Airline Network Seat Inventory Control: Methodologies and Revenue Impacts

by

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Abstract

In the airline industry, it is customary for carriers to offer a wide range of fares for any given seat in the same cabin on the same flight. In order to control the number of seats made available in each fare class, airlines practice what is called seat inventory control. Traditionally, airline seat inventory control has been the process of allocating seats among varies fare classes on a flight leg in order to maximize expected revenues. Reservations for travel on a flight leg are accepted based on the availability of a particular fare class on that flight leg. A passenger's ultimate destination, overall itinerary, or total revenue contribution to the airline is not taken into account. The typical route structure of a large airline, however, is built around a complex network of connecting flights. Maximizing revenues on individual flight legs is not the same as maximizing total network revenues. The objective of this dissertation is to address the seat inventory control problem at the network level, taking into account the interaction of flight legs and the flow of traffic across a network.

Beginning with the traditional network formulation of the seat inventory control problem, practical approaches for actually *controlling* seat inventories at the origin-destination and fare class (ODF) level are first discussed. To avoid problems associated with forecasting ODF itinerary demand, network methods based on aggregated demand estimates are then presented. Taking the network seat inventory control problem one step closer to fit in with current reservations control capabilities, several leg-based heuristics are introduced. These heuristics take into account information about different ODF passenger demand and traffic flows while optimization and control of seat inventories remains at the flight leg level.

In order to effectively measure the revenue potential of the different network seat inventory control methods introduced, an integrated optimization/booking process simulation was developed. Specific issues related to realistically modeling the booking process are discussed and the multi-period, computer-based, mathematical simulation described in detail. With the use of this integrated optimization/booking process simulation, the revenue impacts of the different network seat inventory control methodologies are then evaluated using real airline data for both a connecting hub network and multiple flight leg networks. Overall performance of each method is examined by comparing the revenue obtained with that of current leg-based control approaches and the maximum revenue potential given perfect information.

The performance of the different methods evaluated varies with both the network and the actual demand patterns, however, significant revenue impacts over current seat inventory control approaches can be obtained. One approach which consistently performs well

is a deterministic network approach in which ODF seat allocations are nested by shadow prices. Depending on the network structure, other leg-based OD control heuristics also perform well. The benefits of network seat inventory control are a function of the load factor across a network. Below an average load factor of about 85%, revenue impacts over effective leg-based control are non-existent. However, as the average load factor increases, revenue impacts on the order of 2-4% are obtainable.

Thesis Supervisor: Professor Peter Belobaba
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Glossary

Bid Price: In a network optimization approach, a bid price is the shadow price of a particular capacity constraint. A bid price represents the marginal value of the last seat on a given flight leg. (Pages 90 and 102)

Computer Reservations System (CRS): The CRS is where transactions between an airline and a potential passenger take place. It is the system on which seat availabilities for different fare classes and origin-destination itineraries are stored and displayed and all sale and cancellation activity occurs. (Page 22, Figure 1.5)

Demand Factor: The ratio between demand and capacity.

Leg-Based Optimization: The process of maximizing revenue on a single flight leg independent of all other flight legs in a network.

Network-Based Optimization: The process of maximizing revenue by taking into account the traffic flows over a defined network of interrelated flight legs.

Nested Inventory Structure: An inventory structure in which each inventory (whether it be a fare class inventory, a virtual inventory, or an origin-destination and fare class inventory) is nested, allowing for higher revenue, more-desirable requests to have access to seats allocated to lower revenue, less-desirable demand. The philosophy behind a nested inventory structure is to never reject a higher valued passenger when seats originally allocated to lower valued passengers remain available. (Page 43, Figure 2.9)

Network Seat Inventory Control: The process of managing overall network traffic, allocating seats and limiting sales by origin-destination itinerary as well as fare class. It addresses both the fare class mix and the itinerary control components of seat inventory control and encompasses origin-destination control, segment control, and point-of-sale control. (Page 40)

ODF Itinerary: An origin-destination and fare class routing over a given set of flight legs (path) at a specified departure time on a given airline.

Origin-Destination Control: The process of allocating seats and determining booking limits for each specific origin-destination and fare class within a network of flights. (Page 40)

Overbooking: The practice of accepting reservations in excess of capacity in order to minimize empty seats on a flight due to cancellations and no-shows.

Partitioned/Distinct Inventory Structure: An inventory structure in which seats allocated to an inventory bucket are available solely for demand associated with that inventory bucket while any seats not sold go empty. (Page 43, Figure 2.10)

Point-of-Sale Control: The practice of differentiating between the actual location in which a seat is sold in order to take advantage of differences in net revenues due to commission rates, preferred currencies, etc. (Page 41)

Pricing: The process of determining the number and type of fares to be offered in each origin-destination market. (Page 17)

Reduced Cost: The cost to a network of increasing the nonnegativity constraint by one. A reduced cost is the shadow price of a nonnegativity constraint [1]. (Page 84)

Reservations Control: The process of determining how much of each fare product to sell. It encompasses both overbooking and seat inventory control. (Page 19)

Revenue Management: Commonly known as yield management. The process of determining how many seats to sell at what prices. It is made up of both pricing and reservations control. (Page 15, Figure 1.3)

Seamless Availability/Direct Access: The capability of having requests for a given airline made directly through the reservations system of the airline on a real time basis rather than through the computer reservations systems of other airlines on the basis of predetermined booking limits.

Seat Inventory Control: The practice of allocating seats among various fare levels in an effort to maximize the expected revenue of future scheduled flights. This is done by protecting seats for more desirable, higher revenue passengers while making otherwise empty seats available to less desirable, lower revenue passengers. (Page 19, Chapter 2)

Segment Control: The control of different fare class and *on-flight* itineraries on a multiple leg flight. The origin or destination of passengers *outside* the given multi-leg flight is not taken into consideration. A differentiation is only made between local and through itineraries on a flight. (Page 41)

Shadow Price: The incremental network revenue that would be realized if a given constraint was increased by one unit, all else being held constant [1]. (Page 83)

Virtual Inventory Class: A "hidden" inventory class which is used to forecast, optimize, and/or control groups of ODF itineraries on each flight leg. Virtual inventory classes are not formally recognized classes in which service is offered by an airline. Each virtual class exists only within the seat inventory system itself and is not apparent to the users of a computer reservations system. (Page 65)

Notation

BL	Booking Limit.
C	Capacity of a flight leg.
DBID	Deterministic Bid Price.
DD	Distinct Deterministic.
DP	Distinct Probabilistic.
$EMR_i(S_i)$	The expected marginal revenue of potentially selling the S_ith seat which is simply the probability of selling the S_ith seat times the revenue obtained from selling the seat, i.e. $f_i \cdot \overline{P}_i(S_i)$.
EMSR	Expected Marginal Seat Revenue heuristic developed by Belobaba [2].
LBID	Leg-Based Bid Price.
LBID/BL	Combined Leg-Based Bid Price/Booking Limit.
NDF	Nested Deterministic by Fares.
NDFC	Nested Deterministic by Fare Class.
NDSP	Nested Deterministic by Shadow Prices.
NLBIL	Nested Leg-Based Itinerary Limit.
NPF	Nested Probabilistic by Fares.
NPFC	Nested Probabilistic by Fare Class.
NPSP	Nested Probabilistic by Shadow Prices.
OBL	Optimal Booking Limit approach developed by Curry [3].
ODF	Origin-destination and fare class.
PBID	Probabilistic Bid Price.
$\overline{P}_i(S_i)$	The probability of potentially selling S_i seats, or the probability of having S_i or more requests, r_i, i.e. $P_i[r_i \geq S_i]$.
UPPER	Upper Bound in revenues, or the maximum revenue potential.
VNOC	Virtual Nesting on the "Value Net of Opportunity Cost".

Chapter 1

Introduction

In the airline industry today, it is customary for carriers to offer a wide range of fares for any given seat in the same cabin on the same aircraft. On a nonstop flight from Boston to Orlando in April of 1992, it was possible to find a passenger traveling on a discounted round trip ticket for as low as $278.00, while in the very next seat a passenger could have paid the full coach fare of $1022.00, round trip. Such occurrences have become commonplace since deregulation of the U.S. airline industry.

Prior to deregulation in 1978, the fare structure of the airline industry was relatively simple and static. Airline fares were established collectively, according to an industry average cost, and were determined by a mileage-based structure imposed by the Civil Aeronautics Board (CAB). Carriers that operated at lower than average costs were not permitted to offer lower fares. The CAB also governed each carrier's route structure, permitting only a few carriers to serve an individual market, further limiting competition among carriers.

With the Airline Deregulation Act came the freedom to alter fares at will. The pricing strategies of the entire industry changed dramatically, and airline fares became dynamic and complex. Carriers found rewards in stimulating demand by offering seats, which would otherwise go empty, to passengers at lower fares. By imposing ticketing and travel restrictions on the low fares, diversion of passengers willing to pay higher fares was limited.

While restrictions on the purchase and use of low fare tickets limited the diversion of high fare passengers, airlines were soon faced with another problem. The seats sold to low fare passengers were not necessarily seats which would otherwise be empty. Besides the restrictions on the low fare tickets, capacity controls, or limits, on the number of available seats were needed. It was important to anticipate the number of seats which would not be filled by high fare passengers on a flight leg. These seats could then be made available for low fare passengers early in the booking process, leaving an adequate number of seats for full fare passengers booking closer to departure.

The complexity of airline fares was further compounded by significant changes in the route structures of carriers throughout the industry after deregulation. Without market entry regulations, new low-cost carriers entered major markets, aggressively offering low fares to capture market share. At the same time, smaller regional carriers, who had been limited to feeder-type route structures by the CAB, expanded into high density markets, offering low fares on their multi-stop and connecting flights to compete with the long haul, nonstop flights of trunk carriers.

In order for established airlines to compete with the lower fares being offered by the regional carriers and new entrants, they decided to offer a limited number of seats at these fares. Thus, by offering seats at low fares, established carriers were able to attract discount passengers and maintain a competitive image. At the same time, however, by limiting the number of low fare seats available, these high cost carriers could still retain their regular passengers and cover direct operating costs on a flight. This made the capacity control problem even more complex. Rather than simply determining the number of seats that would otherwise remain empty, the airlines had to provide seats for a range of different low fare passengers, as well as higher fare passengers.

For airlines to operate profitably in this complex and dynamic pricing environment brought on by deregulation, a totally new capability was developed. This capability has become known as yield management, or more appropriately, revenue management. Revenue management is the process of determining how many seats to sell at what prices and, thus, is comprised of both pricing and reservations control. By defining different fare products through the application of restrictions, and then effectively managing the mix of passengers carried, airlines have the potential to increase their total revenues.

Although cost savings are important in the airline industry, major new cost reductions are limited and not always easily attainable. Costs can be contained through improvements in labor productivity, the fuel efficiency of aircraft, and the maintenance capabilities of equipment. The industry as a whole, however, operates at a very high fixed cost. Labor costs make up 25-40% of an airline's operating costs, and fuel prices comprise 15-20% [4, 5]. While the fixed cost of running an airline is high, the marginal cost of carrying an additional passenger is very low. Therefore, the incremental revenue obtained through proper revenue management leads directly to greater profits.

It has been claimed that revenue management can increase an airline's annual revenues 4-5% per year [6, 7]. With operating revenues of major U.S. carriers in 1990 ranging from $1 to $10 billion [8], 4-5% is quite significant. At the same time, virtually all of the incremental revenue provided by revenue management falls to the bottom line. This additional revenue quite often accounts for most, if not all, of an airline's profits. For example, American Airlines (AA) has estimated that revenue management, i.e. overbooking and seat inventory control, has yielded benefits of approximately $1.4 billion over the last three years. In the same three year period, net profits were $892 million [9]. Shown in Figure 1.1 is AA's actual net income over the period 1986 to 1990 compared with the income level if American's estimated 5% in benefits due to revenue management are not included [10]. Similar effects of a hypothetical increase due to revenue management on profits, when compared to doing

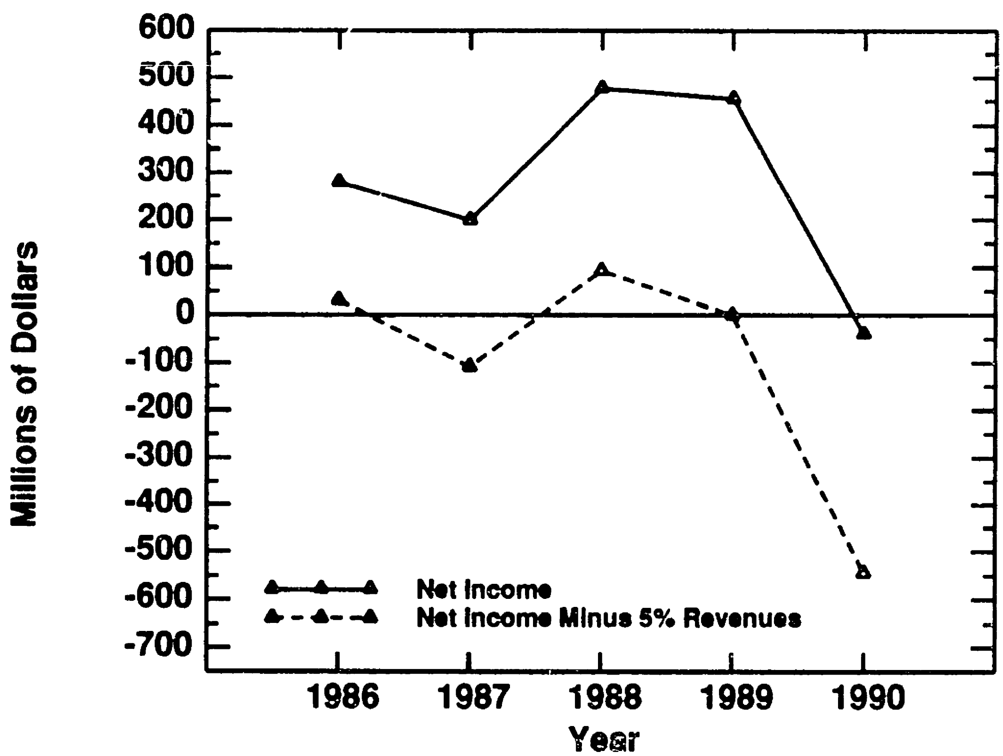


Figure 1.1: The net income of American Airlines over the period 1986 to 1990 compared to the income level without the 5% increase in operating revenues due to revenue management [11].

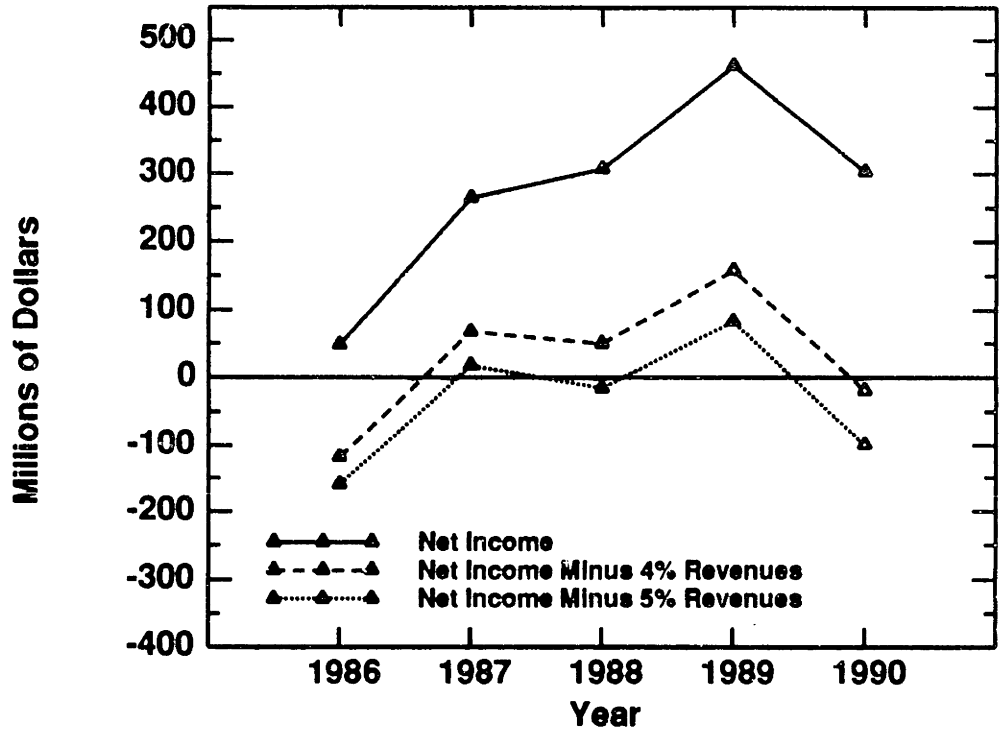


Figure 1.2: Delta Air Line's net income from 1986 to 1990 compared to the net income level minus the estimated 4% or 5% increase in operating revenues due to revenue management [12].

no overbooking or seat inventory control at all, can also be seen for Delta Air Lines in Figure 1.2.

Revenue management involves maximizing total revenues through pricing and reservations inventory control (Figure 1.3). Pricing is the process of determining the number and type of fares to be offered in each market, or more precisely, setting the price and corresponding restrictions of the different fare products offered. Not only do fares vary from market to market, but each market is stratified by a number of fare levels, ranging from full fare to deeply discounted fares. Typically, there can be as many as 10 different coach cabin fare levels, or classes, in a single market, each differentiated from the next by a number of fare restrictions. These restrictions exploit known differences in the travel patterns of time-sensitive versus price-sensitive travelers, i.e. business versus leisure. Common

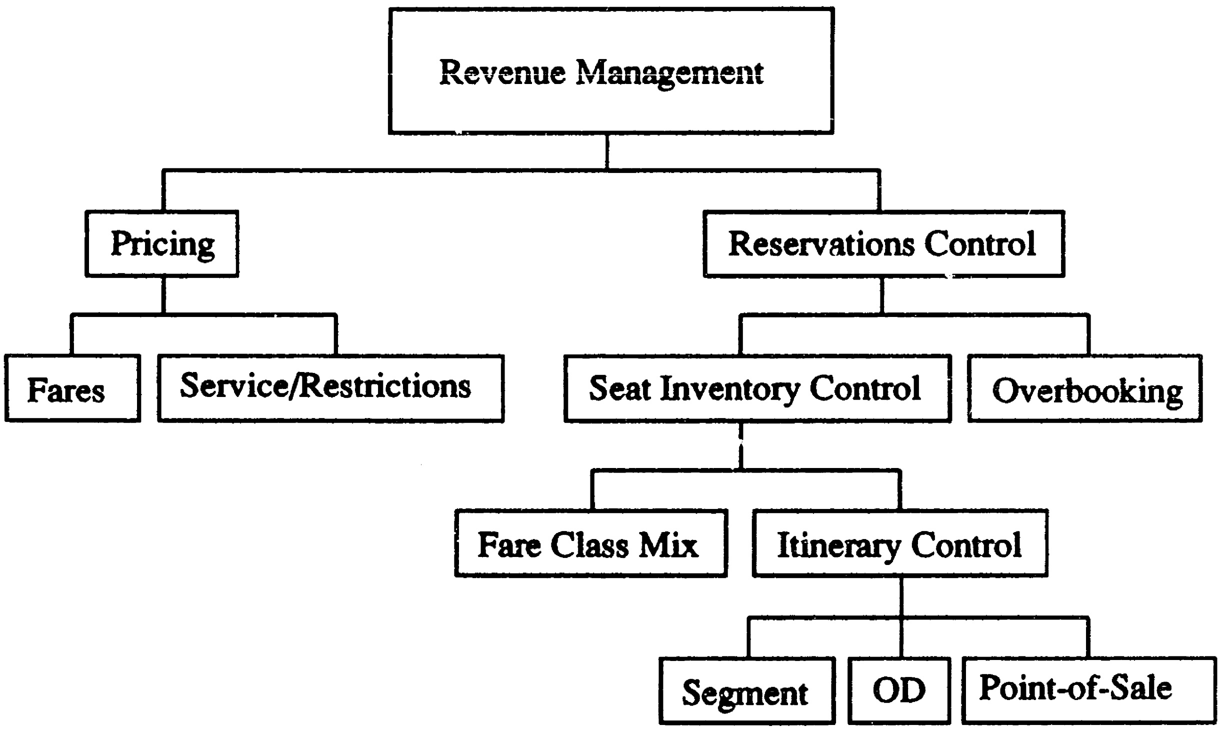


Figure 1.3: Structural diagram of revenue management.

restrictions are advance purchase requirements, cancellation penalties, non-refundability, particular day-of-week travel, round trip purchase, and the Saturday night minimum stay requirement.

Although pricing is an important component of revenue management and has a direct impact on revenue, no airline can influence its own revenue through pricing alone. Fares and restrictions are often determined by what other carriers offer in similar markets. However, an individual airline does have complete control over the reservations control component of revenue management. Within a given pricing structure, airlines can manage total revenues on a departure by departure basis through the use of reservations control. Thus, managing the mix of fares, and not the actual fares themselves, is often the most important and effective part of revenue management.

The purpose of reservations control is to determine how much of each fare product to sell. This is done through overbooking and seat inventory control (Figure 1.3). Overbooking is the process of accepting more reservations than the number of seats on an aircraft in order to compensate for losses due to cancellations. Seat inventory control , on the other hand, is the process of determining the right mix of seats available at different fares on a flight leg in order to maximize revenue. By limiting sales to various passengers and preserving seats for higher revenue, more profitable passengers, revenues can be increased. At the same time, offering otherwise empty seats to passengers at discount fares can also increase revenues.

Effects of such control of seat inventories is evident in the difference of the minimum fare available on flights at different times of the day and different days of the week. For example, on a Friday afternoon flight in a business market it is almost impossible to get a deeply discounted fare due to high full fare demand. However, in the same market in the middle of the day on Wednesday when demand is much lower, yet capacity is essentially the same due to scheduling and routing of aircraft, airlines offer a large percentage of the

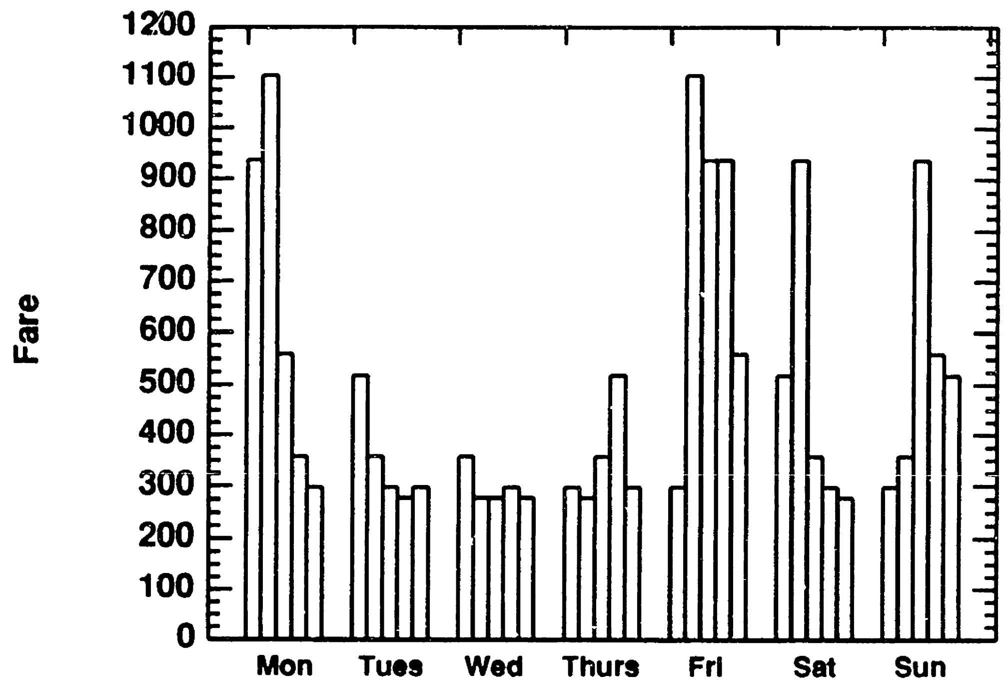


Figure 1.4: The lowest fare available throughout a week in a business market with five flights a day.

seats at discount fares [13]. Figure 1.4 shows an example of the controls placed on fares by an airline by tracing the price of the lowest, or "best", fare available throughout a week in a market with five flights a day [10].

Pricing and reservations control are both part of the marketing process, along with scheduling. In theory, this three phase marketing process should be considered simultaneously. However, the size of such a problem is enormous, with each individual component a difficult problem in itself. Therefore, the process is currently dealt with sequentially, with reservations control being the final step of the process. Given a fixed flight schedule and pricing structure, reservations control techniques are used in an effort to fill the seats on each flight departure in the most profitable manner.

1.1 Goal of Dissertation

In 1989, the average domestic load factor for the major U.S. carriers was 62.7%. With almost 4.7 million domestic departures averaging 158 seats per departure [14], there were over 276 million empty seats being flown. At the same time, 10-20% [15] of these flight departures were fully booked and closed to further reservations. By managing seat inventories and making decisions to accept, reject, or redirect passenger requests, demand can be better balanced and revenues increased.

While there has been much talk on the subject of revenue management and reservations control throughout the airline industry over the last five years, only a few airlines have actually implemented "sophisticated" systems which use automated mathematical decision making techniques. As a whole, airlines have recognized that there are benefits to be had in using statistical tools and mathematical analysis in controlling reservations, and most airlines today have invested in some sort of revenue management system. However, the typical system is simply a huge database and reporting system which retrieves, summarizes and analyzes historical reservations and traffic data.

While revenue management systems were initially developed to be database management and decision support tools, the more advanced systems are evolving into automated optimization systems. Such systems use historical reservations data, along with information on actual bookings, to forecast future demand. These demand estimates are then used as inputs, along with pricing and fare information, in to the reservations control system to determine both booking levels and space allocations. That is, the limit of *total* bookings by flight, i.e. overbooking, and the individual booking limits for different fare classes and itineraries, i.e. seat inventory control.

The reservations control systems today, even the more sophisticated ones, generally break the revenue management issues down into a number of relatively small problems in which bookings are controlled on the flight leg level. Network impacts on traffic flows, such as through and connecting traffic, are usually not considered. Demand, on the other hand, is origin-destination specific and often different from the particular origin and destination of individual flight legs. With rapid changes in fares, the globalization of route structures, and the increasing role of both large and mini hubs, there is a need for better revenue management and reservations control systems which account for the interaction between flight legs in a network [16].

While the need exists for new and improved reservations control systems which permit network control, there are currently many barriers to effective use of such systems. At the individual airline level, data collection and information quality is not always very good, and demand information, for the most part, is not collected and stored at the itinerary level, a necessity in order to manage network flows. At the same time, the basic architecture of most computer reservations systems used today was designed 20-30 years ago when the market environment of the airline industry was much simpler. An airline's computer reservations system (CRS) is where information on the seat availability for different fare classes and itineraries is stored and displayed. It is where all sale and cancellation activity occurs. The actual tactical and strategic seat availability decisions are made through the reservations control system. The computer reservations system simply takes the fare and inventory limit information from pricing and reservations control and makes it available to the reservations agents, travel agents, and other airlines (Figure 1.5).

Existing computer reservations systems allow for control of seat inventories at the fare class and flight leg level, rather than by origin-destination. The number of fare classes available through the computer reservations systems is also limited, not allowing all airlines to control bookings as effectively as they would like. The primary obstacle for reservations control is the dependence of all airlines on computer reservations systems and sales outlets for ticket distribution. No airline is able to completely control its own sales. Therefore

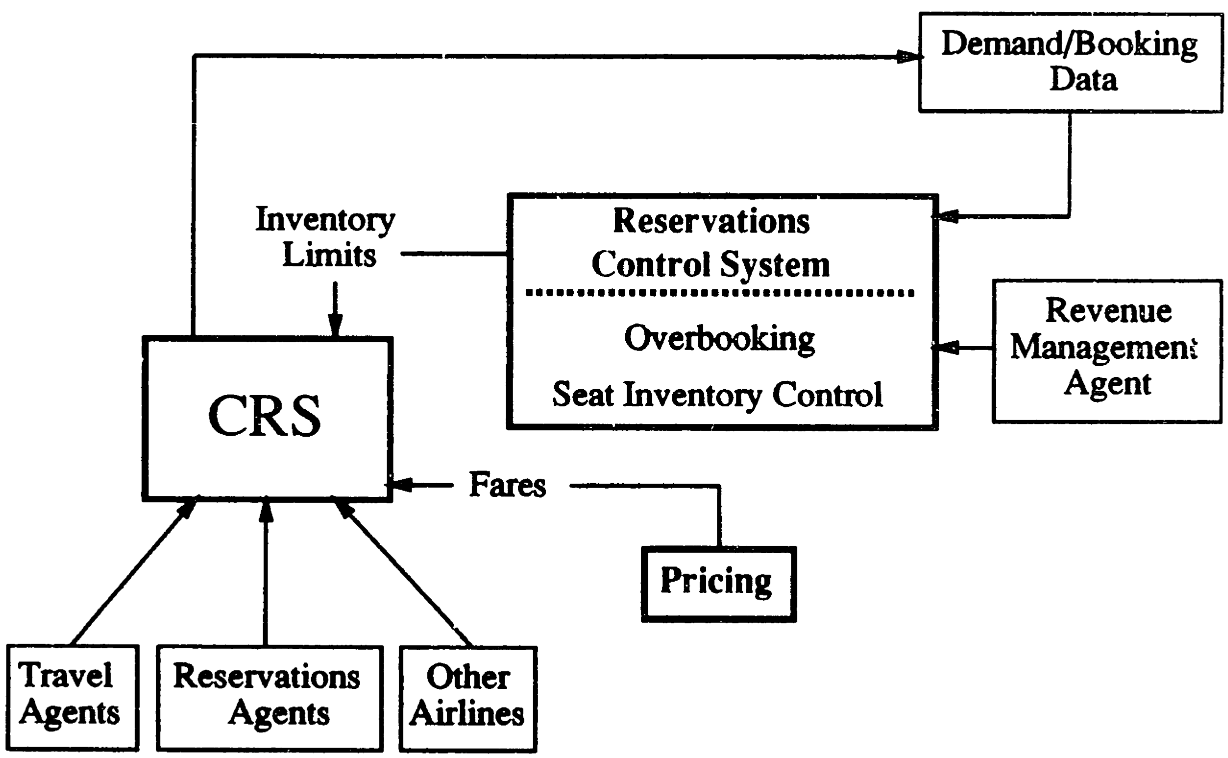


Figure 1.5: Relationship between the computer reservations system (CRS) and pricing and reservations control.

the methods and techniques employed by individual airlines in controlling inventories is not only determined by the capabilities of a carrier's own reservations systems, but by the computer reservations systems of other airlines as well.

These constraints are slowly disappearing. Computing power and data storage capabilities are expanding and becoming less expensive, allowing airlines to collect more detailed and accurate data. Improvements are also being made in the sophistication of optimization methodologies and algorithms. With continuing analytical and technical progress, it will be possible to approach the airline reservations control problem in ways that would have been completely infeasible a few years ago. Most important are the changes and advances which will be made to the computer reservations systems and the eventual introduction of "seamless availability", or direct access. Instead of being constrained by the characteristics and capabilities of other airlines' computer reservations systems, under the concept of seamless availability, requests will be made directly to individual airlines on a real time basis, allowing the airlines to evaluate each request within their own reservations control system.

The objective of this dissertation is to address the problem of reservations control at the network level. The major issues associated with network reservations control are: the availability of data at the itinerary fare class level, demand forecasting at this level, an airline's own system for controlling inventories, the mathematical optimization tools necessary for controlling seats at the network level, and the computer reservations systems. The focus of this research is on the optimization of seat inventories. Different approaches will be introduced and examined, considering a variety of optimization algorithms, as well as control methodologies, i.e. the actual application of the outputs of mathematical models to limit bookings. Together, the combinations of optimization and control techniques will be compared and evaluated on the basis of how they influence an airline's ability to increase revenue through network control.

Once an optimization and control strategy at the network level has been determined, the other issues of reservations control must also be addressed. While the problem of forecasting demand at the itinerary and fare class level has not been solved as of yet, the collection and storage of network demand data is currently possible with a large amount of resources dedicated to the process. By simply investing enough money, an airline can update its own reservations control system, inventory structure, and computer reservations system. Finally, with seamless availability, the computer reservations interdependence problem can be solved.

The intent of this dissertation is to present a comparison of reservations control techniques. A number of factors, such as cancellations, no-shows, the potential for the upgrading of passengers to a higher fare class then originally requested, and the recapture rates of rejected demand on other flights, are not considered in order to isolate the effects of discount and itinerary controls. For further simplification, misconnects (passengers who miss a flight leg out of a hub due to a late or canceled inbound flight) and standbys are also ignored. Such factors must be considered when developing a new reservations control system, but by not including them in this analysis it will be possible to identify more clearly the differences between the actual optimization and control techniques themselves. Although ignoring these factors is restrictive, a solution to the simplified problem can lead to improved sub-optimal approaches for the more realistic problem. By first reducing the complexity of the problem, it may be possible to determine basic insights into the nature of a good solution. Other factors can then be addressed and incorporated.

1.2 Structure of Dissertation

The remainder of this dissertation is divided into six chapters. Chapter Two serves as a formal introduction into seat inventory control, pointing out the specifics of airline operations and practices which contributed to shaping the seat inventory control problem.

The second section of the chapter defines the concept of network seat inventory control and introduces the reservations control functions which it encompasses. The characteristics and complexities of the problem are then defined and discussed, identifying practical issues and constraints which must be considered when solving the problem.

Chapter Three is an overview of previous work, summarizing the two basic approaches taken thus far when addressing the seat inventory control problem, approaches using marginal seat revenues and those using mathematical programming. After reviewing the literature, some of the current practices and approaches used by the airlines to control seat inventories will be described, showing the range of sophistication used in reservations control systems throughout the industry and the need for better and more effective seat inventory control approaches.

With the objective of this dissertation being to develop practical optimization and control methodologies for airline network seat inventory control, Chapter Four begins by introducing basic notation and reviewing the traditional network formulation of the problem. Using the solutions obtained from these mathematical optimizations, practical approaches to controlling seat inventories are discussed. In an effort to avoid problems associated with forecasting at the fully disaggregated origin-destination and fare class level, network methods which use aggregated demand estimates are then presented. Finally, focusing on optimization and control at the flight leg level, simpler leg-based heuristics are described, which incorporate information about the interaction between flights on a network and distinguish between different passenger itineraries.

One of the major interests in the development of new seat inventory control methodologies is the potential for increased revenues. An effective way of estimating the expected revenue generated from different methodologies is through simulation. In Chapter Five, a dynamic and integrated optimization/booking process simulation is described which was designed and implemented to compare revenue impacts from different seat inventory control approaches. Specific issues involved in realistically modeling and simulating the booking process of an airline are then detailed.

Chapter Six presents an analysis and comparison of simulation results. The revenue impacts due to different optimization strategies and control methodologies are evaluated using real airline data from both a connecting hub network and multiple flight leg networks. Different factors, such as protection levels and seat allocations, are examined, and the robustness of the seat inventory control methodologies is investigated through variations in the forecasting accuracy, frequency of revisions, and demand distribution assumptions. The overall performance of different approaches are examined by comparing the revenue obtained with that of a common leg-based seat inventory control methodology, as well as the maximum revenue possible if decisions were made based on perfect information.

Chapter Seven concludes this dissertation, summarizing the research findings and contributions of this work. Discussion will focus on practical issues associated with the implementation of different seat inventory control methodologies. Finally, future research directions stemming from the work presented in this dissertation are outlined.

Chapter 2

Seat Inventory Control

2.1 The Seat Inventory Control Problem

The primary component of reservations control is seat inventory control (Figure 1.3). Traditionally, airline seat inventory control has been the practice of allocating seats to different fare classes in an effort to maximize the expected revenue of future scheduled flights. This is done by protecting seats for higher fare passengers, while at the same time making empty seats available to lower fare passengers. By offering seats at discounted fares, an airline can capture extra passengers who otherwise would not travel, in turn providing additional revenue. Too many seats at lower fares could cause a diversion of potential high fare passengers to available lower fares and may displace some higher fare passengers altogether, lowering total revenues.

The concept of controlling seat inventories by fare category started when airlines found that total revenues could be increased through the practice of differential pricing, i.e. charging different customers different prices for a product. Economically, if only one price, or fare, is offered in a market, that fare would be determined in an attempt to maximize revenue. Therefore, under a simplified demand curve, such as that in Figure 2.1, a trade-off would be made between the demand, i.e. the number of passengers, and the fare until revenues were maximized. In this case, 50 seats would be sold at a fare of $250 for a

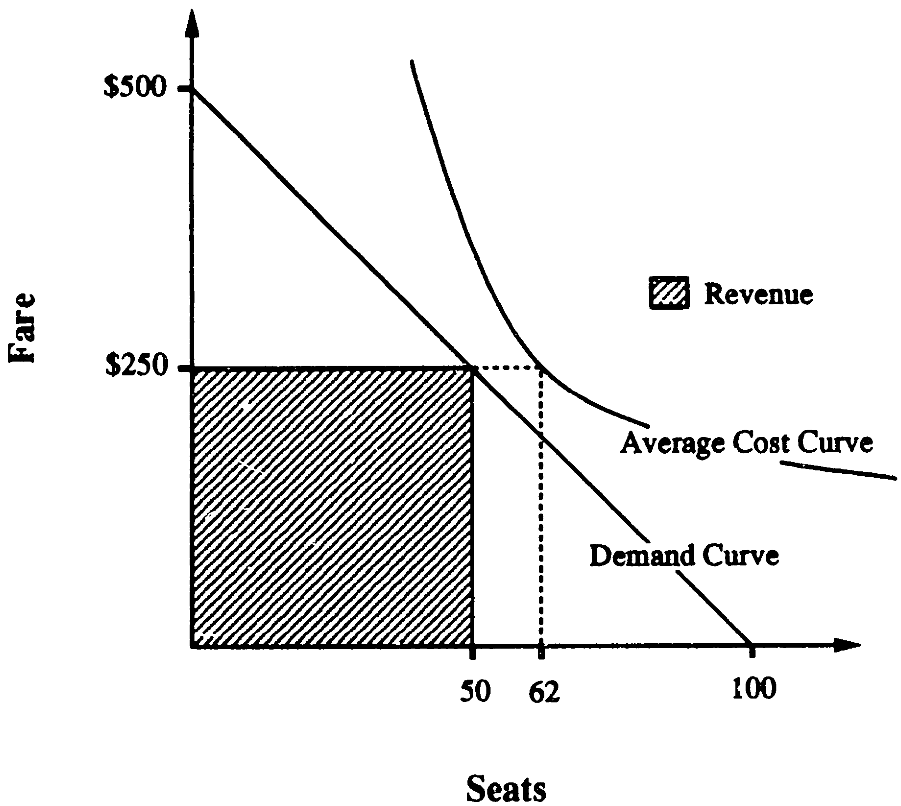


Figure 2.1: Given the assumed demand curve, revenues are maximized for a single fare structure by setting the fare at $250, selling 50 seats, and generating $12,500. However, based on the average total cost curve given for the flight, it would be necessary to sell 62 seats at $250 in order to cover total operating costs.

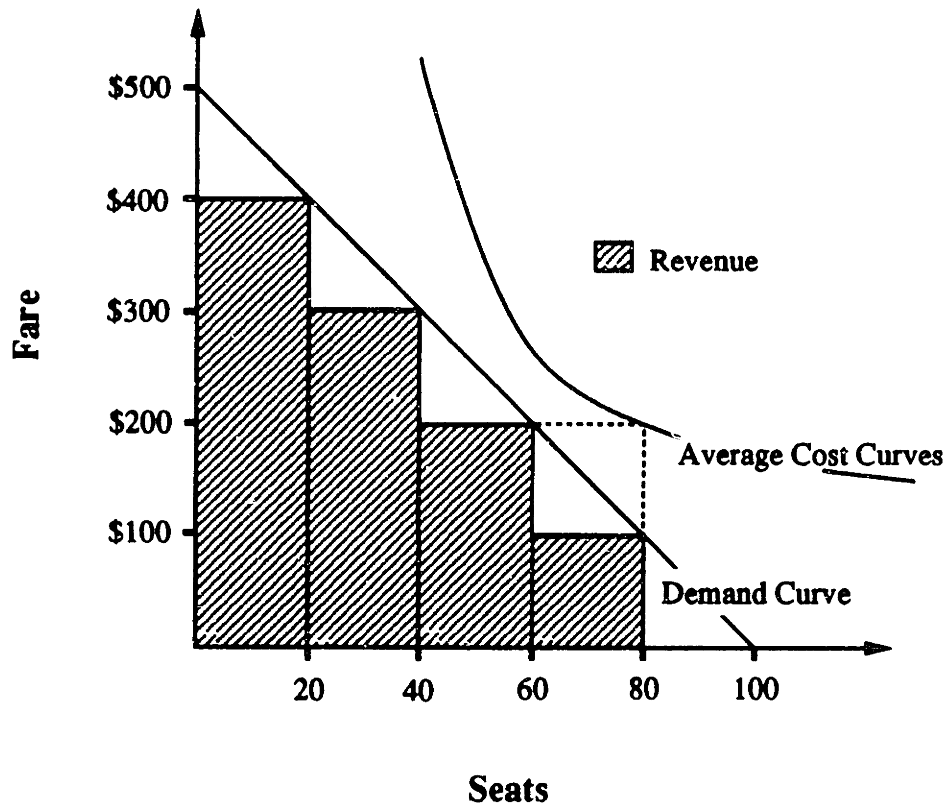


Figure 2.2: By using a differential pricing strategy with four fare levels, total revenues are increased to $20,000 from 80 passengers (20 × $400 + 20 × $300 + 20 × $200 + 20 × $100) and costs are easily covered.

maximum revenue of $12,500.

In practice, there is often not a single fare level at which the revenues generated will cover total operating costs. Figure 2.1 shows an example of an average total cost curve which may be associated with a given flight departure. The closest the demand curve comes to the cost curve is at a fare of $250, where the total cost is $15,500. This would require a demand of 62 passengers, instead of the 50 passengers obtained at a single fare of $250. Under the same demand assumption, if an airline offered seats at four different fare levels, $400, $300, $200, and $100, the airline could attract 80 passengers and generate $20,000 in revenue, as shown in Figure 2.2, assuming diversion of demand willing to pay higher fares does not exist. In this case, not only are revenues increased, but the total cost of carrying 80 passengers is easily covered by this differential pricing strategy. Thus, through a multiple fare structure, airlines are able to refine their pricing so as to cover total costs and maximize revenue.

Although airlines are practicing a form of price discrimination, they are not directly discriminating in price between different passengers for the same product. The differential in price offered by airlines is usually based on differences in "fare products", each of which is uniquely defined by restrictions on their purchase and use for air travel. By recognizing the differences in price elasticities among customers, airlines have been able to segregate market demand into several different product groups. For example, discount fare passengers, who want the lowest fare possible, must be willing to make travel plans in advance, be flexible enough to shift their travel to certain days of the week and times of the day, stay over Saturday night, and are not allowed to change or cancel their travel without penalty. On the other hand, full fare passengers can make last minute plans at peak times, have no restrictions as to when and where they travel, and can change or cancel their plans at any time for no charge. Thus, these restrictions differentiate the quality of service that passengers are buying and justify, to some degree, a difference in price.

In order to obtain the benefits of price discrimination, the relative fare levels and respective restrictions offered in each market must be managed effectively. In addition, the number of seats offered at each fare must then be fine-tuned to capture the most profitable mix from the available passenger demand. For example, looking again at the simple demand assumption used above, if twice as many deeply discounted seats are offered, i.e. 40 seats at $100, with 20 seats still offered at $200, 20 seats at $300, and 20 seats at $400, diversion of demand willing to pay higher fares would result and revenues would only total $14,000, no longer covering the costs of $16,000, (Figure 2.3). On the other hand, if discount seats are limited excessively, the resulting revenue would again be inadequate to cover costs. In Figure 2.4, only $14,000 is realized in revenues when 10 seats, instead of 20, are made available at each of the three discount fares, amounting in a total demand of 50 passengers,

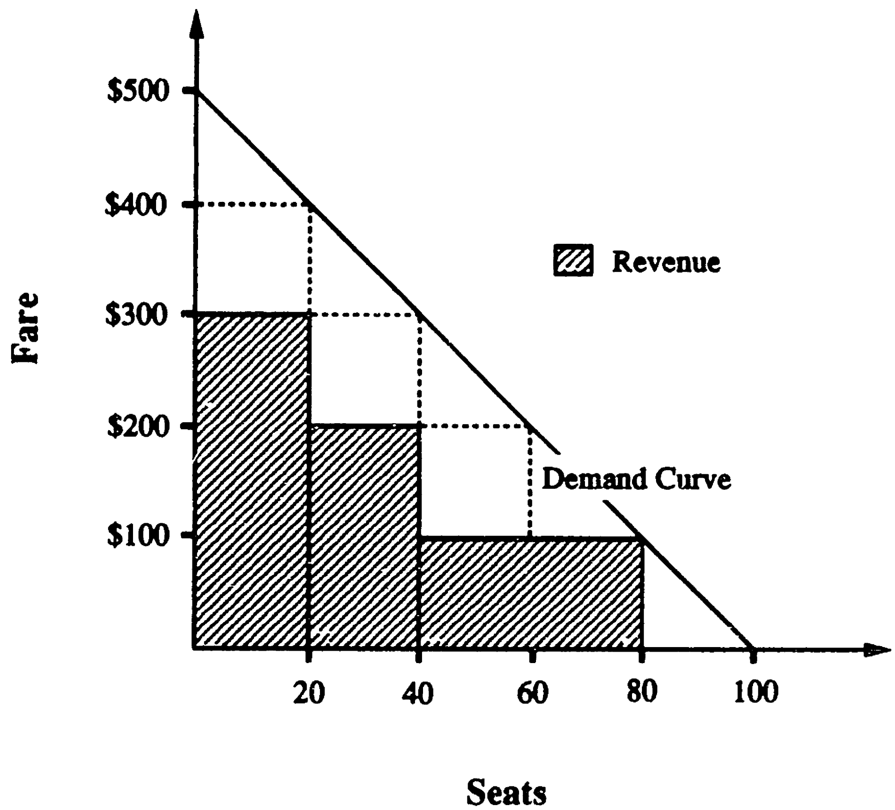


Figure 2.3: By overallocating the deeply discounted seats and allowing as many as 40 seats to be booked at $100, diversion of higher fare demand results. Seats are sold at the $300, $200 and $100 fare levels, rather than at the $400, $300, and $200 fare levels, resulting in significantly lower total revenue.

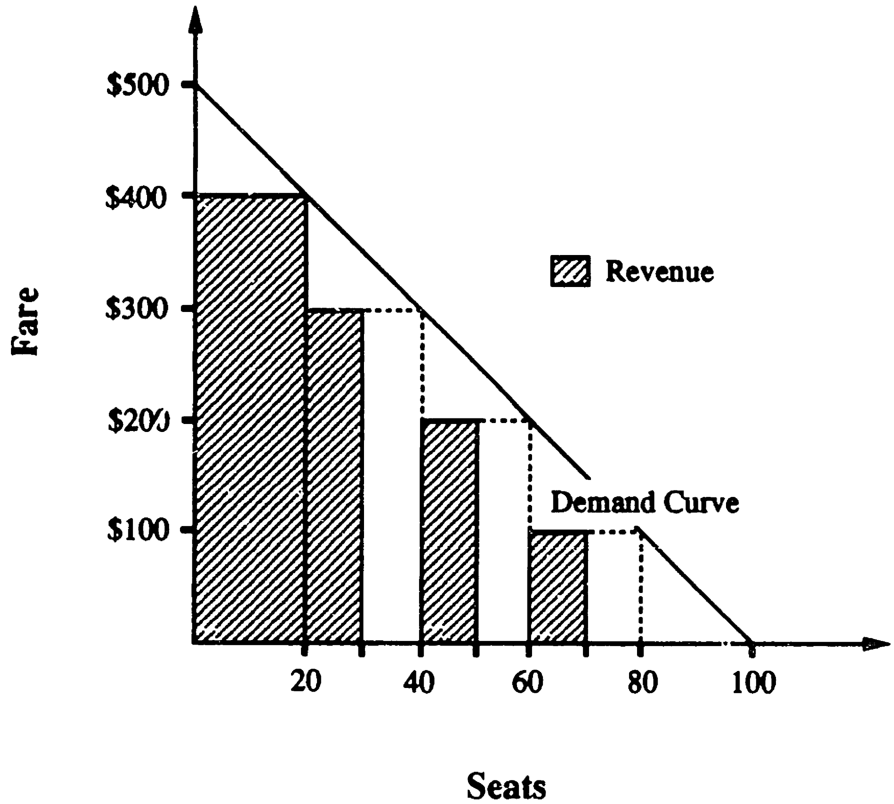


Figure 2.4: By under allocating the number of seats to each of the discount fares and allowing only 10 passengers to book seats at the $300, $200, and $100 fare levels, revenues again drop, costs are not covered, and seats go empty.

while total cost is $15,500 [17].

A seat on any particular flight departure is an extremely perishable commodity. Once the doors close on a plane, the value of any unsold seats is lost forever. With nearly 40% of the airline industry's product perishing, selling a seat to an additional passenger for any price above the marginal cost is advantageous (assuming no diversion from other fares), and the marginal cost of carrying the additional passenger in an otherwise empty seat is very low. Thus, as long as the lowest fares are greater than the marginal cost of carrying the extra passengers, these passengers will be contributing to the fixed costs of operating the flight and to profits. Not only do airlines benefit from carrying low fare passengers, but high fare passengers may benefit as well. With the extra revenue from the additional low fare passengers, airlines may actually be able to reduce the cost of a full fare ticket. Even if this is does not occur, the extra revenue can at least be used towards improvements in service, frequency and equipment, which benefits full fare passengers and their desire for last minute seat availability.

Adjustments to seat inventory allocations can both increase load factors, by filling otherwise empty seats, and increase net yields, by better allocating capacity to higher fare traffic. It can also give an airline the opportunity to participate in competitive discount fare initiatives in order to maintain market presence. The challenge behind seat inventory control is to maximize revenue by best deciding how and when to make trade-offs between the cost of an empty seat, and the loss of a discount fare, and the cost of turning away a full fare passenger, resulting in a loss of revenue equal to the difference between the full and discount fares.

The need for seat inventory control stems from the basic problem that airline supply does not equal demand. In air transportation, supply and demand seldom match exactly. On the one hand, demand for future flights is probabilistic and cannot be forecasted precisely. However, the problem results, to a greater extent, from the actual scheduling of aircraft. Because of the route structure of an airline, the relatively fixed number and size of aircraft, scheduling constraints, and the lack of balance in passenger demands over a network, it is not always possible to schedule exactly the right aircraft for each departure. Therefore, when there is either excess demand or excess capacity, a closer match between supply and demand can be achieved through the use of seat inventory control techniques. The greater the problem of matching the supply of aircraft seats to the demand of different routes, the greater the benefits of effective seat inventory control.

The major difficulty of the seat inventory control problem is the fact that the reservations for discounted fares by leisure travelers have a tendency of being made before full fare, business oriented demand materializes. This occurs both because of the nature of the customers for the respective fares and the early booking restrictions placed on the discount

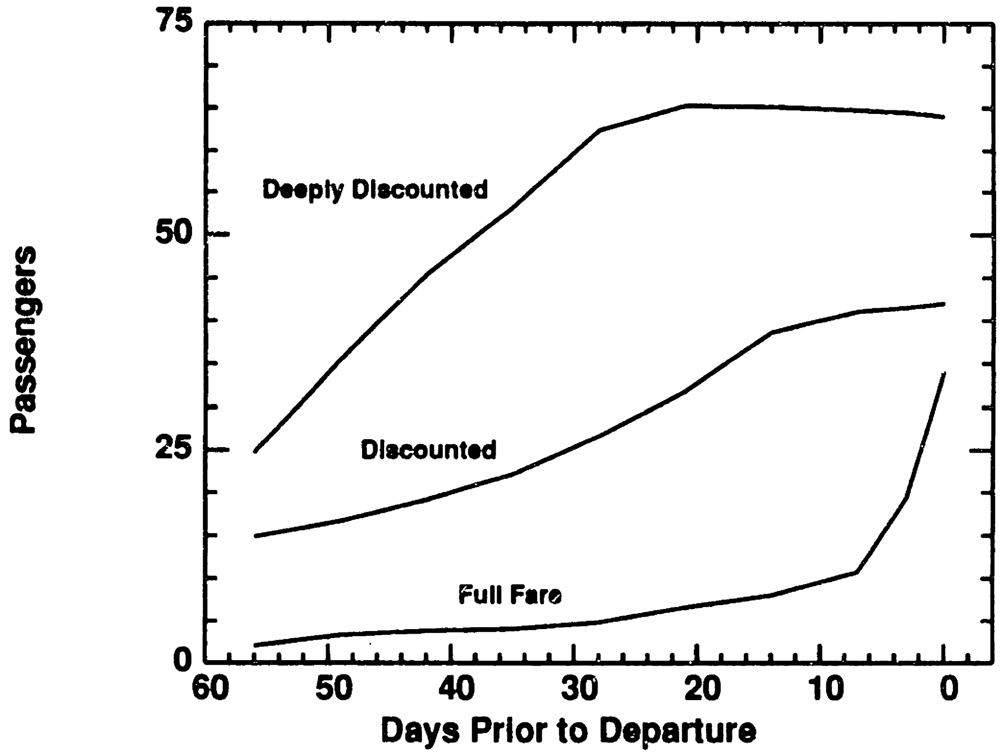


Figure 2.5: The build up of bookings differs with the type of fare being purchased. Deeply discounted tickets are purchased early in the booking process, while full fare demand materializes in the last few days before departure.

fares. If high fare passengers booked before lower fare passengers, the seat inventory control problem would be trivial. Airlines would simply fill seats with the highest fare passengers as they came in until there was no further demand or aircraft capacity was reached. This is not the case, however. Figure 2.5 gives an example of a booking profile, showing actual bookings by groups of fare classes versus the number of days prior to departure. The build up of bookings over time differs with the type of fare being purchased. Thus, seat inventory control is, in essence, the practice of trying to save just the right amount of seats for late-booking, high revenue passengers.

The seat inventory control problem can be approached from a variety of perspectives. Seat inventories can be controlled over individual flight legs, over the entire network of a carrier, or over separate subsets of the network. Most airline reservations systems currently

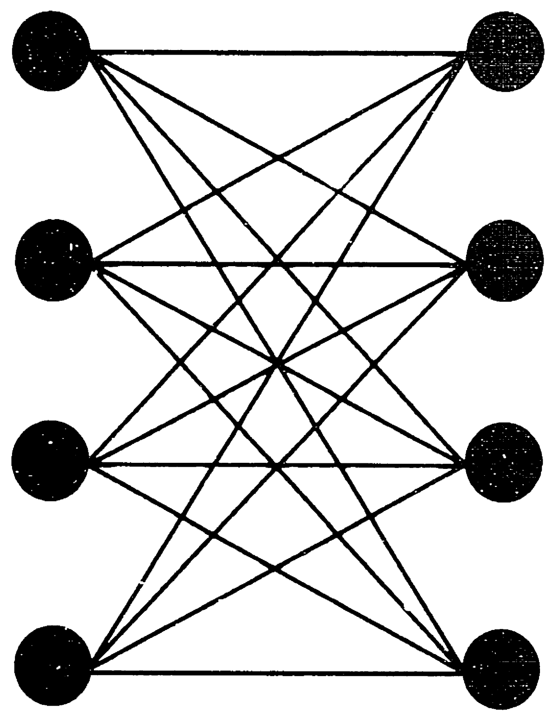


Figure 2.6: Point-to-Point network where each distinct origin and destination is served by a non-stop flight leg.

maintain seats inventories and manage seat availability by fare class. Using such methods, efforts are made to maximize revenue on each individual flight leg. Therefore, reservations for travel on a flight leg are accepted based on the availability of a particular fare class on that flight leg. A passenger's ultimate destination, overall itinerary, or total revenue contribution to the airline is not taken into account.

If the route structure of an airline served each distinct origin and destination market with isolated, non-stop, point-to-point flights, as in Figure 2.6, a flight leg approach to seat inventory control would be all that was necessary. However, the typical major airline route structure is a complex network built around one or more connecting hubs, as shown in Figure 2.7. Instead of aircraft flying between each individual city pair the airline serves, flights from a number of cities converge at one time at a hub, exchange connecting passen-

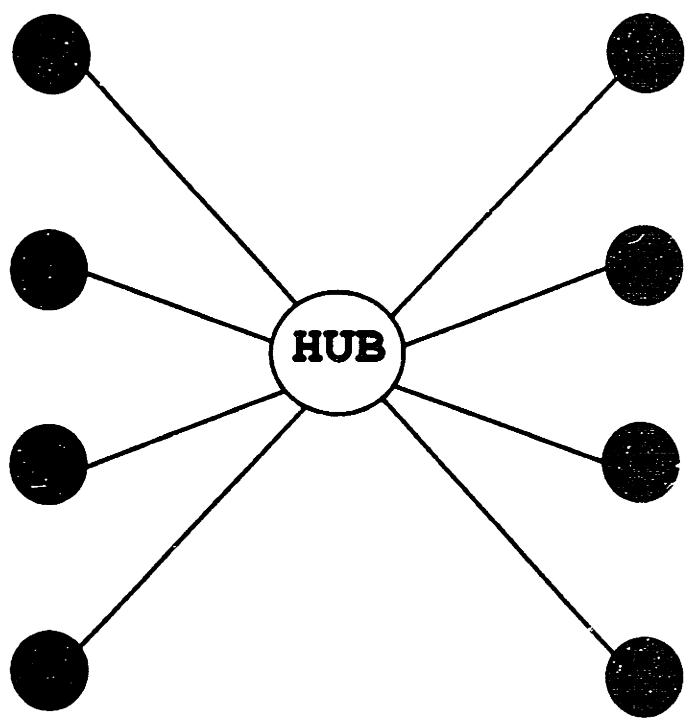


Figure 2.7: Hub-and-Spoke network where each individual market is served by way of a connection through the hub.

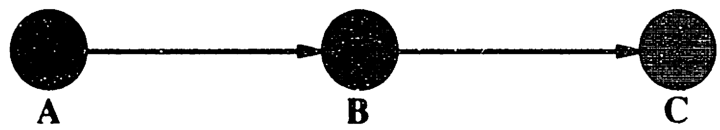


Figure 2.8: A simple multi-leg example with three possible itineraries and one fare class. AB: $100, AC: $150, and BC: $100.

gers, and then depart to a number of other cities. Using such a route structure, an airline can efficiently provide service to many different markets, while minimizing the resources needed to do so.

In a connecting hub environment, maximizing flight leg revenues is not necessarily the same as maximizing total network revenues. This can be illustrated with the following simplified example. Consider a one stop flight from city A to city B to city C. In this example there are two flight legs, A-B and B-C, and there are three possible itineraries, AB, BC, and AC. For a given fare class, the fares for the two short haul itineraries, AB and BC, are each $100. For the long haul AC itinerary, the fare is $150, (Figure 2.8). Under a leg-based inventory structure, a seat on the flight leg A-B could be reserved by either an AB passenger or an AC passenger. Thus, it is possible for flight leg A-B to be filled entirely by AB passengers at $100 each, denying any seats to the higher revenue AC passengers. If demand for local travel from city B to city C is low, seats could go unsold on the B-C flight leg, and a reduced total revenue for the two flight legs combined would result.

In the same example, it could also be possible for all the seats on the A-B flight leg to be booked by AC passengers at the higher revenue of $150. However, if short haul demand is high for *both* the A-B flight leg and the B-C flight leg, total revenues would have been increased by selling the seats on the A-B flight leg to AB passengers. With high local demand on the B-C flight leg, total revenue for a seat from city A to city B and from city B to city C would be $200 when sold to local traffic, versus the $150 received from an AC through passenger.

In the first example, by protecting seats on flight leg A-B for the longer haul AC passengers, total revenue could have been increased. On the other hand, when short haul traffic is high, higher revenue can be obtained by limiting the number of seats for multi-leg passengers. In order to maximize revenue over an entire network, a seat inventory control methodology must take into consideration the interaction between flight legs in the network. Decisions as to which passenger itineraries are most desirable to the network as a whole must be made. At the same time, there are additional complexities of dealing with multiple fare classes in each possible city pair. Should a full fare AB passenger be accepted, or a discounted AC passenger? In actual hub operations, where there are a large number of connections and a multitude of passenger itineraries, decisions among thousands of different origin-destination (OD) and fare class options need to be made.

In 1980, 10% of American Airlines total traffic was connecting traffic, but by the mid-1980's, about two-thirds of the passengers on a typical flight into a hub airport connected to another flight in order to get to their final destinations [9]. Although the control of fare class bookings at a flight leg level was adequate in a network system where each flight leg served a limited number of markets, with large connecting hub-and-spoke systems and an increase in the number and variety of OD combinations served by an individual flight, the effectiveness of controlling seats by fare class and flight leg alone has been reduced. The seat inventory control problem itself is no longer one of simply allocating seats to four, five, six, seven or even ten fare classes on a single flight leg. Today, a single flight leg carries passengers with many different origin-destination itineraries, each of which represents a different revenue contribution to a carrier. Seat inventory control decisions involve not just the number of seats to allocate to each fare class. Decisions of whether a seat should be allocated to a higher yield fare class on a single leg itinerary or to a lower yield fare class with a higher total revenue, on a multi-leg or connecting itinerary, often need to be made.

The complexity of the problem has grown tremendously with the development of large hub-and-spoke operations. On a single flight departure into a major hub, there can be as many as 50 market destinations being served. With some airlines currently offering ten fare classes in the coach cabin alone, there are over 500 OD and fare class itineraries possible on a single flight leg, each having a different level of attractiveness, in terms of revenue, for the airline. At the finest level, decisions need to be made for every unique origin-destination and fare class combination on each departure. With a carrier offering as many as 2500 flights per day, each of which must be considered a member of an interconnected network rather than treated independently, and with the control of seat inventories starting as early as 330 days prior to departure, the problem has become very large.

2.2 Network Seat Inventory Control

Network seat inventory control allows the airline to differentiate between the many types of fares and the variety of itinerary values when determining seat allocations. The purpose of network seat inventory control is not only to manage fare class inventories and limit sales to discounted fare classes, but to manage overall network traffic, limiting sales by origin-destination itinerary, as well as fare class. When there is excess demand competing for too few seats on one or more flight legs of a multi-leg network, greater control over different origin-destination traffic can increase total revenues.

