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# A NEW BEHAVIORAL PRINCIPLE FOR TRAVELERS IN AN URBAN NETWORK

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16. Abstract

A new hypothesis on traveler behavior in a network is described which is based on empirical findings in the theory of travel budgets. It is translated into an assignment principle for characterizing the distribution of travelers, as well as the demand and mode split. A numerical technique is proposed, and it is applied to several examples to illustrate qualitative features. It is applied to a small network to assess the effect of using vehicle-actuated signal control. It is also applied to a rather large network to investiagte traffic control policies and construction policies. Open questions and areas for future research are discussed.

The new hypothesis is significant because it considers all travel decisions -- whether or not to travel, where to go, and what mode to use -- in a single, unified way. The feedback from travel time and money costs on links and modes to traveler behavior is explicitly considered.



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Assignment, Demand, Mode Split, Multi-mode, Behavioral Principle, Equilibrium Travel Budgets, Networks, Planning

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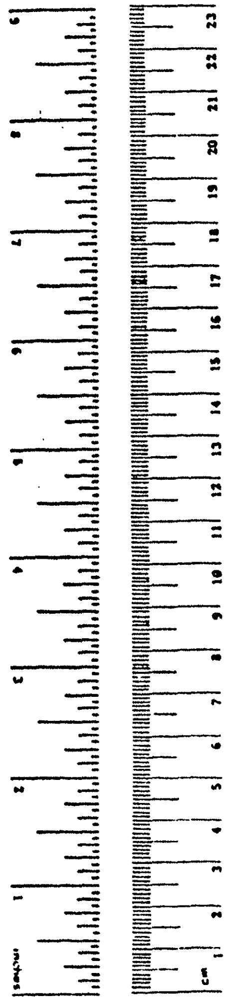
# METRIC CONVERSION FACTORS

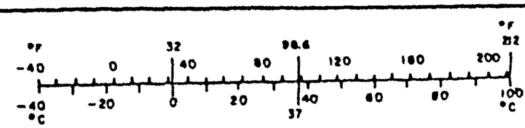
## Approximate Conversions to Metric Measures

| Symbol | When You Know | Multiply by | To Find | Symbol |
|---|---|---|---|---|
| | | **LENGTH** | | |
| in | inches | 2.5 | centimeters | cm |
| ft | feet | 30 | centimeters | cm |
| yd | yards | 0.9 | meters | m |
| mi | miles | 1.6 | kilometers | km |
| | | **AREA** | | |
| $in^2$ | square inches | 6.5 | square centimeters | $cm^2$ |
| $ft^2$ | square feet | 0.09 | square meters | $m^2$ |
| $yd^2$ | square yards | 0.8 | square meters | $m^2$ |
| $mi^2$ | square miles | 2.6 | square kilometers | $km^2$ |
| | acres | 0.4 | hectares | ha |
| | | **MASS (weight)** | | |
| oz | ounces | 28 | grams | g |
| lb | pounds | 0.45 | kilograms | kg |
| | short tons (2000 lb) | 0.9 | tonnes | t |
| | | **VOLUME** | | |
| tsp | teaspoons | 5 | milliliters | ml |
| Tbsp | tablespoons | 15 | milliliters | ml |
| fl oz | fluid ounces | 30 | milliliters | ml |
| c | cups | 0.24 | liters | l |
| pt | pints | 0.47 | liters | l |
| qt | quarts | 0.95 | liters | l |
| gal | gallons | 3.8 | liters | l |
| $ft^3$ | cubic feet | 0.03 | cubic meters | $m^3$ |
| $yd^3$ | cubic yards | 0.76 | cubic meters | $m^3$ |
| | | **TEMPERATURE (exact)** | | |
| °F | Fahrenheit temperature | 5/9 (after subtracting 32) | Celsius temperature | °C |




## Approximate Conversions from Metric Measures

| Symbol | When You Know | Multiply by | To Find | Symbol |
|---|---|---|---|---|
| | | **LENGTH** | | |
| mm | millimeters | 0.04 | inches | in |
| cm | centimeters | 0.4 | inches | in |
| m | meters | 3.3 | feet | ft |
| m | meters | 1.1 | yards | yd |
| km | kilometers | 0.6 | miles | mi |
| | | **AREA** | | |
| $cm^2$ | square centimeters | 0.16 | square inches | $in^2$ |
| $m^2$ | square meters | 1.2 | square yards | $yd^2$ |
| $km^2$ | square kilometers | 0.4 | square miles | $mi^2$ |
| ha | hectares (10,000 $m^2$) | 2.5 | acres | |
| | | **MASS (weight)** | | |
| g | grams | 0.035 | ounces | oz |
| kg | kilograms | 2.2 | pounds | lb |
| t | tonnes (1000 kg) | 1.1 | short tons | |
| | | **VOLUME** | | |
| ml | milliliters | 0.03 | fluid ounces | fl oz |
| l | liters | 2.1 | pints | pt |
| l | liters | 1.06 | quarts | qt |
| l | liters | 0.26 | gallons | gal |
| $m^3$ | cubic meters | 35 | cubic feet | $ft^3$ |
| $m^3$ | cubic meters | 1.3 | cubic yards | $yd^3$ |
| | | **TEMPERATURE (exact)** | | |
| °C | Celsius temperature | 9/5 (then add 32) | Fahrenheit temperature | °F |



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# 1. INTRODUCTION

## 1.1 Purpose

The purpose of this report is to introduce a new model of traveler behavior in a transportation network and to show how it can be used to help make management and design decisions.

In the management or design of a transportation system, decisions must be made frequently. Short range decisions, such as the best signal control; long term decisions, such as whether to build a rapid transit system; and intermediate decisions must be made while considering their consequences. One must always answer the question: if a given action is taken, how will travelers respond to it and what will be the levels of flow of vehicles and travelers throughout the system? This question is often answered with the use of a model or a set of models of traveler behavior.

While traffic engineers have long had computational techniques to assess the effects of transportation system changes on flows (particularly construction rather than control policy modifications), these techniques have not been based on an overall model of traveler behavior.

Some control policies are based on the assumption that travelers have fixed travel patterns that do not change. However, this is a strong assumption; and there are empirical (Stephenson and Teply, 1981) and theoretical (akcelik and Maher, 1975) reasons for doubting it. If a traffic engineer reduces the delay in part of the network, it will attract flow from other parts of the network. This will change flow levels.

Assignment, which is the calculation of flow patterns, is based on the idea that travelers have fixed trips (origins and destinations) amd frequencies of making the trips and that their routes are chosen to minimize travel time. Assignment models are used to predict the effects of major changes in the system, such as roadway or rapid transit construction.

Demand and mode split are often predicted on the basis of a curve-fit procedure whose parameters are determined by calibration with current data. Thus, although existing assignment, demand, and mode split models can be operated simultaneously, there is no existing unified single model to explain and predict the whole range of urban traveler behavior.

The work introduced here differs from this in that a model is proposed which is intended to explain and predict assignment, demand, and mode split. A single unifying idea underlies the model. Furthermore, we expect that it can be extended to cover other phenomena, such as residential choice, without an important alteration of its fundamental assumptions.

## 1.2 Traveler Behavior and Assignment Principles

The models presented in Section 2 are based on the idea that travelers have maximum amounts of time and money that they are willing and able to spend during a day traveling. They may spend less but not more. Within these constraints, they maximize the benefit they obtain from traveling. This benefit, which we call value or utility, is determined by the links and nodes of the network a traveler passes and on the mode he employs.

An important concept is that of a journey. Travelers start and end their journeys at the same point: a residence. They travel in a continuous path, possibly with several loops.

To calculate flows, travel times, and other quantities, the statement about individual behavior must be converted into a statement about flows. This is done in Section 2.

## 1.3 Outline

In Section 2, a model of individual behavior is presented, and two statements about flows -- called assignment principles -- are derived from it. For comparison purposes, this process is reviewed for Wardrop's user optimization principle. A numerical procedure is presented in Section 3 to calculate distributions from one of these principles.

In Section 4, a sequence of networks is analyzed using the results of Sections 2 and 3. These networks are all small, and they illustrate various qualitative features of the model. Section 5 describes another small network in which the benefits of vehicle-actuated traffic signals are assessed.

Sections 6 and 7 discuss a large network. Various traffic control and construction strategies are considered, and their effects are examined. In Section 8, we discuss some easy extensions as well as definite limitations to the models in Secton 2, and in Section 9, future research directions are suggested.

## 2. ASSIGNMENT PRINCIPLE

Traffic Assignment is the computation of vehicle and traveler flows in a transportation network. It is based on data on the travelers such as their origins, possible destinations, and car ownership; and on the physical attributes of the network such as its structure and its link flow capacities. Because travelers make important choices that affect the distribution of flows (such as whether or not to travel, and where to go), all assignment schemes are based, explicitly or otherwise, on a model of human behavior. We refer to this model as a behavioral principle; when it is translated into a statement about flows it is an assignment principle.

The major theoretical advance described in this report is a new assignment orinciple. This assignment principle combines the travel budget theory of Zahavi (1979) and others (Kirby, 1981) with a detailed representation of a network. It extends the current state of the art by combining demand, mode split, and route assignment in an integrated formulation. Because this formulation is based on the travel budget theory, it has solid empirical backing. The discussions of vehicle-actuated signals in Section 5, of control of multi-mode networks in Section 6, and of network planning in Section 7 are based on this principle.

In Section 2.1, we review the user- optimization assignment principle of Wardrop (1952). We show the relationship between an assumption about the behavior of individuals in a transportation network and the widely used statement about flows in the networks on which all modern assignment computation techniques are based. We state a new behavioral principle for individual travelers in a network in Section 2.2. Actually, this principle is not new; it is the basis of Zahavi's (1979) UMOT (Unified Mechanism of Travel) model. Its novelty comes from its new context. The new principle is based on the concept of travel budgets: travelers and potential travelers are assumed to have certain maximum daily expenditures of time and money for transportation. These budgets depend on such attributes as household size and household income. Following the same development as for user-optimization, a set of statements about flows in a network is derived from the individual behavioral principle.

Two versions of the new assignment principle are presented. The first, described in Section 2.2, assumes that budgets are equal for all members of each class of travelers. The second, in Section 2.3, allows budgets to differ for various members of the same class. The second formulation, which is more nearly in agreement with reality, allows the same traveler to choose different travel patterns of different days. It is fortunate that it is easier to analyze this version numerically. Important features of the new principle are discussed in Section 2.4.

The new principle differs from formulations based on Wardrop's principle in many ways. First, daily travel behavior is considered, not single trips. Second, following Zahavi, travel is viewed not as a disutility, but rather as a utility, providing travelers with access to opportunities, and paid for out of limited budgets. Consequently, instead of assuming fixed origins and destinations, we assume a fixed origin, the traveler's residence location. Travelers are assumed to have daily journeys that start and end at that point.

## 2.1 Review of Wardrop's Assignment Principle

### 2.1.1 Assumptions on Individual Behavior

Wardrop (1952) has asserted two principles which are intended to characterize the bahavior of travelers in a transportation network. One, which has since been called user optimization, is based on the premises that

a. Travelers have fixed origins and destinations.

b. Travelers seek paths which minimize their travel time.*

This principle has been of great importance in the development of transportation models. Its evident limitations have provoked many researches to suggest extensions to include elastic demand (Florian and Nguyen, 1974) multiple modes (Leblanc and Abdulaal, 1980) and other modifications.

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*Wardrop has proposed an alternative principle, called system optimization, in which statement 2 is replaced by the assertion that the whole body of travelers seeks a distribution of traffic that minimizes average travel time.

### 2.1.2 Resultant Statements about Flows

To calculate flows, the above statements about individuals must be translated into a set of stameents about flows. For the purposes here, it is coveneient to discuss path flows; these statements can also be expressed in terms of link flows (Dafermos, 1980).

Let i be an index representing an origin node and a destination node in the network. Let $D^i$ be the demand, in vehicles per time unit (most often hours) for that origin-destination pair. That is, $D^i$ travelers per hour wish to go from $a_i$ (the origin) to $b_i$ (the destination). Let j be a path in the network that connects $a_i$ and $b_i$, and let $J_i$ be the set of all such paths. Let $x_j$ be the flow rate of vehicles in path j.

The flow rates satisfy

$$x_j \geq 0 , \quad j \in J_i, \quad \text{all } j, \tag{2.1}$$

$$\sum_{j \in J_i} x_j = D^i \quad . \tag{2.2}$$

Let $t_j$ be the travel time of path j. This travel time is the sum of the travel times of the links that constitute path j. The choice of the shortest path can be represented by considering the flows and travel times of any pair of paths.

Let j, k $\in J_i$. Then,

$$x_j > 0, \quad x_k > 0 \implies t_j = t_k \; . \tag{2.3a}$$

$$t_j > t_k \implies x_j = 0. \tag{2.3b}$$

That is, since travelers choose the shortest path, if both paths have flow, they must have equal travel time. If one path has greater travel time than the other, no travelers will take it.

Path travel time is the sum of link travel times:

$$t_j = \sum_{\ell} A_{j\ell} \tau_{\ell}, \tag{2.4}$$

where $\tau_\ell$ is the travel time of link $\ell$, and $A_{j\ell}$ is the path-link incidence matrix x. That is,

$$A_{j\ell} = \begin{cases} 1 & \text{if link } \ell \text{ is in path } j, \\ 0 & \text{otherwise .} \end{cases} \qquad (2.5)$$

Link travel time is a function of link flow. Let $f_\ell$ be the flow rate of vehicles on link $\ell$. Then,

$$f_\ell = \sum_j A_{j\ell} x_j \ . \qquad (2.6)$$

This is, the flow on a link is the sum of the flows on all paths that pass through that link.

Finally, $\tau_\ell$ is a function of f, the vector of link flows in the network. An important special case is where $\tau_\ell$ depends on $f_\ell$ only. This function is most often assumed to be positive and monotonically increasing with f.

### 2.1.3 Limitations

This principle neglects certain important features of traveler behavior. First, origins and destinations are not fixed; the decision if and where to travel (i.e., the demand) depends on congestion which in turn depends on the demand on the network. Demand also depends on the money cost of travel which is not considered at all.

Ways of treating some of these features have been proposed by many researchers. However the resulting formulations often require parameters that are difficult or impossible to obtain and, because they are based on the fitting of parameters rather than a model of human behavior, they have limited predictive value (Zahavi, 1981).

## 2.2 New Principle - Deterministic Version

### 2.2.1 Assumptions on Individual Behavior

Many authors have observed certain regularities about travelers' behavior. (Kirby, 1981). In developed countries throughout the world, the average traveler spends between 1.0 and 1.5 hours every day in the transportation system. Furthermore, travelers who own cars spend about 11 percent of their income on travel, and those who do not own cars spend 3 to 5 percent (Zahavi, 1979). This must also influence the amount and kind of travel that each individual uses.

It is clear that a behavioral principle that takes these observations into account has a chance of being more realistic than those associated with user optimatization and its extensions. We note, however, that such a principle, to be consistent with the observations, must treat daily travel, not single trips. This is because the travel time budget is an amount of time spend each day. We define a journey to be the path in the network that an individual takes during a day. It is made up of several trips.

In the course of day, most travelers start and end their journeys at the same point: their residences. (A small number of people do not, including travelers starting or returning from business or vacation trips.) These journeys need not be simple loops. For example, some workers go home for lunch; some people travel to work and home, and then out again for shopping or entertainment.

We assume that such journeys are feasible for an individual only if the total travel time is less than his daily budget T, and the total money cost is less than his budget M. The money costs include fixed costs for a car as well as running costs if he drives; or fares if he takes transit or, of course, both if he takes a car for part of his journey (evening shopping or entertainment) and transit for the rest (work trips).

Here, we assume that each traveler has fixed budgets which do not vary from day to day, and which are the same as those of all others in this class. In Section 2.3, we relax these assumptions.

Classes are defined as sets of travelers with the same origin node (residence location) and with the same budgets for time and money to be spent each day for travel. These quantities are determined by such socio-economic indicators as household income and number of travelers per household. (Zahavi (1979) asserts that one's daily travel time and money budgets are influenced by such factors as travel speed; we treat both budgets as fixed at this stage of the analysis.)

Of all the journeys that satisfy these budget contraints, which will be chosen by a traveler? Zahavi suggests that travelers attempt to maximize their access to spatial and economic opportunities. He indicates that the daily travel distance may not be their precise objective, but is quite adequate as a first approximation for a given urban structure and transport network.

Because we are treating a detailed representation of a network, we can represent other objectives. For example, we can assign a value to each link and each node in the network, and add the values encountered on a journey to yield the value of the journey. These values can differ from class to class: that is, the value of a given node (e.g., the location of an extremely expensive shopping mall) can depend on whether the traveler is rich or poor.

To summarize: a traveler in class a chooses a journey p, among all journeys $P^a$ available to him, to

$$\underset{p \in P^a,}{\text{maximize}} \; w_p \text{ , the value or utility of journey p} \tag{2.7}$$

subject to the time and money budget constraints:

$$t_p \leq T^a , \tag{2.8}$$

$$m_p \leq M^a . \tag{2.9}$$

### 2.2.2 Resulting Statement about Flows

Equations (2.7) to (2.9) by themselves are not adequate to characterize network flows. For this purpose, they must be expressed in a form which is analogous to that of Section 2.1.1: a set of equations, inequalities, and logical relations involving flows.

Let $x_p$ be the flow of class of travelers on journey p. Then, $x_p$ must certainly satisfy

$$x_p \geq 0, \tag{2.10}$$

$$\sum_{p \in P^a} x_p = D^a, \tag{2.11}$$

where $D^a$ is the total number of travelers available. As we indicate below, $D^a$ can include people who will choose not to travel. We measure $x_p$ and $D^a$

in units of travelers per day.

To characterize flows, we must specify a set of relations that are consistent with (2.7) to (2.9). That is, if the demand $D^a$ is changed by a small amount, representing one more traveler, the new distribution must be consistent with the behavior of the new traveler.

Assume that for each class a, the journey $p \varepsilon P^a$ are indexed in order of increasing utility. That is, if $p_1 > p_2$, $w_{p_1} > w_{p_2}$. We assume that no two paths have the same utility value and that $D^a > 0$.

The journeys $p \in P^a$ are divided into four sets:

1). Infeasible journeys: are those for which

$$t_p > T^a ,$$

and/or

$$m_p > M^a .$$

If p is an infeasible journey, $x_p = 0$.

2). Constrained journeys are those for which

$$t_p = T^a \quad \text{and} \quad m_p \leq M^a ,$$

or

$$t_p \leq T^a \quad \text{and} \quad m_p = M^a .$$

3). There is at most one special journey p* such that

$$x_{p*} > 0,$$
$$t_{p*} < T^a ,$$
$$m_{p*} < M^a .$$

If no special journey exists, then we define p* as the smallest index of the constrained journeys.

4). Unutilized journeys are those for which

$$p < p^* \quad (\text{i.e., } w_p < w_{p*})$$

and

$$x_p = 0 .$$

To demonstrate that these conditions are consistent with (2.7) to(2.9), add a small increment $\delta D^a$ to the demand $D^a$. Then,the time and money costs associated with whatever journeys p are chosen by the new users comprising $\delta D^a$ are $t_p + \delta t_p$,and $m_p + \delta m_p$ respectively. Assume that $\delta t_p > 0$,and $\delta m_p > 0$. There are two cases to consider.

First, if there exists a special journey p*, all flow due to the new demand is added to $x_p^*$. To see this, consider the effect of $\delta t_p$ and $\delta m_p$ on the four possible sets of journeys:

1). Infeasible journeys remain infeasible since $m_p + \delta m_p > m_p > M_p^a$ and/or $t_p + dt_p > t_p > T_p^a$. Therefore,$\delta x_p = 0$.

2). Constrained journeys may not accept more flow and still remain feasible (under the assumption of monotonically increasing cost functions) $t_p + \delta t_p > T^a$ and $m_p \leq M^a$, or $m_p + \delta m_p > M^a$ and $t_p \leq T^a$. In fact, these journeys may lose flow if their costs increase due to links shared with journeys whose flow increases.

3). The special journey accepts more flow since

$$t_p^* + \delta t_p \leq T^a,$$

$$m_p^* + \delta m_p \leq M^a$$

for sufficiently small $\delta D^a$. Note that $\delta D^a$ may cause the special path to become a constrained path.

4). Unutilized journeys remain unaffected since the newly reduced flow will be assigned to the higher utility, still available, special path.

The second case to consider is really a degenerate case of the situation just described; namely,that there does not exist a special path for the original equilibrated system. That is, all flow is assigned to constrained journeys. When $D^a$ is increased to $D^a + \delta D^a$, it is still true that infeasible journeys and constrained journeys accept no more flow. New flow is then assigned to an unutilized journey which then becomes a special journey.

## 2.3 New Principle - Stochastic Version

While the principle in Section 2.2 captures a greater variety of phenomena than the user optimization principle of Section 2.1, there are several important features with which it cannot deal.

One difficulty is the fact that different members of the same socio-economic class may have different budgets on the same day, and that the same person may have different budgets on different days. This is because circumstances vary from day to day: the amount of food stored at home; over-sleeping (which may lead to a reduced time budget and an expanded money budget); the desire for entertainment, which varies from day to day. Zahavi (1979) shows that there is a consistent coefficient of variation in both budgets among a wide variety of populations.

Therefore, we redefine a class to be a set of people with the same residence location and with a common probability density function for time and money budgets. The probability that an individual of class a will have a time budget between $u$ and $u + \delta u$ and a money budget between $n$ and $n + \delta n$ is

$$f^a(u,n)\,\delta u\ \delta n\ . \tag{2.12}$$

The number of travelers of class a who have time budgets between $u$ and $u + \delta u$ and money budgets between $n$ and $n + \delta n$ is

$$D^a f^a(u,n)\,\delta u\ \delta n\ . \tag{2.13}$$

Let $R$ be a region in $(u,n)$ space. The number of travelers whose budgets fall in that region is

$$D^a \int_R f^a(u,n)\,\delta u\ \delta n. \tag{2.14}$$

To state an assignment principle, we relate flows to integrals of the form (2.14). Let

$$P^a = \{p_1^a,\ p_2^a, \ldots, p_k^a\}$$

be the set of journeys available to travelers in class a and, following the convention stated in Section 2.2, let $p_1^a$ be the least desirable journey and $p_k^a$ be the most desirable. The utilities of all the journeys are assumed to be distinct. Let $x_1^a, \ldots, x_k^a$ be the flows on those journeys and define

$$X_i^a = \sum_{j=i}^{k} x_j^a\ . \tag{2.15}$$

That is, $X_i^a$ is the total demand on journeys $p_i^a$, $p_{i+1}^a, \dots, p_k^a$. Let $t_i^a$ and $m_i^a$ be the time and money costs of journey $p_i^a$.

Since $p_k^a$ is the most desirable journey, the total flow on this journey is the number of travelers whose time budgets are greater than, or equal to, $t_k^a$, and whose money budgets are greater than, or equal to, $m_k^a$. That is,

$$x_k^a = D^a \int_{\substack{u \geq t_k^a \\ n \geq m_k^a}} f(u,n)\,du\,dn \,. \tag{2.16}$$

Define

$$R_k^a = \{(u,n) \mid u \geq t_k^a, \quad n \geq m_k^a\}, \tag{2.17}$$

$$U_k^a = R_k^a \,. \tag{2.18}$$

This region is illustrated in Figure 2.1. Using the new notation, (2.16) can be written

$$X_k^a = D^a \int_{U_k^a} f(u,n)\,du\,dn \,. \tag{2.19}$$

To characterize the rest of the flows, we define

$$R_i^a = \{(u,n) \mid u \geq t_i^a, \quad n \geq m_i^a\}, \tag{2.20}$$

the set of expenditure levels that equal or exceed the required expenditures on path i. The people who can afford journey $p_i^a$ or better are those whose budgets fall in any region $R_j^k$, $i \leq j \leq k$. Define

$$U_i^a = \bigcup_{j \geq i} R_j^a \,. \tag{2.21}$$

Then the number of travelers $X_i^a$ that take journey $p_i^a$ or better is the number whose budgets fall in region $U_i^a$. That is,

$$X_i^a = D^a \int_{U_i^a} f(u,n)\,du\,dn. \tag{2.22}$$

Equations (2.20) to (2.22) are a complete statement of the assignment principle in the stochastic case. The statement is remarkably concise

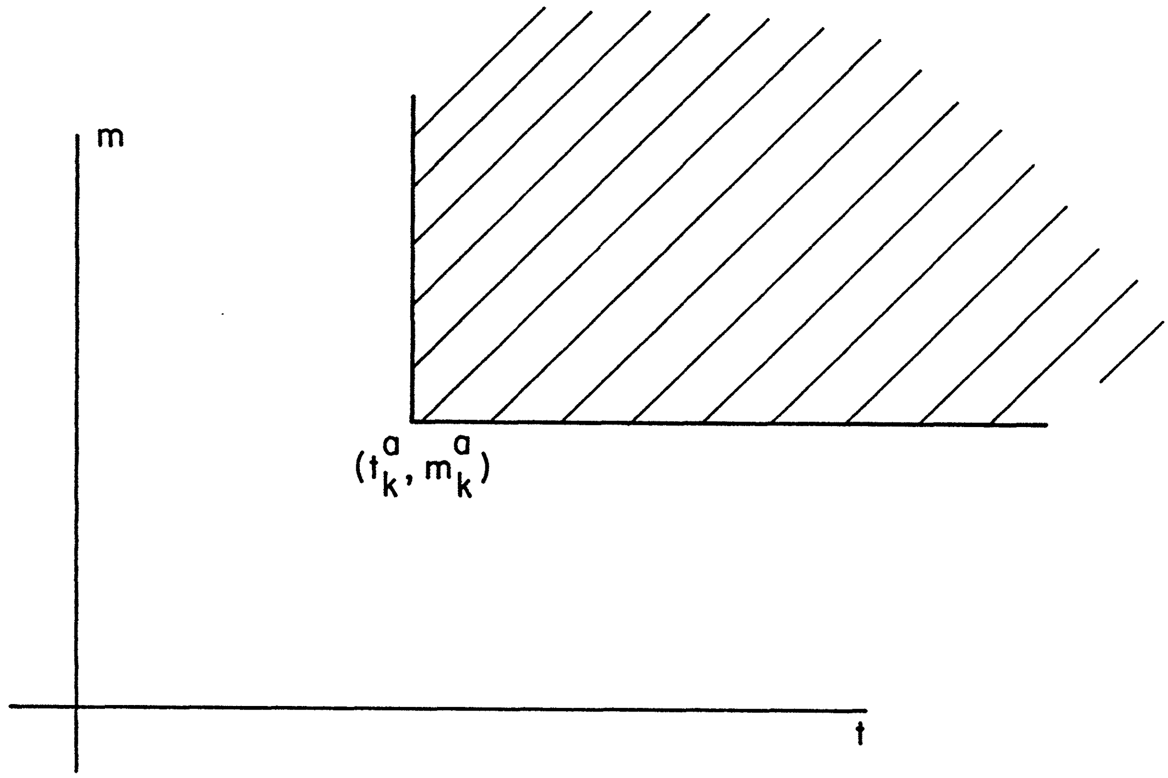



Figure 2.1: Region of Integration for Most Desirable Journey

compared with that required in the deterministic case, especially in light of its greater information content. In addition, a simple numerical technique for solving (2.20) to (2.22) suffices (Section 3) although we have not been able to devise a procedure for the deterministic case. This is fortunate because the stochastic case is more consistent with Zahavi's (1979) empirical findings.

