Lecture 5: Intertemporal Labor Supply

Labor supply is clearly part of a lifetime decision making process. There are some obvious, but important stylized facts about the pattern of wages and employment over the lifecycle:

1) Earnings and wages rise with age after schooling, then decline before retirement.

2) Hours worked generally rises with age, then falls before retirement, then goes to zero at retirement.

3) Individuals have been retiring earlier, on average, over the last 20 years

4) female labor supply has increased dramatically in the last 40 years.

Variations in health status, family composition and real wages – anticipated or otherwise, provide incentives for individuals to vary the timing of their labor market earnings for income-smoothing and taste purposes. But these responses are not adequately captured in the static model. Let me just emphasize here that this does not make the static model useless. Indeed, I think the static model is far more helpful in thinking about labor supply issues than the intertemporal model.

The intertemporal model of labor supply makes the point that there are three types of wage shocks: permanent, transitory, and anticipated, and the supply response (elasticity) will depend on what type of shock occurs. We must be clear what type of shock we are examining when interpreting empirical results that measure the response.

My take on this model is that it is not very testable, and there are a lot of assumptions required to estimate the different types of shocks. Furthermore, the intertemporal model of requires the assumption of time seperability – a dubious assumption.

We modify the utility function to include multiple periods, and allow for uncertainty:

$$U_t \equiv \sum_{t=s}^{T} \beta^{s-t} E\left[u(C_s, H_s, X_s)\right]$$

β is a time preference factor. Note: already we've assumed lifetime utility is time separable: leisure taken in one period does not affect the utility in another period. This is not a realistic assumption, since clearly human capital decisions when young affect wages (price of leisure), and therefore utility in other periods. You might prefer to think of this model beginning after school, since it will have problems explaining labor supply decisions driven by education reasons separable by time period, rather than period t utility dependent on next period's consumption or leisure.

Borrowing and lending is permitted, and so the budget constraint essentially summarizes the time path of wealth (or assets). Note, we've assumed non-labor income is zero. Let's also assume an interior solution, so L<T always.

The model is a bit more involved to solve with uncertainly, but be need uncertainty to derive the more interesting implications. There are several ways to solve. Here's one.

Define expected wealth in period t, $\overline{W}_t$, as the financial assets accumulated through to period t period plus current and future expected income as of date t:

$$\overline{W}_t = A_{t-1}(1+r_{t-1}) + E_t \sum_{s=t}^{T} \left(\frac{1}{(1+r_s)}\right)^{s-t} (w_s H_s - C_s)$$

$$= (1+r_{t-1})\left[\overline{W}_{t-1} + w_{t-1}H_{s-1} - C_{t-1}\right]$$

We use the Bellman equation to solve the dynamic programming problem. The approach is to consider the maximization problem into two parts. From the standpoint of date t, an individual must maximize lifetime utility in date t+1 subject to the future wealth level produced by today's consumption and leisure decisions (If not, the individual could raise

utility by behaving differently after date t). The foregoing property means, however, that a maximizing agent can behave as if $U_t = u(C_t, H_t, X_t) + \beta E_t V(\overline{W}_{t+1})$, where $V(\overline{W}_{t+1})$ is the constrained maximal value of U_{t+1}, subject to wealth in period t+1. Then, the choice of C_t and L_t maximizing lifetime utility is the one maximizing $U_t = u(C_t, H_t, X_t) + \beta E_t V(\overline{W}_{t+1})$ subject to $\overline{W}_{t+1} = (1 + r_t)[\overline{W}_t + w_t H_t - C_t]$. In other words, the Bellman equation is:

$$
\begin{aligned}
V(\overline{W}_t) &= \max_{C_t, H_t} [u(C_t, H_t, X_t) + \beta E_t V(\overline{W}_{t+1})] \\
&= \max_{C_t, H_t} [u(C_t, H_t, X_t) + \beta E_t V((1 + r_t)[\overline{W}_t + w_t H_t - C_t])] \\
&= \max_{C_t, H_t, \lambda_t, \overline{W}_{t+1}} \left[u(C_t, H_t, X_t) + \beta E_t V(\overline{W}_{t+1}) + \lambda_t (\overline{W}_t + w_t H_t - C_t - \frac{\overline{W}_{t+1}}{(1 + r_t)}) \right]
\end{aligned}
$$

