EXPERIENCING MATHEMATICAL *PROVES*

SYNTAX OF AN ASTROLABE

BY

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SUBMITTED TO THE DEPARTMENT OF ARCHITECTURE
IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE DEGREE OF

MASTER OF SCIENCE IN ARCHITECTURE STUDIES

AT THE

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

JUNE 2016

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ABSTRACT

The goal of thesis is discussing the way historical scientific instruments are exhibited in Art or Science Museums. The astrolabe and the related mathematical theories, as developed in the Arabic and Persian tradition between X-XI Century, are taken as emblematic case for this analysis. The proposed solution is the design of museum spaces which translate the language of this instruments through the syntax of the space itself.
The debate has its premise in Benjamin' concept of historical experience which is essential not only for clarifying our approach to the discipline of History of Science but it is also a pivotal point for addressing the question of how we can understand these objects. A historical scientific instrument is the by-product of the scientific knowledge of a specific time and place. It is a synthesis, a representation which concentrate the plurality/multiplicity of knowledge in the materiality of one object, it is the *picture* of Benjamin's *Concept of History*. The knowledge the astrolabe embeds is the scientific knowledge of the Arabic and Persian mathematicians of X-XI century and its construction is a tangible proof of the exactness of mathematical theorems it relies on. Hence, the language of this object has to be the language of mathematics. Its terms and primitives compose the grammar of the axiomatic method (derived from Euclid) and the proof is the syntax of this linguistic system. The design proposes a three-dimensional version of mathematical proofs of some of the theorems used for the construction and functioning of the astrolabe. It is an attempt of bringing the proof from the two-dimension of the paper to the three-dimension of the visitor in order to provide him an experience that is the spatial experience of a proof brought in his three-dimension. The architecture visualize the process of reasoning of the mathematicians by creating a space that looks like a sketch. The sketch is tool we use for visualizing our process of reasoning, hence the design has to follow the "rules" of sketching and materialize its lines.

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ACKNOWLEDGEMENTS

FIRST AND FOREMOST, I WOULD LIKE TO THANK ALL PEOPLE I SHARED MY RESEARCH WITH, STUDENTS, PROFESSORS, FRIENDS AND PEOPLE THAT I BARELY KNEW. MY DESIGN IS THE RESULT OF AN ENDLESS COMBINATION OF THEIR SUGGESTIONS AND THEIR CRITIQUES.

I WOULD ALSO LIKE TO ACKNOWLEDGE MY COMMITTEE. AZRA AKSAMIJA, FOR THE CREATIVITY AND THE ENCOURAGEMENT; GEORGE STINY, FOR THE CONCISENESS AND HIS ABILITY OF UNDERSTAND ME IN TEN SECONDS; ROB SIMCOE, FOR THE BELIEVING IN MY PROJECT SINCE THE VERY BEGINNING AND FOR BEING ABLE TO ENVISION MORE THAN ME.
A SPECIAL THANKS TO ELISABETH CAVICCHI BECAUSE SHE FOLLOWED ME IN THIS TWO YEARS, SHE LISTEN TO ALL MY INFINITE BRAIN STORMING ABOUT THIS PROJECT; THANKS BECAUSE HER ENTHUSIASM AND CURIOSITY HAVE BEEN INSPIRING FOR ME.

I WOULD LIKE TO THANK IRMAK TURAN, FOR BEING ALWAYS PRESENT IN THESE TWO YEARS, FOR SHARING WITH ME MOMENTS OF HAPPINESS AND MOMENTS OF CONFUSION. FOR BEING MY FRIEND, AND THIS WORD IS ENOUGH.

THANKS TO SON JOOHUI FOR BEING MY WISE "BIG BROTHER" DURING THESE TWO YEARS; TO CHO BUMSUK FOR ALL THE CIGARETTES WE SMOKED TOGETHER DISCUSSING ABOUT ARCHITECTURE AND FOR SHARING WITH ME HIS LOVE FOR THE OPERA.

LAST BUT NOT LEAST, THANK YOU TO PIERRE CUVILLIERS FOR HIS OMNIPRESENT PROFESSIONAL HELP AND FOR OUR IMPROVISED DINNERS AND OUR LONG CHATS. AND THANK YOU TO RENAUD DENHAIVE FOR HIS SWEETNESS, FOR OUR LIVELY DISCUSSIONS AND FOR BEING ABLE TO CHEER ME UP ANY TIME I NEEDED. THANKS TO BOTH FOR MAKING THIS LAST SEMESTER SIMPLY AMAZING.

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INTRODUCTION

Knowledge stems from observing and questioning. "Logic is the very foundation upon which all human knowledge rests"[1].

This is an effective synthesis of the essence of the concept of science for the Islamic scholars during the so-called Golden Age of Islamic Science (X-XV century). Answering is not a fundamental step in the process of learning; the main achievement a man should aim is being able to develop a process of understanding based on what al-Ghazzali defines "the necessary stepping stone to eternal bliss"[2], namely logic.