Equation (2.21) can also be written

$$U_i^a = U_{i+1}^a \cup R_i^a \quad , \quad 0 < i \leq k-1 \quad . \tag{2.23}$$

Figure 2.2 illustrates $U_{i+1}^a$, $R_i^a$, and $U_i^a$. Note that the boundary of $U_i^a$ always has a staircase-like structure, with a vertical half-line at the left and a horizontal half-line at the right.

The flow on path i satisfies

$$x_i^a = X_i^a - X_{i+1}^a ,$$

so that

$$x_i^a = D^a \int_{U_i^a - U_{i+1}^a} f^a(u,n)\,du\,dn \quad . \tag{2.24}$$

The region $U_i^a - U_{i+1}^a$ is the oddly shaped rectangular polygon in Figure 2.2 whose lower left-hand corner is $(t_i^a, m_i^a)$.

Note that if $t_i^a$ and $m_i^a$ are sufficiently large, $R_i^a$ falls entirely inside of $U_{i+1}^a$. In that case, $U_i^a - U_{i+1}^a = 0$ and $x_i^a = 0$.

Equations (2.20) to (2.22) have been constructed to be consistent with (2.7) to (2.9). The region $U_i^a$ contains all budgets that are greater than, or equal to, the expenditures on journeys $p_i^a, p_{i+1}^a, \ldots, p_k^a$. Equation (2.22) implies that if an individual has budgets that are in $U_i^a$, he will choose one of these journeys and not journeys $p_1^a, p_2^a, \ldots, p_{i-1}^a$.

In particular (2.24) implies that if his budgets fall in $U_i^a - U_{i+1}^a$, he will take journey $p_i^a$. If he can afford to take journey $p_i^a$, but he cannot afford $p_{i+1}^a, \ldots, p_k^a$, he will choose $p_i^a$. This is exactly what the formulation (2.7) to (2.9) say: $p_i^a$ is the best journey (the ordering convention requires that the utility values of $p_1^a, \ldots, p_{i-1}^a$ are less than that of $p_i^a$) whose costs are less than, or equal to, his budgets.

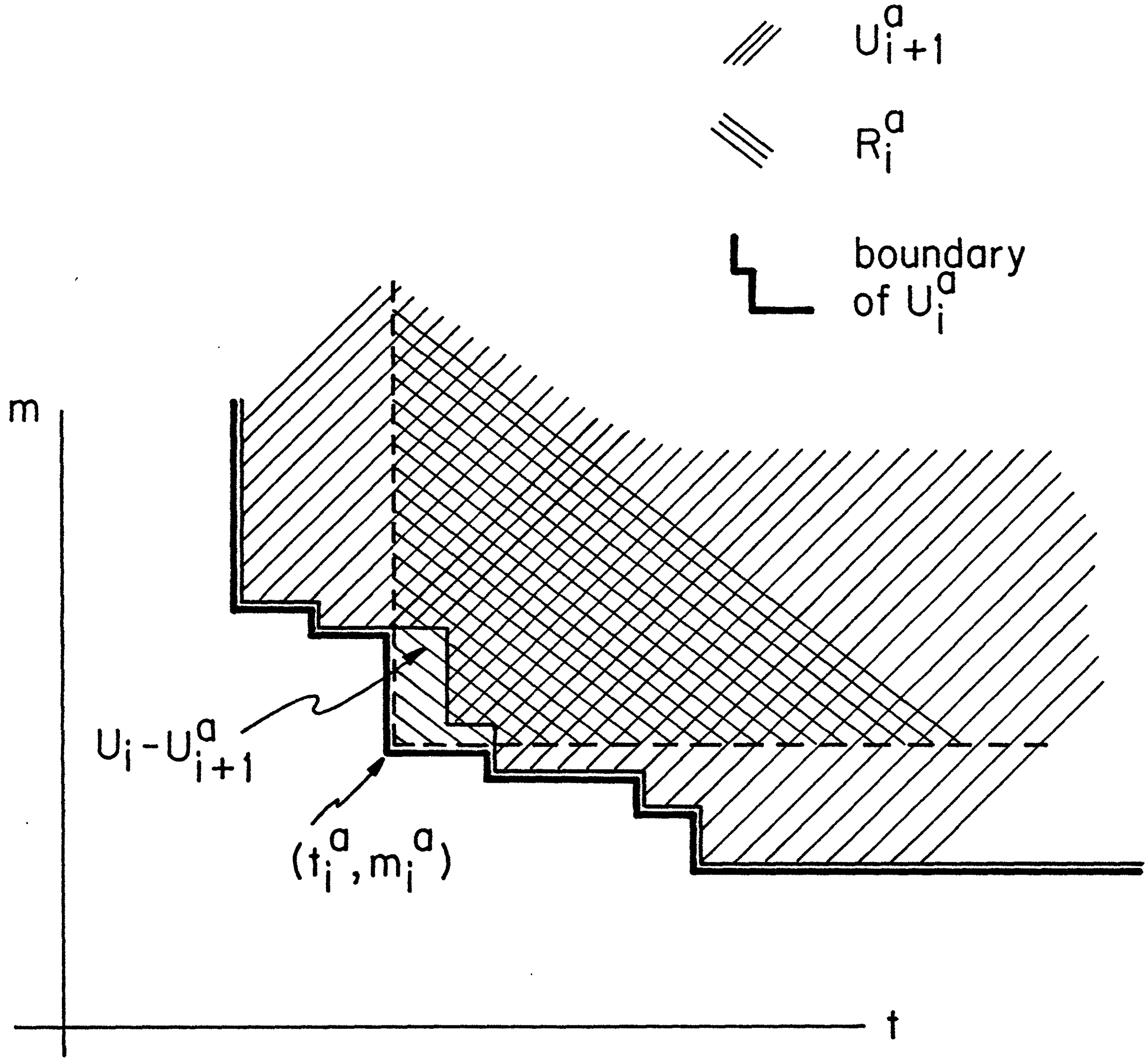



Figure 2.2: Region of Integration for Ordinary Journey

## 2.4 Discussion of New Principles

In this section, we point out some important features of the new assignment principles. Further discussions are found in Section 8, which describes various extensions that broaden the usefulness and applicability of the principles. We also discuss certain limitations which may be the subject of future research.

### 2.4.1 Null Journey

It is convenient to introduce a null journey for each class. This is a journey that requires no travel time or money, that has less utility than any other journey available to its class, and whose flow does not affect the travel or money costs on any other journey in the network. The people who take the null journey are the people who stay home; their budgets are insufficient to allow them to do any traveling at all.

### 2.4.2 Costs

In the numerical examples to follow, the travel time for a journey is the sum of the travel times on its constituent links. The money cost of travel is computed in the same way with an additional term due to either the fixed cost for owning a car or a fare for transit.

### 2.4.3 Utility of Journeys

We have assumed that utility is constant, depending on the geography of the network, and not depending on flows, delays, or money costs. We assume that the utilities of journeys available to the same class are distinct, and that journeys may be ranked in order of utility, with the null journey the worst.

In this model, what is important about a journey is not its utility value, rather its utility ranking. It is important that one journey be better than another; not how much better.

This feature obviates a precise determination of utility. Such a determination may be expensive or impossible. It is a reasonable assumption at least on an individual basis: each individual chooses the best journey he can afford, without being concerned with how much better it is than other journeys.

On the other hand, different individuals may rank journeys differently, particularly if they are similar. For example consider two journeys that involve travel to work and to shopping areas. They are identical except that they include different supermarkets. If the supermarkets are similar, the two journeys should be nearly equally attractive. However, the present model treats them just as it would if they were substantially different.

One way of accounting for the similarities of journeys is to divide the population class into two smaller classes. One class ranks one journey ahead of the other; the other class reverses the order. Thus, if one supermarket is slightly better than the other, we can put, say, 52 percent of the population in classes which rank journeys involving that market ahead of journeys involving the other, and 48 percent in classes which have the opposite preference. It must be noted, however, that this approach can lead to a frightening proliferation of classes, and that a more economical representation of this phenomenon is required.

In the examples discussed below, it is assumed that utility, like travel time, is accumulated as one traverses a journey. A utility value is assigned to each node, and the value of the journey is the sum of the utilities of the nodes passed through.

This choice is made in an effort to build on, and further refine, Zahavi's ideas. Zahavi (1979) suggests that a reasonable approximation to travelers' behavior is to assume that they maximize the distance they cover within their time and money budgets. He points out that distance is only a surrogate for travelers' real objectives,which is to work, shop, be entertained, and generally take advantage of as many of the facilities in the region as possible, within bounds of time and money constraints. The utility value of a node is simply a measure of the number of these facilities that can be found at each location.

It is premature to be dogmatic about the correct way to calculate utility values for journeys. As far as the formulation in this section and the numerical technique in the next section are concerned, all that is important is the ranking of journeys for each class. For example, it is easy to give utility values to links as well as nodes; to treat the same links or nodes as having different utilities for different classes; and to give extra utility to car travel.

We say that utilities are additive if the utility of a journey is the sum of the utility values of the nodes and links through which the journey passes.

## 2.5 Summary

In this section,we have constructed two assignment principles which are based on the travel-budget theory. In the following section, a numerical technique is described to calculate equilibria of the stochastic version of the principle. In later sections, many examples are presented which illustrate the behavior and use of the principle.

## 3. SOLUTION TECHNIQUE FOR STOCHASTIC VERSION OF NEW ASSIGNMENT PRINCIPLE

While equations (2.20) to (2.22) are a compact formulation of the new assignment principle, they do not immediately yield numerical values for flows or other quantities. In this section, we describe a numerical technique that has produced this information for the examples that appear in the following sections.

### 3.1 Statement of Equilibration Procedure

Let x be the vector of all path flows in the network. Define g(x) to be the vector of the same dimensionality as x whose component corresponding to journey i of class a is

$$g_i^a(x) = D^a \int_{U_i^a - U_{i+1}^a} f^a(u,n)\,du\,dn \quad . \tag{3.1}$$

The dependence of g on x is due to the dependence of $U_i^a$ and $U_{i+1}^a$ on x. These sets depend on x because they are determined by the time and money costs on journey $p_i^a$ and all the journeys better than $p_i^a$ for class a. These costs in turn, depend on the costs of the links that make up those journeys, and the link costs depend on the link flows, which are the sums of all the journey flows that pass through those links. It is clear that, in general, $g_i^a$ depends on all the journey flows in the network.

Equations (2.20), (2.21), and (2.24) (which together are equivalent to (2.20) to (2.22)) can be written

$$x = g(x) . \tag{3.2}$$

Thus, we seek a solution to a fixed point problem. It is observed that g(x) is continuous, and that x is restricted by (3.2) to the compact set

$$x_i^a \geq 0 \ , \tag{3.3}$$

$$\sum_i x_i^a = D^a \quad . \tag{3.4}$$

Consequently, at least one solution to (3.2) exists. (Dunford and Schwartz, 1957), Equation (3.2) suggests an equilibration procedure:

$$x(0) \text{ specified,}$$
$$x(q+1) = g(x(q)). \tag{3.5}$$

However, in our experience, (3.5) often fails to converge. This behavior is discussed in Section 3.3. We have done better with:

$$x(0) \text{ specified,}$$
$$x(q+1) = (1-\lambda)x(q) + \lambda g(x(q)), \tag{3.6}$$

where

$$0 \leq \lambda \leq 1 . \tag{3.7}$$

In our experience, (3.6) converges whenever $\lambda$ is sufficiently small. Convergence is considered to have occurred when

$$\max_{a,i} |x_i^a(q) - g_i^a(x(q))| < \epsilon \quad . \tag{3.8}$$

The main effort in executing (3.6) is evaluating the regions $U_i^a$, and then performing the required integrals. This is described in the following section.

## 3.2 Calculation of Integrals

It is convenient to write the iteration process as

$$X_i^a (q+1) = (1-\lambda)X_i^a(q) + \lambda \int_{U_i^a(q)} f^a(u,n)\,du\,dn, \tag{3.9}$$

$$x_k^a(q+1) = X_k^a(q+1), \tag{3.10}$$

$$x_i^a(q+1) = X_i^a(q+1) - X_{i+1}^a(q+1), \tag{3.11}$$

where $X_i^a(q)$ is the class a flow on journey $p_i^a$ or better. The index k refers to the best (highest utility) journey for class a and, as usual, the journey index numbers increase with increased utility.

In Section 3.2.1, we characterize the regions $U_i^a(q)$. In 3.2.2, we demonstrate how to evaluate the integral in equation (3.9).

### 3.2.1 Regions of Integration

Region $U_i^a$ is defined by equations (2.18), (2.20), and (2.23) (where the argument q is suppressed). It always has the characteristic staircase shape of Figure 2.2, of which Figure 2.1 is a special case. To perform the integration in (3.9), a list of the corners of Figure 2.2 is required. The corners of $U_{i+1}^a$ are displayed in Figure 3.1.

The components of a corner of the form $(u_j, n_j)$ are the travel time and travel money costs of some journey of class a better than journey i. Heretofore, they have been listed in order of increasing utility. To perform the integration, they must be listed as they are in Figure 3.1, in order of increasing travel time and decreasing money (or the reverse). Furthermore, the costs of journeys i such that $R_i^a$ is a subset of $U_{i+1}^a$ must not appear in this list. (See the remarks following (2.24).)

Let $L_i^a$ be the list required to perform the integration over $U_i^a$. It is defined inductively from $L_{i+1}^a$ based on (2.23). From (2.17) and (2.18), we define

$$L_k^a = [(t_k^a, m_k^a)]. \tag{3.12}$$

where $p_k^a$ is the best path. Let

$$L_{i+1}^a = [(u_1, n_1), \ldots, (u_r, n_r)], \tag{3.13}$$

where

$$u_1 < u_2 < \cdots < u_r, \tag{3.14}$$

and

$$n_1 > n_2 > \cdots > n_r . \tag{3.15}$$

If, for some j, $1 \leq j \leq r$

$$\left. \begin{array}{l} t_i^a \geq u_j \\ m_i^a \geq n_j \end{array} \right\}, \tag{3.16}$$

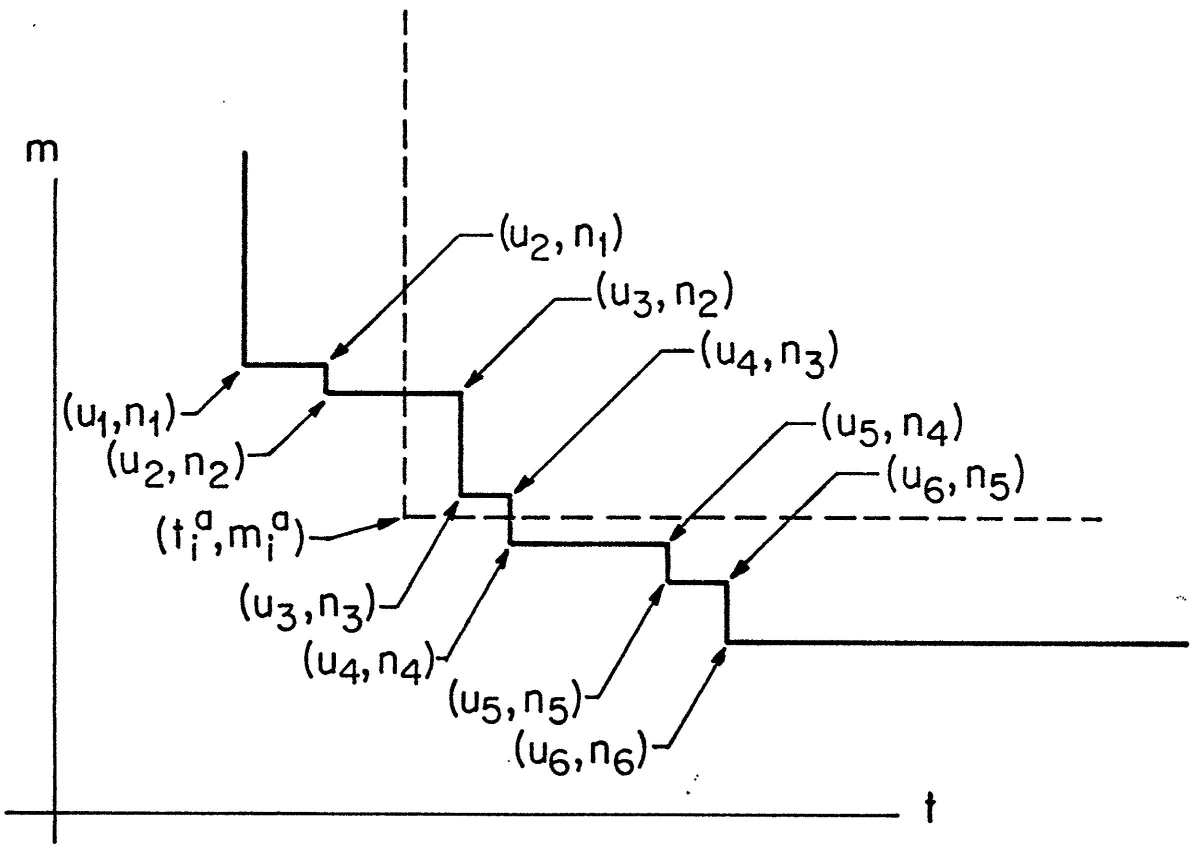



Figure 3.1: Corners of $U_{i+1}^{a}$ and $U_{i}^{a}$

then,

$$L_i^a = L_{i+1}^a \ . \tag{3.17}$$

This is because journey i has lower utility than the journey whose costs are $(u_j, n_j)$. Since its costs are also greater, there is no reason to take this journey. Geometrically, $R_i^a$ is a subset of $U_{i+1}^a$, and $(t_i^a, m_i^a)$ falls inside of $U_{i+1}^a$.

Otherwise, let $\alpha$ be the largest integer such that

$$u_\alpha \le t_i^a , \tag{3.18}$$

and let $\beta$ be the smallest index such that

$$n_\beta \le m_i^a \ . \tag{3.19}$$

(In Figure 3.1, $\alpha = 2$, $\beta = 4$.) Then, the list $L_i^a$ is constructed from list $L_{i+1}^a$ by replacing all the points $(u_{\alpha+1}, n_{\alpha+1}), \ldots, (u_{\beta-1}, n_{\beta-1})$ with $(t_i^a, m_i^a)$. That is, if

$$\begin{aligned} L_{i+1}^a &= [c_1^{i+1}, \ldots, c_r^{i+1}] , \\ L_i^a &= [c_1^i, \ldots, c_s^i ], \end{aligned} \tag{3.20}$$

then,

$$\left.\begin{aligned} c_j^i &= c_j^{i+1}, \quad j = 1, \ldots, \alpha , \\ c_{\alpha+1}^i &= (t_i^a, m_i^a) , \\ c_j^i &= c_{\beta-\alpha-2+j}^{i+1}, \ j = \alpha+2, \ldots, s, \\ s &= r-\beta+\alpha+2. \end{aligned}\right\} . \tag{3.21}$$

In Figure 3.1,

$$L_{i+1}^a = [(u_1, n_1), \ldots, (u_6, n_6)],$$

$$L_i^a = [(u_1, n_1), (u_2, n_2), (t_i^a, m_i^a), (u_4, m_4), (u_5, m_5), (u_6, m_6)] \ .$$

Note that the numbers of corners in $U_i^a$ (i.e., s) may be greater than the number in $U_{i+1}^a$ (i.e., r) by 1, or it may be equal, or it may be fewer if $\beta > \alpha+2$.

### 3.2.2 Calculation of Integrals

Once list $L_i^a$ is determined, the integral in (3.9) can be written

$$\int_{U_i^a} f^a(u,n)\,du\,dn = \sum_{j=1}^{s-1} \int_{S_j} f^a(u,n)\,du\,dn + \int_{Q} f^a(u,n)\,du\,dn, \tag{3.22}$$

where s is the number of corners of $U_i^a$, $S_i$ is the strip,

$$S_j = \{(u,n)\,|\,u_j \leq u \leq u_{j+1},\ n \geq n_j\} \quad , \tag{3.23}$$

and Q is the quadrant,

$$Q = \{(u,n)\,|\,u \geq u_s,\ n \geq n_s\} \quad . \tag{3.24}$$

In all the examples described below, we have assumed that the budgets are independent. As a result,

$$f^a(u,n) = \phi^a(u)\psi^a(n), \tag{3.25}$$

and

$$\int_{S_j} f^a(u,n)\,du\,dn = \left[\int_{u_j}^{u_{j+1}} \phi^a(u)\,du\right]\left[\int_{n_j}^{\infty} \psi^a(n)\,dn\right] \quad , \tag{3.26}$$

$$\int_{Q} f^a(u,n)\,du\,dn = \left[\int_{u_s}^{\infty} \phi^a(u)\,du\right]\left[\int_{n_s}^{\infty} \psi^a(n)\,dn]\right] . \tag{3.27}$$

The integrals in (3.26), (3.27) are particularly easy to calculate when the $\phi$ and $\psi$ density functions are piecewise linear. We feel that numerical results are not sensitive to the detailed shapes of these distributions although they are sensitive to their means and variances.

## 3.3 Algorithm Behavior

In the following sections,we discuss a large number of numerical examples. We restrict our attention there to a discussion of the behavior of the solution of the system (2.20) to (2.22). Here, we present an informal summary of our observations on the behavior of the iteration procedure (3.6).

The convergence properties of the algorithm are sensitive to two major sets of quantities: $\lambda$, and the variances of $\phi^a(\cdot)$ and $\psi^a(\cdot)$.

The larger the variances, the greater the reliability of convergence. When the variances are small, it is very easy for the algorithm to overshoot the equilibrium distribution and oscillate wildly. This becomes a great difficulty in doing Case 5 in Section 4.3 in which the variances were made very small in order to mimic the behavior of the deterministic version of the assignment principle.

We conjecture that this behavior is due to the fact that, when the variance is small, the travelers in a given class are similar to one another. If the cost of a given journey is too high, all the members of that class will react in the same way, and most of the flow will be removed from that journey by the integral in (3.9). If the costs are too low for the present flow on a journey, the integral in (3.9) will tend to redistribute most of the flow of that class onto that journey. If the variances are large, only a small amount of flow will be affected by an error in cost.

The step size $\lambda$ also affects algorithm behavior. When $\lambda$ is small, the change from iteration to iteration is small, and although convergence is likely it is time-consuming. When $\lambda$ is large, overshoot is a danger, and oscillations can be observed.

The amount of computer time that the algorithm requires is also greatly affected by the computation required to evaluate the integrals on the right-hand side of (3.26) and (3.27). Distributions that require a great deal of airthmetic (such as a normal, Erlang, or gamma) require much more total computer time than others (such as the uniform or a piecewise linear density).