Network seat inventory control, as defined here, addresses both the fare class mix and itinerary control components of seat inventory control and encompasses origin-destination control, segment control, and point-of-sale control (Figure 1.3). Full origin-destination control is network seat inventory control at its extreme, where each specific class of service in every origin-destination market, i.e. OD and fare class itinerary, within a network of connecting flights is subject to a booking limit. Seats are allocated and booking limits applied to each individual OD and fare class.

Segment control is essentially a simpler version of network seat inventory control, where a single multiple leg flight is defined as the "network" and the different *on-flight* OD traffic is controlled. Revenue is optimized and seats allocated based on the different segments of the multiple leg flight. The origin or destination of a passenger *outside* of a given multi-leg flight is not important, only the differentiation between the local and through itineraries on the flight. While origin-destination control is important to U.S. domestic airlines with large hub-and-spoke networks, segment control is much more important to many of the international carriers, where route structures tend to be linear, multi-leg networks similar to those of the Canadian Airlines and Air Canada.

Point-of-sale control is the practice of differentiating between the actual location in which a seat is sold. The reason for point-of-sale control arises from the fact that the same type of fare sold at different locations can bring in different net revenues due to such factors as commission rates, fluctuations in currency and exchange rates, and interline prorate agreements with other carriers. By controlling inventories based on point-of-sale, carriers can limit the sales from specific locations to take advantage of non-commission traffic and tickets sold in preferred currencies. Point-of-sale can also be used to attach a priority to a particular country, city or station in conjunction with certain marketing and sales strategies. Although many international carriers have a regulated fare structure, lack direct competition, and do not operate extensive connecting hub-and-spoke networks as in the U.S., the point-of-sale problem is extremely important. Through a network seat inventory control approach, different point-of-sale locations, each of which yield different net revenue contributions, can be viewed in the same way as different OD's on a network and thus, managed and controlled separately.

At first glance, the network, or origin-destination, seat inventory control problem might seem to be a basic network optimization problem. However, the problem is much more complex due to certain airline characteristics and industry constraints. To have a full understanding of network seat inventory control methods and their limitations, the practical complexities of the seat inventory control problem must first be addressed.

One important characteristic of the seat inventory control problem in general is that passenger demand is probabilistic. There always exists some uncertainty as to the number of requests for a future flight, itinerary, or fare class. This uncertainty stems from both cyclical and stochastic variations in demand. Day of week, regional holidays, special events, and seasonal changes throughout the year all create systematic fluctuations in demand. There are also stochastic, or random, variations around an expected demand. Both types of variations can be modeled, and although demand can be represented by statistical probability distributions, there always exist some unpredictable variation associated with future demand.

Demand for a future flight is also dynamic. Passenger behavior, and therefore the optimal seat inventory solutions, vary with time to departure. From one day to the next, the total number of bookings on a flight changes continuously, affecting the entire network of dependent departures. As time passes, the forecasts of demand for each fare class and passenger itinerary also change. Not only are such changes due to the instability of passenger behavior, demand is also affected by schedule and pricing changes. Such effects are not minor and cannot be overlooked given weekly changes in schedules, and an average of over 30,000 price changes per day in the U.S. domestic airline industry alone [18].

Due to the dynamic nature of the problem, it is important to be able to monitor the seat inventory control system, allow for changes in demand and fare assumptions, and make adjustments to seat allocations when needed. Therefore, the solution time of a network seat inventory control model must be small enough to make revisions on an interactive basis. At the same time, most airlines schedule checkpoints during the booking process in order to revise forecasts, seat allocations and booking limits. Thus, the seat inventory control problem becomes a multiple stage problem, in which the optimal current decision can be affected by the knowledge that revisions will be made in the future.

An additional constraint imposed by the practical application of a seat inventory control approach is that recommended seat allocations be integral numbers. Fractions of seats cannot be allocated or sold. The difficulty of such a constraint is that integer solutions usually require a considerable amount of extra processing. Simple rounding of a non-integer solution often does not give the optimal integer solution and can be significantly different from it.

Another complication involved in addressing the network seat inventory control problem is the nested inventory structure which many airlines use. Inventories are nested so that as long as there are seats available, a higher revenue, more desirable request will not be denied. At the flight leg level, each lower fare class inventory is nested within the next higher fare class. For example, in a four fare class structure–Y,B,H,V–with Y being the full coach fare class and V being the lowest discounted fare class, the V-class seat inventory is nested within H-class, and in turn H-class is nested within B, and B within Y, as shown in Figure 2.9. For example, if there were 20 seats allocated to Y-class, 30 to B, 15 to H, and 35 to V-class, there would actually be 100 seats made available to Y-class requests, while up to 80 seats would be available to B-class and 50 seats to H class. V-class availability would remain at the 35 seats which were allocated to it.

In a nested fare class structure such as this, if there is a request for 50 seats on a given flight leg in the highest fare class, Y-class, the passengers would be allowed to book on the flight leg if there were 50 seats still available. The passengers would not be turned away because only 20 seats were actually allocated to Y-class exclusively. On the other hand, in a partitioned fare class inventory structure , such as that in Figure 2.10, only 20 seats could be sold in Y-class, regardless of the extra demand at this high fare. Once a seat has been

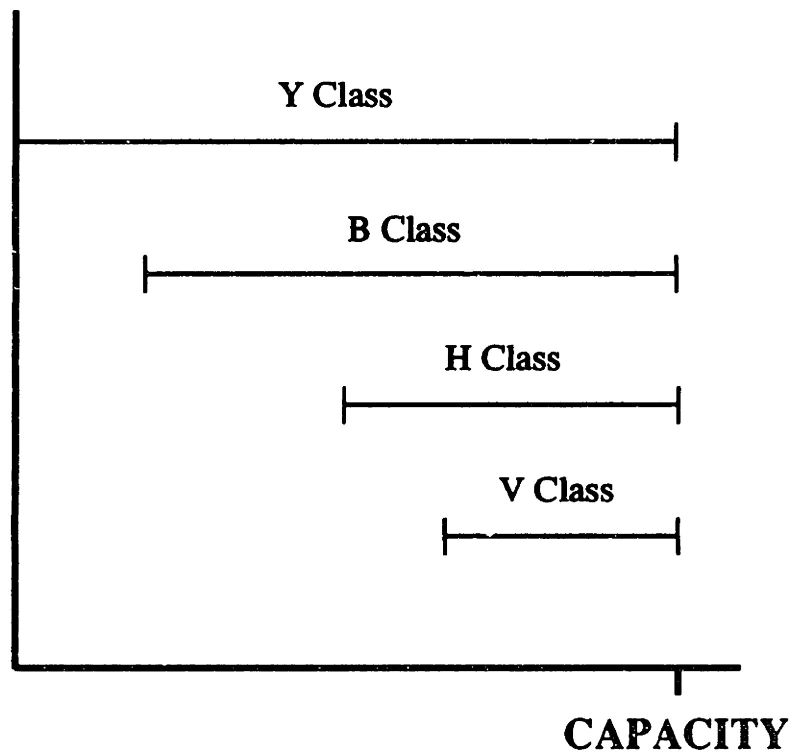


Figure 2.9: Seat inventories for the four different fare classes based on a nested fare class structure.

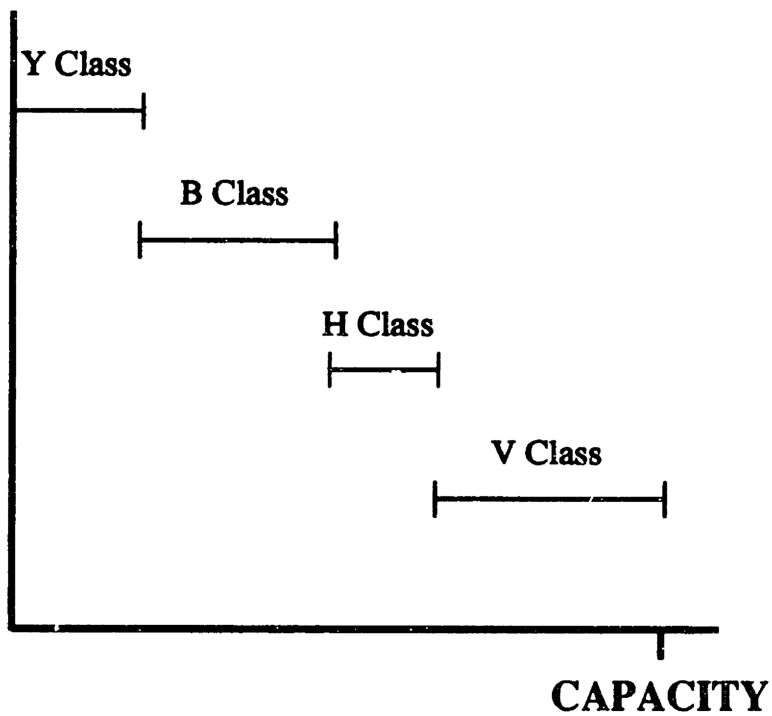


Figure 2.10: Seat inventories based on a partitioned, or distinct, fare class structure.

allocated to a partitioned, or distinct, fare class, it can be booked only in that fare class or remain unsold. By nesting the fare class inventories, if there are requests for the highest fare class, and seats are available, these requests will be accepted and not turned away because of expected lower fare demand. In this way, the impact of errors in the demand forecasts of higher valued classes is reduced. It has been shown that the expected revenue from a nested structure is equal to or greater than that of a partitioned inventory.

The problem with a nested fare class structure is that virtually all traditional network optimization techniques generate solutions for partitioned inventories. Such distinct inventory solutions are generally not the optimal seat allocations for a nested structure. At the same time, the network environment, with different passenger itineraries and fare classes, further complicates nesting. On a single flight leg, the order of nesting inventories is straightforward, with the highest fare at the top and the lowest fare at the bottom. In a network of flights, the highest revenue origin-destination and fare class (ODF) itinerary is not necessarily the most desirable to the network as a whole. With high local demand, it is often the full fare local ODF's which would be more desirable to the network than the higher revenue multi-leg ODF's. Under different fare and demand conditions, the ranking of ODF's, in terms of desirability to the network, changes. Determining *optimal* booking limits for nested fare class inventories at the flight leg level is in itself complex, where the nesting hierarchy already known.

The size and complexity of a theoretical model which attempts to *optimally* solve this large, probabilistic, dynamic, nested network seat inventory control problem make it impractical for an airline to use routinely in its reservations control system. However, with the importance of seat inventory control to airline profitability, emphasis is being placed on finding better and more effective methods of controlling seats than the simple leg-based methods which allocate seats by fare class alone, maximizing revenue on individual flight legs.

Chapter 3

Previous Work and Current Practices

3.1 Literature Review

Prior to 1972, most work in the area of reservations control was devoted to maximizing the number of passengers carried on each flight leg by overbooking. Since then, changes in airline fare and the introduction of advanced purchase excursion (APEX) fares made it increasingly important for airlines to maximize revenue, not just loads. Work in the area of seat inventory control started in the early 1970's with approaches to finding seat allocations on a single flight leg based on equating marginal seat revenues. Research soon evolved into methods which used mathematical programming and network flow analysis in order to incorporate different passenger itineraries, as well as multiple fare classes. While there has been some theoretical research done in this area, there has been a lack in *practical* optimization models for determining the number of seats to allocate to different origin-destination and fare class itineraries in large networks. The following is a review of the published research in both areas.

In 1972, Littlewood [19] introduced the concept of using expected marginal seat revenues to determine whether to accept or reject a reservation for a flight leg according to the fare paid. The expected marginal revenue of potentially selling a seat in a given fare class is simply the probability of being able to fill the seat multiplied by the revenue which would be obtained by selling the seat, i.e. the average fare of the respective fare class. In order to sell S_i seats in fare class i, the number of requests for seats must be greater than or equal to S_i. Therefore, the probability of potentially selling S_i seats is the probability of having S_i or more requests, r_i, in fare class i, or $P_i[r_i \geq S_i]$. For a continuous probability distribution, this probability is:

$$P_i[r_i \geq S_i] = \int_{S_i}^{\infty} p_i(r_i)dr_i = 1 - F_i(S_i), \tag{3.1}$$

where $F_i(S_i)$ is the cumulative distribution function of having S_i or less requests in fare class i. For consistency with previous work in the industry, the notation $\overline{P}_i(S_i)$ will be used to represent the probability of potentially selling S_i seats, i.e. $P_i[r_i \geq S_i]$.

Once the probability of potentially selling the $S_i th$ seat, $\overline{P}_i(S_i)$, is known, the expected marginal revenue of the seat is simply:

$$EMR_i(S_i) = f_i \cdot \overline{P}_i(S_i), \tag{3.2}$$

where f_i is the average fare level of fare class i. Since $EMR_i(S_i)$ is directly dependent on the value $\overline{P}_i(S_i)$, the expected marginal revenue curve is a monotonically decreasing function, similar to that in Figure 3.1. The height of the curve is determined by the scaling factor f_i, while the rate at which the curve decreases is dictated by the mean and standard deviation of the demand distribution. The above notation will continue to be used throughout the review of the literature for consistency.

Under the assumption that lower valued fare classes book before higher valued fare classes, for a simple two class, single flight leg problem, Littlewood [19] suggested that total revenue on a flight leg would be maximized if additional low fare bookings were accepted based on the condition that the certain revenue obtained from each incremental low fare passenger exceeded the expected marginal revenue of saving the seat S_i for a

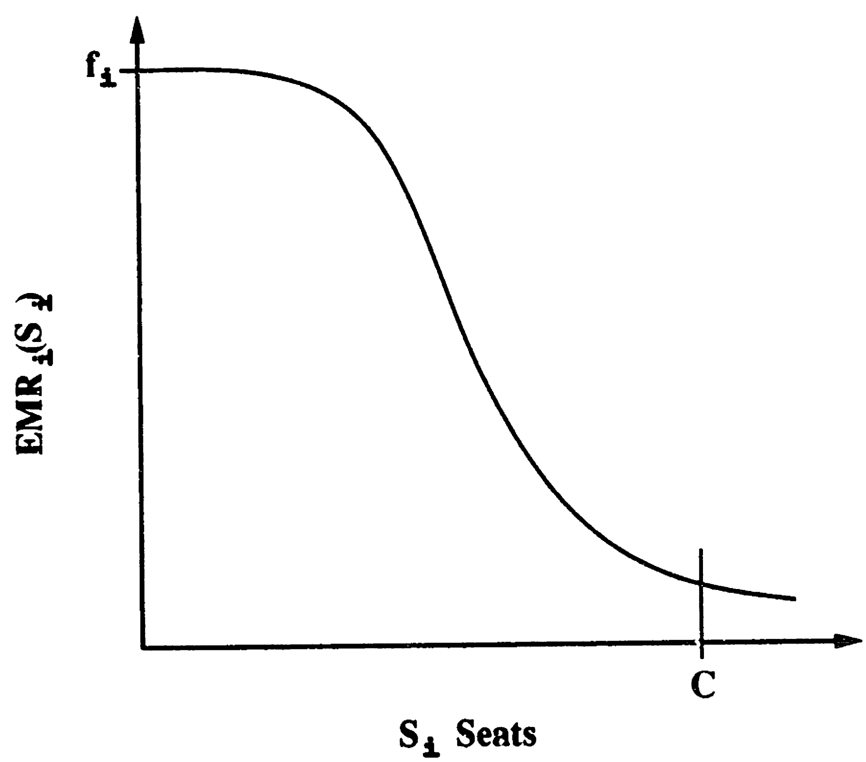


Figure 3.1: The expected marginal revenue curve of potentially selling seat S_i in fare class i.

potential high fare passenger. That is, low fare passengers paying an average fare of f_2 should be accepted if:

$$f_2 \geq EMR_1(S_1) = f_1 \cdot \overline{P}_1(S_1), \tag{3.3}$$

where f_1 is the average value of a high fare passenger. In other words, in order to maximize revenue, low fare passengers are accepted up to the point where the probability of potentially selling all remaining seats to high fare passengers is equal to the ratio of the low fare over the high fare, $f_2/f1$.

Bhatia and Parekh [20] of Trans World Airlines and Richter [21] of Lufthansa proposed variations on the model presented by Littlewood in 1973 and 1982, respectively. Bhatia and Parekh approached the problem by taking the value of the total expected revenue as a continuous function of seats and differentiating to obtain the optimal low fare seat allocation, S_2:

$$\frac{f_2}{f_1} = \int_{C-S_2}^{\infty} f_1(x_1)dx_1, \tag{3.4}$$

where C is the aircraft capacity and $f_1(x_1)$ is the high fare demand distribution.

Richter, on the other hand, looked at the changes in the total expected revenue of a flight leg as additional seats are offered to low fare passengers. He defines the value of the "differential revenue" to be the difference between the additional low fare revenue realized from offering one more seat to a low fare passenger and the high fare revenue lost by displacing a high fare passenger. By using the assumption that demands are independent, Richter is able to derive the probability of an additional low fare passenger displacing a high fare passenger as simply the product of the probability of potentially selling an additional seat at the low fare value, $\overline{P}_2(S_2)$, times the probability that this seat, if not offered at the low fare, is sold to a high fare passenger, $\overline{P}_1(C - S_2 + 1)$. Thus, the differential revenue becomes:

$$DR = f_2 \cdot \overline{P}_2(S_2) - f_1 \cdot \overline{P}_2(S_2) \cdot \overline{P}_1(C - S_2 + 1). \tag{3.5}$$

Initially each additional low fare seat earns more low fare revenue than it loses in expected high fare revenues. As additional seats are allocated to the low fare, the value of DR approaches zero, at which point the optimal low fare seat allocation has been found.

The results obtained from both Bhatia and Parekh and Richter are equivalent formulations of Littlewood's rule for accepting additional low fare requests. Through these extensions, it was shown that this simple rule for accepting low fare requests is not just plausible, but optimal for the nested, two fare class, single leg problem. It was also observed that while the low fare seat allocation, S_2, is a function of the ratio of the fares, the capacity, and the high fare demand distribution, it is not influenced by the low fare demand distribution [21]. For example, if low fare demand is unlimited, seats are protected for potential high fare demand up to the point in which an airline is indifferent to the "certain" revenue of accepting another low fare passenger and the expected revenue of a high fare passenger. At the same time, for the case where there is limited low fare demand, seats are once again protected for high fare demand up to the point in which the expected revenue of a high fare passenger is equal to the revenue of a low fare passenger. If low fare demand does not materialize for those seats not protected for high fare passengers, the expected high fare revenue of the seats will be the same, irrespective of whether the seats were open for low fare bookings or not.

In his doctoral dissertation in 1987, Belobaba [2] presented a heuristic to the nested, multiple fare class problem on a single flight leg called the Expected Marginal Seat Revenue (EMSR) method. In his approach, Belobaba determines the number of seats which should be protected for each higher class i over the lower class j, S_j^i, using expected marginal revenues. He proposes that the number of seats which should be protected for fare class i over fare class j is:

$$EMR(S_j^i) = f_i \cdot \overline{P}_i(S_j^i) = f_j. \tag{3.6}$$

The protection level for the highest fare class, Π_1, is simply S_2^1, such that:

$$EMR(S_2^1) = f_1 \cdot \overline{P}_1(S_2^1) = f_2, \tag{3.7}$$

which is similar to Equation 3.3. However, the total protection level for the two highest fare classes, Π_2, is defined as the sum of the individual protections S_3^1 and S_3^2, determined separately from the relationships:

$$EMR(S_3^1) = f_1 \cdot \overline{P}_1(S_3^1) = f_3 \tag{3.8}$$

and

$$EMR(S_3^2) = f_2 \cdot \overline{P}_2(S_3^2) = f_3. \tag{3.9}$$

In the same manner, the total protection level for the $n-1$ highest fare classes is determined by the combination of the $n-1$ individual protections:

$$\Pi_{n-1} = \sum_{i=1}^{n-1} S_n^i. \tag{3.10}$$

The booking limits, BL_i, or the number of seats available for each fare class i, are then determined by subtracting the number of seats protected for the higher fare classes, Π_{i-1}, from the capacity:

$$BL_i = C - \Pi_{i-1}. \tag{3.11}$$

By calculating and summing each S_j^i separately, the EMSR method obtains optimal booking limits between each pair of fare classes when regarded in isolation. However, the EMSR method does not explicitly consider the fact that the fare classes are sequentially nested within each other and therefore interrelated. Thus the booking limits are not *optimal* limits for multiple nested fare classes. Since the EMSR heuristic does not take into account joint probability distributions, it is able to provide practical formulas which are easily implementable while incorporating information about individual demand densities and average fares.

The extension of optimal booking limits to more than two nested fare classes on a single flight leg, assuming independent demands, was independently solved by Brumelle and McGill [22], Curry [3], and Wollmer [23]. In his optimal booking limit (OBL) approach, Curry addresses the problem assuming a continuous distribution and determines expressions for optimal booking limits in terms of a convolution integral. Based on the slope, SL_i, of the combined expected revenue function from i fare classes, i.e. the joint marginal revenue when classes 1 through i book, the nested protection level for classes 1 through i, Π_i, can be determined. The final result yields a protection level for the highest fare class of Π_1, such that:

$$f_2 = SL_1(0, \Pi_1) = f_1 \int_{\Pi_1}^{\infty} p_1(r_1) dr_1. \tag{3.12}$$

This is equivalent to Littlewood's rule for two fare classes (Equation 3.3). For $i = 2, \ldots, n-1$, the protection levels can be found by recursively solving the optimality conditions:

$$\begin{aligned} f_{i+1} &= SL_i(\Pi_{i-1}, \Pi_i) \\ &= f_i \int_{\Pi_i - \Pi_{i-1}}^{\infty} p_i(r_i) dr_i + \int_0^{\Pi_i - \Pi_{i-1}} dr_i p_i(r_i) SL_{i-1}(\Pi_{i-2}, \Pi_i - r_i), \end{aligned} \tag{3.13}$$

where $\Pi_0 = 0$. The first term of this equation represents the marginal revenue when the number of requests for class i equals or exceeds the protection $\Pi_i - \Pi_{i-1}$. The second term is based on the expected marginal revenue when the number of requests for fare class i is less than the protection $\Pi_i - \Pi_{i-1}$. Figure 3.2 shows an example of the slope of an expected revenue function. The optimal booking limit for each fare class, BL_i, is then calculated as before: $BL_i = C - \Pi_{i-1}$. The relationship between the protection levels and the optimal booking limits is shown in Figure 3.2.

In Brumelle and McGill's work, it is shown that the protection levels which maximize expected revenue can be determined through a simple set of equations based on the partial derivatives of the expected revenue function. These equations can be further simplified, resulting in a set of conditions which relate the joint probability distributions of demand to ratios between the fares. If these joint probability distributions are assumed to be continuous, it is possible to calculate the optimal protection levels, Π_i, from the following

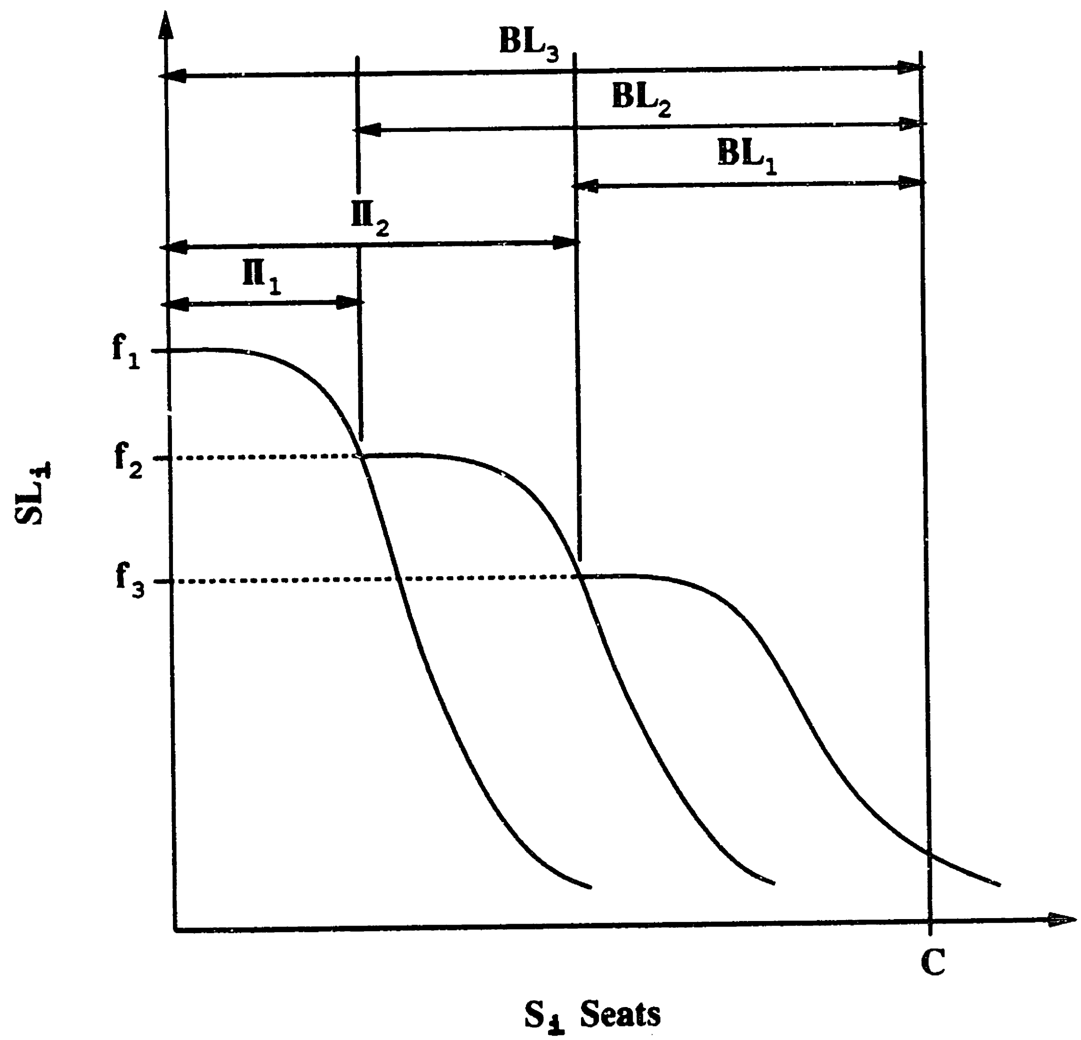


Figure 3.2: The slope, SL_i, of the expected revenue for fare classes $1, \ldots, i$ is used to determine optimal protection levels. The relationship among these protections levels, Π_i, and the optimal booking limits, BL_i, are shown [3].

probability statements:

$$
\begin{aligned}
P[r_1 > \Pi_1] &= \tfrac{f_2}{f_1}, \\
P[r_1 > \Pi_1 \cap r_1 + r_2 > \Pi_2] &= \tfrac{f_3}{f_1}, \\
&\vdots \\
P[r_1 > \Pi_1 \cap r_1 + r_2 > \Pi_2 \cap \ldots \cap r_1 + r_2 + \ldots + r_{n-1} > \Pi_{n-1}] &= \tfrac{f_n}{f_1}.
\end{aligned}
\tag{3.14}
$$

By showing a connection between the seat allocation problem and optimal stopping problems, Brumelle and McGill were able to formally prove that this fixed protection limit policy is optimal. Although the protection levels are based on a simple set of probability statements, multidimensional numerical integration is required to solve the probability statements.

Rather than a continuous distribution of passenger demand, Wollmer concentrates on a discrete demand distribution, deriving optimal booking limits in terms of a convolution sum, rather than an integral. As well as presenting the optimal policy for controlling seats, he also provides an algorithm for computing both the optimal protection levels and the optimal expected revenue. Both Wollmer and Brumelle and McGill make comparisons between their optimal booking limits and Belobaba's EMSR limits. While the booking limits produced by the EMSR method can be significantly different than the optimal limits, the revenues obtained from the EMSR heuristic have, in most cases, been within 1% of that obtained under optimal booking limits [22, 23].

Under the assumption that demand arrives in sequential groups, rather than concurrently, Robinson [24] extended the work done by Brumelle and McGill, Curry, and Wollmer by demonstrating that the optimal booking limits satisfy a simple relationship between the ratio of two fares and a weighted average of the probabilities of filling the aircraft at different points in time. Using this relationship, Robinson shows that good approximations to optimal booking limits can be calculated using Monte Carlo integration. Based on this methodology, he also relaxed the assumption that the fares are monotonically increasing over time and was able to determine optimal booking limits for groups of fares which occur in an arbitrary order, such as in the case of last-minute standby passengers.

In 1990, Brumelle et al. [25] looked at the nested two fare class example and relaxed the assumption required in previous work of statistical independent demands. By generalizing Littlewood's rule, a simple model for determining the optimal discount booking limit was obtained in which the dependency between discount and full fare demands was allowed. Given stochastically dependent demand, as in cases where scheduled events stimulate demand for all fare classes at once or discount fare passengers choose to upgrade to a higher fare when the discount fare class is sold out, the full fare demand distribution must be modified as discount demand occurs. Therefore, by replacing the probability of selling $C - S_2$, or S_1, seats to full fare passengers, $\overline{P}_1(S_1)$, with the conditional probability of selling $C - S_2$ seats to full fare passengers, given S_2 or more discount demand has been observed, the optimality condition becomes:

$$P_1[r_1 > C - S_2 \mid r_2 \geq S_2] \leq \frac{f_2}{f_1}. \tag{3.15}$$

Assuming monotonically associated demands, this expression is relatively simple to solve, resulting in lower discount booking limits for positively correlated demand than that obtained under independent demand assumptions. Similar results were obtained by Pfeifer [26] in 1989 using a different methodology and assuming that demand for both fare classes is taken from a single pool of passengers.

The focus of the research described above has been on expected marginal revenue approaches to seat inventory control which solve the nested fare class problem on a single flight leg. Revenues are maximized for each individual flight leg, but the flow of traffic and the interaction between flight legs is not take into account. Thus, alternative approaches to the seat inventory control problem have been proposed which allow for the control of different origin-destination (OD) itineraries. These approaches have evolved from the leg-based expected marginal revenue approaches to methods which incorporate mathematical programming and network flow techniques.

Ladany and Bedi [27] in 1977 and Hersh and Ladany [28] in 1978 considered the seat allocation problem for a flight with one class of service and one intermediate stop. In both efforts, a sequential decision process was developed which incorporated the time distribution at which reservations and cancellations were actually made, as well as effects due to waitlisted and standby passengers and overbooking. However, the problem addressed was one in which there were no boardings at the intermediate stop, a frequent occurrence in international airline service due to bilateral regulations between countries. Therefore, decisions between a long haul passenger versus combinations of local passengers did not exist, with preference always given to a multi-leg passenger (with a higher revenue contribution) over a single leg passenger.

In 1982, Buhr [29] of Lufthansa looked at the seat inventory control problem of a two leg flight with one fare class, considering both local and through demand on both flight legs. In the same way that the expected marginal revenue of a seat is defined for a given fare class, the expected marginal revenue of seat S_{OD} for a particular OD itinerary is:

$$EMR_{OD}(S_{OD}) = f_{OD} \cdot \overline{P}_{OD}(S_{OD}), \tag{3.16}$$

where f_{OD} is the average fare level of the OD itinerary and $\overline{P}_{OD}(S_{OD})$ is the probability of potentially selling the $S_{OD}th$ seat to a passenger traveling on the given OD itinerary. Using this relationship and assuming demand for each OD itinerary is independent, Buhr postulates that total revenue for a two leg flight A to B to C is maximized by minimizing the value of ΔEMR:

$$\Delta EMR = |\, EMR_{AC}(S_{AC}) - [EMR_{AB}(S_{AB}) + EMR_{BC}(S_{BC})]\,|, \tag{3.17}$$

subject to the capacity constraint. An iterative solution method is used to find the optimal seat allocations for S_{AC} and $S_{AB} = S_{BC}$. For multiple fare class situations, Buhr suggests a two step approach in which optimal allocations are determined for each OD itinerary first and the allocation of seats between fare classes for a given OD itinerary is then determined.

Wang [30] of Cathay Pacific Airways extended Buhr's work in 1983 by addressing the problem of optimizing seat allocations for multi-leg flights with multiple fare classes. By considering that a given seat on a multi-leg flight can be allocated to many different combinations of itineraries and fare classes, Wang developed a method which maximizes the expected marginal revenue of each seat across the multi-leg flight path. The method determines the combination of OD's and fare classes which gives the highest combined expected marginal revenue across the flight path and allocates a seat to that combination. For example, on a two leg flight A-B-C with two classes of service, Y and M, in each OD market, comparisons in the expected marginal revenue between potentially selling a given seat to a Y class AC passenger, an M class AC passenger, a Y class AB passenger and a Y class BC passenger, a Y class AB passenger and an M class BC passenger, and an M class AB passenger and a Y class BC passenger would be made. This process of comparing the expected marginal revenue of the different combinations of OD's and fare classes across a flight path continues, allocating one seat at a time to the highest expected marginal revenue combination, until all seats are allocated. Although this approach is feasible for the problems addressed by Wang, six fare classes and four flight legs, it is not very practical for problems in which the number of OD and fare class itineraries for a given seat becomes very large.

In 1982, Glover et al. [31] formulated the origin-destination and fare class (ODF) seat inventory control problem as a large network flow problem. In an effort to identify the "optimal" passenger mix, a minimum cost/maximum profit network flow model was developed using special side constraints. In the network, two sets of arcs were used, one in the forward direction to represent the different flight legs in the route schedule, and the second set flowing backwards, corresponding to different passenger itineraries and fare classes. While flow on the forward arcs was limited by the aircraft capacity on a flight leg, flow on the back arcs was limited by deterministic ODF demand estimates.

Wollmer [32] of McDonnell Douglas Corporation incorporated probabilistic demands into the large multi-leg multi-class network problem through a mathematical programming framework. In his formulation, Wollmer uses binary decision variables to represent each OD and fare class combination and seat on a flight leg, $x_{ODF,i}$. Associated with each $x_{ODF,i}$ is the value of the expected marginal revenue of potentially selling the ith seat to the OD and fare class combination, $EMR_{ODF}(i_{ODF})$. The objective is to maximize total expected network revenue by choosing the right combination of $x_{ODF,i}$ values to set equal to one, such that the sum of the $x_{ODF,i}$ values associated with an individual flight leg does not exceed the aircraft capacity of the leg.

Wollmer's probabilistic formulation produces an extremely large problem, requiring an $x_{ODF,i}$ variable for every seat available to every ODF itinerary on every flight leg in the network at the extreme. However, Wollmer [33] suggests that while the complete network formulation contains a very large number of $x_{ODF,i}$ variables, or arcs, only a small subset of the arcs need to considered at any one time. Thus, by solving a series of longest path problems on relatively small networks, a solution for the entire problem can be found. D'Sylva [34] of Boeing Aircraft proposed using a piecewise linear approximation of each ODF's expected revenue curve to reduce the size of the linear programming formulation and to make it more manageable.

In 1988, Dror et al. [35] also proposed using a network flow representation of the seat inventory control problem, while incorporating both cancellations and no-shows. A basic network model with gains/losses on certain arcs for seat allocations on a single flight with intermediate stops was first presented. This was then extended to incorporate many such flights, as well as the interrelation between alternate flight choices for passengers over a planning time horizon. For relatively simple problems, where no "switch-over", or connections, are allowed, this representation of the problem is straightforward to solve. However, in order to incorporate passengers connecting between flight legs, a more complex network flow approach with additional constraints is needed. For a large hub operation, such a model cannot be solved practically under current techniques.

The network seat inventory control approaches discussed thus far, which consider multiple flight legs and incorporate different origin-destination itineraries, all have a major shortcoming: the ODF seat allocations produced are distinct, non-nested allocations. As discussed in Chapter 2, a nested structure is a more effective method of controlling seat inventories, providing expected revenues greater than or equal to that of a distinct, partitioned structure. This is particularly true for large networks, where there are many different ODF itineraries, resulting in the average seat allocation being very small. One suggestion made by Boeing [36] to overcome this problem was to aggregate the seat allocations from a network solution into buckets by flight leg. For each leg, the different ODF's which traverse the leg are partitioned according to their fare value. The sum of the seat allocations for each ODF in a bucket is then used as the seat protection for that bucket in a nested inventory structure. The drawback of such an approach is that although a sophisticated seat assignment algorithm is used, the seats are then "clumped" together and nested, resulting in an *ad hoc* control methodology.

In 1990, Curry [3] proposed an optimization approach to allocating seats which captured both fare class nesting and multiple origin-destination itinerary control by combining an expected marginal revenue approach with a mathematical programming approach. Similar to that suggested by Buhr [29], a two step approach is used, however the steps are done jointly rather than in succession. First, based on mathematical programming, distinct itinerary allocations are obtained from the combined optimal expected revenue function of the nested fare classes of the OD. The actual nested fare class allocations are determined within each OD allocation based on Curry's approach using convolution integrals, described in Equations 3.12 and 3.13. Thus, seat allocations are nested by fare class within an origin-destination itinerary, however inventories are not shared between different OD's. While this approach provides allocations which incorporate the nesting of fare classes, this is not the same as finding allocations where inventories are shared between origins and destinations as well. Due to the partitioned nature of inventories between OD's, Curry admits that this approach is not practical for large OD networks which result in small numbers of seats allocated to each itinerary.

3.2 Current Practices in Seat Inventory Control

There is a wide range of approaches employed by airlines to control seat inventories, and the level of sophistication between these approaches varies enormously. Some carriers have allocated huge amounts of resources and money to build computer-based systems which incorporate large databases with mathematical techniques in order to rigorously and systematically determine booking limits on all flight legs in the route structure. Other carriers control seats in an *ad hoc* manner, selecting only certain flight legs and making decisions based on an analyst's judgement and expertise. At the same time, still other carriers offer only a single fare in each market, requiring no seat inventory control capabilities at all.

The simplest approach to seat inventory control is a one-time setting of booking limits, while more complicated approaches use historical data, competitors' actions, and current trends to first set initial booking limits, and then to make adjustments in these limits as bookings materialize. Most carriers which actively control seat inventories have developed or invested in some type of statistical data management and decision support system to assist in this process. These systems collect and store historical reservations data and estimate demand based on historical patterns and forecasting models. This allows an analyst or computer models to respond to changes in booking patterns as departure time approaches.

The typical seat inventory control process has evolved into one of setting initial fare class limits, monitoring actual reservations and making adjustments to the fare class limits [2]. Initial booking limits are set in a variety of ways, the simplest being the use of default booking limits across every flight leg. While such a system does not require much effort, developing a more complex system which differentiates between markets, day of the week, and time of the day can reduce the amount of intervention needed later in the reservations process. A combination of these two methods is often used, depending on the degree of competition and the level of demand for a given flight leg.

Rather than spending a lot of effort in improving the accuracy of initial booking limits, greater benefits can be obtained from developing a reservations monitoring system and adjusting booking limits. A sophisticated monitoring system can offset weaknesses in a simplified initial booking limit approach. One approach used to monitor seat inventories is simply to select and list those flight legs for which reservations are approaching the booking limit for any given fare class. The capability of most major carriers, however, is a bit more advanced, selecting flight legs on the basis of several parameters. Thus, flight legs are flagged in which actual reservations meet or approach a number of different booking limit criteria.

Besides adjusting booking limits when flight legs are flagged, and have therefore become "critical" in some way, many airlines update booking limits on a regular basis. By frequently reassessing booking limits, airlines are able to take advantage of additional information obtained during the booking process, allowing for revisions due to unexpected bookings on hand and changes in forecasted bookings to come. Dates for revisions are often determined not on the basis of an equal amount of time between periods, but by an equal proportion of bookings between periods. For example, revision dates may be scheduled for 85 days before departure, 57, 29, 22, 15, 8, 4, 3, 2, and 1 day before departure [18]. While some airlines update booking limits 15 or 20 times during the booking process, many of the major U.S. domestic carriers make revisions every night.

The most important aspect of the seat inventory control process comes down to actually determining the adjustments which should be made to the booking limits as bookings materialize. Decisions must be made whether to increase the number of seats allocated to a fare class, making it available to additional bookings, or to leave the booking limit as is and allow the fare class to close down. While these decisions are still being made by analysts at some carriers, many airlines now have systems based on mathematical techniques and algorithms which are more consistent and exhaustive.

Most of the seat inventory control systems used by airlines today control seats by flight leg. One common optimization approach is the EMSR method which was discussed above. It is used by a number of major carriers and is in several revenue management packages marketed to the airline industry by software vendors. The reason for the popularity of the EMSR approach is that it is very simple to use and implement, while being relatively easy to understand mathematically for those people responsible for implementing it.

Variations to the EMSR method are used by one vendor in order to offer a more flexible seat inventory control system to different airlines. The basic concepts of the EMSR approach are used to calculate both distinct and nested fare class limits. The amount of each fare class allocation which should be shared with lower classes, nested or partitioned, is then determined. Thus, rather than addressing either a fully nested inventory structure or a completely distinct structure, different hybrid nesting structures are easily accommodated. Such inventory structures are a combination of nested and distinct, where some fare classes are nested within each other and some are partitioned.

Curry's [3] optimal booking limit (OBL) approach for multiple fare classes on a flight leg is also used in several seat inventory control systems. Although this approach is much more complex than the EMSR approach, with the use of numerical approximations, solutions can be found easily and quickly with current computers. Another approach used in the industry, which is not necessarily correct, is simply the nesting of probabilistic, distinct fare class allocations. This method does not explicitly take into account the fact that seats will be shared between inventories when fare class allocations are determined, making it an *ad hoc* approach to determining nested limits.

As stated before, most computer reservations systems control bookings by flight leg, although limited OD control is available in many systems at the segment level. While this segment control has been done for the most part on the basis of on/off, open/close indicators, some systems do allow for actual limits at the segment level. Approaches used for determining segment limits are generally two stage approaches. One such approach is the seat allocation method proposed by Curry which determines optimal nested fare class limits within distinct OD itinerary limits. As discussed previously, the distinct OD limits are determined using mathematical programming, based on the optimal nested fare class expected revenue curve for each OD.

Other segment limit methods first determine probabilistic distinct seat allocations at the ODF level. The total segment limit for each OD is determined by aggregating together the respective fare class allocations. These fare class allocations are then used as protection levels to calculate nested fare class limits within the OD. For example, the distinct ODF allocations for an AB segment may be 10 seats for Y class, i.e. YAB, 5 seats for BAB, 5 seats for MAB and 20 for QAB. The total AB segment limit therefore is the sum of the four AB fare class allocations, or 40. Using the individual seat allocations as fare class protection levels, the segment booking limits become 40 for YAB, 30 for BAB, 25 for MAB, and 20 for QAB.

A slight variation which is also used by airlines is the calculation of the fare class protection levels using the EMSR heuristic. Thus, the probabilistic distinct ODF allocations are determined and summed to obtain the total segment limits. Then, rather that using the distinct fare class allocations for nesting purposes, EMSR protection limits are determined. For the example above, the segment limit for itinerary AB is, once again, 40. However, nested fare class limits are then determined within each OD allocation, based on demand distributions and fare ratios. The results obtained might be a booking limit of 40 for YAB and possibly 34 for BAB, etc.

Another approach, first developed and implemented by American Airlines [37] in 1986, and since adopted by other carriers, is the control of different OD itineraries and fare classes through "virtual" inventories . The "virtual nesting" approach is not an optimization technique, it is a control framework which is implemented at the flight leg level, yet allows for limited OD control. Under this method, ODF itineraries with similar values on a flight leg are assigned to a virtual, or hidden, class. Each virtual class is defined by a dollar range and then nested, high to low. Using a leg-based optimization technique, such as EMSR or OBL, availabilities are determined for the virtual classes.

The inventory structure is labeled "virtual" because the inventory classes themselves are considered to be classes during the booking limit process, but they are not formally recognized classes in that the virtual inventory classes are not offered by the airline. The standard classes of service are offer by the airline, for example Y, M, B, and Q in a four fare class system. Each of these fare classes is in turn assigned to a virtual inventory class according to the respective OD itinerary value. Booking limits are determined for each of the virtual classes on each flight leg, and the seat availabilities for the virtual classes are then related back to the standard fare classes offered in the OD market. The virtual classes exist only within the reservations control system itself; they are not apparent to the users of the computer reservations system.

The first implementations of the virtual nesting approach assigned ODF itineraries to virtual inventory buckets on the basis of total passenger itinerary ticket revenue. Thus, lower revenue itineraries in a high fare class no longer were preferred to higher revenue itineraries in a low fare class, a frequent occurrence under conventional leg-based nesting. However, under this method of grouping ODF's, priority was given to all long haul, higher revenue itineraries over short haul, lower revenue itineraries, which does not necessarily lead to maximum network revenue. To alleviate this problem, it was proposed by both American [37] and United [38] to make virtual class assignments on the basis of "net value". That is, the ticket revenue of a given ODF is adjusted to account for displacement of upline and downline passengers. Using this approach, itineraries which are connecting over two high load factor flight legs are differentiated from situations where only isolated flight legs are constrained and the rest of the flight legs are wide open.

The sophistication in seat inventory control approaches used by airlines varies significantly. While it it obvious that significant improvements can be made very readily to some systems, emphasis is being placed on better and more effective seat inventory control throughout the airline industry. At the same time, however, any changes in optimization strategy and the control mechanisms need to remain consistent with current and anticipated airline reservations systems. Keeping this in mind, the challenge in seat inventory control is a practical approach to tracking passenger itineraries from origin to destination.

Chapter 4

Approaches to Network Seat Inventory Control

Network seat inventory control is the practice of allocating seats among different passenger itineraries and fare classes. When making seat inventory control decisions, focusing on individual flight legs does not guarantee that revenues will be maximized across an entire network of flights. However, by taking into consideration the interaction between traffic flows across flight legs in a network and distinguishing between passenger origin-destination itineraries as well as fare classes competing for the same seats on a flight leg, an effort can be made to maximize the total expected revenue over a network of scheduled flights. The objective of this chapter is to propose and describe several approaches to seat inventory control which step beyond the scope of the individual flight leg in an attempt to capture additional revenue beyond that being realized from effective leg-based seat inventory control.

The discussion begins with traditional mathematical programming formulations of the network seat inventory control problem. Using the solutions from such formulations, several control strategies are detailed. Network methods which use aggregated demand estimates are then presented. Finally, several simpler leg-based OD control heuristics are introduced which incorporate information about the interaction between flight legs on a network and distinguish between different passenger itineraries. While the basic concepts behind some of the methods discussed in this chapter are based on previous work, this chapter presents several extensions and new approaches.

4.1 Network Formulation and Notation

Using traditional operations research techniques, the network seat inventory control problem can be modeled as a mathematical programming problem. In so doing, a "joint" optimization over an entire set of flight legs can be performed in order to maximize total network revenue. The simplest representation of the network problem is a deterministic linear program which can be used to find the "optimal" allocation of seats (for the given representation of the problem as it is formulated) for each individual origin-destination and fare class (ODF) combination in a network of flights.

The standard mathematical program consists of two components, an objective function and a set of constraints, or restrictions, imposed by the nature of the problem. For the deterministic formulation, the objective is to maximize total revenue over the network of flights, subject to certain constraints. There are two types of constraints, the capacity constraints on each flight leg and the demand constraints associated with each passenger itinerary and fare class combination. The demand constraints for this representation are based on deterministic estimates of a maximum demand, D_{ODF}, for each ODF itinerary under the assumption that the values are independent. The deterministic network formulation, therefore, is to maximize the number of seats allocated to each ODF, x_{ODF}, multiplied by the respective fare of the ODF, f_{ODF},

$$\text{Maximize} \sum_{ODF} f_{ODF} \cdot x_{ODF}, \tag{4.1}$$

subject to:

$$\sum_{ODF} x_{ODF} \leq C_j, \qquad \text{for all ODF's on flight leg } j \text{ and all flight legs } j,$$

$$x_{ODF} \leq D_{ODF}, \qquad \text{for all ODF's},$$

where C_j is the capacity on each flight leg j. In order to reflect the most probable anticipated demand level for a particular ODF itinerary, the deterministic maximum demand estimate is usually assumed to be the mean forecasted demand for the ODF, i.e. $D_{ODF} = \mu_{ODF}$.

The solution to this linear programming formulation is a set of seat allocations for each ODF, x_{ODF}, where seats are allocated to an ODF on the basis of total revenue contribution to the network as a whole and the interaction of traffic between connecting flight legs is taken into consideration. However, the solution is a set of *partitioned*, or *distinct*, seat allocations obtained from maximizing total revenue for a non-nested inventory structure. Seats allocated to an ODF are for use solely by that ODF, while any seats not sold to the ODF will remain empty. The solution is also based on the assumption of certainty in demand since the stochastic nature of demand is ignored. Rather than a single estimate of demand for each ODF, it can be important to take into account variations in the demand due to forecasting errors.

Another minor problem with the solution is that it is not necessarily integral, yet fractions of seats cannot be sold. To obtain an integer solution rather than a fractional number of seats for each ODF, the same formulation could be solved as an integer programming problem. On the other hand, due to the nature of the formulation, an integer solution can be obtained with significantly less computational effort by making a simple variation in the demand estimates used as constraints. Under the integrality property of network problems [1], if the upper and lower bounds on the decision variables are integers and the righthand-side values of the flow-balance constraints are integer, the solution will be integer. Thus, by requiring both the demand constraint values and the capacity constraint values to be integer, an integer solution can be obtained.

This requirement can be met by simply truncating the demand estimates. Rather than a constraint of $x_{ODF} \leq 13.15$, the constraint would be $x_{ODF} \leq 13$. This is, in essence, the same as restricting x_{ODF} to be integer. A better solution to the problem at hand may actually be obtained through rounding of the demand estimates. For instance, if the demand constraint on an ODF was 14.9 based on some estimated mean, μ_{ODF}, the maximum number of seats which could possibly be allocated to the ODF under an integer programming technique would be 14, since 15 seats is infeasible under the demand constraint of 14.9. However, by rounding 14.9 to 15, it is possible to obtain a solution of 15. For a mean demand of 14.9 passengers, the incremental probability of obtaining the 15th passenger under a Gaussian distributional assumption is actually greater than the incremental probability of the 14th passenger. By not allocating the fifteenth seat to the ODF, revenue can be lost.

As a side note, a distinction exists between rounding of the "data" in a linear program and rounding of the final solution. While rounding a linear programming solution does not always provide the optimal integer solution, solving a linear program which has integer constraints will yield an *optimal* integer solution to the problem being modeled.

Another representation of the network seat inventory control problem, which incorporates the entire probability distribution of demand, thus taking into account the uncertainty associated with the forecasted demand from each ODF, is a probabilistic mathematical programming formulation. In the probabilistic model, each fare class, OD, and seat combination which is possible over the network of flights is represented. The objective function is to maximize total network *expected* revenue. Based on the expected marginal revenue of selling a given seat i to an ODF, $EMR_{ODF}(i_{ODF})$, decisions are made whether to allocate or not allocate each individual seat to an ODF itinerary.

In this formulation, the objective of maximizing expected revenues is restricted by only the capacity constraints. Forecasted demand estimates, which are used in the deterministic formulation as constraints on the number of seats allocated to each ODF, are incorporated through the objective function itself and are probabilistic in nature. These demand distributions are used to generate the expected marginal revenue of potentially selling each seat i_{ODF} to a given ODF:

$$EMR_{ODF}(i_{ODF}) = f_{ODF} \cdot \overline{P}_{ODF}(i_{ODF}), \tag{4.2}$$

where f_{ODF} is the fare of the respective ODF and $\overline{P}_{ODF}(i_{ODF})$ is the probability of selling the ith seat to the particular ODF itinerary, or equivalently, the probability of having i or more requests for the itinerary.

The probabilistic mathematical programming formulation formally stated is:

$$\text{Maximize } \sum_{ODF} \sum_{i=1}^{C_j} EMR_{ODF}(i_{ODF}) \cdot x_{i,ODF}, \tag{4.3}$$

subject to:

$$\begin{aligned} \sum_{ODF} \sum_{i=1}^{C_j} x_{i,ODF} &\leq C_j && \text{for all ODF's on flight leg } j \text{ and all flight legs } j, \\ x_{i,ODF} &\leq 1 && \text{for all ODF's and } i = 1, 2, ..., C_j. \end{aligned}$$

Each $x_{i,ODF}$ is a binary decision variable for the allocation of the ith seat to the specified ODF. By allowing each $x_{i,ODF}$ to be less than or equal to one, rather than requiring the $x_{i,ODF}$'s to be strictly 0 or 1, the mathematical programming problem is computationally much easier to solve. This simplification is possible due to the monotonically decreasing nature of the probability of selling each additional seat to a given ODF itinerary, and therefore, the decreasing nature of the expected marginal revenue function $EMR_{ODF}(i_{ODF})$ as i_{ODF} increases. This guarantees that a full seat, rather than a fraction of a seat, will be allocated to $x_{i,ODF}$ before any portion of a seat is allocated to the $x_{i+1,ODF}$ variable. Combining this property with the integrality property of a network flow problem, an optimal 0 or 1 integer solution can be obtained through the $x_{i,ODF} \leq 1$ constraint.

In solving the probabilistic network optimization problem, the decision variables, or combination of decision variables, associated with the greatest expected marginal revenues are chosen. By summing the values of each $x_{i,ODF}$ for a given ODF, the total seat allocation for the OD and fare class itinerary can be determined. While this approach to solving the network seat inventory control problem does incorporate probabilistic demand behavior, it is at the expense of a much larger and more complex mathematical program. However, like the deterministic solution, this probabilistic solution is a set of non-nested, partitioned seat allocations.

In the probabilistic formulation, the number of decision variables tends to explode as the size of the network grows due to the number of possible OD pairs and the number of individual seats available to each OD and fare class combination. To avoid this problem, the probabilistic model can be formulated using concave revenue functions for each ODF. Rather than requiring a separate variable for every possible seat i which could potentially be available to an ODF itinerary, only one variable would be necessary for each ODF (similar to the deterministic model). Associated with each ODF variable would be a revenue function which represents the expected revenue of assigning x_{ODF} seats to the respective OD and fare class itinerary, which in essence is the cumulative expected revenue for the ODF.

This cumulative expected revenue function is increasing, yet concave, due to the monotonically decreasing nature of the expected *marginal* revenue curve for an ODF. Although nonlinear problems are intrinsically more difficult to solve than linear programming problems, this disadvantage in using a nonlinear, concave objective function over a linear objective function is compensated by the dramatic reduction in the number of decision variables. Thus, for a flight into a major hub with a capacity of 238 seats, assuming 10 different fare classes, the number of variables associated with a *single* OD itinerary would be reduced from 2380 to 10, one variable for each OD and fare class combination. With the possibility of as many as 50 different OD pairs served by the flight leg, only 500 variables would be needed to represent the ODF combinations on the flight leg, rather than a maximum of 119,000.

Computing capabilities available today have advanced to the point that efficient mathematical programming representations of the network seat inventory control optimization problem can be "solved" with reasonable speed. At the same time, the network seat inventory control problems exhibit a special structure, classifying them as network flow problems. This allows for solutions using network flow algorithms which are even more efficient in finding network solutions than mathematical and linear programming approaches.

4.2 Use of Network Solutions for Control

In the seat inventory control process, it is important to distinguish between the mathematical models used to calculate seat allocations and the control methodologies used to actually limit the number of seats made available to each fare class or ODF itinerary. The first step in the seat inventory control process is the *optimization* step. It is in this step that network flow algorithms and mathematical programming approaches are used to determine "optimal" seat allocations for each ODF in a network of flights. However, the implementation of these seat allocations as booking limits, or the actual *control* of seat inventories, is as important to maximizing revenue through the reservations inventory control process as the optimization.

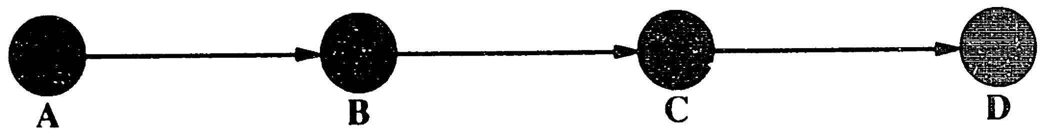


Figure 4.1: A linear three leg network with 6 OD pairs and 4 fare classes.

4.2.1 Partitioned Approaches

Based on the seat allocation solutions derived from the network optimization methods, the seat inventories can be controlled by directly applying the partitioned, or distinct, seat allocations as ODF booking limits. This is the most straightforward control approach given such ODF seat allocations and has been the basic philosophy behind many of the network and mathematical programming approaches proposed in the literature [31, 32, 33, 34].

The linear, multi-leg example in Figure 4.1 is used to illustrate the "optimal" seat allocations and control of these allocations for both the partitioned deterministic and probabilistic network approaches. The example is a three leg network, A to B to C to D, consisting of six different OD pairs, AB, AC, AD, BC, BD, and CD. It is assumed that there are four fare classes of descending revenue value, Y, M, B, and Q, offered in each OD market, making a total of 24 ODF combinations. The mean demand, standard deviation of demand, and fare for each of the ODF itineraries is provided in Table 4.1. For the itinerary ABY, the mean demand is 25.0, with a standard deviation of 7.0 and a fare of \$216.00. For the single leg itinerary AB, the fares range from \$216 in Y class to \$152 in Q class, while the fares for the two leg itinerary AC range from \$519 to \$231, and from \$582 to \$269 for the three leg itinerary AD. Each of these OD's, with their varying fare class revenue values, will be competing for the same seats on the A-B flight leg.

Applying the deterministic network formulation, Equation 4.1, to this example and

	Y	M	B	Q
AB	25.00	3.00	7.00	26.00
	7.00	2.25	5.00	11.50
	216.00	203.00	194.00	152.00
AC	2.00	1.00	4.00	14.00
	1.50	2.00	3.25	6.75
	519.00	344.00	262.00	231.00
AD	3.00	1.00	4.00	2.00
	2.50	1.50	2.50	2.50
	582.00	379.00	302.00	269.00
BC	10.00	22.00	12.00	33.00
	5.00	8.50	6.75	12.50
	440.00	315.00	223.00	197.00
BD	6.00	4.00	5.00	6.00
	4.00	2.75	2.25	4.75
	485.00	340.00	247.00	209.00
CD	19.00	56.00	7.00	6.00
	8.00	20.25	5.50	3.25
	251.00	179.00	164.00	134.00

Table 4.1: Mean demand, standard deviation of demand and fare for each ODF on the three leg A-B-C-D network in Figure 4.1.

	Y	M	B	Q
AB	25	3	7	26
AC	2	1	4	14
AD	3	1	0	0
BC	10	22	12	15
BD	6	0	0	0
CD	19	56	5	0

Table 4.2: Deterministic network seat allocations for the three leg network in Figure 4.1. These seat allocations are equivalent to the partitioned deterministic ODF booking limits for the network.

assuming a capacity of 90 seats on each flight leg, a set of partitioned seat allocations for each ODF can be determined, Table 4.2. Under the partitioned control approach, these *deterministic* seat allocations are then used as booking limits for the (probabilistic) number of requests received for each ODF. Thus, demand for ABY, a high yield, desirable ODF itinerary, are limited by the number of seats allocated to ABY, which in turn, is bounded by the deterministic demand constraint, or the mean demand value of 25. At the same time, the long haul, lower yield ODF itineraries ADB and ADQ, with mean demands of 4 and 2, respectively, are closed to bookings entirely because of the capacity constraint on flight leg B-C. Although bookings for ADB and ADQ are closed on flight leg B-C, due to the differentiation between OD itineraries as well as fare classes in the network approach, bookings are permitted for the itineraries AC and BC in B and Q class.

Using the probabilistic network formulation in Equation 4.3 and assuming that the distribution of demand is normal, or Gaussian, the ODF seat allocations, or the partitioned booking limits, for the multi-leg example are given in Table 4.3. In the probabilistic network optimization, combinations of ODF's are allocated seats based on their *expected marginal revenues* within which is incorporated the demand distribution for each ODF. Thus, the

	Y	M	B	Q
AB	30	5	10	31
AC	3	1	2	5
AD	3	0	0	0
BC	12	23	9	24
BD	6	2	0	0
CD	21	50	5	3

Table 4.3: Probabilistic network seat allocations, or the partitioned probabilistic ODF booking limits, for the three leg network in Figure 4.1.

number of seats allocated to each ODF is not strictly constrained by its mean demand. This allows ODF itineraries which contribute the most to the overall network revenue to be allocated additional seats over their mean demand, such as the ABY itinerary with a seat allocation of 30, 5 more than its mean demand. "Additional" seats have also been allocated to the other fare classes on the single leg AB itinerary. These additional seats are at the expense of seats allocated to such ODF's as the two leg, deeply discounted ACQ itinerary, allocated 14 seats in the deterministic solution but only 5 seats in the probabilistic solution. A common result in both network solutions is the seat allocations for the long haul, low yield ADB and ADQ itineraries which again are allocated no seats in the probabilistic solution.

The problem with implementing the seat allocations derived from network optimization methods directly as partitioned booking limits is that a partitioned, or distinct, inventory control structure can result in significant negative revenue impacts as will be shown in Chapter 6. Controlling seats based on a partitioned inventory structure means that once a seat has been allocated to a given ODF, the seat can only be booked by that ODF or it will remain unsold, even if there is excess demand for the seat from another ODF. Therefore, high valued requests in excess of their allocations can be turned away while seats allocated to other ODF's remain empty when demand fails to materialize.

Some of the problem caused by partitioned ODF booking limits can be overcome through frequent revisions of these limits. However, each revision requires both reforecasting of future demand and the reoptimization of the network problem. Due to the large number of ODF decision variables and the intricacies of different ODF's competing for seats on different flight legs in the network, optimization at the network level is significantly more complex than at the single flight leg level. At the same time, demand estimates are needed for each individual ODF which are both smaller and more variable than demand at the fare class and flight leg level. Thus, even with the capability of solving large network optimizations, the effectiveness of frequent revisions depends on an airline's ability to reforecast relatively small numbers and high variabilities with increasing accuracy as departure time approaches.

4.2.2 Nested Heuristics

In distinguishing between different passenger itineraries and fare classes over a network of flight legs, the number of individual ODF inventory classes, or "buckets", is much greater than at the flight leg level. At the same time, the number of seats allocated to each of these buckets is quite small, and the demand associated with each bucket is subject to large uncertainty. On the small, multi-leg network with 3 flight legs in Figure 4.1, the total number of ODF inventory buckets increases six times over a simple leg-based/fare class seat inventory control approach. At the same time, on the "bottleneck" flight leg B-C the average size of each bucket is reduced four times.

The larger the number of flight legs and fare classes, the smaller the allocations become. For a hub network with 25 flights in and 25 flights out, there can be as many as 26 different OD itineraries on a single flight leg. With 10 different fare classes and an overall average aircraft size of 158 seats [14], the number of seats per ODF is, on average, less than 1. In larger connecting hub networks, this "small numbers" problem can become even more severe. Under a partitioned control approach where seats can only be sold to a particular ODF itinerary or remain empty, the potential for negative revenue impacts can be even greater at the network level than at the flight leg level due to the large number of small allocations, each of which are associated with a high variation in demand.