This statement implies that, according to Islamic *scientific method*[3], knowledge has to be achieved through a consequential logical process in which nothing can remain not proved, or better, nothing can remain not questioned.

All the Islamic scientific production during this period is based on this concept and their consistent outcome in the field of mathematics [4] is a direct consequence of their "scientific method".

Leading a research assuming that any statement has to be doubted and then proved following a consequential chain of reasoning encourages them to find a way to make their result clear and especially repeatable. Namely, anyone would like to re-check the result can follow the logic consequentiality of the mathematical demonstration which was based on the Euclidean axiomatic

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[1] F. ROSHENTAL, p.205

[2] *Ibidem*, quoting Al-Ghazzali, p.206.

[3] The term is historically not applicable to the pre-modern Islamic tradition, although in this paper we are going to cast some doubts on this idea.

[4] Mathematics as a discipline included geometry and astronomy.

method and anyone would like to have a tangible proof of a theorem can use a scientific instruments which has been built according to the theorems axiomatically proved.

The repeatability of a measurement is the base of the scientific intellectual honesty. A scientific community, such as the Arabic and Persian community of the X-IV century, which claims as an assumption of its research method that the exactitude of their reasoning is based on a thorough and constant process of proving cannot help but rely on the use of instruments.

The Arabic and Persian community between the X and the XIV century not only produces and impressive amount of scientific instruments but also developed a consistent *corpus* of theories on the construction of these instruments.

A large part of these instruments is currently preserved and exhibited in Art or Science museums as a tangible or visible synthesis of how they developed their "scientific method".

Herein it is where the past communicate us its story and where we listen. Herein it is where we learn from the past.

Here it is where my inquiry starts and where I met my limits and the limits of our communication strategy, where I challenged those limits and I followed my curiosity. Where I asked to myself if and how an architect can play her role in this convoluted process of past/present communication.

PREMISE

The way historical scientific instruments are nowadays exhibited in the art museums or history of science museums entails a descriptive approach to historical objects. Highlighting aesthetic, chronology or craft of these objects (policy of art museums) or explaining their functioning (policy of science museums) is a procedure that refers to a vision of history fixed in time, a historicist method that is constraining for scientific objects. Museums' historicism describes the past "how it really was"[5] ignoring that, in order to understand a historical scientific instrument, "communicate" with it and translate its language we have to dialectically study how and what made aesthetic or functioning possible or necessary: what is the frame of their formation.

The frame is the *apparatus* of knowledge behind the object and its evolution in the context of history of science. Therefore, the following questions are 1.how can we read this *apparatus*, 2.how can we use museums for making it legible, and 3.what will be the means for doing this? The answer, by recreating a historical experience.

Therefore, the role of the museum should be materializing a historical experience by making this *apparatus* of knowledge embedded in the instrument legible.

Hence, the first step has to be defining what is a historical experience.

[5] W. Benjamin, 1950, VI.

1. THEORY

1.1 PHILOSOPHICAL FRAMEWORK. CREATING AN HISTORICAL EXPERIENCE

First and foremost, the Jetztzeit[6] as *locus* of the historical construction. The present (Jetzt) is the place where History stands as a synthesis and where the historian depicts «an experience with it, which stands alone»[7].

In Benjamin's *On the Concept of History*[8], experience is a meeting between past and present where one renounces to the appropriation of the other; the two terms live in a mutual dimension of distance and proximity where none of them prevails. Hence, the role of the subject, as observer of the past plunged in the perspective of the present, is attenuated by the experience because the subject constructs himself through the experience and is no longer a premise to it.

According to this statement, the past does not enlighten the present, but it is to be held fast «only as a picture, which flashes its final farewell in the moment of its recognizability». Through this picture what has been (the past) join the Jetzt (the here-and-now, the present) creating a constellation, an *unicum* composed by a system of points which have different coordinates in space and time but which come together as a projection on the plane of the curved sky observed by the subject. Missing one of this points would affect the image

[6] Literally, "the time of the present". Walter Benjamin uses this term in the Concept of History (XIV) for comparing the homogeneous and empty time of the historicist and the here-and-now of the historical materialist.
[7] W. Benjamin, 1950, XVI.
[8] As published posthumous in 1950.

of the constellation, that is, the past will disappear as soon as the present will not «recognize itself as meant in it[9]».

As a consequence, the picture can be defined as a dialectic image happening in an instant of time, it is what Benjamin calls the *dialectic of immobility* that we experience dealing with historical objects.

The introduction of Benjamin's discourse in the debate on historical scientific instruments, and in the larger field of History of Science, is not only fundamental for clarifying our approach to the discipline but it is also a pivotal point for addressing the question of how we can understand these objects.

A historical scientific instrument is the by-product of the scientific knowledge of a specific time and place. It is a synthesis which concentrate the plurality and multiplicity of knowledge in the materiality of one object, it is the picture of Benjamin's *On the Concept of History*. Moreover, science itself is dialectic, in the literal sense of being able to διά-λέγειν[10], to be a diachronic infinitesimal addition and revision of concepts which, at a specific time, ends up in a synthesis called invention/discovery. This latter is not an epiphany but it is the result of an always ongoing relation between former scientific studies and current researches.

