14.452: Economic Growth, Fall 2009
Problem Set 4

Instructor: Daron Acemoglu

Due date: December 4, 2009, in recitation.

Exercise 1: Consider the expanding input variety model of Section 13.1 of the textbook, with one difference. A firm that invents a new machine receives a patent, which expires at the Poisson rate ι. Once the patent expires, that machine is produced competitively and is supplied to final good producers at marginal cost.

1. Characterize the equilibrium in this case and show how the equilibrium growth rate depends on ι. [Hint: notice that there will be two different machine varieties supplied at different prices].
2. What is the value of ι that maximizes the equilibrium rate of economic growth?
3. Show that a policy of $\iota = 0$ does not necessarily maximize social welfare at time $t = 0$.

Exercise 2: Consider the following model. Population at time t is $L(t)$ and grows at the constant rate n (i.e., $\dot{L}(t) = nL(t)$). All agents have preferences given by

$$\int_0^\infty \exp(-\rho t) \frac{C(t)^{1-\theta} - 1}{1-\theta} dt,$$

where C is consumption defined over the final good produced as

$$Y(t) = \left(\int_0^{N(t)} y(\nu, t)^\beta \, d\nu \right)^{1/\beta},$$

where $y(\nu, t)$ is the amount of intermediate good ν used in production at time t and $N(t)$ is the number of intermediate goods at time t. The production function of each intermediate is $y(\nu, t) = l(\nu, t)$, where $l(\nu, t)$ is labor allocated to this good at time t. New goods are produced by allocating workers to R&D, with the production function $\dot{N}(t) = \eta N^\phi(t) L_R(t)$, where $\phi \leq 1$ and $L_R(t)$ is labor allocated to R&D at time t. Labor market clearing requires $\int_0^{N(t)} l(\nu, t) \, dv + L_R(t) = L(t)$. Risk-neutral firms hire workers for R&D. A firm who discovers a new good becomes the monopoly supplier, with a perfectly-enforced patent.

1. Characterize the BGP in the case where $\phi = 1$ and $n = 0$, and show that there are no transitional dynamics. Why is this? Why does the long-run growth rate depend on θ? Why does the growth rate depend on L? Do you find this plausible?

2. Now suppose that $\phi = 1$ and $n > 0$. What happens? Interpret.

3. Now characterize the BGP when $\phi < 1$ and $n > 0$. Does the growth rate depend on L? Does it depend on n? Why? Do you think that the configuration $\phi < 1$ and $n > 0$ is more plausible than the one with $\phi = 1$ and $n = 0$?

Exercise 3: Recall that, in the model of subsection 18.3.2 of the textbook, the world technology is endogenized with the following equation

$$N(t) = \frac{1}{J} \sum_{j=1}^{J} N_j(t),$$

which assumes that the contribution of each country to the world technology is the same. Instead of this equation, suppose that the world technology is given by

$$N(t) = G(N_1(t), ..., N_J(t)),$$

where G is increasing in all of its arguments and homogeneous of degree 1.

1. Generalize the results in Proposition 18.5 of the textbook to this case and derive an equation that determines the world growth rate implicitly.

2. Derive an explicit equation for the world growth rate for the specific case in which $N(t) = \max_j N_j(t)$. Interpret this result.