Spider Web Geometry Inspires Long Span Roof Trusses

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Texas Tech University, 2018

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Texas Tech University, 2018

SUBMITTED TO THE DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING IN PARTIAL FULFULLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF ENGINEERING IN CIVIL AND ENVIRONMENTAL ENGINEERING
AT THE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

MAY 2020

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Submitted to the Department of Civil and Environmental Engineering
On May 8, 2020 in Partial Fulfillment of the
Requirements for the Degree of Master of Engineering in
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ABSTRACT

This research explores the methods and results used to learn from spider web geometries and implement them into a practical long span roof truss structure. Specifically, utilizing data and properties of spider webs found in research from Su et al. (Su I. a., 2016). The research uses and implements spider's web design blueprints in present day structural systems. Initially, the size of long span roof truss is determined by the finding the gravity and lateral loads applied an ordinary building structure based on the current building code. Then, the web geometry of a Crytophora citricola's, or tent web spider's web is analyzed and optimized for structural efficiency under loading. The performance of this spider-inspired truss geometry is then compared to a typical truss seen in construction today. This research demonstrates that many web geometries are optimal, or close to it, and are comparable in structural efficiency to the trusses currently used in structures. Therefore, architects and structural engineers can use building code to design irregular spider web-shaped trusses in many instances, for example, in architecturally aesthetic purposes or in reusing old structural materials.

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1. Introduction

Spiders have existed for more than three hundred million years, their evolution has perfected their silks and webs (Edwards, 2012). Spider webs are known to be spun out of their silk, a much stronger material than steel while also elastic (Gu et al., 2016; Su and Buehler, 2016a, 2016b). Many researchers have expanded and applied studies of spider web silk materials from their protein scale to macro-scale two-dimensional (2D) orb webs, however minimal research has been done on resilient three-dimensional (3D) spider web-inspired structures.

While 2D spider webs have a simple geometry composed of radial and spiral threads, only recently researchers were able to describe quantitatively complex 3D spider web architectures. Su *et al.* (Su et al., 2018) developed an automatic method to derive a 3D spider web architecture through image processing of high-resolution images of slices of the web illuminated by a sliding-sheet laser. The original manual method was created by (Luhmann, 2017, 2010; Wulff, 2010). One interesting 3D web architecture is the *Cyrtophora citricola* spider web, that was scanned and modelled in (Su et al., 2018). *Cyrtophora citricola* spiders, also known as tent spiders, are common in wet rainforest areas (Edwards, 2012). In accordance with their common name, the tent spiders create three-dimensional fiber network composed of a horizontal 2D tent confined between irregular tangle regions (Edwards, 2006). Those webs are not sticky and rely on prey penetrating through the tangle barrier into the tent region where the spider is usually located (Edwards, 2006). The 3D barrier also protects the spider from predators (Blackledge et al., 2003; Blamires et al., 2013). The interplay between nonlinear behavior of dragline silk, material distribution and web architecture makes spider web resilient and robust (Cranford et al., 2012; Qin et al., 2015). Spiders webs and building structures share an analogous life cycle that could inspire sustainable and resilient high-performance complex building structures (Su et al., 2020).

Spider webs are fascinating structures to study and replicate. For example a pavilion constructed at the University of Stuttgart in 2014 is an example of human's desire to interact with a complex structure (Brownell, 2015). Also, Tomas Saraceno, artist turned architect, is known for his interactive creations that are widely popular and greatly inspired by spider's webs (Forbes, 2013). While webs are an art form, they can also be studied as a tool for engineering efficiency. In order to go further this idea, the following research compares webs to their most similar existing structural element, trusses. Their efficiency, versatility, and structural capabilities are compared.

There is still much to be learned about web structures; this research is a promising starting point to compare a web structure to current building structures. The results of this project will point the field of structural engineering in the direction of how to create more efficient structures by learning from spiders and comparing nature's evolutionary optimization tools to modern technology's optimization tools.

1. Methods

Methods used in this research project were broken down into three primary steps that were designed to find the optimal truss based on spider web geometry. First, is geometric optimization based on changing roof loading conditions, then topology optimization of a web's geometries, and finally structural analysis of the trusses. For the geometric optimization one truss geometry was chosen from the *Cyrtophora citricola* spider web model from (Su et al., 2018)#and different types of roof loading patterns were compared on the irregular roof structure. The irregular roof structure is based on a section of spiderweb geometry and is characterized as a random network of struts to support the roof loads. The irregular truss was optimized for different loading patterns and each of the optimized trusses were compared. Next, for the topology optimization portion, two different trusses were optimized based on their topology: a control, or regular truss and an irregular truss with geometry based on the web. These trusses were compared based on optimization properties to minimize stiffness. Then, the structural analysis step tested the different types of truss roof structures for comparison. Within the structural analysis three different trusses were compared, one is a control truss shaped like a typical roof joist, the other two are an irregular truss with geometries based off of the geometries of a spiderweb, one has compression struts and the other has interior tension members to simulate more realistic forces in a web.

