2 Different productivity (the Ricardian model)

Two goods, computers and textiles (t-shirts) C and T.

Production in the home country:

$$y_C = \pi_C l_C$$
$$y_T = \pi_T l_T$$

Home consumers/workers have a total endowment of working hours L.

Foreign consumers/workers have L^*. Their productivity is π_C^* and π_T^*.

Assumption:

$$\frac{\pi_C}{\pi_T} > \frac{\pi_C^*}{\pi_T^*}$$

Home has a comparative advantage in the production of computers.

Example: productivities, π, are:

productivity	C	T
home	3	2
foreign	1	1

Suppose labor is allocated as follows:

labor	C	T
home	10	10
foreign	10	10

then output is

output	C	T
home	30	20
foreign	10	10

Now suppose you transfer labor from T to C at home and from C to T in the foreign country and you reach:

labor	C	T
home	15	5
foreign	0	20

Then output is:

output	C	T
home	45	10
foreign	0	20

The world output of textiles is the same, 30, the output of computers has increased from 40 to 45.

Now we turn to market arrangements.

2.1 Demand

Suppose preferences are such that the demand for the two goods is

$$
\begin{aligned}
x_C &= \alpha \frac{wL}{p_C} \\
x_T &= (1-\alpha) \frac{wL}{p_T}
\end{aligned}
$$

the workers spend a fixed fraction of their income in the two goods. Same for the foreign consumers.

Exercise: show that when preferences are

$$U(x_C, x_T) = \alpha \log x_C + (1-\alpha) \log x_T$$

then the demand functions are exactly like that.

2.2 Production

A producer of computers in the home country makes profits:

$$
\begin{gathered}
p_C y_C - w l_C = \\
p_C \pi_C l_C - w l_C
\end{gathered}
$$

so if

$$p_C \pi_C > w$$

then ther is ∞ demand of labor by the computer sector and we cannot have equilibrium on the labor market, if

$$p_C \pi_C < w$$

there is 0 demand of labor by the computer sector, if

$$p_C \pi_C = w$$

then the computer sector is willing to absorb any amount of workers l_C.

Same for other sector, other country.

2.3 Autarky

Now suppose country Home is in autarky.

We need to find prices (p_C, p_T, w) such that the labor market is in equilibrium

$$l_C + l_T = L$$

the goods markets are in equilibrium

$$\begin{aligned} x_C &= y_C \\ x_T &= y_T \end{aligned}$$

and firms and consumers are at their optimal choices.

Equilibrium: consumers consume both computers and t-shirts and prices are

$$\begin{aligned} p_C^a &= \frac{w}{\pi_C}, \\ p_T^a &= \frac{w}{\pi_T}. \end{aligned}$$

What is the demand for the two goods?

$$\begin{aligned} x_C &= \alpha \frac{w}{p_C^a} = \alpha \pi_C \\ x_T &= (1-\alpha) \frac{w}{p_T^a} = (1-\alpha) \pi_T \end{aligned}$$

What is the labor demanded in the two sectors?

Labor inputs in the two sectors

$$\begin{aligned} l_C &= \frac{y_C}{\pi_C} = \alpha \frac{w}{p_C^a} \frac{L}{\pi_C} = \alpha L > 0 \\ l_T &= \frac{y_T}{\pi_T} = (1-\alpha) \frac{w}{p_T^a} \frac{L}{\pi_T} = (1-\alpha) L > 0 \end{aligned}$$

and equilibrium in labor market is satisfied because

$$l_C + l_T = L.$$

Equilibrium relative prices are:

$$\frac{p_C^a}{p_T^a} = \frac{1/\pi_C}{1/\pi_T} = \frac{a_C}{a_T}.$$

Proportional to the labor requirements a_C and a_T.

That's good but... how did we find the equilibrium?

Graphical approach: Relative demand and relative supply (as in KO Fig. 2-3, but in the autarky case for now).

