14.54 International Economics
Handout 4

Guido Lorenzoni

October 31, 2006

1 The Current Account

A country has GDP Y_t, consumption C_t and investment I_t. We can write the trade balance in a given period as

$$Y_t - C_t - I_t$$

where all variables are in dollars. This is the amount of dollars that enters the country to pay for goods.

The country assets are denoted by A_t and the liabilities by L_t.

Suppose assets and liabilities receive the same rate of return r_t. Each period $r_t A_t$ dollars enter the country to pay for the services of the assets that we hold abroad, and $r_t L_t$ exit the country to pay for the services of the assets that foreigners hold at home.

For now, we omit the public sector for simplicity. Then the net amount of dollars entering the country is:

$$CA_t = Y_t - C_t - I_t + r_t A_t - r_t L_t$$

this is the current account surplus.

The current account surplus must go to finance either the accumulation of assets or the decumulation of liabilities:

$$(A_{t+1} - A_t) - (L_{t+1} - L_t) = Y_t - C_t - I_t + r_t A_t - r_t L_t.$$

This is the basic *current account identity.*

Now we can decompose GDP into two parts: non-tradable goods and tradable home-produced goods:

$$Y_t = Y_t^{NT} + Y_t^H.$$

We can decompose consumption and investment in three parts:non-tradable goods, tradable home-produced goods and foreign-produced goods:

$$\begin{aligned} C_t &= C_t^{NT} + C_t^H + C_t^F, \\ I_t &= I_t^{NT} + I_t^H + I_t^F. \end{aligned}$$

For non-tradable goods the following accounting identity holds

$$Y_t^{NT} = C_t^{NT} + I_t^{NT},$$

this means that all non-tradables are used at home (this must be true by definition).

Then we can rewrite the trade balance as:

$$\begin{aligned}&\left(Y_t^H - C_t^H - I_t^H\right) - \left(C_t^F + I_t^F\right) = \\ &EXP_t - IMP_t\end{aligned}$$

and see that the trade balance is equal to exports minus imports.

2 A Two Period Model

Consider two countries, Home and Foreign. The world lasts two periods. There is one commodity, wheat. There are n identical consumers in country Home and n in country Foreign. Consumption of wheat by Home consumers in periods 1 and 2 is denoted c_1 and c_2 and will be determined in equilibrium. The preferences of consumers are described by the utility function

$$u(c_1) + \beta u(c_2), \quad \beta \in [0,1]$$

Consumption by Foreign consumers is denoted c_1^* and c_2^*. Their preferences are identical to Home consumer preferences.

Each consumer at Home owns a farm that produces a given amount of wheat in each period, y_1 and y_2. For the Foreign country we have y_1^* and y_2^*. Wheat cannot be stored. Wheat can be transported at no cost from country Home to Foreign.

The countries can borrow and lend from each other on the international capital market at the interest rate r.

Let b denote the savings of home consumers. Consumers in country Home determine their optimal savings solving

$$\begin{aligned} & \max u(c_1) + \beta u(c_2) \\ s.t.\ b &= y_1 - c_1, \\ c_2 &= y_2 + (1+r)\,b. \end{aligned}$$

If $b < 0$ the consumers are net borrowers, if $b > 0$ they are net lenders.

The budget constraints in periods 1 and 2 can be combined to give the *intertemporal budget constraint*

$$c_1 + \frac{1}{1+r}c_2 = y_1 + \frac{1}{1+r}y_2.$$

Consumer Choice

Consider the case where

$$u(c) = \log c.$$

Then optimal consumption entails

$$\begin{aligned} c_1 &= \frac{1}{1+\beta}\left(y_1 + \frac{1}{1+r}y_2\right), \\ c_2 &= \frac{\beta(1+r)}{1+\beta}\left(y_1 + \frac{1}{1+r}y_2\right). \end{aligned}$$

Consumers take their total lifetime income, in net present value,

$$y_1 + \frac{1}{1+r}y_2,$$

and spend a fraction $1/(1+\beta)$ in the first period of their life and a fraction $\beta/(1+\beta)$ in the second period of their life.

Autarky (Closed Financial Markets)

Market clearing requires

$$b = 0,$$

which implies

$$\begin{aligned} c_1 &= y_1, \\ c_2 &= y_2. \end{aligned}$$

The savings of home country are

$$\begin{aligned} b(r) &= y_1 - c_1 = \\ &= \frac{1}{1+\beta}\left[\beta y_1 - \frac{1}{1+r}y_2\right]. \end{aligned}$$

So if we set $b = 0$ we obtain the following condition:

$$\frac{y_2}{y_1} = \beta(1 + r^a)$$

Market Equilibrium

Put together the consumption choice of Home and Foreign consumers and write the market clearing condition for wheat in period 1

$$b(r) + b^*(r) = 0.$$

Which is the same as:

$$CA_1 + CA_1^* = 0$$

(since they start with zero assets the current account is equal to the assets accumulated at the end of period 1).