At the flight leg level, it has been shown that control of bookings through a *nested* structure of inventory classes generates higher expected revenues than a partitioned, or distinct, structure under the same conditions [2]. Thus, in identifying methods to overcome the partitioned control problem at the ODF level, the focus of this research has been on alternatives that combine the seat allocations derived from network optimization models with the nesting philosophy of leg-based methods. That is, never refuse a higher valued passenger when seats originally allocated to lower valued passengers remain available.

Nesting by Fare Class

Several different methods for nesting the partitioned ODF allocations from either the deterministic or probabilistic network optimization approaches have been examined. The first nesting approach is by fare classes. This simply means that, for each fare class on a flight leg, the respective partitioned, or distinct, ODF allocations from either the deterministic or probabilistic network formulation are summed together. These "total" fare class allocations are then used as the protection levels for each fare class, i.e. the number of seats to be protected for the given fare class from lower fare classes in the hierarchy. Booking limits for each (aggregated) fare class group are determined by subtracting the number of seats protected for all higher fare classes from the capacity of the flight leg. Bookings for individual ODF's are then managed according to their respective fare class booking limit as is currently done at the flight leg level.

	Seats Allocated	Booking Limit
ACY	2	90
ADY	3	90
BCY	10	90
BDY	6	90
ACM	1	69
ADM	1	69
BCM	22	69
BDM	0	69
ACB	4	45
ADB	0	45
BCB	12	45
BDB	0	45
ACQ	14	29
ADQ	0	29
BCQ	15	29
BDQ	0	29

Table 4.4: Booking limits for each ODF on flight leg B-C obtained by nesting the ODF seat allocation solution from the deterministic network formulation by fare classes.

More precisely, the booking limit of the highest fare class, in this case Y class, is set equal to the capacity. Thus, for a given flight leg all seats on the aircraft are available to the Y class OD pairs which use that flight leg. The partitioned network seat allocations of all ODF's on the flight leg associated with Y class are summed together and subtracted from the Y class booking limit to determine the booking limit of the next highest fare class. This is repeated for each fare class on each flight leg. Using the seat allocations from the deterministic network formulation, Table 4.2, the nested by fare class booking limits for each ODF itinerary on flight leg B-C of Figure 4.1 are provided in Table 4.4. In this table, the booking limit for each Y class ODF is equal to the capacity of 90. The total number of seats allocated to the Y class ODF's, $2 + 3 + 10 + 6 = 21$, is then subtracted from the Y class booking limit to obtain the M class booking limit of 69, and so forth. Remember that although the booking limits for each ODF are based on a deterministic seat allocation solution, the actual booking process is probabilistic in nature.

In this "nested by fare class" control approach, partitioned, fully disaggregate network seat allocations are found for each ODF and, in turn, aggregated back to the fare class and flight leg level. While the interaction between flight legs and the flow of traffic across different itineraries on the network is taken into consideration in the optimization stage, the differentiation between passenger itineraries on a flight leg is lost in the control phase. This loss in control leads to inconsistent results in terms of revenues as will be shown in Chapter 6.

Nesting by Fares

The second nesting approach examined which was first proposed by Boeing [36] is based on total itinerary fare value. Under this approach, a nesting hierarchy is determined based on the actual total fare value of each ODF on a flight leg. ODF itineraries are assigned to different inventory buckets according to a fare range associated with the bucket. The ODF's within each bucket are then controlled as a group, with the individual seat allocations aggregated and used as the protection level for the inventory bucket. The larger the number of inventory buckets, the less the ODF aggregation, and the greater the ability to differentiate between OD itineraries and fare class combinations. At the extreme, each inventory bucket on a flight leg can represent a single ODF itinerary.

In this extreme case, the ODF with the highest fare value on a given flight leg is ranked first and given access to all seats on the aircraft. The ODF with the next highest fare value is ranked second, and its booking limit is determined on the basis of the first ODF's booking limit minus the number of seats allocated to the highest ranked ODF. In this manner, the rest of the ODF's are ordered and their booking limits determined. An example of this nesting approach is shown in Table 4.5 where the deterministic network solution of the

	Fare	Seats Allocated	Booking Limit
ADY	$582	3	90
ACY	$519	2	87
BDY	$485	6	85
BCY	$440	10	79
ADM	$379	1	69
ACM	$344	1	68
BDM	$340	0	67
BCM	$315	22	67
ADB	$302	0	45
ADQ	$269	0	45
ACB	$262	4	45
BDB	$247	0	41
ACQ	$231	14	41
BCB	$223	12	27
BDQ	$209	0	15
BCQ	$197	15	15

Table 4.5: The ODF seat allocations of the deterministic network formulation nested by fares for flight leg B-C.

multi-leg A-B-C-D network is nested by fares for flight leg B-C.

By nesting ODF network allocations strictly on fare values, an aspect of "greediness" is introduced where the highest revenue itineraries always receive priority in terms of seat availability. Long haul itineraries will rank above other itineraries within a fare class, with local itineraries nested at the bottom. This can be seen in Table 4.5 where, within each individual fare class, the long haul AD itinerary is nested above the other three itineraries and the local BC itinerary is nested at the bottom of the OD group, i.e. ADY is nested above ACY and BDY, with BCY the lowest ranked ODF associated with Y class. Using this nesting approach, it is possible for multi-leg, low yield itineraries which are not allocated seats in the network optimization to have access to a significant number of seats on the flight leg. Due to the control methodology, it is possible for a multi-leg, low yield itinerary to be nested over a short haul, higher yield, yet lower fare, ODF. This can be observed several times in Table 4.5. One such example is the BDM itinerary which is allocated no seats in the partitioned network solution, yet has access to 67 seats when nested by its fare value.

Nesting by Shadow Prices

The third nesting method examined is based on *shadow prices*. This approach was first introduced by Williamson [39] in 1988, and is an attempt to better reflect the value to the network of each ODF. A shadow price is the incremental network revenue that would be realized if one more seat were made available to a given ODF, all else being held constant. The rationale behind this nesting approach is that ODF itineraries with higher shadow prices have a higher potential value to the network and therefore, should be ranked higher in the nesting hierarchy.

In the process of determining network seat allocations, shadow prices associated with each ODF in the network can be found. For the deterministic formulation, an ODF's shadow price is simply the shadow price for the demand constraint of the respective ODF [1]. ODF's are then ranked in descending order according to the highest shadow price, or marginal value to the network. Once the ODF rankings are determined, booking limits are calculated in the same manner as if nesting was done by fare values. Similar to the nested by fare approach, each ODF can make up an inventory bucket of its own, or inventory buckets can be defined by a range of shadow price values with each ODF being assigned to the appropriate bucket.

Table 4.6 provides the shadow prices from the deterministic formulation for each ODF on flight leg B-C as well as the "nested by shadow price" booking limits for the ODF's. Once again, a shadow price is the additional revenue which could be realized if one more seat was made available to a given ODF. Thus, in the deterministic formulation of the network seat inventory control problem, if the demand constraint on the ACY itinerary was 3 rather than 2, due to the excess capacity on the A-B flight leg, an additional \$322 would be realized simply by allocating an one more seat to the ACY itinerary at the expense of the BCQ itinerary, i.e. $+519 - 197 = 322$. On the other hand, if the demand constraint on the BDQ itinerary was increased by one, from 6 to 7, no additional revenue would be realized. No seats are allocated to BDQ in the current deterministic solution, and by simply increasing the demand constraint by one, zero seats would still be allocated to the BDQ itinerary.

Based on the nested by shadow price approach, the ACY itinerary, with a fare of \$519, is now nested at the top of the hierarchy above the long haul ADY itinerary worth \$582. At the same time, both ACB and ACQ are ranked above the other B class, as well as Q class, ODF combinations. (In the nested by fare approach, it was the long haul ADB and ADQ itineraries which ranked above all other B class and Q class ODF's.) The ACB and ACQ itineraries even rank over two M class itineraries, ADM and BDM, as well. Finally, unlike the nested by fare approach, the ODF itineraries which have no seats allocated to them in the network optimization solution are ranked at the bottom of the nesting hierarchy.

In Table 4.6, a distinction is made between the ODF's with a shadow price of zero and a positive number of allocated seats, eg. BCQ, and ODF's with a shadow price of zero, yet *no* allocated seats. This distinction is made on the basis of the "reduced costs" associated with each ODF. A reduced cost is similar to a shadow price, but it is associated with the (implicit) nonnegativity constraints. Reduced costs can be determined, along with shadow prices and seat allocations, when solving a network problem. The interpretation of a reduced cost is that it is the cost to the network of increasing the nonnegativity constraint on a variable [1], i.e. requiring that the lower bound of a given ODF be 1, rather than 0.

All ODF itineraries with seats allocated to them would not be affected by such a requirement and therefore, have a reduced cost of zero, as shown in Table 4.6. Forcing those ODF's with a zero shadow price which are *not* part of the network solution to be

	Fare	Shadow Price	Reduced Cost	Seats Allocated	Booking Limit
ACY	$519	322	0	2	90
BCY	$440	243	0	10	88
ADY	$582	221	0	3	78
ACM	$344	147	0	1	75
BDY	$485	124	0	6	74
BCM	$315	118	0	22	68
ACB	$262	65	0	4	46
ACQ	$231	34	0	14	42
BCB	$223	26	0	12	28
ADM	$379	18	0	1	16
BCQ	$197	0	0	15	15
BDM	$340	0	-21	0	0
ADB	$302	0	-59	0	0
ADQ	$269	0	-92	0	0
BDB	$247	0	-114	0	0
BDQ	$209	0	-152	0	0

Table 4.6: ODF booking limits on flight leg B-C determined by nesting the deterministic network seat allocations by shadow prices.

allocated at least 1 seat would have a negative impact on the *optimal* network revenue. For example, requiring one seat to be allocated to the BDM itinerary at $340 would result in one less seat available to the BCQ itinerary at $197 on the B-C flight leg *and* one less seat available to the CDB itinerary at $164 on the C-D flight leg, totaling a *loss* in revenues of $21. By using the reduced costs, rather than grouping all ODF's with a zero shadow price into one inventory bucket, a distinction can be made between the different ODF's, allowing **each** ODF to be nested separately, as shown in Table 4.6.

Since there is no demand constraint associated with each ODF in the probabilistic formulation, determining probabilistic ODF shadow prices is not as straightforward as in the deterministic case. One definition for the ODF shadow price is to use the information associated with the "next seat" of an ODF, i.e. the seat following the last seat allocated to an ODF. If n seats are allocated to an ODF, i.e. $x_{1,ODF}, x_{2,ODF}, \dots x_{n,ODF} = 1$, the shadow price or, in this case, the reduced cost associated with the $x_{n+1,ODF}$ variable can be used for nesting purposes. However, this reduced cost value may not always represent the change in revenue associated with "forcing" an additional seat to be allocated to a given ODF at the expense of another ODF or combination of ODF's. The value could actually represent the change in revenue based on displacing the previous, or last seat allocated to the same ODF. In other words, rather than allocating an additional seat to the ODF such that $x_{1,ODF}, x_{2,ODF}, \ldots x_{n+1,ODF} = 1$, the value of $x_{n+1,ODF}$ may be set equal to one while the value of $x_{n,ODF}$ is reduced to zero.

The marginal value to the network of the "next seat" is based on the expected marginal revenue of obtaining an additional request for an ODF, given a forecasted demand distribution. However, the primary purpose behind a nested inventory structure is to allow seats which have been allocated to certain ODF's to be sold to higher yield, more desirable ODF's when unexpected demand materializes. Thus, it would seem appropriate that the incremental revenue of an additional request *above* that which was forecasted for an ODF be reflected in the probabilistic shadow price values used for nesting ODF's.

Rather than using the marginal value of the "next seat", the definition for probabilistic ODF shadow prices proposed and used throughout the remaining of this dissertation is one which is more consistent with the shadow prices obtained in the deterministic formulation. In the deterministic formulation, a shadow price for a given ODF is the incremental revenue to the network if one additional seat is made available to the ODF. This incremental revenue is essentially determined by allowing a unit increase in the demand constraint. However, each ODF variable is constrained by its deterministic demand estimate which is usually assumed to be the mean forecasted demand. Therefore, an increase in the demand constraint is essentially an increase in the mean demand for the ODF. Thus, throughout this dissertation, the shadow prices for the probabilistic model are based on the incremental revenue obtained from individually increasing the mean demand estimate for each ODF by one.

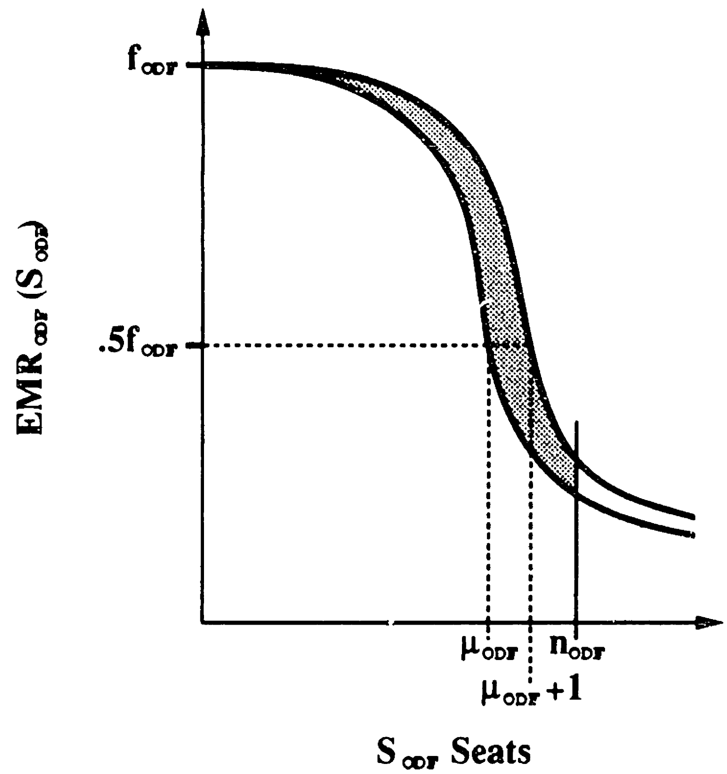


Figure 4.2: For the case where the shift in the expected marginal revenue curve does not alter the optimal seat allocation for the ODF of n_{ODF}, the shaded region between the two curves represents the incremental revenue obtained from increasing the mean demand for an ODF by one. This incremental revenue is used as an estimate for the probabilistic shadow price.

Increasing the mean demand estimate for an ODF will shift the expected marginal revenue curve of the ODF to the right. This shift in the expected marginal revenue curve will not necessarily affect the optimal ODF seat allocation, n_{ODF}. For the case in which the optimal ODF seat allocation, n_{ODF}, does not change, the incremental revenue is simply the difference in the revenue under the original expected marginal revenue curve and the shifted curve from 0 to n_{ODF}. Figure 4.2 shows an example of a shifted expected marginal revenue curve. In this example, the shift in the expected marginal revenue curve does not affect the optimal seat allocation. Therefore, the incremental revenue obtained from increasing the mean forecasted demand of the ODF by one, holding all else constant, is simply the shaded region between the two curves.

For some ODF's, it is possible that the incremental revenue will be generated by a reallocation of seats as well. Because of the shift in the expected marginal revenue curve of an ODF, an additional seat may be allocated to the ODF from another ODF, or combination of ODF's. This would be particularly true for a high yield ODF with a very small standard deviation where the expected marginal revenue curve falls off very quickly, such as in Figure 4.3. Assuming that this is an EMR curve for a fairly desirable ODF with respect to the network as a whole, given an optimal seat allocation for the ODF of $n_{ODF} = \mu_{ODF}$ seats originally, increasing the mean for this ODF by one and resolving the probabilistic network optimization problem would provide a solution of $n'_{ODF} = \mu_{ODF} + 1$ seats for the ODF. The difference in total revenue between the original solution and the new solution is the incremental revenue value obtained from increasing the mean demand for the ODF by one seat. Thus, rather than an incremental revenue equal to the lightly shaded region in Figure 4.3, the incremental revenue would be equal to the entire shaded region (light and dark) between the two curves.

The probabilistic shadow price for each ODF is defined as the change in revenue between the expected revenue of the original problem and the expected revenue obtained from increasing the mean demand of an ODF by one. Each shadow price is determined individually, and using these probabilistic shadow prices, the seat allocations from the original probabilistic solution can be nested in the same manner as the deterministic nested by shadow price approach. Table 4.7 gives an example of the probabilistic network seat allocations nested by shadow prices for the flight leg B-C of Figure 4.1.

4.2.3 Network Bid Prices

Using the same network optimizations, the seat inventory control problem can be approached in an entirely different manner. Instead of using the shadow prices associated with each ODF variable, another approach developed at MIT as part of this research project is

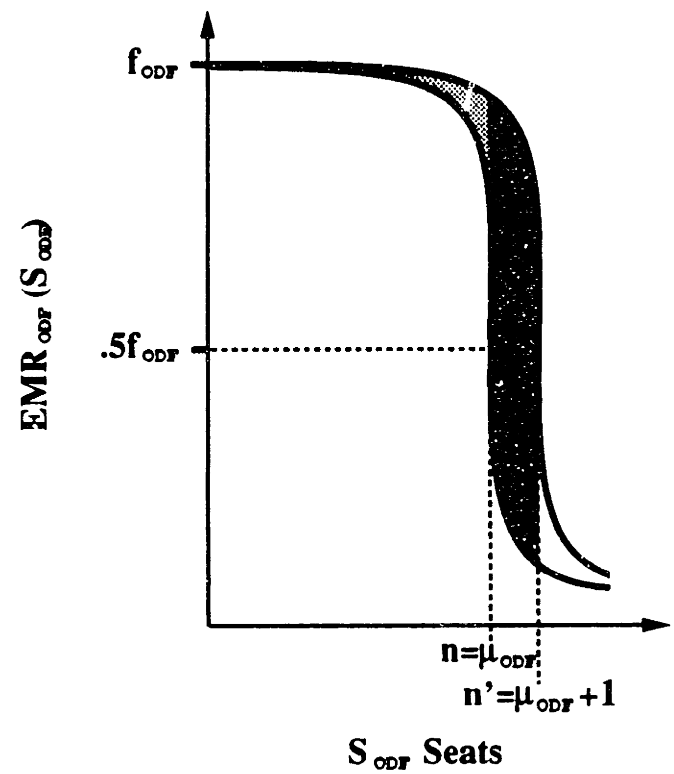


Figure 4.3: An example of an EMR curve for an ODF with a very small standard deviation. Assuming a high yield, desirable ODF, the incremental revenue obtained from increasing the forecasted mean demand not only includes the revenue from 0 seats to μ_{ODF} seats between the original expected marginal revenue curve and the shifted expected marginal revenue curve (region shaded lightly), but it may also include the revenue obtained from allocating an additional seat to the ODF (darker shaded region).

	Fare	Shadow Price	Seats Allocated	Booking Limit
ACY	$519	296.60	3	90
BCY	$440	278.62	12	87
ADY	$582	226.51	3	75
BDY	$485	197.30	6	72
BCM	$315	158.66	23	66
ACM	$344	78.83	1	43
BCB	$223	61.70	9	42
BDM	$340	59.99	2	33
BCQ	$197	41.10	24	31
ACB	$262	35.18	2	7
ADM	$379	20.83	0	5
ACQ	$231	17.37	5	5
ADB	$302	0.00	0	0
ADQ	$269	0.00	0	0
BDB	$247	0.00	0	0
BDQ	$209	0.00	0	0

Table 4.7: ODF booking limits for flight leg B-C based on nesting the probabilistic network seat allocation solution by shadow prices.

an approach based on "bid prices" [40]. A "bid price" is associated with each flight leg and is the shadow price for the capacity constraint, rather than the individual ODF demand constraints. It is the marginal value to the network of the last seat available on a given flight leg. Bid prices are obtained from the same network formulations as the network seat allocations and ODF shadow prices. Since the probabilistic formulation has a defined set of capacity constraints, the problem associated with individual ODF probabilistic shadow prices does not arise.

The idea behind the bid price control approach is to establish a "cutoff" value for each flight leg which can be used to make decisions whether to accept or reject different ODF requests. The difference in the methodology of the bid price approach, when compared to other conventional seat inventory control approaches, is that ODF inventories are either open to bookings or closed, there are no explicit booking limits for different ODF's. For a single leg itinerary, a fare class is open for bookings if the corresponding fare is greater

Bid Price	
A-B:	0
B-C:	197
C-D:	164

Table 4.8: Bid prices for each flight leg of the three leg A-B-C-D network based on the deterministic network formulation of the problem.

than the bid price, or shadow price, for the leg. For a multi-leg itinerary, the total fare must be greater than the sum of the bid prices from the respective flight legs it traverses.

Using the deterministic network formulation, the bid prices for the three leg network in Figure 4.1 are given in Table 4.8. Based on a simple comparison with the fares of each ODF, it can easily be determined which ODF itineraries are open to bookings and which are not. For the local BC itinerary, the bid price, or cutoff value, is simply \$197. Using the fare information from Table 4.1, it can be seen that all four fare classes, with fares ranging from \$440 to \$197, are open to bookings. However, for the local itinerary CD, with a bid price of \$164 on flight leg C-D, the lowest fare class, Q class, at \$134 is closed to bookings. For a multi-leg itinerary, such as the three leg itinerary AD, the cutoff value would be $0 + 197 + 164$, or \$361. Thus, while Y class and M class, with fares of \$582 and \$379, respectively, are open, B class at a fare of \$302 and Q class at a fare of \$269 are closed.

One advantage of the bid price approach is that it is a very simple method of managing seat inventories. Hence, it would be very easy to implement in a reservations system when compared to other OD and fare class approaches. Under a network methodology which uses the traditional control approach of seat allocations, seat protections, and booking limits for each ODF inventory bucket, extensive modifications to the inventory control structure would be required. Implementation of the bid price approach would simply require a comparison between the bid prices for different flight legs and the ODF fares from the existing fare database. Furthermore, the amount of computer storage space necessary to control individual ODF's under the bid price control approach is not as expansive as that needed for a traditional network seat inventory control approach. Rather than having individual booking limits for each ODF on a flight leg, all that is needed is the minimum acceptable bid price for the flight leg.

Although seats are still allocated between ODF's based on a partitioned assumption with no *explicit* recognition of nesting in the optimization, the control mechanism of the bid price approach allows for "*implicit*" nesting of ODF's. Any ODF which is open to bookings has access to any and all unsold seats on a flight leg. Due to the control mechanism, another advantage of the bid price approach is that the integer seat allocation problem is not an issue. Since seat allocations are not needed for controlling different ODF's, it does not matter whether the network solution is integer or not.

The disadvantage of the bid price approach, however, is its open/closed control philosophy. If a given ODF passes the bid price criteria, that ODF remains open to bookings until the bid prices are revised. Therefore, the ODF has access to the entire capacity of the aircraft since there is no limit to the number of bookings which are accepted for the ODF. If, in the "optimal" solution, one seat was allocated to the ODF, the bid price approach would specify the ODF as being open, allowing any and all demand for the ODF to be accepted until the bid prices were revised. Thus, in order for the network bid price approach to be an effective seat inventory control approach, frequent revisions would be necessary, requiring both reoptimization and reforecasting. For a truly *optimal* system, revisions would be necessary on a real-time basis. While frequent reoptimization on a real-time basis is conceivable, producing instantaneous, real-time forecasts at the ODF level is infeasible.

4.3 Aggregated Demand Network Methods

One of the current obstacles to a full network seat inventory control approach is the "small numbers" problem associated with forecasting demand at the ODF level. For the segment control problem on a multi-leg network with three, four, or even five flight legs, the number of OD and fare class combinations does not become unreasonable. However, for origin-destination seat inventory control on a large hub-and-spoke network, the *average* demand for a given OD and fare class combination on a any given flight leg is typically less than one. At the same time, the uncertainty associated with this ODF demand is very large.

One approach to minimizing the small numbers problem involves grouping ODF itineraries together and then forecasting aggregated demand as is currently done at the fare class and flight leg level. Due to the network aspect of the inventory control problem, grouping ODF's at a global level while preserving the differences in the level of attractiveness each ODF has to the network can be difficult. However, from the perspective of an individual flight leg, combinations of ODF's that have the same level of attractiveness, in terms of revenue, can be grouped together without lossing information about the contribution each ODF has to the network as a whole. Using these aggregated ODF "inventory groups", the object is to optimally allocate seats at the network level, rather than the flight leg level.

The first step in this process is formulating the network optimization. An *aggregated* deterministic model can be formulated very similarly to the full deterministic network formulation in Equation 4.1. Seat inventories can then be controlled using many of the approaches discussed above. The objective, as before, is to maximize the sum of $f_{ODF} \times x_{ODF}$ for each ODF, subject to the capacity constraints and the demand constraints. However, instead of having individual demand constraints for each ODF, a demand constraint is used for each aggregated inventory group on each flight leg.

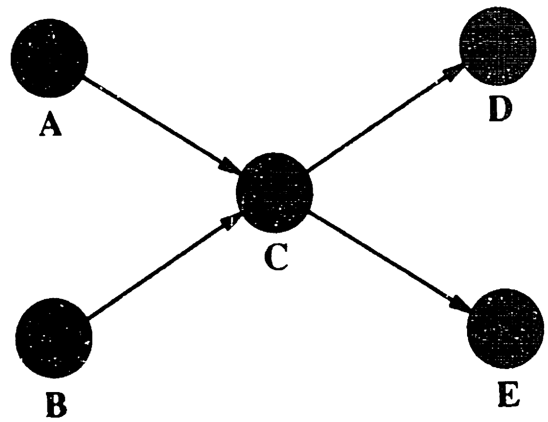


Figure 4.4: Small hub network with two flights in and two flights out.

	Mean Demand	Fare
ACY	40	$100
ADY	30	$150
AEY	40	$150
BCY	30	$100
BDY	10	$150
BEY	20	$150
CDY	20	$100
CEY	50	$100

Table 4.9: Mean demand and fare data for each of the 8 ODF itineraries on the small hub network in Figure 4.4.

Although an aggregated approach is not needed for small networks, a simple hub example with two flights in and two flights out, as in Figure 4.4, is used to illustrate the aggregated deterministic network methodology. Assuming only one fare class, there are eight possible ODF itineraries in this network, four local OD itineraries and four connecting itineraries. The mean demands and fares for each OD itinerary are given in Table 4.9. On each flight leg, ODF's are aggregated into groups according to their revenue value to the airline. In this simple example where the ODF fares on a flight leg are either $100 or $150, two inventory groups are defined on each flight leg. For flight leg A-C, the first inventory group, valued at $100, consists of the local traffic ACY and the second inventory group is comprised of the connecting traffic ADY and AEY, each having a value of $150. Similar aggregations of local and through traffic are made on the other three flight legs.

As in the original deterministic formulation, the aggregated deterministic objective function is:

$$\text{Maximize} \quad 100x_{ACY} + 150x_{ADY} + 150x_{ABY} + 100x_{BCY} + 150x_{BDY} + 150x_{BBY} + 100x_{CDY} + 100x_{CBY}. \tag{4.4}$$

This is subject to a capacity constraint on each of the four flight legs. Assuming a uniform capacity over the network of 100, these constraints are:

$$\begin{aligned} x_{ACY} + x_{ADY} + x_{ABY} &\leq 100, \\ x_{BCY} + x_{BDY} + x_{BBY} &\leq 100, \\ x_{ADY} + x_{BDY} + x_{CDY} &\leq 100, \\ x_{ABY} + x_{BBY} + x_{CBY} &\leq 100. \end{aligned} \tag{4.5}$$

The difference in the aggregated deterministic formulation is the demand constraints. For forecasting purposes, demands are determined for each aggregated inventory group of ODF's on each flight leg. Therefore, on flight leg A-C, demand will be forecasted for the local traffic ACY while demand for the connecting traffic ADY and AEY will be forecasted together. Thus, the mean forecasted demand data available for the optimization would be 40 for ACY and 70 for the ADY and AEY itineraries combined. The aggregated mean demand estimates for the entire network are shown in Table 4.10. Based on the fact that the mean value of the sum of two variables is always equal to the sum of their mean values [41],

$$\mu_{(A+B)} = \mu_A + \mu_A, \tag{4.6}$$

	Mean Demand
ACY	40
ADY, AEY	70
BCY	30
BDY, BEY	30
CDY	20
ADY, BDY	40
CEY	50
AEY, BEY	60

Table 4.10: Aggregated mean demand forecasts for the small, two flights in and two flights out, network in Figure 4.4.

the aggregated demand constraints can be constructed as follows:

$$
\begin{aligned}
x_{AC} &\leq 40, \\
x_{ADY} + x_{AEY} &\leq 70, \\
x_{BCY} &\leq 30, \\
x_{BDY} + x_{BEY} &\leq 30, \\
x_{CDY} &\leq 20, \\
x_{ADY} + x_{BDY} &\leq 40, \\
x_{CEY} &\leq 50, \\
x_{AEY} + x_{BEY} &\leq 60.
\end{aligned}
\tag{4.7}
$$

As in the original deterministic formulation, the solution obtained from this linear program is a set of partitioned seat allocations for each ODF. However, each ODF should *not* be controlled individually based on these allocations. Due to the aggregated demand constraints, it is possible to obtain *individual* ODF allocations which are "non-optimal". For example, one feasible, non-unique solution from the above aggregated deterministic formulation for the four connecting itineraries is:

$$
\begin{aligned}
x_{ADY} &= 10, \\
x_{ABY} &= 50, \\
x_{BDY} &= 30, \\
x_{BBY} &= 0.
\end{aligned}
$$

Using the *disaggregated* mean demand information from Table 4.9 (which would normally not be available), only 40 requests are "expected" for AEY, yet the AEY seat allocation is 50. Consequently, in the aggregated solution 10 extra seats which could have been allocated to other ODF's on the network are protected for AEY on the heavily demanded flight legs A-C and C-E. At the same time, ADY, with a demand of 30, is allocated only 10 seats under the aggregated network solution and BEY, with at demand of 20, is allocated no seats.

By aggregating the seat allocations and managing them in inventory groups, the above problem of "incorrect" allocations can be eliminated. As mentioned in the full network methods nested by fares or nested by shadow prices, each ODF can be assigned to an inventory bucket based on a "value" range associated with the bucket. For the aggregated optimization method, the inventory buckets or groups have already been defined. The same ODF's which are forecasted as a group on each flight leg can be managed as a group. Rather than first solving the network optimization and then aggregating the ODF solutions, ODF's are first aggregated and then the network optimization is performed. For the example and solution given above, the itineraries ADY and AEY on flight leg A-C are forecasted together and therefore, should be managed as a common inventory. The seat allocation for the inventory group is 50 seats from AEY and 10 seats from ADY, or a total of 60. The total estimated demand for the inventory is 70. On flight leg C-E, the combined seat allocation for AEY and BEY is $50 + 0$, or 50, while the estimated mean demand is 60.

In the simple 8 OD's and one fare class example, each flight leg had the same number of inventory groups in which the ODF's on the flight leg were assigned. At the same time, the value, or value range, of the inventory groups was the same on each flight leg ($100 and $150). This does not have to be the case. The number of inventory groups and, more importantly, the value range of each inventory group should be tailored to the individual flight leg in order to achieve maximum control between different ODF itineraries on the flight leg while allowing for reasonable forecasting accuracies. Groups of ODF's on each flight leg also do not have to be divided between local and through traffic. Each aggregated inventory group can be a mixture of short haul and long haul itineraries, as well as full fare and discounted fare ODF itineraries.

A slightly larger example based on the same hub network in Figure 4.4 is used to better demonstrate ODF grouping. For this example, two fare classes are assumed, Y class and B class. As before, the objective function and capacity constraints are the same as in a fully disaggregated network formulation, yet the demand constraints are based on aggregating different ODF's into groups on each flight leg. Using the demand and fare information in Table 4.11, ODF's are assigned to different inventory groups on each flight leg based on their total itinerary fare value, and aggregated demand constraints are then determined. One possible ODF aggregation structure, with the defined revenue ranges for each inventory group and the aggregated demand constraints, is given in Table 4.12.

Actual control of the inventory groups from the aggregated deterministic formulation can be achieved using several of the approaches described above. One of the nicer aspects of the aggregated model is its natural extension to nesting by shadow prices. Associated with each ODF group on each flight leg is a demand constraint. Therefore, associated with each ODF group is also a shadow price. Using the total seat allocation for each ODF group as the seat protection value for the inventory group, the ODF inventory groups on each flight leg can be nested according to their shadow prices. Note also that seats on each flight

	Mean Demand	Fare
ACY	20	$215
ACB	30	$105
ADY	15	$420
ADB	10	$185
AEY	20	$580
AEB	15	$385
BCY	10	$335
BCB	15	$255
BDY	5	$490
BDB	20	$360
BEY	10	$695
BEB	25	$515
CDY	20	$355
CDB	25	$145
CEY	20	$520
CEB	15	$345

Table 4.11: Mean demands and fares for the ODF itineraries on the small hub network in Figure 4.4 with two fare classes.

leg can also be controlled using the network bid price methodology using an aggregated deterministic formulation.

The transition from the regular probabilistic formulation to an aggregated probabilistic formulation is not as straightforward. The difficulty with the probabilistic formulation is that the demand information consists of the entire probability distribution with the solution being determined by the probability of selling incremental seats. In the deterministic method, it was possible to forecast demand at an aggregated level, yet represent each ODF individually based on the identity between the mean value of the sum of two variables and the sum of their means. However, in considering the full probability distribution of demand,

$$\overline{P}_{(A+B)}(i_{(A+B)}) \neq \overline{P}_{(A)}(i_{(A)}) + \overline{P}_{(B)}(i_{(B)}). \tag{4.8}$$

Therefore, an extension from the original probabilistic network model to an aggregated probabilistic model is more complex than that of the deterministic case. Further discussion

	Revenue Range	Demand Constraint
Flight Leg A-C:	\$500-\$700	$x_{ABY} \leq 20$
	\$300-\$499	$x_{ADY} + x_{ABB} \leq 30$
	\$150-\$299	$x_{ACY} + x_{ADB} \leq 30$
	\$100-\$149	$x_{ACB} \leq 30$
Flight Leg B-C:	\$550-\$750	$x_{BBY} \leq 10$
	\$400-\$549	$x_{BDY} + x_{BBB} \leq 30$
	\$300-\$399	$x_{BCY} + x_{BDB} \leq 30$
	\$200-\$299	$x_{BCB} \leq 15$
Flight Leg C-D:	\$475-\$600	$x_{BDY} \leq 5$
	\$376-\$474	$x_{ADY} \leq 15$
	\$275-\$375	$x_{CDY} + x_{BDB} \leq 40$
	\$175-\$274	$x_{ADB} \leq 10$
	\$100-\$174	$x_{CDB} \leq 25$
Flight Leg C-E:	\$650-\$750	$x_{BBY} \leq 10$
	\$551-\$649	$x_{ABY} \leq 20$
	\$450-\$550	$x_{BBB} + x_{CBY} \leq 45$
	\$375-\$449	$x_{ABB} \leq 15$
	\$300-\$374	$x_{CBB} \leq 15$

Table 4.12: ODF groups and their aggregated demand constraints for the four flight legs of the small hub example in Figure 4.4.

and results of the aggregated models will be presented in Chapter 6.

4.4 Leg-Based Methodologies for OD Control

In theory, revenues cannot be truly *maximized* without optimizing over an entire network of connecting flight legs and controlling individual itinerary and fare class combinations. However, it is important to recognize that nesting of "optimal" network ODF seat allocations for control purposes results in a theoretically *sub-optimal* solution. At the same time, full network approaches to seat inventory control present several practical problems. While the interaction of passenger flows across connecting flight legs is taken into account, the formulations necessary for network optimization become very large, particularly when the probabilistic nature of demand is incorporated.

At the same time, network approaches, which differentiate between different passenger itineraries as well as fare classes, require estimates of demand for hundreds, sometimes thousands of ODF itineraries, many of which involve a fraction of a passenger on each flight leg. Methods to forecast such demand accurately have, to date, not been addressed in research. Furthermore, in order to seriously address the ODF forecasting problem, historical data at the ODF level is needed. While a few airlines have begun work on an ODF data base, the current practice among most airlines is to collect and store data at the fare class and flight leg level.

An additional implementation issue is that most computer reservations systems throughout the airline industry do not have the capabilities to control reservations at the OD and fare class level. The few airlines which have such capabilities in their own computer reservations systems are not able to communicate itinerary controls to other leg-based computer reservations systems. Although such limitations currently pose significant problems to controlling seat inventories at the network level, many of these obstacles can or, in the not too distant future, will be overcome. However, airlines are looking for benefits which can be had today, and thus, the current interest throughout the industry is in incorporating network "information" into the simpler leg-based control structure. Therefore, using the concepts and ideas from network optimization, several leg-based heuristics have been developed in this research which take into account information about passenger demand and traffic flows while optimization and control remains at the flight leg level.

4.4.1 Leg-Based Bid Price

Similar to the network bid prices, information at the flight leg level can be used to determine the marginal value of the last seat on a given flight leg. Under current leg-based seat inventory control approaches, such as the Expected Marginal Seat Revenue heuristic [2] or the Optimal Booking Limit approach [3], fare class booking limits are determined for each individual flight leg on the basis of the forecasted demand and the estimated revenue value of each fare class. Using the same mathematical techniques, the expected marginal revenue of each incremental seat offered on a flight leg can be calculated, producing an expected marginal revenue curve for the flight leg, such as that shown in Figure 4.5.

Once the expected marginal revenue curve for a flight leg has been estimated, it is simple to find the expected marginal value of the *last* available seat on the flight leg, $EMR(C)$. For the expected marginal revenue curve given in Figure 4.5, if the capacity of the flight leg is 90, the $EMR(C)$ value would be approximately \$122. For a capacity of 100, the $EMR(C)$ value would be \$94.44. This value of the expected marginal revenue curve at capacity provides a "cut-off" value for the flight leg, representing the minimum fare at which a booking on the flight leg should be accepted. It combines the information about the quantity of demand on the flight leg with the revenue value of this demand, producing an "opportunity cost" associated with using the flight leg.

Based on the fundamental concept behind the network bid price approach, the $EMR(C)$

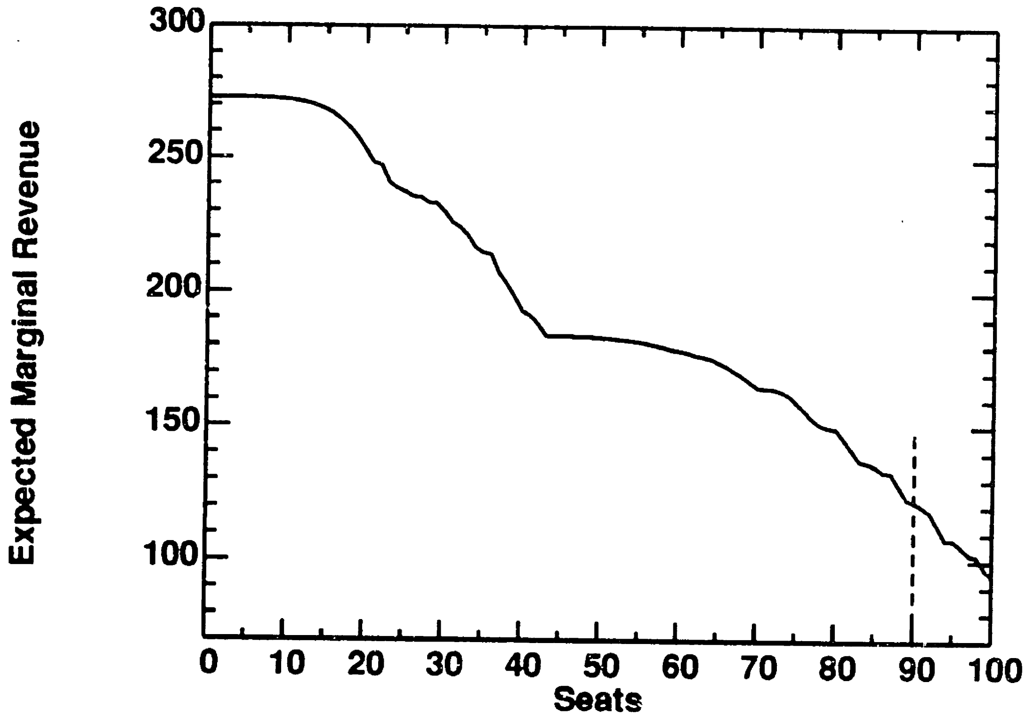


Figure 4.5: Expected marginal revenue curve for each incremental seat on a flight leg.

values can be used as a leg based approximation of the overall contribution an ODF has to the network. For the leg-based bid price approach, the acceptance of different ODF's across a network is determined by comparing the fare of the ODF with the sum of the $EMR(C)$ values of each flight leg traversed by the ODF. Consequently, an ODF request is accepted as long as:

$$f_{ODF} \geq \sum_{j} EMR(C_j), \tag{4.9}$$

for all flight legs j over which the ODF traverses. This decision rule means that a multi-leg itinerary must contribute at least as much in overall revenue as the sum of the marginal value of the last seat on each flight leg traversed by the ODF.

This bid price approach based on $EMR(C)$ values for each flight leg is a leg-based methodology where demand is forecasted at the fare class level and then used to determine the expected marginal revenue curves for each individual flight leg. Yet, in the process of controlling the reservations of seats on each flight leg, information about the total demand on other flight legs is taken into account. While the optimization is performed at the leg level, seats are controlled by combining the estimated value of the last seat on each individual flight leg in an effort to differentiate between ODF's across the network.

When using a traditional leg-based seat inventory control approach, ODF itineraries on a flight leg are usually aggregated together by fare class. The expected marginal revenue of potentially selling seats in each of the fare classes is then determined on the basis of the demand densities and the estimated revenue values of the fare classes. Each fare class is a conglomerate of different OD itineraries with different fare values, therefore the revenue value of each fare class on a flight leg must be determined. While at least one airline assumes a common ratio between the fare class revenue values on each flight leg where the fare for M class might be 85% of the Y fare, the fare for B class 75% of the Y fare, etc., other airlines use a weighted average (based on an estimate of the OD traffic mix) of the fares of each ODF which make up a particular fare class on a flight leg. The ODF values used can be the non-prorated, full ticket value of an ODF or some form of prorated value. In order to incorporate some OD information at the flight leg level, several airlines use a weighted average of the *non-prorated* fare values of each ODF to estimate the revenue value of each fare class.

Using the non-prorated, full ticket value of an ODF in calculating the $EMR(C)$ for each flight leg, the contribution that an ODF has to the entire network of flights will be counted more than once. Under the standard leg-based optimization and control methodologies, this "over-counting" of ODF revenue does not have much effect on the actual fare class booking limits on a flight leg, as shown by Williamson [39]. This is due to the fact that in determining leg based seat allocations and booking limits, it is the ratio of revenue values of each fare class on a flight leg that is important and not the absolute value of each fare. However, under the leg-based bid price approach, decisions on the availability of seats for an ODF is based on summing the expected marginal revenue of the last seat from a set of flight legs over which the ODF traverses. Therefore, it is the *absolute* value of the expected marginal revenues on each flight leg that is important.

"Over-counting" the revenue value of an ODF leads to inflated EMR bid prices on each flight leg. For example, assume there is a two leg flight A to B to C with 1 seat. The only demand for the two flight legs is an AC passenger at a fare of \$100. Focusing at the flight leg level and optimizing on the basis of the full ODF ticket value, for the given capacity of one seat and the certain demand of a single AC passenger at \$100, the $EMR(C)$ value on flight leg A-B is \$100. Similarly, $EMR(C) = \$100$ on flight leg B-C. Based on these $EMR(C)$ values, the total EMR bid price for the AC itinerary is \$200. However, this results in the rejection of the only demand for the two leg flight, the AC demand at \$100, leaving the flight leg empty.

Since optimization in the leg-base bid price approach is done at the flight leg level, the revenue values used for each ODF on an individual flight leg should better represent the contribution of the ODF to the particular flight leg. Therefore, rather than using the full itinerary fare, ODF fares should be prorated across flight legs. There are several methods which can be used for prorating fares, the most suitable method depends on the individual airline and its network and fare structure.

One common proration approach is based on mileage. The fare value of an ODF is divided between flight legs according to the proportion of total itinerary mileage each flight leg represents. For example, the full fare for a Boston-Los Angeles (BOSLAX) itinerary of 2612 miles in April of 1992 was \$792, one way. If a passenger going from Boston to Los Angeles connects through Chicago (ORD), the BOSLAX itinerary is made up of two flight legs, the first being the BOS-ORD leg at 867 miles and the second the ORD-LAX flight leg at 1745 miles. For the BOSLAX itinerary, the mileage prorated fare on the BOS-ORD flight leg is $867/2612 \times \$792$, or \$262.89. At the same time, the prorated fare for the same BOSLAX itinerary on the ORD-LAX flight leg is 1745/2612 × \$792, or \$529.11. In a *non-prorated* fare structure, the fare for the BOSLAX itinerary on both the BOS-ORD flight leg and the ORD-LAX flight leg would be \$792.

A second proration scheme is to apportion ODF fares over their appropriate flight legs based on the ratio of the local full fare, or Y fare, on each flight leg. Therefore, each fare offered for the BOSLAX itinerary through ORD would be divided proportionately based on the local Y fare on the BOS-ORD flight leg of \$447 and the local Y fare on ORD-LAX of \$673. Thus, the prorated BOSLAX Y fare is 447/1120 × \$792, or \$316.09, on flight leg BOS-ORD and 673/1120 × \$792, or \$475.91, on flight leg ORD-LAX. Similarly, for the deeply discounted \$338 fare for the BOSLAX itinerary, using the Y fare prorate method, the value of the itinerary on the BOS-ORD flight leg would be 447/1120 × \$338, or 134.90, and the value on the ORD-LAX flight leg would be 673/1120 × \$338, or \$203.10. Note that the ratio of Y values is applied to all fare classes.

A variation of the Y fare proration method is prorating each ODF fare by the ratio of the local fares of the appropriate fare class. That is, a multi-leg Y class itinerary is prorated on the bases on the ratio of the local Y fares, as in the Y fare prorate method, but a multi-leg B class itinerary is prorated according to the ratio of the local B fares, etc. Therefore, using this proration approach, the prorated BOSLAX Y fare is \$316.09 and \$475.91 on the two respective flight legs, as above. However, the deeply discounted \$338 BOSLAX fare is prorated according to the deeply discounted BOSORD fare of \$178 and the deeply discounted ORDLAX fare of \$288. Thus, the value of the deeply discounted BOSLAX itinerary on the BOS-ORD flight leg would be 178/466 × \$338, or 129.11, and the prorated fare value on the ORD-LAX leg would be 288/466 × \$338, or \$208.89.

The purpose for prorating ODF fares is to obtain some measure of the contribution of a multi-leg ODF to each particular flight leg over which the ODF traverses. However, it is important to recognize that each individual ODF actually has its own correct, or "optimal", proration across relevant flight legs. This "optimal" proration may be very different from the fare proration of other multi-leg ODF's in the network. Due to the fact that the mathematical proration problem is dynamic, nested and probabilistic, it is impractical to determine and use the "optimal" proration for each individual ODF. Therefore, rather than trying to determine the "optimal" proration, the prorate method which can "best" estimate the contribution on each flight leg for all ODF's must be determined.

In many cases, prorated fare values based on mileage or the local fare class revenues are adequate estimates of a multi-leg ODF's contribution to each flight leg it traverses. In other cases, none of the above methods really capture the "correct" contribution of an ODF to a flight leg. If this is the case, other, more complicated, approaches to prorating ODF fares, such as an iterative approach or a network approach, can be used. For instance, the leg-based EMR bid price values may be used iteratively to prorate fares. First, EMR bid price values are determined using a typically prorate approach such as the mileage based approach. This set of EMR bid price values can then be used to prorate the fares for each ODF. Based on these prorated fares, new prorated EMR bid price values can be determined and in turn, used to re-prorate the original ODF fares. After several such iterations, stabilized prorated EMR bid price values should be obtained which may better estimate the correct contribution of an ODF to each flight leg. Another approach is based on using historical ODF data to determine the network bid price for each leg. These values may provide a better representation of the value of each flight leg to the entire network, producing prorated fare values which more accurately capture the contribution of an ODF to a flight leg.

Returning to the three leg example in Figure 4.1, consider a prorated version of the leg-based bid price approach. Based on the mileage prorated method, where the mileage for each of the three flight legs in the A-B-C-D network is 592, 1786, and 426, respectively, the prorated EMR bid prices are given in Table 4.13. Using these prorated EMR bid prices,

Prorated EMR(C)	
A-B:	77.47
B-C:	182.49
C-D:	96.62

Table 4.13: Prorated $EMR(C)$ values, based on a mileage prorate, for each flight leg in Figure 4.1.

	Cut-off Value
AB	\$ 77.47
AC	\$259.96
AD	\$356.58
BC	\$182.49
BD	\$279.11
CD	\$ 96.62

Table 4.14: OD itinerary cut-off values for the multi-leg network in Figure 4.1 using the prorated $EMR(C)$ values of each flight leg.

the acceptance criteria, or cut-off values, for each of the OD itineraries in the network are determined. These cut-off values are provided in Table 4.14. By comparing the respective fare values from Table 4.1 with the appropriate itinerary cut-off value, the open/closed seat availability for each ODF can be determined. The resulting booking status for each ODF is shown in Table 4.15.

Like the network bid price approach, implementation of this leg-based bid price concept requires *real-time* assessment of requests to determine availability. Evaluation based on the cut-off values for an OD pair provides only an open/closed decision with no limit established to the number of seats available to an ODF. However, real-time assessment is well beyond the current capabilities of computer reservations systems. Therefore, rather than simply implementing an open/closed leg-based seat inventory control approach, leg-based limits to the number of bookings accepted for an ODF should be provided as well.

	Y	M	B	Q
AB	Open	Open	Open	Open
AC	Open	Open	Open	Closed
AD	Open	Open	Closed	Closed
BC	Open	Open	Open	Open
BD	Open	Open	Closed	Closed
CD	Open	Open	Open	Open

Table 4.15: Booking status for the multi-leg network in Figure 4.1 based on the mileage prorated EMR bid prices for each flight leg.

4.4.2 Combined Leg-Based Bid Price/Booking Limit Approach

While current seat inventory control approaches determine booking limits for each fare class on a flight leg, the approaches do not make a distinction between OD itineraries within a fare class. However, by combining the current booking limit approach with the EMR bid price acceptance rule, assessment of an individual ODF's approximate contribution to the network can be made while the number of seats available to the ODF are limited. Using a basic leg-based approach, EMSR or OBL, booking limits for each fare class, as well as $EMR(C)$ values, can be determined. Multiple leg itineraries are then analyzed as to whether their fare value is greater than the combined marginal value of the last seat on each flight leg traversed by the itinerary. If the OD itinerary and fare class combination passes the EMR bid price rule, the ODF seat availability is limited by the *maximum* of the respective fare class booking limits of the appropriate flight legs. Thus, if

$$f_{ODF} \geq \sum_j EMR(C_j),$$

for all flight legs j over which the ODF traverses, then the booking limits for each ODF, BL_{ODF}, are:

$$BL_{ODF} = \text{Max}\,(BL_{i,j}), \tag{4.10}$$

for the respective fare class i of the ODF and all flight legs j over which the ODF traverses. Otherwise,

$$BL_{ODF} = 0. \tag{4.11}$$

(Current leg-based approaches determine multi-leg seat availability on the basis of the minimum fare class booking limit.)

Consider a two leg flight A-B-C where the Q class booking limit on the first flight leg, A-B, is 30, but on the B-C flight leg the Q class is closed to bookings. Under the combined leg-based bid price/booking limit approach, local itineraries in Q class are simply limited by the Q class booking limit of the respective flight legs. Thus, the booking limit for ABQ, is 30 while no seats are available to BCQ. If the fare for the multi-leg AC itinerary in Q class is greater than the total EMR bid price of the two flight legs, the maximum of the two Q class booking limits, 30 and 0, is used as the ACQ booking limit, i.e. 30. The *maximum* booking limit of the respective fare class and flight legs is used to allow bookings for an ODF whose value to the network has been verified under the EMR bid price rule, yet the respective fare class on one flight leg is closed. Under current airline leg-based approaches, since no seats are available on flight leg B-C for Q class, bookings for itinerary ACQ would not be accepted.

For the three leg example in Figure 4.1, the prorated EMR bid prices are given in Table 4.13, and the corresponding fare class booking limits for each flight leg are given in Table 4.16. Comparing the total itinerary fare of each ODF with the sum of the respective prorated $EMR(C)$ values, the same open/closed decisions are made as those shown in Table 4.15. However, rather than "unlimited" access to every seat on the flight leg, bookings for each "open" ODF are limited by using the fare class booking limits as overlays. The

	Y	M	B	Q
Flight Leg A-B	90	65	59	52
Flight Leg B-C	90	73	45	27
Flight Leg C-D	90	71	4	0

Table 4.16: Fare class booking limits based on a mileage proration of fare values for each flight leg in the multi-leg network A-B-C-D.

	Y	M	B	Q
AB	90	65	59	52
AC	90	73	59	0
AD	90	73	0	0
BC	90	73	45	27
BD	90	73	0	0
CD	90	71	4	0

Table 4.17: Booking limits for each ODF in Figure 4.1 based on the combined prorated leg-based bid price and fare class booking limit approach.

booking limits from this combined prorated leg-based bid price/booking limit approach are shown in Table 4.17.

In this example, the prorated EMR cut-off value for the AC itinerary is \$259.58. Therefore, Q class, at a fare of \$231, is closed to bookings while Y, M, and B classes are open. Based on the nested fare class booking limits on flight legs A-B and A-C, the booking limit on ACB is 59, the maximum of 59 and 45, and the booking limit on ACM is 73. By limiting the number of seats available to each ODF through the use of fare class booking limits, unexpected demand in lower fare classes can no longer book an "unlimited" number of seats and displace higher fare class demand, as can occur under a straight bid price approach.

4.4.3 Virtual Nesting on the "Value Net of Opportunity Cost"

A problem with aggregating ODF itineraries on a flight leg into fare class buckets is that there often can be a significant amount of overlap in revenue values between fare class buckets. For example, consider the ODF itineraries on the A-B flight leg in Figure 4.1, i.e. itineraries AB, AC, and AD. Based on the fare information in Table 4.1, the fares in Y class range from $216 to $582 while the range of fares in Q class is $152 to $269. Given such significant overlaps, it is possible for the (non-prorated) weighted average fare associated with each fare class on a flight leg to be inconsistent with the nesting structure. That is, if the long-haul demand for ADM at $379 is large compared to the local ABM demand at $203, but the local ABY demand at $216 is significantly more than long-haul ADY demand at $582, then the weighted average fare for M class might be greater than the weighted average Y class fare. When the weighted average fares are not decreasing with respect to the fare class hierarchy, leg-based optimizations are often not very effective since seats are no longer protected for high revenue fare classes from lower revenue fare classes.

Prorating ODF fares can alleviate some of this problem, but inconsistencies with the nesting structure can still occur. One method of assuring that the range in fares within an inventory bucket is not too large and that overlaps in fares between buckets do not exist is by controlling seats through an inventory structure defined on ODF fare values. Rather than aggregating ODF itineraries on a flight leg by their respective fare classes, ODF's are aggregated into virtual inventory buckets according to their actual fares. These virtual inventory buckets can then be maintained and controlled on each flight leg in the same manner as fare class inventories.

As discussed in Chapter 3, the concept of a virtual inventory system, or virtual nesting, was first developed by American Airlines [37] as a means of achieving "some" control over passenger itineraries without a complete network optimization. Each virtual inventory

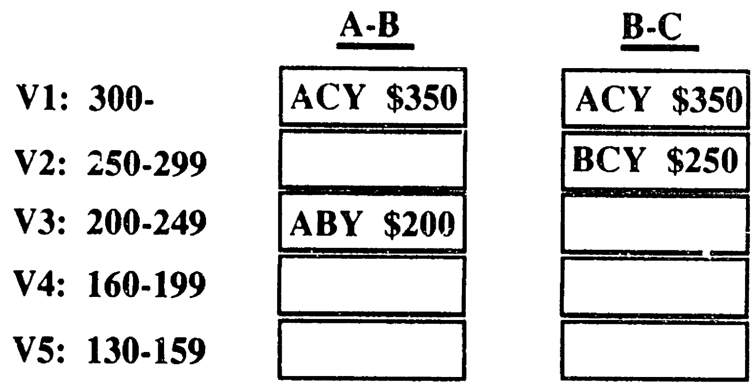


Figure 4.6: Mapping of the Y class itineraries from a two flight leg example on the basis of total itinerary ticket revenue. Each virtual inventory bucket is defined over a range of fares, and ODF itineraries on each flight leg are assigned to the appropriate virtual bucket.

bucket on a flight leg was defined for a range of actual fares, allowing ODF's with similar revenue values on a flight leg to be grouped together for optimization and control purposes. In early descriptions of the virtual nesting approach, ODF's were assigned, or mapped, to virtual inventory buckets based on their total itinerary ticket fare value.

This virtual nesting approach can be illustrated using a two flight leg network, A-B-C. The fares for the Y class itineraries on this network are $200 for ABY, $350 for ACY, and $250 for BCY. For the virtual inventory structure in Figure 4.6, each inventory bucket is defined over a range of fares, with virtual inventory bucket V1 defined over the fare range of $300 and up, virtual bucket V2 over the range of $250 to $299, etc. Based on total ticket fares, on flight leg A-B, ACY at $350 is mapped into virtual bucket V1 and ABY at $200 is mapped into V3. On flight leg B-C, ACY is again mapped into virtual bucket V1, while the local itinerary BCY at $250 is mapped into virtual bucket V2.

The problem with mapping ODF itineraries on each flight leg into virtual buckets on the basis of total itinerary ticket fare value is that such an approach to controlling seat inventories tends to be "greedy", with priority given to long haul, higher revenue itineraries over short haul, lower revenue itineraries. For a network constrained by a only few "bottleneck" flight legs, giving priority to long haul passengers may be beneficial. However, as load factors increase on flight legs across a network, negative revenue impacts from such a "greedy" approach can result as long haul ODF's displace combinations of short haul and local traffic which have a greater *total* revenue value.

As has been proposed by both American [37] and United [38], the actual value of each itinerary to the individual *flight legs* can be used to control seat inventories by adjusting the fares of the different multi-leg itineraries by the displacement cost associated with using other flight legs. One possible estimate of the displacement cost associated with a flight leg is the $EMR(C)$ value. Thus, the value of a multi-leg itinerary to a given flight leg, or the "value *net* of opportunity cost", is determined by adjusting the total ticket fare by the $EMR(C)$ values of all *other* flight legs the itinerary traverses, i.e. the "value net of opportunity cost" on flight leg i for a given ODF itinerary, $VNOC_{i,ODF}$, is:

$$VNOC_{i,ODF} = f_{ODF} - \sum_{j \neq i} EMR(C_j), \tag{4.12}$$

for all flight legs j over which the ODF traverses, where $j \neq i$. Using this "value net of opportunity cost", each ODF is then mapped into virtual inventory buckets.

Consider again the previous two leg example in which the fare for ABY is \$200, ACY is \$350, and BCY is \$250. The displacement cost on each flight leg is assumed to be:

$$\begin{aligned} EMR(C_{A-B}) &= \$150, \\ EMR(C_{B-C}) &= \$100. \end{aligned}$$

Since the local itineraries do not displace passengers from flight legs other than their own local flight legs, the "value net of opportunity cost" for ABY and BCY is the same as the total itinerary fare for each ODF, \$200 and \$250, respectively. However, accepting an ACY passenger on flight leg A-B will displace passengers on flight leg B-C at an estimated value of \$100. Therefore, the "value net of opportunity cost" for ACY on flight leg A-B is $350 - 100$, or \$250. On flight leg B-C, an ACY passenger will displace a passenger

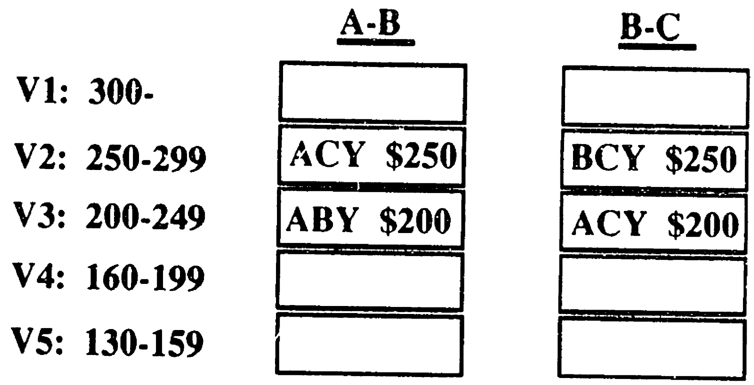


Figure 4.7: Virtual inventory structure of a two leg flight, A-B-C, based on the "value net of opportunity cost" for each ODF.

on A-B worth $150, thus the "value net of opportunity cost" for ACY on flight leg B-C of $200. Mapping the ODF's to virtual inventory buckets according to their "value net of opportunity cost", rather than their total itinerary fare values, results in the virtual inventory structure shown in Figure 4.7. Under this virtual nesting approach, ACY is no longer mapped into virtual bucket V1 on both flight legs. Instead, it is mapped into bucket V2 on flight leg A-B and on flight leg B-C, it is mapped below the BCY itinerary into the virtual bucket V3.

Once ODF's on each flight leg have been assigned to a virtual inventory bucket, each virtual bucket can be considered as a normal leg-based booking class. Demand is forecasted for each virtual class as it is for fare classes, and nested protection levels and booking limits are determined for the virtual classes using a leg-based optimization approach, such as the EMSR heuristic or the OBL method. ODF's are then controlled according to the booking limits of the respective virtual classes on each flight leg, as is done in current fare class inventory structures.