2.1 Geometry Optimization

The problem set up for this test was to design and optimize a three-dimensional roof truss based on a cyrtophora citricola spider's web (Su et al., 2018). This geometry was chosen from a dense segment of web where web elements ran between two close-to-parallel segments of web, essentially where the web looks similar to a joist. Figure 1 and Figure 2 show the section of web analyzed.

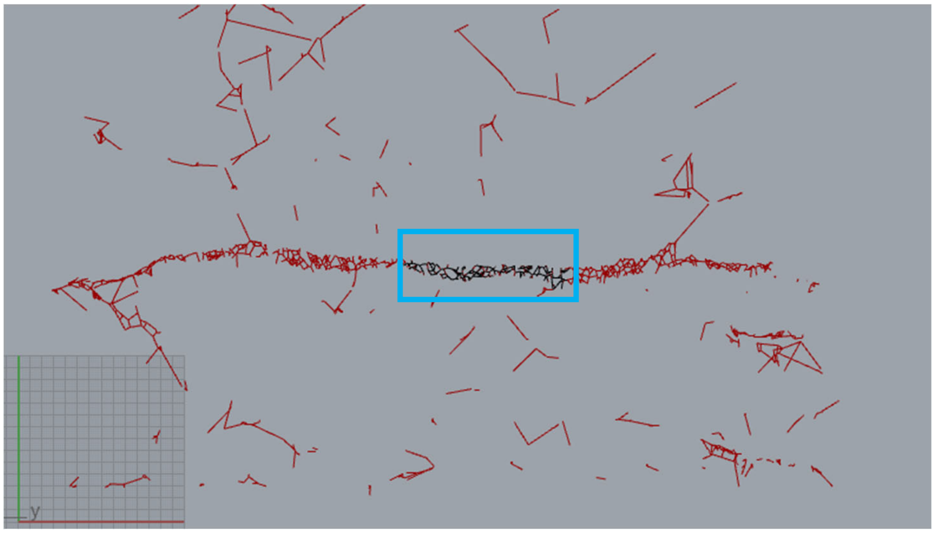

Figure 1. Section of web geometry chosen (using Rhino and Grasshopper).

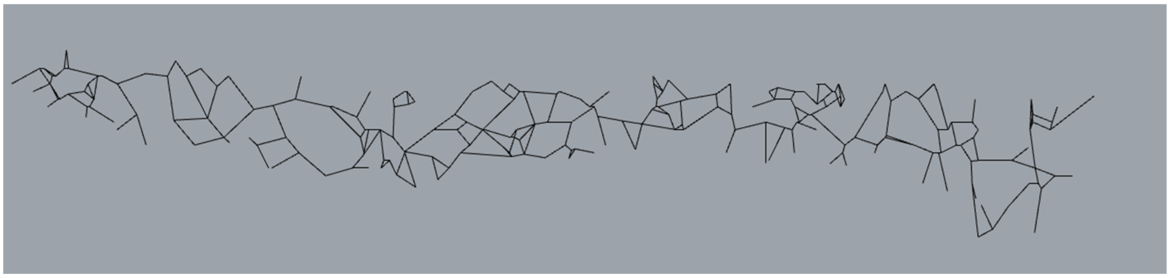

Figure 2. Isolated section of web geometry chosen (using Rhino and Grasshopper).

The final chosen geometry seen in Figure 2, is a three-dimensional segment chosen from the scanned web. Elements were then added around the exterior edges of this geometry, connecting the edge nodes, to close the geometry and act as compression members (see Figure 6).

Then the required loads were determined for a typical building in Cambridge, Massachusetts based off of ASCE 7-10 (ASCE standard, 2010), national loading code and 780 CMR ninth edition, the Massachusetts loading code (Office of Public Safety and Inspection, 2018). To determine the roof loading for a building plan that is seventy feet wide by fifty feet wide and thirty feet tall, the following calculations were performed. The overall roof loading is as follows:

1.2(D)+1.6(LLr, S, R)= 1.2(37psf)+1.6(30psf)= 92.4psf [1]

D= Dead load
LLr= Live Roof Load
S= Snow Load
R= Rain Load

This roof loading is then rounded up to 100psf to remain conservative. Based on a 10ft tributary width (distance between trusses) the calculated distributed load was applied along the upper members.

Next, the asymmetric loads were determined from a person walking across the roof, snow drift, and wind loading. In order to simulate the loading of a person walking across the roof a point load of 200 pounds (the weight of the average American male) was applied at differing points along the roof structure (Gill, 2018). The snow drift loading, where snow piles up against the parapet edges (ASCE standard, 2010), was found to be about 80 psf. The governing lateral force was the wind, which totaled 17psf.

