14.121 Problem Set #6

Due October 26, 2005

General instructions: Use the comparative statics theorems from class where possible. If a question refers implicitly to a set of optimizers then you may simply give results that apply to the highest element of the set.

1. (a) Let $g : \Re \to \Re$ be non-decreasing. Show that if $f : \Re^2 \to \Re$ satisfies increasing differences, then so does the composite function h defined by $h(x, y) = f(g(x), y)$. Interpret this in words. Intuitively, why is this a desirable property when we are thinking about comparative statics?

(b) Let $h : \Re \to \Re$ be twice differentiable. Prove that $f(x, \theta) = h(x - \theta)$ has increasing differences if and only if h is concave.

2. Let $Q(p, \theta)$ be a downward-sloping demand curve, let θ be a parameter of the demand curve, and suppose that Q is differentiable in p and strictly positive in the relevant region of prices. (In this question it may be helpful to note that maximizing a strictly positive function f is equivalent to maximizing $\log(f)$.)

(a) Show that Q becomes more inelastic with θ if and only if $\log(Q(p, \theta))$ has increasing differences.

(b) Suppose the firm solves $\max_p (p - c)Q(p, \theta)$. Derive a sufficient condition for p to increase (weakly) with θ and relate your answer to part (a).

(c) It is often assumed that the demand curve is concave in p, in order to validate the approach based on first order conditions. Show that concave demand is sufficient to ensure that the profit function is concave. Can you give an economic justification for concave demand? Is this necessary for the comparative statics prediction?

3. Suppose there are two inputs to production, technology and the skill level of a single worker who uses the technology. There are two technologies available for production, one which uses computers and one which uses old-fashioned technology. If the firm chooses the computer technology its production function is $G(s)$, while the production function is $H(s)$ if it uses the old-fashioned technology. The total cost of skill is $c(s)$ on the market.

(a) What are sufficient conditions on G and H to ensure that profit-maximizing firms who use the computer-driven technology buy more skill than firms who use the old-fashioned technology? If we wish the result to hold without any structure on $c(s)$, is there a weaker condition which will do? (Hint: Define a function $f(s, \theta)$ where $f(s, \theta^H) = G(s)$ and $f(s, \theta^L) = H(s)$.)

(b) Suppose that the cost of the computer-driven technology falls smoothly over time, starting at a very high level and eventually reaching zero, and that firms choose their technology and level of skill at the same time. Assume whatever conditions you derived in part (a). Do we expect the skill level selected by the firms to change smoothly?