If we follow this reading of historical instruments, it becomes clear that, in order to understand them we have to preserve a dialectic approach to the past and also keep in mind that, if we consider the instrument as a physical

[9] *Ibidem*, V.

[10] From διά , *through* which defines its phenomenological-diachronic path, rejecting any stasis of the formal-logic objectification and λέγειν, *collect, gather, pick up* related with the intention of dialectic method of systematizing by moving from the analytic distinctions of multiplicity to the unification of categories of ideas.

correspondent of Benjamin's picture, its importance has to be something which provide us a dialectic view on the present/past relation. This cannot help but being the scientific knowledge that made its construction possible.

In order to analyze this issue into details I decided to consider the astrolabe, as developed in the XI-XII century in the Arabic and Persian tradition, an emblematic case of study due to the fact that it is one of the main example of this misinterpretation of object's language and failure in communication that both museum of art and science share.

In order to clarify why we can talk about a failure in communication, I will briefly explain what is an astrolabe, how it is composed and what intuitively should be the best way to observe or understand it.

1.2 CASE OF STUDY. THE ASTROLABE

1.2.1 FROM A THREE-DIMENSIONAL MODEL OF THE UNIVERSE TO A BI-DIMENSIONAL SURFACE.

Our understanding of scientific tools is theoretically undermined by the compelling necessity of the use and hence, even though instruments are a material transposition of a theories, when and if they achieve the goal of performing their own function, from the perspective of the user become pure objects and no longer vehicles of explaining, performing and proving physical or mathematical concepts.

Briefly, we use a clock only for the necessity of knowing what time is it and we utterly ignore how it gives us this information.

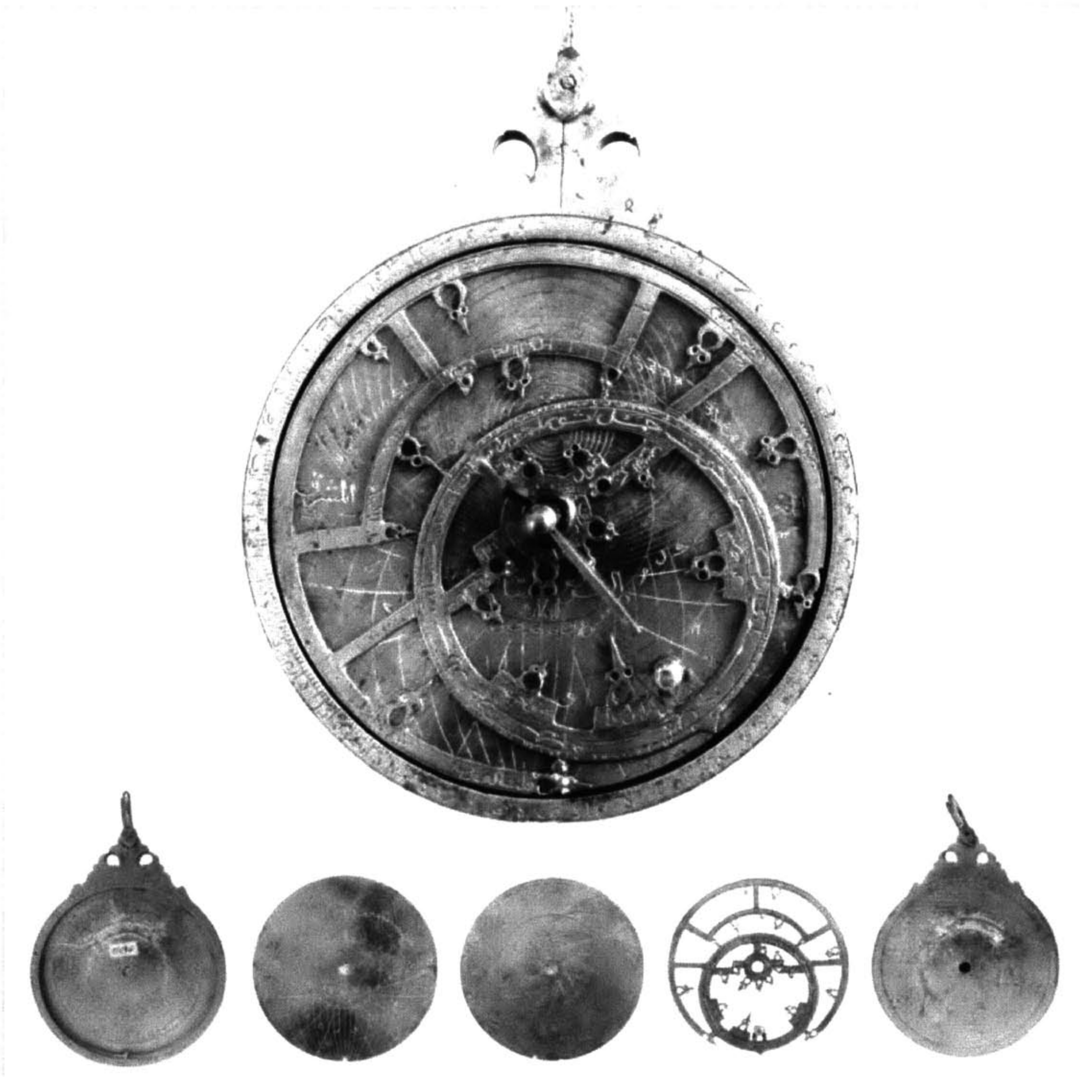
Nevertheless, in this case the straightforward language of mechanic supplies to our negligence reconnecting principle and function. On the contrary, when we use a mathematical instrument we would never be able to intuitively understand it. The abstraction of mathematics prevents us to visually connect what we see with what it means.