After determining the loads applied to the roof, uniform and asymmetric, the structure was modeled in Grasshopper based on the web geometry. For this model the interior web geometry based members were prestrained (2kips) to act solely in tension, as a web would typically. With Karamba (a Grasshopper plugin), the support conditions are modeled as pinned on each corner so that the support conditions will mimic how the web is typically supported in all directions by other web members or by the surroundings that the web is braced to. The loads were then applied individually with Karamba to the roof surface.

Finally, the truss geometry was optimized after the application of each load using Goat (a Grasshopper plugin) to minimize volume of materials (cost) and ideally remove any structural redundancies. The type of optimization used with Goat in Grasshopper, is the local linear approximation. Following the optimizations of each load case, the before and after web geometries were compared. The optimization geometry and loading results were evaluated to determine the structure's efficiency and capacities.

2.2 Topology Optimization

The topology optimization was set up to design a typical (control) roof truss and an irregular two-dimensional roof truss based off of the Cyrtophora citricola spider's web. The control truss geometry was chosen from a typical long span roof joist. Then for the irregular truss, the geometry was chosen from an almost flat portion of web where web elements ran between two parallel segments of web for the second truss (see Figure 3).

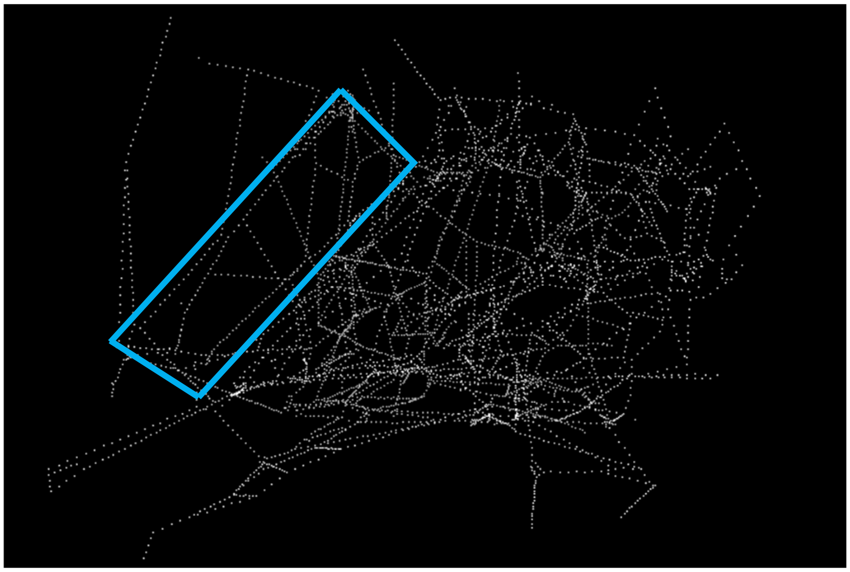

Figure 3. Three dimensional Cyrtophora citricola spider web geometry.

The geometry selected for the irregular truss was scaled to 50ft long and 16ft deep which was originally 25mm long by 8mm deep within the web (see Figure 5). The typical truss geometry was then created to meet these dimensions so the trusses can be easily compared (see Figure 4).

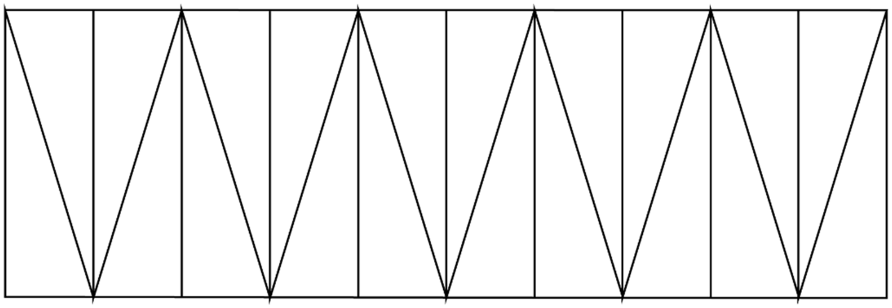

Figure 4. Determined geometry for typical truss.

