67	14.02	11.27
		b'	-0.060	0.269	0.305	0.255	0.087
	1.8	δ'	0.853	2.173	3.657	6.733	5.415
		b'	-0.805	-0.564	0.083	-0.024	-0.151
	3.5	δ'	1.280	2.408	0.096	3.056	11.57
		b'	-1.106	-0.513	-0.258	-0.034	-0.236
	7.0	δ'	2.385	1.370	1.903	4.097	5.714
		b'	-0.989	-0.747	0.218	0.434	0.373
	14.0	δ'	0.504	-0.616	1.766	3.476	5.916
		b'	-0.966	-0.591	-0.298	0.252	0.887

T (Bels)	T (sec)		I (dB SPL) 47	55	63	71	79
3.9	0.2	δ'	5.884	9.089	11.11	15.08	15.78
		b'	0.048	0.633	0.272	0.117	-0.173
	3.5	δ'	1.574	2.947	4.849	4.682	6.711
		b'	-1.075	-0.709	0.115	0.370	0.617
	14.0	δ'	0.065	1.972	3.570	2.727	4.033
		b'	-0.818	-0.575	-0.040	0.488	0.915

R (Bels)	T (sec)		I (dB SPL) 55	59	63	67	71
2.0	0.2	δ'	9.765	8.923	15.40	22.26	18.19
		b'	0.060	0.482	0.275	-0.072	-0.176
	3.5	δ'	2.739	6.372	4.487	9.191	7.006
		b'	-0.715	-0.605	-0.123	0.181	0.387
	14.0	δ'	3.943	3.842	2.304	3.714	7.743
		b'	-1.428	-0.310	0.346	0.304	0.833

R (Bels)	T (sec)		I (dB SPL) 39	51	63	75	87
0.2	0.2	δ'	5.933	7.231	19.39	21.67	25.05
		b'	0.512	0.342	0.436	0.374	0.531
	3.5	δ'					
		b'					
	14.0	δ'					
		b'					

TABLE A2.6

$\hat{\delta}'_2$ and $\hat{b}'$ for Observer JB in Expts. 2 & 2A

R (Bels)	T (sec)		I (dB SPL) 39	51	63	75	87
5.8	0.2	δ'	8.901	11.44	10.16	16.81	17.84
		b'	0.015	-0.037	0.066	-0.050	-0.118
	3.5	δ'	2.929	2.003	2.501	4.650	7.634
		b'	-0.060	-0.001	0.045	0.055	-0.025
	14.0	δ'	2.393	1.966	2.137	3.189	4.756
		b'	0.200	0.100	0.008	-0.079	-0.149

R (Bels)	T (sec)		I (dB SPL) 55	59	63	67	71
2.0	0.2	δ'	12.81	13.80	17.22	22.58	18.27
		b'	0.081	0.157	-0.016	-0.046	-0.254
	3.5	δ'	4.460	4.711	6.008	7.459	8.952
		b'	-0.238	-0.178	0.096	0.106	0.322
	14.0	δ'	3.902	3.740	4.473	5.195	5.799
		b'	0.066	0.052	0.053	-0.060	-0.012

R (Bels)	T (sec)				I (dB SPL) 63		
0.2	0.2	δ'			19.39		
		b'			-0.092		
	3.5	δ'			13.10		
		b'			0.016		
	14.0	δ'			8.617		
		b'			-0.118		

TABLE A2.7

$\hat{\delta}'_2$ and $\hat{b}'$ for Observer KG in Expts. 2 & 2A

R (Bels)	T (sec)		I (dB SPL) 39	51	63	75	87
5.8	0.2	δ'	6.187	6.137	8.282	10.37	9.471
		b'	0.271	0.120	0.182	-0.031	-0.290
	3.5	δ'	2.107	1.699	2.511	4.006	5.048
		b'	-0.114	-0.029	0.023	0.236	0.187
	14.0	δ'	1.497	1.111	1.447	2.684	2.763
		b'	-0.140	-0.105	0.170	0.332	0.271

R (Bels)	T (sec)		I (dB SPL) 55	59	63	67	71
2.0	0.2	δ'	9.120	9.272	10.70	11.33	10.83
		b'	0.571	0.444	0.088	-0.157	-0.379
	3.5	δ'	2.924	4.060	5.286	5.889	4.890
		b'	-0.275	-0.091	0.114	0.138	0.390
	14.0	δ'	1.989	2.376	2.616	3.391	2.790
		b'	-0.166	-0.020	0.168	0.337	0.286

R (Bels)	T (sec)				I (dB SPL) 63		
0.2	0.2	δ'			16.02		
		b'			-0.124		
	3.5	δ'			6.986		
		b'			0.062		
	14.0	δ'			3.160		
		b'			0.126		

TABLE A2.8

$\hat{\delta}'_2$ and $\hat{b}'$ for Observer PN in Expts. 2 & 2A

R (Bels)	T (sec)		I (dB SPL) 39	51	63	75	87
5.8	0.2	δ'	3.104	3.349	6.464	8.999	11.05
		b'	-0.000	-0.249	-0.199	0.153	0.158
	3.5	δ'	1.970	1.489	2.476	3.303	4.580
		b'	-0.189	-0.058	-0.122	0.134	0.164
	14.0	δ'	1.166	1.062	1.044	1.894	2.214
		b'	0.023	0.205	0.238	0.400	0.428

R (Bels)	T (sec)		I (dB SPL) 55	59	63	67	71
2.0	0.2	δ'	7.445	10.03	13.08	16.08	14.00
		b'	-0.046	-0.242	-0.132	-0.023	0.030
	3.5	δ'	2.640	3.613	3.601	4.752	4.300
		b'	-0.310	-0.172	-0.105	0.085	0.221
	14.0	δ'	1.562	1.982	1.714	2.434	2.366
		b'	0.179	0.033	0.247	0.401	0.421

R (Bels)	T (sec)				I (dB SPL) 63		
0.2	0.2	δ'			15.58		
		b'			-0.129		
	3.5	δ'			6.870		
		b'			0.148		
	14.0	δ'			3.317		
		b'			0.585		

TABLE A2.9a

$\hat{\delta}_2'$ and $\hat{b}'$ for Observer JB in Expts. 3 & 3A

T_p = 0.20 sec

R (Bels)	T (sec)		I (dB SPL) 39	51	63	75	87
5.6	0.0	δ'	7.404	9.626	8.573	12.37	12.50
		b'	0.165	0.396	0.642	0.608	0.500
	0.2	δ'	8.886	8.770	11.86	14.13	20.39
		b'	-0.557	-0.245	-0.131	-0.271	-0.611
	3.5	δ'	2.536	2.563	1.578	3.998	14.74
		b'	-0.939	-0.615	-0.062	0.037	0.491
	14.0	δ'	1.641	0.851	2.302	3.177	7.400
		b'	-0.907	-0.404	-0.438	-0.082	0.217

R (Bels)	T (sec)		I (dB SPL) 51	57	63	69	75
2.9	0.0	δ'	9.634	9.154	9.468	12.26	15.33
		b'	0.116	0.245	0.437	0.520	0.709
	0.2	δ'	9.443	10.00	18.30	15.15	17.50
		b'	-0.739	-0.332	-0.693	-0.405	-0.575
	3.5	δ'	2.962	4.766	5.708	7.730	10.27
		b'	-1.458	-0.861	-0.498	0.016	0.496
	14.0	δ'	5.392	3.505	3.558	4.085	4.432
		b'	-1.460	-0.842	-0.255	0.102	0.384

R (Bels)	T (sec)		I (dB SPL) 36	54	63	72	90
0.1	0.0	δ'			11.19		
		b'			0.386		
	0.2	δ'	10.87	13.97		20.40	21.11
		b'	-0.468	-0.461		-0.379	-0.613
	3.5	δ'			9.560		
		b'			-0.191		
	14.0	δ'			8.443		
		b'			-0.075		

TABLE A2.9b

$\hat{\delta}'_2$ and $\hat{b}'$ for Observer JB in Expts. 3 & 3A

T_p = 0.50 sec

R (Bels)	T (sec)		I (dB SPL) 39	51	63	75	87
5.6	0.0	δ'	13.32	16.30	18.76	26.18	31.14
		b'	-0.547	-0.105	-0.122	-0.016	-0.153
	0.2	δ'	9.136	11.22	11.73	17.91	24.53
		b'	-0.833	-0.889	-0.423	-0.411	-0.370
	3.5	δ'	1.962	2.076	3.591	6.881	9.883
		b'	-0.781	-0.626	-0.527	-0.031	0.189
	14.0	δ'	2.802	2.130	2.154	3.662	6.810
		b'	-0.761	-0.668	-0.360	-0.283	0.080

R (Bels)	T (sec)		I (dB SPL) 51	57	63	69	75
2.9	0.0	δ'	17.08	16.52	19.10	26.19	30.42
		b'	-0.253	-0.140	-0.349	-0.316	0.068
	0.2	δ'	9.972	12.44	19.68	19.14	25.40
		b'	-0.951	-0.756	-0.614	-0.410	-0.485
	3.5	δ'	2.928	4.262	5.464	7.297	10.85
		b'	-1.192	-0.435	-0.208	-0.042	0.758
	14.0	δ'	2.680	2.970	4.147	6.029	6.481
		b'	-0.909	-0.701	-0.281	0.100	0.428

R (Bels)	T (sec)		I (dB SPL) 36	54	63	72	90
0.1	0.0	δ'			26.12		
		b'			-0.086		
	0.2	δ'	13.69	14.23		23.97	29.87
		b'	-0.435	-0.373		-0.508	-0.473
	3.5	δ'			10.56		
		b'			-0.072		
	14.0	δ'			10.21		
		b'			-0.191		

TABLE A2.9c

$\hat{\delta}'_2$ and $\hat{b}'$ for Observer JB in Expts. 3 & 3A

$T_p = 1.25$ sec

R (Bels)	T (sec)		I (dB SPL) 39	51	63	75	87
5.6	0.0	δ'	19.91	22.76	30.25	36.42	39.52
		b'	-1.220	-1.015	-0.635	-0.606	-0.200
	0.2	δ'	14.17	14.28	18.91	26.80	33.27
		b'	-0.836	-0.809	-0.587	-0.545	-0.405
	3.5	δ'	2.996	2.306	3.752	7.913	13.46
		b'	-0.538	-0.362	-0.128	-0.006	0.092
	14.0	δ'	3.423	2.042	2.545	3.755	8.084
		b'	-0.421	-0.410	-0.193	-0.193	0.068

R (Bels)	T (sec)		I (dB SPL) 51	57	63	69	75
2.9	0.0	δ'	20.01	30.53	22.88	32.66	30.64
		b'	-0.765	-0.926	-0.533	-0.446	-0.290
	0.2	δ'	13.82	17.47	19.41	28.88	24.30
		b'	-0.690	-0.632	-0.444	-0.609	-0.469
	3.5	δ'	5.056	6.458	9.609	9.316	11.49
		b'	-1.084	-0.540	-0.231	-0.053	0.444
	14.0	δ'	4.277	4.678	5.690	8.036	7.109
		b'	-0.877	-0.297	-0.046	-0.019	0.373

R (Bels)	T (sec)		I (dB SPL) 36	54	63	72	90
0.1	0.0	δ'			28.30		
		b'			-0.722		
	0.2	δ'	15.35	17.13		28.90	32.74
		b'	-0.585	-0.447		-0.370	-0.262
	3.5	δ'			15.77		
		b'			-0.005		
	14.0	δ'			10.39		
		b'			-0.223		

TABLE A2.10a

$\hat{\delta}'_2$ and $\hat{b}'$ for Observer JW in Expts. 3 & 3A

T_p = 0.20 sec

R (Bels)	T (sec)		I (dB SPL) 39	51	63	75	87
5.6	0.0	δ'	3.033	4.120	2.516	4.329	5.134
		b'	-0.909	-0.424	0.118	0.403	0.397
	0.2	δ'	4.200	6.540	7.907	7.001	12.70
		b'	-0.717	-0.094	-0.064	0.165	-0.083
	3.5	δ'	1.895	3.553	3.971	5.667	5.163
		b'	-0.796	0.133	0.428	0.148	0.278
	14.0	δ'	2.414	1.737	2.501	3.464	5.586
		b'	-0.682	-0.134	0.319	0.039	0.449

R (Bels)	T (sec)		I (dB SPL) 51	57	63	69	75
2.9	0.0	δ'	3.506	6.958	5.953	10.28	9.768
		b'	-0.903	-0.427	0.117	0.531	0.632
	0.2	δ'	5.378	5.812	11.73	17.35	14.17
		b'	-0.542	-0.336	-0.196	-0.207	-0.193
	3.5	δ'	3.331	7.118	7.851	7.914	6.939
		b'	-0.560	-0.324	-0.076	0.430	0.483
	14.0	δ'	2.467	4.785	5.541	3.136	4.337
		b'	-0.828	-0.372	-0.092	0.092	0.498

R (Bels)	T (sec)		I (dB SPL) 36	54	63	72	90
0.1	0.0	δ'			10.48		
		b'			0.065		
	0.2	δ'	9.489	9.904		13.93	18.87
		b'	-0.316	-0.309		-0.413	-0.189
	3.5	δ'			8.077		
		b'			0.110		
	14.0	δ'			6.607		
		b'			0.040		

TABLE A2.10b

$\hat{\delta}'_2$ and $\hat{b}'$ for Observer JW in Expts. 3 & 3A

T_p = 0.50 sec

R (Bels)	T (sec)		I (dB SPL) 39	51	63	75	87
5.6	0.0	δ'	11.90	20.00	22.95	31.99	27.48
		b'	-1.106	-0.831	-0.349	0.121	0.034
	0.2	δ'	7.352	9.105	15.64	16.42	19.66
		b'	-0.984	-0.754	-0.523	-0.551	-0.473
	3.5	δ'	3.837	4.301	6.083	6.490	7.817
		b'	-0.922	-0.468	0.153	-0.124	-0.299
	14.0	δ'	1.697	2.129	3.664	4.487	6.277
		b'	-0.490	-0.122	0.027	-0.124	0.182

R (Bels)	T (sec)		I (dB SPL) 51	57	63	69	75
2.9	0.0	δ'	18.89	19.61	30.29	38.29	30.43
		b'	-0.953	-0.704	-0.364	0.021	0.121
	0.2	δ'	9.577	11.28	18.94	19.47	14.96
		b'	-0.684	-0.448	-0.460	-0.395	-0.324
	3.5	δ'	3.303	5.944	7.653	6.618	7.596
		b'	-0.680	-0.354	-0.021	0.009	0.327
	14.0	δ'	1.412	2.749	4.570	5.373	5.019
		b'	-0.855	-0.456	-0.026	0.196	0.734