The gap between physical representation and theoretical construction is truly consistent, insomuch as it not only obstacle our understanding of the instruments but also our possibility to use it. Namely, when we approach to a mathematical instrument, our comprehension, even of its use, is not intuitive (as in mechanical instruments) but it requires a deepened analysis and a previous knowledge of the concepts behind its functioning.

This is a feature common to all the Medieval Islamic instruments, in contrast with the mechanical devices, and this is the reason why in order to answer to our general question (*how can we display a scientific instrument? What is its language and how it "communicates" with us*) is more useful to solve our problem with a mathematical approach: we need to build a frame of reference, to make our assumptions and to use a standard case in order to demonstrate our thesis.

Therefore, we chose the chief of the astronomical and also mathematical instruments used in the Medieval Islam and which embodies, in our opinion, all the contradiction on which we have reflected above: the astrolabe.

We start by examining its approach as mean of communication for science (frame of reference), we briefly define its function and how it works (assumptions) and finally the graphically explain its geometrical construction (demonstration) in order to show where lies its inner contradiction and where how it explains its hypertext character.



Persian Astrolabe of Harvard Collection of Scientific Instruments (photo by Francesca Liuni).

The astrolabe is *de facto* a radical example of this disconnection: between the phase of using and the phase of making (seen as thinking the object) there is a factual disproportion. It cannot be ascribed in the lists of machinery, neither in the list of tools or devices. Even ifs it performs a measure, and at this extent it literally satisfies its role of instrument, it does not "perform its knowledge". At the first approach with the instruments, the apparently confused interpolation of numerous curves on its surface is not able to intuitively communicate the theoretical apparatus behind it. If we observe an astrolabe, assuming that we know that it is used for measuring the altitude of celestial objects and that we also know how to do it, we cannot understand that it is a plane representation of the Celestial Sphere and how this sphere works.

Herein lies its inner contradiction: its complexity, aimed to make it an extremely precise measurer, deceives it preventing its ability to communicate. Metaphorically, it is a class board shown to blind students.

Indeed, the astrolabe is officially a tool/instrument/device whose aim should be simplify and make accessible to everyone its knowledge, or better, explain it to whom want to learn.

An instruments that contradicts its own nature of instrument as democratic simplifier, can be still defined instrument?

From a theoretical perspective the astrolabe is a unique and from an epistemological perspective it belongs to the family of computers, a not simple simplifiers which encapsulates different level of information arranged as a hypertext.

We could easily object that there is a great deal of instruments in the history of science which can be listed in this category. Therefore, to be more precise, what makes the astrolabe unique is not its computational approach to knowledge but rather the fact that it uses this type of approach in the X century[11].

The main achievement of this instrument is *de facto* its being precursor of time, more than its construction and its methodology to explain embodied theories is not its only cutting-edge feature.

Indeed, its inner conflict seems to be a possible reflection of an ongoing contrast in the field of Islamic astronomy in the Middle-Age: the systematization of mathematics as independent discipline and its consequent separation from observational astronomy.

If we consider that "theories express knowledge through the descriptive and argumentative functions of language" and "instruments express knowledge both through the representational possibilities that materials offer and through the instrumental functions they deploy"[12], we can easily notice that the astrolabe express its knowledge by using both methods. Knowing that, in the first definition, the *descriptive function* are the one used by mathematics and the *argumentative* ones can be referred to the language of geometry, we can try to suggest what generates this contradiction: the astrolabe acts as an instrument satisfying its function through assemblage of movable copper pieces, but it uses the synthetic abstractions of mathematics and geometry to explain its knowledge.

[11] The astrolabe were officially "invented" in Greece before this century. However the advancement that it reaches during the Islamic period make our analysis possible.
[12] D. Baird, *Thing knowledge*, p.131.

A visual demonstration of this double identity becomes clear in observing its construction. For this reason the last step of our study is explain the geometrical construction used to trace the curves on the surface of the astrolabe. All the drawings have been developed trying to connect three-dimension and bi-dimension in order to understand the connection between how they visualize the Universe, how they use the stereographic projection as mean of construction and to what this *apparatus* corresponds on the surface of the astrolabe. Moreover, I have added some three-dimensional reproduction of the conic projection which can help the reader to intuitively understand the geometry.