It is clear that these observations can be combined to enhance the efficiency of the algorithm. First, $\lambda$ can be increased as the algorithm shows signs of slowing down; i.e., reaching equilibrium. Second, the variances can be initialized at much larger values than the required variances, and decreased gradually when the algorithm seems to be converging. Finally, if it is necessary that results be computed with difficult-to-calculate distributions, the algorithm can be restarted after first converging with an easy distribution.

### 3.4 Relationship with UMOT (Unified Mechanism of Travel)

The stochastic assignment principle dicussed here bears a complementary relationship with Zahavi's (1979) UMOT model. A fusion of these models can result in an enhanced technique for the calculation of the equilibrium distribution of traffic.

The present model computes x, the flows on all paths. It uses as data, among other things, travel budgets. Ley y be the vector of data required by this model. These data, and other information can be obtained from UMOT. Equations (3.9) to (3.11) can be written

$$x(y;\ q+1) = (1-\lambda)x(y;q) + \lambda g(y,x(y;q)). \tag{3.28}$$

Define

$$z = \begin{pmatrix} y \\ v \end{pmatrix}$$

to be all the information generated by UMOT. The vector v contains information which can also be obtained from our model, such as average speed of a traveler. The UMOT equilibrium process can be written

$$z(q+1) = h(z(q)), \tag{3.29}$$

or

$$y(q+1) = h_1(y(q),\ v(q), \tag{3.30}$$

$$v(q+1) = h_2(y(q),\ v(q). \tag{3.31}$$

Now, consider the integrated equilibrium process which is composed of (3.38) and (3.30); namely,

$$x(q+1) = (1-\lambda)x(q) + \lambda g(y(q), x(q)),$$
$$y(q+1) = (1-\lambda)y(q) + \lambda h_1(y(q), x(q)) \tag{3.32}$$

(in which the convergence rate of (3.30) has been slowed down prudently). Here, the assignment process is influenced by accurate values of budgets and other quantities, and the UMOT process uses a more detailed representation of travel speed. In addition, a potentially more accurate representation of travel utility is now available.

### 3.5 Research Areas

There are several significant questions about this equilibration process that require study.

a. Under what conditions is the process guaranteed to converge? It appears that both $\lambda$ and the budget variances must be within certain bounds, but the relationship between these and other quantities is not understood.

b. Is there a unique equilibrium? Under what conditions is it unique? If not, how many are there? This question can be studied experimentally by using various values for $x(0)$ and observing the limiting values of $x(q)$, but it should also be studied analytically.

c. How can the algorithm be made more efficient for large networks?

d. How can journeys be efficiently generated? At present, we assume that all journeys are specified when the process is initialized. However, it is more realistic to suppose that a small set is chosen at the outset, and more are generated as flow levels, and therefore costs are more precisely determined.

# 4. NUMERICAL EXAMPLES

In this section, we describe several examples of small-size networks that we analyze using the numerical technique of Section 3. These examples illustrate various qualitative properties of the assignment principles of Section 2 and of the iteration process. A network of more realistic size and complexity is presented in Section 6.

## 4.1 Single-Link Network

Figure 4.1 contains the simplest network imaginable: a single link and a single node. Figure 4.2 illustrates the link delay function

$$t(x) = \frac{0.2}{1-x/c} \tag{4.1}$$

which is a function of x, the flow on the journey which traverses that link exactly once and the demand function, d(t), which is the complementary cumulative probability distribution multiplied by the total demand D. This graph is drawn assuming d(t=0) = c although the analysis presented here does not depend on this assumption. There is a null journey, which is taken by all those travelers whose budgets are less than t. Note that only one budget is considered, i.e., money is no object. The distribution is gamma with mean 1.5 and variance 10.0. For this simple network, the function g(x) is simply d(t(x)).

The equilibration process is illustrated with $\lambda=1$, i.e.,

$$x(q+1) = g(x(q)) = d(t(x(q))). \tag{4.2}$$

Note that the parameters are such that convergence is rapid.

The variance is reduced to 4.0 in Figure 4.3. Here, convergence occurs with $\lambda=1$; a more rapid convergence occurs when $\lambda=0.5$.

In Figure 4.4, the variance of the budget distribution is reduced to 1.0. Here, the equilibration (4.3) fails to converge, as illustrated by x(1), x(2), and x(3). If, however, $\lambda$ is reduced, convergence can be achieved, as shown by x'(1), x'(2) and x'(3).

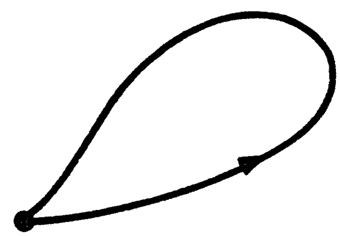


Figure 4.1: Simplest Possible Network

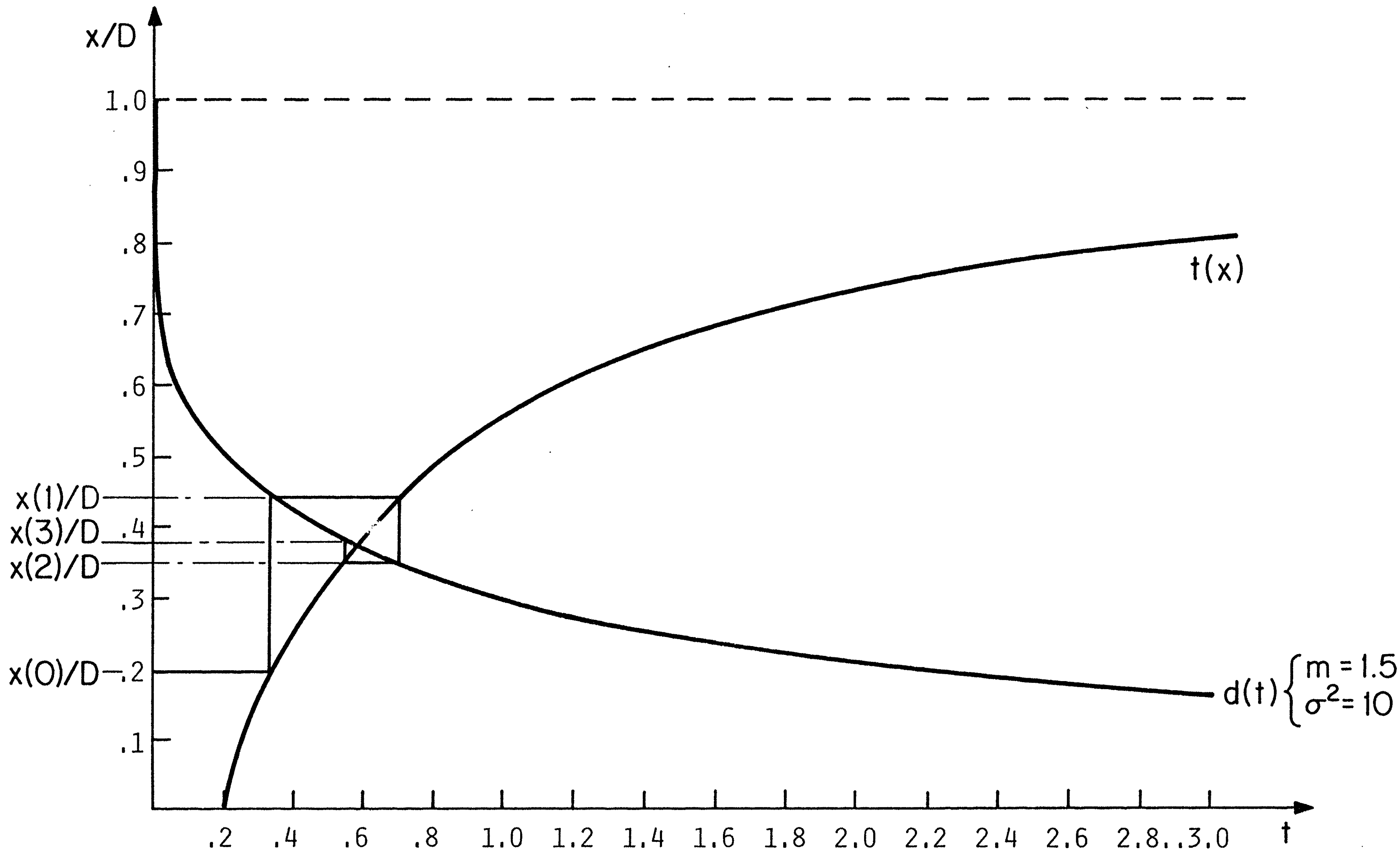



Figure 4.2: Equilibration Process for High Variance

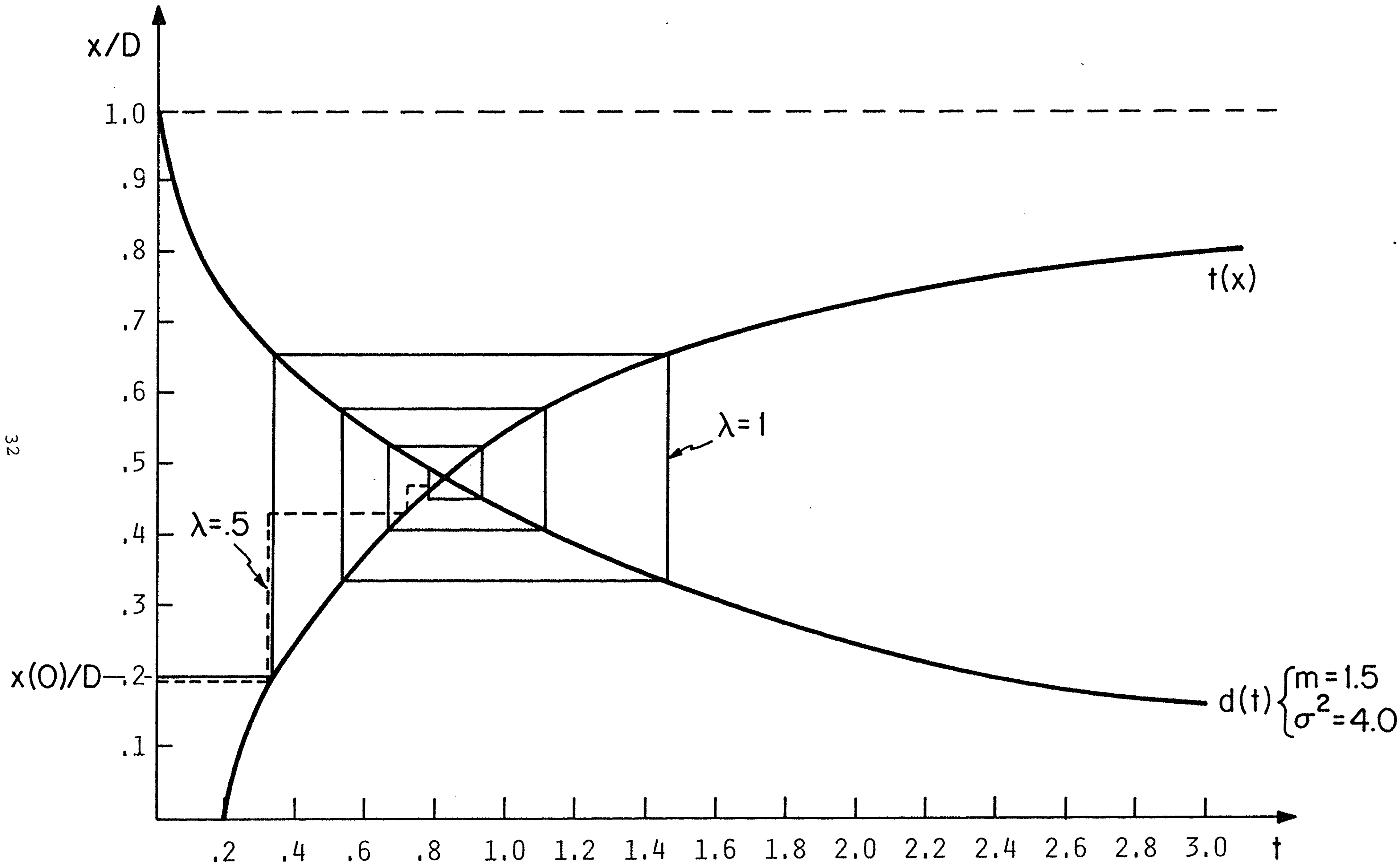



Figure 4.3: Effect of Reducing Variance

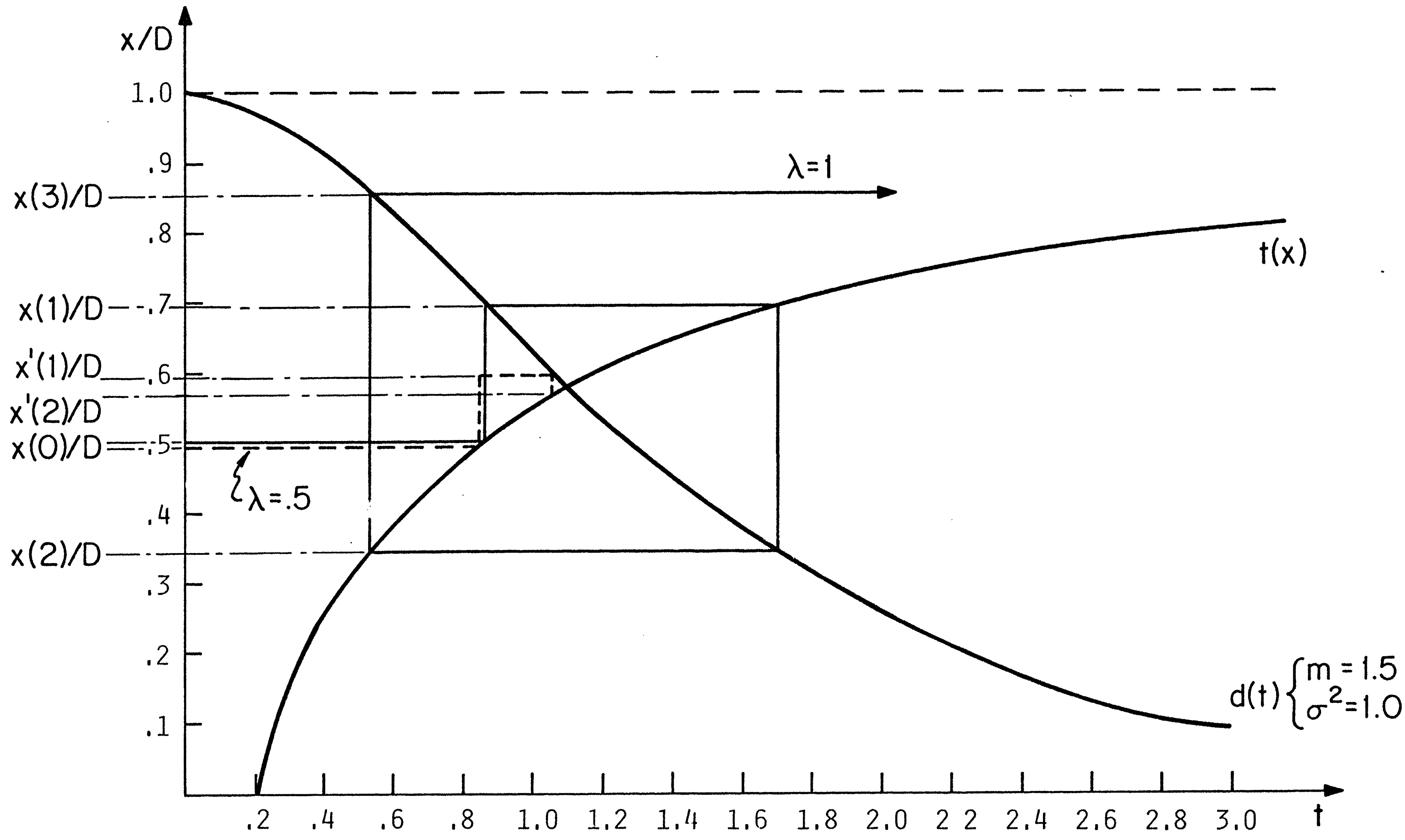



Figure 4.4: Equilibration Process with Reduced Variance

The reason that a reduced $\lambda$ is required is that in Figure 4.4, a small change in t leads to a large change in $x = d(t)$. A low variance means that almost all travelers are affected in the same way by a change in network conditions.

Note that equilibrium is reached when

$$x = d(t(x)), \tag{4.3}$$

which occurs at the intersection of the t(x) (supply) and d(t) (demand) curves. This is an example of elementary economic reasoning.

## 4.2 Four-Link Network

The network in Figure 4.5 is nearly as simple as that of Figure 4.1 There are five journeys in the network: the null journey and the four that each traverse one link once. The flow that takes journey i, which traverses link i, is $x_i$. The flow on the null journey is $x_0$. The four links have identical delay functions given by

$$t_i = .1 + \left(\frac{x_i}{250}\right)^4 . \tag{4.4}$$

The journeys have different utility values: journey 4 is better than journey 3, and so forth.

Figure 4.6 displays the flow levels as a function of total demand when the time budget has a triangle distribution with mean 2.00 and variance 1.95. The triangle density function is displayed in Figure 4.7. Again,money is not considered.

When the demand is near zero, nearly all flow is attracted to journey 4. This is because the delay is small,and so nearly all travelers can afford journey 4. It is the best, so it is chosen.

As the demand increases, the flow on journey 4 increases. However, some travelers are excluded from it since its delay has increased, so they take journey 3. Eventually,journey 3 becomes expensive and travelers move to journey 2, journey 1, and the null journey. In the limit as $D \to \infty$,

$$x_i \to c^*, \tag{4.5}$$

$$x_0 \to D - 4c^*, \tag{4.6}$$

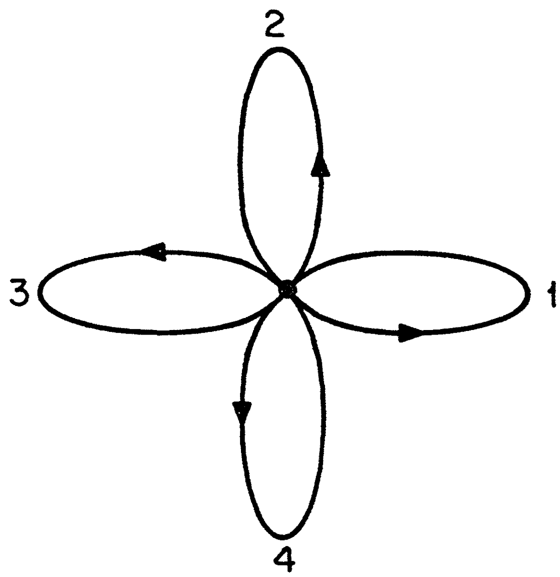



Figure 4.5: Four-Link Network

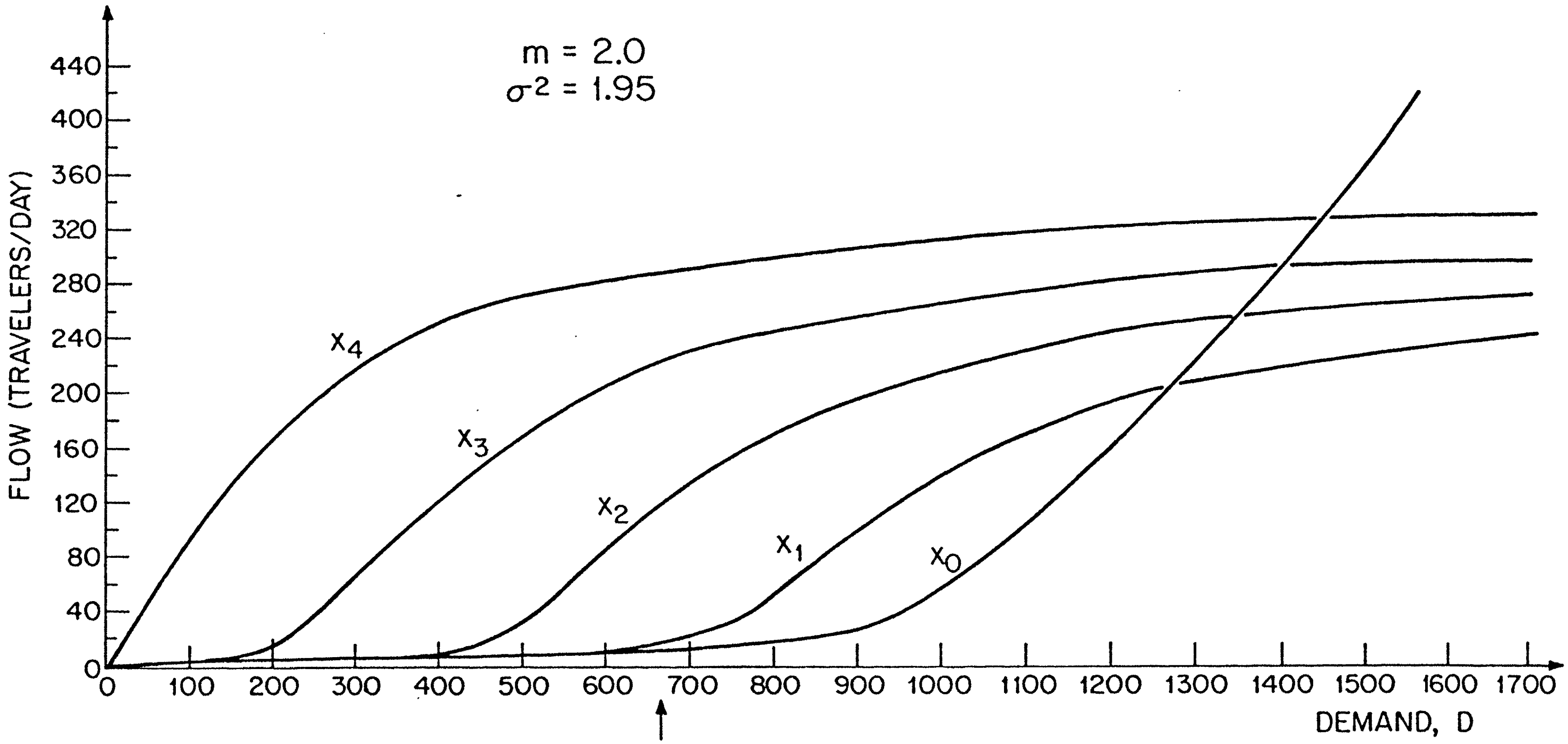



Figure 4.6: Flow vs. Demand -- High Variance

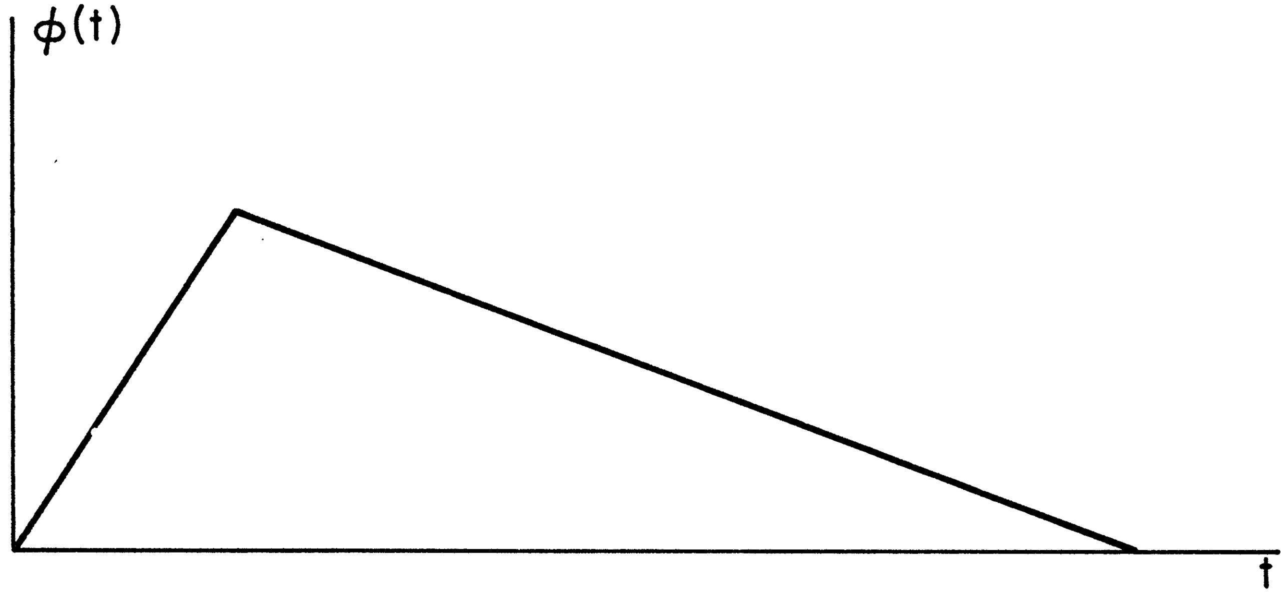



Figure 4.7: Triangle Density Function

where $c^*$ is a limiting flow, and $x_0$ contains all the travel demand that is unmet.

Figure 4.6 illustrates the relationship between the deterministic and stochastic versions of the principle. When the demand D is as indicated by the arrow, two journeys are approximately at budget levels (constrained journeys), one has less travel time than the budget level (special journey), and two have nearly no flow (unutilized journeys).