R (Bels)	T (sec)		I (dB SPL) 36	54	63	72	90
0.1	0.0	δ'			27.05		
		b'			-0.095		
	0.2	δ'	9.756	8.213		23.28	27.90
		b'	-0.202	-0.093		-0.426	-0.187
	3.5	δ'			9.968		
		b'			-0.026		
	14.0	δ'			7.071		
		b'			-0.093		

TABLE A2.10c

$\hat{\delta}_2'$ and $\hat{b}'$ for Observer JW in Expts. 3 & 3A

T_p = 1.25 sec

R (Bels)	T (sec)		I (dB SPL) 39	51	63	75	87
5.6	0.0	δ'	17.48	25.98	44.03	43.98	53.50
		b'	-1.330	-1.050	-1.171	-0.487	0.707
	0.2	δ'	8.943	10.59	17.86	21.21	23.99
		b'	-1.691	-1.137	-1.228	-1.057	-0.608
	3.5	δ'	4.436	4.252	4.556	7.175	10.61
		b'	-0.624	-0.337	-0.292	-0.612	-0.186
	14.0	δ'	3.028	2.649	2.912	3.919	6.613
		b'	-0.389	-0.251	0.001	-0.145	0.033

R (Bels)	T (sec)		I (dB SPL) 51	57	63	69	75
2.9	0.0	δ'	28.69	30.60	38.06	50.91	44.89
		b'	-1.681	-1.178	-0.962	-0.566	-0.044
	0.2	δ'	10.89	12.12	19.67	18.55	19.19
		b'	-1.337	-0.778	-1.326	-0.977	-0.621
	3.5	δ'	6.238	7.584	9.386	9.273	8.159
		b'	-0.916	-0.557	-0.544	-0.216	-0.075
	14.0	δ'	4.188	4.083	5.356	5.462	6.149
		b'	-0.873	-0.547	-0.384	0.005	0.418

R (Bels)	T (sec)		I (dB SPL) 36	54	63	72	90
0.1	0.0	δ'			51.94		
		b'			-0.902		
	0.2	δ'	10.74	14.52		30.68	39.20
		b'	-0.708	-0.233		-0.779	0.155
	3.5	δ'			13.06		
		b'			-0.073		
	14.0	δ'			7.856		
		b'			-0.044		

TABLE A2.11a

$\hat{\delta}_2'$ and $\hat{b}'$ for Observer SK in Expts. 3 & 3A

$T_p = 0.20$ sec

R (Bels)	T (sec)		I (dB SPL) 39	51	63	75	87
5.6	0.0	δ'	8.024	9.812	10.75	19.24	26.17
		b'	-0.821	-0.579	-0.509	0.123	0.093
	0.2	δ'	4.880	5.097	9.352	12.00	22.16
		b'	-0.138	-0.111	-0.071	-0.185	-0.255
	3.5	δ'	2.427	1.773	2.891	5.452	9.468
		b'	-0.563	-0.347	-0.068	0.001	0.087
	14.0	δ'	1.329	0.694	1.981	1.840	5.588
		b'	-0.817	-0.392	-0.032	0.245	0.270

R (Bels)	T (sec)		I (dB SPL) 51	57	63	69	75
2.9	0.0	δ'	7.855	9.683	16.93	24.95	29.12
		b'	-0.660	-0.630	-0.268	-0.161	-0.011
	0.2	δ'	6.915	7.761	13.81	16.21	21.90
		b'	-0.203	0.023	-0.038	-0.150	-0.507
	3.5	δ'	2.243	4.922	4.875	8.040	12.52
		b'	-0.655	-0.479	-0.165	0.422	1.084
	14.0	δ'	1.732	2.918	3.426	3.971	3.415
		b'	-0.835	-0.402	-0.035	0.323	1.109

R (Bels)	T (sec)		I (dB SPL) 36	54	63	72	90
0.1	0.0	δ'			20.06		
		b'			-0.208		
	0.2	δ'	9.455	12.70		19.66	28.59
		b'	-0.069	0.258		-0.198	-0.372
	3.5	δ'			9.297		
		b'			0.479		
	14.0	δ'			6.674		
		b'			0.331		

TABLE A2.11b

$\hat{\delta}_2'$ and $\hat{b}'$ for Observer SK in Expts. 3 & 3A

T_p = 0.50 sec

R (Bels)	T (sec)		I (dB SPL) 39	51	63	75	87
5.6	0.0	δ'	12.13	17.60	19.75	30.99	38.61
		b'	-0.244	0.342	0.103	0.369	0.415
	0.2	δ'	6.847	7.886	11.19	15.83	20.95
		b'	0.120	0.267	0.102	-0.116	-0.071
	3.5	δ'	4.710	2.748	3.117	4.741	7.042
		b'	-0.631	-0.265	0.021	0.019	0.324
	14.0	δ'	1.194	1.967	1.159	2.405	4.764
		b'	-0.885	-0.403	-0.009	0.279	0.501

R (Bels)	T (sec)		I (dB SPL) 51	57	63	69	75
2.9	0.0	δ'	16.23	15.81	24.07	34.21	36.97
		b'	0.351	0.334	0.227	0.052	0.321
	0.2	δ'	10.99	14.92	20.44	21.55	22.50
		b'	0.175	0.313	0.183	0.233	0.203
	3.5	δ'	5.611	5.097	7.386	7.894	10.64
		b'	-0.534	-0.234	0.040	0.654	1.362
	14.0	δ'	2.152	2.304	2.252	3.197	3.287
		b'	-0.691	-0.317	0.108	0.513	1.017

R (Bels)	T (sec)		I (dB SPL) 36	54	63	72	90
0.1	0.0	δ'			32.26		
		b'			0.157		
	0.2	δ'	9.844	14.93		28.78	31.43
		b'	0.166	0.092		0.255	0.221
	3.5	δ'			11.25		
		b'			0.568		
	14.0	δ'			6.476		
		b'			0.210		

TABLE A2.11c

$\hat{\delta}_2'$ and $\hat{b}'$ for Observer SK in Expts. 3 & 3A

T_p = 1.25 sec

R (Bels)	T (sec)		I (dB SPL) 39	51	63	75	87
5.6	0.0	δ'	13.20	23.25	26.62	35.51	37.66
		b'	-0.526	0.198	0.358	0.431	0.397
	0.2	δ'	13.38	13.09	12.02	21.66	23.91
		b'	-0.362	0.246	0.037	0.047	0.038
	3.5	δ'	3.449	3.776	3.484	6.242	8.866
		b'	-0.215	0.129	0.031	0.044	0.280
	14.0	δ'	1.789	1.258	1.662	4.204	6.046
		b'	-0.561	-0.063	0.249	0.384	0.591

R (Bels)	T (sec)		I (dB SPL) 51	57	63	69	75
2.9	0.0	δ'	23.04	22.54	30.32	42.59	38.67
		b'	0.054	0.099	0.139	0.214	0.473
	0.2	δ'	11.02	17.01	27.90	31.35	24.15
		b'	-0.123	0.032	0.058	-0.312	0.149
	3.5	δ'	4.323	6.473	7.180	7.775	10.24
		b'	-0.189	0.172	0.270	0.503	0.858
	14.0	δ'	2.650	2.529	2.633	4.723	5.557
		b'	-0.495	-0.121	0.178	0.611	0.947

R (Bels)	T (sec)		I (dB SPL) 36	54	63	72	90
0.1	0.0	δ'			41.95		
		b'			0.386		
	0.2	δ'	14.40	23.24		38.28	34.26
		b'	-0.213	0.123		0.107	0.263
	3.5	δ'			12.67		
		b'			0.516		
	14.0	δ'			8.427		
		b'			0.358		

TABLE A2.12

$\hat{d}_i'(I_{i+1};I_i)$ and $\hat{b}_i'$ for Observer JB in Expt. 4

R = 5.4		R = 3.6		R = 1.8		R = 0.3	
I_i dB SPL	d' b'	I_i dB SPL	d' b'	I_i dB SPL	d' b'	I_i dB SPL	d' b'
36.0	0.90 0.08	45.0	0.67 -0.20	54.0	0.54 -0.27	61.50	0.24 -0.64
40.5	1.19 0.30	48.0	0.78 0.19	55.5	0.81 0.06	61.75	0.26 -0.53
45.0	0.94 0.03	51.0	0.82 0.15	57.0	0.58 0.01	62.00	0.14 -0.33
49.5	0.77 0.09	54.0	0.82 0.14	58.5	0.60 0.04	62.25	0.25 -0.15
54.0	0.70 0.24	57.0	0.54 0.13	60.0	0.66 0.07	62.50	0.27 -0.13
58.5	0.77 0.30	60.0	0.82 0.07	61.5	0.88 -0.23	62.75	0.04 -0.06
63.0	0.90 0.05	63.0	0.60 -0.10	63.0	0.63 -0.36	63.00	0.32 0.07
67.5	0.76 0.06	66.0	0.89 0.12	64.5	0.72 -0.33	63.25	0.31 0.05
72.0	1.28 0.24	69.0	1.03 -0.01	66.0	0.98 -0.28	63.50	0.30 0.02
76.5	1.62 -0.06	72.0	1.31 -0.09	67.5	0.82 -0.25	63.75	0.22 0.22
81.0	1.79 -0.25	75.0	1.46 -0.38	69.0	0.87 -0.25	64.00	0.18 0.30
85.5	2.53 -0.50	78.0	1.61 0.08	70.5	0.91 0.12	64.25	0.16 0.49
90.0		81.0		72.0		64.50	

TABLE A2.13

$\hat{d}'_i(I_{i+1};\ I_i)$ and $\hat{b}'_i$ for Observer KG in Expt. 4

R = 5.4 I_i dB SPL	d' b'	R = 3.6 I_i dB SPL	d' b'	R = 1.8 I_i dB SPL	d' b'	R = 0.3 I_i dB SPL	d' b'
36.0	0.80 -0.13	45.0	0.69 -0.03	54.0	0.17 -0.15	61.50	-0.07 -0.59
40.5	0.89 -0.19	48.0	0.37 0.13	55.5	0.58 -0.08	61.75	0.17 -0.49
45.0	0.46 -0.39	51.0	0.66 0.03	57.0	0.44 -0.15	62.00	0.17 -0.47
49.5	0.48 -0.31	54.0	0.58 -0.05	58.5	0.37 -0.26	62.25	0.18 -0.52
54.0	0.58 -0.41	57.0	0.57 -0.07	60.0	0.45 -0.31	62.50	0.01 -0.48
58.5	0.27 -0.39	60.0	0.59 -0.24	61.5	0.61 -0.40	62.75	0.14 -0.47
63.0	0.66 -0.37	63.0	0.60 -0.30	63.0	0.30 -0.53	63.00	0.10 -0.48
67.5	0.71 -0.30	66.0	0.78 -0.19	64.5	0.41 -0.48	63.25	0.21 -0.41
72.0	0.98 -0.40	69.0	0.93 -0.46	66.0	0.45 -0.40	63.50	0.11 -0.28
76.5	1.09 -0.65	72.0	0.66 -0.79	67.5	0.59 -0.54	63.75	0.04 -0.21
81.0	1.64 -0.71	75.0	0.89 -0.69	69.0	0.30 -0.65	64.00	0.17 -0.08
85.5	2.06 -0.61	78.0	0.74 -0.31	70.5	0.27 -0.08	64.25	0.05 0.35
90.0		81.0		72.0		64.50	

TABLE A2.14

$\hat{d}'_i(I_{i+1};\ I_i)$ and $\hat{b}'_i$ for Observer PN in Expt. 4

R = 5.4		R = 3.6		R = 1.8		R = 0.3	
I_i dB SPL	d' b'	I_i dB SPL	d' b'	I_i dB SPL	d' b'	I_i dB SPL	d' b'
36.0	0.97 0.47	45.0	0.30 0.76	54.0	0.40 0.19	61.50	0.10 -0.87
40.5	0.30 0.59	48.0	0.38 0.82	55.5	0.30 0.20	61.75	-0.00 -0.57
45.0	0.77 0.60	51.0	0.60 0.59	57.0	0.28 0.23	62.00	0.13 -0.46
49.5	0.40 0.40	54.0	0.50 0.40	58.5	0.24 0.25	62.25	0.05 -0.34
54.0	0.55 0.36	57.0	0.43 0.40	60.0	0.48 0.21	62.50	0.17 -0.14
58.5	0.46 0.19	60.0	0.49 0.21	61.5	0.35 -0.02	62.75	0.13 -0.16
63.0	0.63 0.01	63.0	0.46 -0.01	63.0	0.45 -0.23	63.00	-0.18 0.01
67.5	0.39 0.28	66.0	0.62 -0.15	64.5	0.41 -0.21	63.25	0.17 0.22
72.0	0.79 0.06	69.0	0.26 -0.25	66.0	0.11 -0.17	63.50	0.11 0.26
76.5	0.64 -0.15	72.0	0.58 -0.16	67.5	0.22 0.09	63.75	0.08 0.46
81.0	1.27 0.36	75.0	0.55 -0.49	69.0	0.30 0.33	64.00	-0.03 0.66
85.5	2.08 -0.26	78.0	0.48 -0.69	70.5	0.14 0.50	64.25	0.20 0.79
90.0		81.0		72.0		64.50	

TABLE A2.15

$\hat{d}_i'(I_{i+1};I_i)$ and $\hat{b}_i'$ for Observer PS in Expt. 4

R = 5.4		R = 3.6		R = 1.8		R = 0.3	
I_i dB SPL	d' b'	I_i dB SPL	d' b'	I_i dB SPL	d' b'	I_i dB SPL	d' b'
36.0	1.34 0.23	45.0	0.86 -0.34	54.0		61.50	0.23 -0.58
40.5	1.03 -0.07	48.0	0.21 -0.23	55.5		61.75	0.02 -0.44
45.0	0.60 0.03	51.0	0.36 0.00	57.0		62.00	0.23 -0.22
49.5	0.76 -0.30	54.0	0.39 -0.10	58.5		62.25	0.28 -0.27
54.0	0.72 -0.06	57.0	0.22 0.49	60.0		62.50	0.02 -0.10
58.5	0.70 -0.34	60.0	0.66 0.35	61.5		62.75	0.13 -0.11
63.0	0.47 -0.33	63.0	0.92 -0.18	63.0		63.00	0.12 -0.18
67.5	0.72 0.18	66.0	0.70 0.04	64.5		63.25	0.04 -0.08
72.0	1.03 0.25	69.0	1.06 0.03	66.0		63.50	-0.11 0.17
76.5	1.27 0.38	72.0	1.26 -0.12	67.5		63.75	0.39 0.36
81.0	2.30 0.26	75.0	1.26 -0.36	69.0		64.00	0.72 0.16
85.5	2.82 -0.26	78.0	1.06 -0.08	70.5		64.25	-0.51 0.53
90.0		81.0		72.0		64.50	