In order to clarify our analysis, we are going to describe the tympanum's surface (PLATE 2 and 3) and the rete's surface (PLATE 4).

The introductory image provides a representation of the way Islamic astronomers visualized the Universe during the Middle-Age. The spherical composition is based on translated original texts and late and contemporary commentary cited in the bibliography.

As we know, the shape of the Islamic Universe is strictly based on the Aristotelian and hence Ptolemaic model. The point 0 in the center of the sphere represents the Earth, located at the center of the Universe according to the geocentric theory.

The medieval Islamic idea of Universe is composed by two concentric spheres. The inner smaller sphere is the Earth (0). The outer greater sphere is called Celestial Sphere. The stars (S) are located on this latter and they are considered

as observed from the Earth, which is tied to the geometrical center, that is, it does not rotate on its axis and it does not orbit around the Sun. On the contrary, the external sphere rotates clockwise and hence all the celestial objects, considered fixed on this surface, follow the same movement.
In order to understand how this system we need to explain its frame of reference.

A frame of reference is the set of coordinates which is built as a mathematical model to locate geometrical entities. Nowadays, in astronomy, we distinguish four different frames according to the coordinates chosen to determine the direction or the position of objects in this geometrical space: horizon system, right ascension system, hour angle system, ecliptic system. These information are usually expressed in terms of two orthogonal coordinates or polar coordinates. The first one is reckoned from a primary reference plane and it is usually measured orthogonal to it, whereas the second one is reckoned from a secondary reference plane but it is measured on the primary plane[13].
If we reconsider our double-sphere concentric model, we can identify as primary reference plane the so-called Celestial Horizon (blue plane), geometrically the plane orthogonal to the generative axis of the Celestial sphere which passes through the center and intersect the surface of the sphere tracing a circumference called Celestial Circle. The secondary reference plane is the Observer Celestial Meridian, the plane orthogonal to the primary

[13] I.I. MUELLER, pp.19-26.

whose intersecting circumference passes through the eventual location of a star (considered a point of the Sphere surface), the location of the observer on the Earth (Observer Meridian, that is the circle passing through the geometrical point where the observer is located) and the two poles of the Celestial Sphere. These latter are called Zenith and Nadir, North and South respectively, intersection of the main axis of the sphere with the surface. The intersection circumference is called Vertical Circle.

Therefore, the direction vector of the star is the straight line connecting the observer (O) to the star (S), whose coordinates in the Horizon System are expressed with two parameters: the Altitude and the Azimuth.

The altitude a is the angle between the vector direction 0S and the Celestial Horizon, measured in degrees (0° to 90° because we consider only the portion of celestial sphere that we can see up to the plane of the Horizon, that is half of the sphere, -90° if we consider the Austral Hemisphere) in the plane of the vertical circle through S. Its complementary angle is $z = 90° - a$, called Zenith Angle.

The Azimuth A is the angle between the Vertical Plane of S and the Observer Celestial Meridian, measured from North to East (0° to 360°) on the Celestial Horizon.

The Celestial Equator (red plane), the plane intersecting the Celestial Horizon is a geometrical extension of the Earth's Equator, orthogonal to the North Celestial Pole (NCP) and South Celestial Pole (SCP).

The stereographic projection used by the astrolabe (right image) is, in our case, a projection from the pole P (coincident to the South Celestial Pole) on the Equatorial plane. For this reason we have rotate our model in order to better visualize the projection. The scheme helps to understand how this type of projection works. Briefly, we consider as projected points all the points of intersection between the projective line and the equatorial plane. For example, h is the projection of H and h' of H', horizon.

PLATE 2 represents the construction of the horizon and the *almicantarats* (circles of equal altitude). The horizon is the blue plane and the *almicantarats* the orange parallel plane. We have to notice that the maximum circles which give us the edge of the astrolabe is the stereographic projection of the Tropic of Capricorn. The eccentric circle visible in the projective plane is the Horizon HH' and the concentric circles inside its parallels.

