114.126 (Game Theory) Final Examination

Instructions: This is an open-book exam – you may consult written material but you may not consult other humans. There are five questions, weighted equally. You have 24 hours to complete the exam from the time you first open the envelope. When you have finished, place your answers in the envelope and return it to Muhamet Yildiz.

Please begin your answer to each question on a new page.

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1. This question is about two players trying to divide a dollar. The set of feasible alternatives is $X = \{(x, 1-x) \,|x \in [0,1]\}$. If bargaining breaks down, each gets 0. Each agent $i \in \{1,2\}$ has a concave, continuous, and increasing utility function $u_i : \mathbb{R} \to \mathbb{R}$ with $u_i(0) = 0$. We are interested in how the bargaining outcome changes if the utility function of Player 1 becomes $\hat{u}$ such that $\hat{u}(x) \equiv (u_i(x))^\alpha$ for some $\alpha \in (0,1)$ — when Player 1 becomes more risk averse.

   1. Let x^{NS} and $\hat{x}^{NS}$ be the shares of Player 1 at the Nash bargaining solutions to the bargaining problems $\left(0, \left\{v \in \mathbb{R}^2_+ | \exists x : v \le (u_1(x), u_2(1-x))\right\}\right)$ and $\left(0, \left\{v \in \mathbb{R}^2_+ | \exists x : v \le (\hat{u}(x), u_2(1-x))\right\}\right)$, respectively. Show that
   $$x^{NS} \le \hat{x}^{NS}.$$
   2. Given any common discount rate δ, let x^R be the share of Player 1 in the unique subgame-perfect equilibrium of Rubinstein's (1982) model with alternating offers, where an agent i's payoff from getting share x at time t is $\delta^t u_i(x)$. Define $\hat{x}^R$ similarly for the case when Player 1's utility function is $\hat{u}$. Show that
   $$x^R \le \hat{x}^R.$$
   3. Briefly discuss these results in the context of risk aversion. Explain the result in (b) in terms impatience.

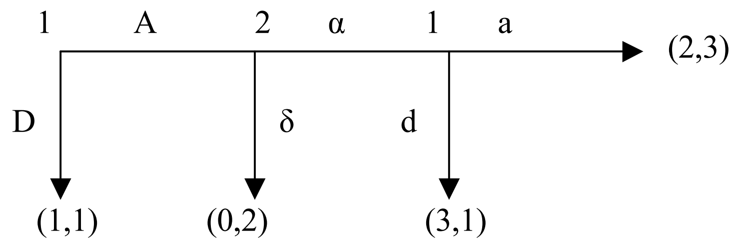
2. Let X, T, and $X \times T$ be complete lattices.

   1. Let $f : X \times T \to X$ be isotone, and $\bar{x}(t)$ be the highest fixed point of $f(\cdot, t)$ for each t. Show that $\bar{x}$ is isotone. [Hint: $\bar{x}(t) = \sup\{x | f(x,t) \ge x\}$.]

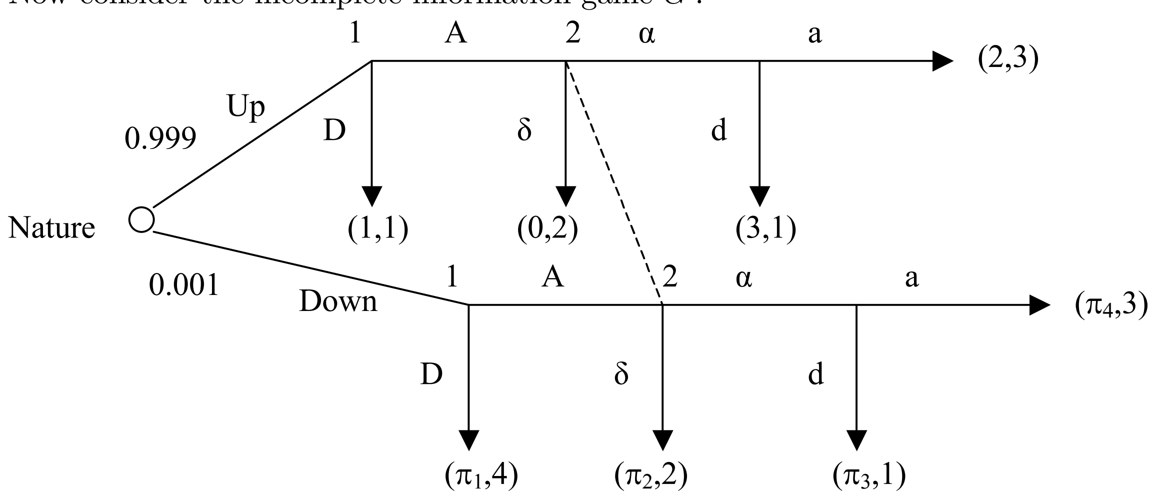
2. Let $B_n(x,t)$ be the largest best reply to x_{-n} for each n in a game G_t with a generic strategy profile $x = (x_1, \ldots, x_N)$. Let also $B(x,t) = (B_1(x,t), \ldots, B_N(x,t))$. Let $\bar{x}(t)$ be the highest Nash equilibrium of G_t. Show (LeChatelier) that, if $t \geq t'$, then

$$\bar{x}(t') \geq B(\bar{x}(t), t').$$

3. Consider the following centipede game G:



   1. Compute all rationalizable pure-strategies.
   2. Now consider the incomplete-information game G':



   Find values of $(\pi_1, \ldots, \pi_4)$ such that, in any sequential equilibrium of G', the set of all terminal nodes that are reached with positive probability conditional on that Nature plays Up is the same as the set of all terminal nodes that are reached at some pure, rationalizable strategy-profile of G.

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