TABLE A2.16

$\hat{d}'_i(I_{i+1};\ I_i)$ and $\hat{b}'_i$ for Observer JM in Expt. 5

I_i dB SPL	I_{ref} → dB SPL	No Ref.	36	49.5	63	76.5	90
36.0	d'	1.42	1.63	1.43	1.36	1.68	1.22
	b'	-0.09	0.05	-0.09	-0.15	1.12	-0.17
40.5	d'	1.22	1.17	1.55	1.47	1.06	1.09
	b'	-0.41	-0.31	0.15	-0.34	0.36	-0.19
45.0	d'	1.06	1.30	1.47	1.06	0.96	0.83
	b'	-0.24	-0.48	0.15	-0.53	0.06	-0.49
49.5	d'	0.85	0.92	1.55	1.07	0.84	0.64
	b'	-0.07	-0.39	-0.15	-0.10	0.08	0.01
54.0	d'	0.72	0.91	1.20	1.39	1.09	0.84
	b'	0.01	-0.53	-0.40	-0.28	-0.10	-0.30
58.5	d'	0.93	0.94	0.99	1.22	0.76	0.68
	b'	0.01	-0.26	-0.43	-0.59	0.01	-0.14
63.0	d'	1.22	0.92	0.97	1.28	0.83	0.58
	b'	-0.25	-0.22	-0.36	-1.01	0.06	-0.31
67.5	d'	0.90	0.93	1.48	1.64	1.24	0.56
	b'	-0.32	-0.06	-0.33	-0.66	0.20	-0.18
72.0	d'	1.32	1.45	1.52	2.17	3.58	0.71
	b'	-0.27	-0.07	0.13	-1.16	0.09	0.34
76.5	d'	2.00	2.38	2.57	2.67	2.90	1.14
	b'	-0.16	-0.33	-0.31	-1.19	-0.11	0.15
81.0	d'	2.34	2.39	2.30	2.91	2.70	2.27
	b'	-0.32	-0.17	-0.21	-2.05	-0.14	-0.17
85.5	d'	3.74	2.87	2.86	3.39	3.83	3.34
	b'	-0.38	-0.21	-0.26	-0.34	-0.20	-0.45

TABLE A2.17

$\hat{d}_1'(I_{i+1}; I_i)$ and $\hat{b}_i'$ for Observer JB in Expt. 5

I_i dB SPL	$I_{ref} \rightarrow$ dB SPL	No Ref.	36	49.5	63	76.5	90
36.0	d'	1.01	1.52	1.36	0.98	1.24	1.21
	b'	-0.13	0.00	-0.05	-0.15	-0.12	0.05
40.5	d'	1.02	1.14	1.58	1.00	1.03	1.08
	b'	-0.04	0.04	0.10	-0.12	-0.36	0.11
45.0	d'	0.68	0.88	1.84	0.89	0.68	0.88
	b'	0.05	-0.08	0.06	-0.19	-0.31	-0.01
49.5	d'	0.88	0.68	0.95	0.98	0.70	0.83
	b'	0.09	-0.21	0.10	-0.16	-0.27	-0.23
54.0	d'	0.40	0.77	1.26	1.39	0.65	0.53
	b'	0.07	-0.26	-0.12	0.32	-0.26	-0.02
58.5	d'	0.67	0.56	1.10	1.92	0.56	0.55
	b'	0.08	-0.21	-0.65	-0.33	0.01	-0.05
63.0	d'	0.62	0.86	0.60	1.53	0.61	0.54
	b'	0.15	-0.15	-0.60	-0.65	0.25	-0.19
67.5	d'	0.89	0.65	0.76	1.54	1.05	0.51
	b'	-0.02	-0.20	-0.16	-0.28	0.62	0.07
72.0	d'	0.97	1.09	0.66	1.43	3.51	0.38
	b'	-0.15	-0.08	0.09	-0.39	-0.02	0.57
76.5	d'	1.13	1.54	1.32	1.30	3.38	1.01
	b'	-0.43	0.18	0.31	0.03	-1.02	0.79
81.0	d'	2.09	2.39	2.04	2.03	2.54	2.50
	b'	-0.70	-0.74	0.44	-0.11	-0.20	0.13
85.5	d'	2.40	2.11	2.61	2.43	2.48	4.52
	b'	-1.68	-0.28	-0.46	-0.08	-0.02	-0.07

TABLE A2.18

$\hat{d}'_i(I_{i+1};\ I_i)$ and $\hat{b}'_i$ for Observer BG in Expt. 5

I_i dB SPL	$I_{ref} \rightarrow$ dB SPL	No Ref.	36	49.5	63	76.5	90
36.0	d'	1.15	0.93	1.28	1.02	1.02	1.17
	b'	0.08	-0.56	-0.40	-0.29	0.03	-0.10
40.5	d'	0.89	0.63	1.25	1.17	1.10	1.02
	b'	-0.11	-0.54	2.01	-0.44	-0.14	-0.20
45.0	d'	0.71	0.57	0.98	0.84	0.61	1.01
	b'	-0.06	-0.29	1.11	-0.42	-0.01	-0.26
49.5	d'	0.47	0.20	0.60	0.85	0.94	0.45
	b'	0.20	0.06	1.44	-0.16	0.02	0.17
54.0	d'	0.51	0.68	0.72	0.97	0.50	0.86
	b'	0.31	0.06	1.44	0.12	0.06	0.19
58.5	d'	0.82	0.48	0.81	0.85	0.80	0.53
	b'	0.16	-0.27	1.38	-0.17	0,11	0.10
63.0	d'	0.75	0.67	0.97	0.96	0.85	0.75
	b'	0.24	0.19	1.04	0.71	0.18	1.31
67.5	d'	0.96	0.81	0.93	1.24	1.62	0.75
	b'	0.16	-0.14	1.10	0.94	0.03	1.07
72.0	d'	0.90	1.12	1.22	1.35	1.68	0.91
	b'	-0.05	-0.25	1.13	0.97	-0.24	1.21
76.5	d'	1.36	1.44	1.59	1.60	1.55	1.69
	b'	0.01	0.15	0.76	0.82	-0.14	0.92
81.0	d'	1.34	2.02	2.42	2.05	2.25	2.33
	b'	0.08	-0.27	0.53	0.24	-0.10	-0.08
85.5	d'	2.54	2.39	2.94	2.88	2.99	2.13
	b'	-0.57	-0.13	-0.05	0.06	0.14	-0.51

TABLE A2.19

$\hat{d}'_i(I_{i+1};\ I_i)$ and $\hat{b}'_i$ for Observer CB in Expt. 5

I_i dB SPL	I_{ref} → dB SPL	No Ref.	36	49.5	63	76.5	90
36.0	d'	1.45	1.63	1.78	1.24	1.61	1.47
	b'	0.19	0.27	0.13	-0.11	0.20	0.09
40.5	d'	1.05	1.61	1.94	1.47	1.33	1.17
	b'	0.11	0.02	-0.12	-0.28	-0.26	0.01
45.0	d'	1.53	1.87	1.57	1.47	0.96	1.36
	b'	-0.15	-0.20	-0.85	-0.22	-0.15	-0.10
49.5	d'	1.06	1.54	1.32	1.60	1.28	0.90
	b'	-0.67	-0.44	-0.55	-0.01	-0.34	-0.37
54.0	d'	0.83	1.15	1.86	1.47	0.71	0.79
	b'	-0.36	-0.51	-1.45	-0.11	-0.45	-0.44
58.5	d'	0.80	1.12	1.80	2.05	0.78	0.54
	b'	-0.06	-0.48	-0.98	-0.81	-0.38	-0.43
63.0	d'	1.20	0.74	1.27	1.65	0.72	0.78
	b'	-0.22	-0.03	1.62	-0.64	0.20	1.06
67.5	d'	1.00	1.07	1.20	3.07	1.49	0.82
	b'	0.27	0.44	2.23	-0.85	0.18	1.14
72.0	d'	2.82	2.94	2.73	2.80	1.35	1.83
	b'	0.38	0.32	1.89	-1.27	-0.44	0.90
76.5	d'	1.87	2.17	2.08	1.61	2.16	1.06
	b'	-0.91	-0.91	0.25	-2.03	-0.54	0.49
81.0	d'	1.97	2.32	1.89	2.86	3.80	2.69
	b'	0.20	-0.23	-0.39	-2.69	-0.67	0.62
85.5	d'	3.06	3.41	2.99	2.89	3.82	3.71
	b'	0.05	-0.03	-1.93	-3.81	-0.20	-0.03

TABLE A2.20

$\hat{d}_i'(I_{i+1}; I_i)$ and $\hat{b}_i'$ for Observer DH in Expt. 5

I_i dB SPL	I_{ref} → dB SPL	No Ref.	36	49.5	63	76.5	90
36.0	d'	1.20	1.21	1.00	1.49	1.23	1.04
	b'	0.26	0.45	0.13	-0.17	-0.07	-0.05
40.5	d'	1.03	1.00	1.16	0.74	0.43	1.06
	b'	0.03	0.12	-0.02	-0.04	-0.05	-0.10
45.0	d'	0.61	1.04	0.83	1.10	0.90	0.47
	b'	-0.02	-0.01	-0.29	-0.01	-0.11	-0.18
49.5	d'	0.94	0.75	1.15	1.31	0.36	0.46
	b'	0.06	-0.07	-0.18	-0.18	0.19	-0.00
54.0	d'	0.55	0.91	1.05	0.83	0.93	0.75
	b'	-0.25	-0.37	-0.55	-0.47	0.03	0.56
58.5	d'	0.74	0.36	0.92	1.09	0.41	0.41
	b'	-0.32	-0.36	-0.41	-0.77	0.71	0.37
63.0	d'	0.57	0.82	0.49	1.16	0.97	0.46
	b'	-0.24	-0.33	-0.36	-0.58	0.53	0.51
67.5	d'	0.79	0.57	1.00	0.97	0.54	0.59
	b'	0.01	-0.07	1.10	-0.50	0.91	0.45
72.0	d'	1.23	1.17	1.34	2.69	1.57	1.09
	b'	0.24	0.18	0.56	-0.72	0.56	0.34
76.5	d'	1.51	2.04	1.49	1.73	2.18	1.33
	b'	0.08	0.39	0.02	-1.41	-0.44	0.18
81.0	d'	2.03	2.31	2.62	2.00	2.52	1.57
	b'	-0.20	-0.38	0.66	-2.09	-1.42	-0.24
85.5	d'	2.74	2.73	2.65	3.21	3.29	2.30
	b'	-0.59	-0.03	0.12	-2.96	-2.33	-0.20

TABLE A2.21

$\hat{d}'_i(I_{i+1};\ I_i)$ and $\hat{b}'_i$ for Observer DM in Expt. 5

I_i dB SPL	I_{ref} → dB SPL	No Ref.	36	49.5	63	76.5	90
36.0	d'	1.40	1.22	1.52	1.24	1.17	1.19
	b'	0.24	0.17	-0.21	-0.05	1.57	0.16
40.5	d'	0.97	1.03	1.36	1.20	1.25	1.00
	b'	0.01	-0.05	-0.23	-0.37	0.44	-0.02
45.0	d'	0.85	0.95	1.07	1.08	1.05	1.12
	b'	0.12	-0.17	2.12	-0.31	0.05	-0.15
49.5	d'	0.83	0.88	1.31	1.37	1.02	0.62
	b'	0.02	-0.15	1.22	-0.11	-0.41	-0.21
54.0	d'	0.66	1.00	1.04	1.75	0.97	0.80
	b'	0.08	-0.29	1.28	-0.05	-0.74	1.24
58.5	d'	0.60	0.44	1.06	1.53	0.93	1.07
	b'	0.13	-0.18	1.11	-0.55	-0.25	0.83
63.0	d'	0.90	0.60	0.54	1.62	0.86	0.64
	b'	0.17	0.03	0.88	2.10	-0.23	0.66
67.5	d'	0.90	0.83	1.15	1.86	1.45	1.19
	b'	0.06	0.28	1.03	0.94	-0.16	0.91
72.0	d'	1.67	1.00	1.23	3.06	2.43	1.20
	b'	0.03	0.14	0.59	0.59	-0.37	0.57
76.5	d'	1.76	1.38	1.60	1.92	1.22	1.59
	b'	0.02	0.23	0.85	-0.61	-0.49	0.41
81.0	d'	1.94	2.19	1.65	1.84	2.25	1.92
	b'	-0.24	-0.13	0.15	-1.53	-0.38	0.38
85.5	d'	2.82	2.71	2.45	2.74	3.06	3.21
	b'	-0.18	-0.24	-0.06	-3.43	-1.04	-0.07