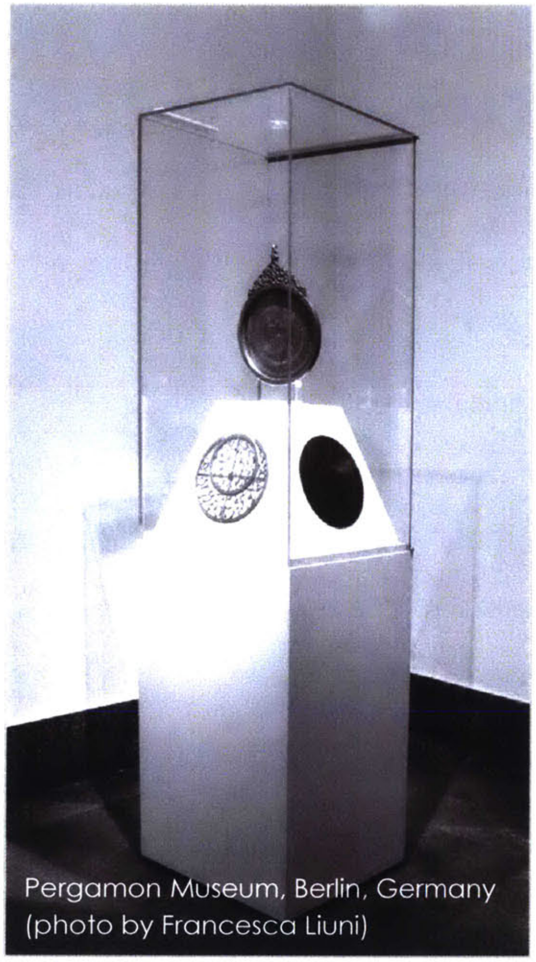
The three-dimensional drawing at the bottom of the page is a representation of how we can obtain this circles on the horizontal plane π by imagining to reason with conics' intersections.

PLATE 3 is the geometrical construction of the projection of the circles of equal azimuth, planes orthogonal to HH', horizon. In this case the geometric construction is not so straightforward. As usual, we need to trace our Tropic of Capricorn as edge of the astrolabe, and the horizon as line which cut the azimuthal circle defining what stars of the *rete* are visible above the horizon at a defined time and what is their angle.

This construction starts from the projection on the Equatorial line of Zenith and Nadir, z and *n*, respectively. The center of the line which connect these two point identifies a straight line orthogonal to the Equatorial plane. On this line we can draw the center of azimuth. In our example we have divided our circumference with 15° angles. Knowing that the $\sin\varphi = ab/ac$, that is the cathetus of the right-angled triangle *abc* (between z' and C_φ) $ab = \sin\varphi\ ac$. Hence, since that they know the values of the sine of angles until 0.5, they could easily trace the length of *ab* and find the center of azimuth $C_{15°}$.
The same process is used for all the centers of azimuth even though we could also divide the main circumference with a goniometer and find the correspondent centers tracing a straight line from z' through the extremity of the arc of circumference drawn by the selected angle.
On the right of the image an upper view of the three-dimensional model of the celestial sphere which can help us to prove that if we were able to observe the Universe from the zenith, what we would see on the Equatorial plane would be a series of azimuthal circles, curved edge of the correspondent meridian planes. In this figure we also show how to measure our azimuth starting from the North (Celestial Meridian) and going toward east.

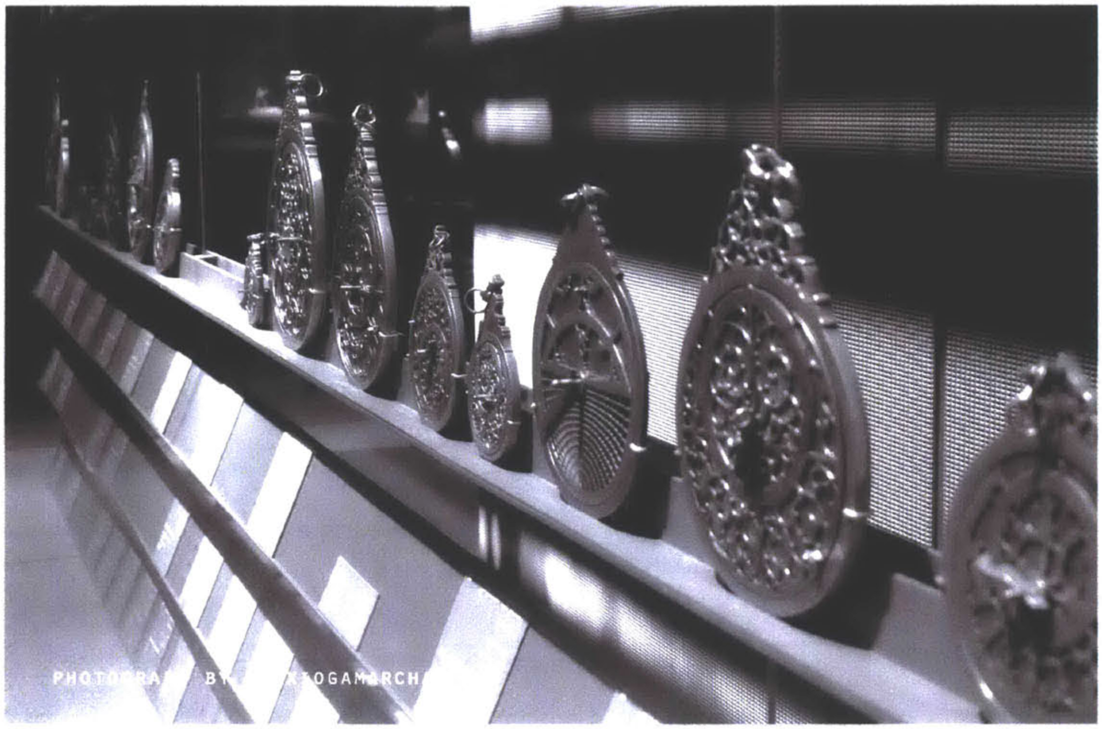
PLATE 4 shows the construction of Ecliptic, the path of the Sun, the Tropic of Cancer and the Tropic of Capricorn. In the case of Tropics we have a perfectly orthogonal conic projection, instead the ecliptic's plane is inclined by 23.5°, hence is projection will be eccentric.

