Vacancy Durations in the Office Market

by

Jiri Sykora

M.A., Applied Economics, 2014
Center for Economic Research and Graduate Education – Economics Institute (CERGE-EI)

B.S., Business Administration, 2013
State University of New York, Empire State College

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Signature of Author____
Center for Real Estate
July 26, 2019

Certified by____
William C. Wheaton
Professor, MIT Center for Real Estate
Thesis Supervisor

Accepted by____
Dennis Frenchman
Class of 1922 Professor of Urban Design and Planning,
Director, Center for Real Estate
School of Architecture and Planning

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Abstract

The durations of other indicators have been researched extensively in real estate studies, primarily the time on market and the duration of residence in housing units. Despite their importance, empirical research on the duration of vacancies is relatively limited and focused mainly on the housing sector. This paper aims to fill this gap and analyze the determinants of vacancy durations in the office sector. The analysis is based on a dataset of individual office suites located in New York City, NY that became vacant between 2012 and 2015.

Vacancy durations are a form of time-to-event data and as such can be examined using survival analysis. We present several parametric and non-parametric survival models. Four key characteristics – unit size, asking rent, building height, and floor number – are found significant across all model specifications. Specifically, vacancy durations are affected the most by unit size and asking rent. Survival probabilities are found to considerably vary over time, which appears to be driven by variations in employment growth.

Thesis Supervisor: William C. Wheaton
Title: Professor, MIT Center for Real Estate

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1. Introduction

Vacancy rates can be expressed as the product of vacancy incidence and duration. Incidence measures the probability that a unit becomes vacant, while duration expresses the length of time a unit remains vacant. Analogous to unemployment rates in the labor market, decomposing vacancy rates into their two components reveals important information about the underlying space market. Higher levels of vacancy incidence might reflect profit-maximizing turnover of rental stock or preference for shorter lease terms. Higher levels of vacancy duration might, in turn, expose structural mismatches between supply and demand in the space market (Gabriel & Nothaft, 2001).

It is generally accepted that real estate markets have “natural” non-zero vacancy rates. Natural vacancy rates, and, therefore, non-zero vacancy durations, exist due to several reasons: (1) time is required for both landlords and tenants to conduct their searches. (2) Time is required for landlords to make any necessary improvements between tenants. (3) Landlords might maximize their total return at a rent that generates a positive vacancy rate. (4) In a market with increasing (declining) rents, landlords (tenants) benefit from waiting to enter into a lease (Smith, 1987).

Despite their importance, empirical research on vacancy durations is relatively limited. This can be partly attributed to a lack of data needed for such analysis, especially in the commercial sector. This paper aims to fill this gap and analyze the determinants of vacancy duration in the commercial sector. Specifically, it focuses on the office sector and utilizes a dataset of office suites located in New York City, NY and collected between 2012 and 2018.

Vacancy durations are a form of time-to-event data and as such can be examined using survival analysis. Survival models are commonly used in bioscience assessing the survival time of biological subjects (for instance, Symmans, et al., 2007), or analyzing unemployment in labor economics (for instance, Gamerman & West, 1987). In real estate studies, survival models are mainly adopted to explore the durations of various housing indicators, such as the duration of residence. We adopt this approach and presents several parametric and non-parametric survival models.

The paper is organized as follows. In the first section, it reviews existing research and lays out the methodology. In the second section, it describes the dataset and presents the results based on a Cox proportional hazards model. Lastly, it presents a parametric survival model allowing to estimate the expected vacancy duration depending on unit-level characteristics, and shows its application in calculating the elasticity of vacancy duration.

2. Literature Review

The durations of other indicators have been researched extensively, primarily the time on market for housing units (Anglin, Rutherford & Springer, 2003; Haurin, 1988; Knight, 2002) and the duration of residence (Deng, Gabriel & Nothaft, 2003; Speare, 1974). Previous research on vacancy durations in the rental market is more limited and focused mainly on the housing sector, where the availability of data is higher.

Guasch & Marshall (1985a) were one of the first researchers to derive a theoretical framework linking residential unit characteristics to vacancy statistics in the rental market. Their framework incorporated the transiency of tenants and proposed that units that attract more transient tenants – typically smaller units and units in larger buildings – display longer vacancy durations. In their subsequent paper (1985b), they extended the framework to account for unit age, which is shown to increase vacancy durations.

Sternberg (1994) built on this research and proposed a hazard model to analyze the effects of additional characteristics on vacancy durations. He confirmed the previous findings that older units experience longer vacancy spells, but found an opposite effect than Guasch & Marshall (1985a) for units in larger buildings. Units that are more atypical are identified to be associated with longer average vacancy durations.

Gabriel & Nothaft (2001) analyzed vacancy duration determinants across different markets. They also supported the hypothesis that heterogeneity among available units increases the mean vacancy duration due to higher tenant search costs and greater differences in reservation prices. Furthermore, vacancy durations were found to be generally higher during times of economic and housing market weakness.

Orr, Dunse, & Martin (2003) analyzed the interaction between vacancy durations in the commercial market and asking and transacted rents. In line with the other studies on vacancy durations (Guasch & Marshall, 1985a) or time on market (Anglin et al., 2003; Trippi, 1977), Orr et al. found a positive relationship between durations and asking rents or prices. Nevertheless, the research did not specifically focus on other determinants affecting vacancy indicators.

3. Methodology

We relied on survival analysis to assess the determinants of vacancy duration. In general, survival analysis is appropriate for data where the dependent variable is the time until the occurrence of a specific event and the data include censored observations. Survival functions, which denote the probability that the event has not occurred by a certain time, form the basis of survival analysis. To identify the determinants of vacancy durations, the effects of various covariates on survival functions were evaluated.

3.1. Survival and Hazard Functions

A continuous random variable *T* is assumed to have a failure density function *f(t)* and cumulative distribution function $F(t) = P\{T < t\}$ at time *t*. Its survival function, which represents the probability that the observed event has not occurred by *t*. In this paper, the survival function characterizes the probability that an office suite has remained vacant until time *t*. It can be described as:

$$S(t) = P\{T \geq t\} = 1 - F(t) = \int_{t}^{\infty} f(x)dx \tag{1}$$

The derivative of *S(t)* becomes:

$$\frac{\partial S(t)}{\partial t} = -f(t) \tag{2}$$

As the derivative is equal to the inverse of the failure density function, $S(t)$ can be derived from $f(t)$ and vice versa.

The hazard rate, sometimes referred to as the failure rate, is denoted as *λ(t)*, and characterizes the instantaneous rate of failure at time *t* given that the event has not occurred by *t*. For the distribution of *T*, the hazard rate is:

$$\lambda(t) = \frac{f(t)}{S(t)} \tag{3}$$

The equation (3) suggests that the hazard rate equals the failure density at time *t* divided by the cumulative survival probability that the event has not occurred by *t*. Combining equations (2) and (3), the hazard rate can be rewritten as:

$$\lambda(t) = -\frac{\partial}{\partial t} logS(t) \tag{4}$$

In other words, the hazard rate can be derived also from $S(t)$ and hence $f(t)$.

Within our context, the hazard rate represents the instantaneous rate that an office suite becomes leased at time *t* given that it has remained vacant until *t*. The hazard rate forms an integral part of proportional hazards models that will be discussed later in this chapter.

3.2. Kaplan-Meier Estimator

The Kaplan-Meier (KM) estimator is one of the simplest non-parametric models used to estimate the survival function. It involves calculating the survival probability at each time interval. Its primary advantage is that it is able to account for right-censored observations. Right censoring occurs when a subject has not experienced failure by the end of the observation period. When no censoring is present, the KM estimator is equivalent to the empirical distribution function. The estimator can be expressed as:

$$S(t) = \frac{n_i - d_i}{n_i} \tag{5}$$

where n_i represents the number of subjects at risk at time t, and d_i the number of failures that occurred at time t. This means that the survival probability is equivalent to the number of subjects surviving longer than time t divided by the total number of subjects at risk at time t. The probability of survival until time t is then calculated by multiplying survival probabilities at all preceding time periods:

$$S_t = \prod\nolimits_{i:t_i \leq t} \left(1 - \frac{d_i}{n_i}\right) \tag{6}$$

We used the KM estimator to perform an initial survival analysis of the office suites in our sample and to identify the likely factors affecting vacancy durations. Seven key unit-level factors were hypothesized to influence vacancy durations – unit size, rent, building height, floor number, building age, lease type, and location variables.