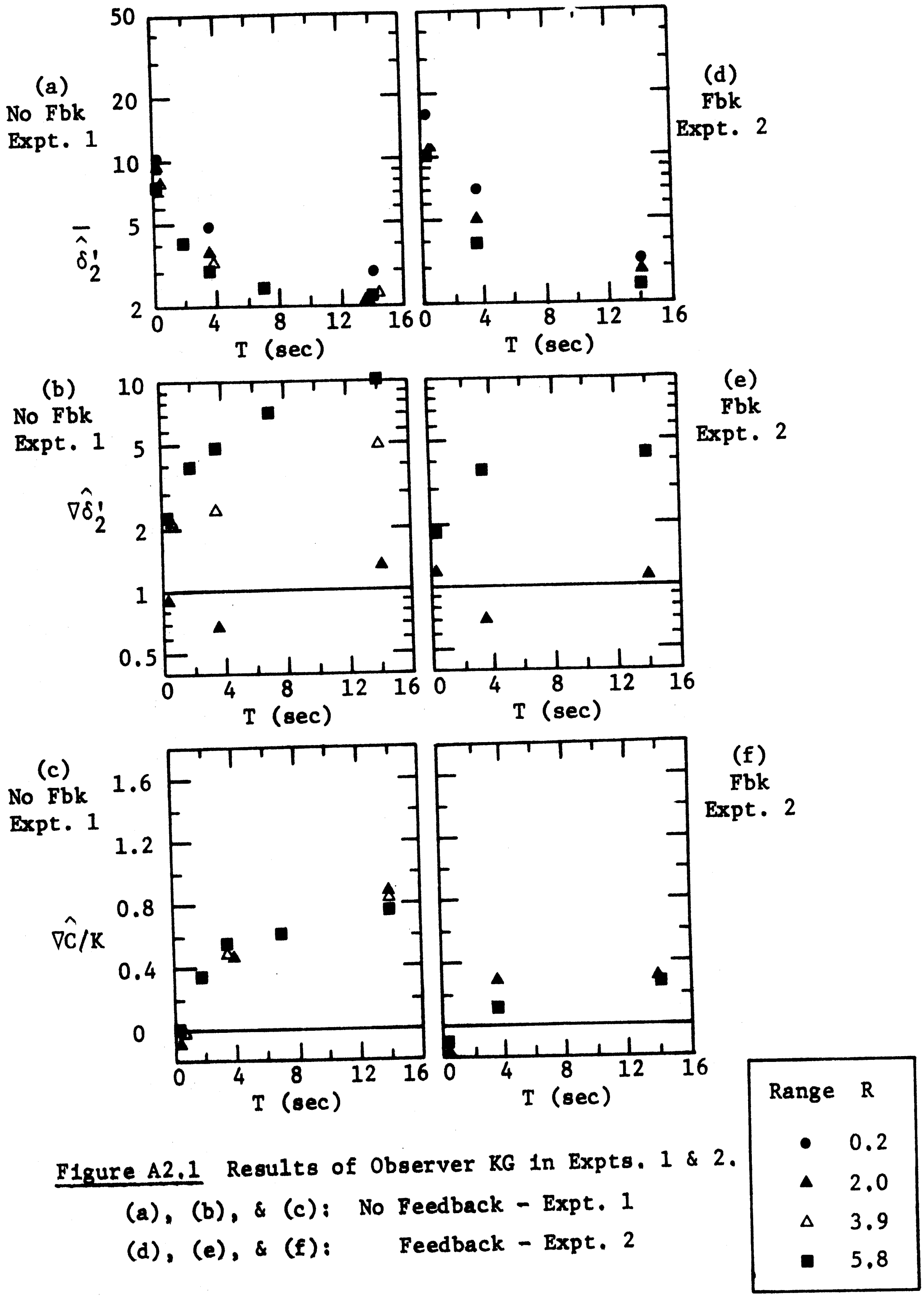
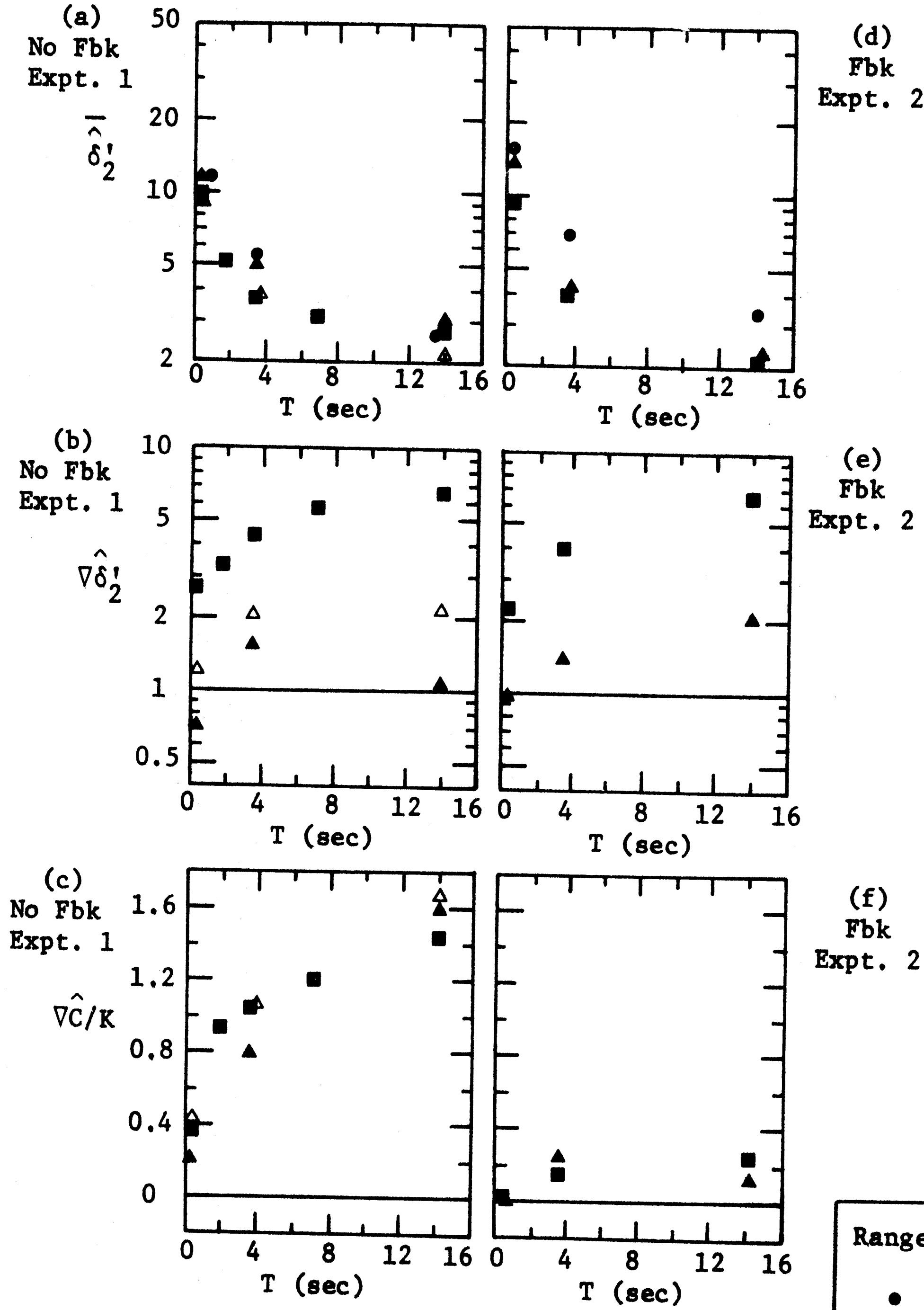


Figure A2.1 Results of Observer KG in Expts. 1 & 2.

(a), (b), & (c): No Feedback - Expt. 1

(d), (e), & (f): Feedback - Expt. 2



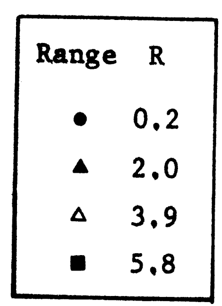


__Figure A2.2__ Results of Observer PN in Expts. 1 & 2.

(a), (b), & (c): No Feedback - Expt. 1

(d), (e), & (f): Feedback - Expt. 2

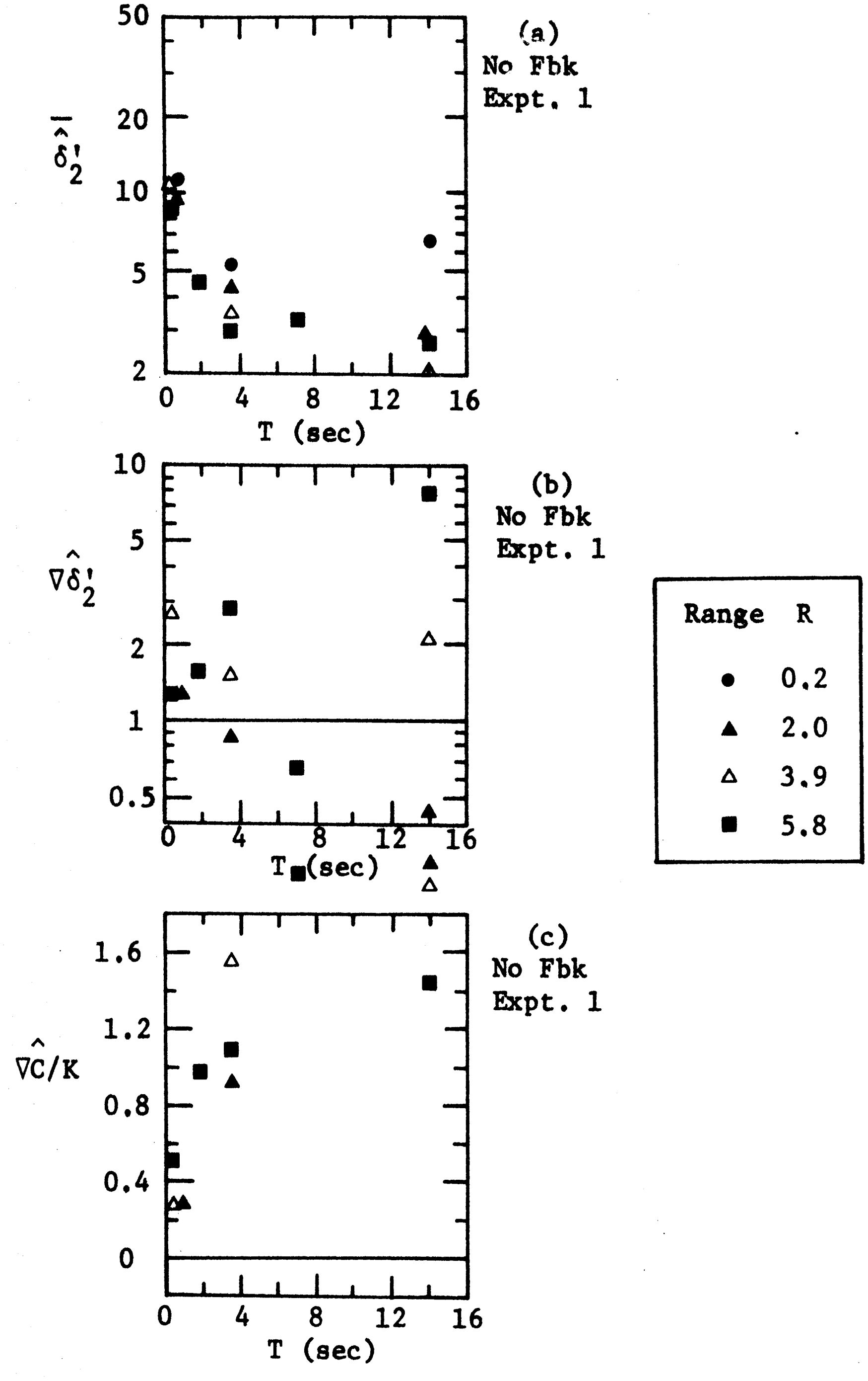


Figure A2.3 Results of Observer PS in Expt. 1.

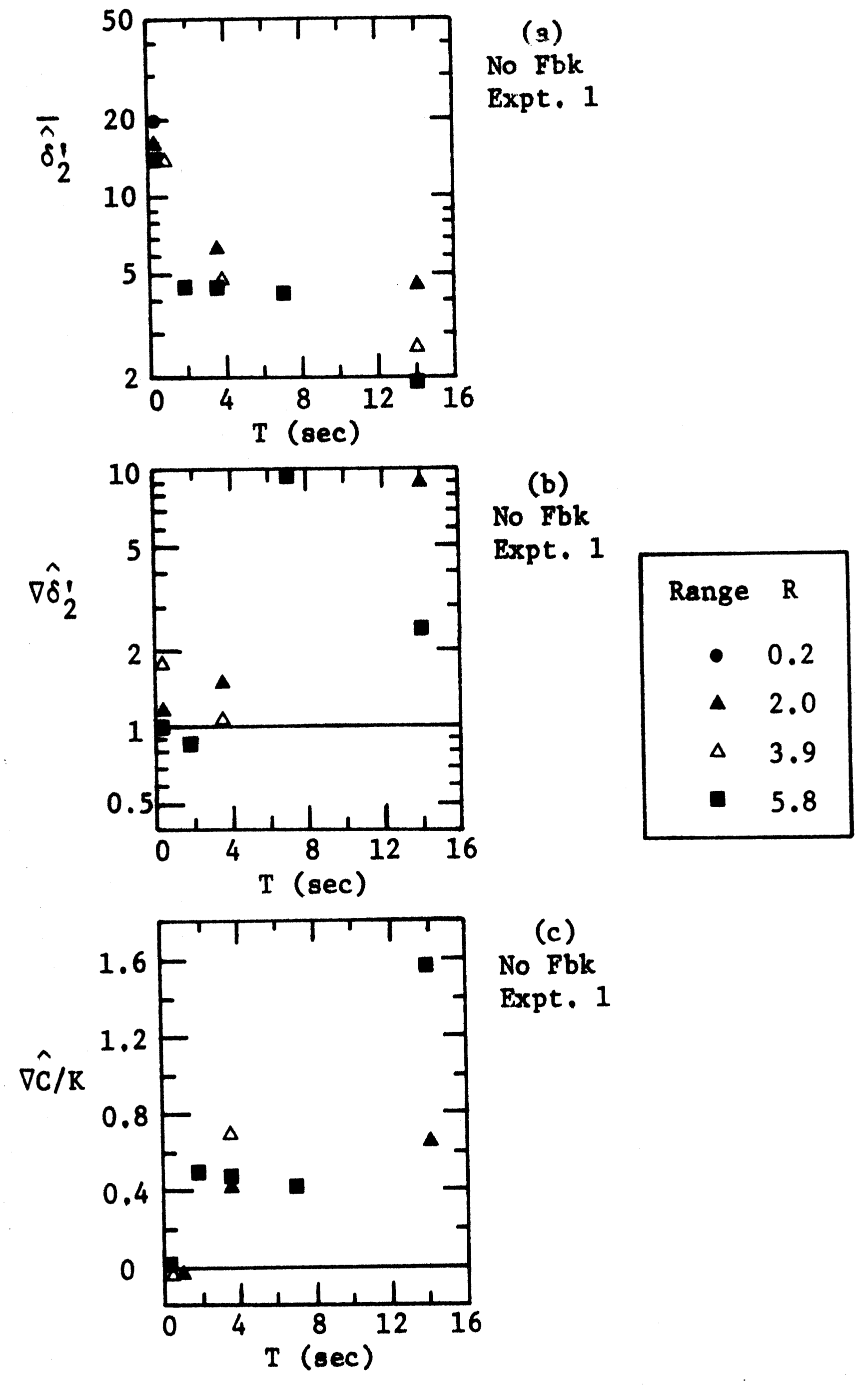


Figure A2.4 Results of Observer BD in Expt. 1.