For the three leg example in Figure 4.1, the first step of the virtual nesting on the "value net of opportunity cost" approach is to determine the displacement costs associated with

"Value Net of Opportunity Cost"

ACY	\$441.53
BCY	\$440.00
ADY	\$407.91
BDY	\$388.38
BCM	\$315.00
ACM	\$266.53
BDM	\$243.38
BCB	\$223.00
ADM	\$204.91
BCQ	\$197.00
ACB	\$184.53
ACQ	\$153.53
BDB	\$150.38
ADB	\$127.91
BDQ	\$112.38
ADQ	\$ 94.91

Table 4.18: The "value net of opportunity costs", for each ODF on flight leg B-C of the network in Figure 4.1 which are determined using the prorated $EMR(C)$values from Table 4.13.

using each flight leg. Based on the prorated $EMR(C)$ values, these displacement costs are \$77.47 for flight leg A-B, \$182.49 for B-C, and \$96.62 for C-D, Table 4.13. Using these displacement costs, the "value net of opportunity cost" for each ODF on flight leg B-C is given in Table 4.18. Each of these values is based on adjusting the total itinerary fares, Table 4.1, by the displacement costs of the other flight legs traversed by the given ODF. Note that while the "value net of opportunity cost" for ACY on flight leg B-C is \$441.53, i.e. $519 - 77.47$, the the value for the same OD itinerary and fare class combination on flight leg A-B is $519 - 182.49$, or \$336.51. Similarly, the ADY itinerary worth \$407.91 on flight leg B-C is worth $582 - 182.49 - 96.62$, or \$302.89, to the A-B flight leg and $582 - 77.47 - 182.49$, or \$322.04, to the C-D flight leg.

Once the "value net of opportunity cost" has been determined for each ODF on each flight leg, the ODF's are mapped into virtual inventory buckets on each flight leg. For instance, on flight leg B-C, virtual inventory bucket V1 could be defined over the revenue value range of \$425 to \$450, resulting in BCY and ACY being assigned to V1. Inventory buckets on different flight legs do not have to be defined over the same revenue ranges. While virtual inventory bucket V1 may be defined over the revenue value range of \$425 to \$450 on flight leg B-C, V1 may be defined for the range of \$300 to \$340 on flight leg A-B.

The results of analysis performed in this research project have demonstrated that selection of the revenue ranges for virtual inventory buckets on each flight leg can have a significant effect on the overall performance of the virtual nesting methodology. Since the purpose here is not to determine the best approach to selecting virtual inventory ranges, but to introduce the basic optimization and control techniques which can be used to manage seat inventories, the revenue range/value of each virtual inventory bucket is defined by a single ODF's "value net of opportunity cost" on the flight leg. Thus, for flight leg B-C in Figure 4.1, the bucket V1 is defined for the revenue value \$441.53 and ACY is assigned to bucket V1, bucket V2 is defined for the value \$440 with BCY assigned to the bucket, etc. Once the mapping of ODF's to virtual inventory buckets on each flight leg is completed, leg-based nested booking limits are calculated for each flight leg based on the forecasted mean and standard deviation of demand and average revenue value of each virtual inventory bucket. The virtual nesting booking limits for flight leg B-C are shown in Table 4.19.

Using the virtual nesting on the "value net of opportunity cost" methodology, the $EMR(C)$ values for each flight leg can be determined from the expected marginal revenue curves of the virtual inventory classes, rather than from a fare class inventory structure. The initial $EMR(C)$ values would need to be determined iteratively, where the $EMR(C)$ values from a straight fare class inventory structure could be used first to determine a virtual inventory structure based on the "value net of opportunity cost", and in turn, new $EMR(C)$ values could be determined from this virtual inventory structure and used to redefine the virtual inventory structure. This iterative process would be continued until

	Booking Limit
ACY	90
BCY	90
ADY	87
BDY	84
BCM	75
ACM	57
BDM	54
BCB	50
ADM	43
BCQ	40
ACB	20
ACQ	1
BDB	0
ADB	0
BDQ	0
ADQ	0

Table 4.19: Virtual nesting booking limits based on the "value net of opportunity cost" for each ODF on flight leg B-C of the three leg network A-B-C-D in Figure 4.1.

the $EMR(C)$ values stabilized. Future "corrections" to the displacement cost associated with each flight leg would be determined directly from the virtual inventory structure. While such corrections will be necessary to capture significant changes in the displacement cost of individual flight legs over time, the benefits of updating the $EMR(C)$ values must be weighed against the costs related to continually changing the composition of individual inventory buckets. Changes in the $EMR(C)$ values will effect the "value net of opportunity cost" for each ODF itinerary, and therefore, the actual mapping of the different ODF itineraries into virtual inventory buckets. In order to forecast demand using historical inventory bucket data, some stability in the contents of an inventory bucket is necessary.

4.4.4 Nested Leg-Based Itinerary Limit

The above leg-based approaches to OD control simply use the value of the *last* seat on each flight leg to differentiate between ODF itineraries on a flight leg. A different idea for controlling ODF itineraries developed in this research is to use information from the *entire* (prorated) EMR curve to determine leg-based ODF itinerary limits on each flight leg. This approach, which is called the "Nested Leg-Based Itinerary Limit" approach, is essentially an extension of the EMR bid price logic, yet decisions are made based on the expected marginal revenue of each seat on a flight leg, not just the last seat. In turn, rather than using the fare class booking limits on each flight leg, actual booking limits can be determined for each ODF on a flight leg.

Under this approach, the entire EMR curves from the respective flight legs of an ODF itinerary are "summed", resulting in a total expected marginal revenue value for each seat of the path traversed by an ODF. Booking limits are then determined based on the point at which the total ODF fare is equivalent to the *total* EMR curve of the itinerary. Thus, for a two leg example, a hypothetical EMR curve for the first flight leg, A-B, with a capacity of 120 is shown in Figure 4.8. The expected marginal revenue of the first seat on the flight leg is estimated to be $272.80. With each additional seat, the expected marginal revenue decreases to a value of $46.63 for the 120th seat on the flight leg. Figure 4.9 shows the hypothetical EMR curve for flight leg B-C where the capacity is assumed to be only 90 seats. The expected marginal revenue of the first seat is $336.20, however, demand is fairly high for the flight leg, and thus, the expected marginal revenue value of the last seat on the flight leg is $163.39.

By summing the A-B flight leg EMR curve in Figure 4.8 and the B-C flight leg EMR curve in Figure 4.9, a total EMR curve for the AC itinerary is determined. Under the assumption that an AC passenger will displace passengers on each flight leg with the lowest expected marginal revenue, rather than the most desirable, highest fare class ODF's on the respective flight legs, the summation is performed beginning with the last seat on each flight leg. For instance, due to the capacity on flight leg B-C, the maximum number of AC passengers would be 90. Therefore, on flight leg A-B, it is the last 90 seats, i.e. seat

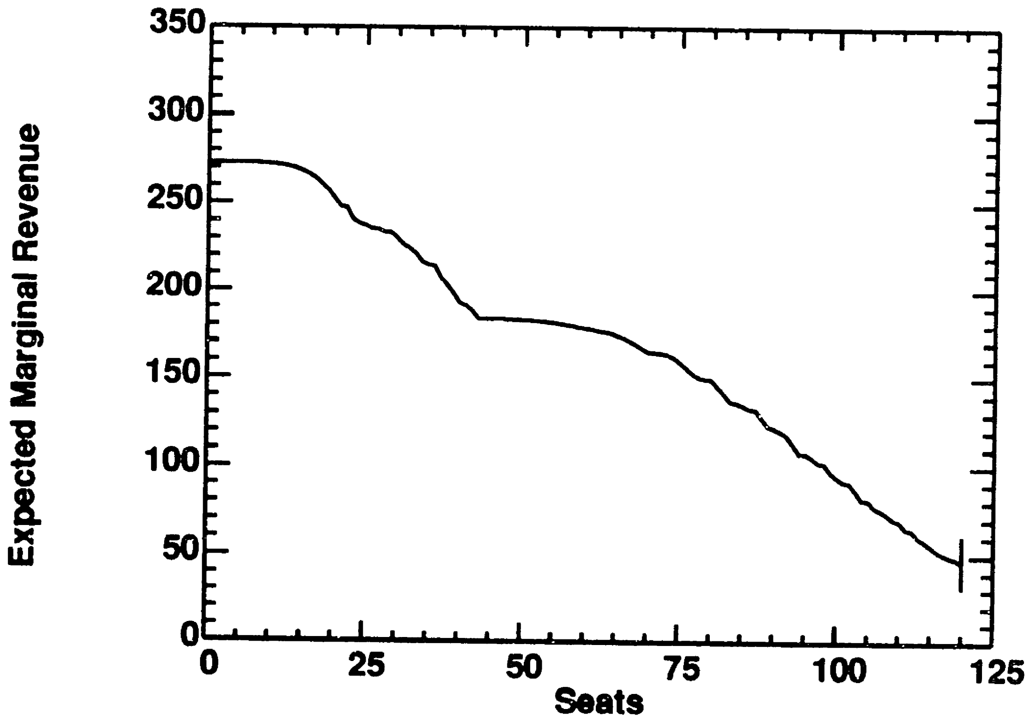


Figure 4.8: The assumed expected marginal revenue curve for flight leg A-B of the two leg example.

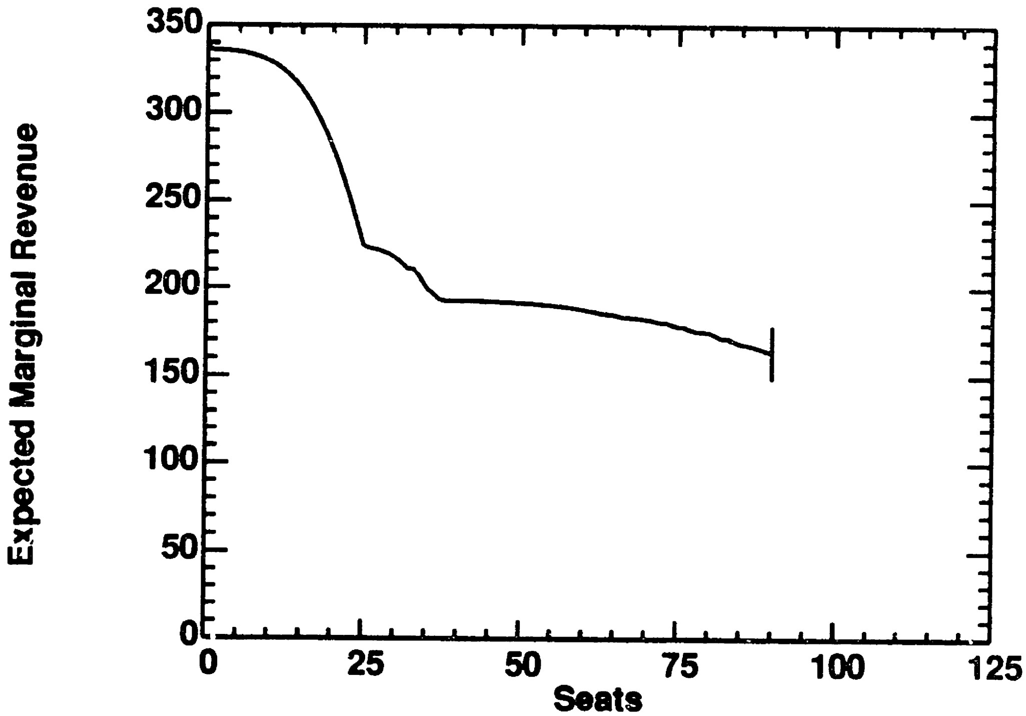


Figure 4.9: The hypothetical expected marginal revenue curve for flight leg B-C of the two leg example.

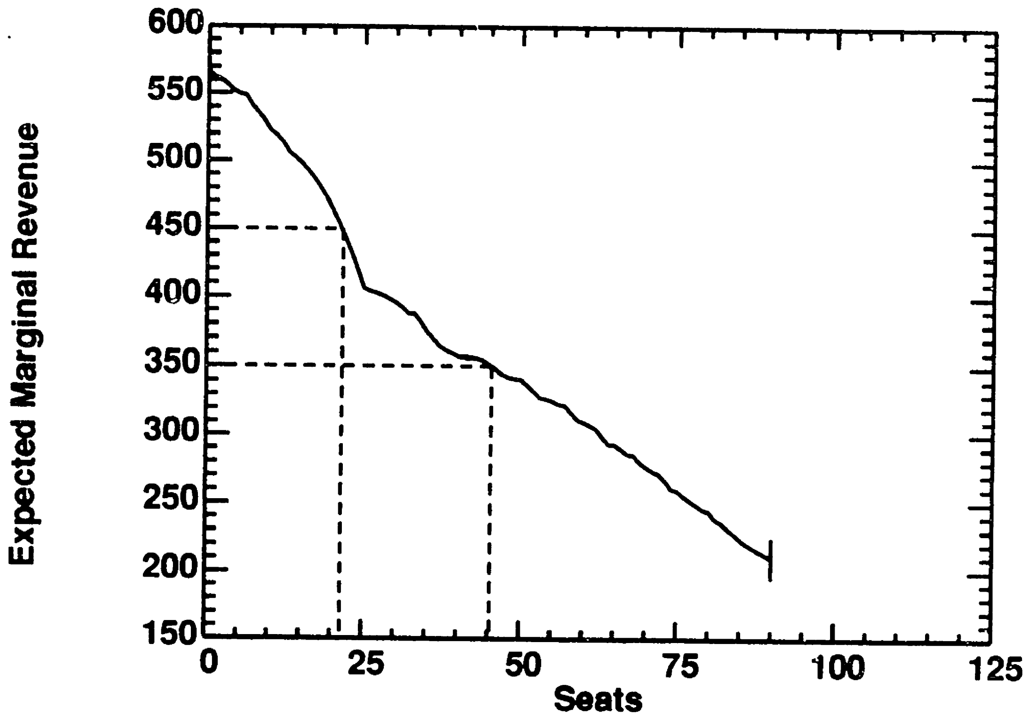


Figure 4.10: The total expected marginal revenue curve for two leg itinerary AC which is determined by summing the EMR curves from flight leg A-B and flight leg B-C in Figures 4.8 and 4.9.

31 to seat 120, which will be used to calculate the EMR curve for itinerary AC. This total AC itinerary EMR curve is shown in Figure 4.10. With the value of the 31st seat on flight leg A-B being \$226.24, the AC EMR curve starts at a value of $336.20 + 226.24$, or \$562.44, and decreases to an estimated value of $163.39 + 46.63$, or \$210.02, for the 90th seat.

Using the EMR curves in Figures 4.8, 4.9, and 4.10, booking limits for each ODF itinerary can be determined by comparing the fare of the ODF, f_{ODF}, with the total expected marginal revenue of each seat on the ODF's path. Based on this comparison, all seats which have an expected marginal revenue value *less* than f_{ODF} are made available to the ODF, and those seats which have a *higher* expected marginal revenue value than the ODF fare f_{ODF} are protected for other ODF combinations which traverse the given flight leg(s), as is the convention of current leg-based seat inventory control (EMSR/OBL) approaches. Thus, the value of the $S_{\overline{ODF}}$th seat is found such that:

$$
\begin{aligned}
f_{ODF} &< EMR(S_{\overline{ODF}}), \\
f_{ODF} &\geq EMR(S_{\overline{ODF}} + 1),
\end{aligned}
\tag{4.13}
$$

where $S_{\overline{ODF}}$ seats are protected for other, more desirable, ODF itineraries while all other seats across the ODF's path are made available to the ODF. Based on the value of $S_{\overline{ODF}}$ for given ODF, the nested booking limit BL_{ODF} of the ODF is:

$$
BL_{ODF} = C - S_{\overline{ODF}}. \tag{4.14}
$$

For the hypothetical two leg example, given an ACY fare of \$450, the nested booking limit for ACY is determined by comparing the \$450 fare to the AC itinerary EMR curve in Figure 4.10. Since 21 seats have a greater expected marginal revenue than an ACY passenger at \$450, these 21 seats are protected for local demand, and 90 – 21, or 69 seats, are made available on each flight leg for ACY demand. Given a fare of \$350 for ACM, 45 seats in Figure 4.10 have a greater expected marginal revenue value than \$350, resulting in a nested booking limit of 90 – 45, or 45 seats for ACM.

Returning to the three leg example in Figure 4.1, the mileage prorated EMR curve for the B-C flight leg is given in Figure 4.11. From this curve, the booking limits for the BC ODF itineraries can be determined using the ODF fares in Table 4.1. For BCY, the \$440 fare is greater than the expected marginal revenue of each seat on the flight leg. Therefore, the BCY booking limit is 90. For BCM, 16 seats have an expected marginal revenue greater than \$315. These seats are protected for BCY passengers and other, more desirable, combinations of ODF passengers, resulting in a BCM booking limit of 74. Similarly, based on the expected marginal revenue curve in Figure 4.11, the nested booking limits for BCB, at a fare of \$223, and BCQ, at a fare of \$197, are 45 seats and 29 seats, respectively.

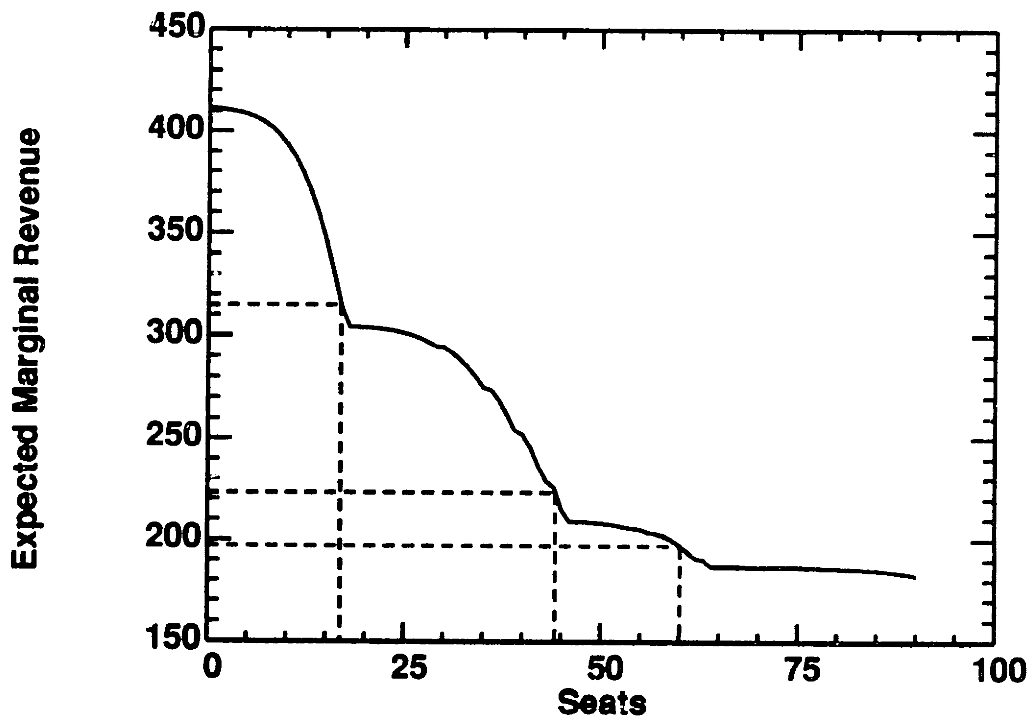


Figure 4.11: The prorated expected marginal revenue curve for B-C flight leg of the three leg network in Figure 4.1.

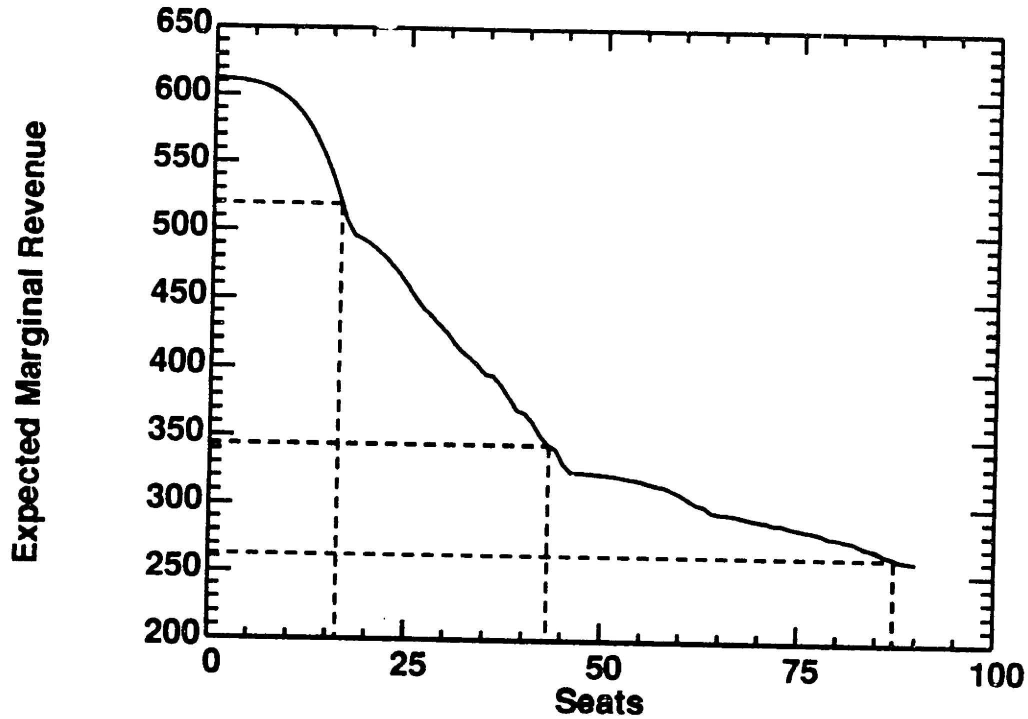


Figure 4.12: The total prorated expected marginal revenue curve for itinerary AC of the three leg network in Figure 4.1. Booking limits for each AC fare class are based on the number of seats whose expected marginal revenue is less than or equal to the respective ODF fares.

For the AC itinerary from the three leg A-B-C-D example, nested booking limits are determined by adding the prorated EMR curve of the B-C flight leg with the prorated EMR curve of the A-B flight leg, producing the *total* AC itinerary EMR curve shown in Figure 4.12. Using this combined EMR curve and the fare information from Table 4.1, the booking limits for each fare class on the AC itinerary can be determined. The nested booking limits for ACY at $519, ACM at $344, and ACB at $262 are 74, 47, and 3, respectively. However, in Figure 4.12, all 90 seats have a higher expected marginal revenue value than an ACQ passenger at a fare of $231, with the value of the 90th seat being $259.80. Therefore, the booking limit for ACQ is 0.

4.5 Summary

Several new approaches to controlling seat inventories at the itinerary level have been introduced, such as the network and leg-based bid price approaches, the aggregated network approaches, and the nested leg-based itinerary limit approach. Extensions to previous methods, such as the nested on shadow price method and the virtual nesting method, have also been discussed in detail. Each of these approaches has its advantages and disadvantages. They differ in the scope of their optimization, from encompassing the entire network of flights to focusing on an individual flight leg; in incorporating or not incorporating the probabilistic nature of demand; in the assumed inventory structure used as the foundation in determining seat allocations and/or booking limits, whether it be nested or partitioned; and in the data requirements necessary, from fare class and flight leg demand forecasts to aggregated ODF demand forecasts to individual disaggregated ODF forecasts.

While the leg-based approaches described above take into account information about other flight legs when controlling seat inventories, the optimization in each approach is performed at the flight leg level. Thus, rather than forecasting demand for each individual ODF, demand is forecasted at the fare class or virtual inventory class level. Another benefit of the leg-based approaches is that nesting between inventory buckets is *explicitly* taken into account in the optimization process. This is not the case in the network approaches. While nesting of network-based seat inventories can be accomplished through the control process by nesting on shadow prices or implicitly nesting through a bid price control approach, the actual network optimization is *partitioned*, or *distinct.*

The difficulty with determining optimal nested seat allocations at the network level is that the ODF hierarchy across the network is not well defined. At the flight leg level, determining the hierarchy of different inventory classes on a particular flight leg is straightforward. The fare classes or virtual inventory buckets with the highest revenue value are the most desirable to an individual flight leg and therefore, are ranked highest in a nesting hierarchy. However, at the network level, the highest fare ODF is not necessarily the most valuable ODF to the network. The value of an ODF to a network is dependent on the amount interaction of the other ODF's across a network and changes as demand for different ODF's materializes. Without first knowing the "correct " nesting hierarchy of different ODF's over a network, a network optimization which explicitly accounts for nesting of ODF's is, to date, both theoretically infeasible and impractical.

None of these approaches is the true "optimal" solution to the real-world probabilistic, dynamic, nested, network seat inventory control problem. Which of these characteristics and properties are the most important in generating "additional" revenue through the management of seat inventories, and how do the revenue benefits compare to the costs associated with actually implementing the seat inventory control methodology? In Chapter 6, an attempt is made at answering these questions.

Chapter 5

Modeling the Booking Process Through Simulation

In evaluating a seat inventory control methodology, it is important to have a complete understanding of the method, its assumptions, and the basis on which seat allocations and booking limits are determined. At the same time, the complexities associated with the actual optimization and control techniques, the support system requirements, and other implementation issues are important to evaluate in depth. However, the driving force behind developing and designing new seat inventory control methods is to capture incremental revenue. Therefore, one of the initial performance measures of a seat inventory control approach is the increase in total revenue it will provide an airline.

Exact levels of revenue obtained from controlling seat inventories through different optimization and control methodologies are difficult to determine analytically due to the probabilistic nature of demand, particularly when nested inventory structures are involved. However, such estimates are very important to measure before implementing a new seat inventory control system since the cost associated with completely redesigning the inventory structure and reservations control system for a major carrier can be as high as tens of millions of dollars [6], if not more. One way to estimate the revenue impacts of different seat inventory control approaches is to simulate the booking process of an airline and the way in which the control methodology will affect the acceptance and/or rejection of booking requests.

A simulation is a procedure in which a computer-based mathematical model of a physical system is used to perform experiments with that system by generating external demands and observing how the system reacts to the demands over a period of time [42]. Simulations have long been a very important tool in decision making. With current computer capabilities, simulations have become easier to perform while the size and complexity of the systems which can be studied effectively has grown. Thus, the practice of using simulations for numerous applications in a wide variety of contexts has become routine, with simulation used as aids in both designing systems and validating the different designs or a system, for troubleshooting, and for insight into the actual operations of a system [43].

A simulation can provide an approach for evaluating different seat inventory control options in a controlled environment which would be difficult to do through real-time experimentation. In the airline industry, there are many factors which contribute to and influence the amount of revenue generated throughout a network of flights, such as marketing strategies, pricing strategies, the economy, competition and reservations control policies. Separating the impacts of each of these factors from one another is extremely difficult. However, by simulating the booking process of an airline, the many factors which influence the revenue potential of an airline can be controlled.

Another advantage of a simulation is that different seat inventory control approaches can be compared under the same demand situations. Not only are there constant changes in the environment under which an airline operates flights each day, but actual demand over the network of flights is probabilistic. Thus, the revenue impacts due to controlling seat inventories on an individual flight departure today are not always directly comparable to the potential revenue impacts on the same flight tomorrow. Through simulation, a variety of different optimization and control techniques can be tested for an *identical* demand pattern, allowing more accurate comparisons to be made between the methods. At the same time, each method can be tested over a range of demand scenarios so that the inventory control actions of a methodology which does well for the demand pattern on one flight departure can be evaluated under a different demand pattern which may occur on a different departure of the "same" flight.

A simulation which realistically models the booking process can provide an airline with reasonably accurate estimates of the potential benefits of a seat inventory control approach. At the same time, a simulation can yield additional insights into the cause-and-effect relationships of a seat inventory control approach and bookings. Thus, besides simply justifying the benefits of a new system, simulations can be used to make adjustments and refinements to a seat inventory control methodology, reducing the risks and costs associated with making major modifications to the reservations control system at a later date.

5.1 Integrated Optimization/Booking Process Simulation

In order to compare the revenue impacts of the different approaches introduced in this research to control inventories of seats at the itinerary level, an integrated optimization/booking process simulation was developed. In this simulation program, the real booking process for a set of interrelated flight departures is imitated and combined with assumed airline reservations control practices in order to estimate the actual performance of different network seat inventory control approaches. In this section, the basic integrated optimization/booking process simulation is outlined. This is followed by a detailed discussion of particular modeling issues relating to the booking process and the simulation as a whole.

The integrated optimization/booking process simulation is a Monte Carlo simulation which was developed to be very flexible. Different combinations of optimization techniques and control structures can be evaluated over a variety of networks ranging from a single

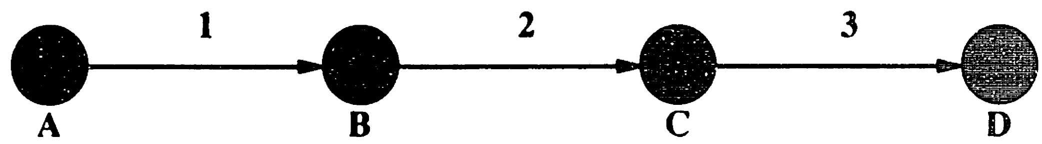


Figure 5.1: Multiple leg network with three flight legs and 6 OD pairs.

Itinerary	Path
AB	1
AC	1,2
AD	1,2,3
BC	2
BD	2,3
CD	3

Table 5.1: The OD itineraries and their paths which are used to define the multiple leg network in Figure 5.1.

flight leg to a multiple leg network to a large connecting hub-and-spoke network. The network over which bookings are simulated is defined by specifying the different OD itineraries served and the path of flight legs over which each traverses. Thus, if a multi-leg network with three flight legs is being considered, such as that in Figure 5.1, the network would be defined by enumerating the 6 possible OD pairs and their respective flight leg "paths". Designating flight leg A-B as Flight 1, leg B-C as Flight 2, and leg C-D as Flight 3, the itineraries and their paths are listed in Table 5.1. For a small hub-and-spoke network with two flights in and two flights out, such as in Figure 5.2, the corresponding OD itineraries and flight leg paths used to define the network are given in Table 5.2, where flight leg A-C is designated as Flight 1, flight leg B-C as Flight 2, leg C-D as 3, and leg C-E as 4.

In addition to the definition of the network over which bookings are to be simulated, other inputs required for the simulation are the aircraft or cabin capacity on each flight leg, the fare classes offered in each OD market pair, and the fares and forecasted demands

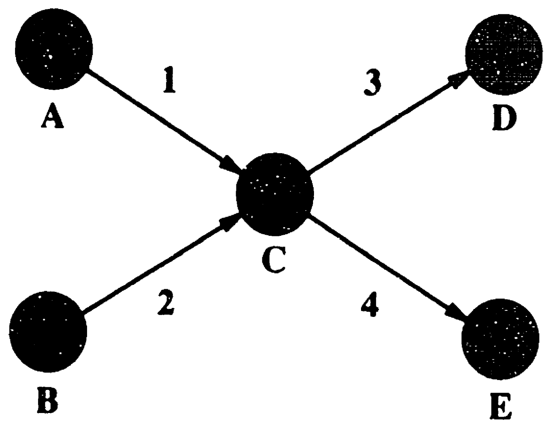


Figure 5.2: Small hub-and-spoke network with 2 flights in, 2 flights out, and 6 OD pairs.

Itinerary	Path
AC	1
AD	1,3
AE	1,4
BC	2
BD	2,3
BE	2,4

Table 5.2: The OD itineraries and the path of flight legs traversed by each which are used to define the small hub-and-spoke network in Figure 5.2, where Flight 1 represents flight leg A-C, Flight 2 represents leg B-C, Flight 3 represents leg C-D, and Flight 4 leg C-E.

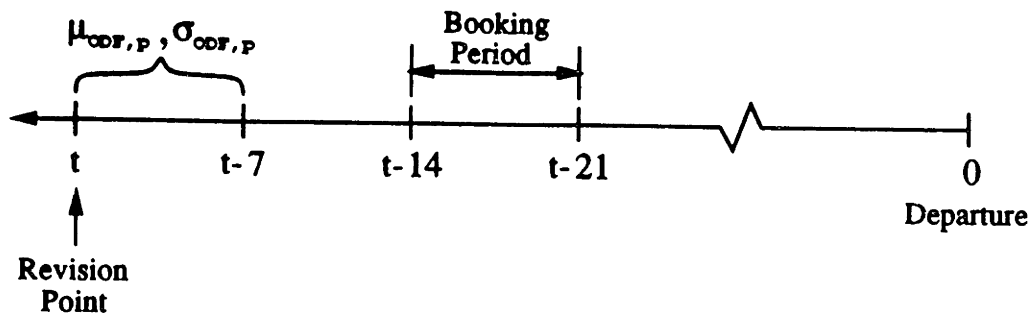


Figure 5.3: Time line of the booking process showing the relationship between revision points and booking periods. To realistically model the booking process, the incremental mean and standard deviation of demand for each ODF, $\mu_{ODF,p}$ and $\sigma_{ODF,p}$, is required for each booking period p between revision points.

for each ODF itinerary. In order to reflect the fact that airlines are continually monitoring and making adjustments to booking limits on a regular basis during the booking process, the simulation is a multi-stage, or dynamic, optimization/booking process. Thus, the seat inventory control optimization approach and the booking process are integrated with scheduled revision points during the booking process at which seat allocations and booking limits can be updated. To realistically model this dynamic optimization/booking process, incremental demand densities for each booking period between revision points are required.

For example, if revision points are scheduled on a weekly basis, as shown in Figure 5.3, an estimate of the mean and standard deviation of the incremental demand for each ODF, $\mu_{ODF,p}$ and $\sigma_{ODF,p}$, is needed for each booking period p between t days prior to departure and $t-7$ days prior to departure, between $t-7$ days prior to departure and $t-14$ days prior to departure, etc. With the use of historical incremental booking data, or total forecasted ODF demand and booking curves, estimates of the incremental demand within each period can be determined. Based on these incremental demand densities, requests between revision points can be randomly generated, allowing for adjustments to be made to seat availabilities at the each revision point on the basis of *current* bookings on hand and forecasted bookings to come.

The frequency of revisions (or the length of the booking periods between revision points) is solely dependent on the availability of incremental booking information, and thus, the ability to differentiate between bookings in a given booking period and forecasted bookings to come. That is, it must be possible to estimate demand between t days prior to departure and $t-7$ days prior to departure, as well as between $t-7$ days prior to departure and departure. At the extreme, if forecasts of the remaining bookings to come can be reestimated after each booking, a "fully" dynamic simulation with revisions after each individual booking is feasible. Although a "fully" dynamic simulation modeling seat inventory control decisions on a real time, seat by seat, basis is technically possible as far as the simulation is concerned, real time forecasting of bookings to come is infeasible. Therefore, the frequency of revisions used in the simulation corresponds to the frequency of revisions in the actual seat inventory control process of an airline which is constrained by the airline's forecasting capabilities.

Once the data requirements have been met, booking limits are calculated using a specified seat inventory control approach. These booking limits are based on the capacity of each flight leg and the number of current bookings in the system, which yields the remaining *available* capacity over the network, and the *total* mean and standard deviation of forecasted demand to come from the current time to the day of departure. Under an assumed probability distribution, demand for each ODF is randomly generated based on the incremental mean and standard deviation estimates for the booking period at hand. Given seats are available, demand is accepted and booked one request at a time and the corresponding availabilities decremented.

When all ODF demand has been booked for a given booking period, subject to the respective booking limits, booking limits are recalculated and the booking process repeated for each subsequent booking period. The complete booking and revision process for a single network of departures is then repeated a large number of times, i.e. 200, 500, 1000, or even 5000 iterations. Once the entire simulation routine is finished, several summary statistics are calculated, the primary one being the expected mean revenue for the entire network. The mean number of bookings and the mean spill, i.e. requests which are not accepted as bookings, for each ODF are also determined in order to evaluate the mix of traffic being accepted under different seat inventory control strategies. Other summary statistics, such as the average load factor on each flight leg, the average network load factor, the average yield per passenger and the average revenue per available seat, is also calculated for comparison purposes. A diagram of the primary components of this integrate optimization/booking process simulation is provided in Figure 5.4.

The simulation program is constructed of several different subroutines which are combined by a main program. There are two primary types of subroutines, those which determine the booking limits for each ODF using one of the seat inventory control approaches discussed and those which randomly generate ODF demand and accepts or rejects this demand in accordance with the booking limits. The size of each subroutine, as well as the main program, varies from approximately 100 lines of code to 500. Input files which provide the ODF data, the general set-up of the network being considered, and basic information about which seat inventory control approach to use, the number of revisions, and the number of iterations are necessary to run the simulation program. The output is a simple file which provides information about what ODF demand was accepted and booked versus what was spilled across the network. The expected mean revenue of the network as well as the summary statistics listed above are also included in the output.

The simulation is programmed in Fortran and can run on a PC, a workstation, or a mainframe. In this research, the simulation was primarily run using a DECstation 3100. The amount of time it took to complete a simulation run depended on the size of the network, the number of revisions, the number of iterations, and the actual seat inventory control approach. For small, multi-leg networks, a typical simulation run would take from 45 seconds for a simple leg-based optimization approach to 3-4 minutes for a network

Iteration:

Single simulation of the entire booking process for a set of flight departures.

Revision Point:

Point in time during the booking process in which revisions to the booking limits are made.

Forecast Demand:

Forecasted demand to come before departure for each ODF is determined by summing the incremental ODF demand for all remaining periods.

Optimization:

Booking limits are calculated based on the capacity on each leg, the number of current bookings on hand, and the total forecasted demand to come.

Booking Period:

Time interval between revision points in a booking process.

Generate Demand:

ODF demand is randomly generated based on the incremental demand forecasts for the given period.

Accept/Reject Bookings:

Demand is either accepted and booked or rejected, depending on the respective booking limit. Availabilities are then decremented.

Next Revision Point/Booking Period

Next Iteration

Summary Statistics

Figure 5.4: A diagram of the primary components of the integrated optimization/booking process simulation.

optimization approach. On a large hub-and-spoke network, the times ranged from 20-30 minutes to 4-6 hours. Remember, these times include both the optimization process and the booking simulation for 15-20 revisions over as many as 500 iterations.

5.2 Modeling Issues

A simulation is a mathematical representation of the way in which a process functions. Thus, in an effort to develop a realistic model of the booking process of an airline rather than an abstract representation several aspects of "ambiguity" relating to the airline booking process should be addressed. In the remaining portion of this chapter, issues involving the simulation methodology required to realistically model the booking process and airline seat inventory control practices will be discussed. At the same time, specific modeling benefits of this integrated optimization/booking process simulation over previous simulations of the airline booking process will be highlighted.

Since the focus of this research is on *network* seat inventory control, the integrated optimization/booking process simulation which was developed as part of this project is a network based simulation. While it was necessary to have a network simulation for the purposes of this research, in order to realistically model the booking process and obtain reliable results, even for individual flight legs within a network, a network simulation is important. Passenger demand is for specific OD itineraries which can often be different from the origin and destination of the flight itself. Therefore, simulating bookings at the flight leg level alone and ignoring cross-leg effects can produce misleading results.

For example, consider a two leg connecting network in which one flight travels from point A to point B and connects to the second flight traveling from point B to C. Assuming two fare classes in each market, Y and M, the demand and fares for each ODF itinerary on each flight leg in the two leg network are given in Table 1.3. Using the leg-based

	ODF	Mean Demand	Standard Deviation	Fare
Flight A-B:	ABY	15	5	$550
	ABM	35	10	$350
	ACY	20	7	$700
	ACM	27	9	$450
Flight B-C:	ACY	20	7	$700
	ACM	27	9	$450
	BCY	3	1	$450
	BCM	2	1	$300

Table 5.3: Mean demand, standard deviation of demand and fare for each ODF on each flight leg of the two leg connecting network.

EMSR heuristic to control seat inventories and simulating the bookings on each flight leg *independently*, the potential revenue on the A-B flight leg for a capacity of 45 is $24916.50, at a load factor of 97%. Looking at flight leg B-C exclusively, the estimated revenue potential is $23748.00, with a load factor of 96%. The corresponding mean number of bookings for each ODF on each flight leg are given in Table 5.4.

By not taking into account the underlying itinerary demands and the capacities on each flight leg, the AC traffic across the two flight legs is not consistent. On flight A-B, 18.61 ACY passengers and 5.45 ACM passengers are "carried" while the number carried on flight B-C is 17.67 and 22.32, respectively. Such inconsistencies in itinerary flows can lead to incorrect conclusions as to the performance of a seat inventory control approach and its effects on loads, yields, and revenues [44].

Based on the independent simulation results for the two flight legs, the total expected revenue for both flights is estimated to be $29371.58 (after adjusting for the fact that the revenue from the AC traffic is double counted by dividing the AC fares between the two flight legs in proportion to the local fare levels and, therefore, allocating 55% of the AC fares to flight leg A-B and 45% to leg B-C). However, while the optimization and control of seat

	ODF	Mean Bookings
Flight A-B:	ABY	12.99
	ABM	6.55
	ACY	18.61
	ACM	5.45
Flight B-C:	ACY	17.67
	ACM	22.32
	BCY	2.22
	BCM	1.12

Table 5.4: Mean bookings under the EMSR seat inventory control policy for each ODF based on *independently* simulating bookings on each flight leg of the two leg connecting network in isolation.

inventories is still performed at the flight leg level using the EMSR heuristic, when bookings are simulated for the two flight legs *simultaneously* and the flow of traffic between flight legs is taken into consideration, the total expected revenue is $26,874. Thus, by simulating each flight leg independently, the revenue is overestimated by 9.29%.

Not only can revenues be estimated inaccurately when individual flight legs are analyzed in isolation, but passenger loads can also be overestimated. When bookings are simulated for the two leg network combined, requests are accepted only if seats are available on *each* flight leg traversed by the ODF. Requiring consistent itinerary flows, AC traffic is limited due to the high demand level and constrained seating capacity on flight A-B. This results in a load factor of only 63% on flight B-C, rather than the 96% load factor determined by independently simulating bookings on flight B-C alone.

The passenger load and the potential revenue of an individual flight leg is not only dependent on the seating capacity of that flight leg, but on the capacities and demand on the other flight legs in the network. Thus, in order to realistically simulate bookings at the flight leg level, as well as across a network, the interaction between other flight legs in the network and the flow of traffic across the network must be incorporated. The integrated optimization/booking process allows this to be done, resulting in much more reliable load, yield and revenue results for both flight legs and networks which rely on and are designed for connecting traffic.

While there have been several simulation models used in the airline industry to evaluate revenue potentials and compare seat inventory control methodologies, the majority of these simulations have been static and based on a single random "draw" of ODF demand from aggregated demand densities. That is, *total* ODF demand requests are generated and each request is processed according to predetermined seat protection levels and booking limits. If a seat is available for an ODF, the request is accepted, otherwise it is rejected. Revenue for the entire booking process is then calculated. As in the integrated optimization/booking process simulation, this process is repeated for a large number of iterations, providing an average expected total revenue [37].

The problem with this type of static simulation is that demand is assumed to "arrive" during a *single* booking period. However, in the actual booking process of an airline, bookings from different ODF combinations are interspersed over time and booking limits are continually being updated. Thus, a multi-period decision process exists which allows airlines to correct seat allocations and booking limits throughout the booking process using up-to-date information about the number of bookings on hand and revised forecasts of bookings to come. The end result is similar to a dynamic programming approach where the problem is divided into several stages, or periods of time, and depending on the state of the problem at a given stage, i.e. the number of seats booked at a given period of time, an optimal policy, in terms of protection levels and booking limits, is determined for the remaining time until departure [45].

To better simulate this multi-period decision process, the integrated optimization/booking process is a dynamic simulation program. At specified revision points which are defined by the incremental demand inputs into the simulation, protection levels and booking limits are updated throughout the booking simulation, thus incorporating the seat inventory control methodology, or the optimization policy, into the booking process. Initial booking limits are determined, ODF demand is randomly generated for a distinct period of time and accepted or rejected depending on the availability of seats across the network, and the booking limits are then revised based on the number of passengers booked.

Not only is a dynamic simulation more realistic, but conclusions reached with respect to the extent to which origin-destination and segment seat inventory control methods generate positive revenue impacts can change dramatically between a static and dynamic simulation. In a multi-period decision process, errors in seat protection levels and booking limits due to probabilistic demand can be corrected in future revisions, even for simple leg-based methods. Thus, rather than the previously claimed revenue improvements of 7%, 10%, or even 13% derived from static simulations, a more realistic estimate of the average revenue impact due to network seat inventory control is on the order of 2-4% over effective leg-based control at *high* load factors, as will be shown in Chapter 6.

The integrated optimization/booking process simulation imitates a real booking process by using probability distributions to randomly generate various booking patterns that occur prior to a set of flight departures. However, in randomly generating ODF demands, questions arise as to the appropriate probability distribution which should be chosen for the model. As a first step in determining the distribution of airline demand, historical bookings and reservations data must be analyzed.

In characterizing actual airline demand, variations in the observed data exist due to differences in passenger behavior as a result of such factors as changes in fares, seasonal patterns, day of week and time of day variations, and the frequency of flights in a market. It is important to isolate and remove such variations from the data where it is possible and then select a uniform sample of data to evaluate. At the same time, when demand exceeds its respective booking limit or the capacity of the aircraft, the observed data sample will

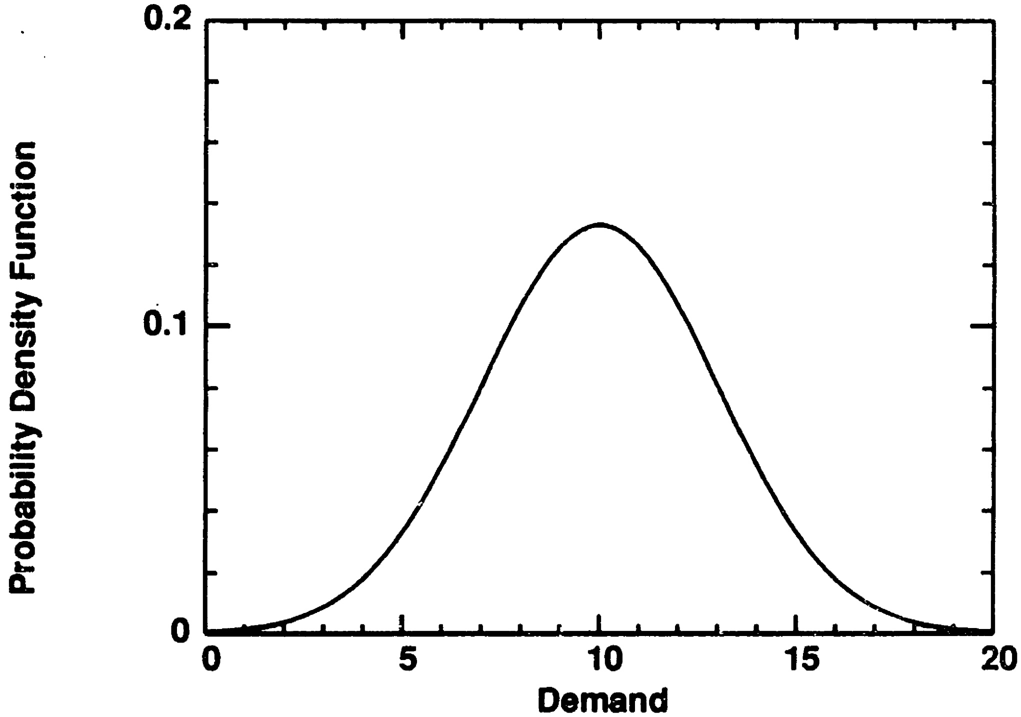


Figure 5.5: An example of a normal distribution with a moderate level of demand.

represent only a portion of the true demand. Thus, based on the observed constrained data sample, the true, unconstrained demand distribution must be estimated.

While there has been a significant amount of analysis done to determine the underlying patterns of airline demand, the results have not led to a consistently accepted distributional assumption for airline demand. In general, several classical parametric distributions tend to statistically fit the data. Thus, the assumed underlying distribution is often left to what an airline may feel is most appropriate based on the characteristics of the particular sample of observed booking data under consideration.

The most common distribution assumption used in the literature [19, 21, 2, 29, 46] and in practice is a normal, or Gaussian, distribution, such as that shown in Figure 5.5. While the normal distribution assumption is reasonable for medium demand levels relative to zero and the booking limit on the demand, at low demand levels, the data shows a significant

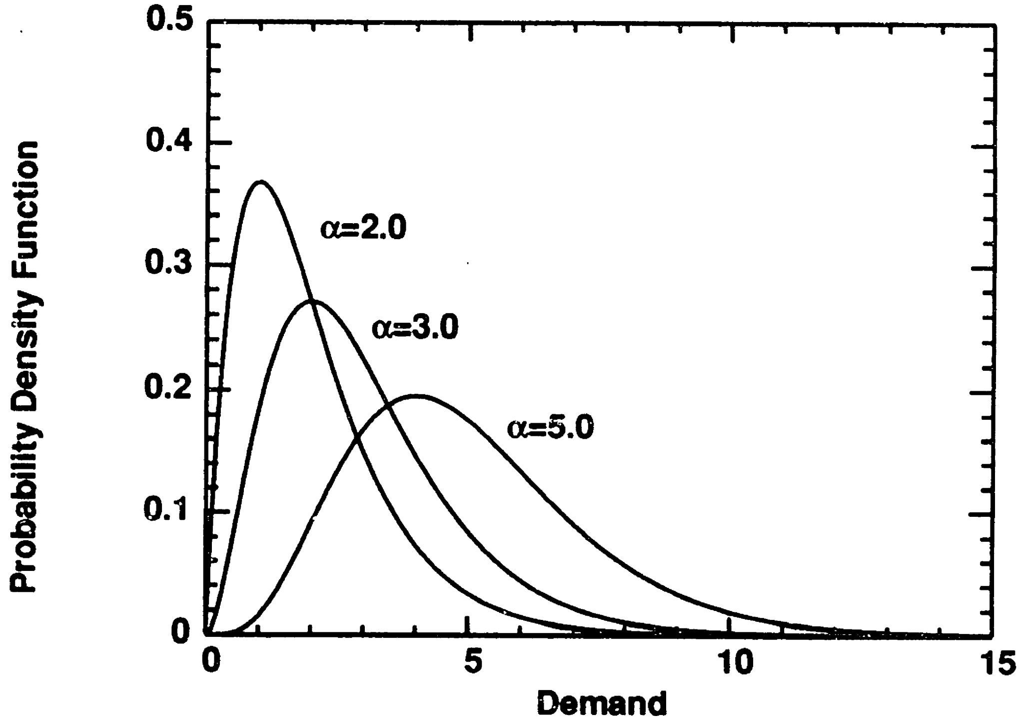


Figure 5.6: An example of the gamma distribution for α = 2, 3, and 5, with $\beta = 1$.

positive skewness [47]. This skewness can be explained by the nonnegative constraint on the *observed* data, while the underlying demand distribution is actually normal. On the other hand, the skewness may be a natural phenomenon of the *true* demand distribution, where the underlying distribution function is undefined for negative quantities while allowing for rare occurrences of extremely high values, leading towards a tendency of positive skewness.

American Airlines [37] suggests that the underlying distribution for airline demand is a gamma distribution. The gamma distribution is a skewed probability distribution which is always nonnegative. An advantage of the gamma distribution is the flexibility in its shape which is determined by different combinations of α and β, the two parameters on which the gamma distribution is defined. An example of the gamma distribution and its flexibility in shape is shown in Figure 5.6.

One inadequacy with the gamma distribution assumption is that the probability den-

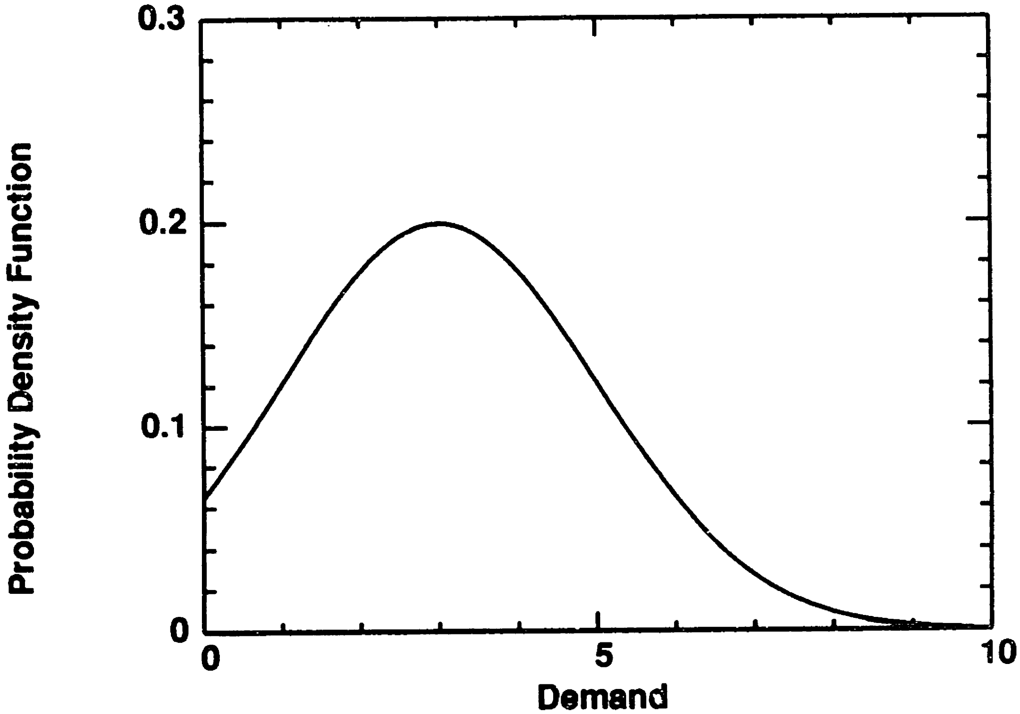


Figure 5.7: An example of a normal distribution truncated a zero. Due to the relatively low mean demand, the value of the probability density function at zero bookings is positive.

sity function is defined only for values greater than zero, thus the value of the gamma distribution at zero is always zero, as seen in Figure 5.6. However, it is entirely possible to have zero bookings, particularly in cases where the mean demand is small, as is the case at the ODF level. Under the normal assumption, when the mean demand is low, the distribution would simply be truncated at zero, allowing for a positive probability of having zero demand, as shown in Figure 5.7.

A primary drawback of both the normal and the gamma distribution assumption is that they are continuous distributions while requests are made in discrete units. By viewing the booking process as a stochastic process, Lee [48] suggests that the bookings can be modeled using a Poisson distribution. The Poisson distribution is a *discrete* distribution which is naturally truncated at zero and demonstrates a significant positive skewness for low mean demands. While the Poisson distribution does not take on negative values, it is

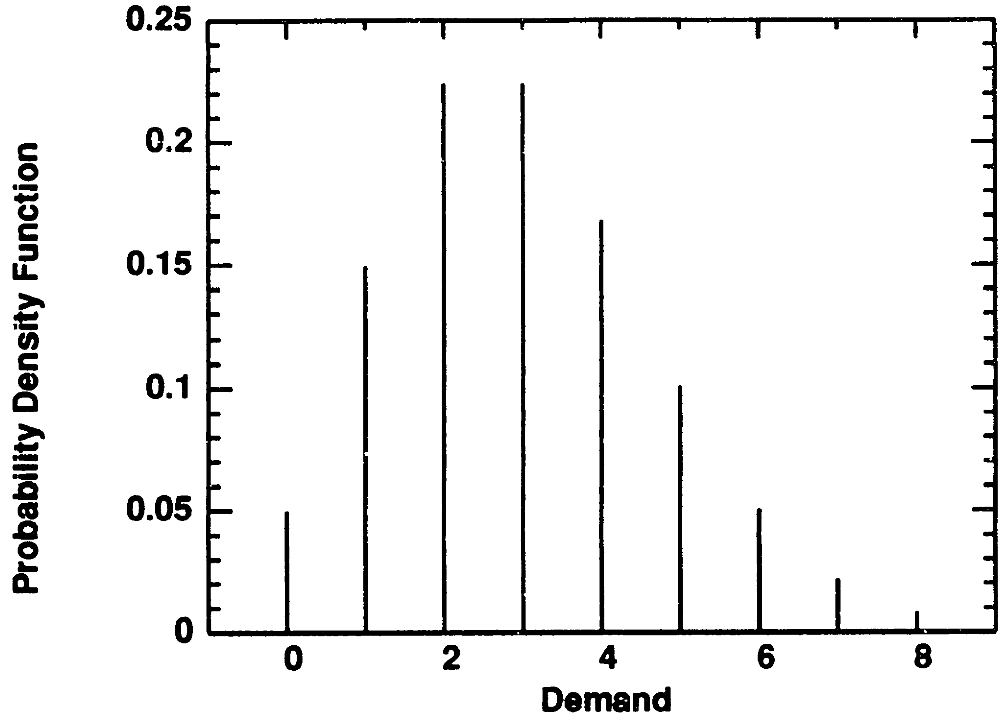


Figure 5.8: An example of a Poisson distribution which is discrete, positively skewed for low mean values, nonnegative, and defined at zero.

defined at zero, unlike the gamma distribution, allowing for the possibility of having zero demand. An example of the Poisson distribution is given in Figure 5.8.

The Poisson distribution, which describes the behavior of arrivals at different points in time, is an intuitive interpretation of the booking process. The Poisson probability function is defined as:

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!} \qquad x = 0, 1, 2, ..., \tag{5.1}$$

where $p(x)$ is the probability that there are exactly x arrivals during a unit of time [41]. The expected value, or mean, of the Poisson probability function is $\mu = \lambda$ and is referred to as the average arrival rate for the process.

Depending on the value of μ, the skewness of the Poisson distribution varies as it does for the gamma distribution. However, the gamma distribution is defined by two parameters, offering flexibility not only in the shape of the distribution, but in the relationship between the mean and standard deviation of the distribution which are both functions of α and β:

$$\begin{aligned} \mu &= \alpha\beta, \\ \sigma^2 &= \alpha\beta^2. \end{aligned} \tag{5.2}$$

The Poisson distribution, on the other hand, is based on the one parameter λ. This parameter is used to define the mean as well as the standard deviation. Under the Poisson assumption, the standard deviation of the distribution is equal to the square root of λ:

$$\sigma = \sqrt{\lambda}, \tag{5.3}$$

which is equivalent to the square root of the mean. Based on empirical studies done as a part of this research and by Lee [48], this property of the Poisson distribution seems to be a reasonable assumption for modeling ODF demand. Yet, unlike the gamma distribution, there is no flexibility in varying the standard deviation with respect to the mean for cases where this assumption does not hold true.

An added benefit of the Poisson distribution is that, like the normal distribution, the sum of two independent Poisson processes is a Poisson process itself. This property is important in terms of the simulation where incremental bookings, bookings to come, and total bookings are all considered. If it is assumed that the incremental bookings are Poisson distributed, it would also be true that bookings to come, which is the sum of future incremental bookings, and total bookings would be Poisson distributed. On the other hand, if incremental bookings are assumed to be gamma distributed, the sum of the incremental bookings, or the total bookings, do not necessarily follow a gamma distribution.

In this research project, the emphasis is on simulating a dynamic booking process over a network of flight legs. Thus, the demand data used in the integrated optimization/booking process simulation is *incremental* demand data at the ODF level which consists of very small numbers. Therefore, the primary distributional assumption used in much of the revenue analysis and evaluation in Chapter 6 is a Poisson distribution. However, the other common distributional assumptions are also tested to determine their effect, if any, on the overall results of the different seat inventory control methodologies.

An extension of the Poisson distribution which has not been considered in this analysis, yet may fit airline demand data very nicely, is the compound Possion distribution. A compound Poisson distribution is a combination of a the usual Poisson process, which describes the arrival rate of demand, with some other distribution which captures information about the total number of bookings made with a given request. Thus, while requests r arrive in accordance with a Poisson process, the actual number of bookings with each request, b_r, does not have to be one, b_r may be 2, 3, or even more [49]. Such a distribution would better characterize the actual behavior of demand where each request for seats on a given ODF itinerary is not always for a single passenger, but may be for a family or a group of colleagues traveling together, etc.

Another issue which should be addressed with respect to historical bookings and reservations data is that of correlation between bookings on hand and bookings to come. While there has been no conclusive evidence of any correlation between the number of bookings and the number of arriving passengers [19], many airline managers intuitively believe that correlation exists. For example, if there is a major event scheduled to take place in a particular city a month from now, it would make sense that bookings would be positively correlated in the market, with higher than normal early bookings and an expected higher than normal number of bookings to come. On the other hand, under circumstances where there is a tendency for people to plan well in advance, higher than normal early bookings can lead to lower than normal late bookings.

Again, due to the lack of solid evidence as to which assumption is accurate, the integrated optimization/booking process simulation was programmed to evaluate seat inventory control methodologies under both assumptions. In general, the more common assumption of no correlation between bookings on hand and bookings to come is used. Thus, incremental demand between revision points is assumed to be independent. Demand is randomly generated for each independent booking period based on the incremental demand data for the period, regardless of the actual demand that has materialized prior to that booking period. In practice, if this independence assumption does not strictly hold, correlation in demand can be captured in the forecasting process. At each revision point, information about the current number of bookings on hand can be used to reforecast the expected bookings to come.

The alternative method to modeling demand which has been evaluated in this research is a case where demand from one booking period to the next is assumed to be correlated. Instead of independently determining demand for each booking period, total demand for the booking process is randomly generated based on the sum of the incremental demand data for an ODF. This total demand is then *randomly* distributed, one request at a time, across the booking periods according to the proportion of demand that historically books in each period. By modeling the demand process in this manner, demand between booking periods is positively correlated, with higher than average demand in the first booking period often followed by higher than average demand in the second booking period, etc. This occurs since the actual level of demand for an ODF is not generated incrementally, but all at one time. Thus, if a total demand of 32 is randomly generated from a demand distribution of 26 ± 7, distributing the higher than average total demand between booking periods gives rise to higher than average incremental demand for the ODF in each booking period.

Once demand is generated, whether independently for each booking period based on incremental demand forecasts or as total demand for the entire booking process, the booking sequence of demand, i.e. the order in which demand is actually accepted as bookings, must be determined. The common assumption used in much of the literature is that lower valued fare classes book before higher valued fare classes [19, 22, 23, 3]. While lower fare class passengers do have a tendency to book earlier than high fare class passengers due to differences in passenger behavior as well as restrictions on discounted fares, the "lowest fare class books first" assumption is not always valid.

For the two fare class case, Titze and Griesshaber of Lufthansa [50] showed that this assumption does not have a major influence on optimal seat allocations. Since early bookings from a higher fare class yield more revenue than low fare bookings, the impact of errors in the actual seat allocations is reduced. However, it can also be seen from the analysis done by Titze and Griesshaber that the actual *level* of revenue generated on a flight *is* influenced by the "lowest class books first" assumption.

When considering a static simulation, the booking sequence assumption can have a significant effect on the level of revenues. More importantly, the effect is not always consistent between seat inventory control methodologies, particularly when comparing partitioned versus nested control approaches. In a partitioned approach, inventories of seats are not shared between ODF's, and therefore, the sequence of bookings within a static booking process does not have any effect on revenues. However, under a nested approach, the "lowest class books first" assumption does not take complete advantage of the benefits of "shared" inventories. Thus, in a high demand situation where the "lowest fare class books first", although higher fare class demand in theory has access to seats allocated to lower fare classes, the lower fare classes which book early will completely fill their seat allocations, leaving no "extra" seats for the high fare classes.