The key disadvantages of the estimator are that it is a univariate model and does not easily extend to numerical variables. For this reason, numerical variables (unit size, initial asking rent, building height, floor number, and building age) were each separated into two subcategories depending on their values and assigned binary values.

Another limitation of the estimator is that it does not allow for a statistical comparison between the total survival functions among different groups, but rather allows a comparison only at individual periods. We addressed this shortcoming by including the log-rank test, which tests the equality of the survival functions. It is based on comparing the difference between the expected and observed number of failures in each period and for each sample group. The null hypothesis is that there is no difference in the survival rate among different groups at any time point.

The log-rank test examines only the significance of the differences in various survival functions – it does not quantify the size of the difference. For this, we turned to the Cox proportional hazards model.

3.3. Cox Proportional Hazards Model

The Cox model (Cox, 1972), commonly used in medical research for analyzing the survival time of patients, is an extension of univariate survival methods and allows incorporating both continuous and categorical variables.

The approach proposed by Cox assumes that the hazard rate for an individual with characteristics x_i at time *t* is:

$$\lambda_i(t|x_i) = \lambda_0(t)e^{\sum_{i=0}^{n}(\beta * X_i)} \tag{7}$$

Where $\lambda_0(t)$ is the baseline hazard function for $X_i = 0$. The term $e^{\sum_{i=0}^{n}(\beta * X_i)}$ denotes a relative increase or decrease in the hazard rate corresponding with non-zero values of X_i. The baseline hazard function is not itself estimated within the Cox model, and $\lambda_0(t)$ is calculated by setting all covariates to zero. This represents one of its main advantages, as there is no need to assume a specific shape of the baseline hazard function. The β coefficients are estimated by maximizing the partial likelihood function and represent the marginal change in the log of the hazard ratio for a one-unit change in X_i, holding other factors constant.

The Cox model is non-parametric with the exception of its assumption of proportionality; the hazard rate in any group is assumed to be a constant multiple of the hazard rate in any other group. In other words, the hazard rate for different characteristics X_i remains constant over time *t*. The assumption can be demonstrated by calculating the ratio of two hazards rates for subjects *j* and *k*:

$$\frac{\lambda_j(t|x_j)}{\lambda_k(t|x_k)} = \frac{\lambda_0(t)e^{\sum_{i=0}^{n}(\beta * X_j)}}{\lambda_0(t)e^{\sum_{i=0}^{n}(\beta * X_k)}} = \frac{e^{\sum_{i=0}^{n}(\beta * X_j)}}{e^{\sum_{i=0}^{n}(\beta * X_k)}} \tag{8}$$

Equation (8) shows that the hazard ratio for subjects *j* and *k* depends on covariates X_j and X_k but not on time *t*.

To test the proportionality assumption, we employed a test of non-zero slope in a linear regression of the scaled Schoenfeld residuals on time (Grambsch & Therneau, 1994). The null hypothesis of zero slope corresponds to testing that the log hazard function is constant over time.

The results of violating the non-proportionality assumption differ among authors. For instance, Allison (1995) suggests that in such cases the estimate can be interpreted as the average effect of the covariate over time. Conversely, Hosmer, Lemeshow and May (2011) claim that alternative models should be used to obtain a more precise interpretation of the estimates.

To err on the side of caution, we also considered three alternative extensions of the Cox model to allow for both covariates and their effects to vary over time. Firstly, we extended the model to allow for time-dependent coefficients:

$$\lambda_i(t|x_i) = \lambda_0(t)e^{\sum_{i=0}^{n}(\beta(t)*X_i)} \qquad (9)$$

An important feature of equation (9) is that the hazard rate is no longer proportional but is assumed to depend on time t. If coefficients $\beta(t)$ are significantly different from zero, the proportionality assumption in the time-independent model is violated. The time-dependence form in this model is parametric and needs to be specified. This, in turn, allows greater flexibility in specifying the model as the form can take any shape.

Secondly, we considered a Cox model stratified on calendar periods to remove any potential bias in the coefficients β resulting from variations in the baseline hazard rate over time. The proportional hazards model with fixed time effects thus becomes:

$$\lambda_{i,g}(t|x_i) = \lambda_{0,\mathrm{g}}(t)e^{\sum_{i=0}^{n}(\beta * X_i)} \tag{10}$$

where g represents different strata (calendar periods). The stratified model assumes equal coefficients β across strata but a different baseline hazard function for each stratum g. This extension of the basic Cox model is typically used to control for a predictor that does not satisfy the proportional hazard assumption. However, as the stratified variable is not included in the model, it is not possible to estimate directly its effects on the likelihood function.

Lastly, we extended the model by including time-dependent covariates to account for the main factors driving potential variations in the hazard rate over time:

$$\lambda_i(t|x_i(t)) = \lambda_0(t)e^{\sum_{i=0}^{n}(\beta * X_i(t))} \tag{11}$$

We hypothesized that the time-varying covariates likely affecting the hazard functions are the market vacancy rate and employment growth in New York City. The vacancy rate reflects market demand and supply factors, while employment growth serves as a proxy for office employment growth and firm expansion.

3.4. *Parametric Model and Its Applications*

The Cox model describes the effects of individual factors on the hazard rate at each period. However, of interest is also the relationship with the expected duration of the vacancy spell. The mean survival time can be calculated as the area under the survival function with boundary conditions $S(0) = 1$ and $S(\infty) = 0$. The expected vacancy duration is defined as the integral of the survival function:

$$\overline{T} = \int_0^\infty S(t)dt \tag{12}$$

As both the Kaplan-Meier estimator and Cox model are non-parametric, their estimated survival functions generally do not reach zero in the presence of censored data. As a result, the expected duration can be calculated only within some time interval. To calculate the expected survival over the entire time interval, we next considered a parametric survival model, which follows an exponential distribution:

$$S(t) = e^{-\lambda_j \mathrm{t}} \tag{13}$$

Assuming the "risk" of a vacant suite becoming leased is constant over time, which approximately reflects the observed distribution, the resulting survival function has an exponential distribution. An advantage of this specification is that equation (12) collapses to:

$$\overline{T} = \frac{1}{\lambda_j} \tag{14}$$

The parameter λ_j is defined as:

$$\lambda_j = e^{\Sigma(\beta * X_j)} \tag{15}$$

Where X_j represents the independent variables affecting vacancy durations. This specification allows to easily calculate the effects of various variables on the expected duration.

Given that the exponential parametric model allows to calculate the expected duration depending on individual suite characteristics, it can also be used to determine the optimal rent that maximizes the landlord's total income. Future rent payments that the landlord will receive can be summarized as an annuity with constant payments, and their present value calculated as:

$$PV_{t=\overline{T}} = r_i\left(\frac{1-(1+y)^{-n}}{y}\right) \tag{16}$$

Where y is the discount rate, n the duration of the lease, r_i the initial rent, and $\overline{T}$ the duration of the vacancy. As the payments are received only after finding a tenant, the present value at *t=0* becomes:

$$PV_{t=0} = \frac{r_i(1-(1+y)^{-n})}{y(1+y)^{\overline{T}}} \tag{17}$$

Next, we extended the present value formula to reflect any holding costs that the landlord incurs during the time the suite is vacant. The present value of these costs is defined as:

$$PV_{t=0} = C\frac{\left(1-(1+y)^{-\overline{T}}\right)}{y} \tag{18}$$

Where *C* is the holding cost of vacant space. Combining equations (17) and (18), the present value of the landlord's total profit becomes:

$$PV_{t=0} = \frac{r_i(1-(1+y)^{-n})}{y(1+y)^{\overline{T}}} - C\frac{\left(1-(1+y)^{-\overline{T}}\right)}{y} \tag{19}$$

By combining equations (14) and (19), we get a function expressing the landlord's profit dependent on individual suite characteristics, including asking rent. For any set of suite characteristics, the equation can be optimized with respect to the rent variable. This potentially provides a tool for landlords to calculate the optimal rent that maximizes their total profit. The applications of this approach are discussed in Chapter 8.