1.2.2 OUTLINE OF THE CURRENT EXHIBITING PRACTICE


Pergamon Museum, Berlin, Germany
(photo by Francesca Liuni)

This choice of the astrolabe as case of study stems from a direct observation or documentary research on astrolabes' collections which led me understand how much my experience as a visitor was constrained by the curatorial practice

The consistent astrolabe's collection of the Doha Museum of Islamic Art, the collection of the Museum of Science and Technology of Istanbul, the Oxford Museum of History of Science are just few examples of what has been defined so far an historicist approach: the astrolabes are exhibited in glass showcases that the visitors can generally observe only from one side; they are labeled with catalogue numbers and information about chronology and material. If the collection own more than five or ten pieces, it is possible to find a panel with a short explanation on the use of the instrument, which most of the time does not go far then "the astrolabe was an instrument used for measuring the altitude of celestial object and locating it in a space/time map traced on the surface of the instrument itself".



Museum of Islamic art, Doha, Qatar (photo by Denxiogamarcha).

This is the policy of the Art museum or museums of History of Science. The value of the object is essentially recognized to be its craft (generally the astrolabe were elegantly decorated with flowers *motif*) or simply its significance as a document of the past. Its role as scientific instrument is utterly omitted.

Conversely, the exhibition strategies of Science Museums go in the opposite direction. Along with the original object is often exhibited a reconstruction with which the visitors can play in order to simulate the actual use of the instrument.

Museum of Natural History, New York (phot credit: Traveling the Silk Road)

This last exhibition strategy better deal with the nature of the object itself by highlighting its value as a scientific instrument more than as an art object. It breaks the historicist approach of the Art museum trying to engage the visitors by let him re-experience the way the instruments was used in the past.

Nevertheless, this methodology causes a total loose of connection with the past. The object is reduced to an old-fashion toy and there is no interest in explaining the actual historical context in which the object has been developed and that is an essential part of it. The visitors interact with a copy of the original object and use it in a copy of the real sky (see fig.), as happen in the Museum of Science and Technology in Islam of Saudi Arabia.

Recreating the context of use of the astrolabe and letting the visitors playing in an interior space (decorated as a celestial sphere or not) is a dangerous attitude: it destroys not only the connection with the history embedded in the object but also the experience of observing the real sky and the relation of the instrument with nature. Trying to understand and systematize nature is the moving factor at the base of the intention of constructing an astrolabe. It is also the moving factor of all the Arabic and Persian scientific community between the X and the XIV century. The reason why the felt the necessity of re-build the Greek prototype of an astrolabe and improve it.

Therefore, on one hand we have the pure historicist approach of the Art museum and on the other hand we have the over-educational flattening strategy of science museum.

What both miss is what I have defined at the beginning of this thesis as the dialectic approach. If we still want to believe that the aim of museums (art or science museums) is using the past to inform the present by creating a communication between the exhibited objects and the visitors, we need to understand what each objects belonging to a collection has to communicate. Then, what "language" is it using for communicating.

I am not going to discuss herein the difference between the language of art and the language of science, but it is necessary to state that a difference exists and that it has its roots in the way they communicate.

Whereas an art object has an intuitive and immediate language, a scientific object requires a slow and complex understanding of all the theories developed for made its construction possible. Therefore, when we observe a scientific instrument we are observing an endless *apparatus* of knowledge synthetized in one instrument.

This is the reason why when we exhibit a historical scientific instrument we should find a way to make this *apparatus* legible.

1.3 MATHEMATICAL FRAMEWORK

The astrolabe, as scientific instrument, is based on mathematical[14] theorems and its construction is a tangible proof of the exactness of its theoretical propositions.

As Frege writes in his *Conceptual Notation* a mathematical proof « tests the validity of a chain of reasoning and expose each presupposition which tends to creep in unnoticed, so that its source can be investigated» [15] Therefore, the goal of a proof are two: 1. giving certainty to the proved proposition; 2. showing the foundations on which it has been proved in order to make the process of proving repeatable and hence objective.

Why defining the concept of proof is essential in the field of Arabic Mathematics and in the case of the astrolabe?