The advantage of a dynamic simulation is that much of the actual booking sequence of passengers is modeled through the multi-period booking process. Rather than having to make an explicit assumption about the booking sequence between fare classes as well as OD itineraries, a close approximation of the booking sequence is given by the incremental ODF demand in each booking period. For example, an integrated, multi-period booking

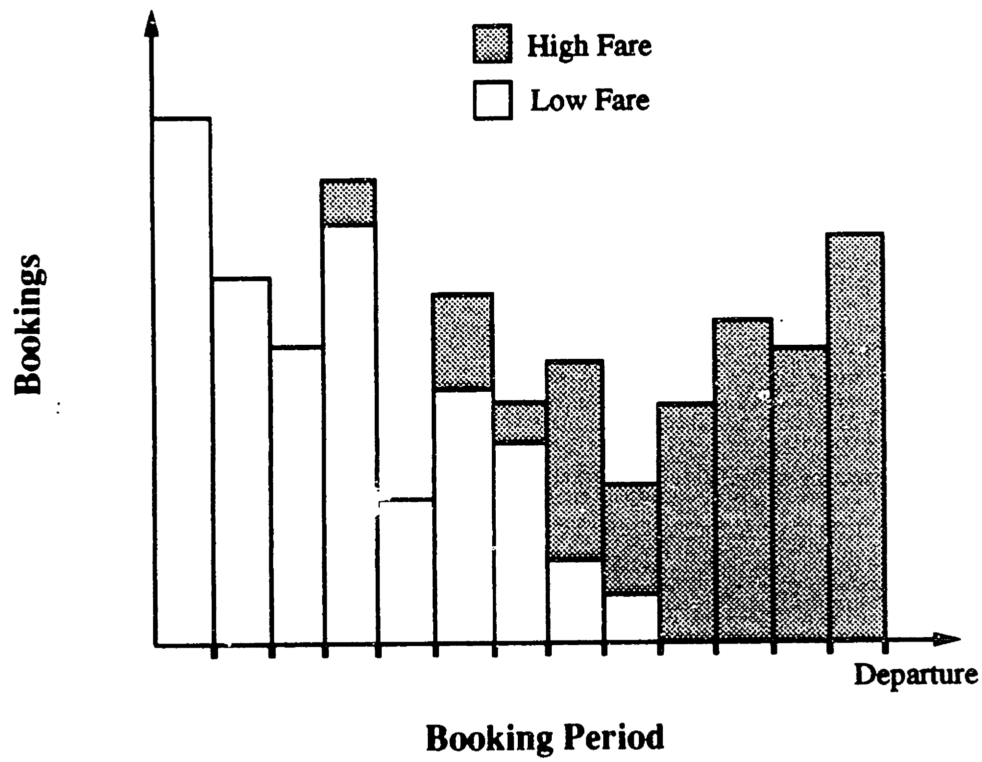


Figure 5.9: An integrated, multi-period booking profile for two fare classes. While the tendency is for low fare passengers to book first, through the multi-period booking process, the overlap between low fare and high fare bookings is modeled.

profile for two fare classes, such as that shown in Figure 5.9, can be modeled, allowing low fare and high fare passengers to book in parallel.

While the general booking sequence is defined by the incremental ODF demand, an assumption of the order in which demand is booked *within* each booking period must still be made. There are several assumptions which have been suggested, ranging from low to high fare class bookings to high to low fare class bookings to booking OD and fare class demand simply at random, with no particular ordering. Under a dynamic simulation, the number of bookings which occur within a given booking period is relatively small due to the frequency of revisions. Rather than determining the booking sequence of 20 Y class requests, 30 M class requests, 15 B class requests and 35 Q class requests, the number of bookings within a booking period may be on the order of 6 to 10. At the same time, much of the ordering is already implicit in the demand patterns of the incremental time periods. Thus, there may be one Y class request, a couple of M class requests, a B class request, and 4 or 5 Q class requests, or 3 Y class requests, 2 M class requests, a B class request and no Q class requests. With so few total bookings and often a rather uneven distribution, the booking sequence assumption within each booking period does not have much effect on the overall simulation results.

Due to the natural tendency of early discount bookings, the primary assumption used in the integrated optimization/booking process is to book demand from low to high fare classes within each booking period. Under this approach, the arrival order of different OD itineraries within a fare class is determined randomly based on the proportion each respective ODF mean demand estimate is of the total fare class demand during the booking period. In order to verify that this particular form of the "lowest class books first" assumption does not yield any biases, a second booking assumption was also tested in this research where all demand, regardless of its itinerary or fare class, is assumed to be constant across the period of time between revisions and therefore, is randomly booked.

A final note about the integrated optimization/booking process simulation is in regard to decrementing seat availabilities. Availabilities are decremented in one of two basic ways depending on the control methodology. In the case of a partitioned, or distinct, approach, the method of decrementing availabilities is fairly straightforward. Upon accepting an ODF request, the availability of the particular ODF, or inventory bucket, is decremented on each flight leg over which the accepted ODF traverses. For example, consider a connecting two flight leg network A-B-C with two fare classes offered on each of the three itineraries AB, AC, and BC. The assumed set of distinct seat allocations, which are equivalent to partitioned booking limits, are given in Table 5.5. Under a partitioned inventory control structure, upon accepting an ABM request, the ABM availability of 10 on flight leg A-B is simple decremented by one while no other availabilities are affected. For an ACY request,

Flight A-B		Flight B-C	
ABY	25	BCY	15
ABM	10	BCM	20
ACY	10	ACY	10
ACM	15	ACM	15

Table 5.5: Partitioned ODF booking limits on each flight leg of a two leg A-B-C network.

Flight A-B		Flight B-C	
ABY	60	ACY	60
ACY	35	BCY	50
ACM	25	ACM	35
ABM	10	BCM	20

Table 5.6: Nested ODF booking limits for the two leg A-B-C network which are determined using the seat allocations in Table 5.5.

the ACY inventory buckets on both flights A-B and B-C are decremented from 10 to 9. As before, no other availabilities are changed.

For a nested inventory structure, availabilities of the respective inventory bucket, as well as all *higher* inventory buckets, are decremented on each flight leg over which the ODF traverses. Thus, based on the partitioned seat allocations in Table 5.5, an example of nested ODF booking limits on each flight leg are shown in Table 5.6. For an ABM booking, since the itinerary is associated with the lowest ranked inventory bucket on flight leg A-B, the ABM availability of 10 is decremented by one. At the same time, due to the nesting structure, the ABY, ACY, and ACM inventories are also decremented, resulting in nested availabilities of 59, 34, 24, and 9. For an ACY booking, on flight leg A-B the availabilities of the ACY inventory bucket and the ABY inventory bucket are decremented by one while the ACM and ABM availabilities are not affected, resulting in availabilities of 59, 34, 25, and 10, respectively. On flight leg B-C, since ACY is the highest nested inventory bucket, the availability of ACY is decremented from 60 to 59, but no other ODF availabilities are altered. On the B-C flight leg, assuming there are no revisions in the forecasts or optimization, as many as 10 ACY bookings would be accepted before the availabilities of other inventory buckets would be affected.

For a nested system, the rationale behind decrementing availabilities of the respective inventory bucket *and* all higher inventory buckets is based on the philosophy behind the optimization techniques evaluated. Seats are allocated to a given ODF based on its forecasted demand. These seats are then nested, protecting the seats from less desirable ODF's while making the seats available to more desirable ODF's. In this manner, a higher revenue, more desirable request will not be denied as long as there are seats available. As in a partitioned control approach, under a nested approach if demand materializes for an ODF, the seats protected, and therefore the availability, of the ODF is decremented. At the same time, since the *same* inventory of seats is made available to inventory buckets nested above the particular ODF, the seat availabilities of those inventory buckets are also decremented. Note, however, that the actual protection levels of the higher nested inventory buckets remain the same.

For example, on flight leg A-B in the previous example, if an ACY request is accepted, the seat protection of 10 is reduced to 9. This seat protection level is based on the forecasted demand density for ACY. Thus, demand which materializes for ACY reduces the expected demand to come and in turn, the number of seats which should be protected for ACY. The revised nested seat availabilities for each inventory bucket can now be determined. Revising the availabilities step by step, with one seat now booked on leg A-B, the remaining capacity is 59, rather than 60. Since ABY is the highest ranked ODF inventory on flight A-B, all remaining seats, or 59, are made available to ABY. Subtracting the 25 seats protected for ABY from 59 gives an availability of 34 for ACY, the second highest nested ODF. With one accepted request in ACY, there are now 9 seats protected for ACY, rather than 10, resulting in an availability of 34 – 9, or 25, for ACM. Finally, subtracting the 15 seats protected for ACM from 25 leaves a seat availability of 10 for ABM. These (decremented) booking limits are the same as those obtained from decrementing the availability of the respective inventory bucket and all higher inventory buckets.

The dynamic, integrated optimization/booking process simulation program developed in this research is believed to be the first of its kind, and the multi-period calculation and revision of booking limits in response to the "actual" bookings simulated has proven to be important. The simulation has been used extensively to test a wide variety of network seat inventory control approaches on an assortment of networks. In many instances, the comparative results and revenue impacts of different methods change dramatically between the static simulation used in the past and the dynamic booking process simulation now available. These results will be shown and analyzed in detail in the next chapter.

Chapter 6

Analysis and Comparison of Network Seat Inventory Control Approaches

As stated earlier, one of the primary interests in developing new and different approaches for controlling seat inventories on a flight leg and across a network of flight legs is the potential for increasing revenue. The purpose of this chapter is to evaluate and compare the revenue impacts of the different optimization and control strategies introduced in Chapter 4. Using actual airline data and the integrated optimization/booking process simulation described in Chapter 5, the different network seat inventory control approaches are compared and contrasted for both linear multiple leg networks and a connecting hub-and-spoke network.

The different network seat inventory control approaches will first be applied to the segment control problem in which seat inventories are controlled and bookings managed by *on-flight* OD and fare class itineraries, or "segments", over a multiple leg flight. The segment control problem is regarded by many airlines as the first step in the development of more advanced network seat inventory control capabilities. At the same time, several airline reservations systems already have the capability to manage seat inventories by different segments and fare classes on multiple leg flights. Once the different network seat inventory control approaches have been analyzed in detail in the context of the segment control problem, a comparison of the more promising approaches will be made on a hub-and-spoke example for which the impacts of full origin-destination network seat inventory control will be examined.

The examples used in this chapter to evaluate the revenue impacts of different seat inventory control approaches are based on real airline data for both the multi-leg flights and hub-and-spoke network. By using historical airline data, a realistic mix of fare classes, as well as the mix of local versus through traffic, is obtained. At the same time, historical data provides a realistic representation of the booking profile of each OD and fare class combination. Thus, by using real data the *actual* bookings for each ODF as a function of time prior to departure can be simulated.

With the use of the integrated optimization/booking process simulation, the same sequence of randomly generated demand is reproduced identically for each alternative seat inventory control strategy. Thus, the differences in the simulation results can be attributed to the optimization and control methodology used rather than to other factors. By varying the capacities on each flight leg, the different seat inventory control approaches are evaluated over a range of demand conditions, allowing for a comparison of revenue impacts to be made as the demand factor (i.e the ratio of demand to capacity), increases, and in turn, the choices between different OD and fare class itineraries across a network become much more critical to the realization of incremental revenue.

6.1 Detailed Analysis and Comparison of Revenue Impacts on Multiple Leg Flights

Throughout this research project, a variety of multiple leg flights have been used to test and evaluate the different network seat inventory control alternatives described in Chapter 4. The results from a sample of those flights will be provided here. While by

Figure 6.1: Flight 31–a three leg flight consisting of a long haul flight leg with two shorter, medium haul flight legs on each end. The bottleneck leg is B-C with 15% more demand than the average over the three leg flight, while the level of demand on flight leg A-B is relatively low.

	A-B	B-C	C-D
Local:	60	78	90
Through:	30	52	30
Total:	90	130	120

Table 6.1: Approximate breakdown of the mean demand for the local versus through traffic on each leg of Flight 31.

no means do these results hold for all cases, the results shown are a representation of the revenue impacts obtained from the set of flights evaluated. A few "non-typical" examples will also be given, showing the type of inconsistencies which can occur.

The revenue impacts for the different multi-leg flights shown throughout this section are based on a simulation of incremental ODF demand data for 15 booking periods and thus, 15 revision points throughout the booking process of a flight. For each multi-leg example, the simulated revenue impacts are based on 500 iterations of the complete booking process. Incremental demand is initially assumed to follow a Poisson distribution, with bookings on hand and bookings to come assumed to be independent. A sensitivity analysis of the revenue impact results with respect to these two assumptions is presented in Appendix A.

Results from four different multi-leg flights will be analyzed initially. The first of these flights is a three leg flight, hypothetically called Flight 31. As shown in Figure 6.1, Flight 31

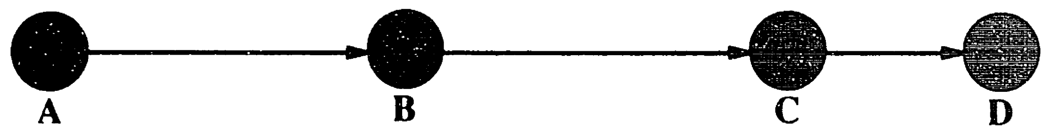


Figure 6.2: Flight 32–a three leg flight consisting of roughly medium haul flight legs. While the bottleneck leg is B-C, the variation in the demand level across all three flight legs is not very large. Multi-leg, through traffic makes up the majority of the demand on flight legs A-B and B-C, while 25% of the traffic across leg C-D is through demand.

	A-B	B-C	C-D
Local:	60	55	75
Through:	40	60	25
Total:	100	115	100

Table 6.2: Approximate breakdown of the mean demand for the local versus through traffic on each leg of Flight 32.

consists of a long haul flight leg B-C, with two shorter, medium haul flight legs A-B and C-D. An approximate breakdown of the mean demands for the local versus through traffic on each flight leg is provided in Table 6.1. As would be expected, the middle leg of the flight is the most constrained, with 15% more demand traversing the B-C leg than the average over the three leg flight. Of this demand, 60% is local while 40% is some form of through traffic, consisting of two leg itinerary demand (AC or BD) and demand for the three leg itinerary AD. The amount of through demand on both flight legs A-B and C-D is approximately the same. However, due to heavy local demand on flight leg C-D, the demand level on the leg is also quite high, leaving flight leg A-B as the relatively low demand leg of the multi-leg flight.

The revenue impacts of a second three leg flight will also be evaluated. This flight,

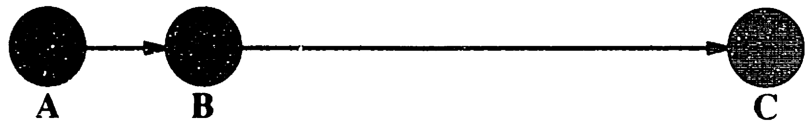


Figure 6.3: Flight 21–a two leg flight consisting of a medium haul flight leg and a long haul leg. Approximately 40% of the average demand over the flight is multi-leg AC requests with flight leg A-B the more heavily traveled leg of the two leg network.

	A-B	**B-C**
Local:	52	40
Through:	30	30
Total:	82	70

Table 6.3: Approximate breakdown of the local versus through demand on each leg of Flight 21.

shown in Figure 6.2, will be referred to as Flight 32. The distance between each point served by Flight 32 is roughly a medium haul trip (averaging 625 miles) with flight leg C-D being a little shorter than legs A-B and B-C. The variation in the demand level between the three flight legs is not as large as that of Flight 31 with a difference in the total mean demand between the three legs of only about 15 (Table 6.2). Once again, the middle flight leg is the bottleneck leg. More than half of the demand across this leg through traffic. The through demand on flight leg A-B is also fairly high, while only 25% of the demand on leg C-D is multi-leg traffic.

A third flight which will be considered is Flight 21, a two leg flight consisting of a medium haul flight leg A-B ($\sim$ 600 miles) and a long haul flight leg B-C as shown in Figure 6.3. The medium haul flight leg is the more heavily traveled leg with nearly 18% more demand on the leg than the long haul B-C leg. A breakdown of the demand across the flight is given in Table 6.3. Overall, Flight 21 is a relatively weak flight in terms of

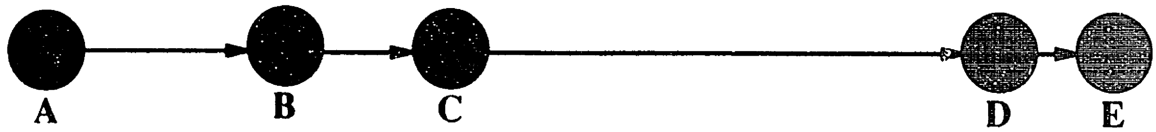


Figure 6.4: Flight 41–a four leg flight made up of two medium haul flight legs, a long haul flight leg, and a short haul flight leg. A range of demand mixes exists across the flight, with 85% of the demand on one leg being through traffic and over 10% more local demand than through on another leg.

	A-B	B-C	C-D	D-E
Local:	32	12	35	45
Through:	46	66	65	40
Total:	78	78	100	85

Table 6.4: Approximate breakdown of the local versus through demand on each leg of Flight 41.

total demand with an *average* demand per flight leg of only 76, 40% of which is multi-leg AC traffic.

The fourth flight for which revenue impacts will be shown is a four leg flight made up of two medium haul flight legs, a long haul flight leg, and a short haul flight leg, Figure 6.4. As shown in Table 6.4, there is quite a variety of demand mixes across the four legs of Flight 41 . On the B-C flight leg, 85% of the demand is through traffic, leaving only 15% as local traffic. Although the majority of the demand on flight leg A-B and C-D is also multi-leg demand, the split between local and through traffic is much closer to half. However, on the short haul D-E flight leg, there is over 10% more *local* demand than through. Flight leg C-D is the bottleneck leg with a mean demand of 18-28% more than that on the other three legs. Legs A-B and B-C each have a relatively low level of demand compared to the average across the flight.

The point behind controlling seat inventories by OD itineraries as well as fare classes is to obtain additional revenue above that which is currently generated using traditional leg-based seat inventory control approaches. Thus, it is the impacts on revenue in comparison to such leg-based approaches that is of interest rather than the absolute revenues obtained or the impacts over no reservations control practices at all. Therefore, the revenue results throughout this chapter will be shown as a function of the percent difference in the expected revenue between the particular network seat inventory control approach and a leg-based control approach.

Two of the more common and effective leg-based approaches used in the industry today are the Expected Marginal Seat Revenue (EMSR) heuristic [2] and the Optimal Booking Limit (OBL) approach [3]. While the revenue difference between the EMSR heuristic and the OBL approach has been shown to be within 1% under a static comparison [22, 23], using the integrated optimization/booking process simulation and making revisions to booking limits throughout the booking process, the difference in the revenues obtained under the EMSR heuristic and the OBL approach is greatly reduced. While in extreme cases these differences are 0.1-0.2%, in most cases the difference is well below this, on the order of hundredths of a percent. A comparison of the revenue potential for the two leg-based approaches is shown in Figure 6.5 for the three leg Flight 31. The expected revenue obtained using the EMSR heuristic and that generated under the OBL approach are almost identical with the slight difference in revenues being statistically insignificant at a 95% confidence level. Such negligible differences have also been found on other flights evaluated in this research project. Since the difference in the expected revenue between the EMSR heuristic and the OBL approach is so small, for the sake of consistency, revenue comparisons throughout this chapter will be made on the basis of the simpler, less computationally intensive EMSR heuristic.

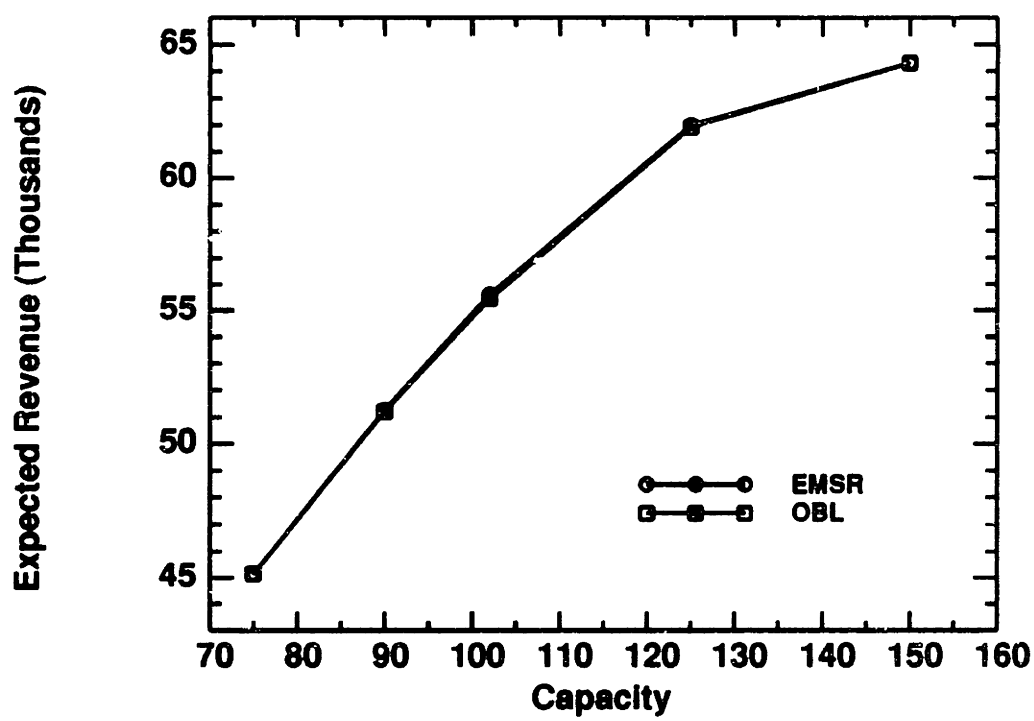


Figure 6.5: Comparison of the expected revenue obtained for Flight 31 under the EMSR heuristic and the OBL approach.

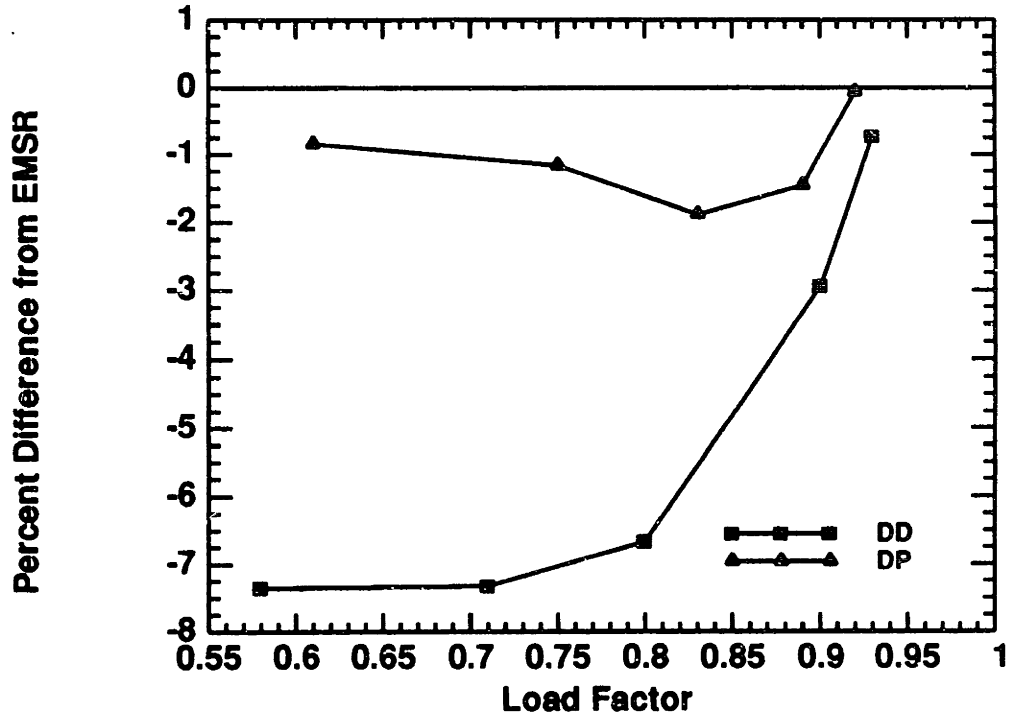


Figure 6.6: Comparison of the Distinct Deterministic (DD) approach and the Distinct Probabilistic (DP) seat inventory control approach for Flight 21.

6.1.1 Partitioned Network Methods

The "optimal" network seat allocations obtained from either the deterministic mathematical programming formulation (Equation 4.1) or the probabilistic formulation (Equation 4.3) can be used directly as partitioned, or distinct, booking limits. However, as suggested in Chapter 4, implementing the seat allocations in this manner can produce significant negative revenue impacts. In Figure 6.6, the Distinct Deterministic (DD) and the Distinct Probabilistic (DP) approaches are compared for the two leg Flight 21. By varying the capacity of the flight, thereby increasing and decreasing the level of demand in relationship to capacity, the revenue impacts over a range of load factors are compared. Due to differences in the actual demand accepted under each specific inventory control method, the average load factor for a given demand level is not always consistent between the two approaches. However, each successive point on the different revenue impact curves is based on the same level of demand.

For the relatively low demand factor of 0.61, it is shown in Figure 6.6 that the DD approach yields 7.4% *less* revenue (at an average load factor of 58%) than the leg-based control approach, while the DP approach has a negative impact on revenues of only -0.8% at an average load factor of 61% for the same demand level. At the highest demand factor of 1.28, the DD approach has a -0.7% impact on revenues at a 93% load factor while the DP approach has essentially no effect on revenues (-0.06%) when compared to the EMSR heuristic with an average load factor of 92% across the two leg flight.

While significant improvements in the revenue impacts occur under the Distinct Deterministic (DD) approach as the level of demand increases, the Distinct Probabilistic (DP) approach performs better than the DD approach over the full range of load factors shown. In both cases, however, the revenue generated using the partitioned network seat allocations as booking limits does not yield positive results. Looking at the three leg Flight 32, Figure 6.7 shows that at *very* high demand levels a positive impact in revenues over the leg based EMSR heuristic can be obtained using the partitioned network approaches. Yet, at lower load factors negative impacts are once again generated by both methods.

While the positive impacts of the Distinct Probabilistic approach are statistically significant at the high load factors, based on statistics from one major carrier in the U.S., only about 7% of all flights have a 95% or higher load factor. At the same time, the overall average load factor for major carriers is between 60 and 65%. Although the impacts on revenues across the entire range of load factors will contribute to the overall potential impact on airline revenues, it is the negative impact at the lower load factor range which is the most significant and will have the greatest effect on total airline revenues. Thus, a network approach based on implementing the "optimal" seat allocations from the *partitioned* network formulations of the problem directly as booking limits is not an effective network seat inventory control method for increasing revenues across a network of flights.

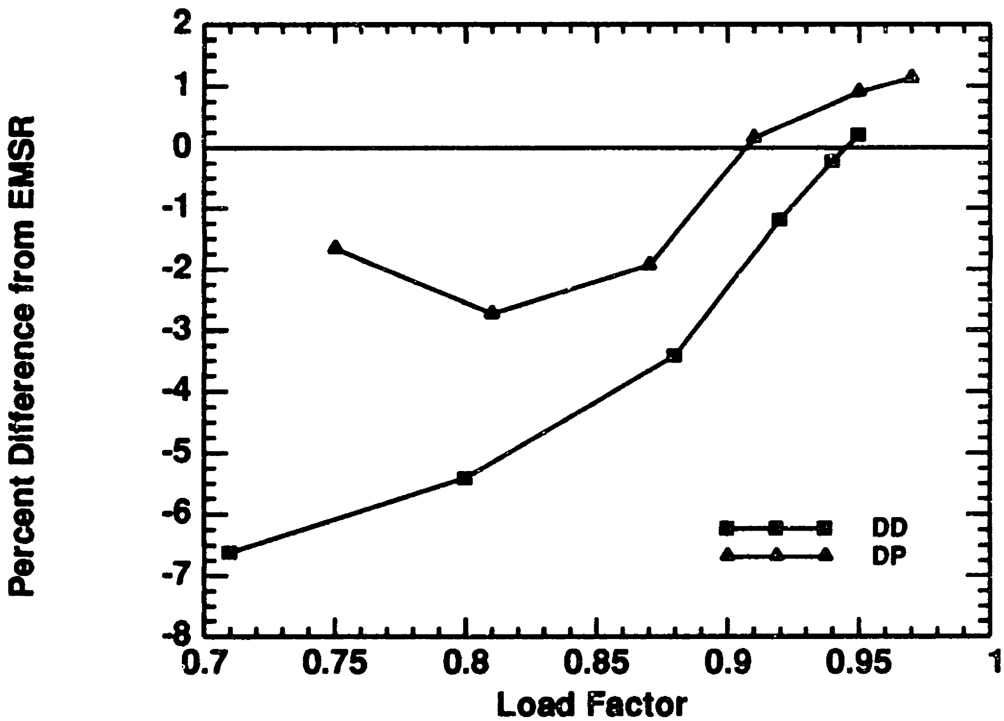


Figure 6.7: Comparison of the Distinct Deterministic (DD) approach and the Distinct Probabilistic (DP) network approach for Flight 32.

6.1.2 Nested Network Heuristics

Although the *assumed* mathematical formulation of the network seat inventory control problem yields an "optimal" solution, it has been shown that implementing such seat allocations as distinct booking limits does not maximize revenue for the *real* problem at hand. Using such a "theoretical" approach can actually cause an airline to lose as much, and more, in revenues than it can make. As discussed in Chapter 4, to overcome this problem and still obtain the benefits of incorporating true network flows, nesting of the ODF seat allocations is considered. Three different nesting strategies were suggested in Chapter 4 and are examined here: nesting by fare class, nesting by fares, and nesting by shadow prices.

Figures 6.8, 6.9, and 6.10 compare the results from nesting the two network solutions by fare class, fares, and shadow prices, respectively, for the three leg Flight 31. In Figure 6.8, it is shown that by nesting the deterministic and the probabilistic seat allocation solutions by fare class, significant positive revenues can be obtained at high load factors when compared to the leg-based EMSR heuristic, with the Nested Deterministic by Fare Class (NDFC) approach outperforming the Nested Probabilistic by Fare Class (NPFC) approach, yielding as much as a 2.1% increase in revenues. Although the NPFC approach results in positive revenue impacts at the higher load factors, it also shows negative impacts at load factors below 90%. While these negative impacts are not as large as that generated under the partitioned approaches (up to -3.2% under the Distinct Probabilistic approach for Flight 31), the negative impact of the NPFC approach is significant.

Figure 6.9 shows somewhat inconsistent results when the greedy method of nesting on fares is used. Nesting the deterministic solution by fares (NDF) leads to small positive revenue impacts for the load factor range between 83% and 96%. However, at the highest load factors where there is a comparatively high demand level and thus, one would not

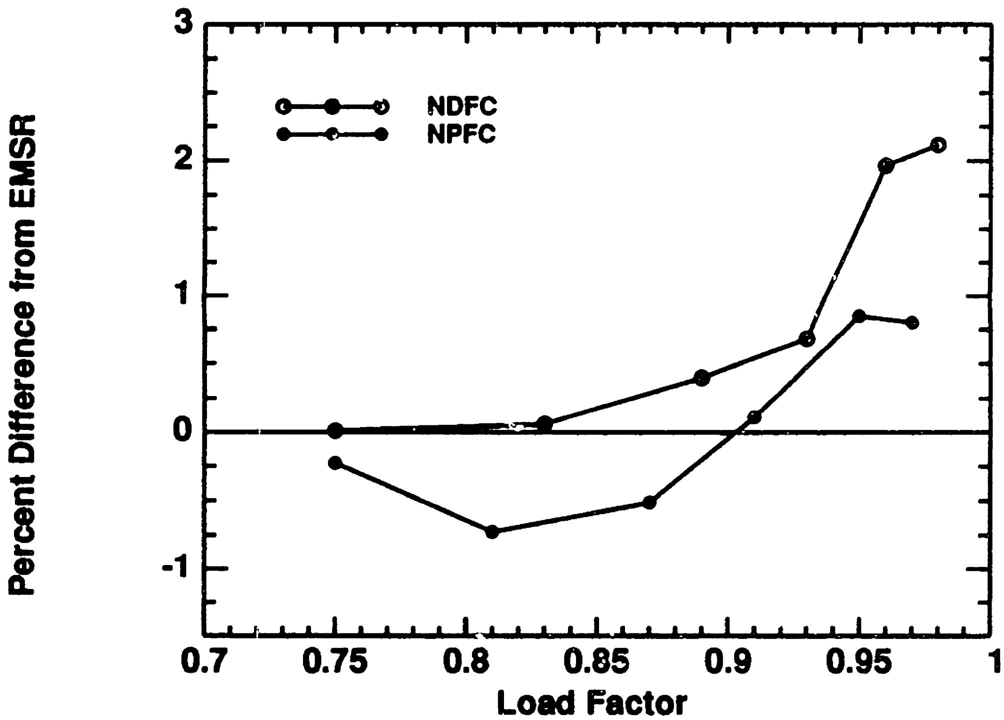


Figure 6.8: Comparison of the Nested Deterministic by Fare Class (NDFC) approach and the Nested Probabilistic by Fare Class (NPFC) approach for Flight 31.

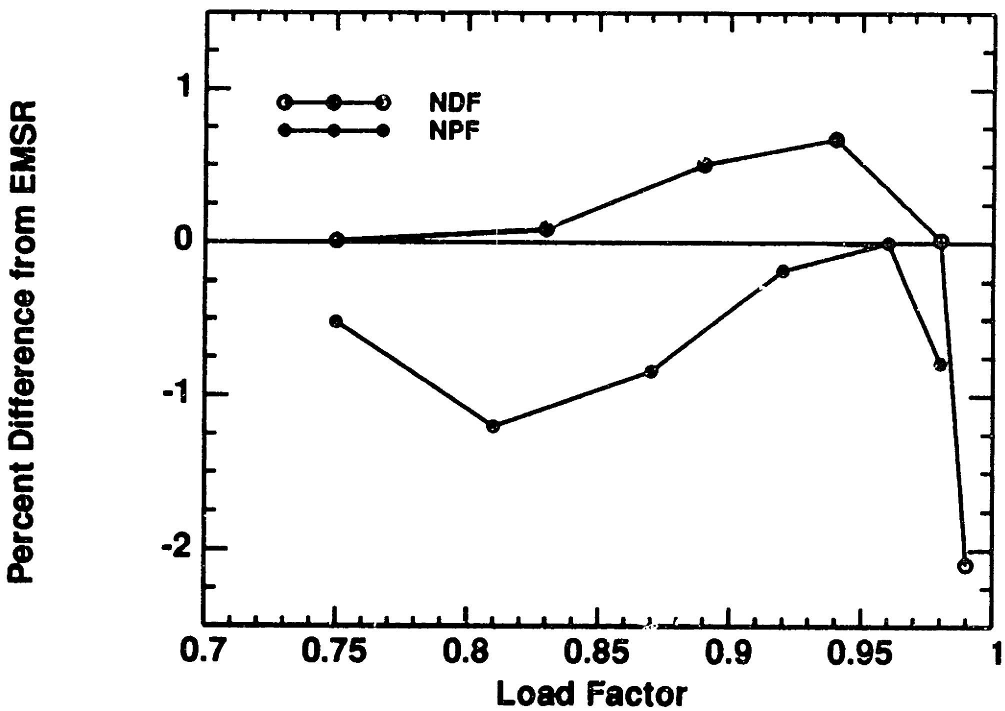


Figure 6.9: Comparison of the Nested Deterministic by Fares (NDF) approach and the Nested Probabilistic by Fares (NPF) approach for Flight 31.

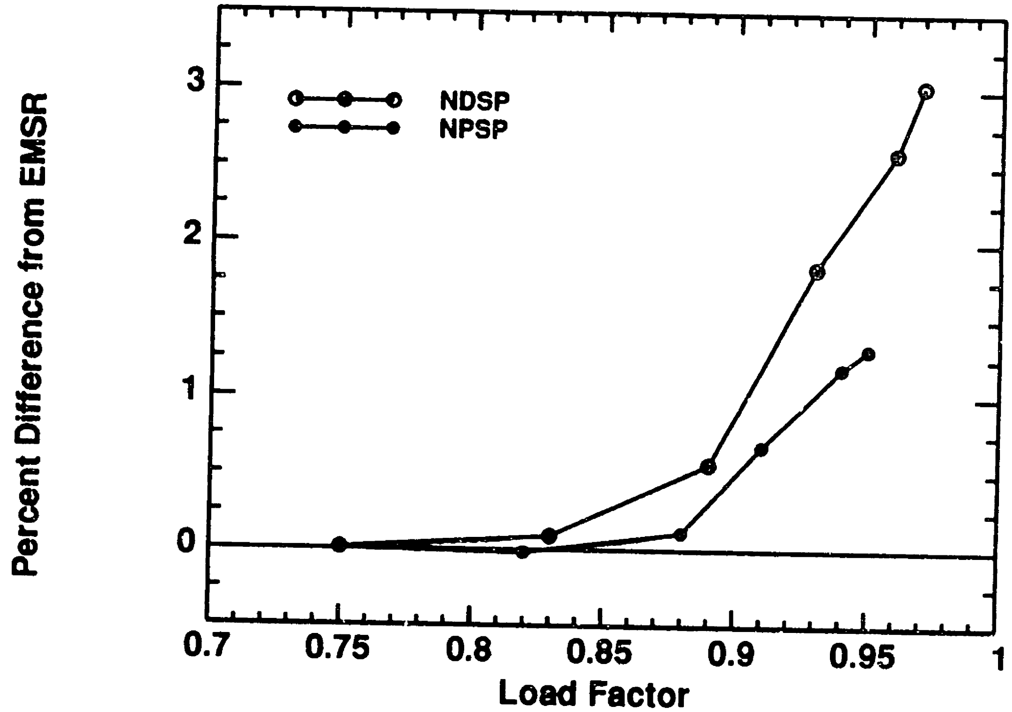


Figure 6.10: Comparison of the Nested Deterministic by Shadow Prices (NDSP) approach and the Nested Probabilistic by Shadow Prices (NPSP) approach for Flight 31.

want to fill up the flight with long haul, low yield demand, the level of revenues generated under the NDF approach falls off drastically. While the percentage of flights above 95% load factor is relatively small, these flights do bring in a disproportionately high percentage of total revenues. Therefore, this extreme drop-off in revenues can be quite significant. For the same flight, Figure 6.9 shows that the Nested Probabilistic on Fares (NPF) approach does not perform well over the *entire* range of load factors.

Nesting by shadow prices, Figure 6.10, does show some promising results. Although there are not significant positive results at the lower load factors, it is important to notice that there are not negative revenue impacts either. At such low load factor levels, the capacity of the flight is able to accommodate most of the demand. Therefore, there is little to no potential for generating additional revenues by differentiating between ODF's across a network of flights. However, as the level of demand increases, and in turn the average load factor increases, the revenue impacts of the two network approaches also increase, yielding a substantial positive impact at the highest load factors. Once again, the nested deterministic method, NDSP, does substantially better than the Nested Probabilistic by Shadow Prices (NPSP) approach, yielding a 3.0% increase over the EMSR heuristic at the highest demand level versus a 1.3% increase under the NPSP approach.

Comparing the three Figures 6.8, 6.9, and 6.10, the network approaches nested by shadow prices perform at least as well and often much better than the network approaches nested by fare class and those nested by fares. Another example of this is provided in Figures 6.11 and 6.12 for the three leg Flight 32. In these two figures, a comparison of the nested deterministic approaches and the nested probabilistic approaches, respectively, are shown. In the deterministic case, the nested by fares (NDF) approach once again drops off sharply at high load factors. Although the nested by fare class (NDFC) approach performs well up to a load factor of 96%, it also drops off at the highest load factor. However, the Nested Deterministic by Shadow Prices (NDSP) approach performs well, even at the highest demand level where the average demand is 1.4 times the capacity of the three leg flight.

For Flight 32, the revenue performance of the NDFC approach is repeated in the probabilistic case. Rather than the NPFC approach performing better at higher load factors, as was the case for Flight 31, the revenue impact of the NPFC approach in Figure 6.12 is significantly worse at the higher load factors. Neither the NPFC approach or the NPF approach result in positive revenue impacts compared to leg-based control at any load factors. The Nested Probabilistic by Shadow Prices (NPSP) approach, on the other hand, does show some positive revenue impacts at the highest demand level and performs consistently better than the NPFC approach and the NPF approach at all load factors.

While the nested by shadow price approaches outperform the other nesting strategies (as expected), but it is interesting to note that although the probabilistic solution performs

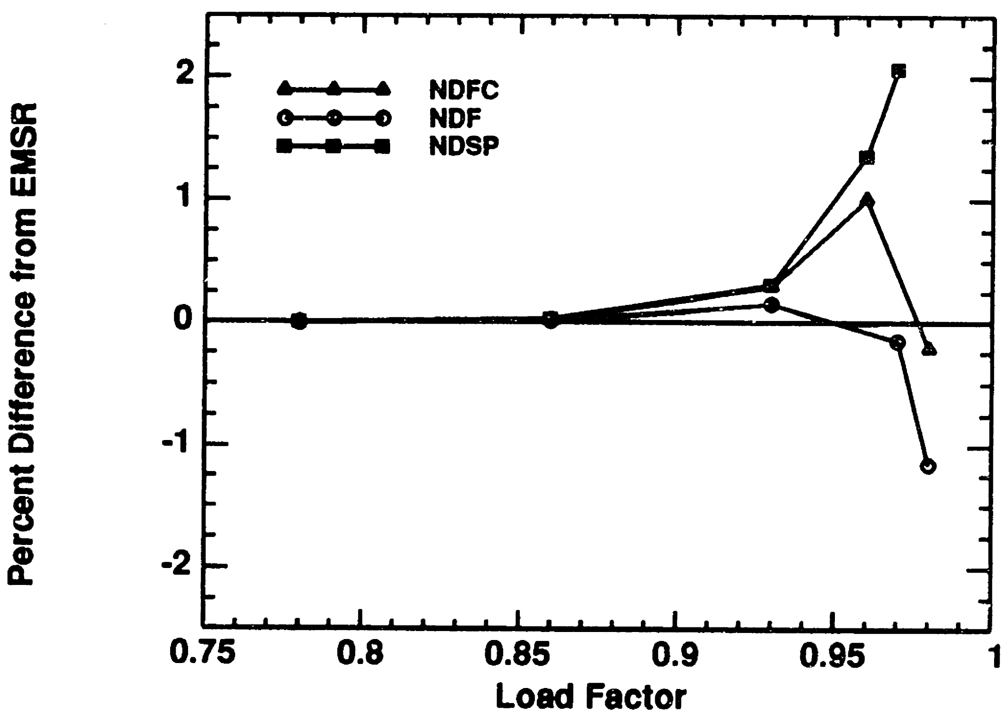


Figure 6.11: Comparison of the Nested Deterministic by Fare Class (NDFC) approach, the Nested Deterministic by Fares (NDF) approach, and the Nested Deterministic by Shadow Prices (NDSP) approach for Flight 32.

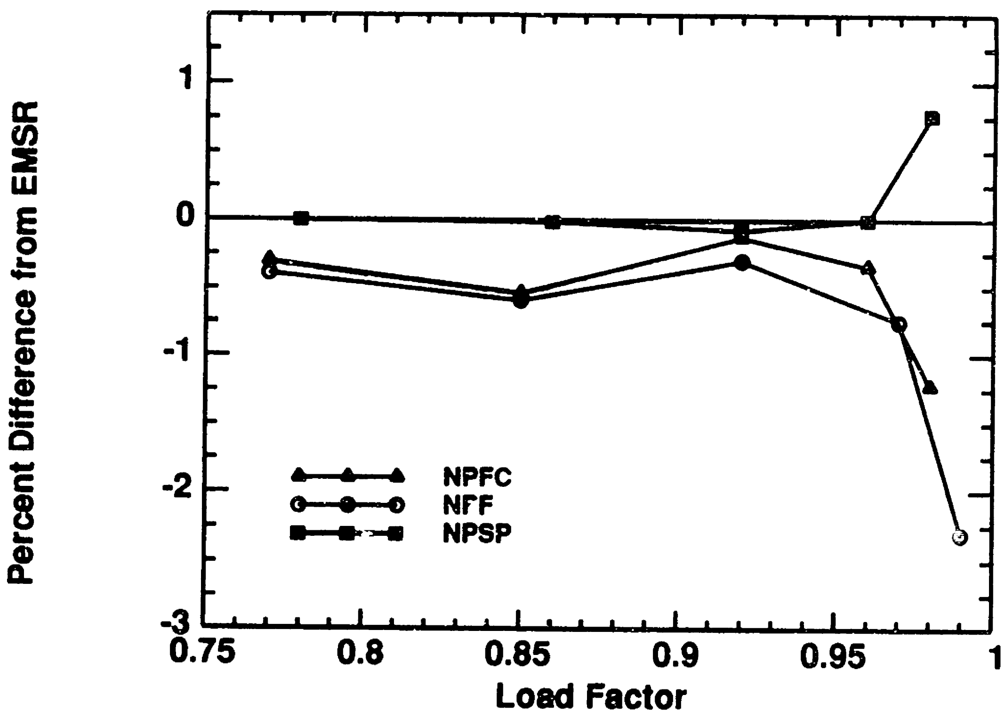


Figure 6.12: Comparison of the Nested Probabilistic by Fare Class (NPFC) approach, the Nested Probabilistic by Fares (NPF) approach, and the Nested Probabilistic by Shadow Prices (NPSP) approach for Flight 32.

	Y	M	B	Q
AB	25	3	7	26
AC	2	1	4	14
AD	3	1	0	0
BC	10	22	12	15
BD	6	0	0	0
CD	19	56	5	0

Table 6.5: Partitioned deterministic ODF booking limits for the three leg example used in Chapter 4, Figure 4.1 and Table 4.1.

better in the partitioned, non-nested case, the nested deterministic methods consistently outperform the nested probabilistic methods. The reason for this is that the probabilistic network solution tends to "overprotect" seats for the more desirable and higher fare class ODF's. For example, consider again the linear three leg network used in Chapter 4, Figure 4.1 and Table 4.1. The partitioned, or distinct, deterministic ODF booking limits for the network are provided in Table 6.5, while the partitioned probabilistic ODF booking limits are shown in Table 6.6.

In this example, the ABY itinerary is allocated 30 seats in the probabilistic solution, or 5 more seats than the mean ABY demand of 25. Due to the stochastic nature of demand, there are times when the demand for itinerary ABY will be greater than 25, and since the ABY itinerary is a fairly desirable ODF to the network as a whole, under the probabilistic formulation *additional* seats are allocated to ABY to take advantage of this potential in "excess" demand, i.e. demand above the forecasted mean. Under a distinct, non-nested control strategy, the benefits of saving these additional seats for potential ABY demand are worth the loss of less desirable ODF demand, such as demand for the ACQ itinerary which is allocated only 5 seats in probabilistic solution versus 14 seats in the deterministic

	Y	M	B	Q
AB	30	5	10	31
AC	3	1	2	5
AD	3	0	0	0
BC	12	23	9	24
BD	6	2	0	0
CD	21	50	5	3

Table 6.6: Partitioned probabilistic ODF booking limits for the three leg network used in Chapter 4, Figure 4.1 and Table 4.1.

solution. However, under a "nested by shadow price" strategy, a more desirable ODF such as ABY is ranked high in the nesting hierarchy, having access to seats allocated to lower ranked ODF's. Thus, rather than *explicitly* allocating additional seats to ABY for those times when there is "excess" demand, any "excess" demand can be accommodated through nesting.

Since "excess" demand of more desirable ODF's has access to additional seats through nesting, the need for allocating extra seats to these ODF's is not as critical. At the same time, while there is the possibility of ABY demand exceeding its forecasted mean of 25, there are also times when ABY demand will actually be less than its mean value and the extra 5 seats allocated to ABY under the probabilistic approach will not be needed. As with a partitioned implementation methodology, these extra 5 seats allocated to the more desirable ABY itinerary, rather than a less desirable itinerary such as ACQ, are often "caught", being nested in the ODF hierarchy at a level in which the less desirable ODF's do not have access to them. Thus, while more desirable ODF's can have access to additional seats through nesting without explicitly allocating such seats to the ODF's, excess seats allocated to the more desirable ODF's are not made available to less desirable ODF's for those cases when the seats are not used.

When nesting the partitioned solutions in Table 6.5 and Table 6.6 by shadow prices, the top three ranked ODF's (ABY, ACY, and ADY) on the A-B flight leg have an additional 6 seats protected under the probabilistic approach than under the deterministic approach where the number of seats allocated is determined on the basis of the mean ODF demands. Looking at the top six ranked ODF's on flight leg A-B, which again includes the same ODF's for both approaches, the probabilistic approach protects an additional 16 seats over that of the deterministic approach. It is this overprotection of seats which causes the *nested* probabilistic approaches to not perform as well as the *nested* deterministic approaches in terms of expected revenues.

This overprotection of seats not only causes the nested deterministic approaches to perform better than the nested probabilistic approaches, but it can cause the Nested Probabilistic by Shadow Price approach to show the same type of inconsistency in revenue impacts as the nested by fare class and the nested by fares approaches. This has been found to be common on two leg flights, such as Flight 21 in Figure 6.13. While the NPSP approach performs well at the high load factor level, substantial negative revenue impacts are shown to occur at the 84% and 90% load factor levels, corresponding to an average demand factor of .85 and 1.02, respectively. At the 93% load factor, corresponding to a 1.28 demand factor, the positive revenue impact over leg-based control is due to the ability of the network approach to differentiate between different OD's within a fare class. However, at the 84% and 90% load factor levels, due to the overallocation of seats to the more desirable, high fare class OD itineraries on the heavily constrained A-B flight leg, low fare class demand is spilled early in the booking process resulting in empty seats at departure time. As was the case for Flight 31 and Flight 32, the Nested Deterministic by Shadow Prices (NDSP) approach performs quite well on the two leg flight, producing significant positive revenue impacts at the highest demand levels.

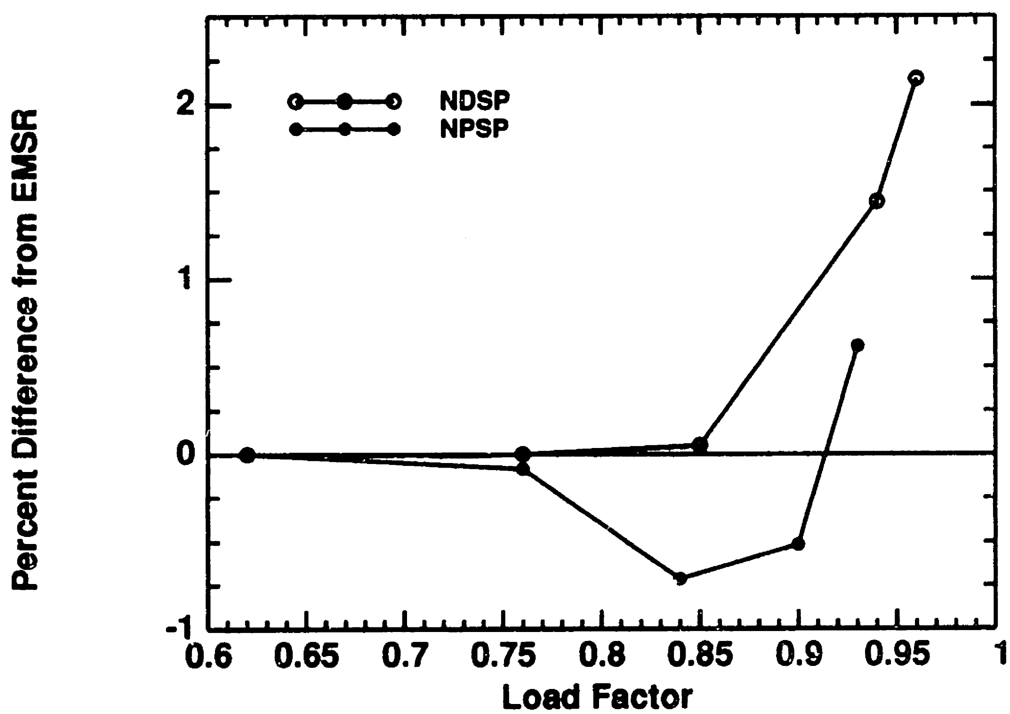


Figure 6.13: Comparison of the Nested Deterministic by Shadow Prices (NDSP) approach and the Nested Probabilistic by Shadow Prices (NPSP) approach for Flight 21.

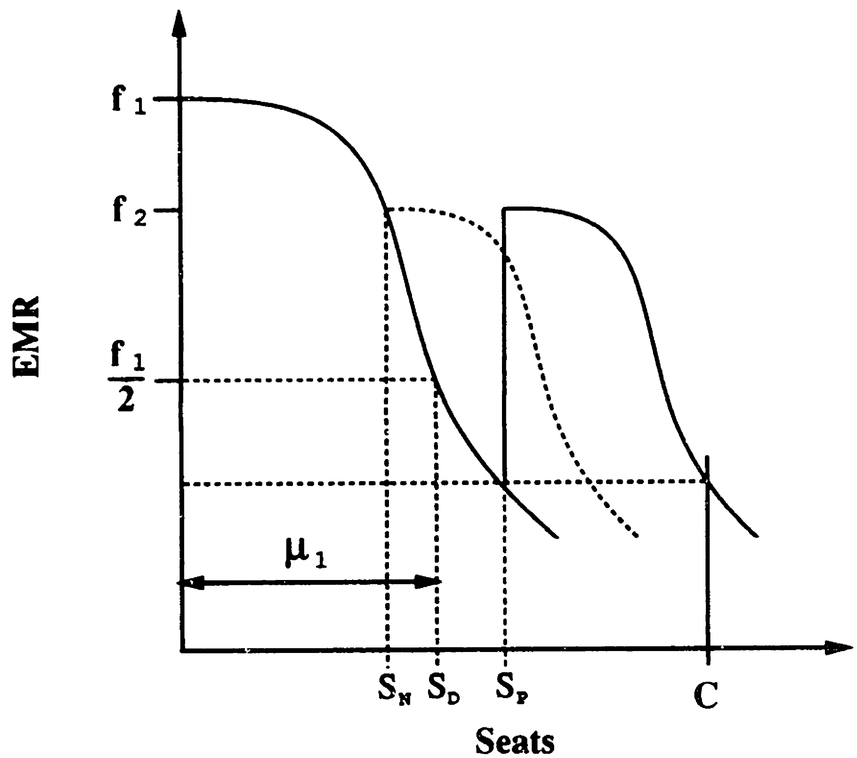


Figure 6.14: Comparison of the seat protection levels from a partitioned deterministic approach, S_D, a partitioned probabilistic approach, S_P, and Littlewood's [19] *optimal* nested seat allocation rule, S_N, for a simple two fare class example.

It is not the case that the deterministic network seat allocations are the "optimal" nested ODF allocations, it is just that the deterministic seat allocations tend to be closer to the optimal nested protection levels than the probabilistic allocations from a static, non-nested formulation of the network seat inventory control problem. For instance, consider a simple two fare class, single itinerary example. According to Littlewood's rule [19], the optimal seat protection level for a nested inventory structure is the point at which the expected marginal revenue of potentially selling an additional seat in the higher fare class is equal to the fare of the lower fare class, i.e. S_N in Figure 6.14. Compare this with the partitioned deterministic seat allocation, S_D, and the partitioned probabilistic allocation, S_P. While the partitioned deterministic seat allocation still overestimates the true nested protection level, it is a much closer estimate than the partitioned probabilistic allocation.

Mean Demand	Deterministic Allocation	Probabilistic Allocation
25.2	25	28
25.1	25	28
24.8	25	28
24.0	24	28
22.8	23	28
22.0	22	26
20.4	20	26
19.3	19	26
16.9	17	25
15.6	16	23
12.3	12	21
9.2	9	19
8.6	9	18
5.9	6	15
2.6	3	11

Table 6.7: Comparison of the partitioned deterministic and partitioned probabilistic network seat allocations over 15 revisions for an ODF itinerary on the three leg Flight 31.

Not only does the partitioned probabilistic network approach initially overprotect seats for the more desirable ODF's, but this overprotection of seats has a tendency of being compounded as the booking process proceeds. An example of this is shown in Table 6.7 for an ODF itinerary on the three leg Flight 31 over the 15 revision points. Notice that as the time prior to departure is reduced, and thus the mean demand to come decreases, the difference between the mean demand and the partitioned probabilistic allocation grows. The reason for this is that, in many cases, the "extra" seats initially allocated to the more desirable ODF's, at the expense of less desirable ODF's, are not used. As time progresses, total ***demand to come*** decreases faster than the *remaining* capacity on the flight due to these unused "extra" seats, leading to a situation in which excess capacity is created by the actual allocation and control process. It is this excess capacity which causes the gap between the mean demand to come and the probabilistic allocations to grow. While the unused "extra" seats can be redistributed as revisions to seat allocations and booking limits are made, the less desirable ODF's often correspond to lower fare class itineraries which are associated with advance purchase restrictions. Thus, after a certain point during the booking process, demand from the lower fare class, less desirable ODF's is cut off, leaving only the more desirable ODF demand. Yet, it is the "extra" seats from these ODF's which need to be redistribution.

Although network optimization models may produce "optimal" allocations in a theoretical sense, it is the method of controlling bookings for different itineraries and fare classes that determines the extent to which revenues can be increased above that realized from simple leg-based control. Using information from the solution of the network optimization to nest ODF itineraries by shadow prices consistently provides better results than nesting by fare class or by fares. At the same time, when basing a nested control strategy on partitioned network seat allocations, it is the deterministic allocations, rather than the probabilistic allocations, which provide the better estimates of the optimal *nested* protection levels. Without knowing in advance the "correct" nesting hierarchy of different ODF's over a network, it is both infeasible and impractical to formulate a network optimization which explicitly takes into account the nesting of ODF's. Such an formulation is needed to determine the optimal nested ODF protection levels.

6.1.3 Network Bid Prices

Based on the same network optimization formulations, bookings can be managed in a completely different manner using a bid price control philosophy. As described in Chapter 4, the bid price approaches are based on the concept of comparing the fare of an ODF itinerary with the marginal value to the network of the last seat on each flight leg over which the ODF traverses. If the fare of the itinerary is greater than the sum of the bid prices of the respective flight legs, requests for the ODF are accepted as bookings, otherwise the ODF is closed to further bookings.

Figure 6.15 compares the bid price control approach based on the deterministic network

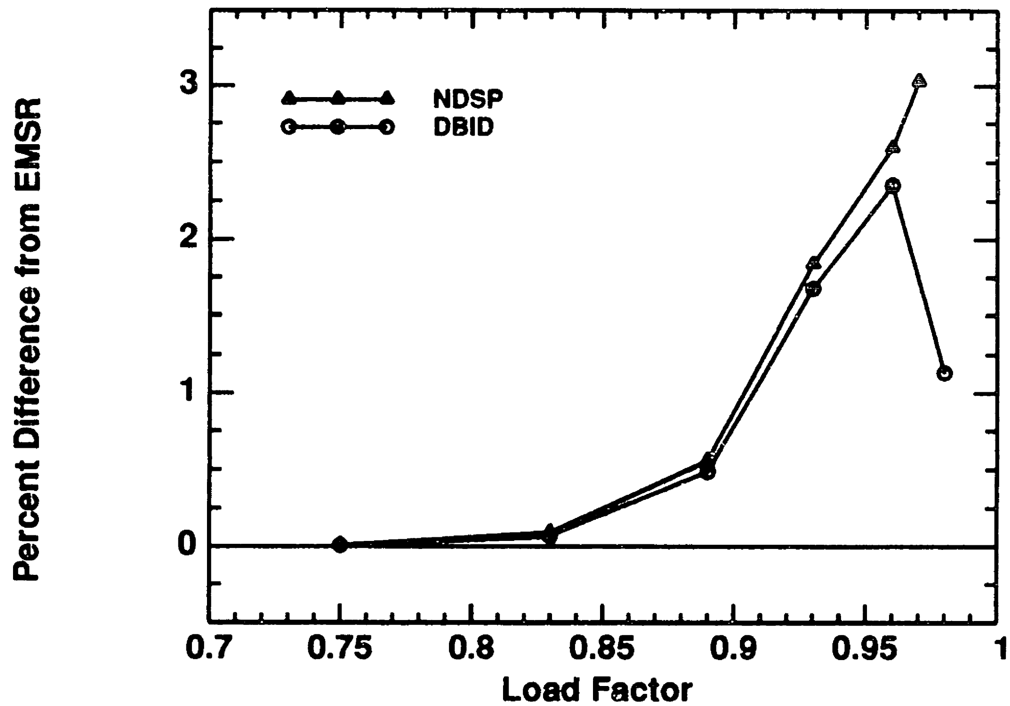


Figure 6.15: Comparison of the revenue impacts for the Deterministic Bid Price (DBID) approach with those of the Nested Deterministic by Shadow Prices (NDSP) approach for Flight 31.

solution (DBID) with the Nested Deterministic by Shadow Price (NDSP) approach for the three leg Flight 31. The revenue impact curve of the Deterministic Bid Price approach follows the NDSP curve very closely up until the highest load factor level (corresponding to a demand factor of 1.5), at which point the DBID approach drops off significantly. This drop-off in revenues is due to the control methodology itself, rather than a problem with the optimization. Under the bid price method, an ODF itinerary is either open for bookings or closed. If it is open, until the aircraft capacity is reached there is no limit to the number of bookings which will be accepted during each booking period. Therefore, if the fare of an ODF exceeds the current bid price for the itinerary, any and all demand for the ODF will be accepted until the bid prices on each flight leg are revised. Thus, the open/closed control philosophy of the bid price approach can result in sub-par performances when there are a limited number of revisions (such as 15), particularly at high demand levels.

The reason the expected revenues of the DBID approach are so close to those of the NDSP approach is that the two approaches use the same deterministic network formulation, combined with the marginal value information from the solution of this formulation to determine if an ODF is open to bookings. The only difference between the two methods is the actual control methodology. Under the NDSP approach, a limit on the number of seats an ODF has access to is established while under the DBID approach, no limit to the number of bookings is determined if an ODF is open.

The performance of the Probabilistic Bid Price (PBID) approach versus the Nested Probabilistic by Shadow Prices (NPSP) approach is shown in Figure 6.16. While the PBID revenue curve is not as close to the NPSP curve as DBID is to NDSP, the trend of the PBID curve is similar to that of the DBID revenue curve with a drop-off in revenues occurring at the highest demand level. The drop-off in expected revenues which occurs under both bid price approaches can be overcome through more frequent revisions. With the bid price on each flight leg being revised more often, there is less time between revisions for "uncontrolled" bookings.