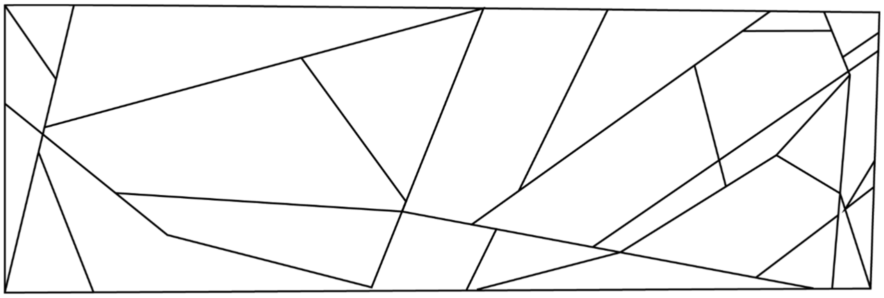

Figure 5. Determined geometry for irregular truss.

The applied roof load was the same 100psf as was found in the Geometric Analysis section (see equation 1).

Using MATLAB, the optimization problem was then set up to minimize the stiffness and maximize the compliance (Carstensen, 2017):

$$
\begin{array}{ll}
\underset{\rho^{e}}{\text{Minimize}} & \mathbf{f=F^{T}d} \qquad [2] \\
\text{Subject to} & \mathrm{h} = \mathrm{K}(\rho^{e})\mathrm{d} - \mathrm{F} = 0 \\
 & \mathrm{G} = \sum \rho^{e} \mathrm{v}^{e} - \mathrm{V} \leq 0 \\
 & \quad e \in \Omega \\
 & \rho\_\mathrm{min}\ \text{^}e \leq \rho\text{^}e \qquad \forall e
\end{array}
$$

The nodes and element locations were specified in MATLAB to define each truss. The modulus of elasticity used was 29,000ksi which is the same as steel; this value is conservative because the modulus of elasticity of a spider web is slightly larger but varies depending on the silk diameter (Ko).

2.3 Structural Analysis

The set up for the structural analysis was the same as the topology optimization method, but with three different trusses: the control truss, and two irregular roof trusses, one using compression struts and the other using tension members.

The uniform roof load, snow drift load, and lateral wind load used was the 100psf, 80 psf, and 17 psf respectively, determined from the Geometry Analysis (see section 2.1).

The structures were modeled in GSA due to its ability to handle complex geometries. GSA is a non-linear structural analysis software used to design complex structures created by a team at ARUP (GSA Analysis- Structural Engineering Analysis Software, n.d.). For the web-based truss using tension members, the interior geometry members were prestrained (2kips) to act only in tension like a web does, similar to the Geometry Optimization (see section 2.1). The loads were then applied to the nodes along the top of each truss by considering the tributary areas.

2. Results and Discussion

3.1. Geometric Optimization

The web-based structure functions adequately under each of the different loading patterns applied. The optimizer was also able to come up with similar geometries needed for each of the trusses under the different loading conditions. The optimization of the web-based structure under uniform roof load, point load, and wind load are shown in Figures 6, 7, and 8 (deflection scaled to 200% for ease of visualization) respectively. The deflected and optimized shape are seen in a lighter color (white) on top of original truss in the darker colors (red and black).

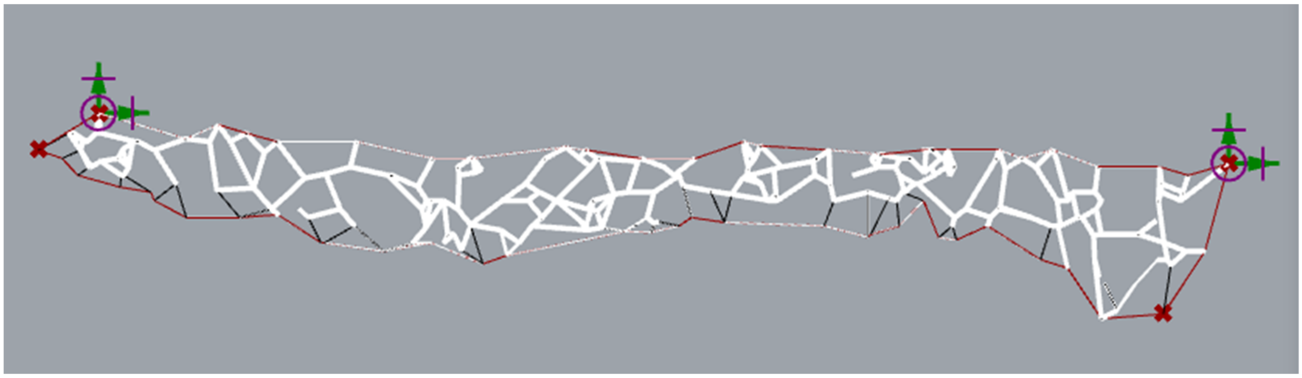

Figure 6. Uniform roof load applied to web geometry structure.