Figure 4.8 is similar to Figure 4.6 except that the variance has been reduced to 0.31. Note that there is less overlap among the curves. The limits (4.5) and (4.6) remain valid although with a different $c^*$. When the demand is as indicated by the arrow, the values of $x_4$, $x_3$ are closer to $c^*$, and the values of $x_1$, $x_0$ are closer to zero. Thus, this is a closer approximation to the deterministic case.

In the deterministic case, Figure 4.9 describes the behavior of the flows. Again, limits (4.5), (4.6) are valid. Here $c^*$ is the solution to

$$T = .1 + \left(\frac{x_i}{250}\right)^4, \tag{4.7}$$

where T = 2.00 is the value of the time budget so that $c^* = 293.5$. Clearly, at the arrow, journeys 4 and 3 are constrained, journey 2 is special, and journeys 1 and 0 are unutilized.

## 4.3 Small Network with Interacting Journeys

Figure 4.10 is a network with three nodes and five links. Node and link numbers are indicated. The link delay functions are given by

$$\tau_\ell(f) = \tau_{\ell 1} + \left(\frac{f}{\tau_{\ell 2}}\right)^4, \tag{4.8}$$

and the link cost functions by

$$\mu_\ell(f) = \mu_{\ell 1} + f^{1.5}, \tag{4.9}$$

where the parameters are listed in Table 4.1.

Several examples are treated that are based on this network. In each, the budget density functions $\phi$ and $\psi$ are uniform.

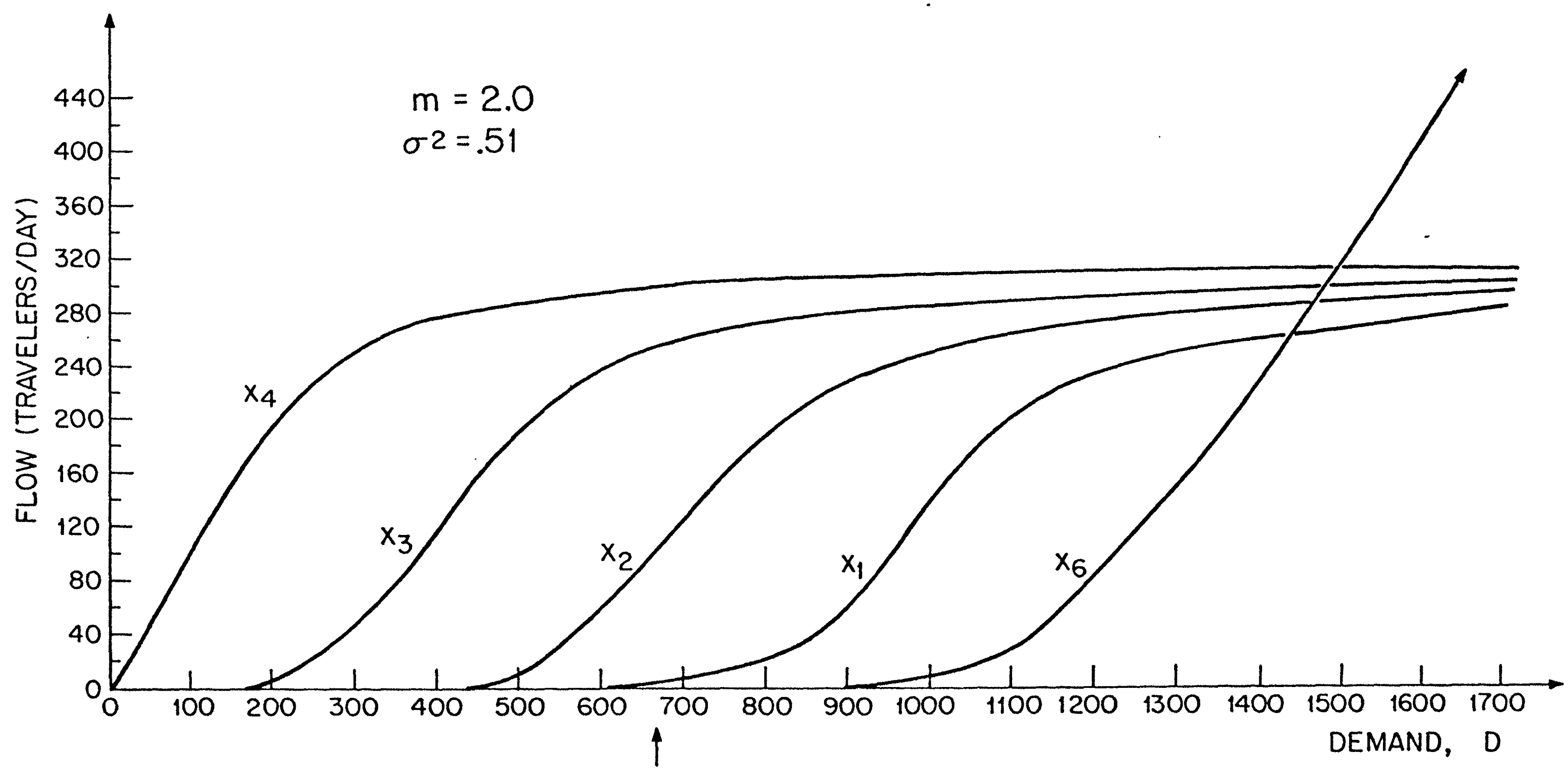



Figure 4.8: Flow vs. Demand -- Reduced Variance

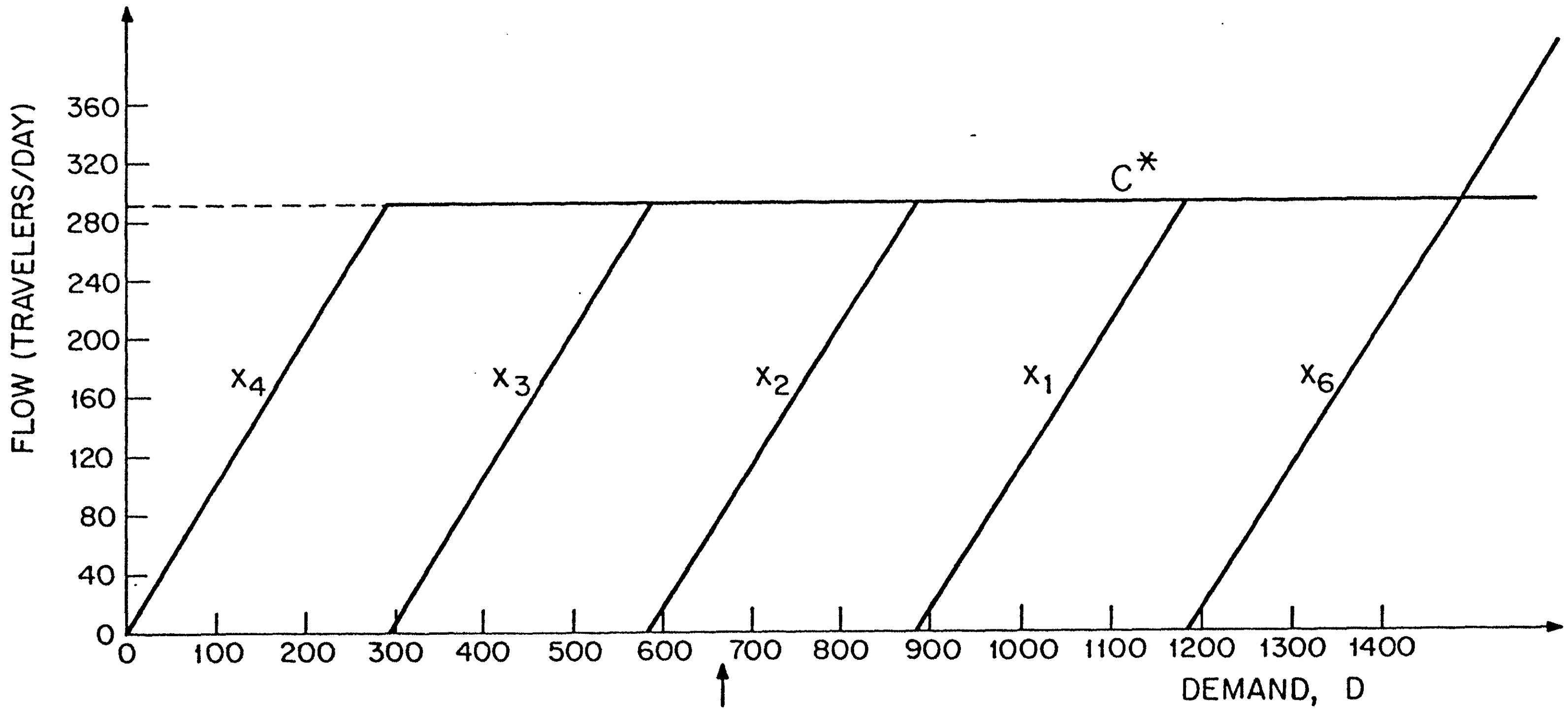



Figure 4.9: Flow vs. Demand -- Deterministic Principle

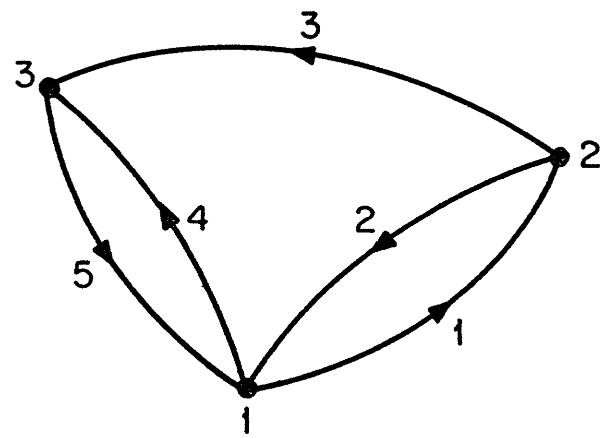



Figure 4.10: Three-Node Network

Table 4.1 - Network Parameters

| link $\ell$ | $\tau_{\ell 1}$ | $\tau_{\ell 2}$ | $\mu_{\ell 1}$ |
|---|---|---|---|
| 1 | 0.25 | 200 | 0 |
| 2 | 0.25 | 200 | 0 |
| 3 | 0.50 | 400 | 0.50 |
| 4 | 0.50 | 200 | 0 |
| 5 | 0.50 | 200 | 0 |

Case 1

The time budget is uniform between 2.0 and 2.5 and the money budget is uniform between 3.0 and 3.5. The total demand is 200. Table 4.2 lists the journeys in increasing order of utility, as well as the equilibrium flow, travel time, and money cost.

Table 4.2 - Results of Case 1

| Journey | Flow | Time | Cost |
|---|---|---|---|
| null (1) | 0 | 0 | 0 |
| 1 - 3 - 1 (2) | 0 | 1.03 | .74 |
| 1 - 2 - 1 (3) | 114.31 | 1.61 | 1.61 |
| 1 - 2 - 3 - 1 (4) | 85.69 | 2.29 | 2.64 |

Journey 4, which is the most desirable, is like a constrained journey in that there are some travelers on it who are spending their entire travel-time budgets. Journey 3 is analogous to the special path of the deterministic formulation since no traveler is spending either of his full budgets on travel. Journeys 1 and 2 are unutilized. It is noted that this interpretation of the relationship between the stochastic and deterministic principles does not always hold. It works here because the uniform density is zero outside of a certain range.

Case 2

Case 2 is the same as Case 1 except that the demand is raised to 300. Results are presented in Table 4.3.

Table 4.3 Results of Case 2

| Journey | Flow | Time | Cost |
|---|---|---|---|
| null (1) | 0 | 0 | 0 |
| 1 - 3 - 1 (2) | 97.12 | 1.20 | .93 |
| 1 - 2 - 1 (3) | 176.24 | 2.16 | 2.29 |
| 1 - 2 - 3 - 1 (4) | 26.65 | 2.46 | 2.87 |

In the previous network, when the demand is increased, it fills up the current "special" path, and then overflows onto the next. Here, all the flow is redistributed. In particular, there is less flow on the best path in Case 2 than in Case 1.

The reason for this is that the time required to traverse journey 4 has increased from 2.29 to 2.46, which is almost at the upper limit of the time-budget distribution. This increase in time is due to the increased flow on journey 3, which shares link 1 with journey 4; and on journey 2, which shares link 5. Again, the stochastic version of the assignment principle mimics the deterministic in that there are constrained, special, and unutilized journeys in the proper order.

Case 3

It is observed that journey 2 has time and money costs which are less than one-half of the upper limits on the budget distributions. Travelers may therefore be expected to go around the loop twice. Consequently we have added an additional journey: 1 - 3 - 1 - 3 - 1. The utility of this journey is greater than that of journey 2 and less than the utility of journey 3.

To our great surprise, no redistribution takes place after the new journey is added. That is, the new equilibrium is such that the flow of the new journey is 0, and the flows on the others are the same as those of Table 4.3. After being satisfied that the computer program is not in error, we realize that the results have told us something. We observe that the time cost of the journey, 2.40, is greater than that of journey 3 (2.16) although its utility is less. (The money costs are irrelevant since they are less than the lower limit of the money budget density function). Consequently, no travelers who are capable of switching from 1 - 2 - 1 to 1 - 3 - 1 - 3 - 1 (i.e., those on 1 - 2 - 1 whose time budgets are greater than 2.40) have any incentive for doing so.

This indicates a marketing-like aspect of travel. When a new travel alternative is offered ( in this case, a new journey), it is not enough to ascertain that it is better than some existing alternatives that are used, and that it is affordable by (i.e., within the budget of) some travelers. The new alternative will be used only when there are some travelers that can afford it, and for whom it is better than their current transportation choice.

This result may be seen in terms of the integration regions $U_i$. Figure 4.11 shows the uniform joint-density distribution of budgets for travelers in the network and the $U_i$ regions for journeys 2, 3, 4, and the new journey. The integrals of regions of the budget joint density are performed in order of utility, with the region for the highest utility journey evaluated first. The new journey we propose has utility greater than journey 2, and less than those of journeys 3 and 4 of the original network. Figure 4.11 shows that by the time the integral of f(.) over the region $U_{new}$ for the proposed flow is evaluated, all potential users of the new journey have already been assigned to journeys 3 and 4. The travelers having budgets in the joint density contained in the region $U_3 - U_2$ are the only candidates still unassigned, and they cannot afford the new journey. Thus, even though a journey with higher utility than a presently employed path is available which has costs in the feasible budget region, it may go unused depending on its value to travelers relative to other journey choices.

Case 4

Class 2 travelers are added to the demand in Case 4. These travelers have a time budget distribution which is uniform between 3.0 and 3.5. The money budget distribution is the same as that for the existing travelers (Class 1). The new travelers are also limited in the journeys available to them. The demand for the new class is 100. The journeys and the equilibrium distribution are listed in Table 4.4.

Although Class 2 seems to be limited by the lesser availability of journeys, in reality it is not. This is because Class 2 travelers would not take journey 3 of Class 1 even if it were available to them since its costs are beyond their

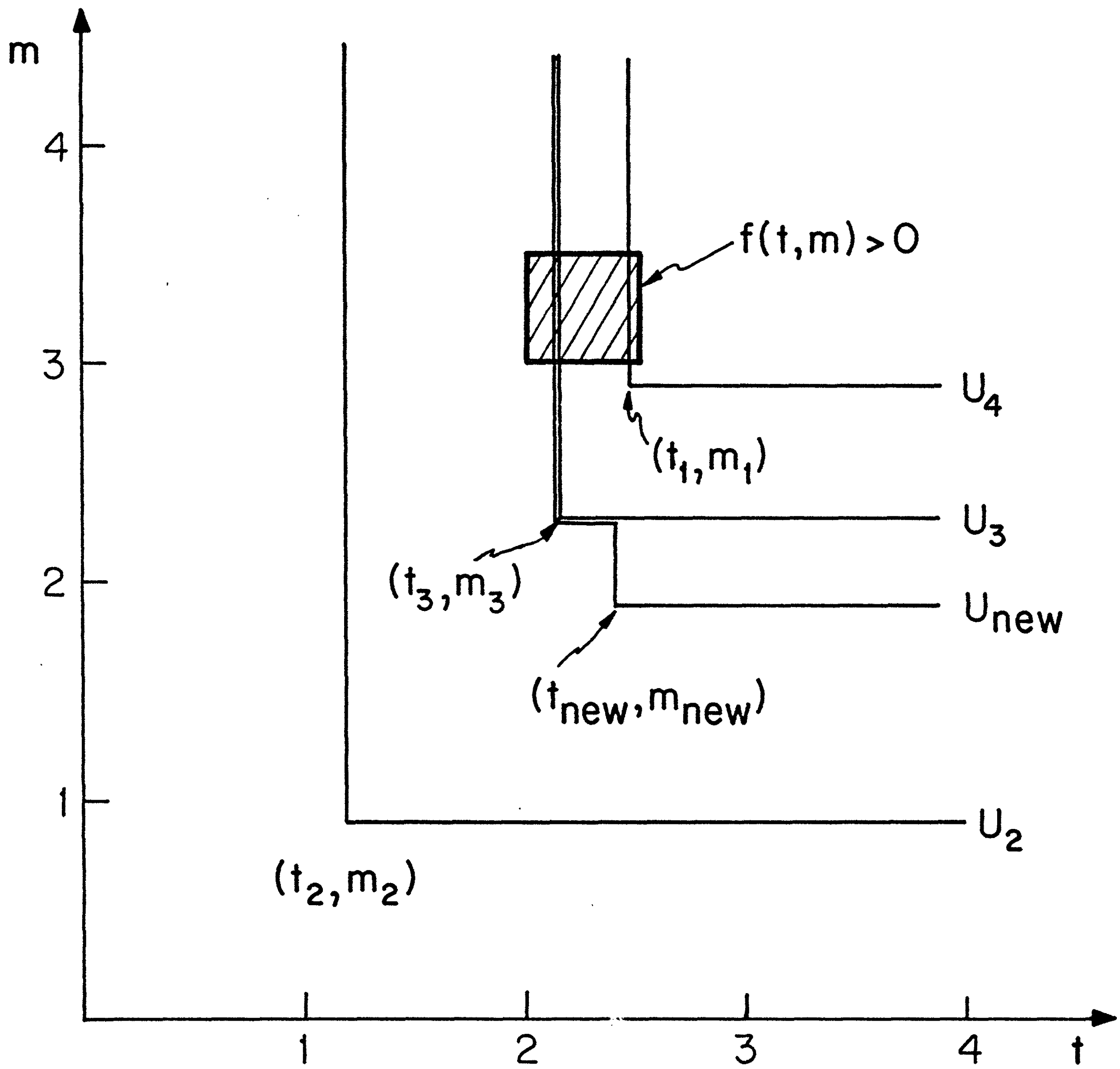



Figure 4.11: Regions of Integration for Cases 2 and 3

Table 4.4. Results of Case 4

| Journey | Flow | Time | Cost |
|---|---|---|---|
| Class 1 | | | |
| null (1) | 34.38 | 0 | 0 |
| 1 - 3 - 1 (2) | 164.46 | 2.06 | 2.09 |
| 1 - 3 - 1 - 3 - 1 (3) | 0 | 4.11 | 4.18 |
| 1 - 2 - 1 (4) | 101.16 | 2.33 | 2.53 |
| 1 - 2 - 3 - 1 (5) | 0 | 2.87 | 3.44 |
| Class 2 | | | |
| Null (1) | 0 | 0 | 0 |
| 1 - 2 - 1 (2) | 88.48 | 2.33 | 2.53 |
| 1 - 2 - 3 - 1 (3) | 11.52 | 2.87 | 3.44 |

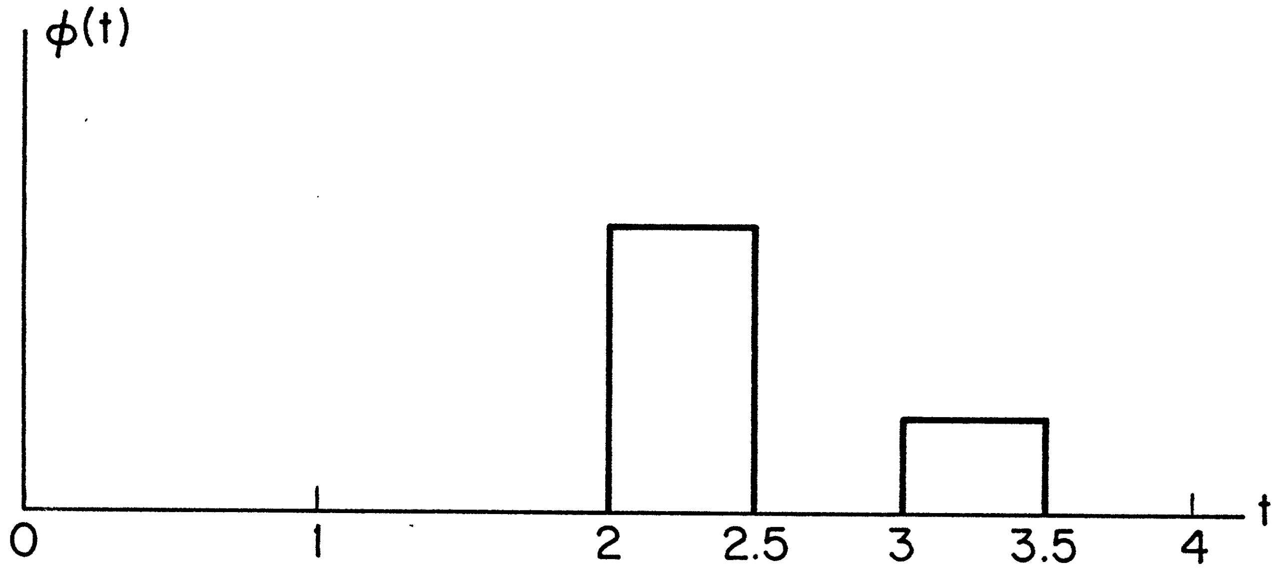
budgets. Also, they would not use journey 2 of Class 1 if they could because their own journey 2 (journey 4 of Class 1) is better and because the travel and money costs of both journeys are below the lower limits of their uniform budget distributions.

By comparing Tables 4.3 and 4.4, we see that the addition of Class 2 travelers is costly to Class 1. Because of their greater willingness to spend time in travel, the new class takes the best journeys and forces Class 1 onto less desirable journeys or out of the network altogether.

Because the two classes are essentially the same except for their time budget distribution, it is possible to think of this example in another way. Consider all travelers as coming from a single class. The money budget distribution is the same as given for the present time classes, and the time budget density function is given in Figure 4.12. The important feature is that it is a bimodal distribution. The total demand is 400. The results are presented in Table 4.5.

Case 5

Case 5 is similar to Case 2 except that the uniform time and money distribution range between 2.0 and 2.2, and between 3.0 and 3.2, respectively. Results




Fiqure 4.12: Density Function for Single Class

Table 4.5: Alternative Interpretation of Case 4

| Journey | Flow | Time | Cost |
|---|---|---|---|
| null (1) | 34.38 | 0 | 0 |
| 1 - 3 - 1 (2) | 164.46 | 2.06 | 2.09 |
| 1 - 3 - 1 - 3 - 1 (3) | 0 | 4.11 | 4.18 |
| 1 - 2 - 1 (4) | 189.64 | 2.33 | 2.53 |
| 1 - 2 - 3 - 1 (5) | 11.52 | 2.87 | 3.44 |

appear in Table 4.6. Here again, the results emulate the behavior of the deterministic principle. Note that the allowable range for flows $x_3$ and $x_4$ is much smaller than in earlier cases because of the narrower range of the uniform distribution.

Table 4.6: Results of Case 5

| Journey | Flow | Time | Cost |
|---|---|---|---|
| null (1) | 0 | 0 | 0 |
| 1 - 3 - 1 (2) | 108.68 | 1.20 | .92 |
| 1 - 2 - 1 (3) | 185.19 | 2.07 | 2.11 |
| 1 - 2 - 3 - 1 (4) | 6.13 | 2.20 | 2.46 |

## 4.4 Circular Network

Figure 4.13 contains a network consisting of 13 nodes and 40 links in the shape of two concentric circles. The links are characterized by delay and money cost functions

$$\tau_\ell = .13 + \left(\frac{f_\ell}{c_\ell}\right)^4,$$

$$\mu_\ell = .065 + 1.39\ \tau_\ell,$$

where $c_\ell$ = 60000 veh/day for the outer links (i.e., those connecting nodes 1-8) and $c_\ell$ = 30000 veh/day on all other links, except where indicated below Travelers originate at nodes 2 and 6.