For the two leg Flight 21, the relationship between the DBID approach and the NDSP approach, Figure 6.17, is similar to that of Flight 31, with the revenue impacts of the DBID approach resembling that of the NDSP approach. However, in this case, the PBID approach actually does better than the NPSP approach for the medium demand levels tested due to the open/closed control methodology of the bid price approach. With the probabilistic solution overprotecting seats for the more desirable, high fare class OD itineraries on the heavily constrained A-B flight leg, very limited availability is left for the less desirable, low fare class demand. Under the NPSP approach, "excess" seats are allocated to the more desirable, higher fare class ODF's and thus, all but a few requests for the less desirable, low fare class itineraries are rejected, resulting in negative revenue impacts. Under the PBID approach, since strict limits are not applied to each ODF, all requests for the low fare class

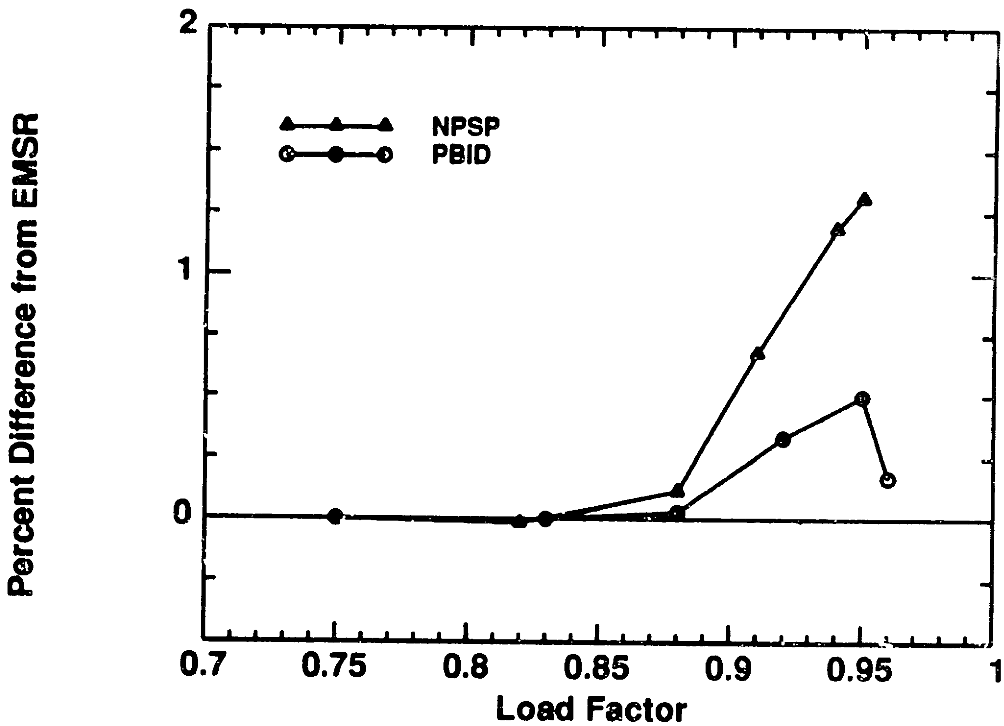


Figure 6.16: Comparison of the revenue impacts from the Probabilistic Bid Price (PBID) approach with those of the Nested Probabilistic by Shadow Prices (NPSP) approach for Flight 31.

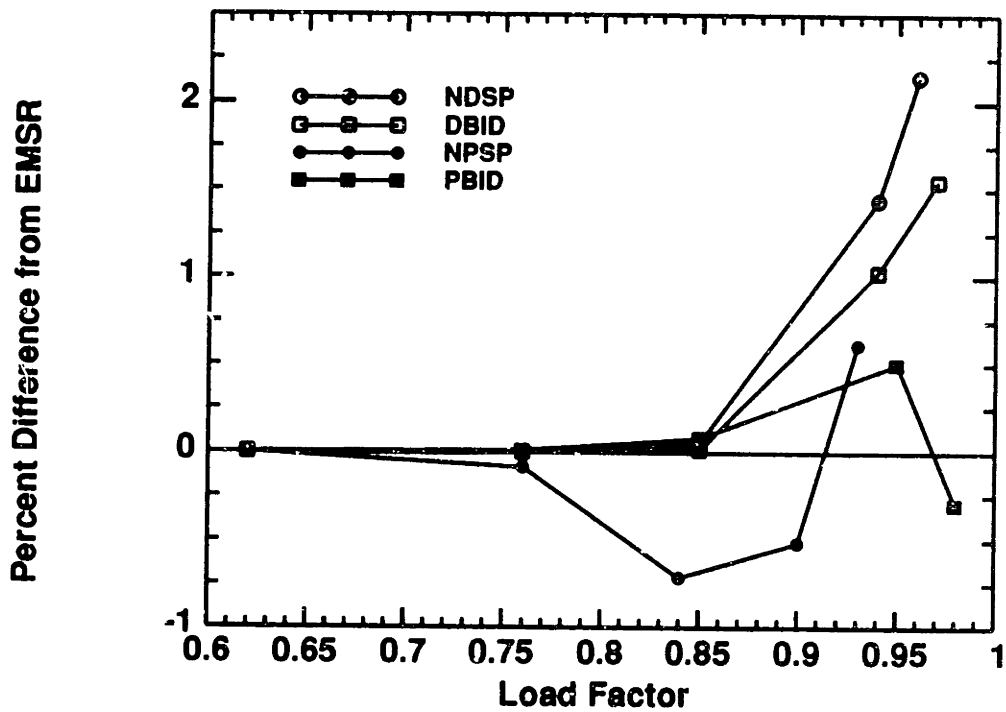


Figure 6.17: Comparison of both the Deterministic and Probabilistic Bid Price (DBID and PBID) approaches and the Nested Deterministic by Shadow Prices (NDSP) and Nested Probabilistic by Shadow Prices (NPSP) approaches for Flight 21.

itineraries on flight leg A-B are accepted early in the booking process until revisions are made to the bid prices. Thus, rather than leaving empty seats at departure time which results in a negative impact on revenues, the PBID approach produces a small positive revenue impact at the .85 and 1.02 demand factor levels, corresponding to the 85% and 95% load factor levels. Similar to that of Flight 32, the PBID approach again drops off at the 98% load factor level due to the high demand level and limited number of revisions.

As was the case for the nested deterministic and nested probabilistic approaches, the bid price approach based on the deterministic formulation of the network seat inventory control problem consistently outperforms the probabilistic bid price approach. Once again, this is due to the partitioned probabilistic network optimization which overprotecting seats for the more desirable ODF itineraries. This overprotection of seats leads to probabilistic bid prices which are often *higher* than the deterministic bid prices and sometimes "too high", particularly on critical flight legs. A graphical example showing why the probabilistic bid price is often higher than the deterministic bid price is provided in Figure 6.18.

Figure 6.18a shows the distinct probabilistic seat allocation solution for a single flight leg where seats are allocated one at a time to the fare class or ODF itinerary with the highest expected marginal revenue of potentially selling the additional seat. When the total number of seats assigned to the different inventory classes reaches capacity, the values of the last seat of *each* inventory class are equal. This common value is the partitioned probabilistic network bid price for the flight leg. By comparing the actual seat allocations under the probabilistic optimization (Figure 6.18a) and the deterministic optimization (Figure 6.18b), an "overallocation" of seats to the highest two inventory classes is found in the probabilistic solution. Thus, the last seat allocated to each inventory class is located on the portion of the expected marginal revenue curve which is beyond the mean demand value of the inventory class. Due to these "extra" seats allocated to the highest inventory classes, the capacity constraint intersects the expected marginal revenue curve of the third

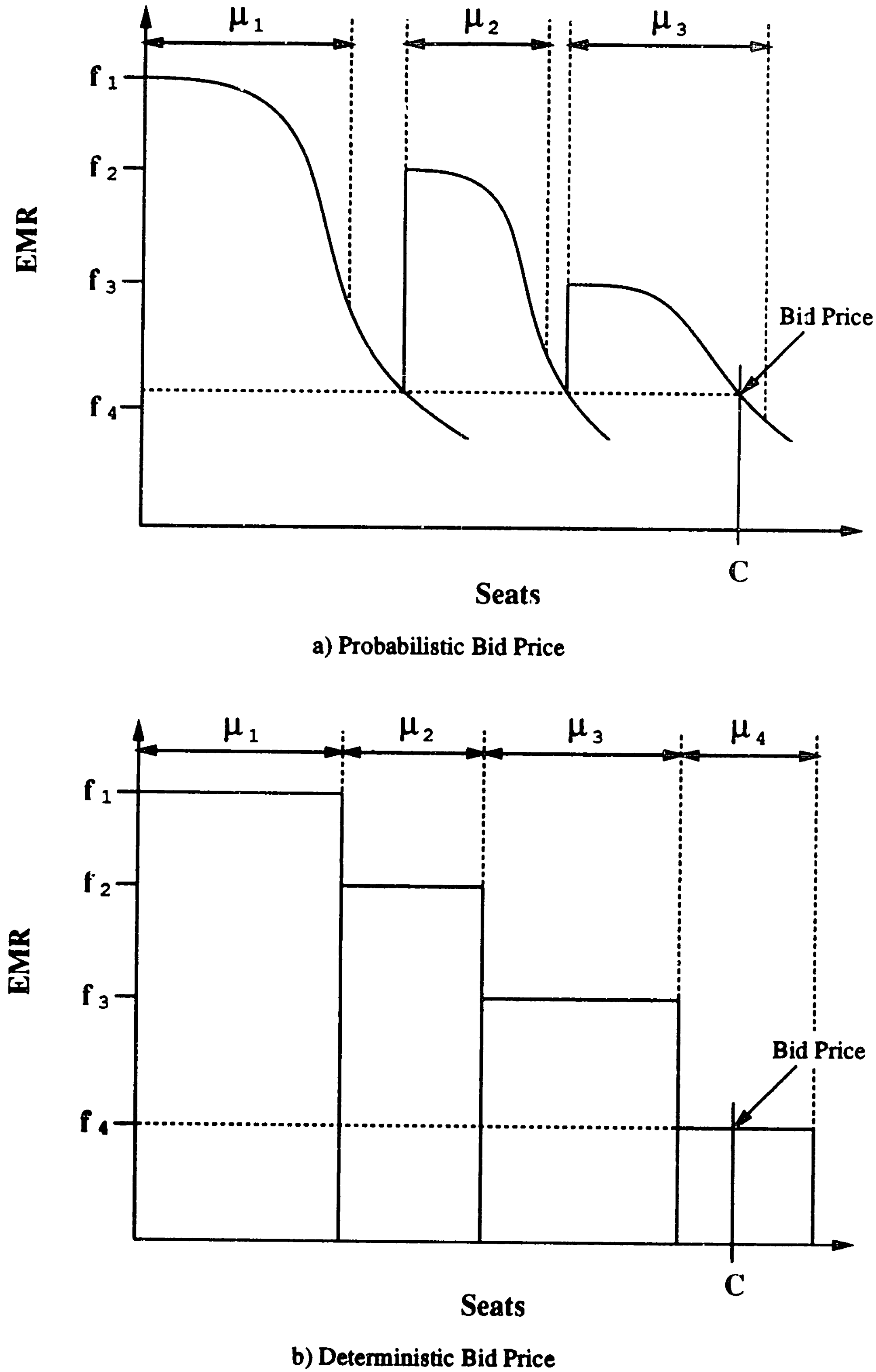


Figure 6.18: A comparison of the level of the probabilistic bid price (a) versus the deterministic bid price (b) for a single flight leg.

class above the mid-point of the curve, allocating less than the mean number of seats to the inventory class. The value at this intersection point is just above the fare value of the fourth inventory class, f_4, and in turn, bookings would not be accepted for the 4th inventory class.

Under the distinct deterministic solution shown in Figure 6.18b, seats are allocated to the highest revenue fare classes, or ODF itineraries, until the estimated mean demand for the fare class or ODF itinerary is met. A *maximum* of μ_i seats can be assigned to each inventory class i, therefore, not only are μ_3 seats allocated to the third inventory class, but before the capacity constraint is reached, seats are made available to inventory class 4. Since the highest inventory classes are strictly constrained in the number of seats which can be allocated to each one, more inventory classes are ultimately allocated seats. With each additional inventory class, the revenue value of the "last seat" decreases. The value of the revenue curve at the point in which the intersection of the capacity constraint occurs, i.e. the deterministic bid price, is the fare value of the fourth inventory class, f_4, which is lower than the probabilistic bid price.

Figures 6.19, 6.20, 6.21, and 6.22 show actual examples of the different probabilistic and deterministic bid price values for each revision point during the booking process of several different flight legs of the different multi-leg flights. The values at each revision point are *average* bid price values for 100 iterations of the booking process. The demand level in each case is on the high side, representing a level at which revenue impacts over leg-based control are obtained while avoiding the very highest demand levels at which significant declines in expected revenues tend to occur. The different examples shown are for flight leg B-C of the two leg Flight 21, legs A-B and B-C of Flight 31, and leg C-D of the three leg Flight 32.

As discussed above, the value of the probabilistic bid price tends to be *greater* than the deterministic bid price over the majority of the booking process in each of the examples.

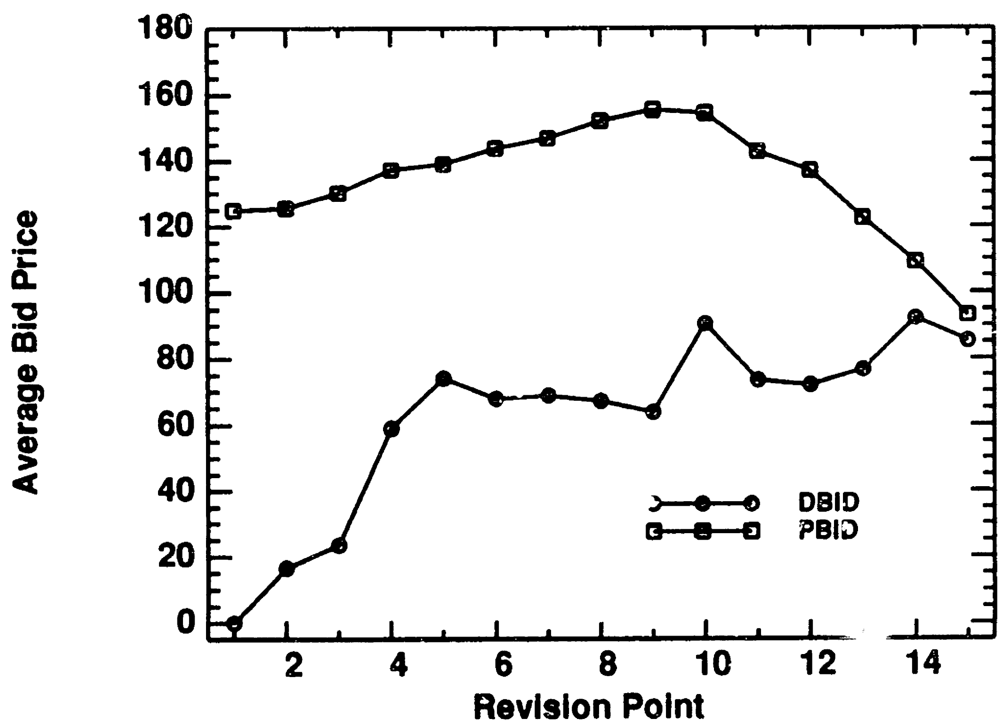


Figure 6.19: Comparison of the average Deterministic Bid Price (DBID) values and the average Probabilistic Bid Price (PBID) values at each revision point throughout the booking process for leg B-C of Flight 21.

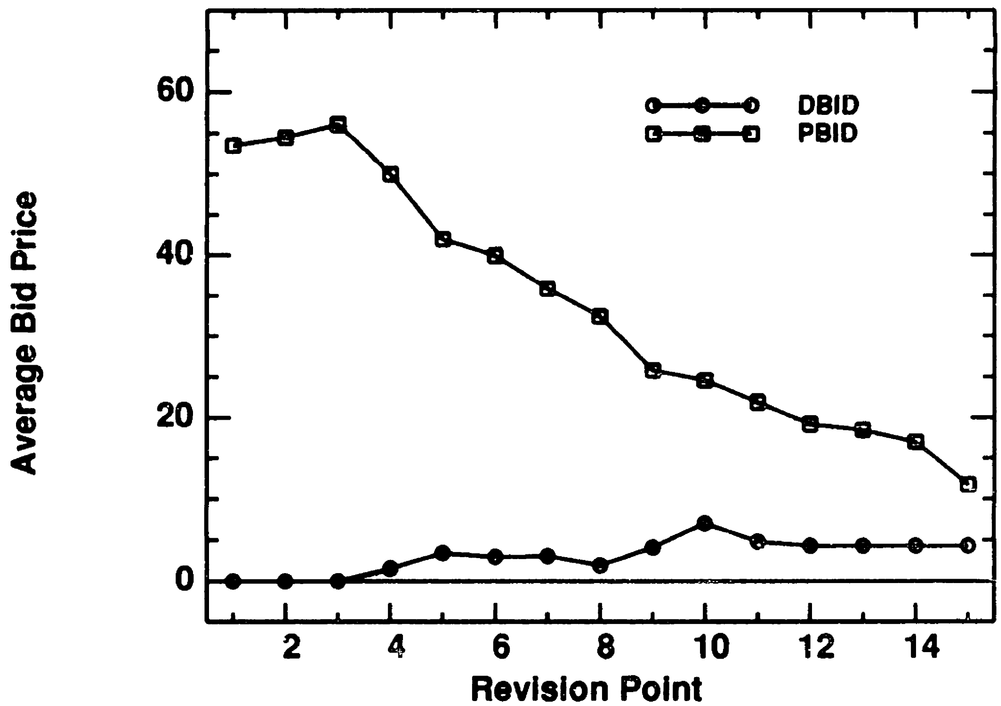


Figure 6.20: Comparison of the average Deterministic Bid Price (DBID) values and the average Probabilistic Bid Price (PBID) values at each revision point throughout the booking process for leg A-B of Flight 31.

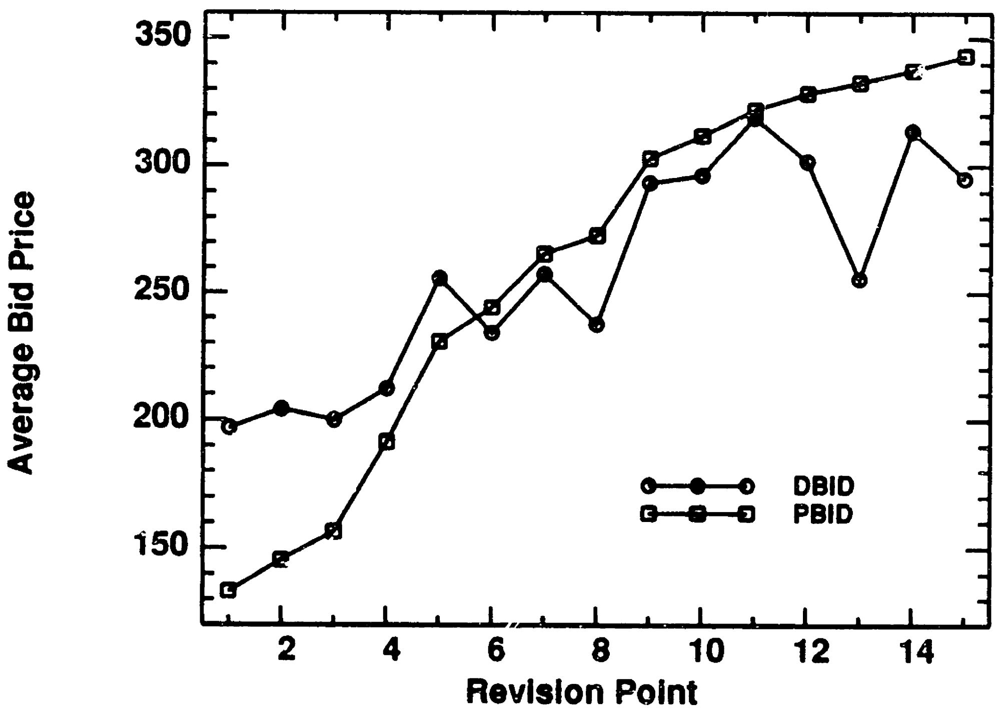


Figure 6.21: Comparison of the average Deterministic Bid Price (DBID) values and the average Probabilistic Bid Price (PBID) values at each revision point throughout the booking process for leg B-C of Flight 31.

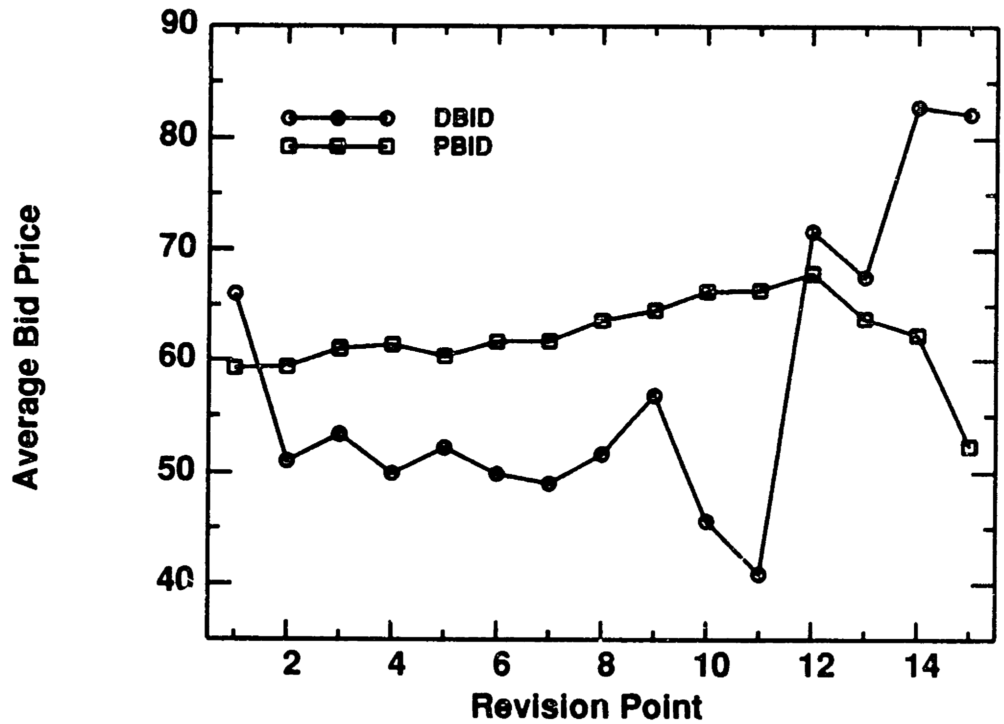


Figure 6.22: Comparison of the average Deterministic Bid Price (DBID) values and the average Probabilistic Bid Price (PBID) values at each revision point throughout the booking process for leg C-D of Flight 32.

While the actual level of the bid price values are very similar at certain times during the booking process, it is interesting to see that at times there is quite a large difference between the values of the deterministic and probabilistic bid prices. As shown in Figures 6.21 and 6.22, changes in the deterministic bid price from one period to the next are not as smooth as that of the probabilistic bid price. This is due to the differences in the revenue function used in both optimizations. Under the deterministic formulation, the revenue function is a step function, as in Figure 6.18b. Thus, as changes in the remaining capacity and the mean demand to come occur, variations in the marginal value of the last seat are determined by this step function. Under the probabilistic formulation, the revenue function is based on the expected revenue curve for each ODF itinerary, such as those in Figure 6.18a, producing a much smoother transition between the expected revenue of one seat to the next.

In using a bid price control methodology, it is important to note that the value of the bid price on each leg does not necessarily steadily increase with time, shutting down the lower fare class, less desirable ODF's one by one and leaving only the higher fare class, more desirable ODF's open. Referring once again to Figure 6.18b, it is not as if the capacity cutoff point slowly approaches zero, each time intersecting the given revenue curve at a higher fare value. The bid price comes from the relationship between demand to come and remaining capacity. Thus, both the remaining capacity and the ODF demand levels which are associated with producing the revenue curve are constantly changing. Therefore, if demand to come decreases faster than capacity, the bid prices will actually *decrease*.

For example, at 60 days prior to departure, total demand to come may be 108, while the capacity is 100, yielding a bid price of, say, \$100. However, at 7 days prior to departure, forecasted demand to come may only be 9, while the remaining capacity is 10 due to the failure of demand materializing. Under such a situation, the bid price at Day 7 is reduced to \$0. Evidence of bid prices actually decreasing with the time to departure is shown by the probabilistic bid price values in Figures 6.19 and 6.20.

Another reason bid prices decrease over time is due to the dependence between different flight legs on a network. The bid price on critical flight legs tends to be the driving force behind the bid prices on other legs. Such is the case for the flight legs A-B and B-C on Flight 31, Figures 6.20 and 6.21. Flight leg B-C is the critical leg, and as the bid price on this flight leg increases, multi-leg itinerary demand is blocked out. As this demand is blocked out, there is less demand remaining for the other non-critical legs on the flight, thus causing the bid price on these flight legs to decrease. This is shown to occur on leg A-B in Figure 6.20.

In a situation where the amount of actual demand remains consistent throughout the booking process, the bid price on each leg should actually remain rather steady. In the initial optimization of a network, it is determined which ODF demand should and should not be accepted and the cutoff values, or bid prices, are set accordingly on each flight leg. As long as the forecasted demand is consistent with bookings, the cutoff level should not change. The only reason the bid prices would change is if there are unforeseen demand occurrences, whether it be expected demand which does not materialize or changes in the forecasted demand to come. This concept has not been well understood by many proponents of the bid price methodology.

With the deterministic bid price approach performing so well with respect to the leg-based EMSR heuristic, serious consideration may be given to controlling seat inventories by bid prices. The philosophy behind the control methodology of the bid price approach is very simple, particularly when compared to other network approaches. Rather than requiring booking limits for each OD and fare class in a network of flights, all that is needed is a simple bid price, or minimum acceptable fare, for each flight leg. Thus, for a four leg flight such as Flight 41, instead of requiring a separate limit for each of the 10 OD itineraries times anywhere from 4 to 10 fare classes (i.e. 40 to 100 booking limits), only 4 bid price values would be needed. Decisions for accepting or rejecting each ODF can then be made based on these 4 values. At the same time, extensions beyond the control of the 10 different on-flight itineraries could also be made quite easily. The bid prices from the four leg network optimization could be used in conjunction with the bid prices of other flight legs, *approximating* the minimum acceptable fare for OD itineraries outside the given segment control problem.

There are certain disadvantages associated with the bid price approach, one of which has to do with the frequency of revisions. As shown in Figures 6.15 and 6.16, at high demand levels a significant drop in expected revenues occurs due to a limited number of revisions. Under an open/closed control philosophy, bid prices must be revised in order to shut down bookings for an ODF, otherwise there is no limit to the number of bookings which can be made. Figure 6.23 gives an example of how the revenue impacts of the Deterministic Bid Price (DBID) approach versus the Nested Deterministic by Shadow Prices (NDSP) approach are affected by the number of revisions made during the booking process for Flight 31 at a demand factor of 1.50.

Using the NDSP approach, a positive revenue impact over leg-based control is obtained whether seats are controlled on a static basis (1 revision point) or dynamically. As the number of revisions increases from 1 to 15, the magnitude of the revenue impact increases significantly, from a 0.6% impact in expected revenue over the EMSR heuristic to a 3.1% impact. However, while the DBID approach produces a positive revenue impact over leg-based control at 15 revisions, significant negative revenue impacts result when revisions are not made this often. Under a static control system, the negative revenue impact of the DBID approach is actually worse at this level of demand than that of a partitioned network control approach due to the "all or nothing" control framework of the bid price approach. As revisions are made, significant improvements in the revenue curve of the DBID approach are made. Even at 15 revisions, the revenue curve of the DBID approach

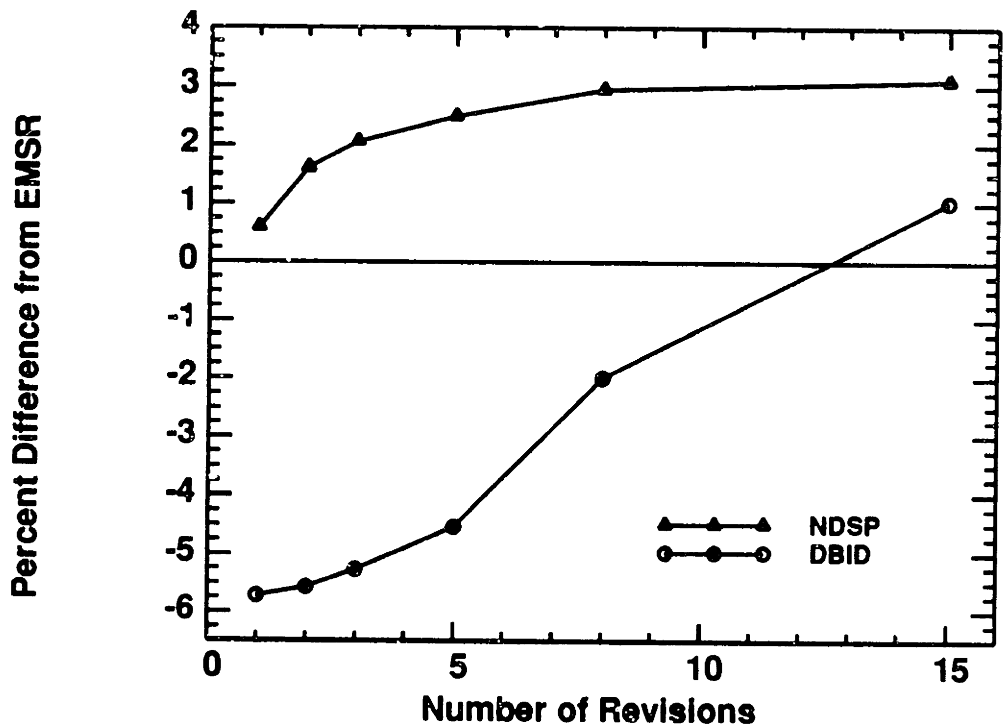


Figure 6.23: Comparison of the revenue impacts as a function of revisions for the Deterministic Bid Price (DBID) approach and the Nested Deterministic by Shadow Prices (NDSP) approach for Flight 31 at the highest demand level tested, i.e. a 1.50 demand factor.

is climbing. Impacts of the NDSP approach, on the other hand, have reached a plateau. Ultimately, the DBID revenue impact curve will meet the NDSP curve at the point where revisions are performed after each booking.

One way of assuring that the *potential* benefits of the deterministic bid price approach are actually achieved is by revising the bid prices on a real time basis. The problem with this solution is that until airlines go to "seamless availability", or direct access, where requests for a given airline are made directly through the reservations system of the airline rather than through the computer reservations systems of other airlines, real time decisions are not possible. Even if it was possible, in order to compute "optimal" bid prices, real time forecasts are also needed. As Robinson [24] notes, while theoretically booking limits and bid prices are dynamic over time due to continual changes in demand, determining such booking limits or bid prices would require knowing the distribution of remaining demand for any ODF itinerary at any particular time. Thus, if the current demand forecast for an ODF is 3 ± 2, once a request is received it must be possible to determine an updated forecast, whether it be 3 ± 1, 2 ± 2, or something else. Using preset revision points, booking limits and bid prices can be determined at distinct periods of time, between which demand distributions can be estimated. Under real time revisions, actual reforecasting of demand is infeasible

Another possible way to make the implementation of a bid price approach feasible is to include with each bid price some type of information about future bid prices. On the one hand, it is not feasible or practical to determine the next "optimal" bid price in advance since it is dependent on the actual bookings which materialize as well as changes in forecasted demand. For example, if the bid price on a flight leg is \$100 and a forecasted request for a seat on the local itinerary at \$600 is made, the "optimal" bid price will most likely not change. Yet, if the request for a \$100 fare on the local itinerary is made and accepted, it is quite possible that the new "optimal" bid price will change, particularly if, based on the actual network solution, the number of seats allocated to the ODF is 1.

Approximations with respect to future bid prices, however, can be made. For instance, the immediate derivative of each bid price can be calculated and used to approximate in advance changes in the cutoff value as bookings are accepted. Thus, if a group of 4 requests is received, it would be possible to have a rough idea of the minimum fare level a bid price might be approaching. At the same time, associated with each bid price can be some value n, where after n bookings, the bid price on the flight leg must be recalculated. Information such as this is obtained from solving mathematical programs.

There are definitely several trade-offs associated with the bid price approach. Operating a reservations control system on the basis of simply an open/closed policy which requires both careful monitoring and frequent adjustments can leave an airline very vulnerable to uncertainties in demand. A bid price approach also makes it much more critical to have reliable systems. Under current control methods, if power failures occur or computers break down, revisions to fare class booking limits cannot be made, yet there is always a limit on the number of bookings accepted in any given fare class. Under a bid price approach, if revisions cannot be made, there is no limit to the number of bookings for any OD or fare class which passes the current "cutoff". While there are certain applications which are well suited for a bid price control structure, to practically implement such an approach in the airline industry would require some type of "safety nets", or limitations on the number bookings accepted at any given time. Given the computer capabilities and the memory capacities available today, the implementation benefits of a bid price system may not be worth the extra day-to-day operating costs and possible risks, especially when the same revenue benefits can be obtained using other control approaches such as nesting the ODF allocations by shadow price.

6.1.4 Leg-Based Methodologies for OD Control

The last set of methods introduced in Chapter 4 are leg-based heuristics for controlling different ODF itineraries across a network of flights. Under these approaches, information about passenger demands and traffic flows is taken into consideration while the optimization and control framework for managing the ODF's remain at the flight leg level. Based on the bid price concept from the network optimization method, an approximation of the marginal value of the last seat on a given flight leg can be determined using the information available at the flight leg level. This leg-based bid price, or $EMR(C)$ value, can be used in several different ways to control bookings.

One such approach is to simply use the leg-based bid prices in the same manner as the deterministic or probabilistic network bid prices, i.e. the Leg-Based Bid Price approach. Decisions whether to accept or reject ODF requests are made based on the sum of the $EMR(C)$ values of each flight leg an ODF itinerary traverses. As discussed in Chapter 4, some type of fare proration is necessary in order to avoid the "over-counting" problem where the full revenue value of an ODF traversing more than one flight leg is counted several times. The three basic proration methods described in Chapter 4 and evaluated here are a mileage based proration, a proration based on the local Y fares of each ODF, and a proration based on the local fares of the respective fare class of each ODF.

These proration methods are not robust. Depending on the which leg of a multi-leg flight is critical as well as the relationship between the local fares and the mileage on each leg of the flight, the most appropriate proration method will vary. For Flight 31, a comparison of the revenue impacts using the three different proration methods for the Leg-Based Bid Price (LBID) approach are shown in Figure 6.24. While there is some variation in the revenue impacts generated under the different proration methods, the variation at each demand level is within 0.35% which is statistically insignificant. It is still worth

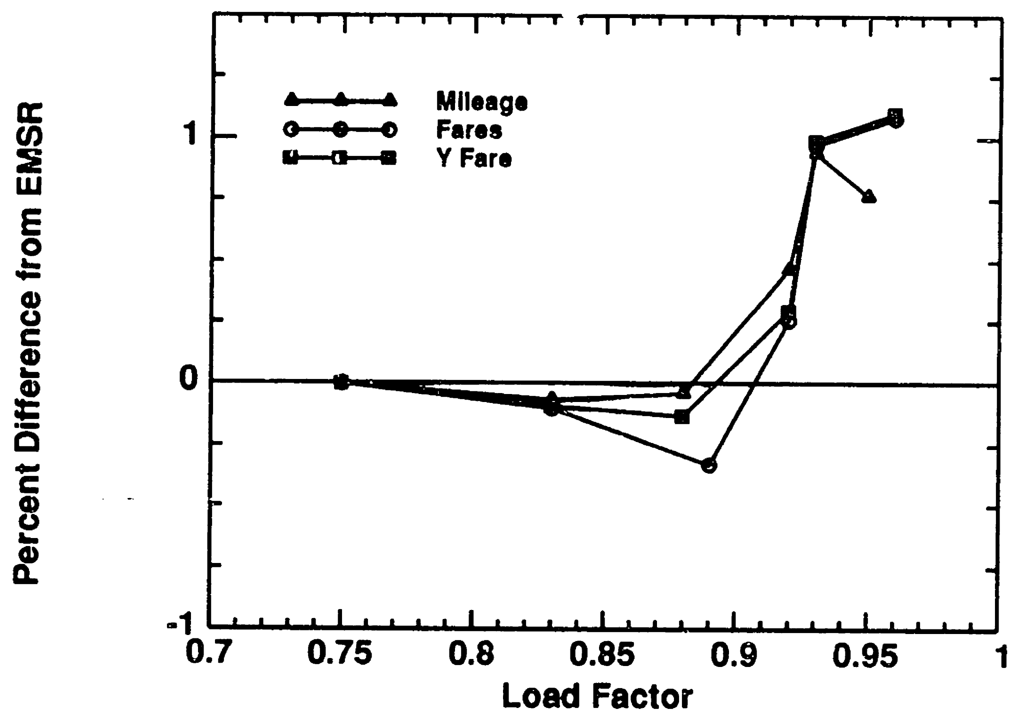


Figure 6.24: Comparison of the revenue impacts for the prorated Leg-Based Bid Price approach on the three leg Flight 31. The three different proration methods evaluated are: a mileage proration of the ODF fares (Mileage), a proration based on the local fares of the respective fare class of each ODF (Fares), and a proration based on the local Y fares (Y Fare).

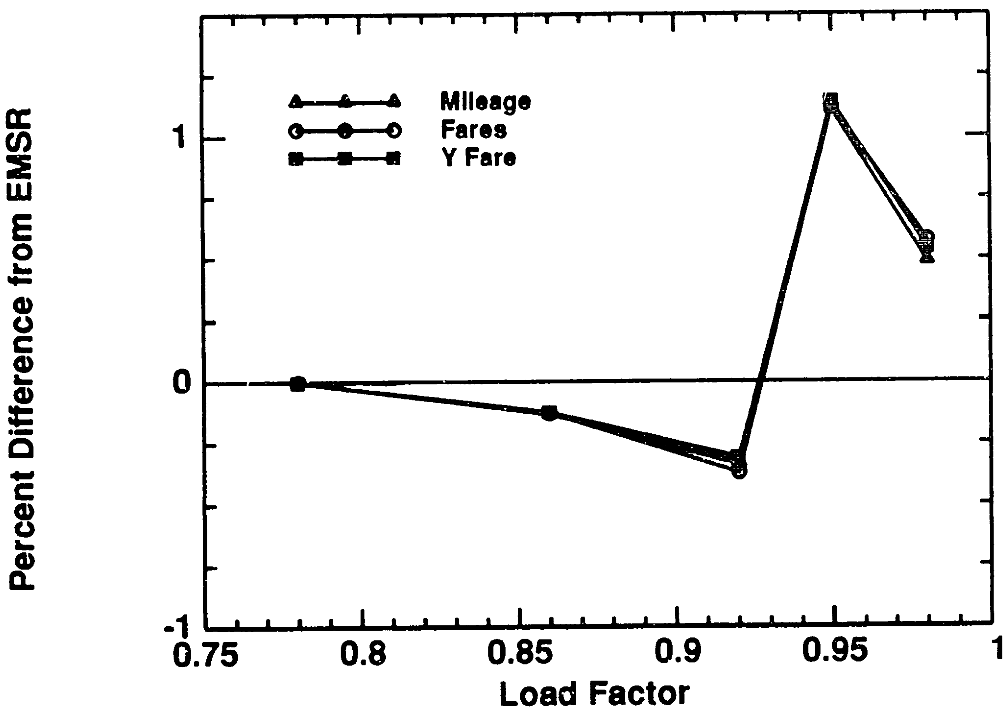


Figure 6.25: Comparison of the revenue impacts for the prorated Leg-Based Bid Price approach on Flight 32. The three different proration methods evaluated are: a mileage proration of the ODF fares (Mileage), a proration based on the local fares of the respective fare class of each ODF (Fares), and a proration based on the local Y fares (Y Fare).

pointing out that the LBID approach based on a mileage proration shows the same trend as the previous bid price approaches, dropping off slightly in expected revenues at high load factors. At the same time, however, the prorated by mileage approach does not show as large a negative impact at the 88/89% load factor level as the other two proration methods, particularly the proration approach based on the local fares of the respective fare class of each ODF.

A similar comparison of the revenue impacts of the Leg-Based Bid Price (LBID) approach using the three different proration methods is provided in Figure 6.25 for Flight 32. For this multi-leg flight, there is almost no variation in revenue impacts between the different proration methods. In general, this was true for most of the multiple leg flights evaluated under the LBID approach. The difference in the proration method also does

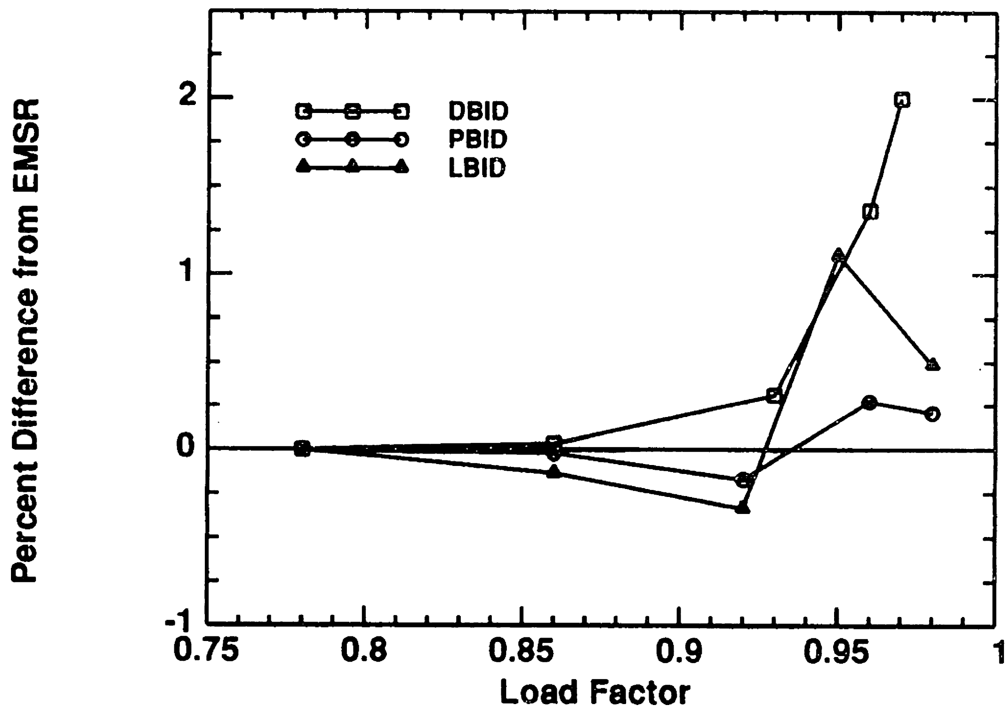


Figure 6.26: Comparison of the revenue impacts for the Leg-Based Bid Price (LBID) approach prorated by mileage, the network Probabilistic Bid Price (PBID) approach, and the network Deterministic Bid Price (DBID) approach for Flight 32.

not affect the performance of the other leg-based control methods which are based on the $EMR(C)$ values of each flight leg, as will be shown later. This is not always the case. However, for the multi-leg flights evaluated, the fare structure and the mileage of the different ODF's were very closely related. Such a condition cannot necessarily be generalized to all airline examples.

When comparing the Leg-Based Bid Price (LBID) approach with the network bid price approaches, the LBID approach actually *outperforms* the network Probabilistic Bid Price (PBID) approach for many of the flights. An example of this is shown in Figure 6.26 for Flight 32. At the lower load factor range, the difference in revenue impacts between the prorated LBID approach and the PBID approach are statistically insignificant. Yet, at the higher load factor levels, the LBID approach yields significant positive revenue impacts compared to the PBID approach. At the 95/96% load factor level, the revenue impacts of the LBID method actually approach those of the DBID method, with the LBID method generating a 1.1% increase in revenues over the EMSR control heuristic and the DBID approach generating a 1.4% revenue impact.

Although the PBID approach is derived from a probabilistic network optimization which explicitly takes into account the flow of traffic over connecting flight legs, this network optimization produces partitioned, or distinct, ODF seat allocations as a solution. As previously discussed, the network probabilistic formulation of the seat inventory control problem tends to *overprotect* seats for the more desirable ODF itineraries, leading to inflated bid prices. While the prorated Leg-Based Bid Price approach is based solely on information available at the flight leg level and does not explicitly take into consideration the interaction between flight legs, nesting of inventory buckets is incorporated in the leg-based optimization. Figure 6.27 provides an example of the optimization framework between two fare classes for a partitioned, or distinct, inventory structure, such as the PBID approach, and a nested inventory structure which is the basis of the LBID optimization approach. The difference in the level of the bid price under the two optimization philosophies is evident, with the value of the partitioned bucket bid price (PBID) being higher than that of the nested bucket bid price (LBID). A truly "optimal" bid price would incorporate in the optimization the nesting philosophy of the LBID approach with the network aspects of the PBID approach. However, without first knowing the correct nesting hierarchy between ODF's on a network, such an optimization combining both nesting and network characteristics cannot be practically formulated.

A comparison of the actual bid price values from the three bid price approaches, LBID, DBID, and PBID, is provided in Figures 6.28, 6.29, and 6.30. The average bid price values at each revision point throughout the booking process are provided for each of the three flight legs of Flight 32 for a demand factor of 1.16, corresponding to the 95/96% load factor

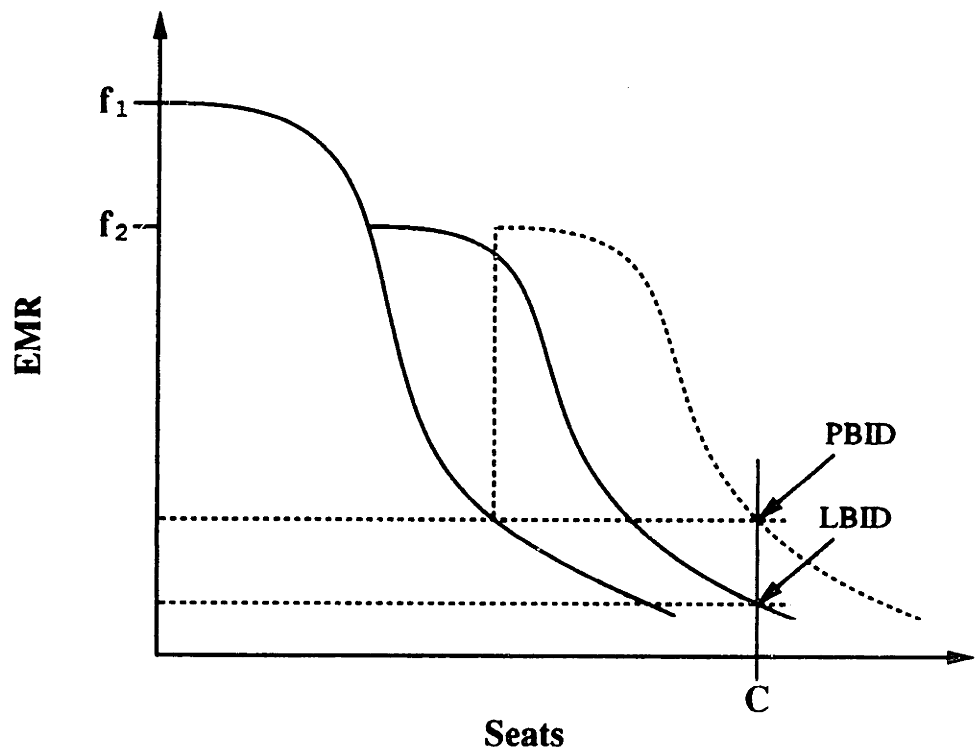


Figure 6.27: A comparison of the bid price levels under a partitioned, or distinct, inventory structure, such as the Probabilistic Bid Price (PBID) approach, and a nested inventory structure as used in the Leg-Based Bid Price (LBID) approach.

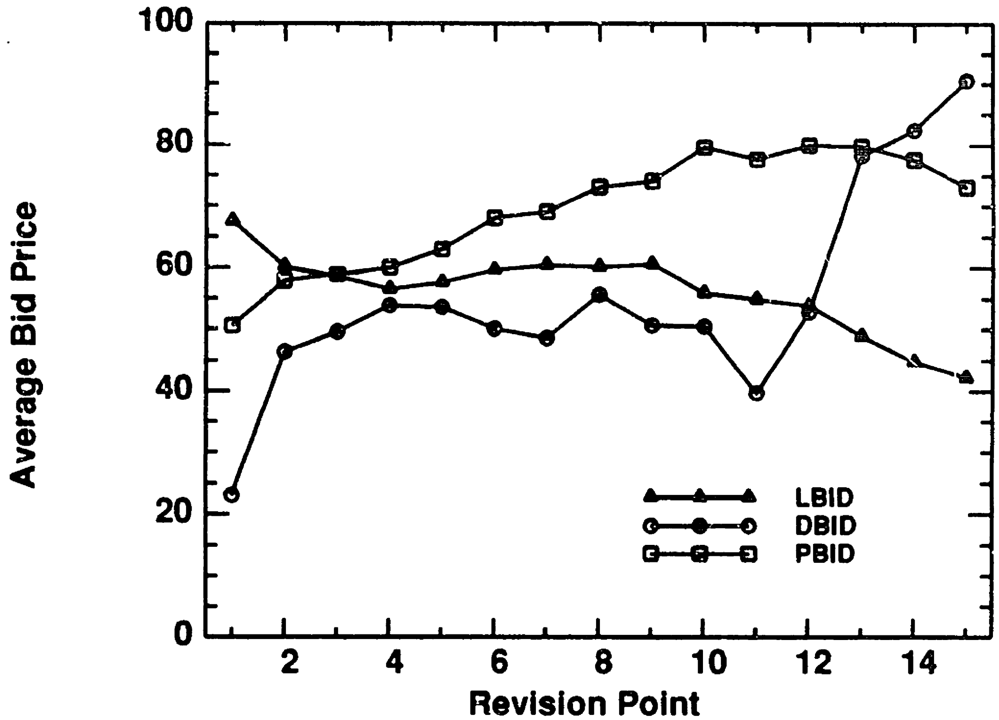


Figure 6.28: Comparison of the average Leg-Based Bid Price (LBID) values, the average network Deterministic Bid Price (DBID) values, and the average network Probabilistic Bid Price (PBID) values at each revision point throughout the booking process for leg A-B of Flight 32.

level. The difference in the network probabilistic bid price values and the leg-based bid price, or $EMR(C)$, values on flight legs A-B and B-C, in Figures 6.28 and 6.29 respectively, become quite significant as the booking process progresses. As was the case with the level of the nested versus partitioned bid price levels in Figure 6.27, the value of the *partitioned* network probabilistic bid price tends to be higher than that of the *nested* leg-based bid price in Figures 6.28 and 6.29. When summing the bid price values from these two legs to determine the cutoff for acceptance of multi-leg traffic, the difference in the levels of the bid prices is compounded.

On flight leg C-D, Figure 6.30, the level of the nested leg-based bid price (LBID) value is slightly higher than the network probabilistic bid price (PBID). However, the two bid price curves are very similar, increasing and decreasing together, rather than converging

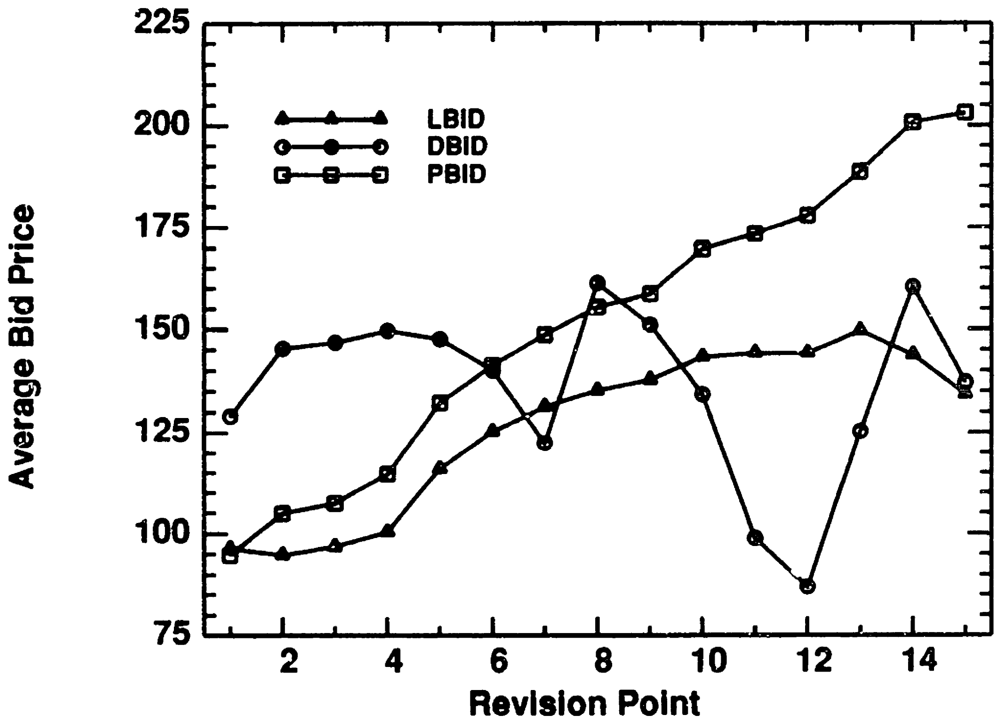


Figure 6.29: Comparison of the average Leg-Based Bid Price (LBID) values, the average network Deterministic Bid Price (DBID) values, and the average network Probabilistic Bid Price (PBID) values at each revision point throughout the booking process for leg B-C of Flight 32.

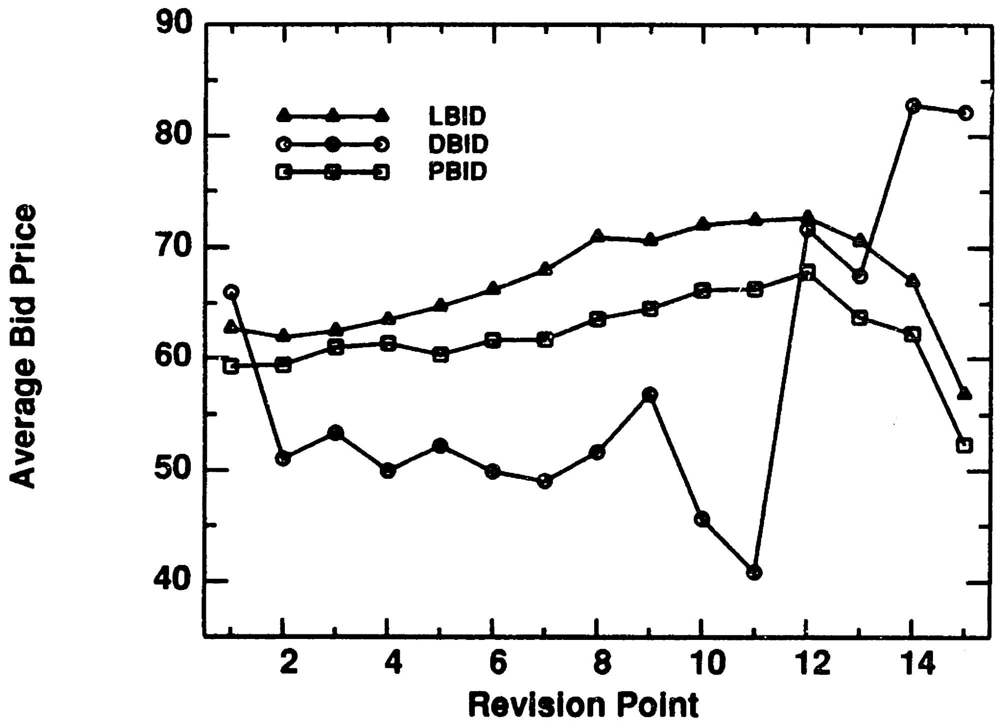


Figure 6.30: Comparison of the average Leg-Based Bid Price (LBID) values, the average network Deterministic Bid Price (DBID) values, and the average network Probabilistic Bid Price (PBID) values at each revision point throughout the booking process for leg C-D of Flight 32.

or diverging as in Figures 6.28 and 6.29. At the same time, the difference between the level of the two bid prices is less than $7.50. This small difference in the two bid prices has very little effect on the actual traffic which is accepted or rejected across the flight leg. Differences in decisions whether to accept or reject multi-leg traffic between the two approaches are based more heavily on the bid price levels of the two more critical flight legs A-B and B-C. This similarity in the bid price curves and the very small differences between the actual bid price values of the LBID and PBID approaches tend to be a common trend on relatively low demand flight legs or the least critical leg of a flight, which is the case for leg C-D.

Referring back to Figure 6.25, the disadvantage of using a totally open/closed control philosophy is again apparent, with a significant drop-off in expected revenues at the highest demand level for Flight 32. A solution to this problem which was suggested in Chapter 4 is to combine the leg-based bid price acceptance rule with the current leg-based booking limit approach. Thus, an assessment of the value of each ODF itinerary to the network as a whole is made using the leg-based bid prices, but the number of seats available to the different ODF's is limited. Using this combined Leg-Based Bid Price/Booking Limit (LBID/BL) approach, the revenue impacts based on the three proration methods are shown in Figure 6.31 for Flight 31. As was the case in Figure 6.24 for the straight LBID approach, the revenue impacts between the different proration methods are not substantially different. By distinguishing between OD itineraries and evaluating the value of each ODF to the three leg network, additional revenue over simply controlling seats using the leg-based EMSR fare class booking limit approach can be obtained. At the same time, by applying booking limits to the leg-based bid price approach, positive revenue impacts of approximately 2% are generated at the highest demand levels, versus the 1% impact under the straight Leg-Based Bid Price (LBID) approach.

A second leg-based ODF itinerary control approach described in Chapter 4 is the Virtual

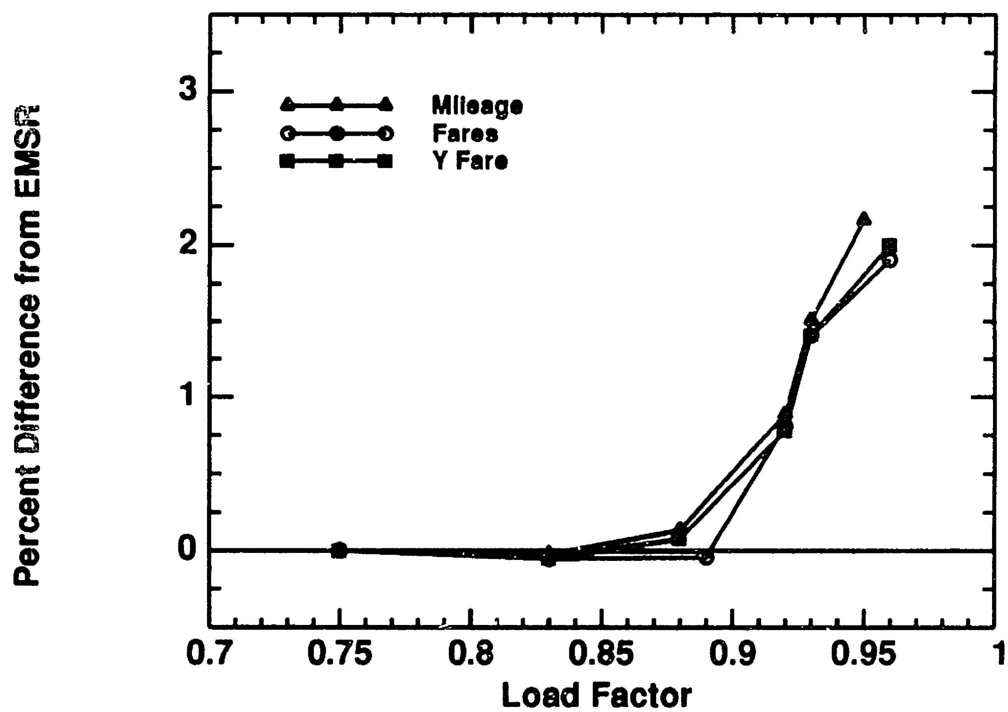


Figure 6.31: Comparison of the revenue impacts for the combined Leg-Based Bid Price/Booking Limit approach using a mileage proration of the ODF fares (Mileage), a proration based on the local fares of the respective fare class of the ODF (Fares), and a proration based on the local Y fares (Y Fare) for Flight 31.

Nesting on the "Value Net of Opportunity Cost" (VNOC) approach. In this method, rather than aggregating ODF combinations into inventory buckets by fare class, control on each flight leg is performed using "virtual " inventory buckets. These virtual inventory buckets are defined according to revenue values. ODF's are then assigned to an inventory bucket on the basis of their *value* to the respective flight leg, taking into consideration the cost of displacing passengers on other flight legs, i.e. "value net of opportunity cost". Using the leg-based bid prices, or $EMR(C)$ values, as an approximation of the displacement cost on other flight legs, the "value net of opportunity cost" of each ODF is determined. Booking limits for each virtual inventory bucket on a flight leg are then determined using current leg-based control approaches such as the EMSR heuristic.

Figure 6.32 illustrates the revenue impacts for the Virtual Nesting on the "Value Net of Opportunity Cost" (VNOC) approach for Flight 31. The small variation between the different proration methods is further reduced when control of seat inventories is performed using the $EMR(C)$ values under a VNOC approach rather than a straight open/closed bid price approach. At the same time, as in the combined LBID/BL approach, the magnitude of the revenue impact obtained at high load factors is significantly increased, generating just over 2% in additional revenues over the simple leg-based EMSR control approach.