4. Data Description

This paper estimates the presented survival models using a unit-level data set of office suites located in Manhattan, New York City. The full data set includes 21,269 observations of office suites that became vacant between 2012 and 2018 and their individual characteristics. Specifically, the data include vacancy durations[1], unit size, initial asking rent[2], number of stories in the building where the suite is located, floor number of the suite, building age[3], lease type[4], and location variables[5].

An important feature of survival models, including the Cox proportional hazard model used in this paper, is that the estimation is based only on subjects who are at risk (i.e. suites that are vacant and available for lease) at any given point in time. As a result, the presence of censored observations in the sample does not bias the results if the censored subjects are not substantially different from the rest of the sample. In other words, it is assumed that survival probabilities are the same for subjects regardless of when they entered the study.

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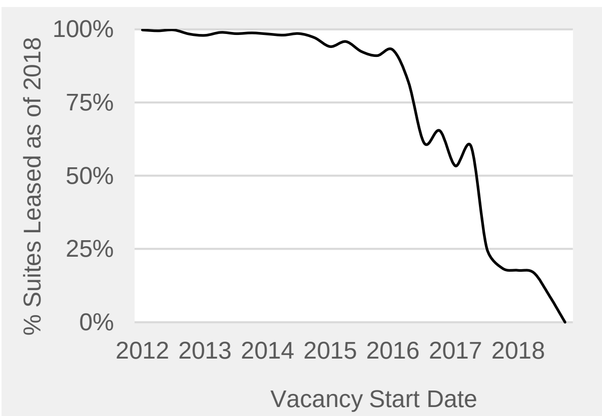
[1] As the data set includes only quarterly observations, vacancy durations are measured as the number of quarters a unit is on the market and available for lease.
[2] Unit's initial asking rent is converted into
[3] Building age measured at the time a unit became vacant.
[4] Specifies whether the space is first generation (new), second generation or later (re-let), offered as a sublease, or an executive suite.
[5] Suites are categorized based on their zip code into four main Manhattan submarket – Midtown, Midtown South, Downtown, and Upper Manhattan. Although the zip codes allow for a more detailed classification, its explanatory power would be relatively low due to small sample sizes in each category.

This assumption is likely violated in our dataset as suites becoming vacant towards the end of the sample period display disproportionally higher survival probabilities than those becoming vacant in earlier years. For instance, 82% of suites that became vacant in Q2 2016 are leased at the end of 2018. However, the share drops to 61% for suites that became vacant only a quarter later. Another similar drop within just one quarter occurs again in 2017.

Figure 4.1: Share of Suites Leased by Vacancy Start Date



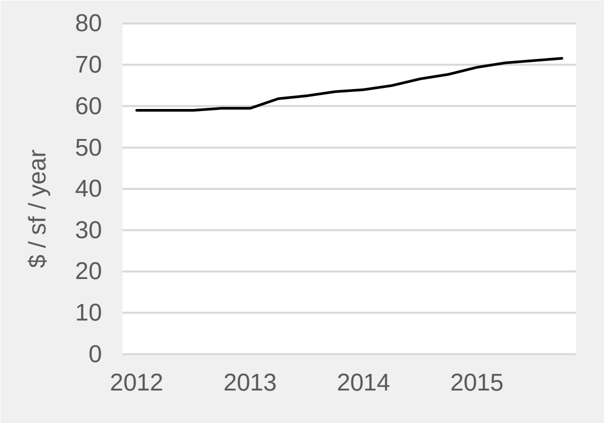
These rapid declines might indicate issues with the collected data rather than actual changes in survival probabilities. To obtain unbiased coefficient estimates even in our models that do not adjust for different calendar periods, we limited the sample period to the years 2012 - 2015. This more conservative approach lowered the sample size to 14,686, but results in relatively homogeneous survival probabilities over time. Descriptive statistics of this dataset are shown in Figure 4.2.

Figure 4.2: Descriptive Statitics (2012 – 2015 Dataset)

	Mean	Median	Std. Dev.	# of Observations
Vacancy Duration (quarters)	3.41	2.00	3.49	14,686
Unit Size (sq.ft.)	8,394.55	4,368.00	13,345.96	14,686
Rent ($ / sq.ft. / year)	48.56	46.00	16.16	5,123
Rent Ratio	.76	.71	.24	5,123
Building Age (years)	76.01	85.30	27.91	14,686
Number of Stories	26.88	23.00	14.85	14,686
Floor Number	13.68	11.00	10.47	14,422
Lease Type				
New Space				421
Re-Let Space				10,396
Sub-Let Space				3,668
Executive Suite				201
Submarket				
Midtown				8,883
Midtown South				3,653
Downtown				2,000
Upper				150

As Figure 4.3 shows, the average market rent in Manhattan during the period increased substantially from $59 per square foot in 2012 to over $71 at the end of 2015. Using nominal asking rents would result in biased coefficients as it does not account for the change in underlying market rents. Therefore, we instead used as a ratio of a suite's rent divided by the average market rent in that particular period. A ratio greater than one suggests that a suite has a higher rent than the market average, while a ratio of less than one indicates that it is priced lower than the average.

Figure 4.3: Average Market Rent[6]



In the survival model with time-dependent covariates, we include the market vacancy rate and employment growth. Figure 4.4 illustrates that the vacancy rates of offices in Manhattan increased from 9.1% in 2012 to a high of 11.0% in 2013 and displayed a downward trend from 2014 onwards. Similarly, New York City employment growth rebounded from a low of 2.0% year-over-year in 2013 to above 3.0% in 2014, and has remained above 2.5% since that time.

Figure 4.4: Market Vacancy Rate[7]

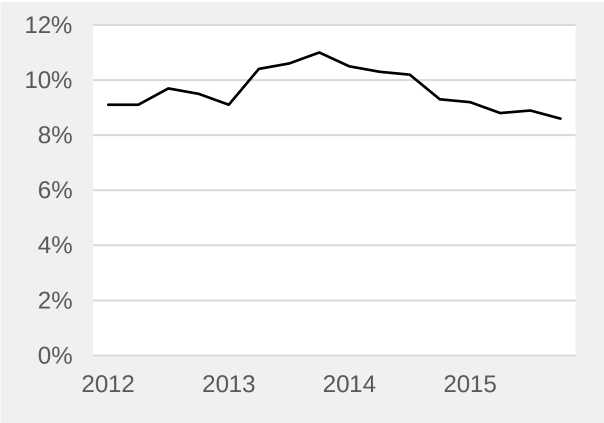
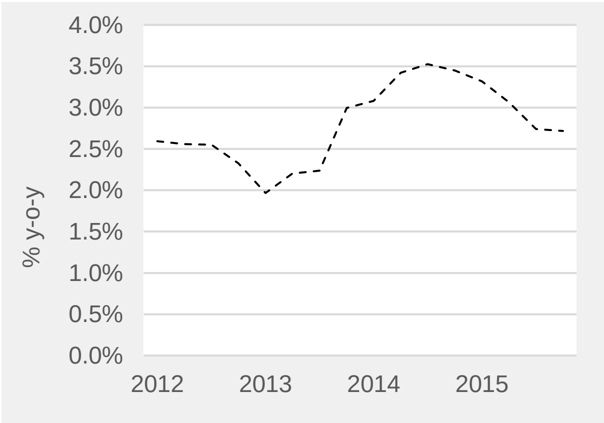


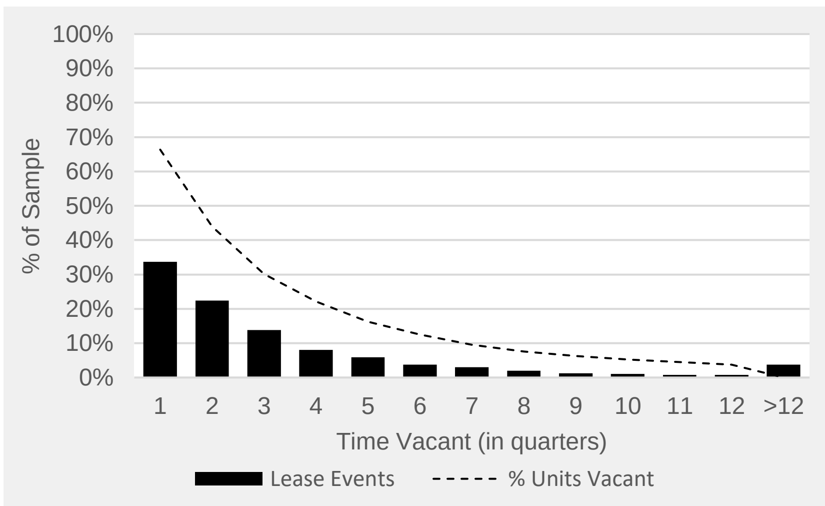
Figure 4.5: Employment Growth[8]



[6] Source: Cushman & Wakefield Manhattan Office Snapshot.
[7] Source: Cushman & Wakefield Manhattan Office Snapshot.
[8] Total non-farm seasonally adjusted employment growth in New York City, NY. Source: New York State Department of Labor.