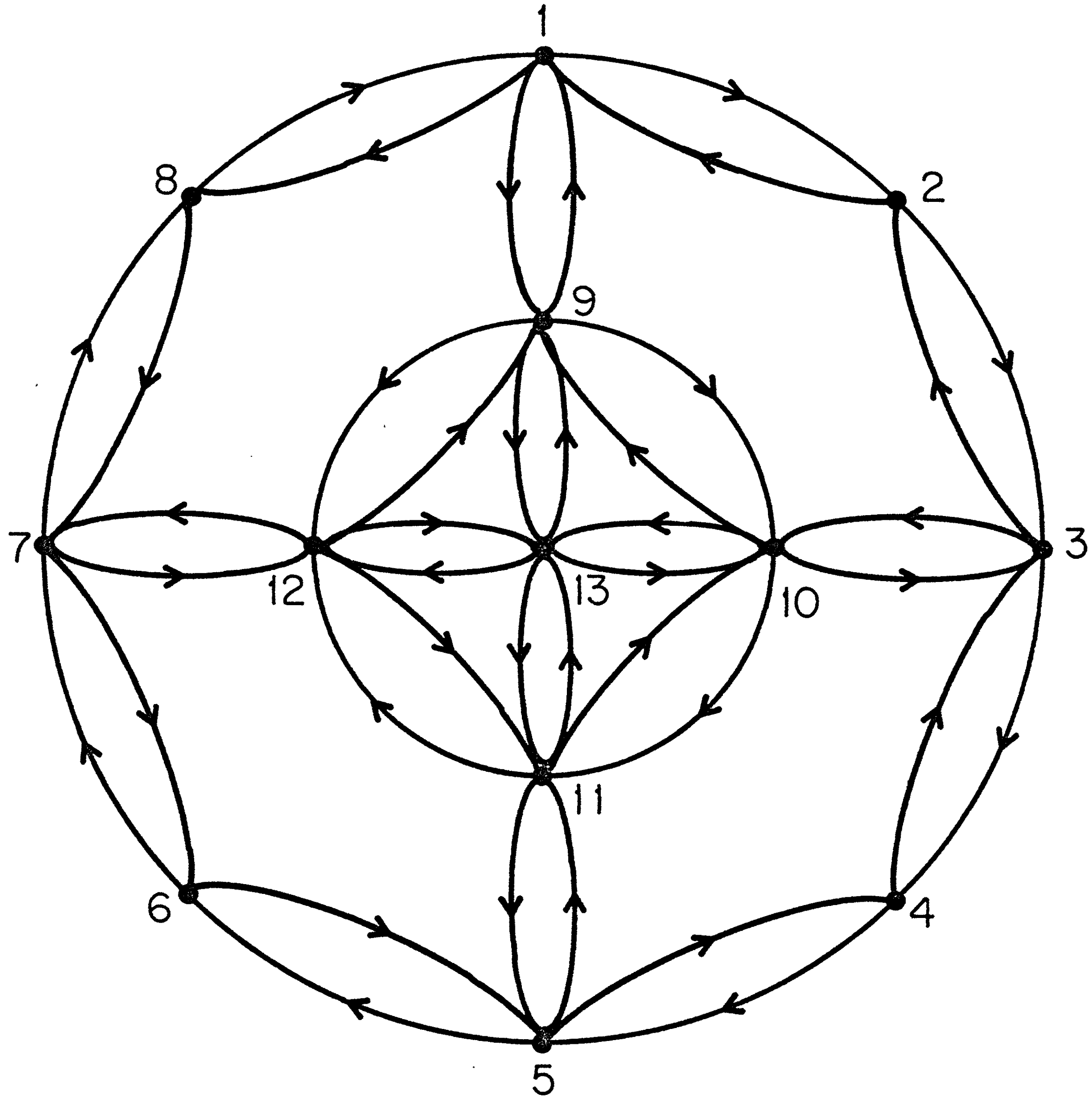



Figure 4.13: Circular Network

Case 1

With demand of 35000 at each origin node, two runs were performed: one with 4 journeys and the other with 5. The budgets were uniformly distributed between 0.025 and 2.47 hours and between 0.88 and 5.12 dollars (i.e., with means 1.25 hours and 3.00 dollars and variance 0.5 $(\text{hour})^2$ and 1.5 $(\text{dollars})^2$. The results for the two runs are indicated in Table 4.7. In the case with four journeys, the results are perfectly symmetric. With 5 journeys, the new journey for Class 1 goes through a link that has reduced capacity (link 8-7: 15000 veh/day).

Note that the addition of the new journeys increases the total number of people traveling (i.e., reduces the flow of the null journey). Class 2 benefits more because its new journey has greater capacity.

The flows on journey 3 and 4 for both classes have been decreased by the addition of journey 5, and the flow on journey 2 has been increased. This is evidently because the new journey has more links in common with journeys 4 and 5 than with journey 2.

Case 2

Figure 4.14 is a set of graphs of flows and demand for the two classes considered with the circular network (Figure 4.13). Here, each class is allowed eight journeys (including the null journey), and asymmentry is introduced by allowing the journeys to be different and reducing the capacity of a few links in the outer loop. Journeys 6, 7, and 8 are the same for the two classes; however, journey 5 for Class 2 has no counterpart for Class 1. Journey 5 for Class 1 is the same as journey 4 for Class 2.

Budgets are distributed uniformly between 1.00 and 1.50 hours and between 2.75 and 3.25 dollars. Only the time budget is effective here; all journeys are cheaper than 2.75 dollars.

Flow on corresponding journeys are nearly the same, and differ only when the demand is greater than 35000. The null journey for Class 1 begins to accumulate flow when demand is 32696, and the null journey for the other class has flow when demand is 33123.

Two important observations can be made from Figure 4.14. The flow need not increase monotonically as demand increases, and the flow level on one journey

Table 4.7 Case 1

| Journey | Flow (4) | Flow (5) | Time (4) | Time (5) | Money Cost (4) | Money Cost (5) |
|---|---|---|---|---|---|---|
| Null - class 1 (1) | 17011.57 | 15161.27 | 0 | 0 | 0 | 0 |
| 2-1-9-13-9-1-2 (2) | 1131.99 | 3375.80 | .92 | .83 | 1.68 | 1.54 |
| 2-3-10-13-10-3-2 (3) | 2547.58 | 150.17 | .99 | 1.01 | 1.76 | 1.79 |
| 2-1-9-13-10-3-2 (4) | 14308.86 | 7688.31 | 1.13 | 1.02 | 1.96 | 1.80 |
| 2-1-8-7-12-13-10-3-2 (5) | — | 8624.45 | — | 1.47 | — | 2.57 |
| Null - Class 2 (1) | 17011.57 | 14747.98 | 0 | 0 | 0 | 0 |
| 6-5-11-13-11-5-6 (2) | 1131.99 | 3541.84 | .92 | .81 | 1.68 | 1.51 |
| 6-7-12-13-12-7-6 (3) | 2547.58 | 227.85 | .99 | .99 | 1.76 | 1.77 |
| 6-5-11-13-12-7-6 (4) | 14308.86 | 5331.29 | 1.13 | 1.01 | 1.96 | 1.99 |
| 6-5-4-3-10-13-12-7-6 (5) | — | 11151.04 | — | 1.29 | — | 2.32 |

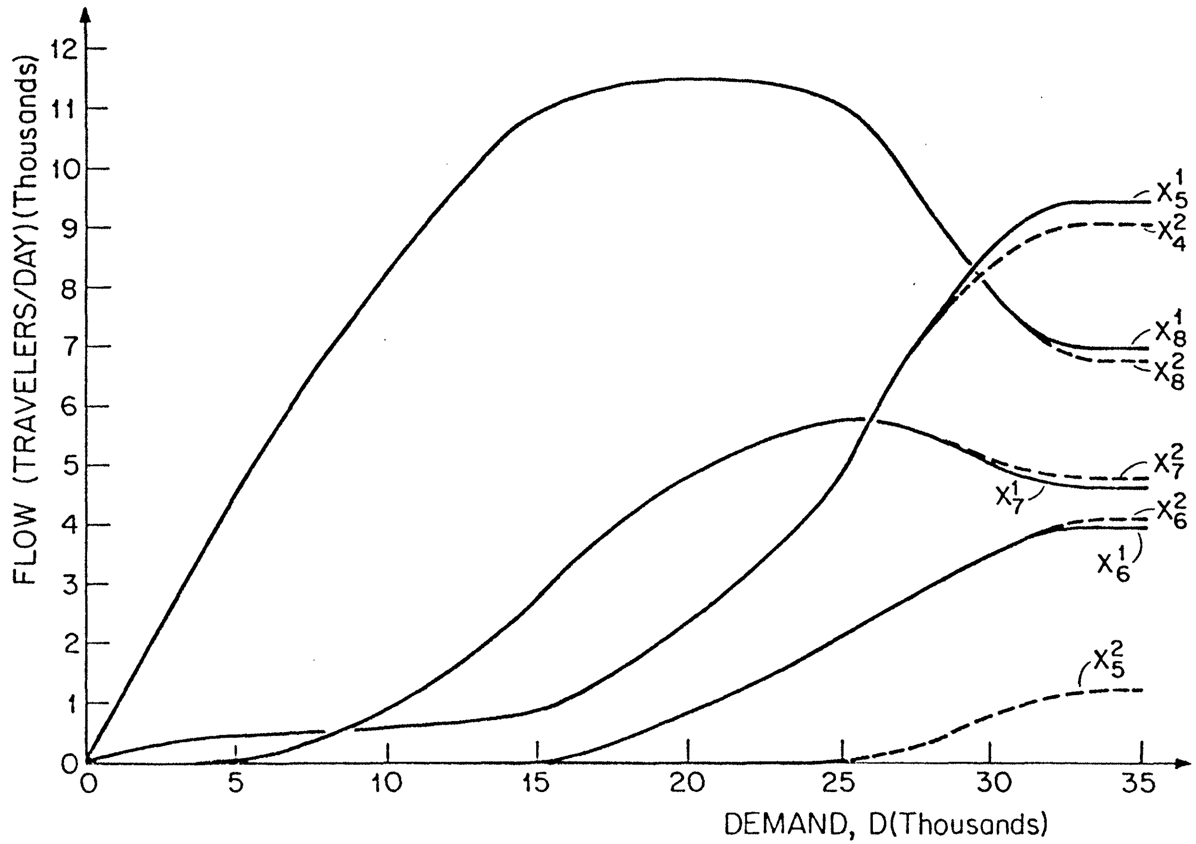



Figure 4.14: Flow vs. Demand for Case 2

can overtake the flow on another as demand inceases.

Case 3

In Case 3, the effect of the shape of the distribution is investigated. Two runs are compared that are identical -- 5 journeys for each of two classes; time budgets means are 1.25 hours and variances 0.5 $(\text{hour})^2$; money budgets means are 3.00 dollars and variances 1.5 $(\text{dollars})^2$-- in all respects except one. One run has gamma distributions and the other has uniform distributions. See Table 4.8.

The resulting flows are close to one another. If larger variances were used, we assume that the flows would be even closer. (Larger, more realistic, variances could not be used because one of the distributions was uniform. There is a limit on the variance of an uniform distribution whose mean is specified and whose lower limit is required to be zero or positive.) This assumption follows from the observation that the journey costs are similar in the two cases and from our experience that large variances tend to reduce the sensitivity of the distribution of flows to journey costs.

## 4.5 Conclusion

In this section, we have described a sequence of increasingly complicated examples for the purposes of characterizing the two versions of the new assignment principle and the equilibration technique. We have emphasized qualitative features. We conclude that the model behaves in a reasonable way.

In the following section, we use the model to draw conclusions about traffic-control strategies. In Section 6 and 7, a network of much greater detail and realism is analyzed and perturbed. The model continues to behave reasonably, and leads to the expectation that it can provide useful information for transportation planners and managers.

Table 4.8. Case 3

| Journey | Flow (gamma) | Flow (uniform) | Time (gamma) | Time (uniform) | Money Cost (gamma) | Money Cost (uniform) |
|---|---|---|---|---|---|---|
| null | 13405.39 | 14773.79 | 0 | 0 | 0 | 0 |
| 2-1-9-13-9-1-2 | 4802.91 | 3651.29 | .84 | .81 | 1.55 | 1.52 |
| 2-3-10-13-10-3-2 | 678.12 | 0 | 1.01 | 1.01 | 1.79 | 1.80 |
| 2-1-9-13-10-3-2 | 6441.36 | 5326.32 | 1.04 | 1.00 | 1.83 | 1.78 |
| 2-1-8-7-12-13-10-3-2 | 9672.22 | 11248.59 | 1.29 | 1.29 | 2.31 | 2.31 |

# 5. VEHICLE-ACTUATED CONTROL EXAMPLE

In this section, we describe the application of the new assignment principle to a network containing an intersection controlled by a vehicle-actuated (VA) signal light.

## 5.1 Description of Vehicle-actuated Control Strategy

The motivation for VA signals is the desire to reduce transportation delays in a network due to travelers encountering a red light at an intersection which is not at the time carrying flow in a conflicting direction. The strategy used in such a situation is: present a green signal to travelers approaching the intersection whenever the intersection is not occupied by travelers in the cross street. There needs to be an upper limit set on the length of time traffic in any direction sees green to prevent usurpation of the intersection by any one direction's travelers. A lower limit is also necessary to prevent undesirably rapid cycling of the signal.

To use the new assignment principle to study the effects of using a VA signal, we need a formula which will relate delay along a link to flow through the intersection. Such a formula, as an extension of the work of Courage and Papapanou (1977), has been suggested by Gartner in private communications with the authors. Since it is based on the widely used formulas introduced by Webster (1958) for use with fixed-cycle signals, a review of Webster's equations is in order.

## 5.2 Webster's Delay Formulas

Assuming that traffic is not modulated by any nearby control devices, the average delay per vehicle, $d^w$, approaching an intersection may be thought of as comprised of a component which would result from uniform flow, $d_d$, and a component due to stochastic effects, $d_s$.

$$d^w = d_d^w + d_s^w. \tag{5.1}$$

Webster (1958) gives the following approximate formulas:

$$d_d^w = \frac{0.45C(1-G^*/C)^2}{(1-f/s)}\ , \tag{5.2}$$

$$d_s^w = \frac{0.45\ x^2}{f(1-x)} \quad , \tag{5.3}$$

where we define the quantities

C = signal cycle time (sec)

G = Effective green time for the approach (sec)

f = arrival flow on approach (veh/sec)

s = saturation flow at signal stop line (veh/sec)

x = f/gs; ratio of flow to maximum possible flow under setting g.

g = G/C; proportion of cycle which is effectively green

The optimal splits of green time for approach j is given by

$$G_j^* = \frac{(C^*-L)y_j}{Y}, \tag{5.4}$$

where

$$y_j = \max(q_i/s_i) \tag{5.5}$$

over all roads i having simultaneous right of way during phase j.

$$Y = \sum_j y_j, \tag{5.6}$$

L = lost time at intersection.

The minimum cycle time, which will serve all phases operating at saturation levels, is

$$C_{min} = \frac{L}{1-Y}. \tag{5.7}$$

Webster shows that the fixed cycle time which minimizes delay is well approximiated by

$$C^* \cong 2C_{min}. \tag{5.8}$$

The lost time L is a fixed period of relative inactivity during each cycle of the signal due to driver reaction times and the disbursement of traffic as lead cars pull away from the stop line. It is L which will guide our selection of a minimum allowed cycle time for VA signals. If L = 10 seconds, a cycle time of 20 seconds will only allow a very few drivers from each phase of a two-phase intersection to pass for each cycle.

Notice that for all except very asymetric settings, the $d_s$ component of d dominates with increasing flow levels: its denominator approaches zero as f approaches gs($0 \leq g \leq 1$). It is the principal aim of VA signals dynamically to vary g as a function of demand so as to reduce $d_s$.

## 5.3 Vehicle-actuated Signal Delay Function

The formula we use for delay at a VA signal is an intuitively appealing modification of Webster's formula. Webster's C* optimal cycle setting is in fact a compromise value. If traffic flow here truly uniform, delay would be minimized in(5.1)by using $C_{min}$ in(5.2)with $d_s^w = 0$. Random fluctuations in flow necessitate the addition of the $d_s^w$ term. Such randomness would be better accommodated by selection of a longer cycle time than $C_{min}$ in the calculation of(5.3). The selection of control parameters for the VA signal is a straightforward application of these ideas. One selects a minimum cycle time to handle anticipated steady state flows,and allows extensions of green time in a given direction, up to a fixed limit, so as to allow complete depletion of the incoming queue in that direction.

This procedure suggests a modification to Webster's equation.

Let

$$d^v = d_d^v + d_s^v, \qquad (5.9)$$

$$d_d^v = \frac{0.45\, C_{min}(1 - G/C^*)}{(1 - f/s)}\,, \qquad (5.10)$$

where G is calculated as in (5.4), using $C_{min}$ instead of C*.

The form of $d_s^v$ is the same as in (5.3), the x term is now defined as

$$x^v = \frac{f}{(\frac{G}{C^*})\, s}\,, \qquad (5.11)$$

where G is calculated as in (5.4), using $C_{max}$ instead of C*.

$$C_{max} = G_{1\,max} + G_{2\,max} + L, \qquad (5.12)$$

where $G_{1\,max}$ and $G_{2\,max}$ are the maximum green times allowed in directions 1 and 2,respectively. As expected, when traffic volume gets high enough to cause the VA control to show green for the maximum time in both directions, the delay

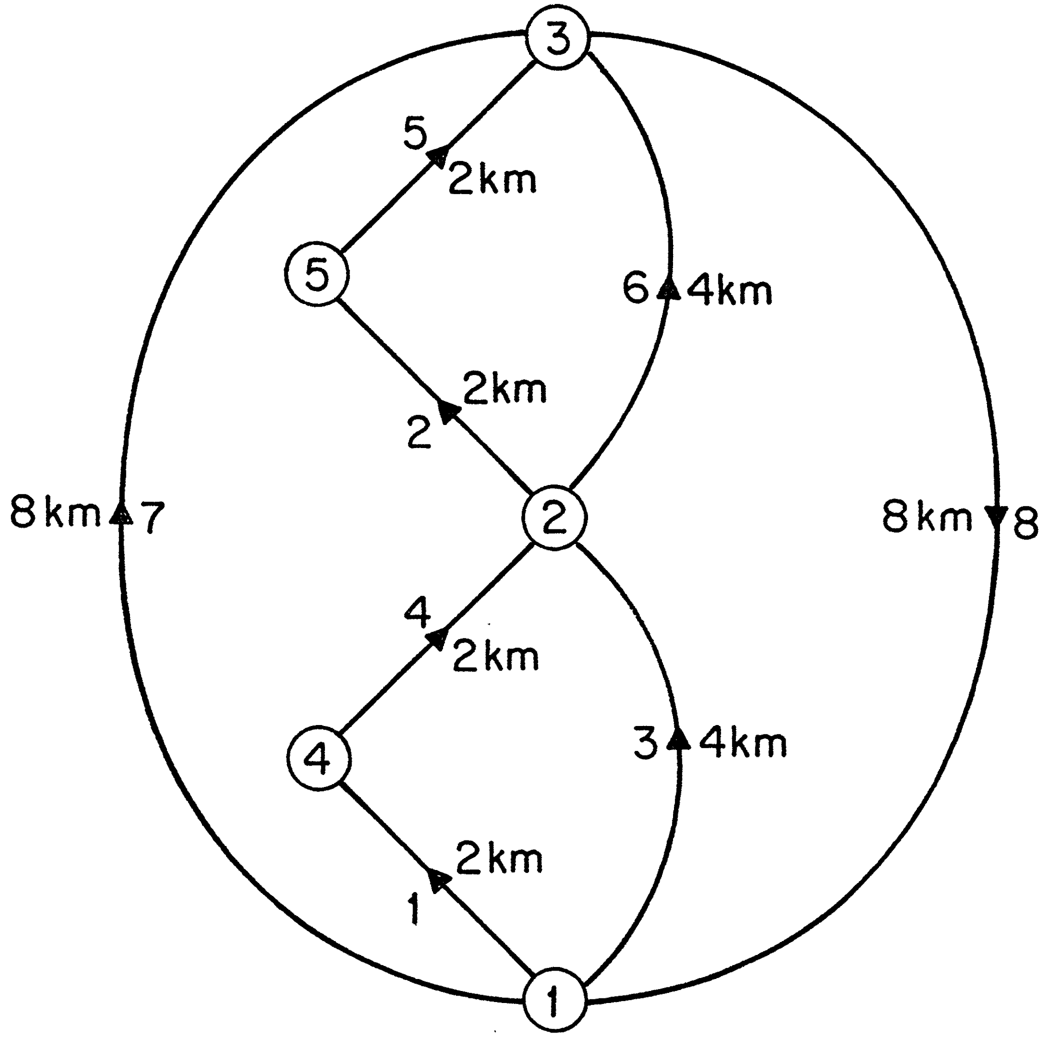



Figure 5.1: Example Network for Vehicle-actuated Control

is dominated by the second term and

$$d^V \cong d^W , \quad C \to C_{max} , \tag{5.13}$$

Figure 5.1 shows a small network used to exhibit properties of VA control. The intersection at node 2 is signalized and the functions used to calculate delays on links 1-2 and 4-2 are from equations (5.1) and (5.9). All other link delays are calculated using the fourth power formula.

$$d = \left[\frac{\text{link length}}{\text{50 kph}}\right]\left[1 + 0.15 \left(\frac{f}{\text{capacity}}\right)^4 .\right] \tag{5.14}$$

Link dollar costs are calculated as

$$m = \text{(length) (0.069/km) + (1.45/hour) (link time)}. \tag{5.15}$$

All link capacities are taken to be 2000 veh/day except link 3 - 1 which has a capacity of 5000. All traffic is one way, and no turns are permitted at node 2. All free-flow journey times are the same. There are two classes of travelers on the network. The only distinction between the two classes is their utility ranking of the four available journeys. The rankings of the journeys are presented in ascending order of preference for both classes.

Class 1

1. 1 - 1 (null)
2. 1 - 3 - 1
3. 1 - 2 - 5 - 3 - 1
4. 1 - 4 - 2 - 3 - 1

Class 2

1. 1 - 1 (null)
2. 1 - 3 - 1
3. 1 - 4 - 2 - 3 - 1
4. 1 - 2 - 5 - 3 - 1

## 5.4 Assignment with Fixed-Cycle Times

The network has first been investigated by using Webster's formulas for delays on links entering node 2. We begin by attempting to update our estimate of optimal cycle times and green splits after each iteration of the assignment algorithm. The flows on the network have converged to equilibrium in such a way that one journey or the other passing through node 2 carriers no flows. All flow through node 2 is carried by the journey assigned the larger starting flow, regardless of how small the difference between starting flows is made to be. This will always happen with such a procedure because in equation (5.4) we see that the larger the percentage saturation (i.e., flow or links with equal capacity) a link carries, the larger percentage of the control cycle it will be afforded green. This positive feedback situation is hopelessly unstable without a system management involving some sort of hybrid optimization (Tan and Gershwin, 1979).

Due to the symmetries of the network and the class demands and journey preferences, we have assumed that the flow volumes on links 1 -2 and 4-2 will be equal. We therefore have fixed $g = 0.5$. The value of $C^*$ is updated after every iteration of the assignment procedure. Table 5.1 shows the equilibrium flow distribution by link and by journey.

## 5.5 Assignment with Vehicle-actuated Cycle Times

Table 5.2 shows equilibrium-flow distribution by link and journey when a VA control scheme is employed. The first thing we note is that there is very little effect on flows through the signalized intersection as a result of the different signal strategy. There is almost no change at all in the total number of travelers on the network.

The effect of the change in control may only be seen in the delays introduced at node 2 as a function of congestion. The times for traversing a link given in Tables 5.2 and 5.3 include a free-flow delay as well as the delay due to conditions at intersection. The free-flow delays on links 1-2 and 4-2 are 0.078 and 0.039 hours, respectively. To see the effects of the VA signal on intersection delay, we subtract the free-flow delay from the link times and compare the Webster and VA numbers. We then find that delay at the intersection has been reduced by 4 percent on link 3 and by 58 percent on link 4 by introducing the VA control scheme. This information is reflected, although not so strongly, in the average speeds on the links.