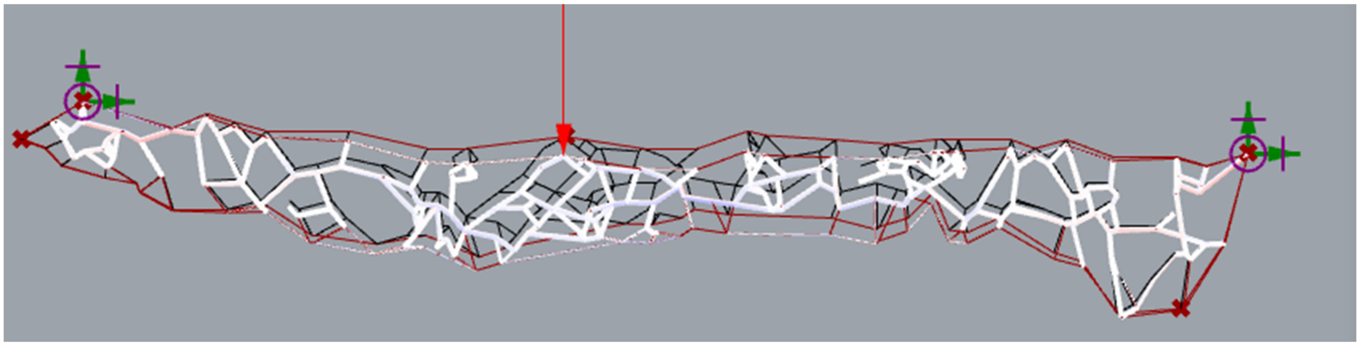

Figure 7. Person point load at location “C” applied to web geometry structure (worst case).

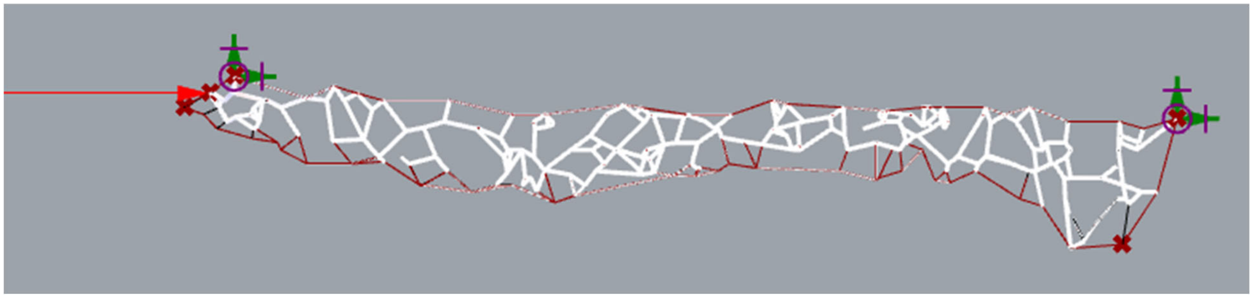

Figure 8. Lateral wind load applied to the left side of the web geometry structure.

Since each loaded truss was optimized from the same initial geometry (the chosen web geometry in Figure 2), they have the same local minimum (optimal shape) with an equal total volume, as expected. As each optimal configuration is same geometry, it proves that this geometry is optimized for any type of roof loading patterns. The optimizer removed members with no axial load transferred through them to the supports, which is the only type of load these members can handle based off of grasshopper loading and connection conditions. The members (black and red), located at base of the truss, are unused and therefore eliminated by the optimizer because the load applied to the truss never goes through these members to be transferred out at the pins.

In addition, Figures 6, 7, and 8 illustrate the change of deflections due to the differing loading patterns. These deflections are summarized in Table 1 where it is evident that the person point load in location “C” causes the greatest deflection. This makes sense because this point load occurs almost in the middle of the truss with a large load.

Table 1: summary of maximum deflections.

Loading	Loading Location	Deflection (in)
Uniform Roof Load	-	0.00512
Person Point Load	A	0.0965
	B	0.324
	C	0.651
	D	0.774
	E	0.529
	F	0.187
Snow Drift Load	-	0.344
Lateral Wind Load	Right	0.097
	Left	0.0189

All of these deflections are allowable because the structure is about fifty feet long. A typical structure of this length has an allowable deflection of 1.67in which is determined in the following equation (ASCE standard, 2010):

Length (in)/360= [50ft(12in/ft)]/ 360= 1.67in [3]

Based on current building code the deflection of all the trusses is well within the required amount.

These proposed structures prove to be very efficient in this orientation, where the loads are in plane with the majority of the structural elements. Other loading orientations were tried, for example applying the loads perpendicularly to the structure, but the original orientation proved to have the most structural capacity to take the load. This is understandable because the deeper a structural member, typically the more load the member can handle therefore using the deepest axis provides the greatest strength.