Before estimating survival models, we started with a simple empirical distribution of vacancy durations in the sample. Figure 4.6 shows that 66.3% of suites remained vacant after one quarter, while conversely, 33.7% of suites leased within one quarter. The share of vacant suites decreased to 44.0% within two quarters, 30.1% within three quarters, and 22.1% within one year. After two years, only 7.6% of all suites remained vacant. Over the 2012 – 2015 sample period, only 3.7% of all suites remained vacant for three years or longer.

Figure 4.6: Empirical Distribution of Vacancy Durations

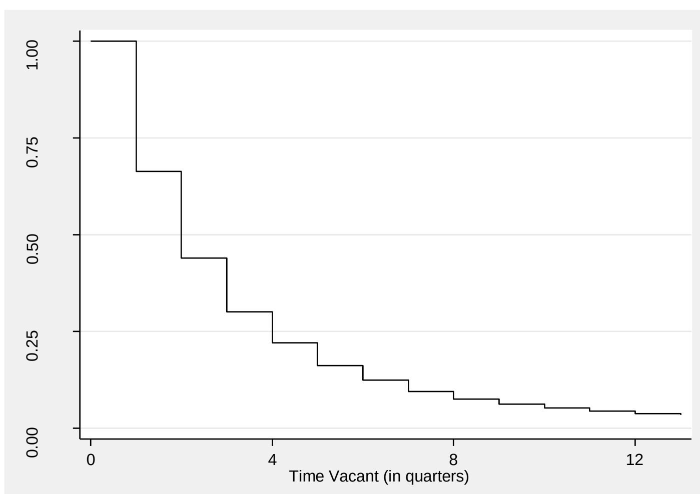


However, a primary disadvantage of the empirical distribution is that it does not account for censorship. In other words, it does not distinguish between the suites that are still vacant as of the end of the sample period and those that are leased. For this reason, we turn to the KM estimator to obtain more precise survival function estimates.

5. Kaplan-Meier Analysis

Figure 5.1 shows the KM survival function for our sample. It illustrates that 66.4% of office suites leased after one quarter, 44.0% after two quarters, 30.1% after three quarters, and 22.1% after one year. The probability of remaining vacant decreases to 7.6% after two years and 3.8% after three years. These estimates differ from the empirical distribution by less than 0.1 percentage points and are essentially identical. This can be attributed to the fact that only 2.5% of suites remained vacant at the end of the sample period. In other words, the effect of censorship was negligible.

Figure 5.1: KM Survival Function



Next, we analyzed the effects of the variables expected to influence vacancy durations – unit size, rent, building height, floor number, building age, lease type, and location – by estimating separate survival functions.

First, the sample was split into large and small suites. Large suites were defined as having an area of at least 8,400 square feet.[9] Figure 5.2 shows that the survival probability for large suites was substantially higher than for smaller ones. In summary, large offices stayed vacant longer. Nevertheless, the difference between the survival functions does not appear to be constant over time, which suggests a potential violation of the proportionality assumption.

Second, the suites were divided into two categories, depending on whether their initial asking rents were lower or higher than the average market rent at the time they become vacant. Suites with higher rents (rent ratio higher than one) appeared to have higher survival probabilities and remained vacant longer. Similarly as for the unit size variable, the difference between the survival functions appeared to decline over time.

Figure 5.2: KM – Unit Size

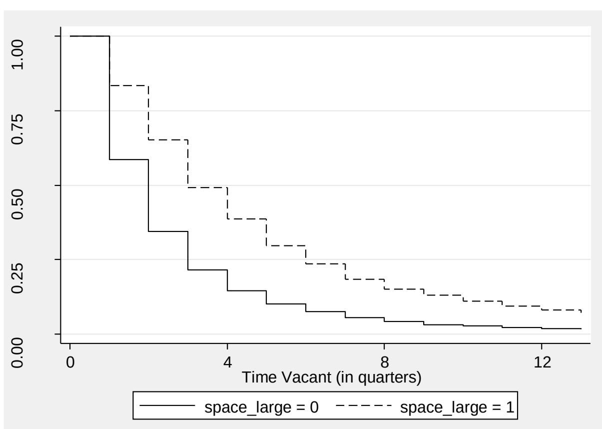
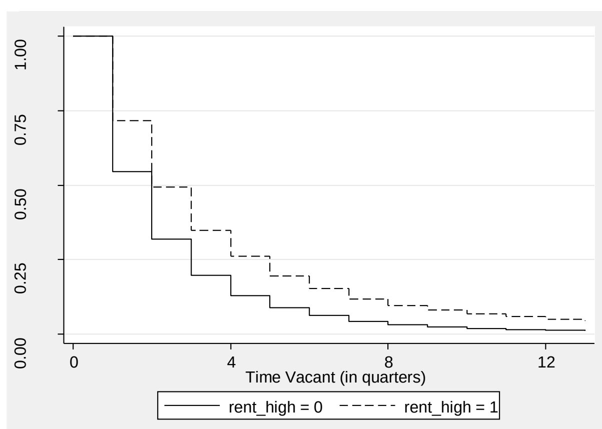


Figure 5.3: KM – Rent Ratio



[9] The mean area in the sample is 8,395 square feet.

In the next step, buildings were categorized into three groups according to the number of stories. Mid-rises include buildings with less than 12 stories, high rises are those between 12 and 40 stories, and skyscrapers are above 40 stories. Figure 5.4 shows that the difference in survival probabilities for mid and high rises is relatively small and likely not statistically significant. Conversely, skyscrapers seem to display substantially higher survival probabilities (longer vacancy durations). Building height is intrinsically linked to a floor number. Figure 5.5 shows that the survival probability for suites on a 12th or higher floor is also slightly higher than for the ones on lower floors.

Figure 5.4: KM – Building Height

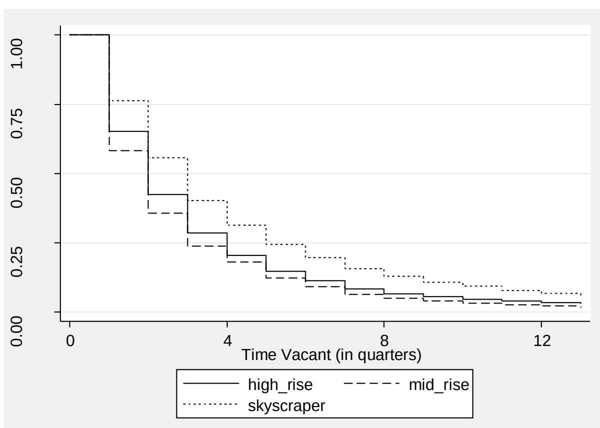
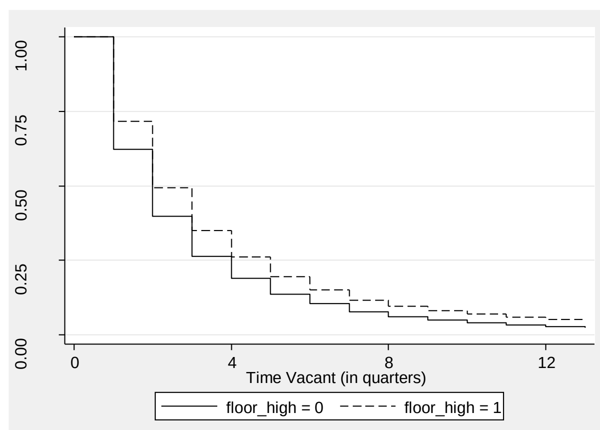


Figure 5.5: KM – Floor Number



We then divided the sample based on the building age. A building is defined as old if it is 15 years old or more. Interestingly, older office suites display lower survival probabilities, i.e. lower vacancy durations, than newer space.