Table 5.1 Fixed-Cycle Equilibrum Data

| LNK | START | END | TIME | DOLLARS | FLOW | SPEED |
|---|---|---|---|---|---|---|
| 1 | 1 | 4 | 0.039 | 0.19 | 570.85 | 31.97 |
| 2 | 2 | 5 | 0.039 | 0.19 | 570.88 | 31.97 |
| 3 | 1 | 2 | 0.099 | 0.42 | 570.88 | 25.24 |
| 4 | 4 | 2 | 0.060 | 0.22 | 570.85 | 20.82 |
| 5 | 5 | 3 | 0.039 | 0.19 | 570.88 | 31.97 |
| 6 | 2 | 3 | 0.078 | 0.39 | 570.85 | 31.97 |
| 7 | 1 | 3 | 0.091 | 0.68 | 71.42 | 32.00 |
| 8 | 3 | 1 | 0.091 | 0.68 | 1213.15 | 31.98 |

CLASS 1

| JOUR | FLOW | TIME | DOLLARS |
|---|---|---|---|
| 1 | 893.42 | 0.00 | 0.00 |
| 2 | 35.71 | 0.18 | 6.36 |
| 3 | 0.02 | 0.27 | 6.49 |
| 4 | 570.85 | 0.27 | 6.49 |

CLASS 2

| JOUR | FLOW | TIME | DOLLARS |
|---|---|---|---|
| 1 | 893.42 | 0.00 | 0.00 |
| 2 | 35.71 | 0.18 | 6.36 |
| 3 | 0.00 | 0.27 | 6.49 |
| 4 | 570.87 | 0.27 | 6.49 |

Table 5.2 Vehicle-actuated Equilibrium Data

| LNK | START | END | TIME | DOLLARS | FLOW | SPEED |
|---|---|---|---|---|---|---|
| 1 | 1 | 4 | 0.039 | 0.19 | 580.02 | 31.97 |
| 2 | 2 | 5 | 0.039 | 0.19 | 579.98 | 31.97 |
| 3 | 1 | 2 | 0.079 | 0.39 | 579.98 | 31.81 |
| 4 | 4 | 2 | 0.039 | 0.19 | 580.02 | 31.61 |
| 5 | 5 | 3 | 0.039 | 0.19 | 579.98 | 31.97 |
| 6 | 2 | 3 | 0.078 | 0.39 | 580.02 | 31.97 |
| 7 | 1 | 3 | 0.091 | 0.68 | 53.27 | 32.00 |
| 8 | 3 | 1 | 0.091 | 0.68 | 1213.28 | 31.98 |

CLASS 1

| JOUR | FLOW | TIME | DOLLARS |
|---|---|---|---|
| 1 | 893.42 | 0.00 | 0.00 |
| 2 | 26.64 | 0.18 | 6.36 |
| 3 | 0.08 | 0.25 | 6.46 |
| 4 | 579.92 | 0.25 | 6.46 |

CLASS 2

| JOUR | FLOW | TIME | DOLLARS |
|---|---|---|---|
| 1 | 893.42 | 0.00 | 0.00 |
| 2 | 26.64 | 0.18 | 6.36 |
| 3 | 0.10 | 0.25 | 6.46 |
| 4 | 579.90 | 0.25 | 6.46 |

## 5.6 Conclusions

Introduction of a VA signal in this particular simple example network has negligible effect on the total number of class members who avail themselves of travel opportunity. There is very little redistribution of travelers from one journey to another and even on the journeys directly passing through the VA signal, there is very little flow change.

Although the temptation is great to conclude that VA signals may not be of any great value, we must remember that the particular network used in this example introduces a certain measurement bias. On the one hand, it points out the fact that local improvements may have little effect on total network equilibrium. On the other hand, its geometry and parameters are such that considerable local improvement is unduly overshadowed by a large surrounding framework. If, for example, the network consists of a large number of intersections connected by short links which carry only medium to light flow, the delay saving at intersections will overshadow the fixed free-flow delays in the network, and we expect an improvement in system performance as measured by percentage delay decrease. Such a situation exists in an urban center during non-peak periods.

It is also worth nothing that even in the network in (5.1), approximately 570 people each on links 4-2 and 1-2 experience a time savings of 0.02 hour each day for a total savings of 23 work-hours/day. This kind of savings can be important when considering networks of realistic size and complexity.

# 6. EFFECTS OF TRAFFIC MANAGEMENT POLICIES

In this section, we present a network of much greater detail than any discussed thus far. We show that a system of realistic size and complexity can be treated using the technique described here, that it yields reasonable results, and that it can be used to assess certain traffic management policies. These policies include changing transit fares, adding tolls, and prohibiting cars from certain links. In Section 7, we consider changes to the network that involve construction.

The purpose of Sections 6 and 7 is to demonstrate only the kinds of results that can be obtained using the model described in this report. In particular, we show that a detailed network can be analyzed. We do not intend to deduce any general lessons on real cities or on which network modifications can be expected to yield greatest benefits.

Relatively crude models for mode delays and costs are used. For example, parking fees are not included. Only a single kind of car is represented, and a single global average of 1.5 occupants per car is used for all economic classes and all residential locations. Walking, which is an important transportation mode in the dense downtown area, is not considered.

In spite of this, the results are reasonable. They indicate that, with some additional research devoted to more exact mode models and with accurate socio economic data about a real city, the kinds of uses illustrated here and in Section 7 can yield important, useful information.

## 6.1 Discussion of System

### 6.1.1 Network

The network is presented in Figure 6.1. It has 144 nodes and 264 links. It is symmetric about its vertical and horizontal axes. Its total length is 343.2 kilometers. Lengths of links are indicated in the figure. Roads are most dense near the city center and become less dense toward the periphery. All streets are considered to be one-way.

We calculate the utility of a journey by counting the number of nodes that the journey passes through. This count is multiplied by 1.1 for car journeys to represent the attractiveness of a car over that of a bus. All nodes are

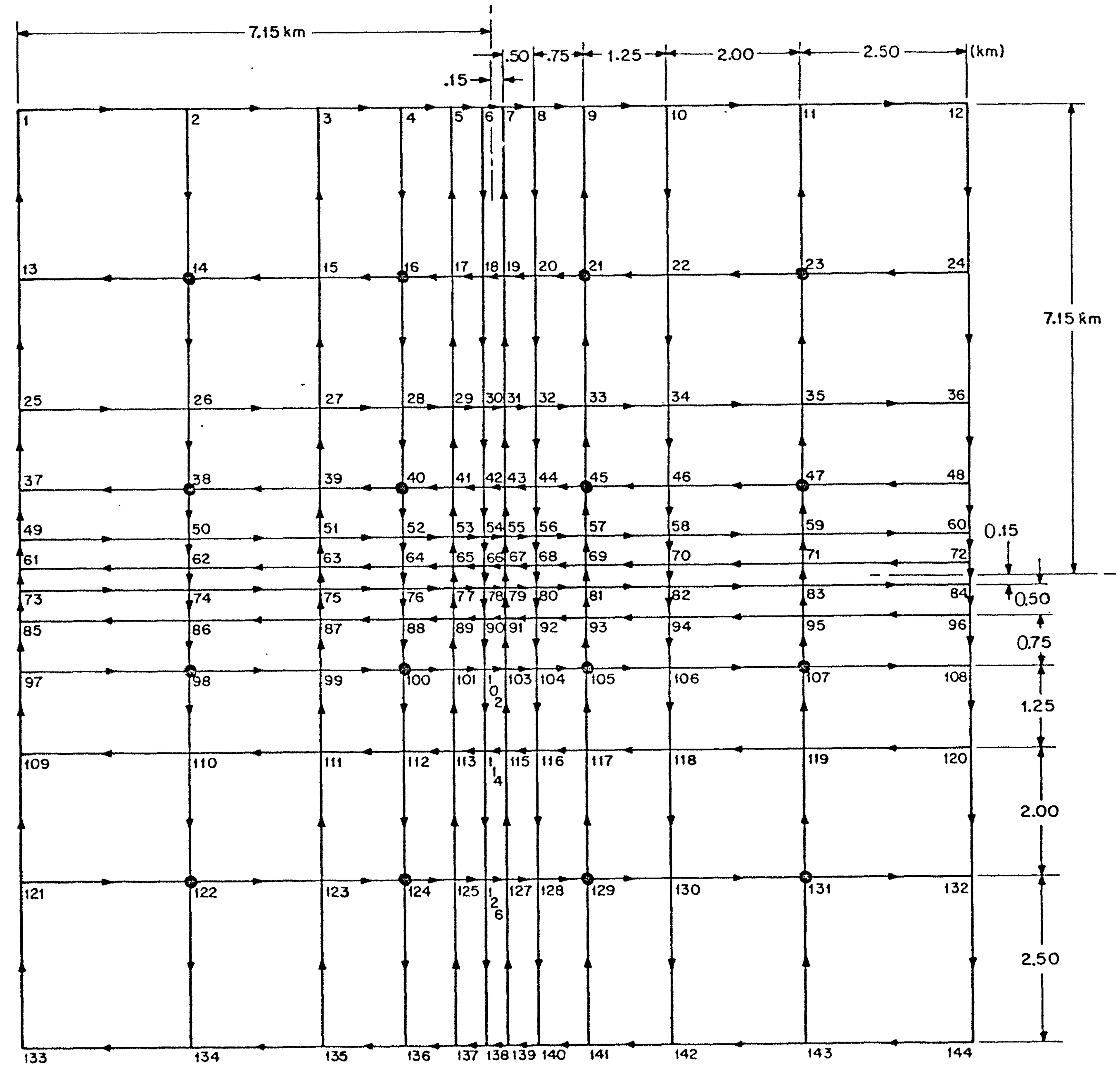



Figure 6.1: Large Network

equally desirable. Because of the greater density of nodes near the city center, journeys going inward are more valuable than those going outward.

Journeys are paths in the network that start and end at the same residential node. Between 76 and 81 journeys are made available to each class. (See Table 6.1) No mixed-mode journeys have been considered.

Car journeys are allowed to go anywhere in the network as long as path-continuity and link directions are respected. Bus journeys are more restricted. They have the same freedom as cars in the inner city; that is, among nodes 40, 45, 100, and 105. However, outside of that region they may travel only on certain links. These are the links that connect the following nodes: 13-24; 37-48; 97-108; 121-132; 2, 14, 26,...,134; 4, 16, 18,...,136; 9, 21, 33,...,141; 11, 23, 35,...,143.

6.1.2 Population

The population consists of 300,000 people divided into 2 income groups and 16 residential locations. The wealthier group has a household income of 35,000 dollars per year and the poorer has a household income of 15,000 dollars per year. Households are assumed to contain 3 people, on the average; and years have, on the average, 320 days of regular travel, so these incomes are 36.46 dollars per day per person and 15.63 dollars per person respectively.

The locations at which people live and their class designations are listed below. Classes 1 to 16 are the wealthier people; classes 17 to 32 are the poorer. The number of people in each class is 9375.

| Node | Class |
|---|---|
| 14 | 1, 17 |
| 16 | 2, 18 |
| 21 | 3, 19 |
| 23 | 4, 20 |
| 38 | 5, 21 |
| 40 | 6, 22 |
| 45 | 7, 23 |
| 47 | 8, 24 |
| 98 | 9, 25 |
| 100 | 10, 26 |
| 105 | 11, 27 |
| 107 | 12, 28 |
| 122 | 13, 29 |
| 124 | 14, 30 |
| 129 | 15, 31 |
| 131 | 16, 32 |

Because of the great size of this system, techniques have been sought to reduce computation time. A very effective technique is to use a triangle density function (Figure 4.7) rather than a gamma or other distribution. Calculations involving the triangle are simple and, as we argued in Section 4.4, probably accurate when variances are large.

The time budget distribution for all classes is triangular with mean 1.1 hours and variance 0.43 $(\text{hours})^2$ for each traveler, which yields a coefficient of variation of 0.6. This is consistent with Zahavi's (1979) empirical findings. The money budget is also triangular, with means of 11 percent, or 6.00 and 2.55 dollars per traveler. The variances are 17.50 $\text{dollars}^2$ and 3.10 $\text{dollars}^2$, respectively.

### 6.1.3 Time Costs

The time for a journey is the sum of the times for its constituent links. In the case of bus journeys, an additional delay of 0.2 hours per loop is added to represent the time spent waiting for a bus.

Link travel time for cars is given by

$$\tau_\ell = \tau_{\ell 1}\left(1 + 0.15\left(\frac{f_\ell}{c_\ell}\right)^4\right), \qquad (6.1)$$

where $\tau_{\ell 1}$, and $c_\ell$ are constants. The capacity, $c_\ell$, is 24000 vehicles per day. The free-flow time, $\tau_{\ell 1}$, is the length of each link (as indicated in (Figure 6.1) divided by the free-flow speed, which is assumed to be 50 km/h. Link travel time for buses is assumed to be twice that for cars.

The flow, $f_\ell$, is given as

$$f_\ell = \alpha_c f_\ell^c + \alpha_b f_\ell^b, \qquad (6.2)$$

where $f_\ell^c$ and $f_\ell^b$ are the total car-traveler flow and bus-traveler flow through link $\ell$. That is,

$$f_\ell^c = \sum_a \sum_{j \varepsilon car} A_{j\ell}^c x_j^a, \qquad (6.3)$$

$$f_\ell^b = \sum_a \sum_{j \varepsilon bus} A_{j\ell}^b x_j^a, \qquad (6.4)$$

where the second sum in (6.3) is over all car journeys, and the second sum in (6.4) is over all bus journeys. The quantities $\alpha_c$ and $\alpha_b$ represent the impact of a single traveler on the total car-equivalent flow.

We choose

$$\alpha_c = 1/1.5$$

since we assume a car occupancy of 1.5 travelers, and

$$\alpha_b = 0.125.$$

This is obtained by dividing the bus equivalency factor (2) by an average bus occupancy of 16.

### 6.1.4 Money Costs

The cost of most bus journeys is a fare of 1.00 dollars. There are a small number of journeys that involve multiple loops. That is, some nodes, other than the residence node, are visited more than once. We assume that travelers taking such journeys are making stops that do not allow free transfers, so they are charged one dollar per loop.

The cost of a car journey is given by

$$m_j = \frac{1}{1.5} (m^F + m^D + m^T t_j), \tag{6.5}$$

in which $m^F$ is the fixed cost of owning a car and is assumed to be 5.00 dollars/day. This value has been chosen to account for the cost of purchasing a car as well as other fixed costs such as insurance. The divisor 1.5 is the average car occupancy and reflects the occupants sharing the car's costs. The other terms are due to the empirical findings of Evans and Herman (1978) who found that the fuel cost of a car is linear in the distance and the time it travels. We have increased their coefficients to account for other running costs such as repairs and maintenance. We assume $m^D$ is 0.0672 dollars per kilometer times the length of a journey, and that $m^t$ is 2.322 dollars per hour.

## 6.2 Basic Network

Table 6.1 lists the journeys available to the classes that originate at nodes 14, 16, 38, and 40, and their utilities. Because of the symmetry of the network, this is all that is required to describe all the journeys for all classes. (Note that a journey of the form 14-14 is a null journey for classes at node 14.)

As noted earlier, only two modes are considered here. Additional modes such as walking can be easily included. Walking is particularly easy to include since the time for a path is independent of flow and its money cost is negligible.

Table 6.2 lists summary results for this network. Note that more wealthy people take cars than buses; more poor people take buses. More wealthy people travel. Note that car behavior is similar for all people who take cars, and bus behavior is the same for all bus riders; the major difference in behavior is due to the different fractions of each class who take each mode.

Note that more people travel in the inner city (i.e., more of those who originate at nodes 40, 45, 100, and 105) and more stay at home in the suburbs.

Wealthy people gain more benefits, i.e., utility, from travel, than poor people. They spend more money, and more of them take cars. Poor people spend more time traveling. Note that all times and speeds are door-to-door times and speeds.

Certain links are particularly congested. For example, links 40-52, 52-64, and 43-42 have flows in excess of 20,000 vehicles per day, and links 42-41 and 41-40 have flow over 16,000 vehicles per day. (The same statements are true of 3 other sets of links that are 90, 180, and 270 degree rotations away from these.) Similarly, links 21-20, 20-19, 19-18, 18-17, and 17-16 have flows over 26,000 vehicles per day.

In the remainder of Section 6, and in all of Section 7, changes are made in the network description. All results reported in the tables should be compared with those of Table 6.2.

## Table 6.1 List of Journeys

| PATH | ORIGINS | UTIL | PATH SPECIFICATIONS | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 14 | 2.2 | 14 | 14 | | | | | | | | | | | |
| 2 | 16 | 2.2 | 16 | 16 | | | | | | | | | | | |
| 3 | 38 | 2.2 | 38 | 38 | | | | | | | | | | | |
| 4 | 40 | 2.2 | 40 | 40 | | | | | | | | | | | |
| 5* | 40 | 5.0 | 40 | 52 | 53 | 41 | 40 | | | | | | | | |
| 6 | 14 | 5.5 | 14 | 26 | 27 | 15 | 14 | | | | | | | | |
| 7 | | 5.5 | 67 | 66 | 78 | 79 | 67 | | | | | | | | |
| 8 | | 5.5 | 53 | 54 | 66 | 65 | 53 | | | | | | | | |
| 9 | 40 | 5.5 | 41 | 40 | 52 | 53 | 41 | | | | | | | | |
| 10 | 40 | 5.5 | 27 | 28 | 40 | 39 | 27 | | | | | | | | |
| 11 | 14 | 5.5 | 14 | 13 | 1 | 2 | 14 | | | | | | | | |
| 12 | 14 | 5.5 | 11 | 12 | 24 | 23 | 11 | | | | | | | | |
| 13 | | 5.5 | 5 | 6 | 18 | 17 | 5 | | | | | | | | |
| 14 | 16 14 | 9.9 | 14 | 26 | 27 | 28 | 29 | 17 | 16 | 15 | 14 | | | | |
| 15 | 38 14 | 9.9 | 14 | 26 | 38 | 50 | 51 | 39 | 27 | 15 | 14 | | | | |
| 16 | | 9.9 | 53 | 54 | 55 | 56 | 68 | 67 | 66 | 65 | 53 | | | | |
| 17 | | 9.9 | 5 | 6 | 7 | 8 | 20 | 19 | 18 | 17 | 5 | | | | |
| 18* | 40 | 13.0 | 40 | 52 | 64 | 76 | 77 | 78 | 79 | 67 | 55 | 43 | 42 | 41 | 40 |
| 19* | 40 | 13.0 | 40 | 52 | 64 | 76 | 88 | 100 | 101 | 89 | 77 | 65 | 53 | 41 | 40 |
| 20 | 38 16 14 | 14.3 | 14 | 26 | 38 | 50 | 51 | 39 | 27 | 28 | 29 | 17 | 16 | 15 | 14 |
| 21 | 38 16 14 | 14.3 | 14 | 26 | 38 | 50 | 51 | 52 | 53 | 41 | 29 | 17 | 16 | 15 | 14 |
| 22 | | 14.3 | 53 | 54 | 55 | 56 | 68 | 80 | 92 | 91 | 90 | 89 | 77 | 65 | 53 |
| 23 | 40 | 14.3 | 40 | 52 | 64 | 76 | 77 | 78 | 79 | 67 | 55 | 43 | 42 | 41 | 40 |
| 24 | | 14.3 | 27 | 28 | 29 | 30 | 42 | 54 | 66 | 65 | 64 | 63 | 51 | 39 | 27 |
| 25 | 40 | 14.3 | 40 | 52 | 53 | 54 | 55 | 56 | 57 | 45 | 44 | 43 | 42 | 41 | 40 |
| 26 | 40 38 16 | 14.3 | 3 | 4 | 16 | 28 | 40 | 39 | 38 | 37 | 25 | 26 | 27 | 15 | 3 |
| 27* | 40 | 17.0 | 40 | 52 | 64 | 76 | 88 | 100 | 101 | 102 | 103 | 91 | 79 | 67 | 55 |
| | | | 43 | 42 | 41 | 40 | | | | | | | | | |
| 28 | 38 16 14 | 18.7 | 14 | 26 | 38 | 50 | 51 | 39 | 27 | 28 | 29 | 30 | 31 | 19 | 18 |
| | | | 17 | 16 | 15 | 14 | | | | | | | | | |
| 29 | 38 16 14 | 18.7 | 14 | 26 | 38 | 50 | 62 | 74 | 75 | 63 | 51 | 39 | 27 | 28 | 29 |
| | | | 17 | 16 | 15 | 14 | | | | | | | | | |
| 30 | 38 16 14 | 18.7 | 14 | 26 | 38 | 50 | 51 | 52 | 53 | 54 | 55 | 43 | 31 | 19 | 18 |
| | | | 17 | 16 | 15 | 14 | | | | | | | | | |
| 31 | 38 16 14 | 18.7 | 14 | 26 | 38 | 50 | 62 | 74 | 75 | 76 | 77 | 65 | 53 | 41 | 29 |
| | | | 17 | 16 | 15 | 14 | | | | | | | | | |
| 32 | 40 16 | 18.7 | 1 | 2 | 3 | 4 | 16 | 28 | 40 | 52 | 64 | 63 | 62 | 61 | 49 |
| | | | 37 | 25 | 13 | 1 | | | | | | | | | |
| 33 | 40 | 18.7 | 40 | 52 | 64 | 76 | 77 | 78 | 79 | 80 | 81 | 69 | 57 | 45 | 44 |
| | | | 43 | 42 | 41 | 40 | | | | | | | | | |
| 34 | 40 | 18.7 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 46 | 45 | 44 | 43 | 42 |
| | | | 41 | 40 | 39 | 27 | | | | | | | | | |
| 35 | 40 | 18.7 | 3 | 4 | 5 | 6 | 7 | 8 | 20 | 32 | 44 | 43 | 42 | 41 | 40 |
| | | | 39 | 27 | 15 | 3 | | | | | | | | | |
| 36* | 40 | 21.0 | 40 | 52 | 64 | 76 | 88 | 100 | 101 | 89 | 77 | 65 | 53 | 54 | 55 |
| | | | 56 | 57 | 45 | 44 | 43 | 42 | 41 | 40 | | | | | |
| 37* | 40 | 21.0 | 40 | 52 | 64 | 76 | 88 | 100 | 101 | 102 | 103 | 91 | 79 | 80 | 81 |
| | | | 69 | 57 | 45 | 44 | 43 | 42 | 41 | 40 | | | | | |
| 38* | 40 | 21.0 | 40 | 52 | 64 | 76 | 77 | 78 | 90 | 102 | 103 | 91 | 79 | 80 | 81 |
| | | | 69 | 68 | 67 | 55 | 43 | 42 | 41 | 40 | | | | | |
| 39* | 40 | 21.0 | 40 | 52 | 64 | 76 | 77 | 78 | 90 | 102 | 103 | 91 | 79 | 80 | 81 |
| | | | 69 | 57 | 45 | 44 | 43 | 42 | 41 | 40 | | | | | |
| 40* | 40 | 21.0 | 40 | 52 | 64 | 76 | 88 | 100 | 101 | 102 | 103 | 104 | 105 | 93 | 81 |
| | | | 69 | 57 | 45 | 44 | 43 | 42 | 41 | 40 | | | | | |
| 41 | 38 16 14 | 23.1 | 14 | 26 | 38 | 50 | 62 | 74 | 75 | 76 | 77 | 65 | 53 | 54 | 55 |
| | | | 43 | 31 | 19 | 18 | 17 | 16 | 15 | 14 | | | | | |
| 42 | 38 16 14 | 23.1 | 14 | 26 | 38 | 50 | 62 | 74 | 75 | 76 | 77 | 78 | 79 | 67 | 55 |
| | | | 43 | 31 | 19 | 18 | 17 | 16 | 15 | 14 | | | | | |
| 43 | 40 | 23.1 | 40 | 52 | 64 | 76 | 88 | 100 | 101 | 102 | 103 | 104 | 105 | 93 | 81 |
| | | | 69 | 57 | 45 | 44 | 43 | 42 | 41 | 40 | | | | | |
| 44 | | 23.1 | 27 | 28 | 29 | 30 | 31 | 32 | 44 | 56 | 68 | 80 | 92 | 91 | 90 |
| | | | 39 | 88 | 87 | 75 | 63 | 51 | 39 | 27 | | | | | |
| 45 | | 23.1 | 1 | 2 | 3 | 4 | 5 | 6 | 18 | 30 | 42 | 54 | 66 | 65 | 64 |
| | | | 63 | 62 | 61 | 49 | 37 | 25 | 13 | 1 | | | | | |
| 46 | | 23.1 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 46 | 58 | 70 | 69 | 68 |
| | | | 67 | 66 | 65 | 64 | 63 | 51 | 39 | 27 | | | | | |
| 47 | 40 | 23.1 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 22 | 34 | 46 | 45 | 44 |
| | | | 43 | 42 | 41 | 40 | 39 | 27 | 15 | 3 | | | | | |
| 48 | | 23.1 | 3 | 4 | 5 | 6 | 18 | 30 | 42 | 54 | 66 | 65 | 64 | 63 | 62 |
| | | | 61 | 49 | 37 | 25 | 26 | 27 | 15 | 3 | | | | | |
| 49* | 40 38 16 | 25.0 | 40 | 52 | 64 | 76 | 88 | 100 | 101 | 102 | 103 | 104 | 105 | 93 | 81 |

Table 6.1 <u>List of Journeys</u> (cont'd)