3.2. Topology Optimization

As shown in Figures 9 and 10 the optimizer removes any redundant members in the truss with a negligible stiffness under the uniform roof load, so the resulting truss is optimal under the loading conditions. In Figure 9 and 10 the initial geometry is shown on the right and the optimized geometry on the left. The control truss in Figure 9 was able to remove many members, where as in Figure 10 no members needed to be removed. This result demonstrates that the topology of the web geometry truss was already an optimal solution of stiffness because it no changes were implemented into the geometry and the initial volume was smaller than required.

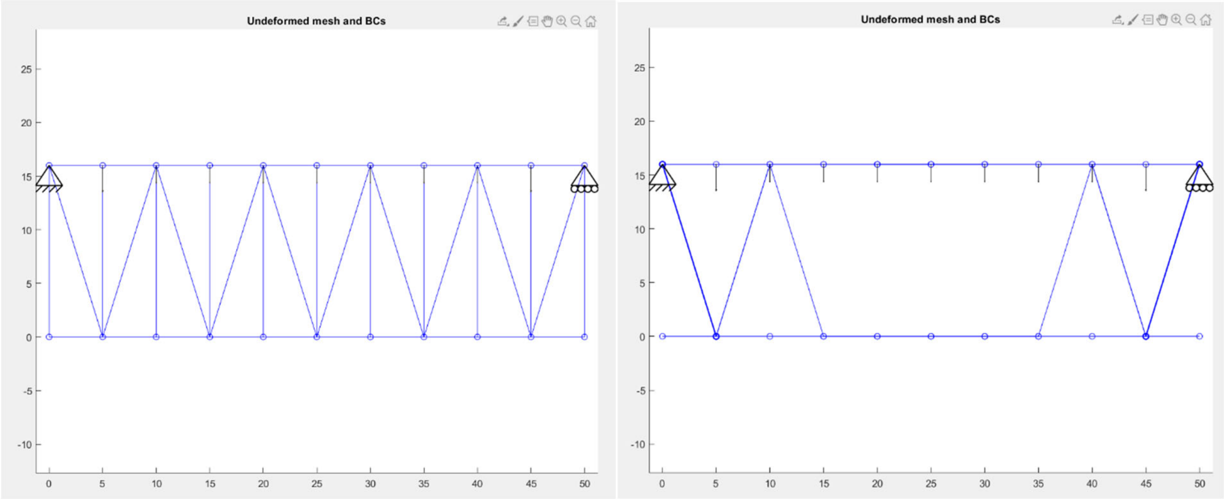

Figure 9. Initial Truss Regular Geometry vs Optimized Truss Regular Geometry.

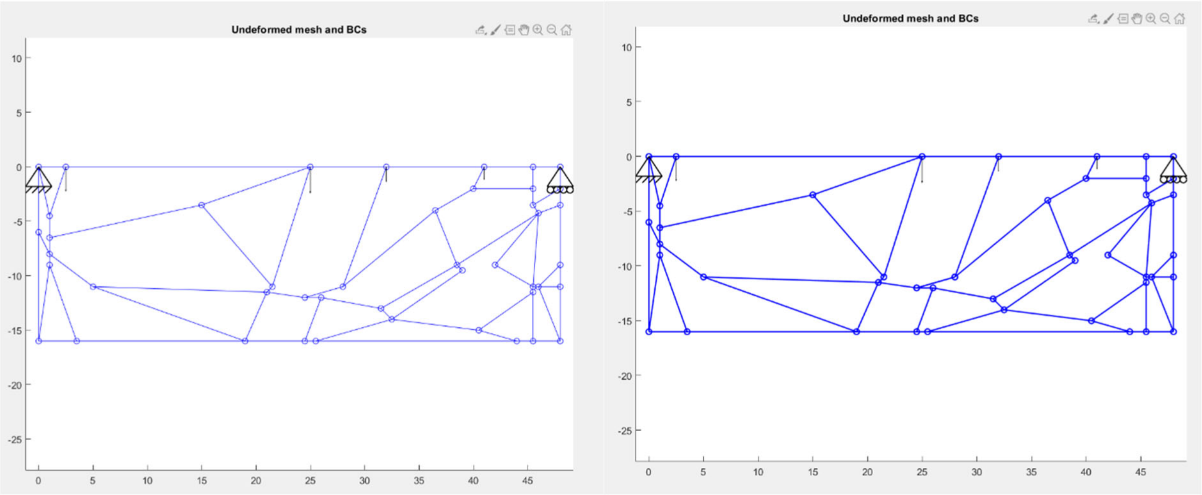

Figure 10. Initial Truss Web Geometry vs Optimized Truss Web Geometry.