Figure 5.7 shows the survival probabilities for different lease types. They exhibit considerably larger differences than other variables. Additionally, re-lets and sub-lets display significantly lower survivals (lower vacancy durations) than first-generation space. This conclusion is the same as for other attractive features, such as being located in a skyscraper, on a higher floor or in a newer building. A potential explanation is likely collinearity between these features and rent, which results in the individual survival probabilities of these variables being imprecise.

Figure 5.6: KM – Building Age

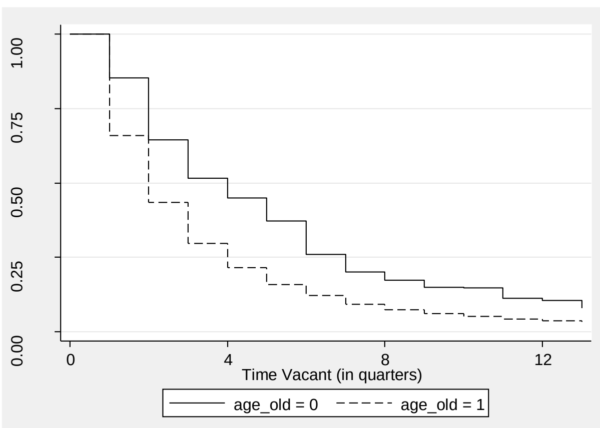
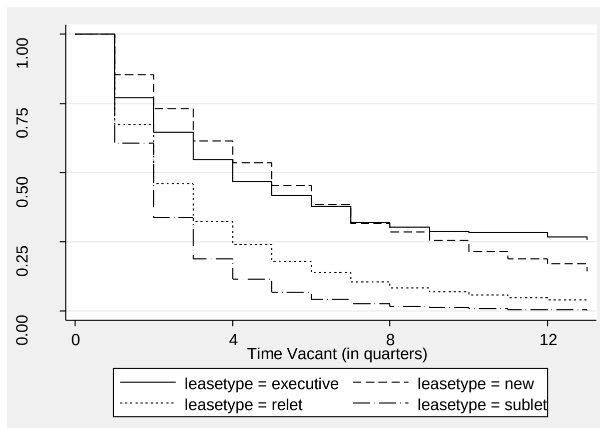
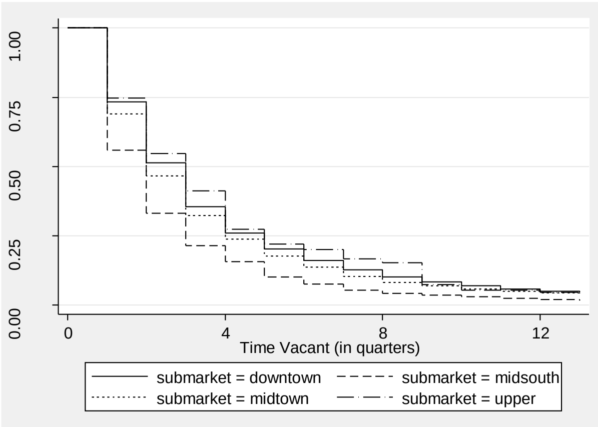


Figure 5.7: KM – Lease Type



Lastly, Figure 5.8 shows survival functions depending on the submarket. There appears to be only a negligible difference between offices located in Downtown, Midtown or Upper Manhattan. However, suites in the Midtown South submarket, which includes Chelsea, Tribeca, Hudson Square, SoHo, Greenwich Village, and Gramercy Park, display lower survival probabilities. As these are currently considered the trendiest office destinations in Manhattan, their popularity seems to translate into lower vacancy durations.

Figure 5.8: KM – Submarket



In order to formally check the differences between the estimated survival functions, the log-rank test was performed for the same binary groups. Results are shown in Figure 5.9. All of the analyzed differences were highly significant and confirmed the results obtained from the KM survival functions. The very high X^2 statistic of unit size suggests that it is one of the primary determinants of survival, and hence vacancy duration. Although still highly significant, the effect of building age appeared to be weaker than other variables.

Figure 5.9: Log-Rank Test

	X^2	P-Value
Unit Size	1280.73	0.0000
Rent Ratio	579.83	0.0000
Building Height	258.80	0.0000
Floor Number	177.85	0.0000
Building Age	65.07	0.0000
Lease Type	775.83	0.0000
Submarket	274.55	0.0000

6. Results

6.1. Time-Independent Model

One of the primary disadvantages of the KM estimates or the log-rank test are their inability to estimate survival adjusted for other covariates and to quantify the size of their effect on survival. The Cox Proportional Hazards Model addresses these shortcomings.

Firstly, we fit a simple proportional hazard model as described in equation (7), with only one variable – unit size – which is expected to have the largest effect. To reduce the highly positive skew in the unit size variable, it was logarithmically transformed. Figure 6.1 shows the regression output. The hazard ratio of 0.71 suggests that a 10% increase in a unit's square footage is associated with a 3.2% decrease[10] in the hazard rate – the risk of a unit getting leased at any particular point in time.

Figure 6.1: Cox PH Model (1)

No. of observations	14686	LR chi2(1)	2101.53
No. of failures	14321	P-Value	0.000

Variable	Haz. Ratio	Std. Err.	P-Value
ln Unit Size	.7098389	.0052125	0.000

[10] The hazard ratio of .7098389 translates into a coefficient of -0.3427 $\left(\hat{\beta} = \ln(.7098389)\right)$. The effect of a 10% increase in the unit size is calculated as $e^{-0.3555*\ln(1+10\%)} - 1 = -0.0321$.

Secondly, we added the rent ratio, which is defined as a suite's rent divided by the average market rent in a particular period, to the regression model. The interpretation of the rent ratio is less straight forward, as it depends on both the unit's rent and the average market rent. For instance, a suite that is priced at 200% of the market rent is 36.0%[11] less likely to lease at any point than one priced at 100% of the market rent. It is also important to note that for this analysis the sample size was substantially decreased, as unit rent was is available for only a third of the overall sample. Despite the decrease in sample size, the overall model remains highly significant.

Figure 6.2: Cox PH Model (2)

No. of observations	5123	LR chi2(2)	861.32
No. of failures	5080	P-Value	0.000

Variable	Haz. Ratio	Std. Err.	P-Value
ln Unit Size	.6892161	.0091549	0.000
Rent Ratio	.6395043	.0385941	0.000

Next, we added the building height and floor number to the model. Each additional floor in building height decreased the "risk" of getting leased by 1.0%[12], while an increase in the floor number increased the hazard by 0.4%[13].

[11] Calculated as $0.6395043 - 1 = -.3605$.
[12] Calculated as $.989599 - 1 = -.0104$.
[13] Calculated as $1.004242 - 1 = .0042$.

Although building height and floor number are intrinsically related, the negative effect of building height outweighs the positive effect of the floor number. Consequently, offices in high-rise buildings display, on average, longer vacancy duration.

Figure 6.3: Cox PH Model (3)

No. of observations	5022	LR chi2(4)	918.62
No. of failures	4981	P-Value	0.0000

Variable	Haz. Ratio	Std. Err.	P-Value
ln Unit Size	.7016622	.0096235	0.000
Rent Ratio	.6663167	.0426237	0.000
Building Height	.989599	.0014800	0.000
Floor Number	1.004242	.0022524	0.059

Figure 6.4 shows the impact of adding building age at the start of the vacancy spell into the model. The effect of building age on survival is not statistically significant, which is contrary to the substantial difference identified by the KM estimator (Figure 5.6).

Subsequently, the lease type variable was added to the model (Figure 6.5). As expected, executive suites have the same hazard rate compared to the baseline category (newly constructed suites). Conversely, offices offered as sub-leases are 43.0% more likely to become occupied than new ones. The hazard rate for second generation space or later is 26.7% higher but significant only at the 10% level.