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 69 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 | 16 | 28 | 40 | |
| 50 | 40 38 16 14 | 27.5 | 14 | 26 | 38 | 50 | 62 | 74 | 75 | 76 | 77 | 65 | 53 | 54 | 55 |
| | | | 56 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | |
| 51 | 40 38 16 14 | 27.5 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 89 | 77 |
| | | | 65 | 53 | 54 | 55 | 43 | 31 | 19 | 18 | 17 | 16 | 15 | 14 | |
| 52 | 40 38 16 14 | 27.5 | 14 | 26 | 38 | 50 | 62 | 74 | 75 | 76 | 77 | 78 | 79 | 80 | 81 |
| | | | 69 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | |
| 53 | 40 38 16 14 | 27.5 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 91 | 79 | 67 | 55 | 43 | 31 | 19 | 18 | 17 | 16 | 15 | 14 | |
| 54 | | 27.5 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 46 | 58 | 70 | 82 | 94 |
| | | | 93 | 92 | 91 | 90 | 89 | 86 | 87 | 75 | 63 | 51 | 39 | 27 | |
| 55* | 40 38 16 14 | 29.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 104 | 105 | 93 | 81 | 69 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 |
| | | | 16 | 15 | 14 | | | | | | | | | | |
| 56* | 40 38 16 14 | 29.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 89 | 77 |
| | | | 65 | 53 | 54 | 55 | 56 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 |
| | | | 16 | 15 | 14 | | | | | | | | | | |
| 57* | 40 38 16 14 | 29.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 91 | 79 | 80 | 81 | 69 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 |
| | | | 16 | 15 | 14 | | | | | | | | | | |
| 58* | 40 38 16 14 | 29.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 91 | 79 | 80 | 81 | 69 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 |
| | | | 16 | 15 | 14 | | | | | | | | | | |
| 59 | 40 38 16 14 | 31.9 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 91 | 79 | 80 | 81 | 69 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 |
| | | | 16 | 15 | 14 | | | | | | | | | | |
| 60 | 40 38 16 14 | 31.9 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 104 | 105 | 93 | 81 | 69 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 |
| | | | 16 | 15 | 14 | | | | | | | | | | |
| 61 | | 31.9 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 46 | 58 | 70 | 82 | 94 |
| | | | 106 | 118 | 117 | 116 | 115 | 114 | 113 | 112 | 111 | 99 | 87 | 75 | 63 |
| | | | 51 | 39 | 27 | | | | | | | | | | |
| 62 | | 31.9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 20 | 32 | 44 | 56 | 68 |
| | | | 80 | 92 | 91 | 90 | 89 | 88 | 87 | 86 | 85 | 73 | 61 | 49 | 37 |
| | | | 25 | 13 | 1 | | | | | | | | | | |
| 63 | | 31.9 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 22 | 34 | 46 |
| | | | 58 | 70 | 69 | 68 | 67 | 66 | 65 | 64 | 63 | 62 | 61 | 49 | 37 |
| | | | 25 | 13 | 1 | | | | | | | | | | |
| 64 | | 31.9 | 3 | 4 | 5 | 6 | 7 | 8 | 20 | 32 | 44 | 56 | 68 | 80 | 92 |
| | | | 91 | 90 | 89 | 88 | 87 | 86 | 85 | 73 | 61 | 49 | 37 | 25 | 26 |
| | | | 27 | 15 | 3 | | | | | | | | | | |
| 65 | 40 38 16 14 | 31.9 | 14 | 26 | 38 | 50 | 62 | 74 | 75 | 76 | 77 | 65 | 53 | 54 | 66 |
| | | | 65 | 53 | 54 | 55 | 56 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 |
| | | | 16 | 15 | 14 | | | | | | | | | | |
| 66 | 38 16 14 | 31.9 | 14 | 26 | 33 | 50 | 62 | 74 | 75 | 63 | 51 | 39 | 27 | 28 | 29 |
| | | | 30 | 42 | 54 | 66 | 65 | 64 | 63 | 51 | 39 | 27 | 28 | 29 | 17 |
| | | | 16 | 15 | 14 | | | | | | | | | | |
| 67 | 40 38 16 14 | 31.9 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 89 | 77 |
| | | | 65 | 53 | 41 | 40 | 52 | 53 | 54 | 55 | 43 | 31 | 19 | 18 | 17 |
| | | | 16 | 15 | 14 | | | | | | | | | | |
| 68 | 38 16 14 | 31.9 | 14 | 26 | 38 | 50 | 62 | 74 | 75 | 76 | 77 | 65 | 53 | 54 | 55 |
| | | | 56 | 68 | 67 | 66 | 65 | 53 | 54 | 55 | 43 | 31 | 19 | 18 | 17 |
| | | | 16 | 15 | 14 | | | | | | | | | | |
| 69* | 40 38 16 14 | 33.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 104 | 105 | 106 | 107 | 95 | 83 | 71 | 59 | 47 | 35 | 23 | 22 | 21 |
| | | | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | | | | | |
| 70* | 40 38 16 14 | 33.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 89 | 77 |
| | | | 65 | 53 | 41 | 40 | 52 | 53 | 54 | 55 | 56 | 57 | 45 | 33 | 21 |
| | | | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | | | | | |
| 71 | 40 38 16 14 | 36.3 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 91 | 79 | 80 | 81 | 82 | 83 | 71 | 59 | 47 | 35 | 23 | 22 | 21 |
| | | | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | | | | | |
| 72 | 40 38 16 14 | 36.3 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 110 | 122 | 123 | 124 | 125 |
| | | | 126 | 127 | 115 | 103 | 91 | 79 | 80 | 81 | 69 | 57 | 45 | 33 | 21 |
| | | | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | | | | | |
| 73 | 40 38 16 14 | 36.3 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 104 | 105 | 106 | 107 | 95 | 83 | 71 | 59 | 47 | 35 | 23 | 22 | 21 |
| | | | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | | | | | |
| 74 | 40 38 16 14 | 36.3 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 110 | 122 | 123 | 124 | 125 |
| | | | 126 | 127 | 128 | 129 | 117 | 105 | 93 | 81 | 69 | 57 | 45 | 33 | 21 |
| | | | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | | | | | |
| 75 | | 36.3 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 24 |
| | | | 36 | 48 | 60 | 72 | 71 | 70 | 69 | 68 | 67 | 66 | 65 | 64 | 63 |

Table 6.1 List of Journeys (cont'd)

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 62 | 61 | 49 | 37 | 25 | 13 | 1 | | | | | | |
| 76* | 38 16 14 | 37.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 110 | 122 | 123 | 124 | 125 |
| | | | 126 | 127 | 128 | 129 | 130 | 131 | 119 | 107 | 95 | 83 | 71 | 59 | 47 |
| | | | 35 | 23 | 22 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | |
| 77* | 40 38 16 14 | 37.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 110 | 122 | 123 | 124 | 125 |
| | | | 126 | 127 | 128 | 129 | 117 | 105 | 106 | 107 | 95 | 83 | 71 | 59 | 47 |
| | | | 35 | 23 | 22 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | |
| 78* | 40 38 16 14 | 37.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 110 | 122 | 123 | 124 | 125 |
| | | | 126 | 127 | 128 | 129 | 117 | 105 | 106 | 107 | 95 | 83 | 71 | 59 | 47 |
| | | | 46 | 45 | 33 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | |
| 79* | 40 38 16 14 | 37.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 110 | 122 | 123 | 124 | 125 |
| | | | 126 | 127 | 128 | 129 | 130 | 131 | 119 | 107 | 95 | 83 | 71 | 59 | 47 |
| | | | 46 | 45 | 33 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | |
| 80 | 40 38 16 14 | 40.7 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 110 | 122 | 123 | 124 | 125 |
| | | | 126 | 127 | 128 | 129 | 117 | 105 | 106 | 107 | 95 | 83 | 71 | 59 | 47 |
| | | | 35 | 23 | 22 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | |
| 81 | 38 16 14 | 40.7 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 110 | 122 | 123 | 124 | 125 |
| | | | 126 | 127 | 128 | 129 | 130 | 131 | 119 | 107 | 95 | 83 | 71 | 59 | 47 |
| | | | 35 | 23 | 22 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | |
| 82 | | 40.7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 22 | 34 | 46 |
| | | | 58 | 70 | 82 | 94 | 106 | 118 | 117 | 116 | 115 | 114 | 113 | 112 | 111 |
| | | | 110 | 109 | 97 | 85 | 73 | 61 | 49 | 37 | 25 | 13 | 1 | | |
| 83 | | 40.7 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 24 |
| | | | 36 | 48 | 60 | 72 | 84 | 96 | 95 | 94 | 93 | 92 | 91 | 90 | 89 |
| | | | 88 | 87 | 86 | 85 | 73 | 61 | 49 | 37 | 25 | 13 | 1 | | |
| 84 | | 40.7 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 22 | 34 | 46 | 58 | 70 |
| | | | 82 | 94 | 106 | 118 | 117 | 116 | 115 | 114 | 113 | 112 | 111 | 110 | 109 |
| | | | 97 | 85 | 73 | 61 | 49 | 37 | 25 | 26 | 27 | 15 | 3 | | |
| 85 | 38 16 14 | 40.7 | 14 | 26 | 38 | 50 | 51 | 39 | 27 | 28 | 29 | 30 | 31 | 32 | 44 |
| | | | 56 | 68 | 80 | 92 | 91 | 90 | 89 | 88 | 87 | 75 | 63 | 51 | 39 |
| | | | 27 | 28 | 29 | 30 | 31 | 19 | 18 | 17 | 16 | 15 | 14 | | |
| 86 | 40 38 16 14 | 40.7 | 14 | 26 | 38 | 50 | 62 | 74 | 75 | 76 | 77 | 78 | 79 | 67 | 55 |
| | | | 43 | 42 | 41 | 40 | 52 | 64 | 76 | 77 | 65 | 53 | 54 | 55 | 56 |
| | | | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | |
| 87 | 38 16 14 | 40.7 | 14 | 26 | 38 | 50 | 51 | 39 | 27 | 28 | 29 | 30 | 31 | 32 | 33 |
| | | | 34 | 46 | 58 | 70 | 82 | 94 | 93 | 92 | 91 | 90 | 89 | 88 | 87 |
| | | | 75 | 63 | 51 | 39 | 27 | 28 | 29 | 17 | 16 | 15 | 14 | | |
| 88 | 38 16 14 | 40.7 | 14 | 26 | 38 | 50 | 62 | 74 | 75 | 76 | 77 | 65 | 53 | 54 | 55 |
| | | | 56 | 68 | 67 | 66 | 65 | 53 | 54 | 55 | 56 | 68 | 67 | 66 | 65 |
| | | | 53 | 54 | 55 | 43 | 31 | 19 | 18 | 17 | 16 | 15 | 14 | | |
| 89* | 40 38 16 14 | 41.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 89 | 77 |
| | | | 78 | 79 | 67 | 55 | 43 | 42 | 41 | 40 | 52 | 64 | 76 | 77 | 65 |
| | | | 53 | 54 | 55 | 56 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 | 16 |
| | | | 15 | 14 | | | | | | | | | | | |
| 90* | 40 38 16 14 | 41.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 91 | 79 | 67 | 55 | 43 | 42 | 41 | 40 | 52 | 64 | 76 | 77 | 78 |
| | | | 79 | 80 | 81 | 69 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 | 16 |
| | | | 15 | 14 | | | | | | | | | | | |
| 91 | | 45.1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 24 |
| | | | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 119 | 118 | 117 | 116 | 115 |
| | | | 114 | 113 | 112 | 111 | 110 | 109 | 97 | 85 | 73 | 61 | 49 | 37 | 25 |
| | | | 13 | 1 | | | | | | | | | | | |
| 92 | 40 38 16 14 | 45.1 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 110 | 122 | 123 | 124 | 125 |
| | | | 126 | 127 | 128 | 129 | 117 | 105 | 106 | 107 | 95 | 83 | 71 | 59 | 47 |
| | | | 35 | 23 | 11 | 12 | 24 | 23 | 22 | 21 | 20 | 19 | 18 | 17 | 16 |
| | | | 15 | 14 | | | | | | | | | | | |
| 93 | 16 14 | 45.1 | 14 | 26 | 27 | 28 | 29 | 17 | 5 | 6 | 7 | 8 | 9 | 10 | 22 |
| | | | 34 | 46 | 58 | 70 | 69 | 68 | 67 | 66 | 65 | 64 | 63 | 62 | 61 |
| | | | 49 | 37 | 25 | 13 | 1 | 2 | 3 | 4 | 5 | 6 | 18 | 17 | 16 |
| | | | 15 | 14 | | | | | | | | | | | |
| 94* | 40 38 16 14 | 49.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 91 | 79 | 80 | 81 | 69 | 68 | 67 | 55 | 43 | 42 | 41 | 40 | 52 |
| | | | 64 | 76 | 77 | 78 | 90 | 102 | 103 | 104 | 105 | 93 | 81 | 69 | 57 |
| | | | 45 | 33 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | | |
| 95* | 40 38 16 14 | 49.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 91 | 79 | 80 | 81 | 69 | 57 | 45 | 44 | 43 | 42 | 41 | 40 | 52 |
| | | | 64 | 76 | 77 | 78 | 90 | 102 | 103 | 104 | 105 | 93 | 81 | 69 | 57 |
| | | | 45 | 33 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | | |
| 96* | 40 38 16 14 | 49.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 89 | 77 |

Table 6.1 List of Journeys (cont'd)

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 65 | 53 | 54 | 55 | 56 | 57 | 45 | 44 | 43 | 42 | 41 | 40 | 52 |
| | | | 64 | 76 | 88 | 100 | 101 | 102 | 103 | 104 | 105 | 93 | 81 | 69 | 57 |
| | | | 45 | 33 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | | |
| | 16 14 | 49.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 91 | 79 | 80 | 81 | 69 | 68 | 67 | 55 | 43 | 42 | 41 | 40 | 52 |
| | | | 64 | 76 | 77 | 78 | 90 | 102 | 103 | 91 | 79 | 80 | 81 | 69 | 57 |
| | | | 45 | 33 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | | |
| | | 49.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 91 | 79 | 80 | 81 | 69 | 57 | 45 | 44 | 43 | 42 | 41 | 40 | 52 |
| | | | 64 | 76 | 77 | 78 | 90 | 102 | 103 | 91 | 79 | 80 | 81 | 69 | 57 |
| | | | 45 | 33 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | | |
| | | 49.5 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 24 |
| | | | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 | 143 | 142 | 141 |
| | | | 140 | 139 | 138 | 137 | 136 | 135 | 134 | 133 | 121 | 109 | 97 | 85 | 73 |
| | | | 61 | 49 | 37 | 25 | 13 | 1 | | | | | | | |
| | 16 | 49.5 | 1 | 2 | 3 | 4 | 16 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 |
| | | | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 | 143 | 142 | 141 |
| | | | 140 | 139 | 138 | 137 | 136 | 135 | 134 | 133 | 121 | 109 | 97 | 85 | 73 |
| | | | 61 | 49 | 37 | 25 | 13 | 1 | | | | | | | |
| | 40 16 | 49.5 | 1 | 2 | 3 | 4 | 16 | 28 | 40 | 52 | 53 | 54 | 55 | 56 | 57 |
| | | | 58 | 59 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 | 143 | 142 | 141 |
| | | | 140 | 139 | 138 | 137 | 136 | 135 | 134 | 133 | 121 | 109 | 97 | 85 | 73 |
| | | | 61 | 49 | 37 | 25 | 13 | 1 | | | | | | | |
| | 40 16 | 49.5 | 1 | 2 | 3 | 4 | 16 | 28 | 40 | 52 | 64 | 76 | 77 | 78 | 79 |
| | | | 80 | 81 | 82 | 83 | 84 | 96 | 108 | 120 | 132 | 144 | 143 | 142 | 141 |
| | | | 140 | 139 | 138 | 137 | 136 | 135 | 134 | 133 | 121 | 109 | 97 | 85 | 73 |
| | | | 61 | 49 | 37 | 25 | 13 | 1 | | | | | | | |
| 103 | 40 38 16 | 49.5 | 1 | 2 | 3 | 4 | 16 | 28 | 40 | 52 | 64 | 76 | 88 | 100 | 101 |
| | | | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 120 | 132 | 144 | 143 | 142 | 141 |
| | | | 140 | 139 | 138 | 137 | 136 | 135 | 134 | 133 | 121 | 109 | 97 | 85 | 73 |
| | | | 61 | 49 | 37 | 25 | 13 | 1 | | | | | | | |
| 104 | 40 38 16 14 | 49.5 | 1 | 2 | 3 | 4 | 16 | 28 | 40 | 52 | 64 | 76 | 88 | 100 | 112 |
| | | | 124 | 125 | 126 | 127 | 128 | 129 | 130 | 131 | 132 | 144 | 143 | 142 | 141 |
| | | | 140 | 139 | 138 | 137 | 136 | 135 | 134 | 133 | 121 | 109 | 97 | 85 | 73 |
| | | | 61 | 49 | 37 | 25 | 13 | 1 | | | | | | | |
| 105 | | 49.5 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 24 | 36 | 48 |
| | | | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 | 143 | 142 | 141 | 140 | 139 |
| | | | 138 | 137 | 136 | 135 | 134 | 133 | 121 | 109 | 97 | 85 | 73 | 61 | 49 |
| | | | 37 | 25 | 26 | 27 | 15 | 3 | | | | | | | |
| 106 | 40 38 16 14 | 49.5 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 91 | 79 | 67 | 55 | 43 | 42 | 41 | 40 | 52 | 64 | 76 | 77 | 78 |
| | | | 79 | 80 | 81 | 82 | 83 | 71 | 59 | 47 | 35 | 23 | 22 | 21 | 20 |
| | | | 19 | 18 | 17 | 16 | 15 | 14 | | | | | | | |
| 107* | 40 38 16 14 | 53.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 89 | 77 |
| | | | 65 | 53 | 54 | 55 | 56 | 57 | 45 | 44 | 43 | 42 | 41 | 40 | 52 |
| | | | 64 | 76 | 88 | 100 | 101 | 102 | 103 | 104 | 105 | 106 | 107 | 95 | 83 |
| | | | 71 | 59 | 47 | 35 | 23 | 22 | 21 | 20 | 19 | 18 | 17 | 16 | 15 |
| | | | 14 | | | | | | | | | | | | |
| 108* | 40 38 16 14 | 53.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 104 | 105 | 93 | 81 | 69 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 |
| | | | 16 | 28 | 40 | 52 | 64 | 76 | 88 | 100 | 101 | 89 | 77 | 65 | 53 |
| | | | 54 | 55 | 56 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 | 16 | 15 |
| | | | 14 | | | | | | | | | | | | |
| 109* | 40 38 16 14 | 53.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 104 | 105 | 93 | 81 | 69 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 |
| | | | 16 | 28 | 40 | 52 | 64 | 76 | 88 | 100 | 101 | 102 | 103 | 91 | 79 |
| | | | 80 | 81 | 69 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 | 16 | 15 |
| | | | 14 | | | | | | | | | | | | |
| 110* | 40 38 16 14 | 57.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 89 | 77 |
| | | | 65 | 53 | 54 | 55 | 56 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 |
| | | | 16 | 15 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 |
| | | | 102 | 103 | 104 | 105 | 93 | 81 | 69 | 57 | 45 | 33 | 21 | 20 | 19 |
| | | | 18 | 17 | 16 | 15 | 14 | | | | | | | | |
| 111* | 40 38 16 14 | 57.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 91 | 79 | 80 | 81 | 69 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 |
| | | | 16 | 15 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 |
| | | | 89 | 77 | 65 | 53 | 54 | 55 | 56 | 57 | 45 | 33 | 21 | 20 | 19 |
| | | | 18 | 17 | 16 | 15 | 14 | | | | | | | | |
| 112* | 40 38 16 14 | 61.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 104 | 105 | 93 | 81 | 69 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 |
| | | | 16 | 15 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 |
| | | | 102 | 103 | 104 | 105 | 106 | 107 | 95 | 83 | 71 | 59 | 47 | 35 | 23 |

Table 6.1 <u>List of Journeys</u> (cont'd)

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 22 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | | | |
| 113 | 40 38 16 14 | 62.7 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 89 | 88 |
| | | | 87 | 86 | 85 | 73 | 61 | 49 | 37 | 25 | 13 | 1 | 2 | 3 | 4 |
| | | | 5 | 6 | 7 | 8 | 20 | 32 | 44 | 56 | 68 | 80 | 92 | 91 | 90 |
| | | | 89 | 77 | 65 | 53 | 41 | 40 | 52 | 53 | 54 | 55 | 43 | 31 | 19 |
| | | | 18 | 17 | 16 | 15 | 14 | | | | | | | | |
| 114 | 38 16 14 | 62.7 | 14 | 26 | 38 | 50 | 51 | 39 | 27 | 28 | 29 | 17 | 16 | 28 | 29 |
| | | | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 48 | 60 | 72 | 84 | 96 | 108 |
| | | | 120 | 132 | 144 | 143 | 142 | 141 | 140 | 139 | 138 | 137 | 136 | 135 | 134 |
| | | | 133 | 121 | 109 | 97 | 85 | 73 | 61 | 49 | 37 | 25 | 13 | 1 | 2 |
| | | | 3 | 4 | 16 | 15 | 14 | | | | | | | | |
| 115* | 40 38 16 14 | 65.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 110 | 122 | 123 | 124 | 125 |
| | | | 126 | 127 | 128 | 129 | 130 | 131 | 119 | 107 | 95 | 83 | 71 | 59 | 47 |
| | | | 35 | 23 | 22 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | 26 | 38 |
| | | | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 | 104 | 105 | 93 |
| | | | 81 | 69 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 |
| 116 | 40 38 16 14 | 67.1 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 110 | 122 | 123 | 124 | 125 |
| | | | 126 | 127 | 115 | 103 | 91 | 79 | 80 | 81 | 69 | 57 | 45 | 33 | 21 |
| | | | 20 | 32 | 44 | 56 | 68 | 80 | 92 | 91 | 90 | 89 | 88 | 87 | 86 |
| | | | 85 | 73 | 61 | 49 | 37 | 25 | 13 | 1 | 2 | 3 | 4 | 5 | 6 |
| | | | 7 | 8 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | | | |
| 117 | 40 38 16 14 | 67.1 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 104 | 105 | 106 | 107 | 95 | 83 | 71 | 59 | 47 | 35 | 23 | 22 | 34 |
| | | | 46 | 58 | 70 | 69 | 68 | 67 | 66 | 65 | 64 | 63 | 62 | 61 | 49 |
| | | | 37 | 25 | 13 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | | | 22 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | | | |
| 118* | 40 38 16 14 | 69.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 91 | 79 | 80 | 81 | 69 | 68 | 67 | 55 | 43 | 42 | 41 | 40 | 52 |
| | | | 64 | 76 | 77 | 78 | 90 | 102 | 103 | 91 | 79 | 30 | 81 | 69 | 57 |
| | | | 45 | 44 | 43 | 42 | 41 | 40 | 52 | 64 | 76 | 77 | 78 | 90 | 102 |
| | | | 103 | 104 | 105 | 93 | 81 | 69 | 57 | 45 | 33 | 21 | 20 | 19 | 18 |
| | | | 17 | 16 | 15 | 14 | | | | | | | | | |
| 119* | 40 38 16 14 | 69.0 | 14 | 26 | 38 | 50 | 62 | 74 | 36 | 98 | 99 | 100 | 101 | 89 | 77 |
| | | | 65 | 53 | 54 | 55 | 56 | 57 | 45 | 44 | 43 | 42 | 41 | 40 | 52 |
| | | | 64 | 76 | 88 | 100 | 101 | 102 | 103 | 91 | 79 | 30 | 81 | 69 | 57 |
| | | | 45 | 44 | 43 | 42 | 41 | 40 | 52 | 64 | 76 | 88 | 100 | 101 | 102 |
| | | | 103 | 104 | 105 | 93 | 81 | 69 | 57 | 45 | 33 | 21 | 20 | 19 | 18 |
| | | | 17 | 16 | 15 | 14 | | | | | | | | | |
| 120* | 40 38 16 14 | 69.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 91 | 79 | 80 | 81 | 69 | 68 | 67 | 55 | 43 | 42 | 41 | 40 | 52 |
| | | | 64 | 76 | 77 | 78 | 90 | 102 | 103 | 91 | 79 | 30 | 81 | 69 | 57 |
| | | | 45 | 44 | 43 | 42 | 41 | 40 | 52 | 64 | 76 | 77 | 78 | 90 | 102 |
| | | | 103 | 91 | 79 | 80 | 81 | 69 | 57 | 45 | 33 | 21 | 20 | 19 | 18 |
| | | | 17 | 16 | 15 | 14 | | | | | | | | | |
| 121 | 40 38 16 14 | 71.5 | 14 | 26 | 38 | 50 | 62 | 74 | 75 | 76 | 77 | 65 | 53 | 54 | 55 |
| | | | 43 | 31 | 19 | 18 | 17 | 16 | 28 | 40 | 52 | 53 | 54 | 55 | 56 |
| | | | 57 | 58 | 59 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 | 143 | 142 |
| | | | 141 | 140 | 139 | 138 | 137 | 136 | 135 | 134 | 133 | 121 | 109 | 97 | 85 |
| | | | 73 | 61 | 49 | 37 | 25 | 13 | 1 | 2 | 3 | 4 | 16 | 15 | 14 |
| 122* | 40 38 16 14 | 73.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 89 | 77 |
| | | | 65 | 53 | 54 | 55 | 56 | 57 | 45 | 44 | 43 | 42 | 41 | 40 | 52 |
| | | | 64 | 76 | 88 | 100 | 101 | 102 | 103 | 91 | 79 | 80 | 81 | 69 | 57 |
| | | | 45 | 44 | 43 | 42 | 41 | 40 | 52 | 64 | 76 | 88 | 100 | 101 | 102 |
| | | | 103 | 104 | 105 | 106 | 107 | 95 | 83 | 71 | 59 | 47 | 35 | 23 | 22 |
| | | | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | | | | | |
| 123 | 40 38 16 14 | 75.9 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 91 | 79 | 80 | 81 | 82 | 83 | 71 | 59 | 47 | 35 | 23 | 22 | 34 |
| | | | 46 | 58 | 70 | 82 | 94 | 106 | 118 | 117 | 116 | 115 | 114 | 113 | 112 |
| | | | 111 | 110 | 109 | 97 | 85 | 73 | 61 | 49 | 37 | 25 | 13 | 1 | 2 |
| | | | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 22 | 21 | 20 | 19 | 18 |
| | | | 17 | 16 | 15 | 14 | | | | | | | | | |
| 124 | 40 38 16 14 | 80.3 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 91 | 79 | 80 | 81 | 69 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 |
| | | | 16 | 28 | 40 | 52 | 64 | 76 | 77 | 78 | 79 | 80 | 81 | 82 | 83 |
| | | | 84 | 96 | 108 | 120 | 132 | 144 | 143 | 142 | 141 | 140 | 139 | 138 | 137 |
| | | | 136 | 135 | 134 | 133 | 121 | 109 | 97 | 85 | 73 | 61 | 49 | 37 | 25 |
| | | | 13 | 1 | 2 | 3 | 4 | 16 | 15 | 14 | | | | | |
| 125 | 40 38 16 14 | 84.7 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 110 | 122 | 123 | 124 | 125 |
| | | | 126 | 127 | 128 | 129 | 117 | 116 | 115 | 114 | 113 | 112 | 111 | 110 | 109 |
| | | | 97 | 85 | 73 | 61 | 49 | 37 | 25 | 13 | 1 | 2 | 3 | 4 | 5 |
| | | | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 24 | 36 | 48 | 60 | 72 | 84 |

