14.381: Statistics

Problem Set 3

1. Let $X_1, \ldots, X_n$ be a random sample from a Poisson distribution with parameter λ

$$P\{X = j\} = \frac{e^{-\lambda}\lambda^j}{j!} \quad j = 0, 1, \ldots$$

   (a) Find the MLE of λ and its asymptotic distribution.

   (b) Assume that we are interested in estimating probability of a count of zero $\theta = P\{X = 0\} = \exp\{-\lambda\}$. Find the MLE of θ and its asymptotic distribution. *Hint:* you may use the delta-method.

   (c) Show that ΣX_i is a sufficient statistic.

   (d) Find an unbiased estimator of θ. *Hint:* $\theta = P\{X = 0\} = E\mathbb{I}\{X = 0\}$.

   (e) Is the estimator in (d) a function of a minimal sufficient statistics? Modify the estimator to make sure it is a function of a minimal sufficient statistics.

2. Consider a sample $X_1, \ldots, X_n$ from pdf

$$f(x \mid \theta) = \begin{cases} e^{-x+\theta}, & x \geq \theta \\ 0, & \text{otherwise} \end{cases}, \qquad \theta > 0.$$

   (a) Find Fisher information and establish Rao-Cramer bound for the variance of an unbiased estimator of θ.

   (b) Find the distribution of the statistic $X_{(1)} = \min_i X_i$.

   (c) Find $EX_{(1)}$ and $Var(X_{(1)})$.

   (d) Show that $\hat{\theta} = X_{(1)} - \frac{1}{n}$ is unbiased. Find its variance.

   (e) Why does it violate the Rao-Cramer bound?

   (f) Find the MLE.

(g) Does the theorem about MLE asymptotics hold? Why?

3. Let $X_1, \ldots, X_n$ be a sample from the following distributions. In each case find the asymptotic variance of the MLE.

   (a) $f(x \mid \theta) = \theta x^{\theta-1}, \quad 0 < x < 1, \quad 0 < \theta < \infty.$

   (b) $f(x \mid \theta) = \frac{1}{\theta} \exp\left\{-\frac{x}{\theta}\right\}, \quad 0 < x < \infty, \quad 0 < \theta < \infty.$

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