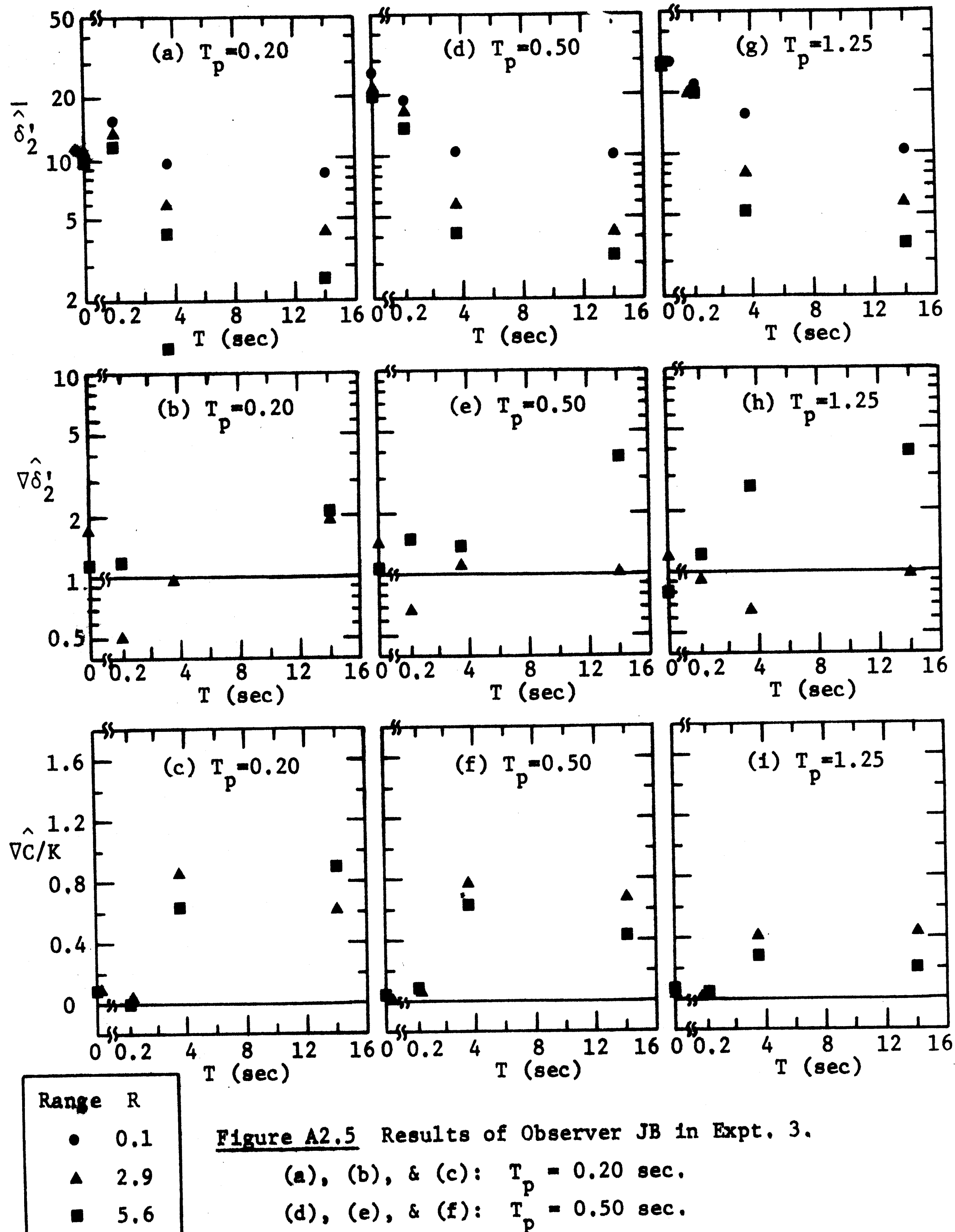
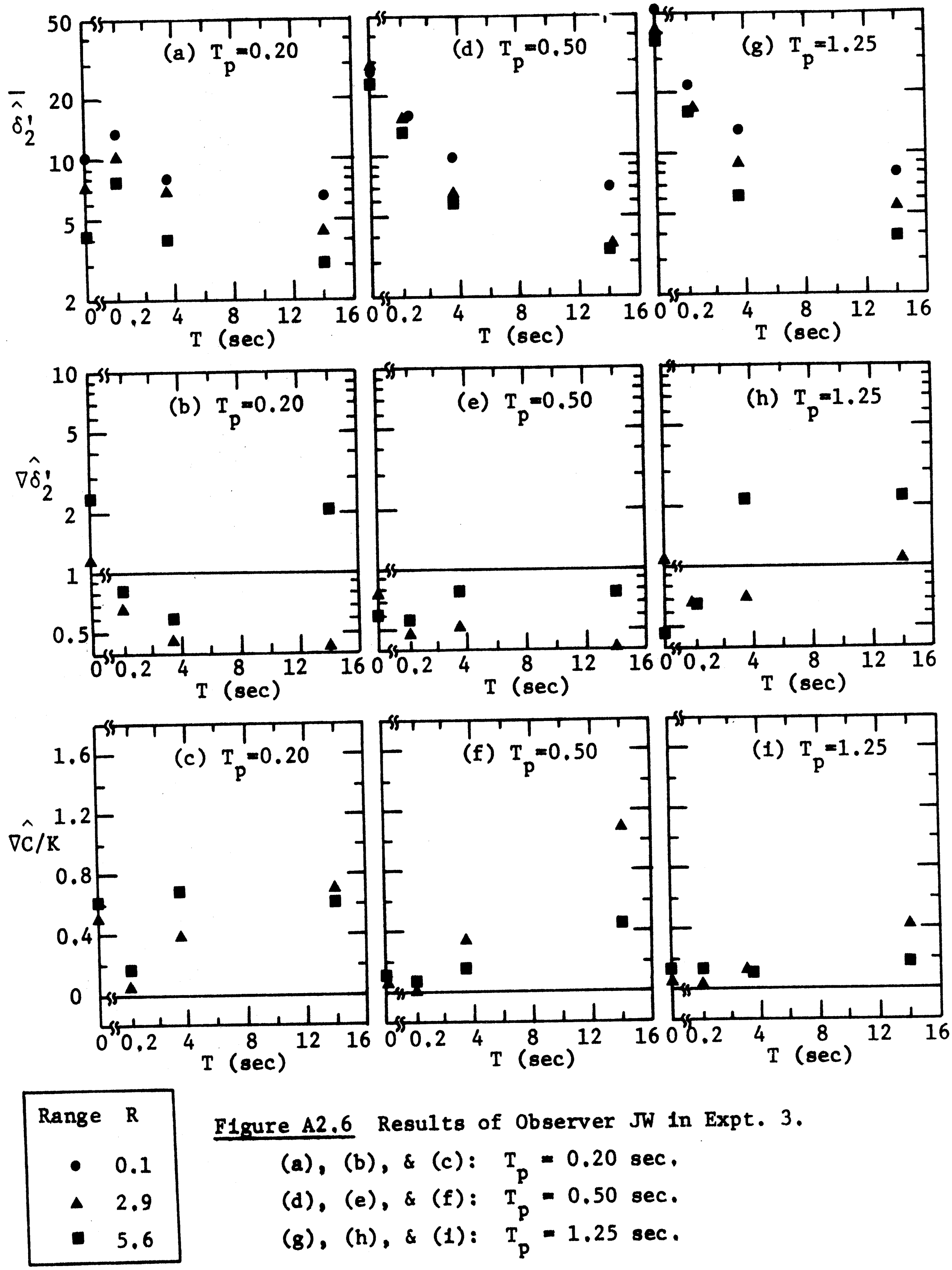


Figure A2.5 Results of Observer JB in Expt. 3.

(a), (b), & (c): T_p = 0.20 sec.

(d), (e), & (f): T_p = 0.50 sec.

(g), (h), & (i): T_p = 1.25 sec.

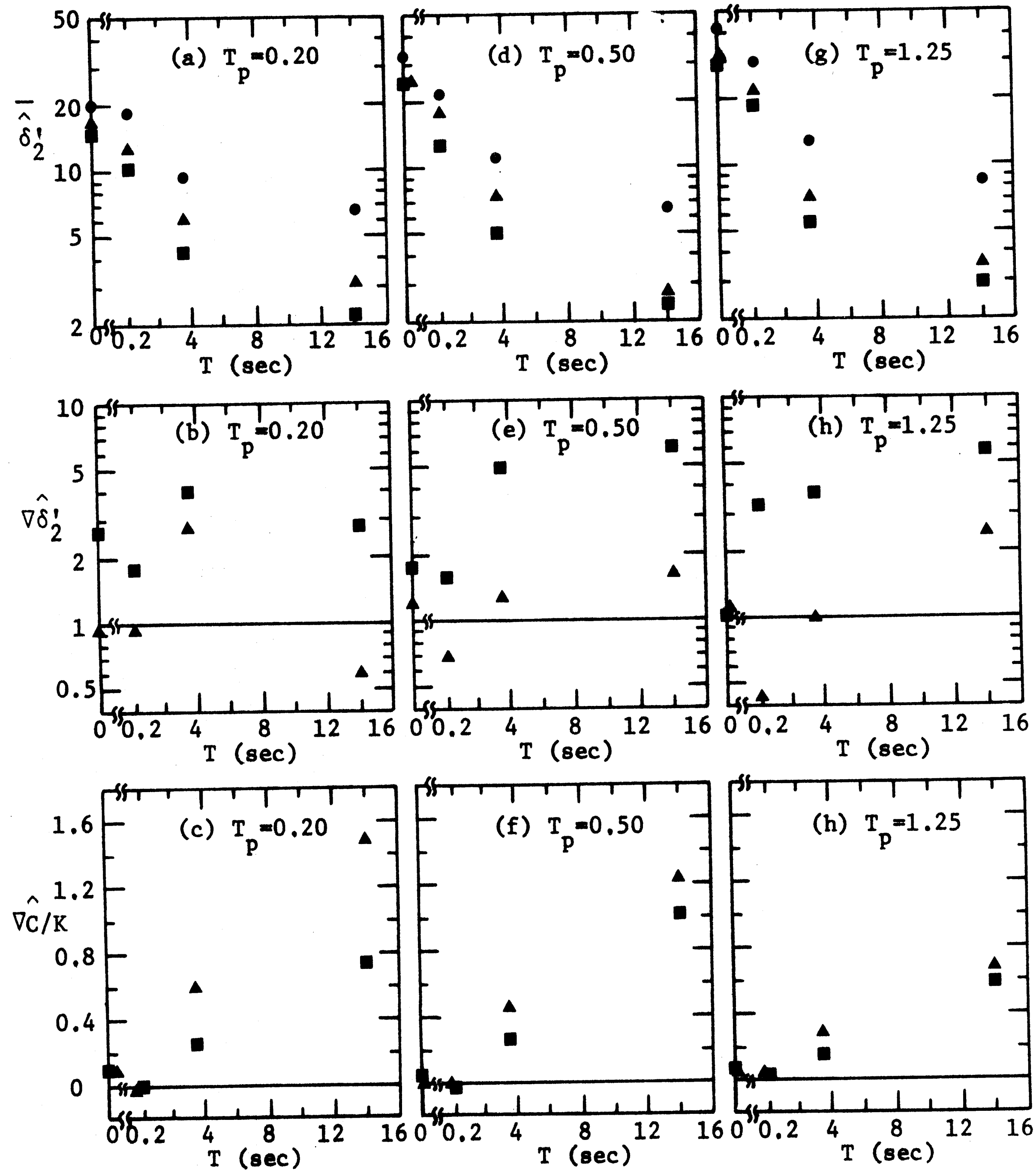


<u>Figure A2.6</u> Results of Observer JW in Expt. 3.

(a), (b), & (c): T_p = 0.20 sec.

(d), (e), & (f): T_p = 0.50 sec.

(g), (h), & (i): T_p = 1.25 sec.



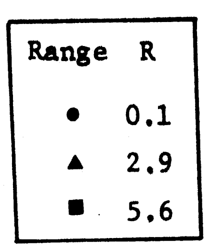
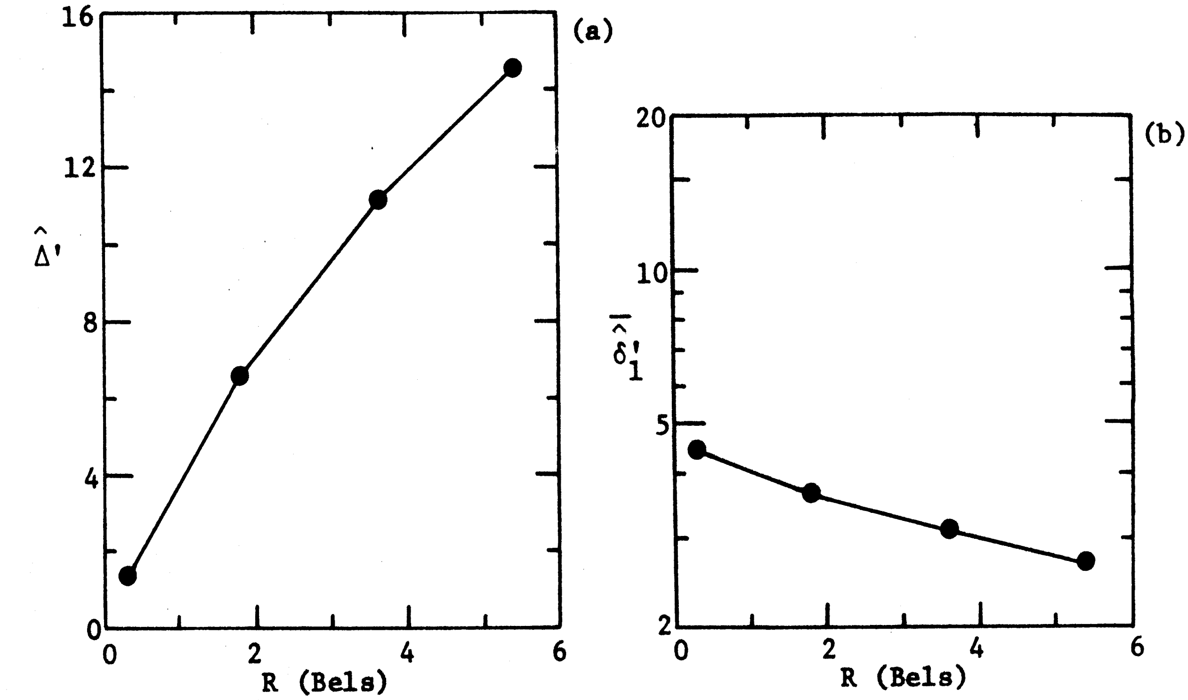


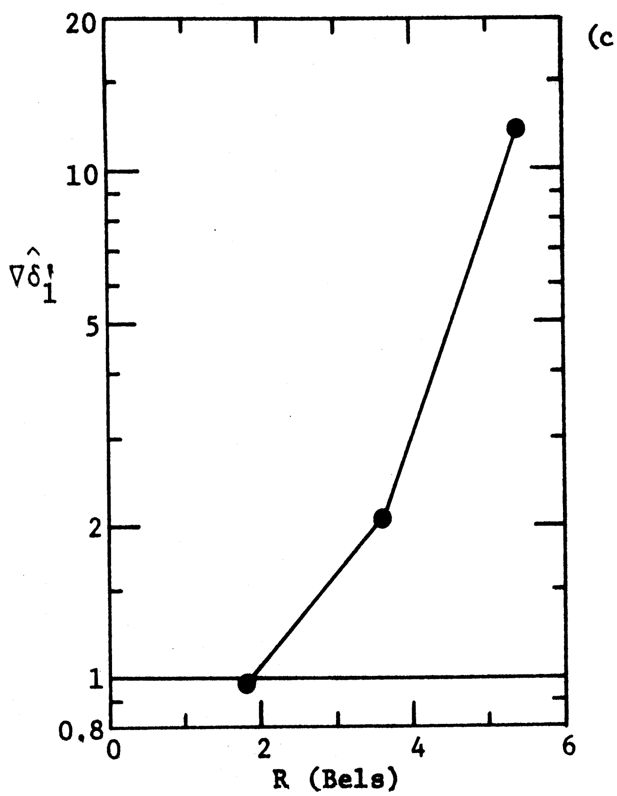
Figure A2.7 Results of Observer SK in Expt. 3.