Table 6.1 List of Journeys (cont'd)

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | 96 | 108 | 120 | 119 | 118 | 117 | 105 | 106 | 107 | 95 | 83 | 71 | 59 |
| | | | 47 | 35 | 23 | 22 | 21 | 20 | 19 | 18 | 17 | 16 | 15 | 14 | |
| 126 | 40 38 16 14 | 89.1 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 91 | 79 | 67 | 55 | 43 | 42 | 41 | 40 | 52 | 64 | 76 | 77 | 78 |
| | | | 79 | 80 | 81 | 82 | 83 | 71 | 59 | 47 | 35 | 23 | 22 | 34 | 46 |
| | | | 58 | 70 | 82 | 94 | 106 | 118 | 117 | 116 | 115 | 114 | 113 | 112 | 111 |
| | | | 110 | 109 | 97 | 85 | 73 | 61 | 49 | 37 | 25 | 13 | 1 | 2 | 3 |
| | | | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 22 | 21 | 20 | 19 | 18 | 17 |
| | | | 16 | 15 | 14 | | | | | | | | | | |
| 127 | 40 38 16 14 | 89.1 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 110 | 122 | 123 | 124 | 125 |
| | | | 126 | 127 | 128 | 129 | 117 | 105 | 106 | 107 | 95 | 83 | 71 | 59 | 47 |
| | | | 35 | 23 | 22 | 21 | 20 | 19 | 18 | 17 | 16 | 28 | 40 | 52 | 64 |
| | | | 76 | 88 | 100 | 101 | 102 | 103 | 104 | 105 | 106 | 107 | 108 | 120 | 132 |
| | | | 144 | 143 | 142 | 141 | 140 | 139 | 138 | 137 | 136 | 135 | 134 | 133 | 121 |
| | | | 109 | 97 | 85 | 73 | 61 | 49 | 37 | 25 | 13 | 1 | 2 | 3 | 4 |
| | | | 16 | 15 | 14 | | | | | | | | | | |
| 128 | 40 38 16 14 | 89.1 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 110 | 122 | 123 | 124 | 125 |
| | | | 126 | 127 | 128 | 129 | 117 | 105 | 106 | 107 | 95 | 83 | 71 | 59 | 47 |
| | | | 35 | 23 | 22 | 21 | 20 | 19 | 18 | 17 | 16 | 28 | 40 | 52 | 64 |
| | | | 76 | 88 | 100 | 112 | 124 | 125 | 126 | 127 | 128 | 129 | 130 | 131 | 132 |
| | | | 144 | 143 | 142 | 141 | 140 | 139 | 138 | 137 | 136 | 135 | 134 | 133 | 121 |
| | | | 109 | 97 | 85 | 73 | 61 | 49 | 37 | 25 | 13 | 1 | 2 | 3 | 4 |
| | | | 16 | 15 | 14 | | | | | | | | | | |
| 129* | 40 38 16 14 | 97.0 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 | 102 | 103 |
| | | | 91 | 79 | 80 | 81 | 69 | 57 | 45 | 33 | 21 | 20 | 19 | 18 | 17 |
| | | | 16 | 15 | 14 | 26 | 38 | 50 | 62 | 74 | 86 | 98 | 99 | 100 | 101 |
| | | | 89 | 77 | 78 | 90 | 102 | 103 | 91 | 79 | 80 | 81 | 69 | 68 | 67 |
| | | | 55 | 43 | 42 | 41 | 40 | 52 | 64 | 76 | 77 | 78 | 90 | 102 | 103 |
| | | | 91 | 79 | 80 | 81 | 69 | 57 | 45 | 44 | 43 | 42 | 41 | 40 | 52 |
| | | | 64 | 76 | 77 | 65 | 53 | 54 | 55 | 56 | 57 | 45 | 33 | 21 | 20 |
| | | | 19 | 18 | 17 | 16 | 15 | 14 | | | | | | | |

All are car journeys, except those indicated by *, which are bus journeys. Also indicated are utilities and the set of nodes each path goes through. Of the 129 journeys, 78 are available to Classes 1 and 5 (who originate at node 14), 81 can be used by Classes 2 and 6 (node 16), 76 are for Classes 3 and 7 (node 38), and 80 are available to Classes 4 and 8 (node 40).

Table 6.2: Summary of Basic Network Results

| Class | Number of Travelers | Distance Per Traveler, km | Time Per Traveler, hours | Money Per Traveler, dollars | Utility Per Traveler, | Velocity Per Traveler, km/h | Mode |
|---|---|---|---|---|---|---|---|
| Wealthy | 76,700 | 25.39 | 0.80 | 6.19 | 45.56 | 31.89 | Auto |
| Poor | 20,609 | 15.63 | 0.44 | 4.91 | 27.94 | 35.22 | |
| Wealthy | 35,453 | 16.75 | 1.47 | 1.47 | 34.62 | 11.37 | Bus |
| Poor | 57,535 | 16.24 | 1.41 | 1.39 | 32.93 | 11.49 | |
| Wealthy | 112,153 | 22.65 | 1.01 | 4.70 | 42.10 | 22.44 | Both |
| Poor | 78,143 | 16.08 | 1.16 | 2.32 | 31.61 | 13.89 | |

| Class | Traveling | Fraction by Car | Fraction by Bus | Area |
|---|---|---|---|---|
| Wealthy | 0.87 | 0.62 | 0.38 | inner city |
| Poor | 0.71 | 0.21 | 0.79 | |
| Wealthy | 0.71 | 0.71 | 0.29 | outer city |
| Poor | 0.46 | 0.29 | 0.71 | |

## 6.3 Fare Change

Table 6. 3 summarizes the behavior of the network when bus fares are raised by 50 percent. Bus riders are profoundly affected, particularly those of lower income. Car commuters are almost unaffected. It is important to see that, in this network, when fares are raised, people do not switch from buses to cars (except for a very small number of the poor). Instead, they switch from traveling in buses to not traveling at all.

The opposite result can be expected when fares are lowered; car travelers do not become bus travelers. Non-travelers become travelers.

## 6.4 Tolls

An experiment has been run to assess the effect of adding tolls on certain links entering the inner city. The intention is to discourage the use of cars in the inner city, so only cars are subject to the fee. The links are 45-44, 100-101, 40-52, 105-93, and the amount charged is 1.00 dollar.

As Table 6.4 indicates; the desired effect is achieved. Fewer people travel by car, and those who do drive tend to drive less. The effect of tolls is greater on the wealthy than on the poor since it is the wealthy that drive more. The average daily money expenditure of wealthy car drivers goes up, while that of all others is nearly constant. The automotive travel of the poorer people who do use cars, however, is affected much more than that of the wealthier. Bus ridership increases enough so that there is only a small decrease in total travel.

## 6.5 Prohibitions

Table 6.5 displays the effects of forbidding cars from traveling on certain links. The links are the same as those studied in Section 6.4. (In fact, the method used is to impose a 50 dollar toll on those links, which very effectively limits their use.)

Table 6.3 Summary of Effects of Fare Change

| Class | Number of Travelers | Distance Per Traveler, km | Time Per Traveler, hours | Money Per Traveler, dollars | Utility Per Traveler, | Velocity Per Traveler, km/h | Mode |
|---|---|---|---|---|---|---|---|
| Wealthy | 76,606 | 25.43 | 0.79 | 6.19 | 45.68 | 31.99 | Auto |
| Poor | 20,539 | 15.65 | 0.44 | 4.91 | 28.02 | 35.35 | |
| Wealthy | 31,161 | 16.66 | 1.47 | 2.10 | 34.40 | 11.30 | Bus |
| Poor | 48,165 | 16.08 | 1.41 | 1.95 | 32.43 | 11.43 | |
| Wealthy | 107,766 | 22.89 | 0.99 | 5.01 | 42.42 | 23.10 | Both |
| Poor | 68,704 | 15.95 | 1.12 | 2.83 | 31.11 | 14.26 | |

Table 6.4 Effects of Tolls

| Class | Number of Travelers | Distance Per Traveler, km | Time Per Traveler, hours | Money Per Traveler, dollars | Utility Per Traveler, | Velocity Per Traveler, km/h | Mode |
|---|---|---|---|---|---|---|---|
| Wealthy | 73,609 | 25.27 | 0.77 | 6.66 | 44.29 | 32.78 | Auto |
| Poor | 18,615 | 16.61 | 0.42 | 4.93 | 26.08 | 39.24 | |
| Wealthy | 39,043 | 16.78 | 1.47 | 1.50 | 35.55 | 11.43 | Bus |
| Poor | 60,337 | 16.16 | 1.39 | 1.39 | 33.27 | 11.61 | |
| Wealthy | 112,652 | 22.33 | 1.01 | 4.87 | 41.26 | 22.05 | Both |
| Poor | 78,952 | 16.26 | 1.16 | 2.23 | 31.57 | 13.98 | |

The effect of prohibiting travel is only a little greater than that of imposing tolls on the same links. Most of the numbers in Table 6.5 are quite close to those of Table 6.4. One significant difference is the average daily expenditure of wealthy car riders. When a 1.00 dollar toll was imposed, enough of them were willing to pay it that the expenditure went up. However, a 50.00 dollar toll was unreasonable and they refused. Because they lost opportunities to spend their money, their expenditure decreased.

It seems reasonable to conclude that as the toll increases from zero (the base case in Section 6.2) the expenditure of wealthy car riders will increase, reach a maximum, and decrease. The maximum expenditure is limited by the budget distributions. Additional study is required to verify this conclusion and to make it precise.

Table 6.5 Link Prohibitions

| Class | Number of Travelers | Distance Per Traveler, km | Time Per Traveler, hours | Money Per Traveler, dollars | Utility Per Traveler, | Velocity Per Traveler, km/h | Mode |
|---|---|---|---|---|---|---|---|
| Wealthy | 72,707 | 25.27 | 0.76 | 6.08 | 40.67 | 33.32 | Auto |
| Poor | 18,323 | 16.43 | 0.43 | 4.87 | 25.33 | 38.13 | |
| Wealthy | 39,990 | 16.84 | 1.47 | 1.52 | 36.02 | 11.47 | Bus |
| Poor | 60,923 | 16.17 | 1.38 | 1.39 | 33.45 | 11.68 | |
| Wealthy | 112,697 | 22.28 | 1.01 | 4.46 | 39.02 | 22.05 | Both |
| Poor | 79,246 | 16.23 | 1.16 | 2.20 | 31.62 | 13.94 | |

# 7. EFFECTS OF NETWORK CHANGES

In this section, we make changes in network links and compare traveler behavior with that of Section 6.2. This illustrates the use of the behavioral principle described here in network planning.

It is important to reiterate that this section demonstrates only the kinds of experiments and results that are feasible with the technique presented here, rather than any representative results for a real city. The model limitations may have a profound affect on these results: first, residences are not distributed throughout the network. They are concentrated in a small number of locations. This tends to concentrate flow on the adjacent links, rather than in the city center where it might be expected. Second, no residential choice model is included. Consequently, there is no mechanism to depict the movement of dwellings to areas in the network of greater accessibility.

## 7.1 Inner Loop

An inner loop has been created by increasing the capacities of links 40-52, 52-64, 64-76, 76-88, 88-100, 100-101, 101-102, 102-103, 103-104, 104-105, 105-93, 93-81, 81-69, 69-57, 57-45, 45-44, 44-43, 43-42, 42-41, and 41-40 from 24000 to 48000 vehicles per day. The effect of this change is summarized in Table 7.1.

The links which are widened are among those observed to be congested in Section 6.2. Although a significant effect was expected, none was observed. Travelers spend approximately the same time and money, go a little further, and get slightly greater utility rewards. The number of travelers is nearly unchanged.

## 7.2 Outer Loop

Here the links on the square connecting nodes 14, 122, 131, and 23 have been expanded to a capacity of 48000 from 24000.

As shown in Table 7.2, the number of car travelers is nearly the same as in Section 7.1. However, those travelers clearly benefit more here then when the inner loop was considered (as measured by increased distance, velocity, and utility). Bus riders, however, gain substantially. Not only do all measures of transportation service indicate improvement over the base case (Table 6.2) but there are now more bus travelers.

Table 7.1 Effect of Inner Loop

| Class | Number of Travelers | Distance Per Traveler, km | Time Per Traveler, hours | Money Per Traveler, dollars | Utility Per Traveler, | Velocity Per Traveler km/h | Mode |
|---|---|---|---|---|---|---|---|
| Wealthy | 76,378 | 25.45 | 0.80 | 6.20 | 45.71 | 32.01 | Auto |
| Poor | 20,381 | 15.66 | 0.44 | 4.92 | 28.06 | 35.41 | |
| Wealthy | 36,204 | 16.77 | 1.46 | 1.48 | 35.07 | 11.46 | Bus |
| Poor | 58,540 | 16.26 | 1.40 | 1.39 | 33.22 | 11.60 | |
| Wealthy | 112,581 | 22.66 | 1.01 | 4.68 | 42.29 | 22.44 | Both |
| Poor | 78,920 | 16.10 | 1.15 | 2.30 | 31.89 | 13.96 | |

## 7.3 Inner and Outer Loops

Table 7.3 summarizes the effects of expanding both loops. The trends observed earlier continue: car travelers are somewhat better off, although their numbers decrease somewhat. Bus riders also benefit, and their numbers increase substantially when compared with Table 6.2.

## 7.4 Conclusions

A series of numerical experiments have been performed to assess the effects of both traffic control policy changes in Section 6 and roadway expansions in Section 7. These experiments illustrate the kinds of results that can be obtained and the kinds of uses to which this technique can be put. Becuase the models of the modes and the distribution of population in the network are crude, general conclusions on the behavior of cities should not be drawn from these results. They do show, however, that the technique described here is capable of being extended to actual urban networks.

Table 7.2 Effect of Outer Loop

| Class | Number of Travelers | Distance Per Traveler, km | Time Per Traveler, hours | Money Per Traveler, dollars | Utility Per Traveler, | Velocity Per Traveler, km/h | Mode |
|---|---|---|---|---|---|---|---|
| Wealthy | 76,365 | 25.97 | 0.78 | 6.24 | 47.37 | 33.37 | Auto |
| Poor | 20,230 | 15.87 | 0.43 | 4.91 | 28.96 | 37.32 | |
| Wealthy | 37,240 | 17.27 | 1.47 | 1.50 | 36.62 | 11.75 | Bus |
| Poor | 59,855 | 16.70 | 1.40 | 1.40 | 34.66 | 11.92 | |
| Wealthy | 113,605 | 23.12 | 1.01 | 4.69 | 43.84 | 23.00 | Both |
| Poor | 80,086 | 16.49 | 1.16 | 2.29 | 33.22 | 14.28 | |

Table 7.3 Effect of Both Loops

| Class | Number of Travelers | Distance Per Traveler, km | Time Per Traveler, hours | Money Per Traveler, dollars | Utility Per Traveler, | Velocity Per Traveler, km/h | Mode |
|---|---|---|---|---|---|---|---|
| Wealthy | 75,933 | 26.04 | 0.78 | 6.25 | 47.58 | 33.54 | Auto |
| Poor | 19,940 | 15.89 | 0.42 | 4.91 | 29.12 | 37.60 | |
| Wealthy | 38,211 | 17.29 | 1.46 | 1.52 | 37.23 | 11.84 | Bus |
| Poor | 61,12[illegible] | 16.70 | 1.39 | 1.41 | 35.06 | 12.04 | |
| Wealthy | 114,144 | 23.11 | 1.01 | 4.66 | 44.12 | 22.99 | Both |
| Poor | 81,060 | 16.50 | 1.15 | 2.27 | 33.60 | 14.35 | |

# 8. EXTENSIONS AND LIMITATIONS

In Section 8.1, we outline extensions of the assignment principle described in Section 2, and of the numerical technique presented in Section 3. These extensions enhance the usefulness of the formulation, they appear to be conceptually straightforward, and they are compatible with the approach and structure of the technique described here. Section 8.2, describes some difficulties that tend to limit its applicability, and thus, suggest directions for further research.

## 8.1 Extensions

### 8.1.1 Addition of New Journeys

Suppose that after equilibrium is reached with one set of journeys, it is decided that a certain new journey must also be considered. It is easy to include the new journey. The initial flow vector, $x(0)$, for the new equilibration process can be chosen as the final converged vector of the old process, enhanced with an initial guess for the flow on the new path. That guess can be zero. In our experience, relatively few iterations are required to reach the new equilibrium.

A more serious difficulty is the generation of new journeys. For the examples described here, we are willing to generate the journeys by hand. As networks grow, however, the number of journeys required can make this impractical.

### 8.1.2 Time-stratified Travel

As Stephenson and Teply (1981) show, changes in network conditions can lead to changes in the time of day that people travel. Consequently, the variation of the distribution of traffic in a network as a function of hour of the day can be important.

In principle, this can be treated using the approach introduced here. Journeys now become paths in space-time and not merely in space. That is, to describe a journey, one must not only list the links and nodes that the journey passes through, but the time of day that each link and node is reached.

Two journeys that pass through the same points in the network are now considered different if they reach those points at different times. Utilities may be assigned according to when a visit to a node or link takes place. (This is important: food shopping after work is worth much more than shopping before work if there is no intervening stop at home). Link costs vary with the time of day according to the time-varying congestion.

The difficulty, of course, is that the number of distinct journeys will become considerably larger than it is now. It will be necessary to discretize time, but even if this discretization is coarse, the number of different space-time paths that correspond to the same space path can be enormous.

### 8.1.3 Additional Modes

As we have demonstrated in Section 6, the equilibration process is not affected by the presence of more than one mode. What is required is a complete statement of utilities and costs of each journey on each mode, and how traffic on one mode (such as buses) affects costs on another (such as cars). For example, if carpools are considered as a distinct mode from single-passenger cars, there are certain details that have to be treated carefully, including how money costs are shared, and how much extra delay is accrued.

In addition to treating rapid transit, buses, and carpools as modes that are distinct from cars, this approach can be applied to predicting the distribution of cars of different prices. Different types of cars are treated as separate modes in which the time costs are the same. The money costs can cover a wide range.

### 8.1.4 Subjective Utility Ranking

In the present model, the number of people in a class that choose a journey depends on the journey's costs, and on the desirability ranking of all the joruneys. The utility values of the journeys are not considered, except to determine the ranking.

This is reasonable when journeys are not similar. However, when they are nearly the same, we can expect that different people will have different opinions on relative rankings. Therefore, the model should be changed to represent the effects of having journeys with nearly equal utilities.

One approach to this problem is to divide the class into subclasses, in which each subclass has a homogeneous ranking of journeys. Although this is conceptually simple, it can again lead to a dangerous proliferation of journeys.

A more practical approach is to incorporate into the density function $f^a(.,.)$ an indication of ranking. That is, if there are k journeys available to class a let $\pi$ be a permutation of $\{1,...,k\}$, which represents a possible ranking of the journeys. Then let

$$D^a f^a(u,n,\pi)du\, dn,$$

be the number of people in class a with time and money costs between u and u + du and n and n + dn, respectively, and who have preference order $\pi$. The analysis that has led to equations (2.20) to (2.22) must be repeated for each possible preference order. If, as we suspect, there are not many likely permutations, the increase in complexity that this causes will be limited and manageable.

### 8.1.5 Residential Choice

In this report, we have considered only some of the choices available to travelers that affect their travel behavior. In particular, we have not allowed them the freedom to choose their residences; we assume that their residential locations are fixed. A more realistic and comprehensive model would include a representation of this freedom.

The choice of residential location is greatly influenced by the transportation system: when a household considers a new residence, it carefully considers how much access it has to locations of work, shopping, and where other needs are fulfilled. Other factors must be considered as well, such as housing cost. Housing cost itself is determined in part by these same transportation considerations.

It is desirable to include residential choice in the model presented here to assess long-term effects of changes in the transportation system. To do this, the following modifications of the method must be considered:

a. Classes no longer have fixed origins but rather can choose journeys anywhere in the network. Some of the nodes on each resulting journey become residential locations.

b. A precise model of individual behavior incorporating both residential and travel decisions is required. This may be an extension of (2.7) to (2.9). It is hoped that the utility can be extended to include the intrinsic utility (i.e., independent of transportation considerations) of a house or apartment, and that the money budget can be enlarged to include housing as well as travel costs. It is also hoped that such a model can be transformed into a principle that describes flows and demands for housing which is analogous to (2.20) to (2.22).

c. A model of housing cost is required. The cost of owning or renting a house or apartment depends on its physical characteristics (such as its size, the size of the plot of land it is on, its state of repair), and on market factors such as interest rates. It also depends on the demand for residences in its vicinity which in turn depends on transportation considerations. These modifications will extend the model described here. Travelers will be represented as making both travel decisions and residence decisions. Both phenomena must be considered simultaneously if long-term changes in urban structure are to be understood.

### 8.1.6 Toll Roads and Bridges

This is an easy extension on all links in which tolls are collected, money costs are increased. Because of the finite capacities of toll collectors and the necessity for drivers to stop, delays are also increased. A queuing theory analysis is required to estimate the amount of delay due to tolls.

## 8.2 Limitations

The major limitation to the new assignment principle is in its computational complexity and large storage requirements when networks of reasonable size are treated. This difficulty is exacerbated by some of the extensions described above. Further research to accelerate the equilibration process, or to replace it altogether, will help mitigate this difficulty. Of greater value will be a procedure for aggregating the network so that a reduced network can be used which will produce approximately equivalent results.

The ultimate purpose of this anlaysis is to help design transportation-system changes. We are not yet ready to suggest optimization procedures based on the new assignment principle although the examples described earlier serve to illustrate directions for further study in this area.

# 9. FURTHER RESEARCH

The demand/distribution/assignment principles described in Section 2, promise to be valuable tools for the understanding and prediction of travel behavior in a city. Additional research can enhance the usefulness of these tools. This research is of two types: analytical and empirical.

## 9.1 Analytical Research

Modeling extensions have already been described in Section 8. The limitations listed in Section 8 must also be studied, particularly the difficulties associated with a large problem size. An efficient technique -- exact or heuristic -- is required to generate journeys.

There are important qualitative issues that must be understood, such as the number of equilibria and what influences their number. The convergence properties of the algorithm need to be understood, and acceleration techniques investigated.

We have briefly touched on the effect of the shape of the budget distributions when the means and variances are specified. This question cna be formalized and other sensitivity issues, such as the effect of the shape of the link delay functions, can be considered. Aggregation methods will be useful here, as in any study of large-scale systems.

If the full benefits of symmetry are used, very large networks can be treated, particularly concentric circle networks, such as in Figure 4.13. Although such studies will have no direct application, they will be useful for the study of the behavior of travelers in large abstract city networks. That is, general principles for understanding transportation systems can be developed by considering such special cases.

Optimization techniques based on this model are required. Such techniques will seek the best of a class of network modifications or the best of a class of control policies. They will minimize such costs as total funds expended, aggregate delay, energy consumption, or maximize such objectives as mobility while taking into account travelers' responses to the change. These optimization techniques will be of great value to system planners and managers.

As suggested in Section 8, a more complete model of travel behavior can be formulated when certain issues concerning residential cost and residential location choice are understood and incorporated. Such an extended model will also offer insights into the process of urban growth and development. This is because travel behavior and residential location are closely interrelated. If the distribution of residence in an urban region is altered (for example by a new housing development) the prevailing travel patterns will change. If the travel patterns are modified (by the construction of a new freeway, for example) the characteristics of the residences will eventually change.

## 9.2 Empirical Research

Various assumptions have been made in formulating the examples. Most important may be the assumption that utility is additive along journeys. Such assumptions must be tested and verified or modified.

A study can be undertaken to apply the methods described here to a real city. The city, of course, should be small, and well-documented, in order to assess the predictive power of the model.

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## APPENDIX -- REPORT OF NEW TECHENOLOGY

There are no inventions or other patentable items in this work. However, advances in the formulation of behavioral models for travelers are described in Section 2. These models can be used as the basis for the improvement of methods of calculating transportation demand, mode split, and the distribution of travelers in the network. A numerical technique is proposed in Section 3, and illustrations of its use to assess proposed changes in system management and network structure are described in Sections 6 and 7.