Figure 6.4: Cox PH Model (4)

No. of observations	5022	LR chi2(5)	919.80
No. of failures	4981	P-Value	0.0000

Variable	Haz. Ratio	Std. Err.	P-Value
ln Unit Size	.7027707	.009692	0.000
Rent Ratio	.6754746	.0440169	0.000
Building Height	.9901105	.0015515	0.000
Floor Number	1.004156	.0022544	0.065
Building Age	1.00077	.0007112	0.279

Figure 6.5: Cox PH Model (5)

No. of observations	5022	LR chi2(8)	930.83
No. of failures	4981	P-Value	0.0000

Variable	Haz. Ratio	Std. Err.	P-Value
ln Unit Size	.705919	.0098641	0.000
Rent Ratio	.6808302	.0443564	0.000
Building Height	.9903338	.0015569	0.000
Floor Number	1.003726	.0022575	0.098
Building Age	1.000807	.0007137	0.258
Lease Type [14]			
Executive Suites	1.025946	.2764057	0.924
Re-Lets	1.267279	.1644253	0.068
Sub-Lets	1.429788	.1955316	0.009

[14] Lease Type = 0 for newly constructed buildings.

As the last step in constructing the base Cox model with no time-dependent coefficients or covariates, we added the sub-market variables. The results presented in Figure 6.6 offer slightly different conclusions than the KM estimator. When adjusted for other covariates, offices in the Downtown and Upper Manhattan submarkets are 33.7% and 57.7% respectively, less likely to lease than comparable offices in Midtown. On the other hand, Midtown South appears to be the most popular submarket and the hazard rate is 14.3% higher than in Midtown.

Figure 6.6: Cox PH Model (6)

No. of observations	5022	LR chi2(11)	1143.67
No. of failures	4981	P-Value	0.0000

Variable	Haz. Ratio	Std. Err.	P-Value
ln Unit Size	.7049870	.0100017	0.000
Rent Ratio	.4378216	.0346429	0.000
Building Height	.9921973	.0016074	0.000
Floor Number	1.005573	.0022767	0.014
Building Age	.9988757	.0007733	0.146
Lease Type [15]			
Executive Suites	1.074328	.2894814	0.790
Re-Lets	1.300351	.169123	0.043
Sub-Lets	1.431648	.1963057	0.009
Sub Market [16]			
Downtown	.6633815	.0282077	0.000
Midtown South	1.143338	.0400507	0.000
Upper Manhattan	.4225273	.047931	0.000

[15] Lease Type = 0 for newly constructed buildings.
[16] Sub Market = 0 for Midtown.

The estimated hazard ratios appear to be relatively robust, as they remain stable across different model specifications (Figure 6.7). The only significant change is the increase in the negative effect of rent when adjusted for different submarkets. This is attributable to the collinearity between the rent and submarkets variables – rents in the attractive submarkets tend to be on average higher, and vice versa. Without controlling for different submarkets, the rent ratio coefficients largely represent the combined effect of both rent and submarket attractiveness.

Figure 6.7: Cox PH Model Summary

Model	(1)	(2)	(3)	(4)	(5)	(6)
No. of obs.	14686	5123	5022	5022	5022	5022
No. of failures	14321	5080	4981	4981	4981	4981
LR chi2	2102 ***	861 ***	919 ***	920 ***	931 ***	1144 ***

Variable	Hazard Ratios					
ln Unit Size	.710 ***	0.689 ***	0.702 ***	0.703 ***	0.706 ***	.705 ***
Rent Ratio		0.640 ***	0.666 ***	0.675 ***	0.681 ***	.438 ***
Building Height			0.990 ***	0.990 ***	0.990 ***	.992 ***
Floor Number			1.004 *	1.004 *	1.004 *	1.006 **
Building Age				1.001	1.001	.999
Lease Type						
Executive					1.026	1.074
Re-Lets					1.267 *	1.300 **
Sub-Lets					1.430 ***	1.432 ***
Sub Market						
Downtown						.663 ***
Midtown South						1.143 ***
Upper						.423 ***

The main assumption of the Cox model is the hazard proportionality. To test this assumption, we employed a test based on the Schoenfeld residuals on time (Grambsch and Therneau, 1994). The null hypothesis corresponds to a zero slope of residuals over time, which indicates proportionality. The test results shown in Figure 6.8 indicate that the null hypothesis must be rejected, and the overall model displays non-proportional hazards. Specifically, the largest contributor to the non-proportionality is unit size. As a result, the basic Cox model provides a reasonable estimate of the average effects over time but more complex specifications are needed to account for specific time fixed effects.

Figure 6.8: Proportionality Test

	X^2	P-Value
ln Unit Size	41.22	0.0000
Rent Ratio	3.69	0.0546
Building Height	0.51	0.4760
Floor Number	5.61	0.0179
Building Age	4.98	0.0256
Lease Type		
Executive Suites	0.02	0.8926
Re-Lets	0.14	0.7096
Sub-Lets	0.01	0.9182
Sub Market		
Downtown	8.74	0.0031
Midtown South	0.34	0.5615
Upper	10.13	0.0015
Global Test	87.08	0.0000

6.2. Time-Dependent Coefficients

The first estimated alternative model follows equation (9) and incorporates time-dependent coefficients. This specification allows the effects of variables on survival to vary over the duration of vacancy. In other words, it allows variables to have different effects at the start and towards the end of the vacancy spell. The primary advantage of this specification is that hazard rates are no longer required to be proportional.

The results of the survival model with time-dependent coefficients are reported in Figure 6.9. They are consistent with the findings of the proportionality test, showing that unit size is the main factor driving the variation in effects over time, while the effect of the other variables is less significant. The unit size hazard ratio at $t = 0$ equals .633, which suggests that a 10% increase in a suite's square footage is associated with a 4.3% decrease[17] in the hazard rate. The decrease is slightly higher than under the proportional hazards model (7) that results in a 3.3% decrease given a 10% increase in a suite's size. However, with each quarter, the unit size effect increases (becomes less significant) by 4.4%[18]. The total effect of a 10% increase in a unit's size is a 3.9% decrease in the hazard after one quarter[19], 3.5% after two quarters, 3.1% after three quarters, etc.

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[17] The hazard ratio of .633 translates into a coefficient of -0.457 $\left(\hat{\beta} = \ln(.633)\right)$. The effect of a 10% increase in the unit size is calculated as $e^{-.457*\ln(1+10\%)} - 1 = -.043$.

[18] The hazard ratio for the interaction of unit size with time is 1.043974. The effect is calculated as $1.043974 - 1 = .0439$.

[19] The hazard ratio at $t = 1$ becomes $.633 * 1.044 = .661$. The effect of a 10% increase in the unit size is then $e^{\ln(.661)*\ln(1+10\%)} - 1 = -.039$.

Figure 6.9: Survival Model with Time-Dependent Coefficients (7)

No. of observations	5022	LR chi2(11)	1207.47
No. of failures	4981	P-Value	0.0000

Variable	Haz. Ratio	Std. Err.	P-Value
ln Unit Size	.6331831	.0138083	0.000
Rent Ratio	.4099188	.0460023	0.000
Building Height	.9910342	.0024399	0.000
Floor Number	1.01001	.0035158	0.004
Building Age	1.001213	.0011897	0.308
Lease Type			
Executive Suites	1.117549	.3018055	0.681
Re-Lets	1.322996	.1728501	0.032
Sub-Lets	1.455143	.2003395	0.006
Sub Market			
Downtown	.667364	.028708	0.000
Midtown South	1.130477	.0397265	0.000
Upper Manhattan	.4314963	.0489649	0.000
Interaction Variable [20]			
ln Unit Size	1.043974	.0068617	0.000
Rent Ratio	1.023609	.03191	0.454
Building Height	1.000214	.0005451	0.694
Floor Number	.9987622	.0008732	0.157
Building Age	.9992320	.0003208	0.017

[20] Variables interacted with a linear function of time (vacancy duration).

The diminishing effect of unit size on survival can also be illustrated graphically. Figure 6.10 shows the survival functions of two hypothetical suites – a smaller suite (1,000 square feet) and a larger one (10,000 square feet)[21]. The survival function for the smaller suite is consistently below the larger suite, which reflects the smaller suite's greater probability to lease at any point, i.e. a shorter vacancy duration. However, the difference between these two functions shrinks over the duration of the vacancy and becomes negligible after three years.