(a), (b), (c): T_p = 0.20 sec.

(d), (e), (f): T_p = 0.50 sec.

(g), (h), (i): T_p = 1.25 sec.





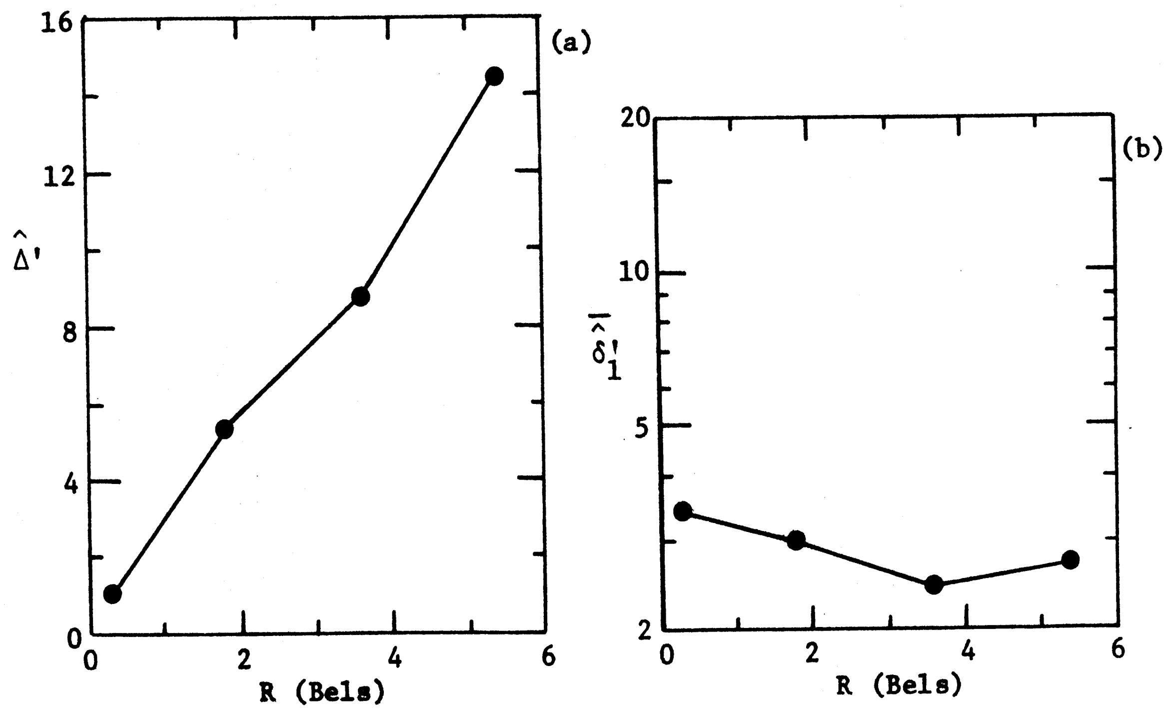
<u>Figure A2.8</u>

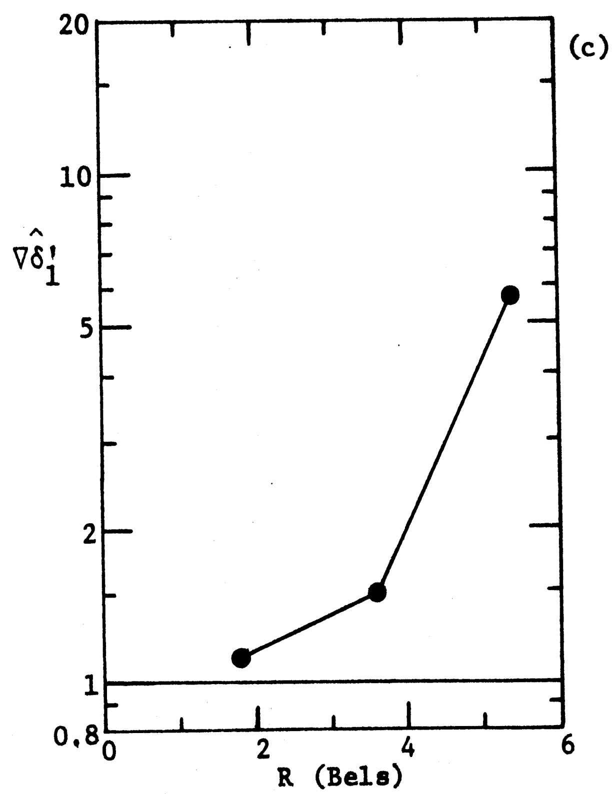
Results of Observer KG in Expt. 4.

(a): $\hat{\Delta}'$ versus R

(b): $\overline{\hat{\delta}_1'}$ versus R

(c): $\nabla\hat{\delta}_1'$ versus R





<u>**Figure A2.9**</u>

Results of Observer PN in Expt. 4.

(a): $\hat{\Delta}'$ versus R

(b): $\hat{\bar{\delta}}'_1$ versus R

(c): $\nabla\hat{\delta}'_1$ versus R

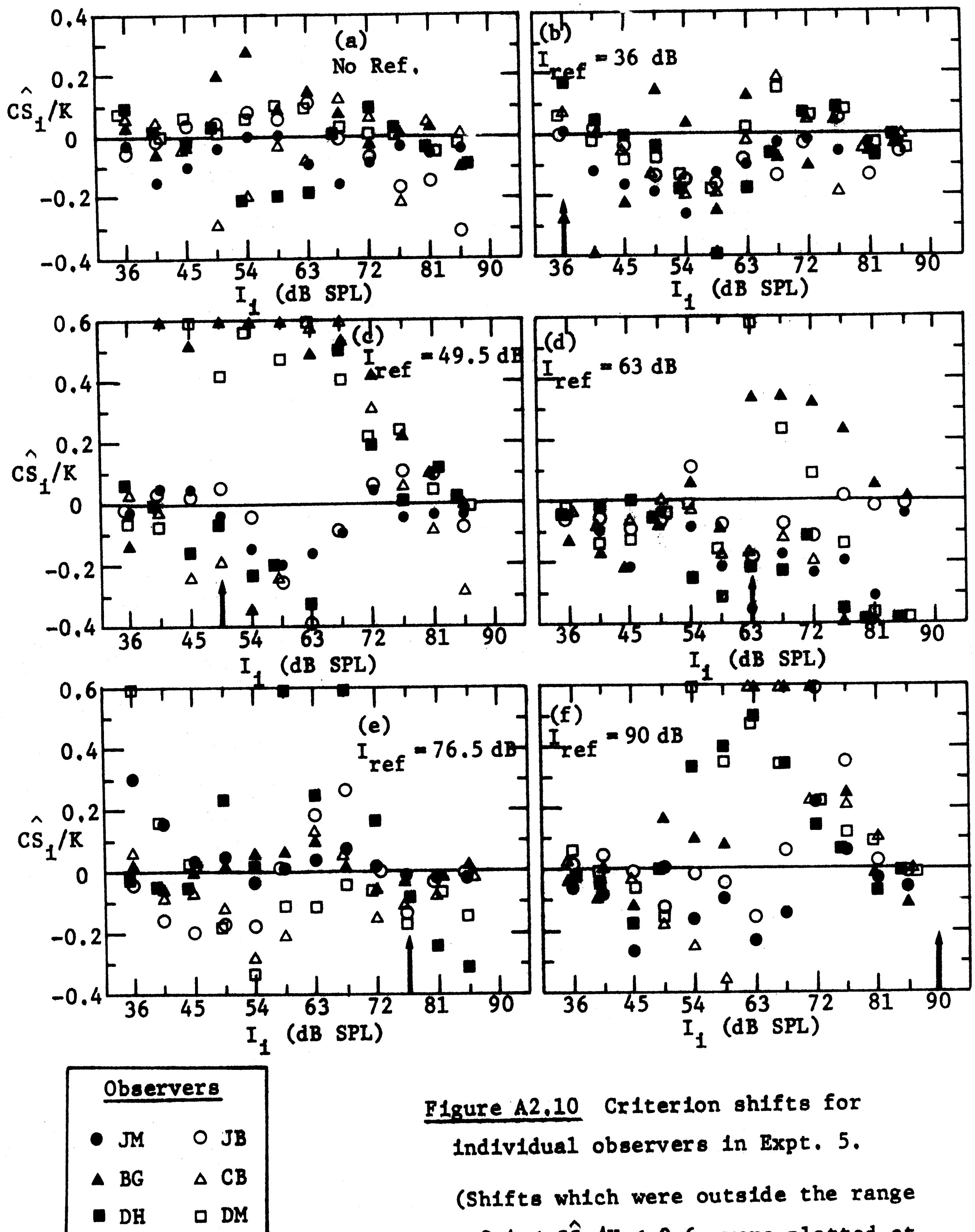


Figure A2.10 Criterion shifts for individual observers in Expt. 5.

(Shifts which were outside the range $-0.4 < C\hat{S}_i/K < 0.6$, were plotted at the edges of the graphs.)

APPENDIX III. TWO THEOREMS

In part A of this appendix, a theorem is proven concerning a log likelihood-ratio decision variable. In part B, a theorem is proven concerning a maximum likelihood estimate.

A. Log Likelihood-Ratio Theorem

Let ζ_1 and ζ_2 be two events, and let X_T and X_Q be jointly Gaussian random variables with:

$$E(X_T|\zeta_1) = - E(X_T|\zeta_2) = \varepsilon \ , \tag{A3.1}$$

$$E(X_Q|\zeta_1) = - E(X_Q|\zeta_2) = \varepsilon/\kappa \ , \tag{A3.2}$$

$$\mathrm{Var}(X_T|\zeta_1) = \mathrm{Var}(X_T|\zeta_2) = \sigma_T^2 = 2(\beta^2 + AT) \ , \tag{A3.3}$$

$$\mathrm{Var}(X_Q|\zeta_1) = \mathrm{Var}(X_Q|\zeta_2) = \sigma_Q^2 = 2(\beta^2 + \pi^2)/\kappa^2 \ , \tag{A3.4}$$

and

$$\mathrm{Cov}(X_Q, X_T|\zeta_1) = \mathrm{Cov}(X_Q, X_T|\zeta_2) = \mu_{Q,T} = 2\beta^2/\kappa \ . \tag{A3.5}$$

In this section, it is proven that there exists a decision variable X_c, proportional to the log of the likelihood-ratio,

$$X_{Q,T} = \ln[p(X_Q, X_T|\zeta_1)/p(X_Q, X_T|\zeta_2)] \ , \tag{A3.6}$$

with the properties:

$$E(X_c|\zeta_1) = - E(X_c|\zeta_2) = \varepsilon \ , \tag{A3.7}$$

and

$$\mathrm{Var}(X_c|\zeta_1) = \mathrm{Var}(X_c|\zeta_2) = 2\beta^2 + \frac{1}{1/2AT + 1/2\pi^2} \quad . \qquad \text{(A3.8)}$$

Furthermore, X_c is shown to be a linear combination of X_Q and X_T, so that it too has a Gaussian probability density.

Proof:

Since X_Q and X_T are jointly Gaussian, $X_{Q,T}$ is given by:

$$X_{Q,T} = -\frac{1}{2(1-r^2)}\left\{\left[\frac{(X_T-\varepsilon)^2}{\sigma_T^2} - \frac{2r(X_T-\varepsilon)(X_Q-\varepsilon/\kappa)}{\sigma_T\sigma_Q} + \frac{(X_Q-\varepsilon/\kappa)^2}{\sigma_Q^2}\right]\right.$$

$$\left. - \left[\frac{(X_T+\varepsilon)^2}{\sigma_T^2} - \frac{2r(X_T+\varepsilon)(X_Q+\varepsilon/\kappa)}{\sigma_T\sigma_Q} + \frac{(X_Q+\varepsilon/\kappa)^2}{\sigma_Q^2}\right]\right\} , \qquad \text{(A3.9)}$$

where $r = \mu_{Q,T}/\sigma_Q\sigma_T$ is the correlation coefficient. After some simplification, this becomes:

$$X_{Q,T} = \frac{2\varepsilon}{\sigma_Q^2\sigma_T^2 - \mu_{Q,T}^2}\left[\sigma_Q^2(X_T) - \mu_{Q,T}(X_T/\kappa + X_Q) + \sigma_T^2(X_Q/\kappa)\right] \quad . \qquad \text{(A3.10)}$$

Substituting the expressions for σ_Q^2, σ_T^2 and $\mu_{Q,T}$ and simplifying a bit further $X_{Q,T}$ becomes:

$$X_{Q,T} = \frac{2\varepsilon/\kappa^2}{\sigma_Q^2\sigma_T^2 - \mu_{Q,T}^2}\left[X_T(2\pi^2) + X_Q\kappa(2AT)\right] \quad . \tag{A3.11}$$

Now define X_c as:

$$X_c = \frac{X_{Q,T}\left(\dfrac{\sigma_Q^2\sigma_T^2 - \mu_{Q,T}^2}{2\varepsilon/\kappa^2}\right)}{2\pi^2 + 2AT}$$

$$= \frac{X_T(2\pi^2) + X_Q\kappa(2AT)}{2\pi^2 + 2AT} \quad , \tag{A3.12}$$

proportional to $X_{Q,T}$, and a linear combination of X_Q and X_T.