Table 2: Results from MATLAB for each truss

Truss	Number of Nodes	Number of Elements	Stiffness ($f=F^Td$)	Amin (in^2)	Total Volume ($v=A*L$)(in^3)
Control	22	41	1.3483	0.15	500
Web 1	45	71	-1.2806E+20	0.99	399

Even though the stiffness solution for the web truss was a large negative number, this does not affect the optimal shape of the truss because the negative number is due to the decreased stress and increased strain resulting in material instability which is how this geometry would act if the material was spider silk (Sonnerlind, 2016). This was caused by the optimizer finding a local minimum at this stiffness point. In future tests it would be ideal to change the assumptions to try to get a more reasonable positive stiffness for a man-made structure. Also

MATLAB did respond with an error message about a matrix being singular and badly scaled, this error message resulted from the program due to the irregularity of the web truss geometry. Therefore, while there were errors due to the web geometry truss, it proved to be most optimal (minimal geometry change) and more efficient than the typical roof truss because the web truss resulted in a smaller total volume.

3.3. Structural Analysis

In the following Figures (11-13) each of the trusses analyses are shown with applied loads in purple and reactions at the supports in green. The applied prestrain is also shown in dark green in Figure 13.

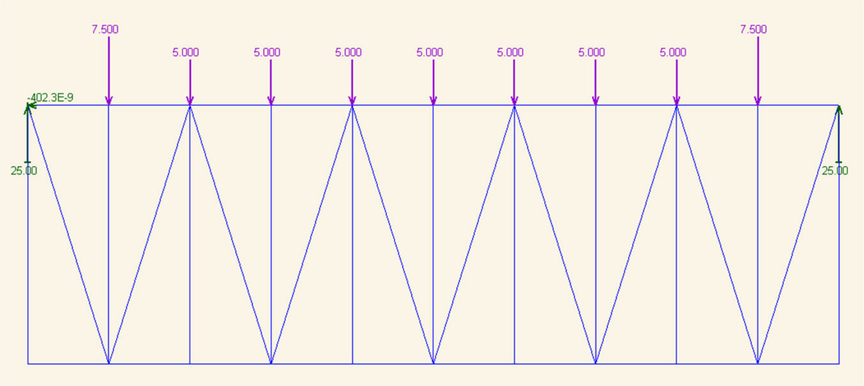

Figure 11. Typical truss analysis in GSA.

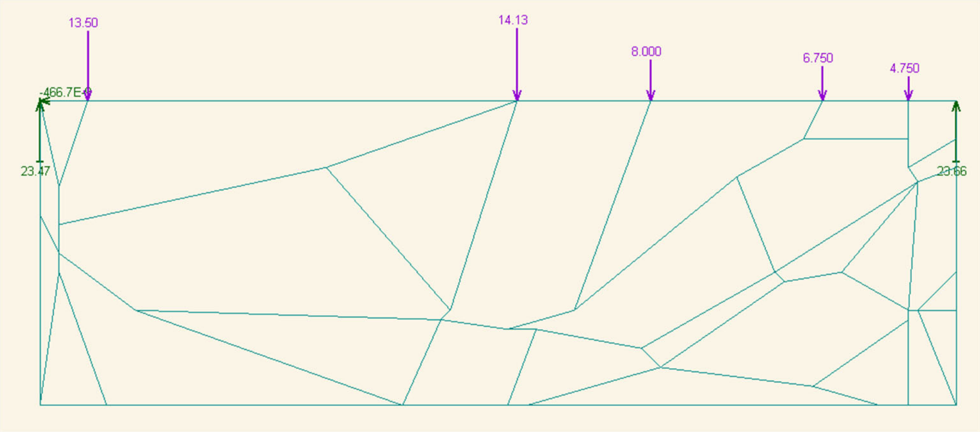

Figure 12. Web truss analysis in GSA.

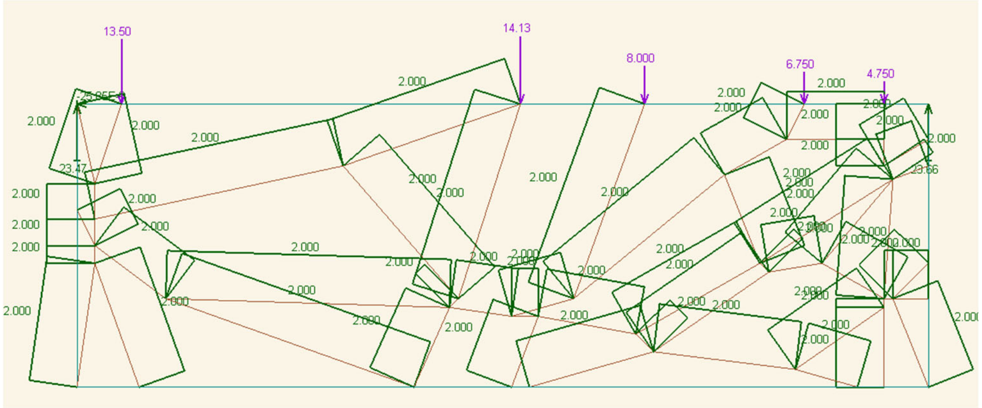

Figure 13. Web truss in tension analysis in GSA.