Relative Supply

Supply of the two goods. Just from firms optimal behavior and equilibrium in the labor market.

$$y_C = \begin{cases} 0 & \text{if } \frac{p_C}{p_T} < \frac{1/\pi_C}{1/\pi_T} \\ [0, \pi_C L] & \text{if } \frac{p_C}{p_T} = \frac{1/\pi_C}{1/\pi_T} \\ \pi_C L & \text{if } \frac{p_C}{p_T} > \frac{1/\pi_C}{1/\pi_T} \end{cases} \tag{3}$$

$$y_T = \begin{cases} \pi_T L & \text{if } \frac{p_C}{p_T} < \frac{1/\pi_C}{1/\pi_T} \\ [0, \pi_T L] & \text{if } \frac{p_C}{p_T} = \frac{1/\pi_C}{1/\pi_T} \\ 0 & \text{if } \frac{p_C}{p_T} > \frac{1/\pi_C}{1/\pi_T} \end{cases} \tag{4}$$

Putting them together we get

$$\frac{y_C}{y_T} = \begin{cases} 0 & \text{if } \frac{p_C}{p_T} < \frac{1/\pi_C}{1/\pi_T} \\ [0, \infty) & \text{if } \frac{p_C}{p_T} = \frac{1/\pi_C}{1/\pi_T} \\ \infty & \text{if } \frac{p_C}{p_T} > \frac{1/\pi_C}{1/\pi_T} \end{cases}$$

This relations already incorporate equilibrium in the labor market.

For example, consider the case $\frac{p_C}{p_T} < \frac{1/\pi_C}{1/\pi_T}$. This means that

$$p_C \pi_C < p_T \pi_T$$

suppose that $w < p_T \pi_T$, then there would be ∞ demand of labor by the textile industry and no equilibrium is possible. Suppose that $w > p_T \pi_T$, then both sectors would demand 0 labor and no equilibrium is possible. Then we need $w = p_T \pi_T$, at this wage the computer sector is inactive and all labor must be employed by the textile sector. Therefore in this case $y_C = 0$ and $y_T = \pi_T L$.

Exercise: go through same reasoning for the other cases.

Relative Demand

Remember the two demand curves

$$\begin{aligned} x_C &= \alpha \frac{wL}{p_C} \\ x_T &= (1 - \alpha) \frac{wL}{p_T} \end{aligned}$$

and we get the relative demand

$$\frac{x_C}{x_T} = \frac{\alpha}{1 - \alpha} \frac{1}{\frac{p_C}{p_T}}.$$

Nice property: the relative demand does not depend on the wages, so we don't need to worry about that.

Equilibrium

$$\frac{y_C}{y_T} = \frac{x_C}{x_T}$$

you can show that only equilibrium is

$$\frac{p_C^a}{p_T^a} = \frac{1/\pi_C}{1/\pi_T}.$$

2.4 Trade

Now we allow them to trade. We get total supply of the two countries.

Remember that

$$\frac{1/\pi_C}{1/\pi_T} < \frac{1/\pi_C^*}{1/\pi_T^*}$$

Total Relative Supply

$$\frac{y_C + y_C^*}{y_T + y_T^*} = \begin{cases} 0 & \text{if } \frac{p_C}{p_T} < \frac{1/\pi_C}{1/\pi_T} \\ [0, \frac{\pi_C L}{\pi_T L^*}] & \text{if } \frac{p_C}{p_T} = \frac{1/\pi_C}{1/\pi_T} \\ \frac{\pi_C L}{\pi_T L^*} & \text{if } \frac{1/\pi_C}{1/\pi_T} < \frac{p_C}{p_T} < \frac{1/\pi_C^*}{1/\pi_T^*} \\ \infty & \text{if } \frac{p_C}{p_T} > \frac{1/\pi_C^*}{1/\pi_T^*} \end{cases}$$

(Exercise: redo the steps to derive this relation like in class.)

Total Relative Demand is just:

$$\frac{x_C + x_C^*}{x_T + x_T^*} = \frac{\alpha}{1-\alpha} \frac{1}{p_C/p_T}.$$

World equilibrium: See Figure 2-3 in KO and discussion there.

2.5 Gains from trade

What happens to wages?

Real wages: in terms of the two goods.

$$\frac{w}{p_T} \geq \frac{w^a}{p_T^a}$$

we know that

$$w^a = p_T^a \pi_T$$

so

$$\frac{w^a}{p_T^a} = \pi_T.$$

Now we know that

$$w \geq p_T \pi_T$$

so

$$\frac{w}{p_T} \geq \pi = \frac{w^a}{p_T^a}.$$

Exercise: show that

$$\frac{w}{p_C} \geq \frac{w^a}{p_C^a}.$$

Show that in the case of full specialization these inequalities are strict, the workers are strictly better off under trade.