Finally, the mean and variance of X_c are calculated:

$$E(X_c|\zeta_1) = - E(X_c|\zeta_2) = \frac{\varepsilon(2\pi^2) + (\varepsilon/\kappa)\kappa(2AT)}{2\pi^2 + 2AT}$$

$$= \varepsilon \quad , \tag{A3.13}$$

$$\mathrm{Var}(X_c|\zeta_1) = \mathrm{Var}(X_c|\zeta_2) = \frac{\sigma_T^2(2\pi^2)^2 + 2\mu_{Q,T}(2\pi^2)\kappa(2AT) + \sigma_Q^2\kappa^2(2AT)^2}{(2\pi^2 + 2AT)^2}$$

$$= \frac{2\beta^2(2\pi^2 + 2AT)^2 + 2AT\ 2\pi^2(2\pi^2 + 2AT)}{(2\pi^2 + 2AT)^2}$$

$$= 2\beta^2 + \frac{1}{1/2AT + 1/2\pi^2} \quad . \tag{A3.14}$$

B. Maximum Likelihood Estimate Theorem

Let $\{X_i\}$ be a set of n jointly Gaussian random variables with:

$$E(X_i|\Gamma) = \Gamma + \varepsilon_i \ , \tag{A3.15}$$

$$Var(X_i|\Gamma) = \gamma^2 + \sigma_i^2 \ , \tag{A3.16}$$

and

$$Cov(X_i, X_j|\Gamma) = \gamma^2 \ , \tag{A3.17}$$

and define $\hat{\Gamma}_{ml}$ as the maximum likelihood estimate of Γ, based on the $\{X_i\}$. In this section, it is proven that $\hat{\Gamma}_{ml}$ is the following linear combination of the $\{X_i\}$:

$$\hat{\Gamma}_{ml} = \frac{\sum_{i=1}^{n} (X_i - \varepsilon_i)/\sigma_i^2}{\sum_{i=1}^{n} 1/\sigma_i^2} \ . \tag{A3.18}$$

Consequently, $p(\hat{\Gamma}_{ml}|\Gamma)$ is Gaussian. In addition, it is proven that:

$$E(\hat{\Gamma}_{ml}|\Gamma) = \Gamma \ , \tag{A3.19}$$

so that $\hat{\Gamma}_{ml}$ is an unbiased estimate of Γ, and that

$$Var(\hat{\Gamma}_{ml}|\Gamma) = \gamma^2 + \frac{1}{\sum_{i=1}^{n} 1/\sigma_i^2} \ . \tag{A3.20}$$

Proof:

First, we define the observation vector $\underline{X} = (X_1, X_2, \ldots, X_n)$, and the bias vector $\underline{\varepsilon} = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)$, and note that the covariance matrix $[\underline{\mu}]$ has the form:

$$[\underline{\mu}] = \begin{bmatrix} \gamma^2 + \sigma_1^2 & \gamma^2 & \cdots & \gamma^2 \\ \gamma^2 & \gamma^2 + \sigma_2^2 & \cdots & \gamma^2 \\ \vdots & \vdots & & \vdots \\ \gamma^2 & \gamma^2 & \cdots & \gamma^2 + \sigma_n^2 \end{bmatrix} . \qquad \text{(A3.21)}$$

In vector form, the joint conditional probability density function $p(\underline{X}|\Gamma)$ is:

$$p(\underline{X}|\Gamma) = \frac{1}{\left\{(2\pi)^n \det[\underline{\mu}]\right\}^{1/2}} \exp\left\{- \frac{1}{2}[\underline{X} - (\Gamma+\underline{\varepsilon})][\underline{\mu}]^{-1} [X - (\Gamma+\underline{\varepsilon})]^t\right\} , \qquad \text{(A3.22)}$$

where $\det[\underline{\mu}]$ is the determinant of $[\underline{\mu}]$, and $[\underline{\mu}]^{-1}$ is the inverse covariance matrix.

Before proceeding to find $\hat{\Gamma}_{ml}$, we must compute $\det[\underline{\mu}]$ and $[\underline{\mu}]^{-1}$. From Eq. A3.21, $\det[\underline{\mu}]$ is evaluated in several steps:

$$\det[\underline{\mu}] = \det\begin{bmatrix} \sigma_1^2 & 0 & \cdots & -\sigma_n^2 \\ 0 & \sigma_2^2 & \cdots & -\sigma_n^2 \\ \vdots & \vdots & & \vdots \\ \gamma^2 & \gamma^2 & \cdots & \gamma^2 + \sigma_n^2 \end{bmatrix}$$

$$= \gamma^2\sigma_n^2 \det\begin{bmatrix} \sigma_1^2 & 0 & \cdots & -1 \\ 0 & \sigma_2^2 & \cdots & -1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & (1/\gamma^2+1/\sigma_n^2) \end{bmatrix}$$

$$= \gamma^2\sigma_n^2 \det\begin{bmatrix} \sigma_1^2 & 0 & \cdots & -1 \\ 0 & \sigma_2^2 & \cdots & -1 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & (1/\gamma^2+1/\sigma_n^2+1/\sigma_1^2+1/\sigma_2^2+ \cdots + 1/\sigma_{n-1}^2) \end{bmatrix}$$

$$= \gamma^2\left(\prod_{i=1}^{n} \sigma_i^2\right)\left(1/\gamma^2 + \sum_{i=1}^{n} 1/\sigma_i^2\right)$$

$$= \left(\prod_{i=1}^{n} \sigma_i^2\right)(1 + \gamma^2 s) \ , \qquad \text{(A3.23)}$$

where $s = \sum_{i=1}^{n} 1/\sigma_i^2$.

To compute $[\underline{\mu}]^{-1}$, we note that it is given by:

$$[\underline{\mu}]^{-1} = \frac{1}{\det[\underline{\mu}]} [cf]^t , \tag{A3.24}$$

where [cf] is the matrix of cofactors of the matrix $[\underline{\mu}]$. The elements cf_{ij} are given by:

$$cf_{ij} = (-1)^{i+j} M_{ij} , \tag{A3.25}$$

where M_{ij} is the minor of row i and column j of the matrix $[\underline{\mu}]$. The elements on the diagonal, $cf_{jj} = M_{jj}$, are computed first, by analogy with $\det[\underline{\mu}]$:

$$cf_{jj} = M_{jj} = \frac{1}{\sigma_j^2}\left(\prod_{i=1}^{n} \sigma_i^2\right)(1 + \gamma^2 s - \gamma^2/\sigma_j^2) . \tag{A3.26}$$

Thus, the elements on the diagonal of $[\underline{\mu}]^{-1}$, $(\underline{\mu}^{-1})_{jj}$, are:

$$(\underline{\mu}^{-1})_{jj} = cf_{jj}/\det[\underline{\mu}]$$

$$= \frac{1}{\sigma_j^2} - \frac{\gamma^2}{\sigma_j^4(1 + \gamma^2 s)} . \tag{A3.27}$$

Next, the terms off the diagonal of $[\underline{\mu}]^{-1}$, $(\underline{\mu}^{-1})_{jk}$, are found. The analogy with $\det[\underline{\mu}]$ no longer applies, so that we must compute directly the minors M_{jk} where $j \neq k$. M_{jk} is defined as the determinant of the matrix $[\underline{\mu}]$ with row j and column k deleted. In this case not only are the terms $(\gamma^2 + \sigma_j^2)$ and $(\gamma^2 + \sigma_k^2)$ missing from the matrix, but one row (k') and one column (j') contain all γ^2. When $j < k$, $j' = j$ and $k' = k-1$; when $j > k$, $j' = j-1$ and $k' = k$. In either case, $j'+k' = j+k-1$. Subtracting row k' from each row, and factoring out γ^2, we obtain:

$$M_{jk} = \gamma^2 \det \begin{array}{l} \qquad\qquad\qquad\quad \text{column } j' \\ \left[\begin{array}{cccccc} \sigma_1^2 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & & \vdots \\ 1 & 1 & \cdots & 1 & \cdots & 1 \\ \vdots & \vdots & & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & \sigma_n^2 \end{array}\right] \begin{array}{l} \\ \\ \\ \longleftarrow \text{row } k' \\ \\ \\ \end{array} \end{array} \quad . \tag{A3.28}$$

Expanding the determinant by the j'^{th} column, we find:

$$M_{jk} = \frac{\gamma^2 (-1)^{j+k-1} \left(\prod_{i=1}^{n} \sigma_i^2 \right)}{\sigma_j^2 \sigma_k^2} \quad , \tag{A3.29}$$

so that

$$cf_{jk} = (-1)^{j+k} M_{jk}$$

$$= \frac{-\gamma^2 \left(\prod_{i=1}^{n} \sigma_i^2 \right)}{\sigma_j^2 \sigma_k^2} \quad . \tag{A3.30}$$

Thus the elements $(\underline{\mu}^{-1})_{jk}$, where $j \neq k$, are:

$$(\underline{\mu}^{-1})_{jk} = cf_{kj}/\det[\underline{\mu}]$$

$$= \frac{-\gamma^2}{\sigma_j^2\sigma_k^2(1+\gamma^2 s)} \quad . \tag{A3.31}$$

Now we can compute $\hat{\Gamma}_{ml}$, which is defined as the value of Γ that maximizes $p(\underline{X}|\Gamma)$, or equivalently maximizes $\ln[p(\underline{X}|\Gamma)]$, for a particular $\underline{X}$. From Eq. A3.22, we compute:

$$\ln[p(\underline{X}|\Gamma)] = -\frac{1}{2}\ln\left\{(2\pi)^n \det[\underline{\mu}]\right\} - \frac{1}{2}[\underline{X}-(\Gamma+\underline{\varepsilon})][\underline{\mu}]^{-1}[\underline{X}-(\Gamma+\underline{\varepsilon})]^t$$

$$= -\frac{1}{2}\ln\left\{(2\pi)^n \det[\underline{\mu}]\right\}$$

$$-\frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}[X_j-(\Gamma+\varepsilon_j)](\underline{\mu}^{-1})_{jk}[X_k-(\Gamma+\varepsilon_k)] \quad . \tag{A3.32}$$

Differentiating with respect to Γ, we obtain:

$$\frac{\partial \ln[p(\underline{X}|\Gamma)]}{\partial\Gamma} = \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}\{[X_j-(\Gamma+\varepsilon_j)] + [X_k-(\Gamma+\varepsilon_k)]\}(\underline{\mu}^{-1})_{jk} \quad . \tag{A3.33}$$

For convenience we define $Y_j = X_j - (\Gamma+\varepsilon_j)$, so that

$$\frac{\partial \ln[p(\underline{X}|\Gamma)]}{\partial\Gamma} = \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n}(Y_j + Y_k)\,(\underline{\mu}^{-1})_{jk} \quad . \tag{A3.34}$$

Substituting the values of $(\underline{\mu}^{-1})_{jj}$ and $(\underline{\mu}^{-1})_{jk}$ obtained previously, we get:

$$\frac{\partial \ln[p(\underline{X}|\Gamma)]}{\partial\Gamma} = \frac{-\gamma^2}{2(1+\gamma^2 s)}\sum_{j=1}^{n}\sum_{k=1}^{n}\frac{(Y_j+Y_k)}{\sigma_j^2\sigma_k^2} + \sum_{j=1}^{n}\frac{Y_j}{\sigma_j^2} \quad . \tag{A3.35}$$

Breaking up the double summation, and recalling the definitions $s = \sum_{i=1}^{n} 1/\sigma_i^2$ and $Y_j = X_j - (\Gamma+\varepsilon_j)$, the derivative becomes:

$$\frac{\partial \ln[p(\underline{X}|\Gamma)]}{\partial\Gamma} = \frac{-\gamma^2}{2(1+\gamma^2 s)}\left(2s\sum_{j=1}^{n} Y_j/\sigma_j^2\right) + \sum_{j=1}^{n} Y_j/\sigma_j^2$$

$$= \frac{1}{1+\gamma^2 s}\sum_{j=1}^{n} Y_j/\sigma_j^2$$

$$= \frac{1}{1+\gamma^2 s}\sum_{j=1}^{n}[X_j - (\Gamma+\varepsilon_j)]/\sigma_j^2 \quad . \tag{A3.36}$$

Setting the result equal to zero, we obtain the likelihood equation, the solution of which is $\hat{\Gamma}_{ml}$:

$$\frac{1}{1+\gamma^2 s}\sum_{j=1}^{n}[X_j - (\hat{\Gamma}_{ml}+\varepsilon_j)]/\sigma_j^2 = 0. \tag{A3.37}$$

Thus:

$$\hat{\Gamma}_{ml} = \frac{\sum_{j=1}^{n} (X_j - \varepsilon_j)/\sigma_j^2}{\sum_{j=1}^{n} 1/\sigma_j^2} \quad . \tag{A3.38}$$

Now the mean and variance of $\hat{\Gamma}_{ml}$ are computed. From equations A3.15 and A3.38, it is clear that:

$$E(\hat{\Gamma}_{ml}|\Gamma) = \Gamma \quad . \tag{A3.39}$$

The second derivative of the log of the likelihood is used to calculate the variance of the estimate. Differentiating Eq. A3.36 we get:

$$\frac{\partial^2 \ln[p(\underline{X}|\Gamma)]}{\partial \Gamma^2} = \frac{-s}{1+\gamma^2 s} \quad . \tag{A3.40}$$

Thus the variance is:

$$\mathrm{Var}(\hat{\Gamma}_{ml}|\Gamma) = -\left(\frac{\partial^2 \ln[p(\underline{X}|\Gamma)]}{\partial \Gamma^2}\right)^{-1}$$

$$= \gamma^2 + 1/s \ . \tag{A3.41}$$

APPENDIX IV. FOOTNOTES

1. See Miller (1956).

2. See Neisser (1967) and Pollack (1959).

3. In addition, the observers were allowed to indicate a degree of confidence in their responses. The confidence rating response set included the following six responses: "Very sure H,L;" "Medium sure H,L;" "Unsure H,L;" "Unsure L,H;" "Medium sure L,H;" and "Very sure L,H."

4. These studies indicate that resolution decreases when the paradigm is changed from fixed- to roving-level. Harris (1952) studied frequency, while Pollack (1955 and 1956) studied intensity.

5. For a more complete and rigorous description of the preliminary theory, see Durlach and Braida (1969).

6. In a more general form of the theory, the variance of $p[\overline{Y}(T)|Y]$ at time $t = T$ is:

$$\mathrm{Var}[\overline{Y}(T)|Y] = \rho^2(T) = \int_0^T f^2(t)\,dt ,$$

where $f^2(t)$, the interference function, depends on the detailed nature of the interference during the interval $0 \leq t \leq T$. In situations where $f^2(t)$ is a constant (something we would expect, for example, when the only interference is the passage of time), $\rho^2(T)$ becomes a linear function of T. The restricted form of the trace mode presented in the text, where $\rho^2(T) = 2AT$ (the factor of 2 is included to simplify some formulas derived later), is equivalent to the diffusion model of Kinchla and Smyzer (1967).

7. To allow for the fact that the observers' criteria were found to vary over the different intensity levels, C should be written C_i. More is said about this in Chapter VIII.

8. Referring to the theorem proven in Appendix III, part A, let events ζ_1 and ζ_2 be (I_i, I_i^*) and (I_i^*, I_i), respectively. Define

$\varepsilon = K \log(I_i/I_i^*)$, $\kappa = 1$, $\pi^2 = G^2R^2$, and note that $\mathrm{Cov}(X_Q, X_T) = 2\beta^2$. The results stated in the text then follow directly from the theorem.