Table 3: Values for uniform roof loading

Maximum Quantities	Typical Truss	Web Truss	Web Truss in Tension
Deflection	0.1681 in	0.2363 in	90 in
Reactions	25 kip	23.6 kip	23.6 kip
Axial	26.2 kip	23.66 kip	9.5 kip
Shear	0	10.45 kip	23.9 kip
Moment	0	10 kipft	145 kipft

Table 4: Values for lateral wind loading

Maximum Quantities	Typical Truss	Web Truss	Web Truss in Tension
Deflection	0.0032 in	0.03 in	1.2 in
Reactions	1 kip	1 kip	1 kip
Axial	1 kip	1 kip	3.2 kip
Shear	0	0.086 kip	0.17 kip
Moment	0	0.2635 kipft	28.9 kipft

Table 5: Values for snow drift loading

Maximum Quantities	Typical Truss	Web Truss	Web Truss in Tension
Deflection	0.1305 in	0.35 in	156 in
Reactions	20 kip	40 kip	40 kip
Axial	20.95 kip	27.27 kip	16.35 kip
Shear	0	18 kip	40 kip
Moment	0	18 kipft	2 45 kipft

The results specified in the tables show that the typical roof truss is the most efficient truss because it receives lower values for the most part in the truss members. However, the web geometry is a comparable truss, only receiving slightly larger results as well as moment and shear. With moment shear values this truss does not act perfectly axially which is a downfall in the design. The prestrained web truss preforms the worst because the tension and moment capacities are astronomical compared to typically loaded trusses. The deflection for the web truss with tension members is at a maximum 156in which is extremely excessive because the building code allows the truss to deflect only 2in (ASCE standard, 2010). The difference between the trusses is most likely due to the fact that the tension cables cannot take any compression which greatly reduces where the GSA software is able to apply the load and track the load through a logical path. Since spider webs are typically tensioned from all directions and anticipate loads from any direction it is probable that this truss with tension should be supported differently and loaded differently to act as efficiently as a web in tension typically would as well as be made of spider silk and not steel cables. However, proving that the web-based truss is more optimal (based on the results from the topology optimization portion) and can compete in structural efficiency may prove that irregular trusses are the way to maintain structural requirements and decrease cost by reducing the amount of materials needed.

4. Conclusion

Since the creation of spiders, they have been adapting their webs, this is why their webs are known for advanced structural capabilities including surviving prey capture, wind, and predators with minimal repairs required. Based on the results of this research it is determined that spider-web inspired structures can be used for human structures as well. Geometry optimization determined that the spider web-based truss can handle all of the loads required by code including asymmetrical loads. This proves that tension trusses are a sufficient replacement for long span roof trusses and can be optimized in order to minimize typical building materials. However, the volume of these web geometry truss structures is slightly greater, about 12%, than a typical roof truss of this shape that is two dimensional. Where the volume isn’t as ideal as a typical roof truss it makes up for it in its capacity in many directions due to its three-dimensional quality. Overall the geometric optimization proves that the irregular spider web geometry is comparable to a typical roof truss because it can easily handle the required loads under code. For the topology optimization the geometry of the web truss changed minimally where the regular truss changed greatly and the total member volume was less than the regular truss. When the initial and optimal geometries of the web truss did not change at all, it confirms that in this location this web was optimized by the spider for maximum stiffness and minimal material. While other areas of the web may not be optimized for stiffness the one illustrated in this research was, most likely due to exterior location of the chosen geometry; spiders need the edges of their web as malleable as possible. Structural analysis of the trusses demonstrated that spider web-based geometries are not typically loaded and supported the way that roof truss in current structures are. However, it does prove that irregular geometries in trusses could follow building code with proper analysis and testing. This research opens the door for different choices of truss configurations desired to fit a certain architectural or design aesthetic.

The direction this research could go in is to 3D print some of these optimal geometries with a strain-sensitive conductive material to verify loaded members. Another direction would be to use all the web data generated in (Su et al., 2018) for a machine learning model to generate new webs, and compare them to real webs structurally. These results can then inform the direction of spider web structural research for human applications and if these results can be reapplied in a more efficient or conservative way.

5. Acknowledgements:

Acknowledgments go to Isabelle Su for the work she has done and is still doing to create and collect accurate data from the spider webs, to Markus Buehler for the guidance and inspiration to pursue this topic, to the entire MIT CEE staff for all of their knowledge and support, and to my fellow 2020 Masters of Engineering CEE classmates for all of the help, comradery, and friendship.

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