14.126 GAME THEORY

PROBLEM SET 1

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Due by Wednesday, February 24, 5pm

Question 1

Consider the following game. Each of 15 students simultaneously announces a number in the set $\{1, 2, \ldots, 100\}$. A prize of \$1 is split equally between all students whose number is closest to 1/3 of the class average.

(a) Compute the sequence of sets of pure strategies $S_i^1, S_i^2, \ldots$ defining the process of iterated elimination of strictly dominated strategies for every student i.

(b) Show that the game has a unique Nash equilibrium (in pure or mixed strategies).

Question 2

Provide an example of a 2-player game with strategy set $[0, \infty)$ for either player and payoffs continuous in the strategy profile, such that no strategy survives iterated deletion of strictly dominated strategies ($S^\infty = \emptyset$), but the set of strategies remaining at every stage is nonempty ($S^k \neq \emptyset$ for $k = 1, 2, \ldots$).

Question 3

In the normal form game below player 1 chooses rows, player 2 chooses columns, and

A

	L	R
U	9	0
D	0	0

B

	L	R
U	0	9
D	9	0

C

	L	R
U	0	0
D	0	9

D

	L	R
U	6	0
D	0	6

player 3 chooses matrices. We only indicate player 3's payoff. Show that action D is not a best response for player 3 to any independent belief about opponents' play (mixed strategy for players 1 and 2), but that D is not strictly dominated. Comment.

Question 4

A game (N, S, u) with $N = \{1, 2, \ldots, n\}$ $(n < \infty)$ is symmetric if $S_i = S_j$ for all $i, j \in N$ and $u_i(s_1, \ldots, s_n) = u_{\pi(i)}(s_{\pi(1)}, \ldots, s_{\pi(n)})$ for every $i \in N$ and all permutations π of N. A (mixed) strategy profile σ is symmetric if $\sigma_i = \sigma_j$ for all $i, j \in N$. A symmetric Nash equilibrium is a Nash equilibrium in symmetric strategies.

(a) Prove that every symmetric game has a symmetric Nash equilibrium.

(b) For some compact set X, consider a family of symmetric games $(N, S, u^x)_{x \in X}$, where $u_i^x(s)$ is continuous in x (over X) for every $i \in N$. Show that the correspondence that maps each $x \in X$ to the set of symmetric Nash equilibria of (N, S, u^x) is upper-hemicontinuous.

Question 5

Each of two players $i = 1, 2$ receives a ticket with a number drawn from a finite set Θ_i. The number written on a player's ticket represents the size of a prize he may receive. The two prizes are drawn independently, with the value on i's ticket distributed according to F_i. Each player is asked simultaneously (and independently) whether he wants to exchange his ticket for the other player's ticket. If both players agree then the prizes are exchanged; otherwise each player receives his own prize. Find all Bayesian Nash equilibria (in pure or mixed strategies).

Question 6

Consider the infinitely repeated prisoners' dilemma with stage payoffs given below. Assume

	C	D
C	$1, 1$	$-1, 2$
D	$2, -1$	$0, 0$

that both players discount payoffs by δ. The "tit-for-tat" strategy is formulated as follows.

Start out by playing C. After that, choose the action that the other player used in the previous period. For what values of $\delta \in (0, 1)$ does the strategy profile where both players use tit-for-tat constitute a subgame perfect equilibrium?

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