The first order conditions are:

$$
\begin{aligned}
&u_c(C_t, H_t, X_t) = \lambda_t \\
&u_H(C_t, H_t, X_t) = -w_t \lambda_t \\
&\overline{W}_{t+1} = (1 + r_t)[\overline{W}_t + w_t H_t - C_t] \\
&(1 + r_t)\beta E_t V'(\overline{W}_{t+1}) = \lambda_t
\end{aligned}
$$

Now we have to figure out what $V(\overline{W}_t)$ is. For an optimizing individual, an increment to wealth on any date has the same effect on lifetime utility regardless of the use to which the wealth is put – consumption, saving, or leisure. At time t, a marginal value of consumption greater or less than the marginal value of saving can't be optimal. Since the marginal value of saving is the marginal value of wealth, the implication is that: $V'(\overline{W}_t) = u_c$. Thus, $u_c = \lambda_t = V'(\overline{W}_t)$ is the marginal utility of wealth. (perhaps a more detailed definition is the marginal utility of wealth, evaluated at time t).

More formally:

$$V(\overline{W}_t) = \left[u(C_t(\overline{W}_t), H_t(\overline{W}_t), X_t) + \beta E_t V(\overline{W}_{t+1}(\overline{W}_t)) + \lambda_t(\overline{W}_t + w_t H_t(\overline{W}_t) - C_t(\overline{W}_t) - \frac{\overline{W}_{t+1}(\overline{W}_t)}{(1+r_t)}) \right]$$

$$V'(\overline{W}_t) = u_c \frac{\partial C}{\partial \overline{W}_t} + u_h \frac{\partial H}{\partial \overline{W}_t} + \beta E_t V'(\overline{W}_{t+1}) \frac{\partial \overline{W}_{t+1}}{\partial \overline{W}_t} + \lambda_t + \lambda_t w_t \frac{\partial L}{\partial \overline{W}_t} - \lambda_t \frac{\partial C}{\partial \overline{W}_t} - \frac{\lambda_t}{(1+r_t)} \frac{\partial \overline{W}_{t+1}}{\partial \overline{W}_t}$$

$$= \lambda_t$$

(using the first order conditions, or recognizing this is just an application of the envelope theorem)

which means also for the first order conditions:

$$(1+r_t)\beta E_t(\lambda_{t+1}) = \lambda_t$$

In sum, we have:

$$u_c(C_t, H_t, X_t) = \lambda_t$$

$$u_H(C_t, H_t, X_t) = -w_t \lambda_t$$

$$\lambda_t = \beta(1+r_{t+1})E(\lambda_{t+1})$$

$$u_c = \lambda_t = V'(\overline{W}_t)$$

Can use first order conditions to solve implicitly:

$$C_t = C(\lambda_t, w_t, X_t)$$

$$H_t = H(\lambda_t, w_t, X_t)$$

These are called the Frischian demand functions. Whereas the marshallian function holds income constant and hicksian function hold utility constant, frisch functions hold the marginal utility of wealth constant.

What's interesting is that current period labor supply can be decomposed into only 3 components: X, w, and lamda, which summarizes the relevant information from all other periods. Variables such as future wealth, wages, or personal characteristics affect consumption and labor supply decisions only by changing the value of lambda.

Note, with concavity in preferences, first order conditions imply:

1) $\dfrac{\partial \lambda_0}{\partial A_0} \leq 0$ declining MU of wealth

2) $\dfrac{\partial \lambda_0}{\partial w_t} \leq 0$ wages generate wealth effects

3) $\dfrac{\partial H_t}{\partial w_t}\Big|_{\lambda_0 = \text{constant}} \geq 0$, effect of a movement along wage profile

4) $\dfrac{\partial H_t}{\partial \lambda_0} \geq 0$, effect of change in marginal utility of wealth

As a consequence of the additive structure of preferences, the effects of asset income and future wages are completely summarised by the value of λ_t. With perfect foresight and constant read interest rates, $U_c(C_t, H_t, X_t) = \lambda_t$ implies $\lambda_t = [\beta(1+r)]^{-t} \lambda_0$, then apart from taste changes and a geometric trend, the life-cycle profile of labor supply is completely determined by the profile of wages.