The final leg-based OD itinerary control approach discussed in Chapter 4 is the Nested Leg-Based Itinerary Limit approach. Under the Nested Leg-Based Itinerary Limit (NLBIL) approach, rather than differentiating between ODF combinations on the basis of the value of the *last* seat on each flight leg, the entire expected marginal revenue curve of each flight leg is used to determine different OD and fare class itinerary limits. Similar to the bid price approach, the value of an individual seat on an itinerary's path is determined by summing the expected marginal revenues of the respective seat on each flight leg the OD traverses. However, the value of *every* seat across the itinerary's path is calculated, creating an entire EMR curve for the OD itinerary. ODF itinerary booking limits are then determined by

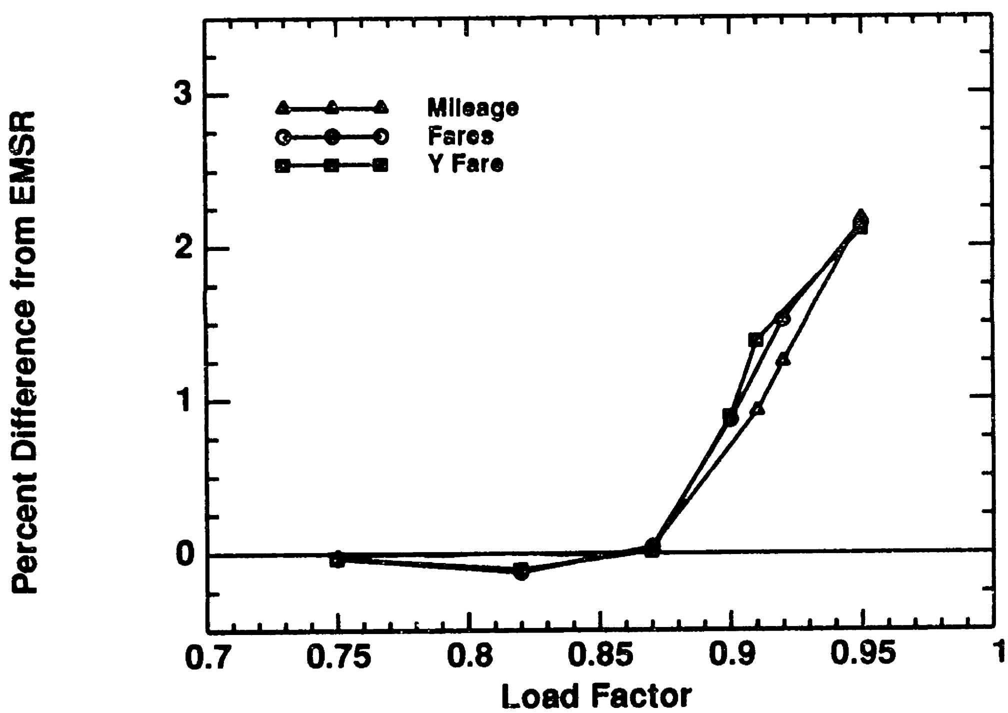


Figure 6.32: Comparison of the revenue impacts for the Virtual Nesting on the "Value Net of Opportunity Cost" approach where the opportunity cost of each flight leg is determined using a mileage proration of the ODF fares (Mileage), a proration based on the local fares of the respective fare class of each ODF (Fares), and a proration based on the local Y fares (Y Fare).

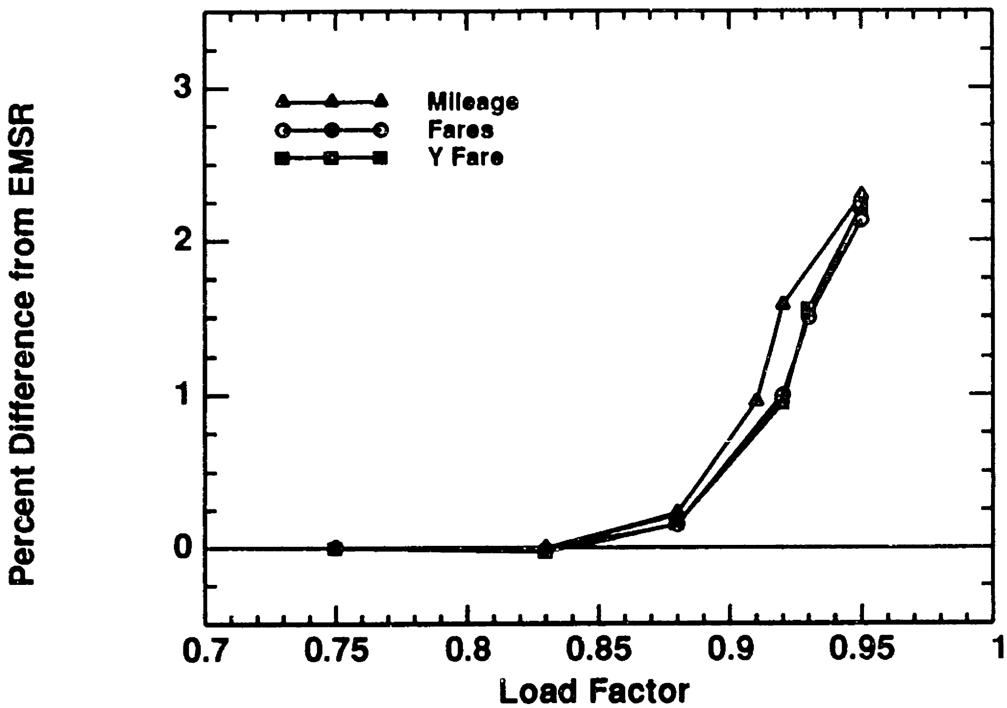


Figure 6.33: Comparison of the revenue impacts for the Nested Leg-Based Itinerary Limit approach where the expected marginal revenue curve for each flight leg is determined using a mileage proration of the ODF fares (Mileage), a proration based on the local fares of the respective fare class of each ODF (Fares), and a proration based on the local Y fares (Y Fare).

comparing the fare of the ODF to the *total* expected marginal revenue curve of the itinerary.

Based on this Nested Leg-Based Itinerary Limit (NLBIL) approach, the revenue impacts for the three different proration methods are shown in Figure 6.33 for Flight 31. In comparing Figure 6.33 with Figures 6.31 and 6.32, not only is the variation in revenues between the different proration methods small for each of the three approaches, but the revenue impacts between the different leg-based OD control methodologies are very similar across the range of load factors. For all three leg-based OD control approaches, significant positive revenue impacts are obtained at load factor levels above ~ 88%, corresponding to a demand factor of .94, with as much as 2% additional revenue generated over the simple leg-based EMSR heuristic at the highest demand level.

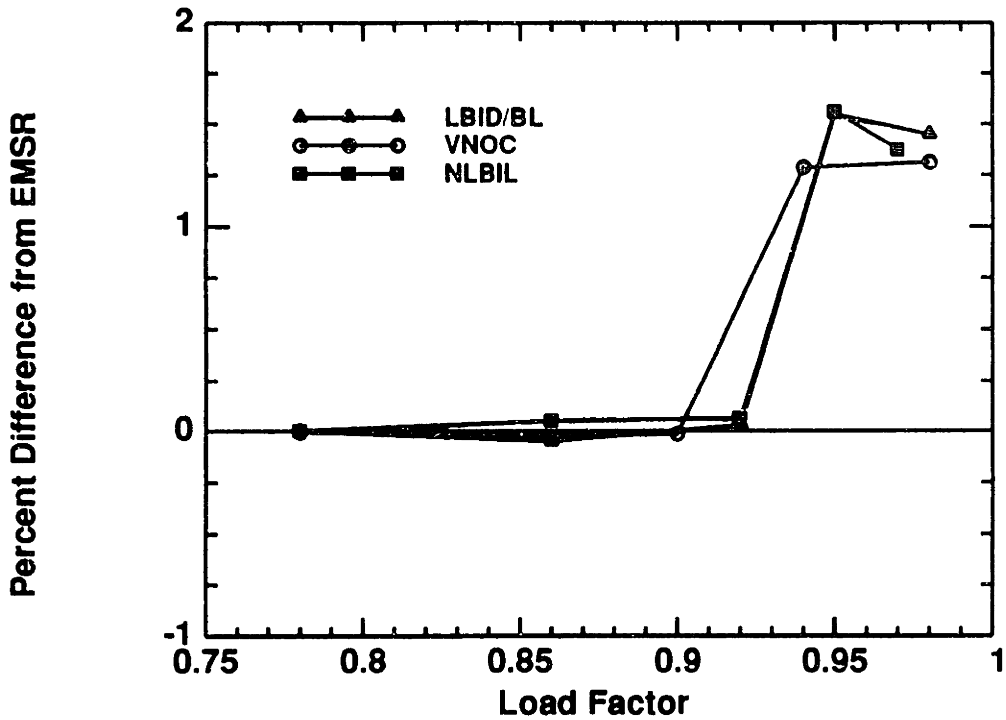


Figure 6.34: Comparison of the revenue impacts for the combined Leg-Based Bid Price/Booking Limit (LBID/BL) approach, the Virtual Nesting on the "Value Net of Opportunity Cost" (VNOC) approach, and the Nested Leg-Based Itinerary Limit (NLBIL) approach for Flight 32 based on a mileage proration of ODF fares.

Similar results are shown for the three leg Flight 32 in Figure 6.34 where revenues at high demand levels are about 1.5% more than that obtained using the EMSR heuristic. While the magnitude of the revenue impacts varies from flight to flight, the similarity between the different leg-based methods for controlling ODF itineraries is fairly consistent for most of the two and three leg flights evaluated. However, as the number of flight legs increases, the performance of the Virtual Nesting on the "Value Net of Opportunity Cost" approach begins to degrade. This is evident for the four leg Flight 41 in Figure 6.35. As before, the LBID/BL approach and the NLBIL approach perform consistently well across the range of load factors, generating significant positive revenue impacts at the highest demand levels. While the VNOC approach also produces significant positive revenue impacts at the highest demand levels, the magnitude of these impacts are not as great.

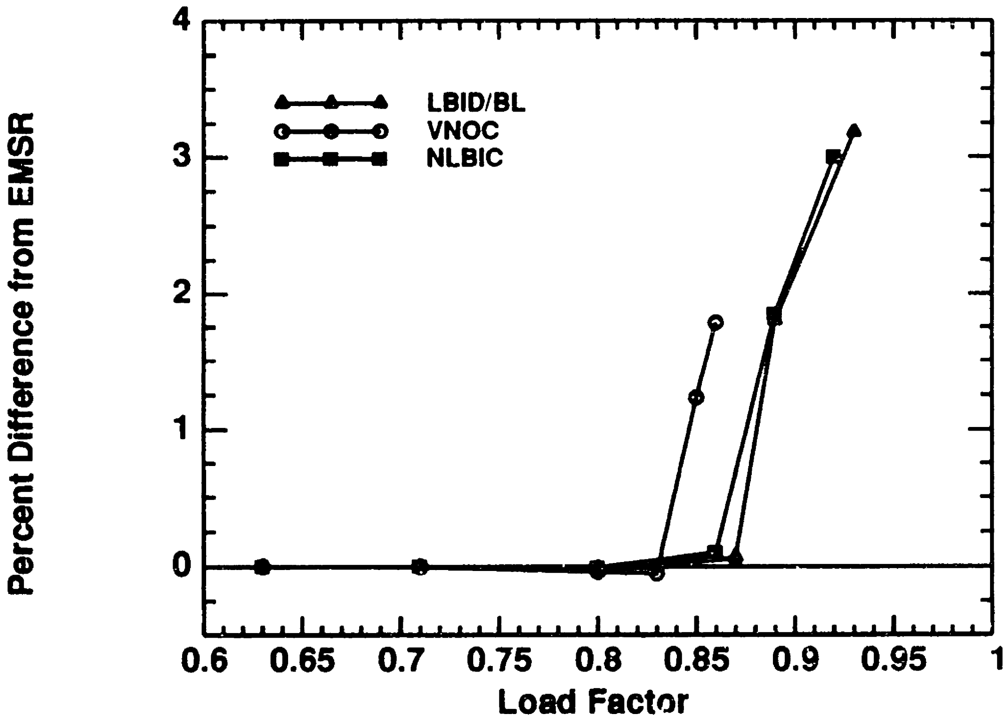


Figure 6.35: Comparison of the revenue impacts for the combined Leg-Based Bid Price/Booking Limit (LBID/BL) approach, the Virtual Nesting on the "Value Net of Opportunity Cost" (VNOC) approach, and the Nested Leg-Based Itinerary Limit (NLBIL) approach for Flight 41 based on a mileage proration of ODF fares.

The reason for the relatively "poor" performance of the VNOC approach is that on the highly constrained flight legs there tends to be a bias *against* long haul traffic, resulting in less overall revenue as well as lower load factors. For a three leg flight, there is only one OD itinerary extending over more than two flight legs, and typically the amount of demand for the three leg OD itinerary is disproportionately smaller than the demand for other OD itineraries, thus limiting the effect of this negative bias. However, with a four leg flight, there are two different three leg OD itineraries as well as a four leg itinerary. Even if there is a relatively small proportion of demand on *each* of these multi-leg itineraries, the total demand affected by this bias begins to become much more significant. For flights with an even greater number of legs, the problem is compounded further.

6.1.5 Summary Comparison of Multiple Flight Leg Revenue Impacts

To summarize the performance of the different seat inventory control alternatives for the segment control problem on multiple leg flights, a direct comparison of the network optimization approaches to the leg-based OD control approaches is provided here. Rather than comparing every method reviewed, a representative approach from each group of methods is shown. The approaches compared are: 1) the Nested Deterministic on Shadow Prices (NDSP) and the Nested Probabilistic on Shadow Prices (NPSP) approaches, control methods which consistently outperform both the nested by fare class and the nested by fares approaches; 2) the Deterministic Bid Price (DBID) approach which is very similar to the NDSP approach (except at high demand levels when the frequency of revisions is limited) and which consistently performs better than the Probabilistic Bid Price approach; and 3) the prorated Nested Leg-Based Itinerary Limit (NLBIL) approach which provides an indication of the revenue impacts which can be obtained using one of the leg-based OD control methods, performing equally as well as the prorated Leg-Based Bid Price/Booking Limit approach and as well as, if not better than, the Virtual Nesting on the "Value Net of Opportunity Cost" approach for the multiple leg flights.

As before, the different approaches are compared on the basis of the percent difference in expected revenues from that obtained using a simple leg-based approach, i.e. the EMSR heuristic. In this manner, the benefits of implementing a network seat inventory control approach which differentiates among both OD's and fare classes can be determined. Also included in this comparison is the level of the maximum revenue potential for each flight. This maximum revenue potential, or "upper bound" in terms of revenues, provides a measure of how well the different seat inventory control approaches perform in relation to the *total* revenue opportunity which could feasibly be obtained given perfect information.

The upper bound (UPPER) in revenues is based on making decisions and determining ODF seat allocations and booking limits in hindsight. In determining the revenue impacts of the different seat inventory control approaches, booking limits for the different ODF itineraries are calculated on the basis of a forecasted demand distribution. Requests are then randomly generated with the use of the integrated optimization/booking process simulation. However, in determining the upper bound in revenues, *all* requests are randomly generated for the full booking process and the *optimal* combination of ODF requests which maximizes revenue over the network are then actually booked. Thus, the "upper bound" represents the maximum possible revenue for a particular set of requests across a flight.

The summary revenue comparisons for each of the four multiple leg flights, Flight 31, Flight 32, Flight 21, and Flight 41, are provided in Figures 6.36, 6.37, 6.38, and 6.39, respectively. In each case, the Nested Deterministic by Shadow Prices (NDSP) approach consistently performs as well as, and often significantly better than, the other network seat inventory control approaches evaluated. However, the leg-based OD control heuristic, NLBIL, does surprisingly well relative to the NDSP approach and actually outperforms the probabilistic network approach, NPSP, a majority of the time.

The benefits of using a network seat inventory control approach is clearly a function of the average load factor of a multiple leg network. Below an average load factor of 85%, the

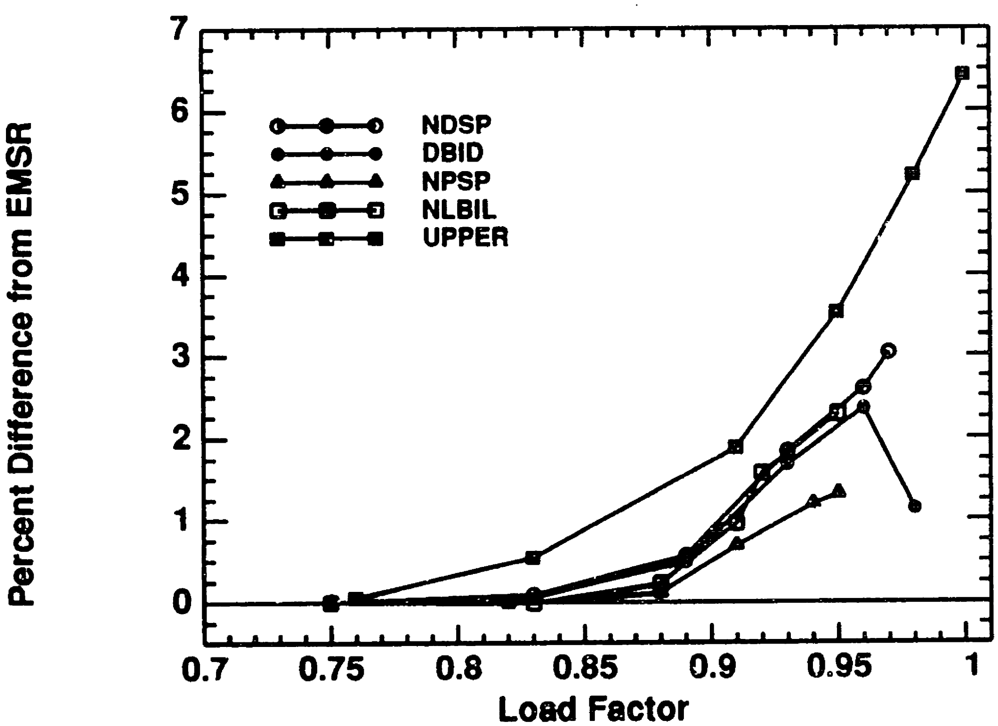


Figure 6.36: Comparison of the revenue impacts for the Nested Deterministic by Shadow Prices (NDSP) approach, the Deterministic Bid Price (DBID) approach, the Nested Probabilistic by Shadow Prices (NPSP) approach and the prorated Nested Leg-Based Itinerary Limit (NLBIL) approach and their relationship to the maximum revenue potential, or upper bound (UPPER), for Flight 31.

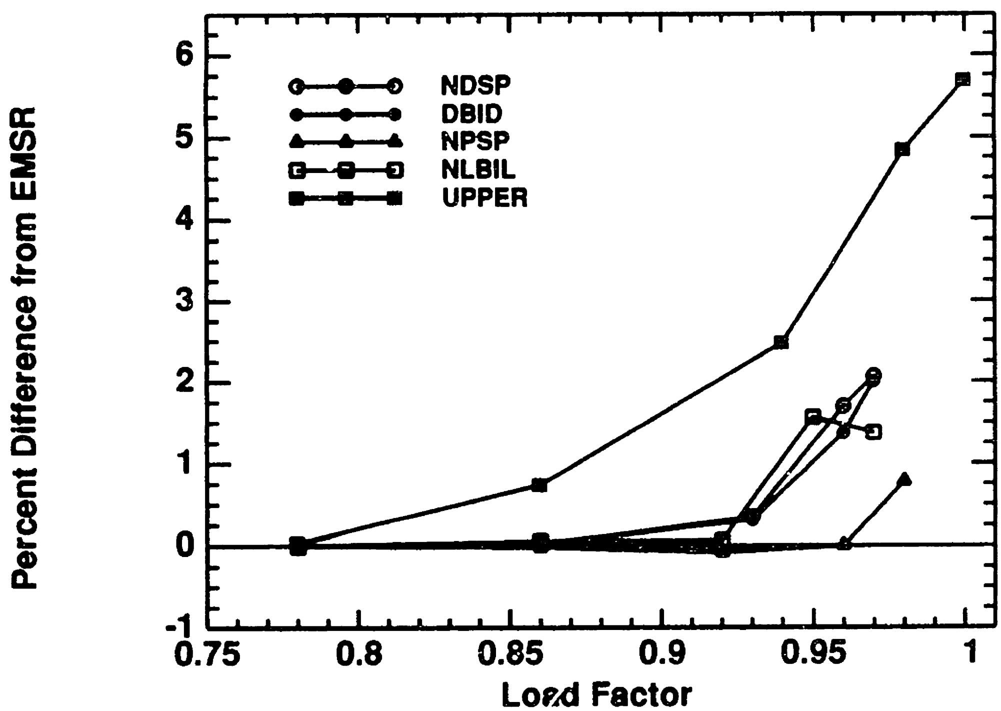


Figure 6.37: Comparison of the revenue impacts for the Nested Deterministic by Shadow Prices (NDSP) approach, the Deterministic Bid Price (DBID) approach, the Nested Probabilistic by Shadow Prices (NPSP) approach and the prorated Nested Leg-Based Itinerary Limit (NLBIL) approach and their relationship to the maximum revenue potential, or upper bound (UPPER), for Flight 32.

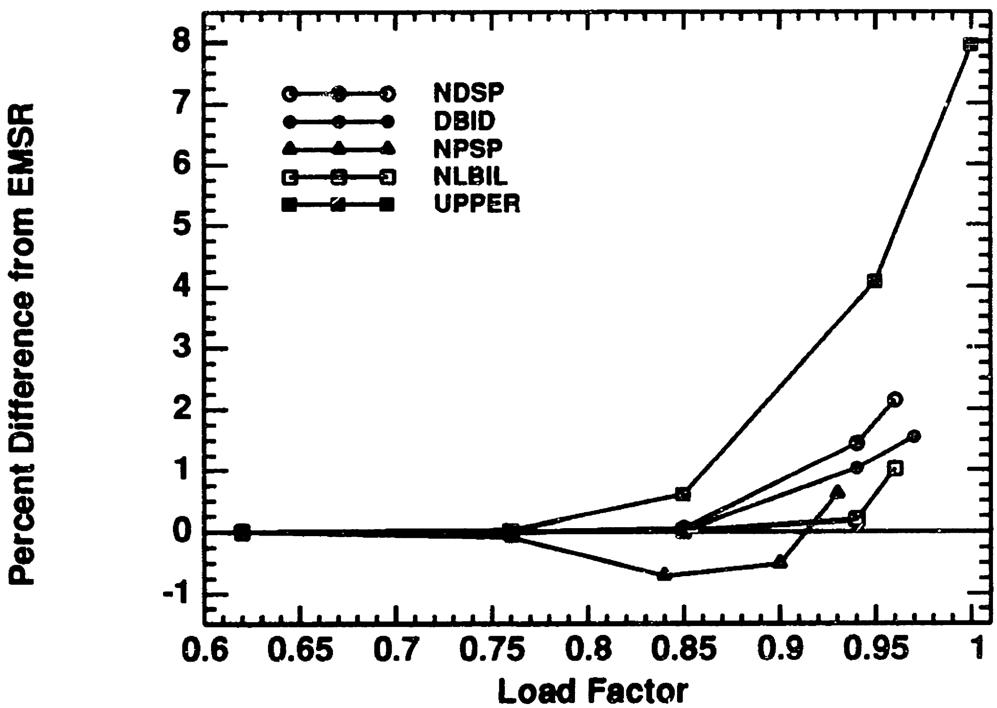


Figure 6.38: Comparison of the revenue impacts for the Nested Deterministic by Shadow Prices (NDSP) approach, the Deterministic Bid Price (DBID) approach, the Nested Probabilistic by Shadow Prices (NPSP) approach and the prorated Nested Leg-Based Itinerary Limit (NLBIL) approach and their relationship to the maximum revenue potential, or upper bound (UPPER), for Flight 21.

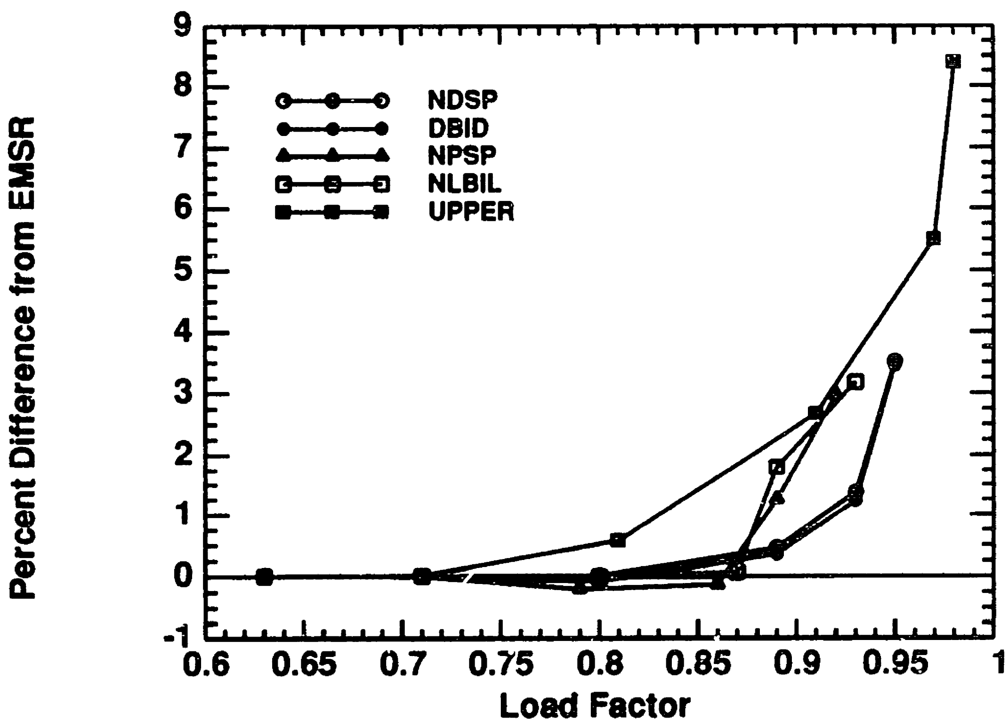


Figure 6.39: Comparison of the revenue impacts for the Nested Deterministic by Shadow Prices (NDSP) approach, the Deterministic Bid Price (DBID) approach, the Nested Probabilistic by Shadow Prices (NPSP) approach and the prorated Nested Leg-Based Itinerary Limit (NLBIL) approach and their relationship to the maximum revenue potential, or upper bound (UPPER), for Flight 41.

impacts of segment control are non-existent relative to a simple leg-based control approach. However, at very high load factors the benefits of an effective network seat inventory control approach typically range between 2-4%. Benefits exceeding the 2-4% impact over leg-based control are rare and are clearly not on the order of the 10-13% previously claimed by some in the airline industry given that the maximum revenue potential, or upper bound in revenues (UPPER), is only on the order of 6-9% for most multi-leg flights.

By controlling different ODF itineraries using the Nested Deterministic by Shadow Prices (NDSP) approach, close to one-half of the total revenue potential between the EMSR approach and the upper bound in revenues can be achieved on average, while the leg-based OD control heuristics, such as the NLBIL approach, achieve revenue benefits on the order of one-third of the maximum revenue potential at load factor levels corresponding to demand factors greater than 1.0. For two leg flights, such as Flight 21 in Figure 6.38, the overall revenue opportunity achieved by controlling different ODF itineraries is not as substantial as that of the longer multi-leg flights. Rather than making decisions between a *variety* of OD itineraries on each leg, the choices on a two leg flight are simply between local and through traffic. Thus, only about a quarter of the total revenue potential between using an effective leg-based control approach and the upper bound on revenues is obtained. However, this 1.0% to 2.1% in revenue impacts at the highest demand level is still quite significant.

There are definite trade-offs in the costs and benefits between using a network approach, such as the NDSP approach or possibly the DBID approach, and a leg-based OD control method, such as the NLBIL approach, the LBID/BL approach, or even the VNOC approach. Forecasting for a leg-based OD control method is done at the fare class (or virtual class) and flight leg level as is currently the case for simple leg-based approaches. Implementing a full network optimization approach requires individual ODF forecasts. However, at load factors just over 85%, the NDSP approach provides almost 0.5% in additional revenue impacts over the leg-based EMSR heuristic than the NLBIL approach, as shown in Figure 6.36 for the three leg Flight 31. This gap in the revenue impacts between the two approaches increases to as much as 1.0% at higher load factors, where the NLBIL approach provides revenue impacts on the order of 1.6% and 2.3% at the 92% and 95% load factor levels, respectively, while the revenue impacts of the NDSP approach are 2.6% and 3.1% at the same demand levels (corresponding to a 96% and 97% load factor level). Similar differences of 0.5% to 1.0% in additional revenues are also the case for the other three flights.

This additional 0.5% to 1.0% in revenues can be quite substantial for an entire network of flights. For multi-leg flights, the transition from flight leg to OD itinerary forecasting is not totally infeasible. For a two leg flight, rather than forecasting demand for each fare class on two individual flight legs, demand forecasts for each fare class of only three different OD itineraries would be necessary. For a three leg flight, the number of forecasts increase by only a factor of two, from three flight legs to 6 OD itineraries. However, as the number of legs on a flight becomes larger, the forecasting problem does become more significant.

While the above results are a fair representation of the revenue impacts which can be obtained using different network seat inventory control approaches, there are certain flights for which the results are not as consistent as those presented. One such example is shown in Figure 6.40 for another two leg flight. In this example, it is not the case that the results are necessarily poor, yet *substantial* positive revenue impacts over the simple leg-based EMSR control methodology are not generated. Even at a demand factor of 1.5, the impact on revenues from both the NDSP approach and the NLBIL approach is only around the 0.5% level, rather than the 2-4% obtained in the previous examples. Yet, 33% of the demand over the two leg flight is through traffic. Thus, there is both local and through traffic on each flight leg which could theoretically be managed more effectively using some type

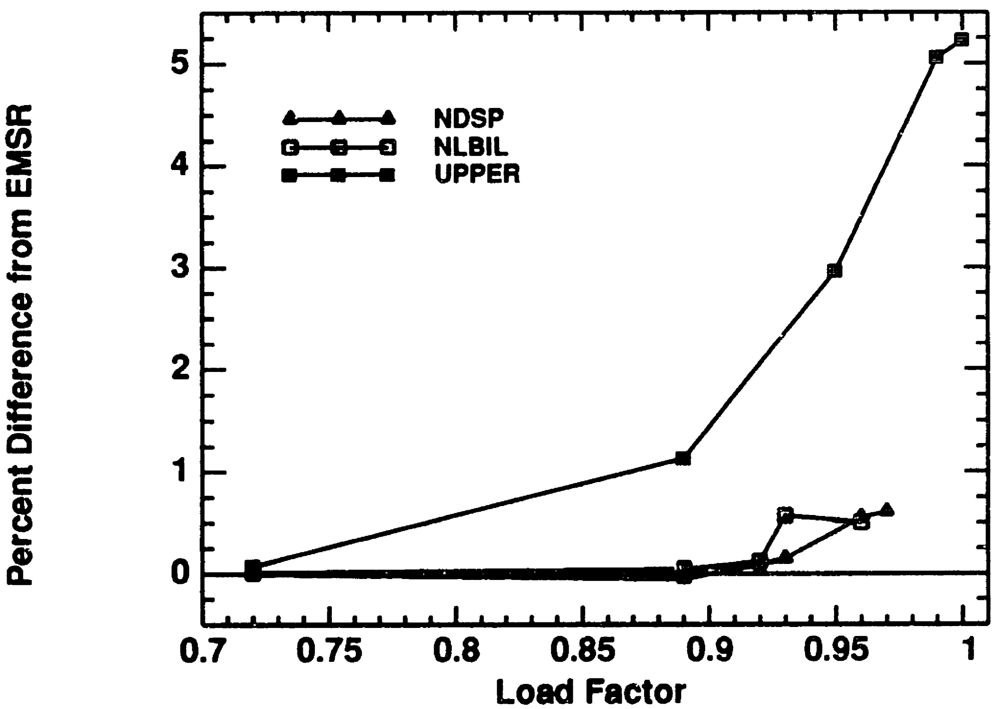


Figure 6.40: Comparison of the revenue impacts for the Nested Deterministic by Shadow Prices (NDSP) approach and the mileage prorated Nested Leg-Based Itinerary Limit (NLBIL) approach and their relationship to the maximum revenue potential, or upper bound (UPPER), for a two leg flight.

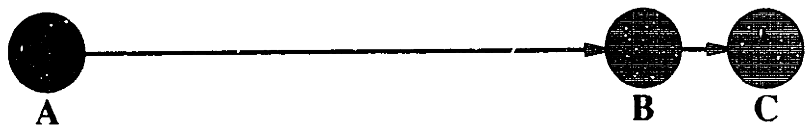


Figure 6.41: A two leg flight consisting of a long haul and a short haul flight leg. The demand on the long haul flight leg A-B is significantly more than that of the short haul B-C leg.

of network control approach. However, only a limited amount of the maximum revenue potential is gained using the "best" of the network approaches. This raises the issue of determining the nature of multi-leg flights where benefits will be achieved.

A second example which should be noted is a two leg flight in which the demand on one leg is significantly greater than the demand on the second leg. The flight consists of a very long haul leg followed by a short haul leg as shown in Figure 6.41. The total demand on the long haul A-B flight leg is over 2.3 times that of the B-C flight leg, making for a very heavily constrained first leg and a relatively empty second leg. In general, the characteristics of this two leg flight are very similar to a international transatlantic or transpacific flight where an airline may not have the rights to carry local traffic between foreign countries (B to C).

This two leg flight is noted here because of the rather poor results obtained when applying the leg-based OD control approaches to this flight, Figure 6.42. As the load factor increases, the leg-based OD control approaches generate negative revenue impacts when compared to the simple leg-based EMSR heuristic, particularly the LBID/BL approach and the VNOC approach. It should be noted that the highest average load factor shown, i.e. 79%, corresponds to a fairly high demand level. However, due to disproportionate level on demand over the two flight legs, the load factor on one leg ranges from 85-100% as the level of demand versus capacity varies while the load factor on the second leg ranges from

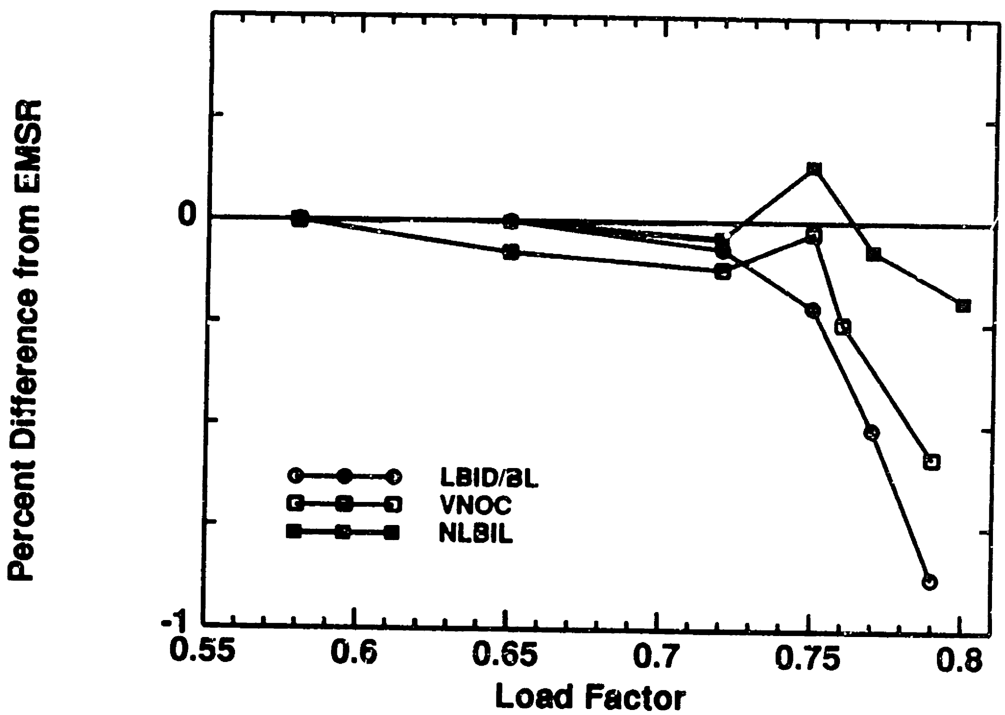


Figure 6.42: Comparison of the revenue impacts for the prorated Leg-Based Bid Price/Booking Limit (LBID/BL) approach, the Virtual Nesting on the "Value Net of Opportunity Cost" (VNOC) approach, and the Nested Leg-Based Itinerary Limits (NLBIL) approach for the two leg flight in Figure 6.41.

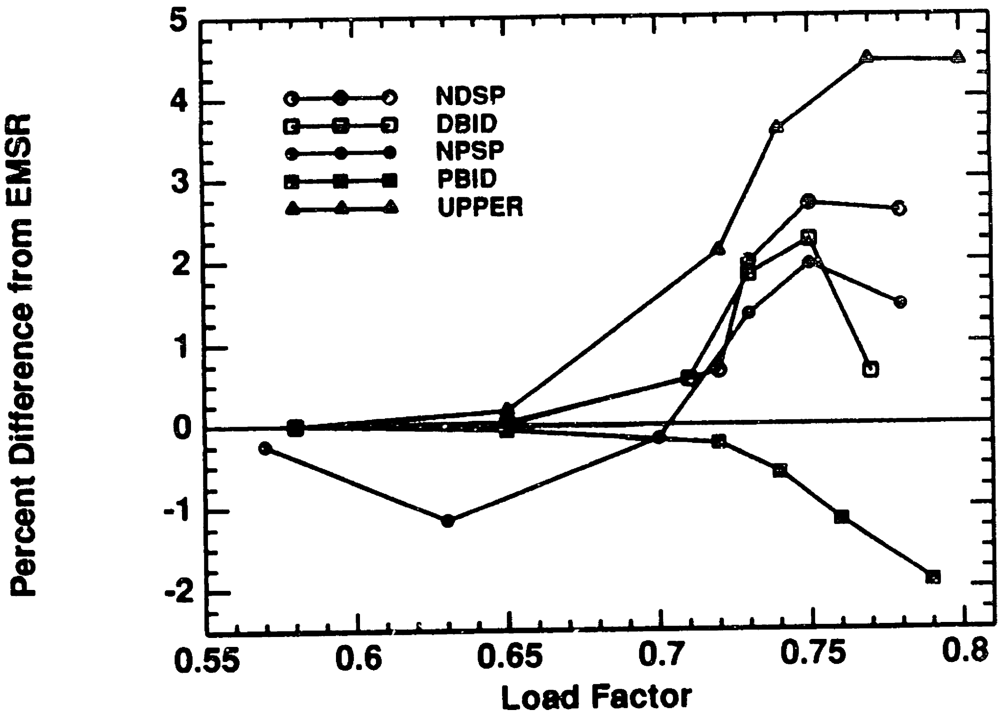


Figure 6.43: Comparison of the revenue impacts for the Nested Deterministic by Shadow Prices (NDSP) approach, the Deterministic Bid Price (DBID) approach, the Nested Probabilistic by Shadow Prices (NPSP) approach and the Probabilistic Bid Price (PBID) approach and their relationship to the maximum revenue potential, or the upper bound (UPPER), for the two leg flight in Figure 6.41.

30-60%, resulting in a much lower *average* load factor than was typically the case for the previous flights evaluated.

Although the performance of the leg-based OD control methods for the two leg flight in Figure 6.41 is not very good, the same is not true for the deterministic network approaches. Figure 6.43 provides a comparison on the network methods for the flight along with the upper bound (UPPER) on the potential revenue impacts under perfect information. As shown in previous cases, the revenue impacts from the probabilistic network approaches, NPSP and PBID, are inconsistent, with significant negative revenue impacts resulting from the PBID approach as the load factor increases. However, as has consistently been the case thus far, the NDSP approach performs very well, generating revenue impacts of 2.6% at high load factors.

6.2 Comparison of Revenue Impacts on a Hub Network

Based on the detailed analysis in the previous section, a comparison of the more promising approaches will be made here on a hub-and-spoke network where the *full* origin-destination seat inventory control problem is considered. As with the multi-leg analysis, this example is based on actual airline data, providing both a realistic mix of the ODF traffic across the network as well as the actual booking profile of each ODF. For the hub example, incremental ODF demand data was provided for 20 different booking periods, allowing for 20 revision points during the booking process.

The hub network used to evaluate the different network seat inventory control alternatives is made up of 32 different flight legs consisting of 16 flights in and 16 flights out of the hub. A variety of different flight legs is represented over the hub network, ranging in distance from short haul to long haul, as portrayed in Figure 6.44. Across the network of connecting flights, demand exists for 196 of the possible 272 OD pairs. With 10 different fare classes offered in each OD market, there are a total of 1960 different ODF itineraries. There exists a wide range in the level of demand in relationship to the individual flight leg capacities over the network. In the base case which corresponds to the actual demand data and the historical scheduled flight capacities, the demand factor on the different flight legs varies from 0.56 to 1.46, with an overall average demand factor of 0.95. By increasing and decreasing the capacity on each flight leg, the different network seat inventory control approaches are evaluated over a network average demand factor range of 0.80 to 1.39.

As shown in the multiple flight leg analysis, the network probabilistic approaches over-protect seats for the more desirable ODF itineraries, leading to both inconsistent and poor performances, even when ODF's are controlled on the basis of a nested by shadow prices

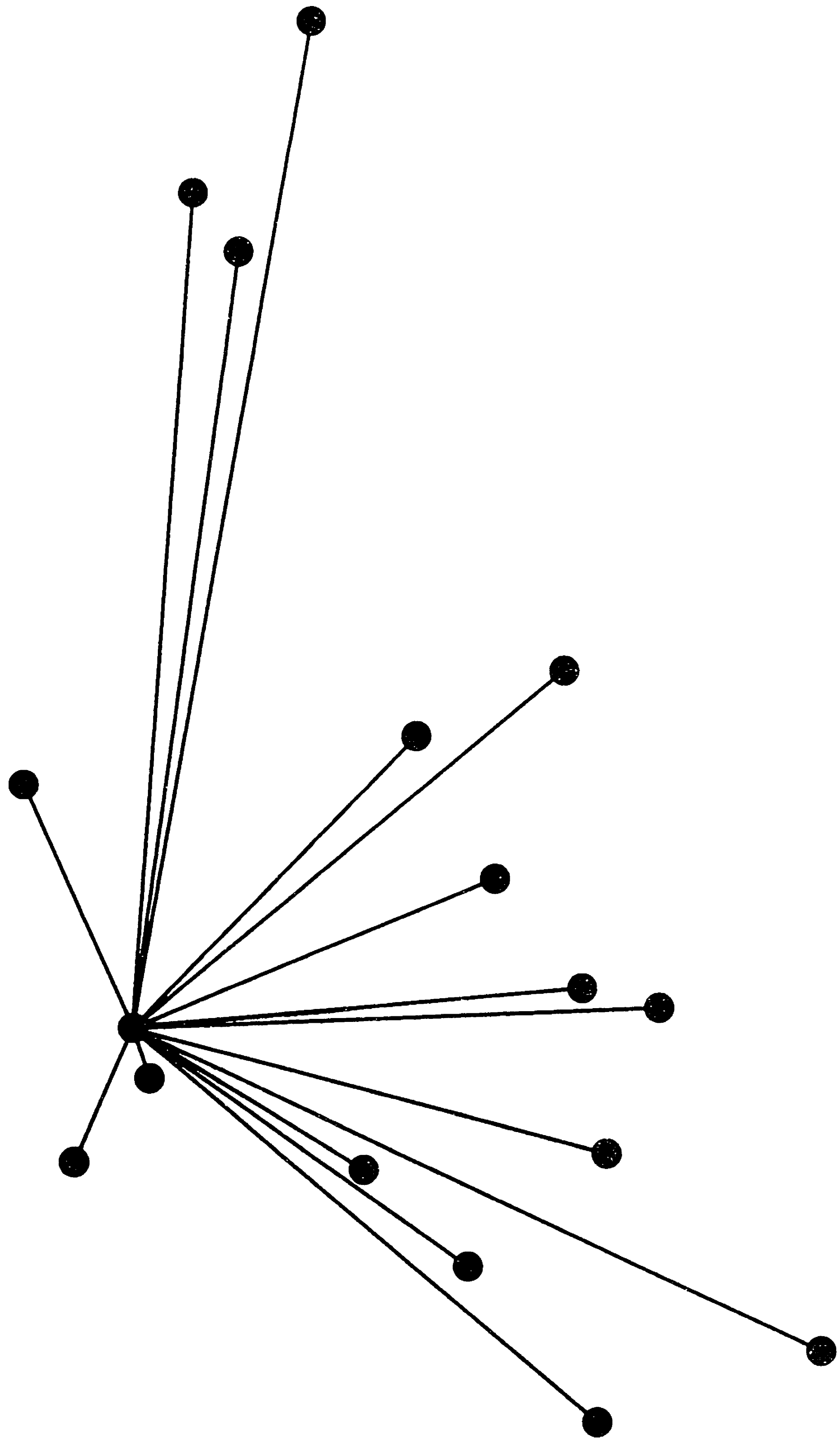


Figure 6.44: A hub-and-spoke network connecting 272 different OD pairs through a connecting complex of 16 flights in and 16 flights out.

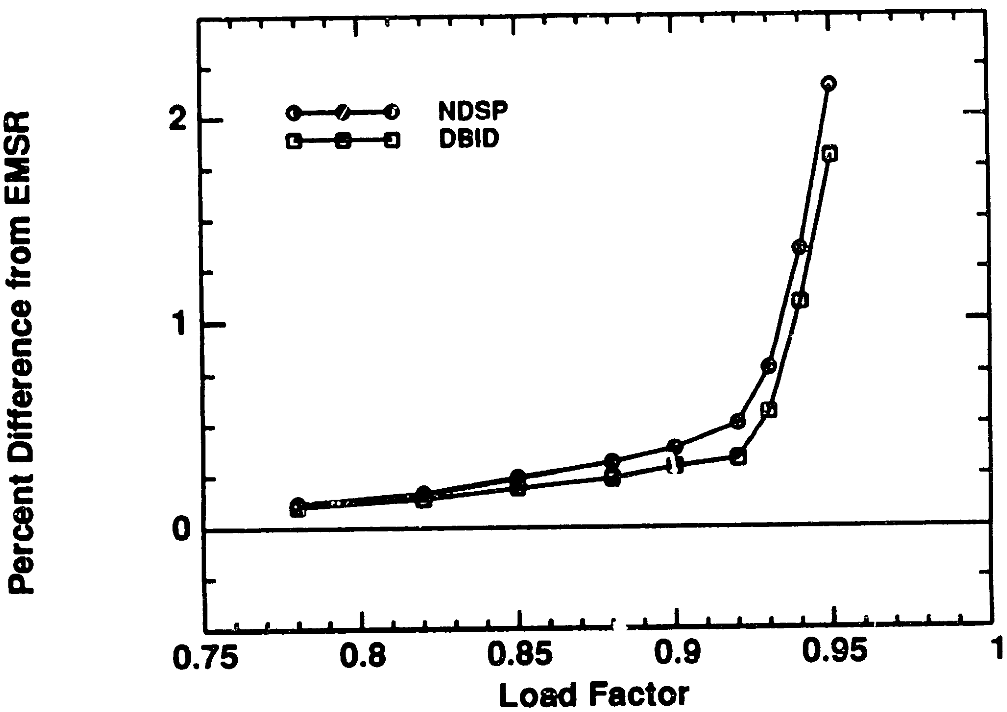


Figure 6.45: Comparison of the revenue impacts from the Nested Deterministic by Shadow Prices (NDSP) approach and the Deterministic Bid Price (DBID) approach for the hub-and-spoke network in Figure 6.44.

methodology or a bid price strategy. However, by using the deterministic network formulation and nesting the solution by shadow prices, significant positive revenues were obtained over the leg-based EMSR control approach. Similar results were also obtained under the Deterministic Bid Price (DBID) approach, however the expected revenue tended to drop-off at the highest demand levels due to the limited frequency of revisions.

Applying the Nested Deterministic by Shadow Prices (NDSP) approach and the DBID approach to the full origin-destination seat inventory control problem on the hub network, Figure 6.45, results once again in very positive revenue impacts. As before, the NDSP approach performs slightly better than the DBID approach, with an increase in revenues over the leg-based EMSR heuristic of as much as 2.1% at high load factors. Even for the base case demand level, corresponding to the 88% load factor level in Figure 6.45, the

NDSP approach yields a 0.3% improvement in revenues. Although this does not seem like a large improvement, for this particular example the 0.3% revenue impact is statistically significant at a 95% confidence level. In terms of absolute revenue, the difference amounts to $7500 which over the course of a year provides $2.7 million dollars in additional revenue over a simple leg-based control approach for this small 32 leg complex, assuming a constant demand level.

While the full deterministic network optimization has consistently been shown to yield positive revenue impacts under an effective control methodology, i.e. nesting by shadow prices or the bid price strategy, problems arise at the hub-and-spoke level with the need to forecast individual ODF demand. For a multi-leg network, the difference in the number of demand estimates between forecasting demand by fare class versus origin-destination itinerary *and* fare class may be a factor of two, four, or even six times. However, even for the small hub-and-spoke network in Figure 6.44, the number of forecasts needed on each flight leg is increased by as much as 16 times. Thus, rather than a demand forecast on a given flight leg of say 20 for Y class, the average OD Y class demand would be 1.25. Typically associated with such small mean demands are large variations in the demand. Forecasting such small numbers with any accuracy has, to date, been both theoretically infeasible and impractical.

One solution to this problem is to group ODF demand together so that forecasting can be performed at a more aggregate level. This is the basis behind leg-based approaches where forecasts are generated at the leg level with ODF demand aggregated into fare classes or virtual classes. Another aggregated demand approach which was introduced in Chapter 4 is the aggregated network method. Similar to leg based approaches, ODF demand on each flight leg is aggregated and forecasted by groups based on similar itinerary fare values. These aggregated demand forecasts are then used in a *network* formulation of the seat inventory control problem rather than optimizing at the flight leg level. In this manner, the interaction between flight legs and the flow of traffic across the network is *explicitly* taken into consideration.

As discussed in Chapter 4, formulating the seat inventory control problem as an aggregated probabilistic network optimization is not a straightforward extension of the original probabilistic formulation. Based on the full probabilistic network approaches, using partitioned probabilistic network solution leads to inconsistent, and even negative, revenue impacts due to the "overprotection" of seats for certain ODF's at the expense of other ODF's across the network. However, by using a partitioned deterministic solution as the basis for some type of (implicit or explicit) nested control strategy, better results are consistently achieved. Making the transition from a full deterministic network model to the aggregated optimization is straightforward.

Using an aggregated deterministic network formulation, seats can be controlled either by nesting the aggregated ODF allocations or by the bid price approach. As explained in Chapter 4 and demonstrated in the multiple flight leg analysis, nesting by shadow prices consistently performs the best out of the three nesting strategies considered. Using the hub-and-spoke example and aggregating the demand on each flight leg into groups, or buckets, based on similar itinerary fare value, an Aggregated Nested Deterministic by Shadow Prices (ANDSP) approach and an Aggregated Deterministic Bid Price (ADBID) approach have been tested. The revenue impacts of these two aggregated approaches over the leg-based EMSR heuristic are compared to the regular Nested Deterministic by Shadow Prices (NDSP) approach in Figure 6.46.

The aggregated methods do not perform quite as well as the full NDSP approach, but the revenue impacts are very similar, with significant positive impacts at high load factors. While the impacts of the NDSP approach are statistically significant (with 95% confidence) for load factors of 85% and above, the impacts from the ANDSP approach and the ADBID approach do not reach statistical significance until load factor levels of 92%

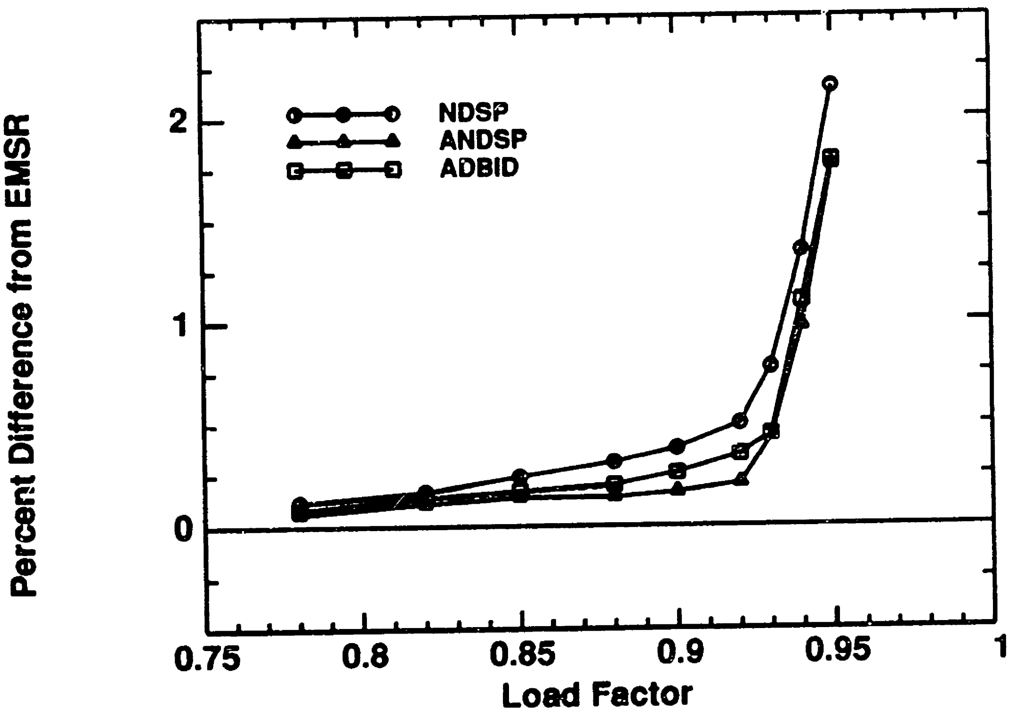


Figure 6.46: Comparison of the revenue impacts for the Nested Deterministic by Shadow Prices (NDSP) approach versus the Aggregated Nested Deterministic by Shadow Prices (ANDSP) approach and the Aggregated Deterministic Bid Price (ADBID) approach for the hub-and-spoke network in Figure 6.44.

and 90%, respectively, corresponding to demand factors of 1.09 and 1.01.

Under the aggregated methodology, the bid price approach actually performs *slightly* better than the ANDSP approach, although not significantly. The reason for this is that, while the aggregation of demand significantly affects individual ODF allocations as well as relative ranking of ODF's in the shadow price nesting hierarchy, there is much less of an impact on the value of the last seat of each flight leg. As shown in the small hub example in Figure 4.4 and Table 4.9 of Chapter 4, the difference in the allocation of seats to each individual ODF between an aggregated network approach and the full network approach can be quite substantial. However, individual differences in the allocation of seats among ODF's of similar revenue value does not have a large effect on the marginal value of the last seat of a flight leg. While the ADBID approach generates slightly more revenue than the ANDSP approach in this example, without adequate revisions the ADBID approach is still subject to the potential of the expected revenue dropping off at high demands due to its open/closed philosophy.

Although positive revenue impacts can be obtained using an aggregated network approach, as shown in Figure 6.46, the potential for such revenue impacts is very sensitive to the actual aggregation of ODF demand. In Figure 6.47, the total expected revenue of the ANDSP approach under several different methods of aggregating demand is compared with that of the leg-based EMSR heuristic and the full NDSP approach. In this figure, the expected revenue bar labeled 'ANDSP' corresponds to the Aggregated Nested Deterministic by Shadow Price approach in Figure 6.46 which generated positive revenue impacts over the EMSR heuristic. Under this approach, the aggregation of ODF demand on each flight leg is fine tuned so that there is very little variation in the fares of ODF's aggregated together while the variation in the fares between buckets is quite large. Thus, if there are several different ODF's with fares between \$220 and \$230 and an ODF with a fare of \$245, the ODF's with the fare values between \$220 and \$230 would be aggregated together while

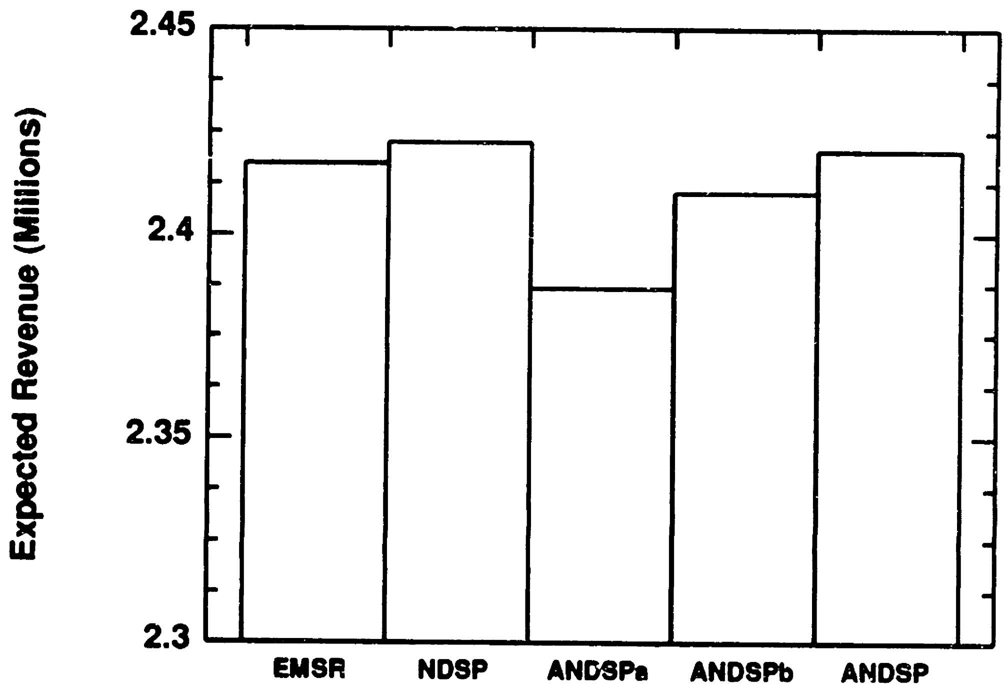


Figure 6.47: Comparison of the expected revenue generated from several Aggregated Nested Deterministic by Shadow Prices (ANDSP) approaches based on different ODF demand aggregations versus the expected revenue of the leg-based EMSR heuristic and the full Nested Deterministic by Shadow Prices (NDSP) approach.

the $245 ODF would be assigned to a separate bucket.

The other two aggregated network approaches shown in Figure 6.47 perform very poorly, generating negative revenue impacts when compared to the leg-based EMSR heuristic. The ANDSPa approach is based on aggregating ODF demand on each flight leg into 15 buckets. While the different buckets are defined based on the same principle as the ANDSP approach with a limited difference in the itinerary fares within a bucket and a larger variation in the fares between buckets, aggregating the ODF's into just 15 buckets does not offer enough differentiation between individual ODF's. Thus, using a simple leg-based approach which explicitly incorporates nesting into the optimization performs better than this distinct, non-nested network optimization solution, as was the case for the probabilistic network approaches.

Under the ANDSPb approach, the aggregation of ODF demand on each flight leg is performed based on $20 increments. Aggregating ODF's together in this manner generates significantly more revenue than under the ANDSPa aggregation approach with just 15 buckets on each flight leg since a larger differentiation between ODF's is made. However, the expected revenue is still less than that obtained by simply controlling seats on the basis of the leg-based EMSR approach. Using such an arbitrary grouping policy, situations arise when an ODF with a fare of $201 is aggregated with an ODF of $219, rather than possibly aggregating the $201 fare with the more closely related fare of $199. The performance of the ANDSPb approach shows how sensitive the aggregated deterministic network method is to the aggregation of demand. Even when the difference between the fares of ODF's aggregated together is only $20, without fine tuning the revenue ranges to properly differentiate between ODF's, a negative revenue impact can result instead of the positive impacts of the ANDSP approach in Figure 6.46.

The last set of methods tested on the hub-and-spoke network are the leg-based heuristics for controlling OD itineraries. In the multiple flight leg analysis it was found that several of the leg-based OD control methods performed quite well, in many cases yielding significant positive revenue impacts over the simple leg-based EMSR approach. For most of the two leg and three leg flights evaluated, the impacts of the combined Leg-Based Bid Price/Booking Limit (LBID/BL) approach, the Virtual Nesting on the "Value Net of Opportunity Cost" (VNOC) approach, and the Nested Leg-Based Itinerary Limits (NLBIL) approach were very similar. As the number of legs on the multi-leg flights increased, the revenue impacts of the VNOC approach began to fall off in comparison to the LBID/BL and the NLBIL approaches.

When applying the three leg-based OD control approaches to the hub network in Figure 6.44, ODF fares are once again prorated to avoid "over-counting". For this example, prorating the fares based on the local Y fare of each flight leg actually resembles the fare structure across the network better than the mileage based proration method, providing slightly better results. Therefore, based on the Y fare proration method, the revenue impacts from the leg-based OD control approaches are provided in Figure 6.48. In general, the overall performance of the leg-based heuristics is quite different from that of the multi-leg examples. Although the VNOC approach does perform well, providing significant positive revenue impacts as the level of demand increases, the combined LBID/BL approach and the NLBIL approach perform very poorly, not just in relationship to the VNOC approach, but compared to the simple leg-based EMSR approach as well.

The poor performance of both the LBID/BL approach and the NLBIL approach is due to a variety of factors. The underlying optimization method used for both leg-based OD control approaches is based on a fare class inventory structure. Thus, on each flight leg an assortment of different OD itineraries with various revenue values are aggregated into a single fare class bucket at a *single* (average) revenue value. Because of the combination of both very long haul and very short haul flight legs in the hub-and-spoke example, within each fare class bucket there is a wide range of revenue values, even after prorating

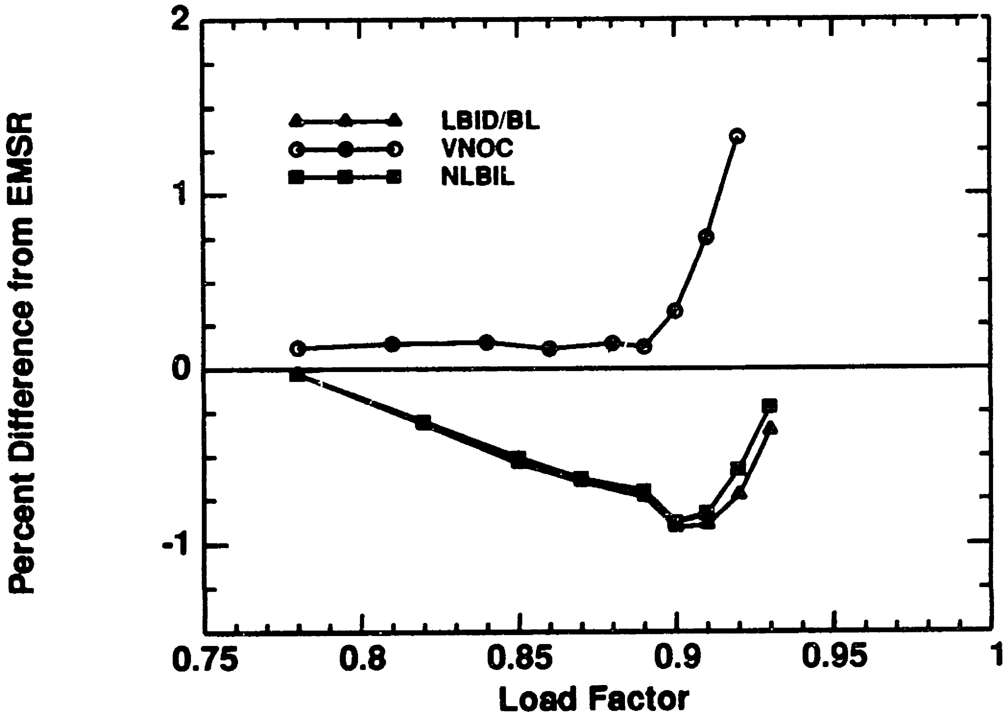


Figure 6.48: Comparison of the prorated Leg-Based Bid Price/Booking Limit (LBID/BL) approach, the Virtual Nesting on the "Value Net of Opportunity Cost" approach, and the Nested Leg-Based Itinerary Limit approach for the hub-and-spoke network.

