14.01, Fall 2007

Problem Set 4
Due October 5, 2007

1. Please write your name, the name of your TA, and your section/recitation time (e.g. MWF 10am, or F 1 pm) on top of your solutions.

2. Problem sets are due IN SECTION/RECITATION. Late Problem sets will not be accepted under any circumstances.

Questions

1. Are the following true, false or uncertain? Explain why.

a. (7 points) Many people buy insurance against their home burning down and also purchase lottery tickets for large prizes. (The insurance and lottery tickets both have negative expected value.) These people cannot be maximizing the expected utility of their wealth.

b. (7 points) Jenna owns $5000 worth of Microsoft stock and no other stocks. She is deciding whether to invest another $5000 from her chequing account in General Electric stock. If Jenna is risk averse, she should buy the General Electric stock in order to diversify her portfolio.

2. Zack is an MIT student who lives in Harvard Square. Each morning he takes the train or the bus to school. Riding the bus takes 5 minutes with probability $1/4$ and 20 minutes with probability $3/4$. Riding the train takes 7 minutes with probability $1/2$ and 12 minutes with probability $1/2$.

a. (5 points) If Zack is risk neutral and only cares about minimizing his travel time, does he take the train or the bus? If he values his time at $\$0.25$ per minute, how much more is he willing to pay for his preferred method of transportation?

b. (10 points) Assume Zack values his time at $\$0.25$ per minute. He can purchase an MBTA report each morning that tells what the actual travel time will be for both the bus and the subway. How much is he willing to pay for this information? (Travel times on the bus and subway are independent.)

3. Tim is a fan of the New England Patriots; he is risk averse, and he wants to maximize his expected utility. After the Patriots play a game, Tim's utility will be $u(W + P)$ if the Patriots win and $u(W - P)$ if the Patriots lose, where W is his wealth.

a. (7 points) This week the Patriots are playing against the Pittsburgh Steelers; they have a $1/2$ probability of winning the game. Tim can bet any amount B on either team. Letting W_0 be Tim's wealth before the game, he will have wealth $W_0 + B$ if the team he bets on wins and wealth $W_0 - B$ if the team he bets on loses. How much should Tim bet, and on which team?

b. (7 points) Next week the Patriots will be playing against the Philadelphia Eagles; they have a $2/3$ probability of winning that game. If Tim bets B on the Patriots, he will gain $0.5B$ when the Patriots win and lose B when the Patriots lose. If Tim bets B on the Eagles, he will lose B when the Patriots win and gain $2B$ when the Patriots lose. How much should Tim bet, and on which team?

4. Jane is a farmer who can plant either potatoes or corn on her land. Both choices are risky, and her profits will depend on the Fall weather according to the following table:

Fall Weather	Potatoes	Corn
Cold	4000	1000
Normal	8000	8000
Warm	6000	11,000

There is a $1/4$ probability that the Fall will be cold, a $1/2$ probability that the Fall will be normal, and a $1/4$ probability that the Fall will be warm.

a. (8 points) Calculate Jane's expected profits and standard deviation for each crop.

b. (8 points) If Jane's utility function over this year's income is $u(y) = \sqrt{y}$ and she has $2000 of income from other sources (so her total income is crop profits plus $2000), which crop would she rather plant?

c. (9 points) Now suppose that Jane can plant a fraction α of her land with Potatoes and a fraction $1 - \alpha$ with Corn. That is, if $\alpha = .6$, then Jane would get 60% of the Potato profits and 40% of the Corn profits for whatever type of weather occurred. What value of α will Jane choose?

5. Emre has an income of $10,000 per year. There is a $1/5$ probability that he will need mayor dental surgery in the next year, which would cost $5000 but wouldn't otherwise affect his well-being. He has utility function $u(W) = \ln(W)$ over his wealth.

a. (5 points) An insurance company offers Emre a dental policy where he pays a price $p * C$ for C dollars of coverage. That is, Emre pays

the company $p * C$ whether he needs surgery or not, and the company pays him C if he ends up requiring the surgery. What is the actuarially fair price for one dollar of coverage (p)?

b. (9 points) If the insurance company offers Emre a policy at the actuarially fair price, how much coverage will Emre buy?

c. (9 points) Suppose the insurance company doesn't offer him the actuarially fair price–instead, they take a profit of $\$0.05$ per dollar of coverage (so if the actuarially fair premium is p_f, they charge $p = p_f + 0.05$). How much insurance will Emre buy now?

d. (9 points) Now suppose that Emre's income is y (not necessarily $\$10,000$) and that the insurance company charges some premium p. The probability of needing surgery is still $1/5$, and the cost of surgery is still $\$5000$. Find Emre's optimal level of coverage for arbitrary $y > 0$ and any p greater than the actuarially fair rate from part (a). (Note: pay attention to the possibility of a corner solution where Emre buys no insurance.) Is the amount of insurance Emre buys increasing or decreasing in y?