Note also, if $\beta(1+r) > 1$, marginal consumption decreases over time, so consumption increases (individual is relatively impatient), and vice-versa.

Example

$$u(C_t, H_t, X_t) = \frac{C_t^{1-\alpha}}{1-\alpha} - d\frac{H_t^{1+\delta}}{1+\delta}$$

$$u_c = C_t^{-\alpha}$$
$$u_h = -dH_t^{\delta}$$

$$\log H_t = \frac{1}{\delta}\left(\log\frac{-u_h}{d}\right)$$
$$= \frac{1}{\delta}\left(\log\frac{\lambda_t w_t}{d}\right)$$
$$= \frac{1}{\delta}\left(\log w_t + \log\lambda_t - \log d\right)$$

$$\log H_t = A + \eta\log w_t + \gamma\log\lambda_t$$

η is the elasticity of labor supply in period t with respect to wages in t, holding constant the marginal utility of wealth. γ is the elasticity of labor supply with respect to the marginal utility of wealth. Note, in this example, $\gamma = \eta$, but other utility functions lead to different values for the elasticities (e.g. see Card).

Consider the change in hours between periods t-1 and t:

$$\lambda_t = [\beta(1+r)]E_t(\lambda_{t+1})$$
$$\log\lambda_t = \log[\beta(1+r)] + \log E_t(\lambda_{t+1}) \text{ (ignoring Jensen's inequality) (e.g. if distribution of wealth}$$
shocks log normal. See MaCurdy when not allowing this simplification)
$$\log\lambda_t = \log[\beta(1+r)] + \log\frac{\lambda_t}{\beta(1+r)} \text{ (which is just a tautology)}$$
$$\log\lambda_{t-1} = \log[\beta(1+r)] + \log E_{t-1}(\lambda_t)$$
$$\log\lambda_t - \log\lambda_{t-1} = -\log[\beta(1+r)] + \log\lambda_t - \log E_{t-1}(\lambda_t)$$

$$\Delta \log H_t = \eta \Delta \log w_t - \log[\beta(1+r)] + \gamma[\log(\lambda_t) - \log E_{t-1}(\lambda_t)]$$

The change in labor supply consists of a component due variation in wages, differences between real interest rate and time preference, and component due to any updating in log of marginal utility of wealth.

In stochastic environment, response of individual hours to observed change in wages has two parts – the first is $\eta \Delta \log w_t$, as in the perfect foresight model. The second is the change in labor supply generated by the change in the marginal utility of wealth. The realization of wages provides new information that generates an update in the distribution of future wages and brings about a revision in the forecast of lambda. Unfortunately there are no closed form expressions for λ_t in an uncertain environment. Thus, the component of the change in labor supply attributable to wealth effects is usually treated as a 'nuisance', and is eliminated by an instrumental variables procedure.

3 Alternative Sources of Wage Variation:

(Draw Graph)

1) Evolutionary wage changes (which are anticipated) generate just substitution effects, as they hold λ_0 constant (along anticipated wage profile).

$$\frac{\partial \log H_t}{\partial \log w_t} = \eta > 0$$

2) Transitory wage changes (which are unanticipated) generate substitution and income effects, but note that income effect is distributed over entire life cycle, through impact on λ_t

$\dfrac{\partial \log H_t}{\partial \log w_t} = \eta + \dfrac{\partial \log \lambda_t}{\partial \log w_t}\gamma$. Since the wage change is temporary, the effect on wealth, $\dfrac{\partial \log \lambda_t}{\partial \log w_t} < 0$, may be small and most likely the effect is positive.