The only other variable whose effect over time is found to be significant is the building age. Nevertheless, as its base coefficient is not significant, the time interaction term has only a weak impact on survival (Figure 6.11). The remaining base coefficients do not change considerably compared to the proportional hazards model. These findings indicate that non-proportionality is attributable primarily to unit size, while the other coefficients appear robust.

Figure 6.10: Unit Size Function

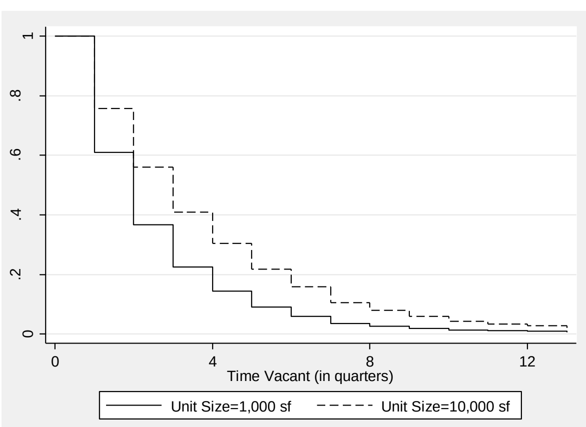
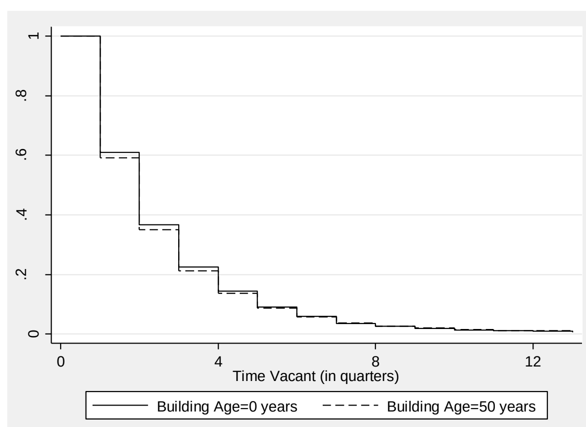


Figure 6.11: Building Age Function



[21] The other characteristics are assumed to be the same for both suites. Specifically, it is assumed that the offices are in a new building with 10 stories, priced at the market average rent and located in Midtown Manhattan.

6.3. *Stratified Survival Model*

Although the model with time-dependent coefficients addresses the issue of non-proportionality, it implicitly assumes that the baseline hazard is the same for all time periods during the duration of the study. As our sample includes suites that became vacant between 2012 and 2015, the model assumes that the baseline probability of getting leased did not change throughout this period.

We introduced a survival model stratified on calendar periods. This relaxes the assumption of constant hazards and allows the baseline hazard, and hence vacancy durations, to vary over time. The specification assumes a different baseline hazard at each stratum (calendar period), which can be interpreted to reflect the combined effects of unobserved time-dependent variables on vacancy durations. The effects (coefficients) of suite-specific variables are assumed to stay constant over the different strata.

Figure 6.12 shows that the estimated coefficients remained virtually unchanged compared to the base model with constant hazards over time (6). The results indicate that the time fixed effects do not have a considerable impact on the coefficients of the other independent variables. In other words, their effects do not appear substantially different in periods with higher or lower market demand and supply. Instead, different periods seem to uniformly affect survival probabilities across all suite characteristics.

Figure 6.12: Stratified Survival Model (8)

No. of observations	5022	LR chi2(11)	1133.16
No. of failures	4981	P-Value	0.0000

Variable	Haz. Ratio	Std. Err.	P-Value
ln Unit Size	.6989084	.0100966	0.000
Rent Ratio	.4077532	.0334942	0.000
Building Height	.9922883	.0016323	0.000
Floor Number	1.006202	.0023055	0.007
Building Age	.9985376	.0007859	0.063
Lease Type			
Executive Suites	1.080132	.294475	0.777
Re-Lets	1.374352	.1813336	0.016
Sub-Lets	1.464198	.2038176	0.006
Sub Market			
Downtown	.6696573	.0287637	0.000
Midtown South	1.153744	.0408853	0.000
Upper Manhattan	.4310614	.0496252	0.000

6.4. Time-Dependent Covariates

One of the primary disadvantages of the stratified model is that it does not allow to directly observe the impact of the time-dependent covariates. To address this shortcoming, we extend the base proportional hazards model to include two main time-dependent covariates – the market vacancy rate and employment growth. It was hypothesized that changes in these two covariates were the main drivers of variations in the hazard rate over time.

Figure 6.13 indicates that employment growth market vacancy rate has a positive effect on the hazard rate, which suggests that vacancy duration is lower in periods with high employment growth. It suggests that a 1 percentage point increase in the year-over-year employment growth increases the probability of getting leased by 14.0%. Conversely, market vacancy was found to have no impact on the hazard rate during the observation period.

Figure 6.13: Survival Model with Time-Dependent Variables (9)

No. of observations	5022	LR chi2(13)	1163.85
No. of failures	4981	P-Value	0.0000

Variable	Haz. Ratio	Std. Err.	P-Value
ln Unit Size	.703794	.0100111	0.000
Rent Ratio	.4260358	.0339573	0.000
Building Height	.9922358	.0016088	0.000
Floor Number	1.005845	.0022773	0.010
Building Age	.9987181	.0007735	0.098
Lease Type			
Executive Suites	1.059283	.2856193	0.831
Re-Lets	1.283458	.1670116	0.055
Sub-Lets	1.396614	.191691	0.015
Sub Market			
Downtown	.6629584	.0281847	0.000
Midtown South	1.144707	.0400731	0.000
Upper Manhattan	.4264626	.0484041	0.000
Market Vacancy	1.000439	.0202229	0.983
Employment Growth	1.140027	.0335223	0.000

6.5. Parametric Model

Figure 6.14 shows the results of estimating the exponential parametric model. In line with the previous results, unit size, rent ratio and building height have a highly negative effect on hazard – higher values result in longer vacancy durations. The effect of floor number is less pronounced but still significant at the 5% level. Offices on higher floors display shorter vacancy spells. The negative coefficient of building age suggests that suites in older buildings might experience longer vacancy durations, but the effect is relatively small and significant only at the 10% level.

Figure 6.14: Parametric Model (10)

No. of observations	5022	Wald chi2(11)	1161.97
No. of failures	4981	P-Value	0.0000

Variable	Coefficient	Robust Std. Err.	P-Value
ln Unit Size	-.278242	.0120909	0.000
Rent Ratio	-.6272015	.0765517	0.000
Building Height	-.0068808	.001517	0.000
Floor Number	.0041236	.0019892	0.038
Building Age	-.0010482	.0006131	0.087
Lease Type			
Executive Suites	.1705677	.2426152	0.482
Re-Lets	.2309308	.1045888	0.027
Sub-Lets	.3260999	.1107898	0.003
Sub Market			
Downtown	-.3037686	.0356884	0.000
Midtown South	.0967476	.0282379	0.001
Upper Manhattan	-.6786591	.0914888	0.000
_cons	1.788928	.1678158	0.000

7. Expected Vacancy Duration

Next, we used the estimated coefficients reported from the parametric survival model to calculate the expected vacancy duration depending on suite characteristics. For instance, for a suite with average characteristics located in Midtown, the expected duration is calculated as:

$$\hat{T} = \frac{1}{e^{-.278*\ln(size)-.627*rent-.007*height-.004*floor-.001*age+2.020}} \tag{20}$$

$$\hat{T} = 3.25\ quarters = 0.81\ years \tag{21}$$

The expected survival time of 3.25 quarters is similar to the mean vacancy duration of 3.41 quarters for the entire sample, which suggests that the exponential parametric model fits the data relatively well.

The estimated coefficients can be thought of as the elasticity of vacancy duration with respect to the individual variables. The elasticities are plotted and shown in Figures 7.1 – 7.5. The expected duration appears to be the most elastic with respect to unit size and asking rent. As unit size increases from 1,000 to 10,000 square feet, the duration increases from 0.6 years to 1.1 years. If the rent ratio increases from 0.5 to 1.5, the duration increases from 0.9 to 1.6 years.