9. See Torgerson (1954 and 1958), as well as Thurstone (1927).

10. Wickelgren (1969) considered a "higher-lower" model as well as a "same-different" model. These two kinds of models make contrasting predictions about the ROC's obtained in "higher-same-lower" experiments. The frequency discrimination data collected by Wickelgren tend to support the "same-different" model as opposed to the "higher-lower" model.

11. For a discussion of the validity of Pollack's computation of "equivalent information transmission," see Durlach and Braida (1969) or Braida (1969).

12. These anomalous results in magnitude estimation, and the explanation proposed by Braida and Durlach, appear possibly related to the anomalous results obtained from two of the observers in this thesis. This point is discussed in Chapters VII and VIII.

13. For a discussion of the consistency of some of the parameter estimates, see Braida and Durlach (1972).

14. See Chapter V, especially the review of Rabinowitz (1970).

15. The ranges, R, in these experiments depended on the increment, q. In Expt. 1 R = 5.4 + q, 3.6 + q, or 1.8 + q; and in Expt. 1A R = q. The values given in the text, and in Table 6.1, were determined by the average value of q in each case.

16. In Expt. 2 R = 5.4 + q, or 1.8 + q; and in Expt. 2A R = q.

17. In Expt. 3 R = 5.4 + q, or 2.7 + q; and in Expt. 3A R = q.

18. Observer BD completed only 20% of Expts. 1 and 1A, and did not participate in Expts. 2, 2A, or 4. Observer PS completed only 50% of Expts. 1 and 1A, did not participate in Expts. 2 or 2A, and completed only 25% of Expt. 4.

19. Virtually all the results for resolution per bel were plotted logarithmically mainly for two reasons. First, in examining the experimental results, the primary interest was in observing the percent changes of $\hat{\delta}'_1$ and $\hat{\delta}'_2$ as functions of the various experimental parameters; the absolute values or arithmetic changes of $\hat{\delta}'_1$ and $\hat{\delta}'_2$ were considered of secondary importance. In a logarithmic plot, uniform percent changes appear as uniform differences. Second, it was found that the results for resolution per bel differed primarily by a constant multiplicative factor from one observer to another. Consequently, although linear plots of resolution per bel showed substantial differences among the observers, logarithmic plots minimized the effects of these differences, displaying them merely as constant vertical shifts.

20. See Chapter IV for a description of these predictions.

21. The performance of these observers appears related to the performance of observers in the magnitude estimation experiment reported by Braida and Durlach (1972). This relation is discussed further in Chapter VIII.

22. Previous studies have indicated that simple discrimination of pitch improves as T_p is increased from 0.1 sec to values between 0.8 and 1.6 sec [König (1957)], and that pitch memory improves as T_p is increased from 2 to 8 sec [Wickelgren (1966)].

23. This result was obtained only for Observers JB and JW, not for SK.

24. A ratio of δ's was chosen for this measure, rather than a difference, for the same reasons that the results for resolution per bel were plotted logarithmically. See footnote 19.

25. All the observers but one, Observer JW in Expt. 3, exhibited similar resolution edge-effects. See the selected graphs for individual observers in Appendix II.

26. This measure was chosen instead of the more obvious

$$\frac{\hat{\delta}_1'(I_2;\ I_1)\ \hat{\delta}_1'(I_{13};\ I_{12})}{\hat{\delta}_1'(I_6;\ I_7)\ \hat{\delta}_1'(I_7;\ I_8)}$$

because, by including data from more intensity levels, it was less noisy. In addition, $\nabla\hat{\delta}_1'$ was not plotted for very small R (R = 0.3) because it tended to be rather noisy. This was the case since $\hat{d}_1'(I_{i+1};\ I_i)$ was nearly zero when R was small.

27. The Weber's law normalization was not done for the results from Expt. 5 because the necessary fixed-level data had not been collected for these observers.

28. In future revisions, the observer may also use the trace from the previous trial as an anchor.

29. As with the preliminary theory, some revision may be needed here. See Chapter IX.

30. See the discussion of renewal processes in Cox and Miller (1965), p. 342-343.

31. When the sensation being coded, Y, is very near the anchor, ψ_a, the Gaussian prediction as well as the expressions for the mean and variance of the verbal code will be in error. For simplicity, however, this problem is ignored in the current discussion.

32. This result is the main significance of the counting model; I regard it as more important than the details of the counting process.

33. The choice of the lower edge is arbitrary. The results are essentially unchanged if, instead, either the upper edge or the reference are used.

34. Referring to the theorem proven in Appendix III, part B, define $\Gamma = [\alpha(I) - \alpha(I_{\psi_1})]/\mu W$, $X_i = Q_i$, $\varepsilon_i = [\alpha(I_{\psi_1}) - \alpha(I_{\psi_i})]/\mu W$, $\gamma^2 = \beta^2/\mu^2 W^2$, $\sigma_i^2 = |\alpha(I) - \alpha(I_{\psi_i})|\sigma_s^2/\mu^3 W + \beta_\psi^2/\mu^2 W^2$, and $\hat{\Gamma}_{ml} = \overline{Q}$. Thus the conditions $E(X_i|\Gamma) = \Gamma + \varepsilon_i$, $Var(X_i|\Gamma) = \gamma^2 + \sigma_i^2$, and $Cov(X_i, X_j|\Gamma) = \gamma^2$ are satisfied. Assuming that the observer knows the $\{\varepsilon_i\}$, the set of numerical representations of the mean distances from the various anchors to the lower edge of the context (quantities which remain fixed during an experimental run), the results stated in the text then follow directly from the theorem.

35. We now refer to $p(\overline{Q}|I, \{I_{\psi_a}\})$ as $p(\overline{Q}|I)$, since the $\{I_{\psi_a}\}$ are constant for a given experiment.

36. This corresponds to using the point on the ROC where $P_d = 1 - P_f$ to define d'.

37. Referring to the theorem proven in Appendix III, part A, let events ζ_1 and ζ_2 be (I_i, I_i^*) and (I_i^*, I_i), respectively. Define $\varepsilon = \alpha(I_i) - \alpha(I_i^*)$, $\kappa = \mu W$, $2\pi^2 = 1/\sum_a 1/\sigma_a^2(I_i) + 1/\sum_a 1/\sigma_a^2(I_i^*)$, and note that $Cov(X_Q, X_T) = 2\beta^2/\mu W = 2\beta^2/\kappa$. The results stated in the text then follow directly from the theorem.

38. It turns out that the predictions of the revised theory depend somewhat on N, the number of overall intensity levels. This dependence, although slight, has not yet been investigated. For this preliminary discussion of one-interval paradigms, N has been fixed at 13, as in the actual experiments.

39. See footnote 38. For this preliminary discussion of two-interval paradigms, N has been fixed at 10, as in the actual experiments; and the analysis has been performed by averaging together adjacent pairs of levels, as in the actual data analysis.

40. See footnote 38.

41. See footnote 38.

42. See footnote 39.

43. Taylor (1972) reports a preliminary investigation in which identification experiments were conducted with and without feedback, with a random standard, and with a fixed standard. Among other things, the results incidate that feedback may do more than just increase the utility of the previous trial as an anchor. It may, for example, help reduce the variability of the anchors from the edges of the range, by improving the observer's notion of the context.

44. Because the shapes of the ROC's are strongly dependent on the form of the underlying decision densities, slight departures from Gaussian densities may severely alter the shapes of the ROC's. For example, if on a small fraction of the trials (say about 4%) the observer's concentration is reduced, increasing the effective variances of the decision densities on those trials, the net densities (averaged over all the trials) will be non-Gaussian, and the ROC's will be curved. See Lippmann (1973) for a further discussion of this situation.

45. The assumption is made that the statistics of the steps are determined by the context of a particular experimental run, rather than by the context of the entire experiment as are the anchors, primarily because the resulting dependence of resolution per bel on range is in agreement with the data. The intuitive basis of this assumption is that while the choice of anchors is a tactical decision made by the observer, and might be influenced by instructions, the steps are thought of as arising from recent sensations and are not chosen by the observer.

46. The idea of substitution errors producing response bias was proposed by Rimm (1967).

47. Another possibility is that the formation of the trace from the sensation may involve substitution errors, thus producing the effect for small T.

48. In the limit as $J \to \infty$ (so that the bias term $\to 0$), setting $s^2/\Delta t = 2A$ makes the model equivalent to the trace mode of the preliminary theory. See footnote 50.

49. $\Phi(t)$ may be recognized as the Ornstein-Uhlenbeck process. See Cox and Miller (1965), p 225-227.

50. Note that $\lim_{J\to\infty} \delta_2' = \frac{2K}{\sqrt{2\beta^2 + 2AT}}$, identical with the prediction from the trace mode of the preliminary theory.

51. Referring to the theorem proven in Appendix III, part B, define $\Gamma = \alpha(I_1)$, $X_1 = \overline{Y}_1(T)$, $X_2 = Y_{Q1}$, $\varepsilon_i = 0$, $\gamma^2 = \beta^2$, $\sigma_1^2 = 2AT$, $\sigma_2^2 = 2G^2R^2$, and $\hat{\Gamma}_{m1} = Y_{Q,T}$. Thus the conditions $E(X_i|\Gamma) = \Gamma + \varepsilon_i$, $Var(X_i|\Gamma) = \gamma^2 + \sigma_i^2$, and $Cov(X_i, X_j|\Gamma) = \gamma^2$ are satisfied. The results stated in the text then follow directly from the theorem.

52. See footnote 22. In addition, Wickelgren (1966) presents a model for the consolidation of the memory trace for pitch.

APPENDIX V. BIBLIOGRAPHY

1. Berliner, J.E., and Durlach, N.I. (1973). "Intensity Perception. IV. Resolution in Roving-Level Discrimination," J. Acoust. Soc. Amer. 53, 1270-1287.

2. Braida, L.D., (1969). "Intensity Perception in Audition," Ph.D. thesis, Dept. of Elec. Eng., Mass. Inst. Technol.

3. Braida, L.D., and Durlach, N.I. (1972). "Intensity Perception. II. Resolution in One-Interval Paradigms," J. Acoust. Soc. Amer. 51, 483-502.

4. Campbell, R.A., and Lasky, E.Z. (1967). "Masker Level and Sinusoidal-Signal Detection," J. Acoust. Soc. Amer. 42, 972-976.

5. Cox, D.R., and Miller, H.D. (1965). The Theory of Stochastic Processes (John Wiley & Sons, Inc., New York).

6. Durlach, N.I., and Braida, L.D. (1969). "Intensity Perception. I. Preliminary Theory of Intensity Resolution," J. Acoust. Soc. Amer. 46, 372-383.

7. Eijkman, E., Thijssen, J.M., and Vendrick, A.J.H. (1966). "Weber's Law, Power Law, and Internal Noise," J. Acoust. Soc. Amer. 40, 1164-1173.

8. Green, D.M., and Swets, J.A. (1966). Signal Detection Theory and Psychophysics (John Wiley & Sons, Inc., New York).

9. Harris, J.D. (1952). "The Decline of Pitch Discrimination with Time," J. Exp. Psychol. 43, 96-99.

10. Helson, H. (1964). Adaptation-Level Theory (Harper and Row Publishers, Inc., New York).

11. Kinchla, R.A., and Smyzer, F. (1967). "A Diffusion Model of Perceptual Memory," Perception Psychophys. 2, 219-229.

12. König, E. (1957). "Effect of Time on Pitch Discrimination Thresholds under Several Psychophysical Procedures; Comparison with Intensity Discrimination Thresholds," J. Acoust. Soc. Amer. 29, 606-612.

13. Lippmann, R.P. (1973). "The Effect of Payoffs on Sensitivity and Bias in Absolute Identification," S.M. thesis, Dept. of Elec. Eng., Mass. Inst. Technol.

14. Luce, R.D. (1963). "Detection and Recognition," in R.D. Luce, R.R. Bush, and E. Galanter (Eds.), Handbook of Mathematical Psychology, Vol. 1 (John Wiley & Sons, Inc., New York). 103-189.

15. McGill, W.J., and Goldberg, J.P. (1968). "Pure-Tone Intensity Discrimination and Energy Detection," J. Acoust. Soc. Amer. 44, 576-581.

16. Miller, G.A. (1956). "The Magical Number Seven, Plus or Minus Two: Some Limits on our Capacity for Processing Information," Psychol. Rev. 63, 81-97.

17. Neisser, U. (1967). Cognitive Psychology (Appleton-Century-Crofts, Inc., New York).

18. Pollack, I. (1954). "Intensity Discrimination Thresholds under Several Psychophysical Procedures," J. Acoust. Soc. Amer. 26, 1056-1059.

19. Pollack, I. (1955). "Sound Level Discrimination and Variation of Reference Testing Conditions," J. Acoust. Soc. Amer. 27, 474-480.

20. Pollack, I. (1956). "Identification and Discrimination of Components of Elementary Auditory Displays," J. Acoust. Soc. Amer. 28, 906-909.

21. Pollack, I. (1959). "Message Uncertainty and Message Reception," J. Acoust. Soc. Amer. 31, 1500-1508.

22. Pynn, C.T., Braida, L.D., and Durlach, N.I. (1972). "Intensity Perception. III. Resolution in Small-Range Identification," J. Acoust. Soc. Amer. 51, 559-566.

23. Rabinowitz, W.M. (1970). "Frequency and Intensity Resolution in Audition," S.M. thesis, Dept. of Elec. Eng., Mass. Inst. Technol.

24. Rimm, D.C. (1967). "The Effect of Interpolated Stimulation on Short-Term Memory for Tones," Psychol. Record 17, 429-435.

25. Taylor, J.W. (1972). "A Comparison of Standards and Feedback in Intensity Perception," S.B. thesis, Dept. of Elec. Eng., Mass. Inst. Technol.

26. Thurstone, L.L. (1927). "A Law of Comparative Judgment," Psychol. Rev. 34, 273-286.

27. Torgerson, W.S. (1954). "A Law of Categorical Judgment," in L.H. Clark, Consumer Behavior (New York University Press, Washington Square) 92-93.

28. Torgerson, W.S. (1958). Theory and Methods of Scaling (John Wiley & Sons, Inc., New York).

29. Viemeister, N.F. (1972). "Intensity Discrimination of Pulsed Sinusoids: The Effects of Filtered Noise," J. Acoust. Soc. Amer. 51, 1265-1269.

30. Wickelgren, W.A. (1966). "Consolidation and Retroactive Interference in Short-Term Recognition Memory for Pitch," J. Exp. Psychol. 72, 250-259.

31. Wickelgren, W.A. (1969). "Associative Strength Theory of Recognition Memory for Pitch," J. Math. Psychol. 6, 13-61.

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