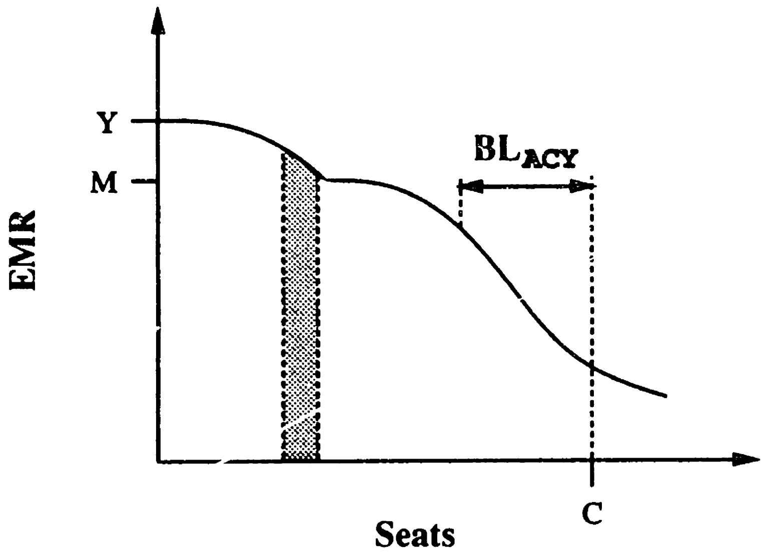
ODF fares across each flight leg. At the same time, there is significant overlapping in revenue values between buckets. Due to this rather arbitrary aggregation of ODF's with their significantly different revenue values on each flight leg, an OD control strategy based strictly on this fare class structure does not allow for an adequate distinction between different ODF itineraries across a network. On the other hand, using virtual inventories defined on the basis of revenue ranges, as in the VNOC approach, a much better distinction between the revenue *contributions* of different ODF's can be made.

Although a leg-based/fare class optimization does not provide the ability to *finely* differentiate between ODF's, that alone does not explain the poor revenue performance of the LBID/BL approach or the NLBIL approach, particularly in relationship to the leg-based EMSR heuristic which is also based on a fare class inventory structure. While the underlying optimization of the two approaches is based on the arbitrary aggregation of ODF's into fare classes, another factor contributing to the poor performance of the LBID/BL approach and the NLBIL approach is the actual OD control methodology of each approach. By combining a leg-based fare class optimization with the control strategies of these two approaches, ODF booking limits do not *directly* correspond with their underlying fare classes and the relationship between the demand in each fare class on *each* flight leg. On the other hand, the VNOC approach explicitly links ODF demand estimates (aggregated into virtual classes) with their respective booking limits, and availability for different ODF's across the network is based on the relationship between demand and the booking limits from *each* flight leg traversed.

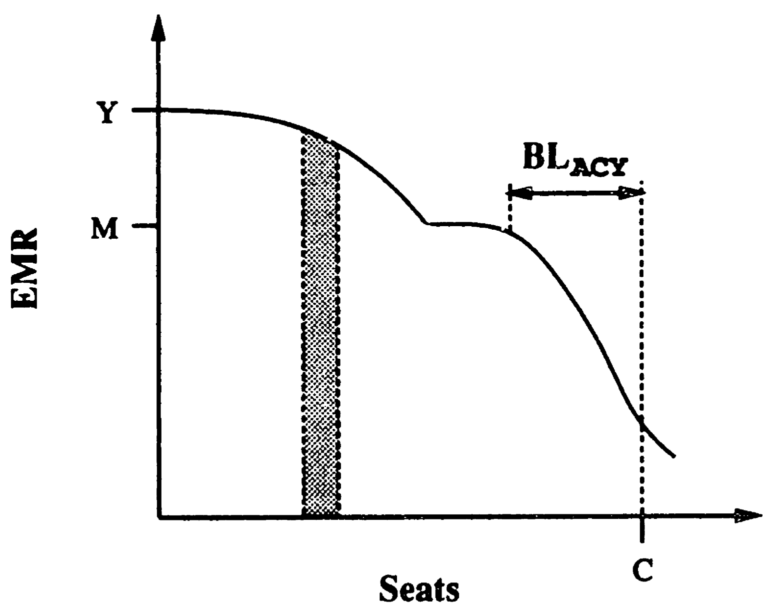
Under the LBID/BL approach, *total* fare class demand on each flight leg is used to determine the bid price "cutoff" value for each ODF. However, in determining the booking limits for an ODF, the level of *individual* fare class demand on *each* flight leg is not taken into consideration. Bookings for a given ODF are limited by the maximum of the respective fare class booking limits of each flight leg traversed by the ODF. Thus, while the relationship between different fare class demand on the leg corresponding to the maximum fare class booking limit is considered, the relationship between demand for each fare class on the other flight legs traversed by the ODF is not considered. With both the wide range in the level of demand on each flight leg across the network and the wide range and overlapping of revenue values in each fare class, the leg-based bid price and the *maximum* fare class booking limit do not provide a very strict limit on the number of bookings for an ODF. Thus, on the more heavily constrained flight legs, lower revenue, less desirable ODF bookings have a tendency of filling the aircraft at the expense of the higher revenue, more desirable ODF's.

Although the control methodology of the NLBIL approach is very different from that of the LBID/BL approach, the NLBIL approach also suffers from the fact that the booking limits for different ODF's do not necessarily correspond to the underlying fare class structure and its forecasted demand. For example, on a two leg flight A-B-C, the multi-leg ACY itinerary is aggregated into the Y fare class inventory bucket on each flight leg. Using a leg-based optimization approach, seats on each flight leg are protected from other fare classes for the aggregated Y fare class demand. However, after summing the expected marginal revenue curves on each flight leg, it is possible for the fare of the ACY itinerary to correspond to a point on the expected marginal revenue curves which is associated with, say, M class demand. Under the NLBIL approach, the nested booking limit for ACY is determined by this point, yet on each flight leg seats are actually being *protected* for the ACY demand in Y class. This inconsistency between the ACY booking limit and the Y fare class protection for ACY demand is shown in Figure 6.49. One potential solution to this problem is more frequent revisions.

An opposite situation can also occur where the fare of an ACM itinerary may correspond to a point on the expected marginal revenue curves which is associated with M class demand on the A-B flight leg, but Y class demand on the B-C flight leg, as shown in Figure 6.50.



Flight Leg A-B



Flight Leg B-C

Figure 6.49: Under the NLBIL approach, the ACY booking limit is determined by comparing the ACY fare with the sum of the expected marginal revenue curves of flight legs A-B and B-C. This example shows how it is possible for the ACY booking limit to correspond with the M class protection on each flight leg while the seats which are actually protected for ACY demand is shown by the shaded region in Y class.

Under such a situation, bookings excepted for the ACM itinerary on flight leg B-C may cut into seats which, based on the fare class optimization, are protected for Y class demand. This can lead to the potential of not enough seats for the typically later booking, high fare Y class demand.

While the fare class aggregation of very large differences in ODF revenue values and the inconsistent relationship between booking limits and fare class protections on each flight leg can potentially lead to poor revenue results under the LBID/BL approach and the NLBIL approach, these problems are further compounded due to the hub-and-spoke network. On a multiple leg flight network of two, three, four, or even five flight legs, the total number of OD itineraries is relatively small. At the same time, on any given flight leg of the network, a large percentage of the OD itineraries are represented. For example, on a three leg flight, 3 of the 6 OD itineraries across the flight traverse the first leg of the flight and are, therefore, represented within a leg-based optimization on the flight leg through the aggregated fare class demand estimates. On the second leg of a three leg flight, 4 of the 6 OD itineraries are represented on the flight leg. However, for the hub-and-spoke example with 16 flights in and 16 flight out, on a single flight leg only 16 of the possible 272 OD pairs are represented in a leg-based optimization on the flight leg. At the leg level, a majority of the OD itineraries are not taken into consideration. Combined with this is the fact that from one flight leg to another there is only *one* common OD itinerary, while there are often a few OD itineraries in common out of the relatively small number of itineraries on a multiple leg flight. Thus, on a multi-leg flight, a leg-based optimization on a single leg of the flight incorporates much of the information about the entire network. Yet, on a hub-and-spoke network, very little about the network as a whole is captured within a leg-based optimization.

Combining the limited information of a full hub-and-spoke network used by a leg-based optimization technique with the arbitrary fare class aggregation and the fact that the ODF

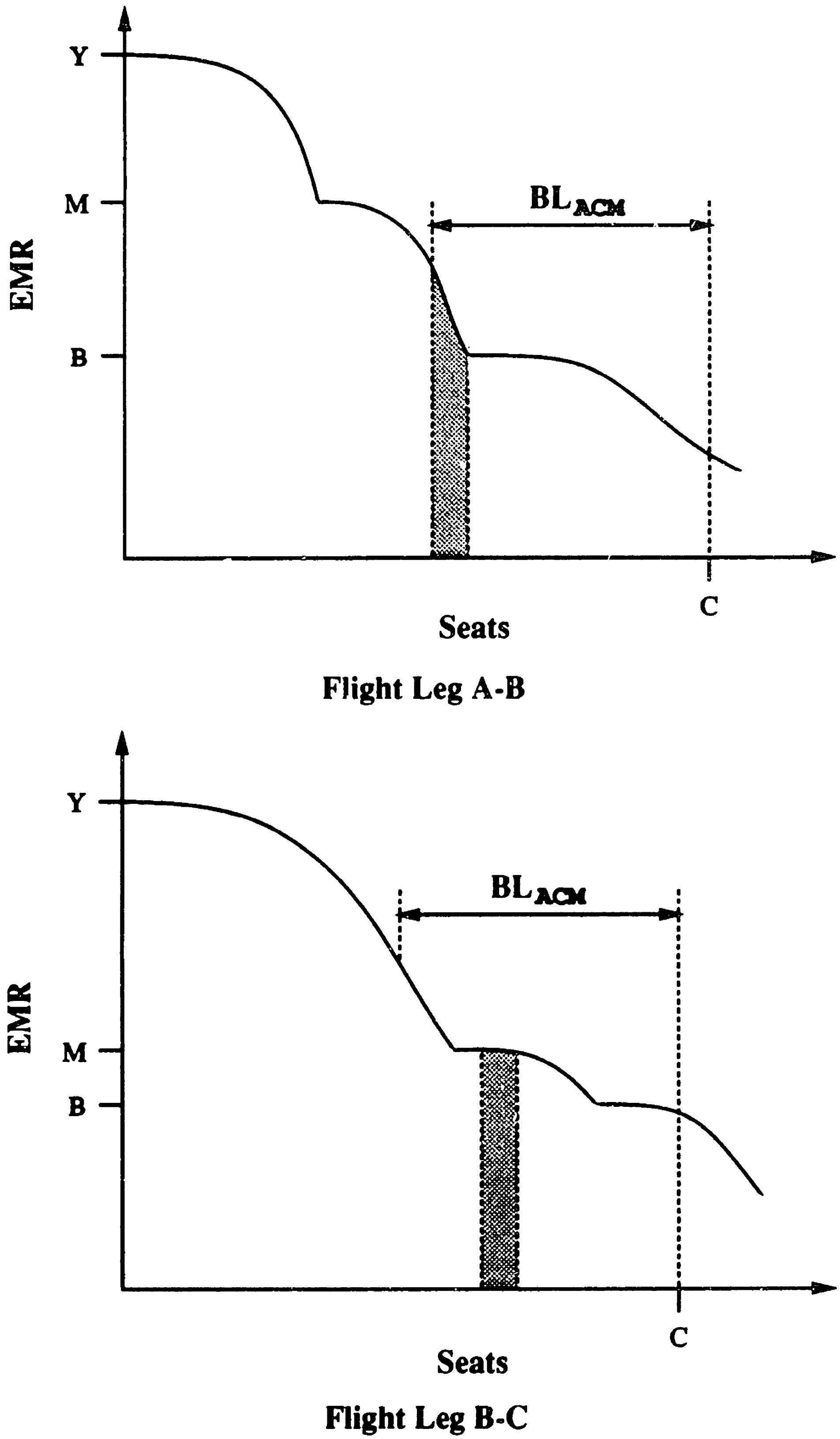


Figure 6.50: In this example, the ACM booking limit, determined by the sum of the two expected marginal revenue curves for the NLBIL approach, is associated with M class on flight leg A-B, but Y class on flight leg B-C. Thus, it is possible for ACM bookings to take seats protected for forecasted Y class demand, resulting in future high fare Y class demand being spilled.

booking limits do not correspond directly with forecasted demand estimates leads to the significant negative revenue impacts for the LBID/BL and the NLBIL approaches shown in Figure 6.48. One solution to this problem may be to use a virtual inventory structure as the basis in determining the expected marginal revenue curves and the leg-based bid prices for each flight leg. Under such an inventory structure, more specific information about the revenue value of different ODF's can be obtained, making it possible to better differentiate between ODF itineraries as demonstrated by the performance of the VNOC approach.

6.2.1 Summary Comparison of Hub Network Revenue Impacts

To summarize the revenue impacts of different network seat inventory control approaches on the hub-and-spoke network, a direct comparison of the potential revenue impacts from the different network optimization approaches and the leg-based heuristic approaches is provided in Figure 6.51. While the Nested Deterministic by Shadow Prices (NDSP) approach once again performs the best, generating an additional 2% in revenues over the leg-based EMSR heuristic at high demand levels, using an aggregated network optimization approach such as the ANDSP approach or a leg-based OD control method such as the VNOC approach, significant positive revenue impacts can be obtained. While statistically significant positive revenue impacts where obtained under the NDSP approach for the base case of a 0.95 average demand factor (corresponding to the 88% load factor level for NDSP), significant revenue impacts are not obtained under the ANDSP and VNOC approaches until the average demand factor is over 1.09 and 1.15, respectively, corresponding to the 92% load factor and the 90% load factor.

Also shown in Figure 6.51 is the maximum revenue potential, or upper bound (UPPER), for the hub network. While the maximum revenue potential at high demand levels was on the order of 6-9% for the multi-leg flights evaluated, due to the wide range in the level of demand across the hub-and-spoke network, the maximum revenue potential is on the

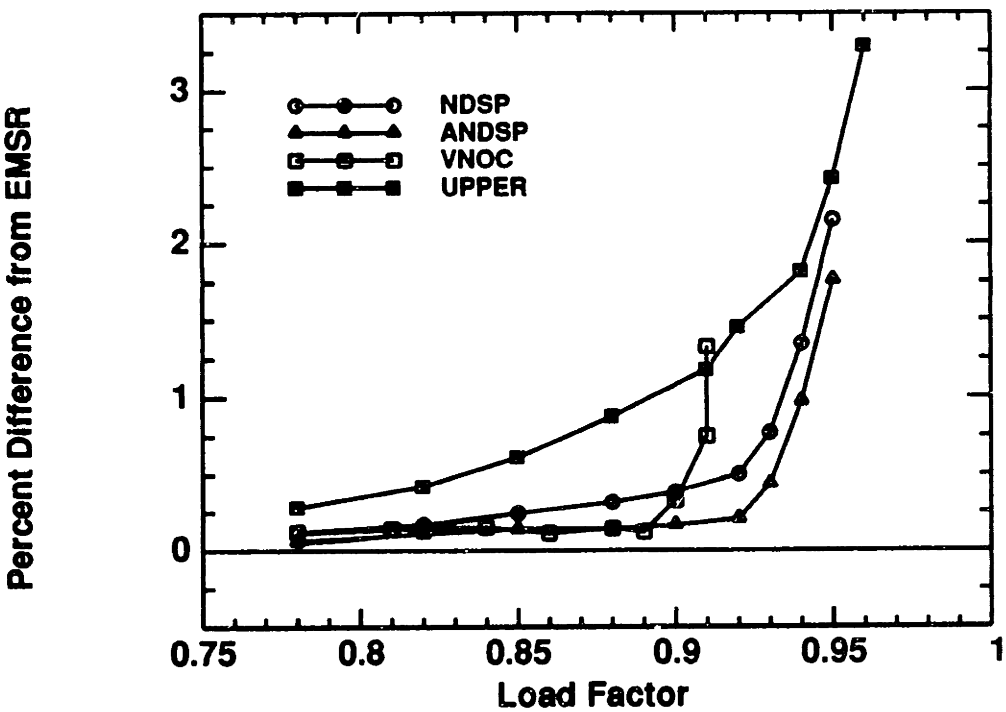


Figure 6.51: Summary comparison of the revenue impacts of the Nested Deterministic by Shadow Prices (NDSP) approach, the Aggregated Nested Deterministic by Shadow Prices (ANDSP) approach, and the Virtual Nesting on the "Value Net of Opportunity Cost" approach and their relationship to the maximum revenue potential, or the upper bound (UPPER), for the hub-and-spoke network.

order of 3.5% at the highest demand level. It has been thought by some that the benefits found in segment control will increase as a full origin-destination seat inventory control strategy is implemented across an entire network of connecting flights. As illustrated in this hub-and-spoke example, not only is the *upper bound* in revenues not even close to the 10-13% claimed to be obtainable through controlling seat inventories by OD's as well as fare classes, but it also shows that, as the size of the network increases, the potential revenue benefits do not necessarily increase.

6.3 Summary

In this chapter, the revenue impacts of the different network seat inventory control methodologies introduced in Chapter 4 have been evaluated for both a variety of multiple flight leg networks and a connecting hub-and-spoke network. It was shown that the direct implementation of the network seat allocations derived from the both the deterministic and the probabilistic network optimization models as *partitioned* ODF booking limits can result in significant negative revenue impacts when compared to an effective leg-based control approach. However, by combining these "optimal" network seat allocations with the nesting philosophy used in leg-based methods, positive revenue impacts can be obtained.

Several different strategies for nesting the ODF seat allocations are examined: nesting by fare class, nesting by fares, and nesting by shadow prices. Under the nesting by fare class approach, disaggregated ODF seat allocations are aggregated back to the fare class and flight leg level where the differentiation between passenger itineraries is lost. This loss leads to inconsistent revenue results. By nesting ODF network seat allocations by fare values, an aspect of "greediness" is introduced where the highest revenue itineraries always receive priority in terms of seat availability. This results in significant negative revenue impacts, particularly at high demand levels. Using a shadow price nesting strategy, an attempt is made to better reflect the value of each ODF to the network as a whole in the nesting hierarchy. Based on such a control approach, very promising results are found with as much as 2-4% positive revenue impacts obtained at high load factors under the Nested Deterministic by Shadow Price (NDSP) approach for both the multi-leg flights and the connecting hub network.

Contrary to what is typically thought to be the case, when the partitioned network seat allocation solutions are used as the basis of some type of nested control strategy, the deterministic optimization models consistently outperform the probabilistic models. The reason for this is that the partitioned probabilistic network solution tends to "overprotect" seats for the more desirable and higher fare class ODF's. Not only does the partitioned probabilistic network approach initially overprotect seats for more desirable ODF's, but this overprotection of seats has a tendency of being compounded throughout the booking process. It is important to emphasize that the network seat allocation solutions are based on a simplified representation of the real world problem which does not include, among other things, nesting. Due to the "incorrect" representation of the problem, methods for nesting these so called "optimal" solutions are themselves no longer optimal. Within these network *heuristics*, it is the deterministic solution which provides a better *input* for nesting.

Based on the same network optimization models, a completely different approach developed as a part of this research project is to manage bookings using a bid price control strategy. Revenue impacts of the Deterministic Bid Price (DBID) approach are very similar to that of the NDSP approach except at high demand where, due to the open/closed control philosophy of the bid price approach, the level of the expected revenues tends to drop-off significantly under a limited revision policy. As in the nested network optimization approaches, the DBID approach outperforms the Probabilistic Bid Price (PBID) approach. In the same manner that the partitioned probabilistic optimization model overprotects seats for more desirable ODF's at the expense of less desirable ODF's, the probabilistic bid prices are often higher than deterministic bid prices, closing down lower fare ODF's prematurely.

While forecasting demand at the ODF level on a multi-leg network of three, four, or even five flight legs is not impractical, an obstacle to a full origin-destination seat inventory control approach on a large hug-and-spoke network is the "small numbers" problem associated with forecasting demand at the ODF level. Through an aggregated deterministic network optimization model, combinations of ODF demand which have the same level of attractiveness on each flight leg are aggregated while the allocation of seats is performed at the network level, rather than the flight leg level. Using either a nested by shadow price control strategy or a bid price control strategy, revenue impacts similar to, although not quite as high as, a full NDSP approach can be obtained. Although positive revenue impacts can be obtained using an aggregated network approach, the potential for such revenue impacts is very sensitive to the actual aggregation of ODF demand.

Currently, there are several obstacles within the airline industry which pose significant problems with controlling seat inventories at the network level. Therefore, several leg-based heuristics have been evaluated which take into account information about passenger demands and traffic flows across a network while the actual optimization and control of ODF itineraries remains at the flight leg level. The overall performance of the different leg-based OD control approaches is not as consistent as that of the NDSP approach. Depending on the actual network, the market conditions, and the relationship between local and connecting ODF demand and fares, the "best" leg-based OD control approach varies. However, it has been shown that incremental revenue impacts can be obtained over a straight leg-based and fare class seat inventory control approach through the use of leg-based OD control approaches.

Lastly, a comparison of the different approaches to the maximum revenue potential, or upper bound in revenues, has been presented. Over most multi-leg flights considered, this upper bound in revenues is on the order of 6-9% at very high demand levels, i.e. average

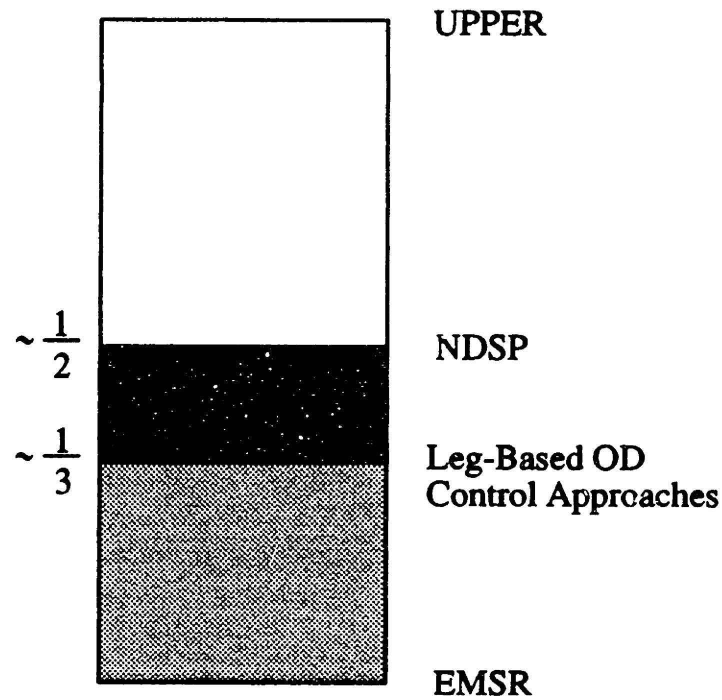


Figure 6.52: A summary of the approximate revenue opportunity achieved under the Nested Deterministic by Shadow Prices (NDSP) approach and the leg-based OD control approach.

demand factors of 1.3 to 1.5. For the hub-and-spoke example, the upper bound was 3.5% for a network average demand factor of 1.39 Comparing the different network seat inventory control approaches to these upper bounds in revenues, it is shown that, on average, approximately 1/2 of the total revenue opportunity between current leg-based control and the maximum revenue potential is achieved by the Nested Deterministic by Shadow Prices approach for both multi-leg flights and the hub-and-spoke network. Using a leg-based OD control heuristic, as much as approximately 1/3 of the total revenue opportunity is achieved. The relationship of these revenue potentials is illustrated in Figure 6.52.

Chapter 7

Conclusion

7.1 Summary of Research Findings

The emphasis of this dissertation has been on the airline seat inventory control problem at the network level, taking into account the interaction of flight legs and the flow of traffic across a network. It is important to note that there are two basic components to the problem of maximizing revenue through seat inventory control, the first being the actual determination of seat allocations in an optimization algorithm and the second being the application of these seat allocations in the form of booking limits in such a way that the potential of increased revenues is obtained. With this in mind, combinations of a variety of optimization algorithms and control methodologies have been developed and evaluated in detail for both multiple leg flights and a hub-and-spoke network. In an effort to isolate the effects of the different network seat inventory control techniques, the analysis was based on a rather simplified representation of the seat inventory control problem, with such factors as cancellations, no-shows, and passenger upgrades not included.

The driving force behind developing and designing new approaches to managing and controlling seat inventories is to capture additional revenue beyond that being realized from current leg-based seat inventory control approaches. In order to effectively measure the revenue potential of the different network seat inventory control approaches introduced in this dissertation, an integrated optimization/booking process simulation was developed as a part of this research. The simulation is a multi-period, computer-based, mathematical program which realistically models the booking process and airline seat inventory control practices. A sensitivity analysis was performed to test the key assumptions made in modeling the airline booking process.

Through an "upper bound" analysis, the true revenue potential of better seat inventory control approaches has been obtained. While it has been claimed that controlling seats at the origin-destination and fare class level can provide revenue impacts of as much as 10-13% over current leg-based seat inventory control approaches, the absolute *maximum* potential has been shown to typically range from 6-9% for multiple leg flights of two, three, and four legs at very high demand levels. On a hub-and-spoke network, the upper bound in revenues at high demand levels is approximately 4%.

Through the integrated optimization/booking process simulation, it has been shown that direct application of traditional network solutions to seat inventory control yields significant negative revenue impacts when compared to current leg-based approaches, particularly at load factor levels below 90%. However, by using information from the dual of the network optimization model, network solutions can be adapted to the seat inventory control problem to provide significant positive revenue gains. On average, the revenue potential of these network optimization approaches is about 1/2 of the maximum potential. At very high load factors, these incremental revenues have consistently been found to be in the 2-4% range. Although the revenue impacts are not on the order of the 10-13% previously claimed, a 2-4% increase over leg-based control on high demand flights can translate into a 0.3-0.5% increase in *total* airline revenues which is not a small benefit in absolute terms.

While effective control of origin-destination and fare class itineraries at the network level can provide significant revenue benefits, there are several practical obstacles to the successful implementation of full OD and fare class control at the network level. However, by using concepts from the network optimization approaches to develop leg-based heuristics which incorporate information about passenger demands and traffic flows across a network of flights, as much as 1/3 of the revenue potential between current leg-based approaches and the upper bound in revenues can be obtained.

7.2 Contributions

There are several contributions made in this research in terms of both theoretical developments in the area of airline revenue management and practical approaches for carriers to gain incremental revenue through network seat inventory control. From a theoretical standpoint, perhaps one of the most important contributions of this research is the demonstration that direct applications of traditional network seat allocation solutions provide significant *negative* revenue impacts when compared to an effective leg-based seat inventory control approach. The notion that any kind of itinerary control will increase total flight revenues is clearly not true.

The foundation of many of the contributions of this research is due to the development of the integrated optimization/booking process simulation. In this research, the theory for a realistic representation of the interaction between airline reservations control and the booking process was developed. This was then modeled through a dynamic, computer-based mathematical simulation model. This simulation has been used extensively to test a wide variety of network seat inventory control approaches on an assortment of networks, from a single flight leg to multiple leg flights to a hub-and-spoke network. In many instances, the comparative results and revenue impacts obtained from the dynamic booking process simulation have been significantly different from those obtained previously under a static simulation.

By combining network optimization with nested control strategies, several new approaches to network seat inventory control have been introduced which can provide significant positive revenue impacts over current leg-based approaches. Contrary to what is intuitively thought to be the case, it has been shown that using a *partitioned* probabilistic network solution as the basis for nested control applications is not as effective as a deterministic network solution.

Using concepts from the network optimization, several leg-based heuristics to OD control have also been introduced. These methods take into account both passenger demands and the interaction between different flight legs across a network while the optimization and control of ODF itineraries remains at the flight leg level. Using such leg-based OD control methods, incremental revenue gains over current leg-based and fare class approaches are obtained.

Finally, using the integrated optimization/booking process simulation, realistic estimates of the revenue impacts obtained from controlling seat inventories at the network level have been generated. These revenue impacts typically range between 2-4%, but only in very high demand situations. Benefits exceeding the 2-4% impact over leg-based control are rare and are clearly not on the order of the 10-13% previously claimed within the airline industry.

7.3 Future Research Directions

While significant progress into the network seat inventory control problem has been made through this research, there is much work which can be done as a direct extension to the evaluation performed here, as well as with regards to controlling seats on the basis of origin-destination and fare class in general. As stated initially, a number of factors, such as cancellations, no-shows, and passenger upgrades, are not considered, making it easier to identify differences in the actual optimization and control of origin-destination and fare class itineraries. However, before developing and implementing a new reservations control system, it is important to address and incorporate such factors.

Many of the approaches introduced and discussed in this dissertation require some type of aggregation into virtual inventories for forecasting and optimization purposes as well as simply for control purposes. Based on analysis done through this research project, the particular revenue ranges for virtual inventory buckets on each flight leg can have a significant effect on the performance of a seat inventory control methodology. Since the purpose of this research has been to introduce the basic optimization and control techniques, for much of the analysis (with the exception of the evaluation of the aggregated network methods on the hub-and-spoke example), virtual inventory buckets were defined on the basis of a single ODF itinerary. Future work is needed to determine the "best" value definitions, or ranges, for virtual inventory buckets as well as the "optimal" number of virtual buckets which provides enough aggregation of ODF demand for forecasting purposes while allowing for enough differentiation between ODF itineraries to obtain revenue benefits.

With respect to the leg-based OD control heuristics introduced, there are two additional areas which should be addressed in future work. One of these has to do with the proration of ODF fares across individual flight legs. Several *basic* proration approaches have been evaluated in this research, but these approaches are not robust and their effectiveness varies with an airline's network and fare structure. Through some combination of such factors as mileage, fares, and demand levels, it might be possible to determine a prorate method which consistently captures the "correct" contribution of an ODF to a flight leg.

Another area for future work is in the extension of several of the leg-based heuristics to a virtual inventory structure. As alluded to in Chapter 6, it is possible that a virtual inventory structure, rather than a fare class inventory structure, would provide better information about the revenue value of different ODF's on a flight leg, particularly for a hub-and-spoke network. In order to determine if this is true, evaluation of the leg-based OD control heuristics using a virtual inventory framework is necessary.

While several network optimization approaches have been introduced through this research, there has been no work done in determining the most efficient mathematical algorithms to use. Work in this area is needed to answer such questions as: Which optimization algorithms work the fastest? How fast can a solution be obtained from a cold start? How fast can a solution be found from a warm start? Within the multi-revision process, how good is the previous network solution as a warm start for the next revision?

Additional analysis with respect to the effects on revenue impacts of forecasting and controlling reservations at the network level would be interesting. If it is possible to forecast demand at the flight leg level within 10% of the true demand, yet forecasting demand at the ODF level can only be done within 50% of the true demand, how does this effect the the relative revenue impacts between a network optimization approach and a leg-based optimization approach? At the same time, if an airline is able to control 40% of its own reservations at the network level, but all other reservations are controlled by other airlines at the leg level, what effect does this have on the overall potential revenue benefits of network seat inventory control approaches?

Other areas for future work: Including the first class cabin in a network seat inventory control solution, taking into account differences in the level of service which must be provided while allowing seats to be shared between first class and coach compartments; incorporating overbooking at the network level; modeling airline demand using a compound Poisson distribution; developing *practical* network optimization approaches which will take into account the fact that future adjustments to seat allocations are made, such as a dynamic programming approach; and finally, considering the full integration of reservations control with pricing and scheduling, rather than treating each as separate functions of the marketing process.

Another important issue to consider in greater detail is the implementation requirements of each network seat inventory control alternative in an airline's reservations system. This has been discussed briefly, but not to the extent of its importance. There are still outstanding questions with respect to the feasibility of forecasting demands for each ODF itinerary, the potential effects on revenue impacts of implementing network seat allocations in an actual inventory control structure, and the need to communicate availability for each OD and fare class to other CRS's. At the same time, the development costs associated with the different approaches and the ability to adapt a particular network seat inventory control system to changes in the constantly evolving airline industry are both important issues which should be addressed.

Compatibility with an airline's current practices, the size of the carrier, the overall route structure of an airline, the number and type of markets served, the demand densities over the network, and the competitive environment in which an airline operates can all dictate the specific approach which is best for a given airline. However, the potential benefits of an effective network seat inventory control approach to an airline are not insignificant. Not only is origin-destination and fare class control an important tool for revenue improvement; it will become a necessity in the increasingly competitive environment of the airline industry.

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Appendix A

Sensitivity Analysis

As discussed in Chapter 5, there is much uncertainty when it comes to characterizing airline demand. Thus, in modeling the booking process of an airline, several assumptions were made in the integrated optimization/booking process simulation regarding the underlying patterns of ODF demand. In this appendix, several of these assumptions will be reviewed using the multi-leg flight examples in Chapter 6 to determine what effects, if any, the different assumptions have on the performance of the network seat inventory control approaches.

The simulation results shown in Chapter 6 are based on the assumption that incremental demand follows a Poisson probability distribution with no correlation between bookings on hand and bookings to come. For the multiple leg flights, ODF requests are simulated using incremental demand data for 15 different booking periods. Through this incremental demand data, the general relationship in the booking sequence between different OD's and fare classes is represented. However, within each booking period the booking order is assumed to be from low to high fare class, with the arrival of different OD itineraries within a fare class determined randomly.

The first assumption which will be analyzed is the demand distribution assumption. Based on analysis done in this research and by Lee [48], a Poisson distribution with its

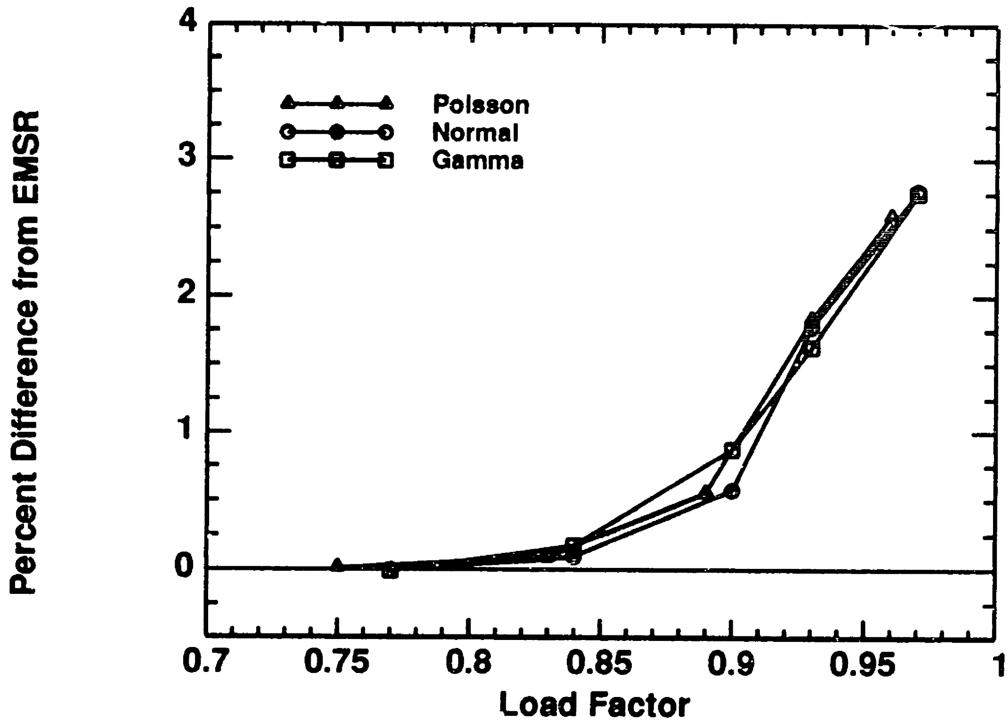

Figure A.1: Comparison of the revenue impacts under a Poisson, a normal, and a gamma demand distribution assumption for the Nested Deterministic by Shadow Prices approach for Flight 31.

fundamental properties of discreteness, positive skewness at low means, truncation at zero, and even the strict relationship between the mean and standard deviation seems to be a reasonable assumption for modeling airline demand, particularly at the ODF level. However, other demand distributions which also seem to statistically fit the data and are used throughout the industry are the normal distribution and the gamma distribution. Using the three leg Flight 31 from Chapter 6 and varying the demand factor from 0.75 to 1.25, simulation results based on these three different distribution assumptions, the Poisson, the normal and the gamma, are compared in Figures A.1, A.2, A.3, A.4, and A.5 for the NDSP approach, the DBID approach, the LBID/BL approach, the VNOC approach, and the NLBIL approach, respectively.

In all five figures there is very little variation in the revenue impacts from the different

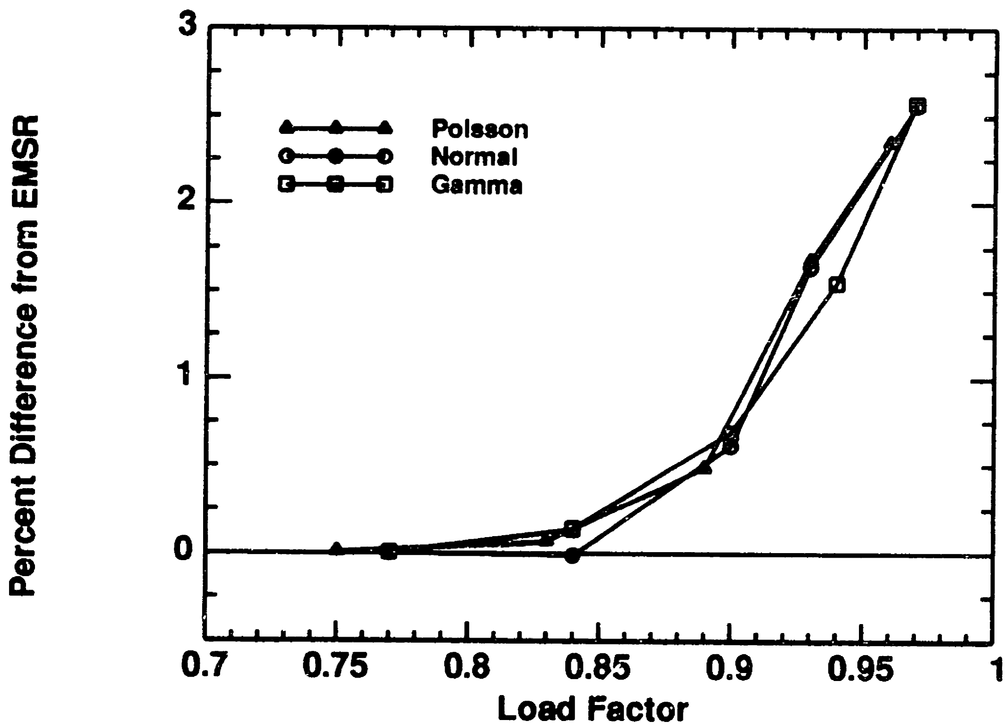

Figure A.2: Comparison of the revenue impacts under a Poisson, a normal, and a gamma demand distribution assumption for the Deterministic Bid Price approach for Flight 31.

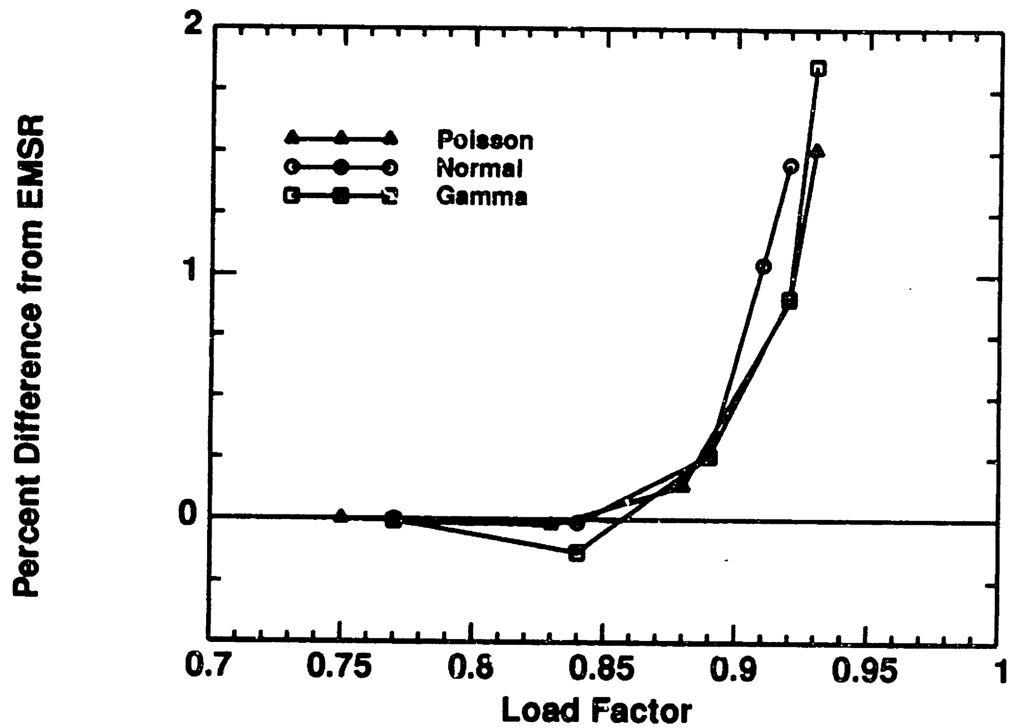

Figure A.3: Comparison of the revenue impacts under a Poisson, a normal, and a gamma demand distribution assumption for the mileage prorated combined Leg-Based Bid Price/Booking Limit approach for Flight 31.

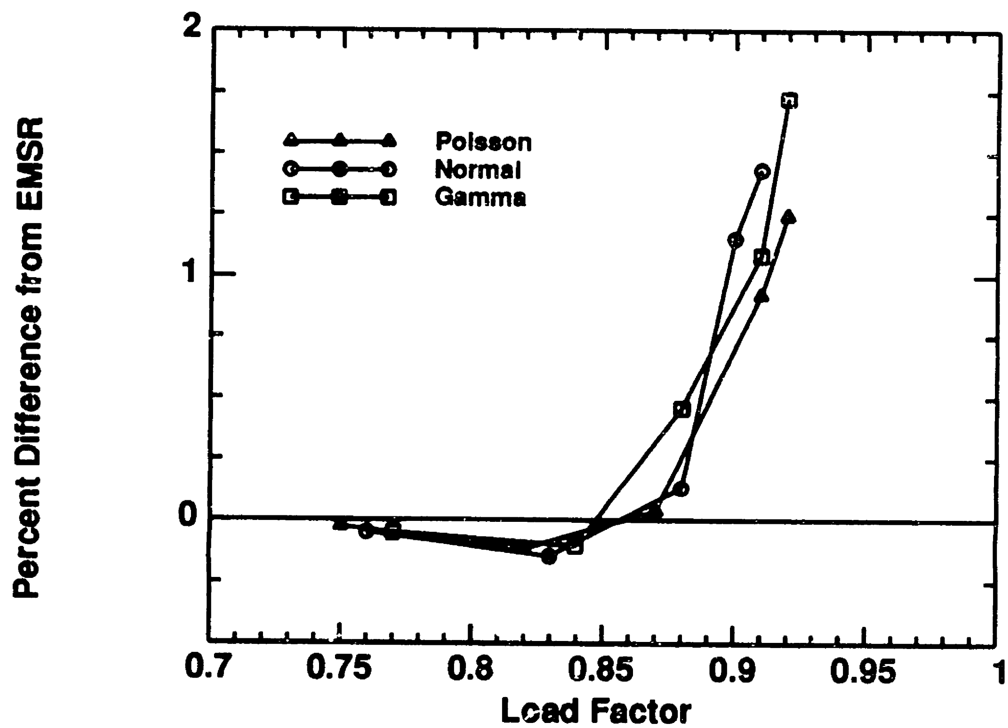

Figure A.4: Comparison of the revenue impacts under a Poisson, a normal, and a gamma demand distribution assumption for the mileage prorated Virtual Nesting on the "Value Net of Opportunity Cost" approach for Flight 31.

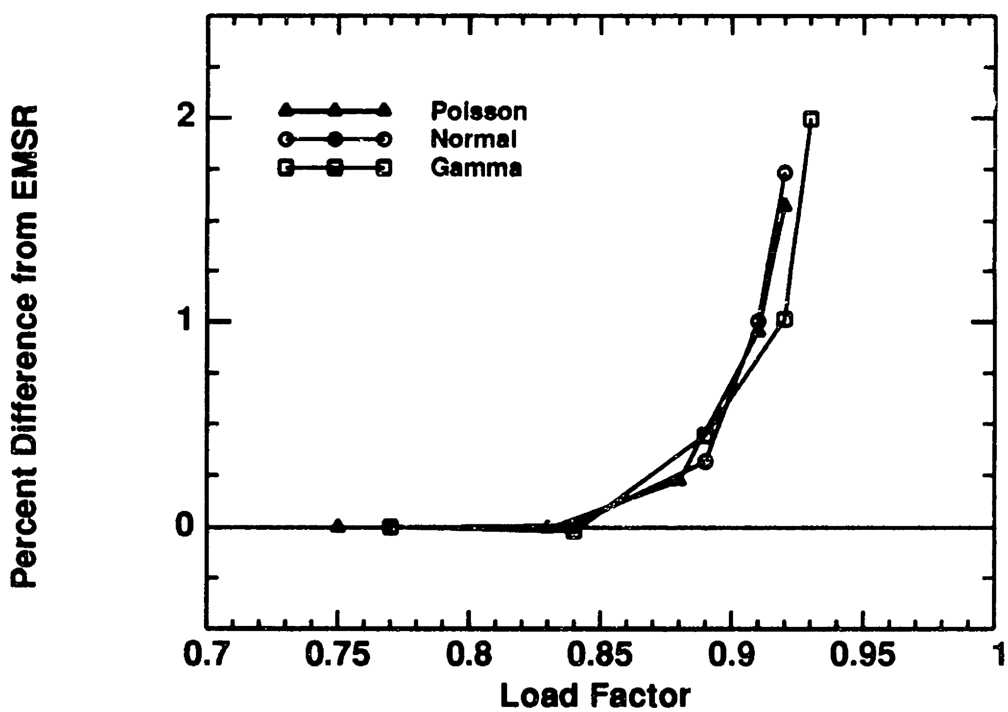

Figure A.5: Comparison of the revenue impacts under a Poisson, a normal, and a gamma demand distribution assumption for the mileage prorated Nested Leg-Based Itinerary Limit approach for Flight 31.

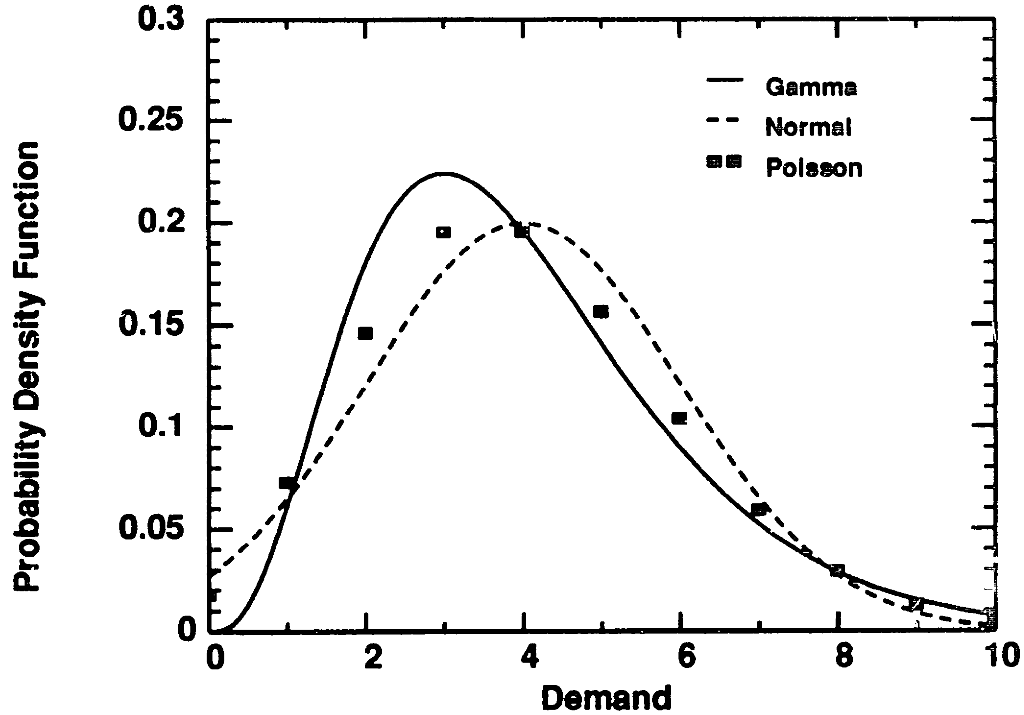

Figure A.6: Comparison between a Poisson, a normal, and a gamma distribution for a mean demand of 4 and a standard deviation of 2.

demand distributions. More importantly, for a given distributional assumption the relationship between the different seat inventory control approaches has not changed. While there are some basic fundamental differences between the three distributions considered, the underlying distributions are quite similar. This is shown in Figure A.6 where the three distributions are compared for a mean demand of 4 and a standard deviation of 2. It is this similarity which leads to the inconsistency throughout the industry in the actual distribution which is accepted and used by different airlines. Regardless of the *true* underlying distribution of demand, the final revenue impacts of a network seat inventory control approach over a simple leg-based control approach do not seem to be significantly affected.

In order to analyze the sensitivity of the different approaches with regards to the correlation between demands as well as the booking sequence, a modified simulation was developed. Under this modified simulation, demand from one booking period to the next

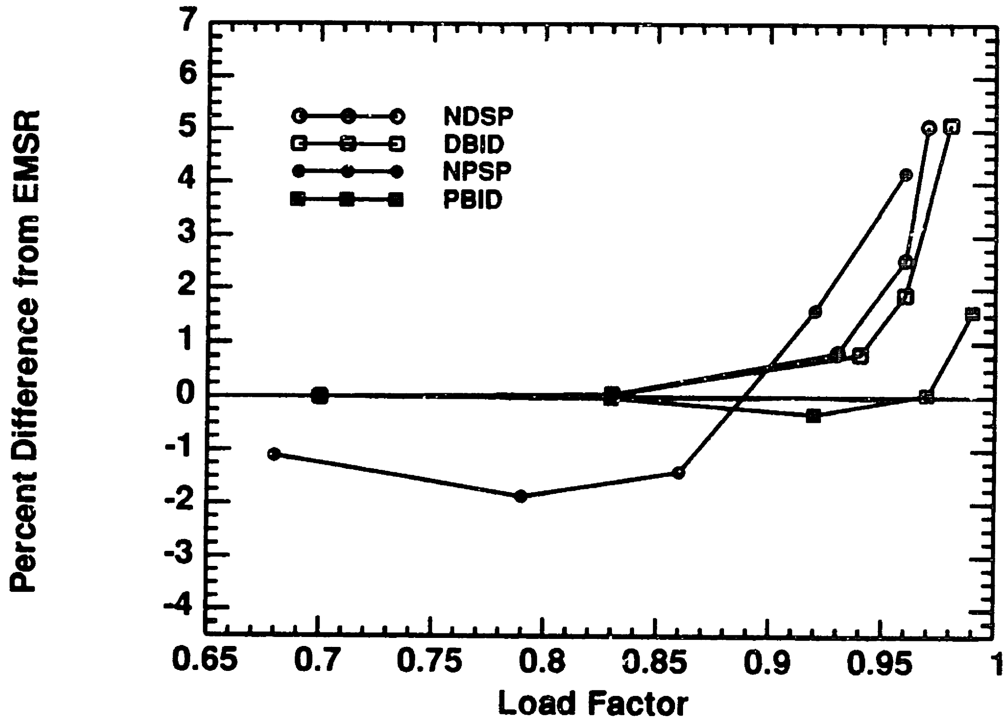

Figure A.7: Revenue impacts of the different network approaches, i.e. the Nested Deterministic by Shadow Prices (NDSP) approach, the Deterministic Bid Price (DBID) approach, the Nested Probabilistic by Shadow Prices (NPSP) approach, and the Probabilistic Bid Price (PBID) approach, under the modified simulation where ODF demand within each booking period is booked *randomly* and a *positive correlation* between bookings on hand and bookings to come is assumed.

is assumed to be correlated. Thus, instead of independently determining demand for each booking period, total demand for an ODF is determined on the basis of sum of the incremental demand data. This total demand is then randomly distributed between booking periods according to the proportion of ODF demand which historically books in each period, reflecting a positive correlation between bookings on hand and bookings to come. At the same time, the booking sequence between periods in the modified simulation is different. Rather than a "lowest class books first" assumption, all ODF demand within a booking period is booked at random.

Using a variation of the three leg Flight 32 in Chapter 6, the effect of this modified simulation is shown in Figures A.7, A.8, A.9, and A.10. Figures A.7 and A.8 compare the

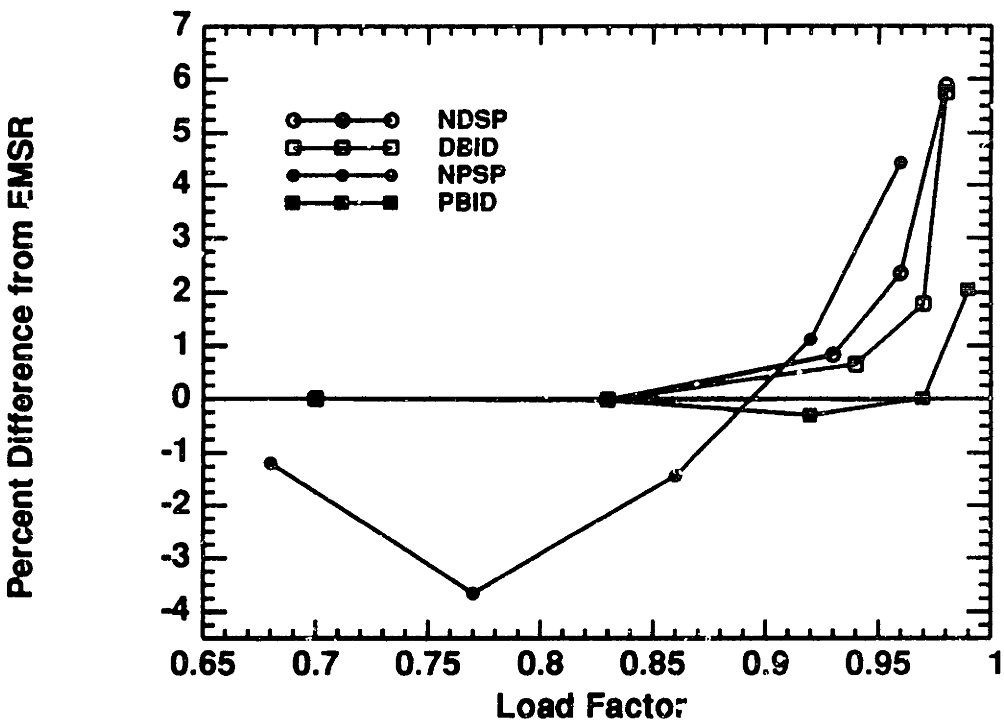

Figure A.8: Revenue impacts of the different network approaches, i.e. the Nested Deterministic by Shadow Prices (NDSP) approach, the Deterministic Bid Price (DBID) approach, the Nested Probabilistic by Shadow Prices (NPSP) approach, and the Probabilistic Bid Price (PBID) approach, under the original integrated optimization/booking process simulation where ODF demand between booking periods is assumed to be *independent* and ODF demand within a booking period is booked from *low to high* fare class.

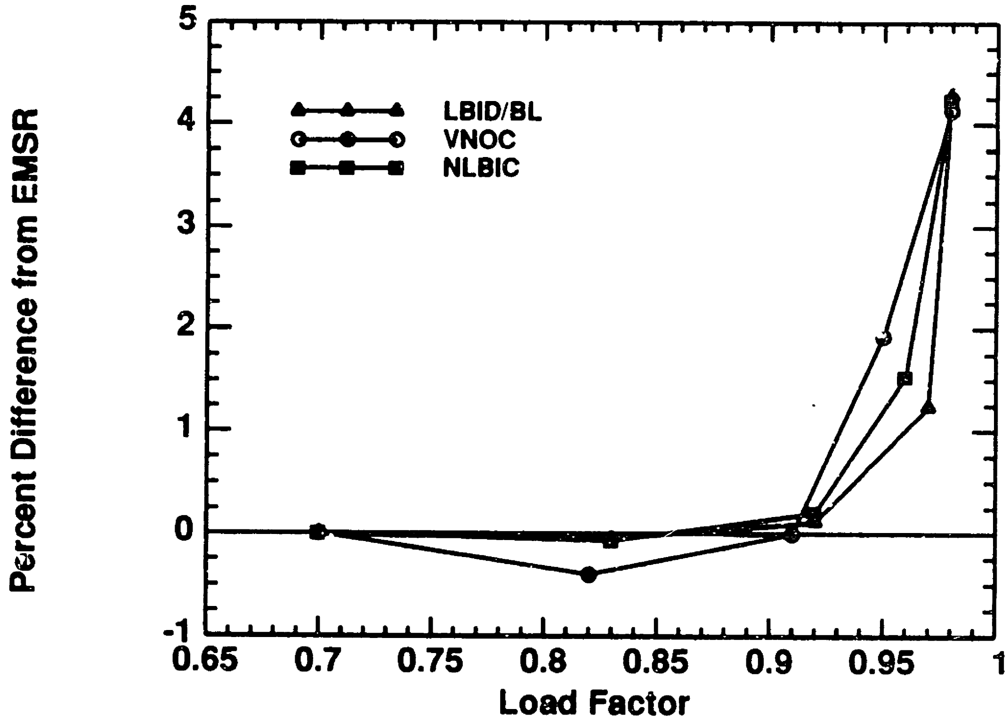

Figure A.9: Revenue impacts of the different prorated leg-based OD control approaches, i.e. the combined Leg-Based Bid Price/Booking Limit (LBID/BL) approach, the Virtual Nesting on the "Value Net of Opportunity Cost" (VNOC) approach, and the Nested Leg-Based Itinerary Limit (NLBIL) approach, under the modified simulation where ODF demand within each booking period is booked *randomly* and a *positive correlation* between bookings on hand and bookings to come is assumed.

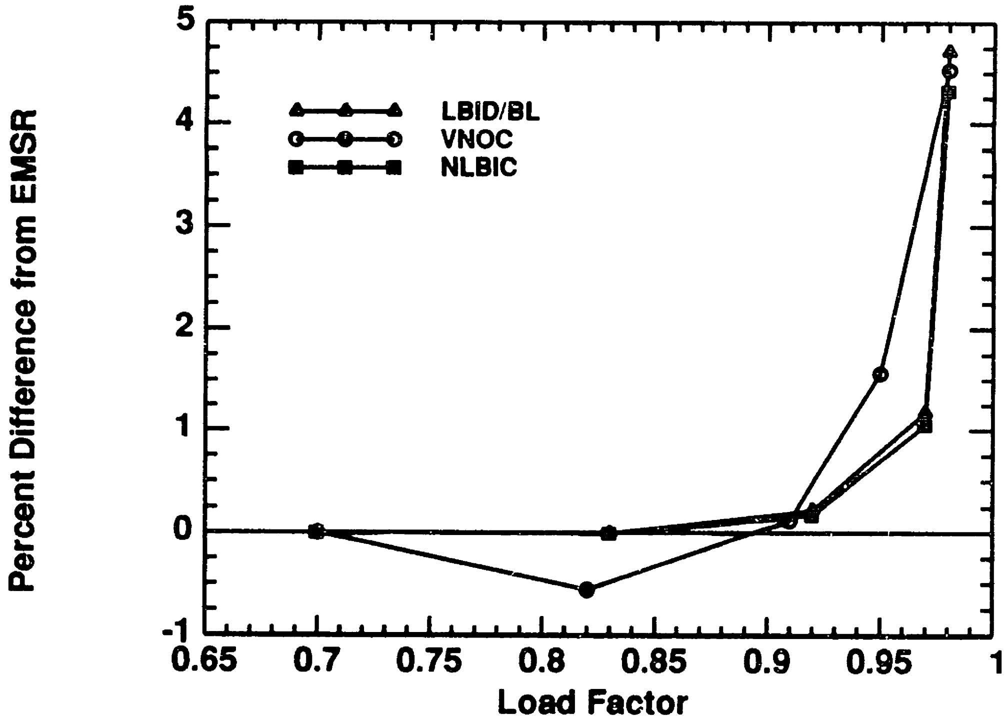

Figure A.10: Revenue impacts of the different prorated leg-based OD control approaches, i.e. the combined Leg-Based Bid Price/Booking Limit (LBID/BL) approach, the Virtual Nesting on the "Value Net of Opportunity Cost" (VNOC) approach, and the Nested Leg-Based Itinerary Limit (NLBIL) approach, under the original integrated optimization/booking process simulation where ODF demand between booking periods is assumed to be *independent* and ODF demand within a booking period is booked from *low to high* fare class.

different network methods under the modified simulation with the original integrated optimization/booking process simulation, respectively. The same comparison for the leg-based OD control approaches are shown in Figures A.9 and A.10. Once again, the results are very similar with little difference between the revenue impacts and the relative performance between the different seat inventory control approaches.

Although the basic assumptions used in integrated optimization/booking process simulation for modeling airline demand may not be absolutely correct, the different network seat inventory control methods are very robust. Thus, if demand really follows a gamma distribution rather than a Poisson distribution, or if demand between different booking periods is really correlated, the overall conclusions of this research are still applicable. While the actual magnitude of the revenue impacts may vary a little, the relative performance of the different approaches is not affected significantly.