Figure 7.1: Mean Duration – Unit Size[22]

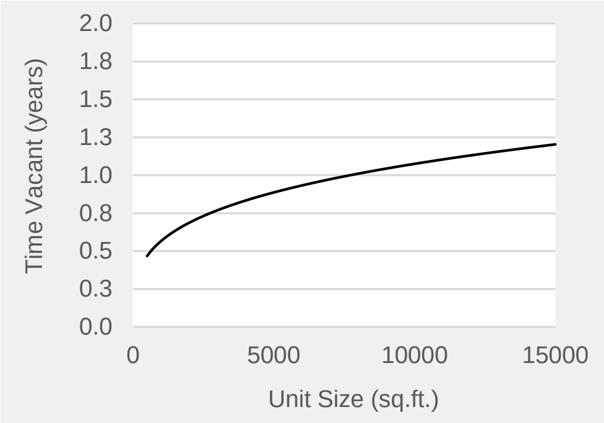


Figure 7.2: Mean Duration – Rent Ratio

Figure 7.3: Mean Duration – Height

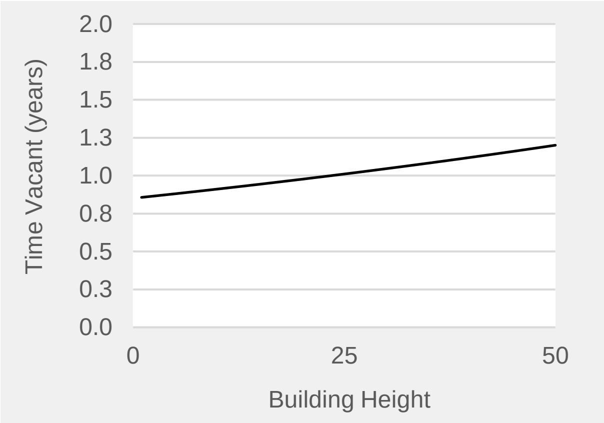


Figure 7.4: Mean Duration – Floor

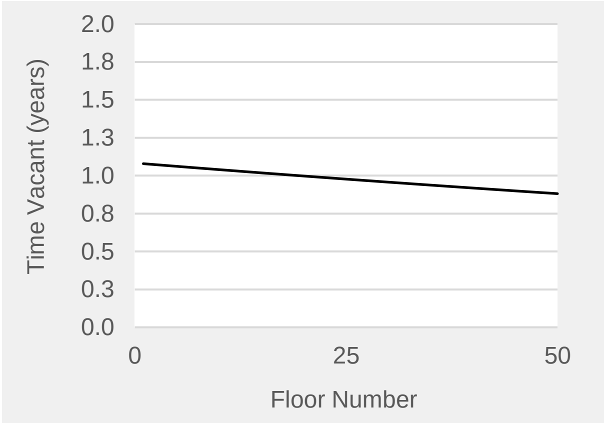
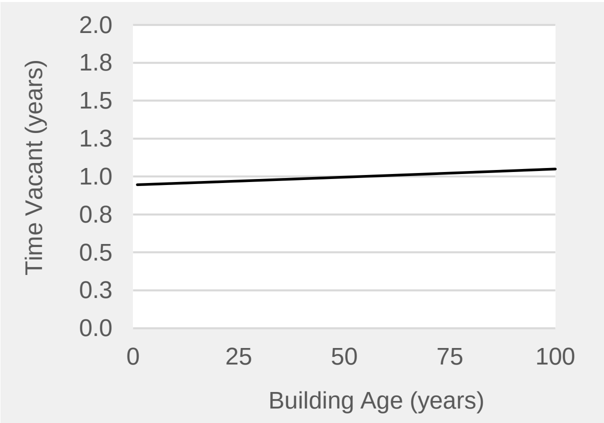


Figure 7.5: Mean Duration – Age



[22] In Figures 7.1 – 7.5, the remaining variables are assumed to be fixed at their mean values.

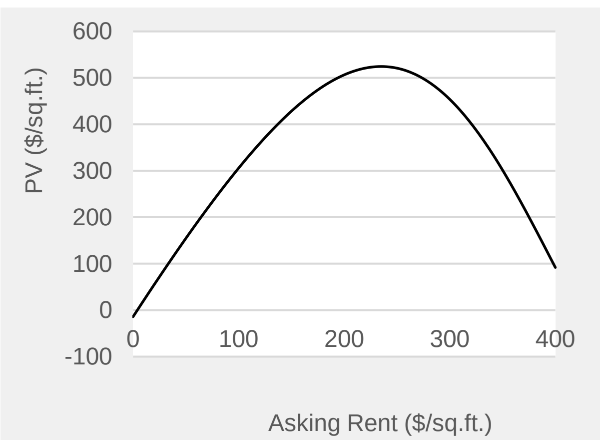
8. Rent Optimization

As the last step, we illustrated the potential application of the survival model in setting an optimal asking rent depending on suite characteristics. Using the estimated coefficients from the parametric survival model, the expected vacancy duration for a suite with average characteristics[23] and unknown asking rent is defined as:

$$\overline{T} = \frac{1}{e^{-0.7024 - .627 * \frac{r_i}{r_m}}} \tag{22}$$

Where r_m represents the current average market rent. By combining equations (19) and (22), we get an equation expressing the present value of the landlord's profit as a function of the asking rent. This function[24] is graphed in Figure 8.1. The optimal rent for an average suite is \$235 per square foot, which is associated with the vacancy duration of 3.8 years.

Figure 8.1: Present Value of Landlord Profit



[23] Average unit size 8,395 sq.ft., number of stories 26.88, floor number 13.68, building age 76.01 years. The average unit is assumed to be located in Midtown and offered as a re-let.
[24] It is assumed that $y = 10\%$, $n = 5$ *years*, $C = \$30$, and $r_m = \$72.5$.

The optimal rent is considerably higher than the current average market rent of $72.5 per square foot. This has several possible explanations. 95% of all suites in the dataset have rent ratios (asking rent divided by market rent) between 0.5 and 1.2 times the market rent. Extrapolating the relatively inelastic demand in this range to higher rent values might yield unrealistic results. Furthermore, the parametric survival model also assumes that the relationship between expected duration and asking rent is exponential. The true relationship might follow some other form, possibly with a steep change in the slope after a certain rent value. Lastly, the true profit-maximizing rent might be considerably above the current market rent, but other constraints, such as financing covenants, prevent landlords from waiting for several years to find a tenant.

9. Discussion

The paper aimed to fill the gap in existing research by identifying the main determinants of vacancy durations in the office sector. It applied survival analysis to offices that experienced a vacancy between 2012 and 2015. First, the paper presented a Cox proportional hazards model and several of its extensions introducing various forms of time dependency. Four key characteristics – unit size, asking rent, building height, and floor number – were significant across all model specifications.

Unit size was found to be positively related to vacancy durations. The conclusion is contrary to previous studies in the housing sector (Guasch & Marshall, 1985a), where smaller units have been found to display longer durations. A potential explanation is that smaller offices are more homogeneous than larger ones. This decreases tenant search costs and, hence, vacancy durations.

We found asking rents to have a positive effect on vacancy durations, which supports the results of Orr et al. (2003) for commercial, or Guasch & Marshall (1985a) for residential units. These results are in line with the search theory proposed in previous research. The theory suggests that landlords who are motivated to transact quickly discount asking rents, while high-priced suites attract fewer potential tenants and trade in a thinner market.

Building height was found to increase average durations, while suites on higher floors lease faster. This could be attributable to the fact that the transiency of office tenants is higher in larger buildings, which in turn results in longer durations. However, the negative impact of building height is partially mitigated by the attractiveness of offices located on higher floors.

Survival probabilities were found to contain significant time fixed effects. These time effects seem to be driven predominantly by employment growth – vacancy durations are generally lower during the times of rapid growth in employment, which is accompanied by increased demand for office space. The overall market vacancy rate was found not to affect durations. Nevertheless, a longer observation period is needed to obtain a more robust conclusion.

Lastly, the paper presented an exponential parametric survival model. It further confirmed the effects unit size, asking rent, building height, and floor number as identified by the previous survival models. The estimated coefficients were used to calculate the expected vacancy duration and to show its elasticity with respect to the main characteristics. Vacancy durations appear to be the most elastic with respect to unit size and asking rent. Their elasticity with respect to other variables is less pronounced.

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