You obtain

$$\begin{aligned} b(r) &= -b^*(r) \\ y_1 - \frac{1}{1+\beta}\left(y_1 + \frac{1}{1+r}y_2\right) &= -\left[y_1^* - \frac{1}{1+\beta}\left(y_1^* + \frac{1}{1+r}y_2^*\right)\right] \end{aligned}$$

Let y_t^w denote the world per capita endowment of wheat, that is

$$y_t^w = \frac{1}{2}\left(y_t + y_t^*\right),$$

and let

$$1 + g = \frac{y_2^w}{y_1^w}$$

be the world groth rate.

Then we obtain

$$\frac{1}{1+\beta}\left(y_1^w + \frac{1}{1+r}y_2^w\right) = y_1^w,$$

and finally we obtain the equilibrium interest rate on the world market

$$1 + r = \frac{1}{\beta}(1+g).$$

The equilibrium interest rate *does not* depend on the income profile of individual countries, only on the common discount factor β and on the world-wide growth rate g.

Graphically:

Implications

Suppose

$$y_1 < \frac{1}{1+\beta}\left(y_1 + \frac{\beta}{1+g}y_2\right),$$

or

$$\frac{y_2}{y_1} > 1 + g.$$

Then the home country will borrow in period 1 and repay in period 2.

Proof:

$$\frac{c_1}{y_1} = \frac{1}{1+\beta}\left(1 + \frac{1}{1+r}\frac{y_2}{y_1}\right) > \frac{1}{1+\beta}\left(1 + \frac{1+g}{1+r}\right) = 1$$

and we obtain

$$\frac{c_1}{y_1} > 1 \Rightarrow b < 0$$

- The country growing faster borrows from the country growing slower, to finance early consumption.

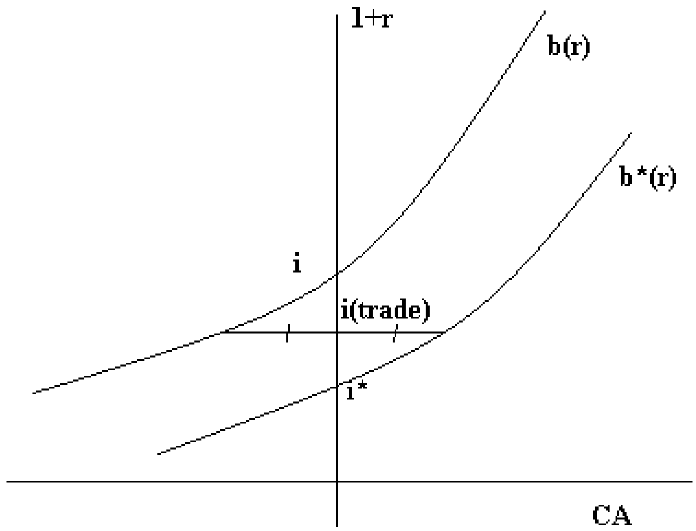

Figure 1:

Notice that writing

$$\frac{y_2}{y_1} > 1 + g$$

is the same as writing

$$y_1 < \frac{1}{1+\beta}\left(y_1 + \beta \frac{y_2}{1+g}\right)$$

the right hand side is a the net present value of income at the equilibrium world interest rate. We call it "permanent income." So this means that:

- The country with income in period 1 lower than its permanent income borrows from the country with income in period 1 higher than its permanent income.

The growth rate of consumption will be equal for the two countries and equal to the world growth rate. You can get it straight from the first order condition

$$\frac{1}{c_1} = \beta \left(1 + r\right) \frac{1}{c_2}$$

and from the equilibrium interest rate. This gives us

$$\frac{y_2^*}{y_1^*} < \frac{c_2^*}{c_1^*} = 1 + g = \frac{c_2}{c_1} < \frac{y_2}{y_1}$$

- International capital markets induce a more similar consumption path across countries, i.e. they make the path of consumption less responsive to local changes in output and more responsive to world changes in output.

Exercise 1 *Suppose that the two countries have different discount rates $\beta \neq \beta^*$.*
(i) Derive the equation that determines market clearing in the goods market at date 1;
(ii) Derive the equilibrium interest rate in the case $y_1 = y_1^* = y_2 = y_2^*$;
(iii) Derive the pattern of borrowing and lending in this case;
(iv) In the general case show that if $\frac{1}{\beta^*}\frac{y_2^*}{y_1^*} < \frac{1}{\beta}\frac{y_2}{y_1}$ the equilibrium interest rate has to satisfy

$$\frac{1}{\beta^*}\frac{y_2^*}{y_1^*} < 1 + r < \frac{1}{\beta}\frac{y_2}{y_1}$$

(Hint: a graphical argument should work fine).