3) Permanent wage changes (which are unanticipated) generate income and sub effects, so sign is ambiguous

Here the effect is larger.

4) Windfall: win the lottery: only affects λ_t. Unambiguous fall in labor supply.

When estimating labour supply effect, need to be clear what type of variation we are looking at), and what estimate can be used for in predicting other responses.

Estimating labor supply

We can try to estimate

$$\log H_{ti} = A_{it} + \eta \log w_{ti} + \delta \log \lambda_{ti} + e_i$$

Where now A_{it} are individual background controls, e is the error term. But we need a measure for λ_{ti}. If we have data over time, we can take differences (like before):

$$\Delta \log H_{ti} = \eta \Delta \log w_{ti} + \Delta \log[\beta(1+r)] + \delta[\log(\lambda_{ti}) - \log E_{t-1}(\lambda_{ti})] + v_{it}$$

But looking at changes in log labor supply requires positive hours worked in both periods. Some studies restricted to looking at prime age males: not very interesting.

In general, we need instrument for wages, or exogenous variation in wages where we know what type of wage variation occurs.

Some older notes on estimation

A typical model for estimating labor supply model is:

$$\log H_{ti} = \beta_0 + \beta_1 \log w_{ti} + \beta_2 Q_{it} + v$$

Q are controls. The specification implies restrictions on preferences. The value of β_1 determines substitution effect associated with the response of labor supply changes in wages. As discussed above, the interpretation of this substitution effect varies according to precisely which controls one includes in the vector Q and which of these controls are treated as exogenous.

Static Specifications: add taste shifters and non-labor income controls:

$$\beta_2 Q_{it} = \rho X_{it} + \theta Y_{it}$$

This static specification appropriate only if static model is correct. Consumers must behave myopically, or capital markets completlley constrained, so impossible to transfer capital. Then we are estimating β_1, which is uncompensated substitution elasticity given income Y (Marshallian wage elasticity in the static model). It's estimate involves instrumental variables for wages, because unobservables affecting both W and H.

But, if consumers adjust behaviour to account for factors in future periods, the coefficient on log wage lacks economic meaning: no matter what econometric methods are applied. That is, if the labor supply decision has any lifecycle elements, static regressions confuse shifts of wage profiles with movements along wage profiles and thus yield parameters that lack economic interpretation.

$$\beta_2 Q_{it} = \rho X_{it} + \theta Y^c_{it},$$

now instead, try to control for wealth, or position in age/wage profile.

Easier to estimate changes, which lessons need to worry about wealth.

$$\Delta \log H_{ti} = \beta_0 + \beta_1 \Delta \log w_{ti} + \beta_2 \Delta X_{it} + v_i$$

β_1 is intertemporal substitution elasticity, which holds marginal utility of wealth constant, and describes how changes in wages induced by movements along an indivudal's wage profile influence hours of work. Individuals fully anticipate these changes, and this is why wealth remains fixed. This is evolutional wage changes we are looking at.

To look at unanticipated shifts of individual wage profiles, must complete model and provide empirical specification of evolution of wages.

Generally, to do this, anticipate wage profile using average profiles of individuals in group:

$$\beta_2 Q_{it} = \theta_0 A_0 + bt + \rho X_t + \sum_{j=0}^{T} \gamma_j E_o(\log w_j)$$

So looking at deviations around this.

Most appropriate elasticity to examine for policy evaluation, since most tax and benefit reforms like once-and-for-all unanticipated shifts in net-of-tax real wages today and in the future.

Note, if preferences equal interest rate, only taste shifters or unanticipated shocks change labor supply.

Criticisms with this model:

See Card's critique: model doesn't hold up well when looking at the data and comparing model's implications. (But see Heckman for a defense of Card's critique).

Time separation assumption seems unlikely.

Liquidity constraints also may limit usefulness of these models.

Model ignores training/firm relationships that see wages changing over tenure (e.g. unions).

Many assumptions required to estimate all relevant parameters.

We've simplified to look at interior solutions, but most of action seems to be coming from whether individuals work or don't (see Heckman).