The first statement of most of the treatises of mathematics developed in the historical context we are discussing is based on a sort of methodological *dictat*: proving again and in different ways what the Greeks had developed during the previous centuries. Accordingly, Arabic mathematics has its ontology in the concept of proof which ends up to be a sort of methodology of approaching to every mathematical problem. It is utterly based on the concept of proof

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[14] We have to consider that even is the astrolabe is an instrument for astronomical observations, at that time astronomy was part of mathematics. Moreover, according to the classification of F. Charette there were different types of astronomical instruments developed during the XI-XII century in the Islamic area: mathematical, graphical, trigonometric and observational instruments. The astrolabe belongs to the first group (explaining why would involve a larger discussion on scientific instruments that is not part of our research).

[15] G. Frege, *Conceptual Notation*, p.104.

probably seen as a way to challenge the undisputable preeminence of the Greek science.

Therefore, proving becomes a recurring statement for Arabic mathematicians who thoroughly translate, apply and extend the Greek model of theorems' demonstration. Moreover, between the X and the XIV century (during the so-called "Islamic Golden Age of Science") mathematics become an overwhelming presence in the field of science. The great importance Arabic mathematicians gave to the concept of proof has its roots in a sort of constant "obsession" for calculating everything and then using logically developed proofs for demonstrating the exactness of their calculations. The astrolabe is an example of this effort of "mathematizing" the space: it is technically a representation (by stereographic projection) of the Universe (or what was their idea of Universe) on a bi-dimensional surface. A universe where everything had its place in a perfect geometrical construction and hence everything could be calculated.

This strictly structured system finds in mathematical proofs its means for checking and justifying the correctness of the system itself. In order to guarantee the precision of their construction they needed a method of proving that was as constrained as their mathematical model that is the axiomatic method.

The mathematics the astrolabe refers to inherits the Euclidean axiomatic system (due to the Greek legacy[16]) which is, as described by Hilbert at the

[16] The Arabic and Persian science has been developed in continuity with the Greek one. Scientists translated all the Greek texts in Arabic and Persian and started to develop their own science.

beginning of the XX century, composed by terms and primitives combined to create axioms. The axioms contain the fundamentals proprieties of the objects that constitute the primitive concepts of the theorem itself.

Nevertheless, Euclid axiomatic method also includes what Hilbert calls intuitions; Euclid used proprieties belonging to objects of which he had knowledge of (for example line, points, etc.) and his system worked perfectly only as long as the not-defined terms (primitive terms) satisfied the axioms.

The structure of this system based on terms not to be defined and established as "conventions"[17] necessary for create meaningful propositions resembles a linguistic system (indeed it has been theorized by Frege and Hilbert in the attempt of defining a language for mathematics). Therefore, if the astrolabe is based and constructed on mathematical theorems using an axiomatic method, and these theorems are structured as a linguistic system, then the language the astrolabe uses and that we have to interpret is the language of axiomatic geometry.

This mathematical grammar structures the language of the theorems the astrolabe relies on. Hence, understanding its language is a process which embeds the necessity of translating the language of an axiomatic system. Still, whereas the axiomatic system is the grammar defined through terms and primitives, the mathematical proof is the syntax of this language.

[17] The Aristotelic *sythéke*.

2. THEORY IMPLEMENTATION

After outlining the theoretical framework we need to define how we can implement it, that is how we can address the request of the materialist historian and at the same time follow the instructions of the mathematician.
In which way does architecture manage this issue? What a designer can do in order to translate the theory in practice? The answer is designing a space which follow the rules of the mathematicians and visualize his theories by shaping the space according to his rules. In this way we can obtain a space which represent the process of reasoning of the mathematician and therefore allows the visitors to have what we have defined so far a dialectic approach to history: the visitor does not observe the object but he observes the generative process the object, that his the evolution of mathematical concept which made the construction of the object possible.
Hence, we have defined that we are going to implement our theoretical framework using architecture as a tool. The design methodology will be divided in two parts:

1) THE PRELIMINARY DESIGN PROCESS which involves defining what we are *de facto* going to materialize through architecture, which mathematical concepts and which rules are we going to follow in order to recreate a perfect correspondence between mathematics as a theory and its material translation into space.

2) DESIGN PRINCIPLES. Which principles are we going to follow in composing the architectural space, which material are we going to use and why; how the visitor move in this space.

2.1. CONVEYING HISTORICAL EXPERIENCE

If we need to recreate an historical experience which dialectically connects past and present in the picture (in the Benjamin sense of the word) of the astrolabe, we have to materialize a historical experience that enables the dialogue among exhibited object and observing subject.

As we have stated in the first part of this thesis, recreating an historical experience means making the *apparatus* of knowledge in which the object has been developed and which allowed its construction legible.

In our case the *apparatus*, as we has already explained, is composed by all the mathematical[18] theories developed by the lively community of Arabic scholars.

I decided to pick four of the main mathematical concepts as explained by Ibn al-Haytham in the translated collection of Roshi Rashed[19] related with the astrolabe and represent them by dividing the museum space in four parts. The concepts selected are:

- Calculating the volume of the sphere
- The solid angle
- The concept of sine and cosine
- The stereographic projection

These concept are organized according to a precise path which lead the visitors: 1. the general concept of the volume of the sphere, intended as the

[18] It has to be noticed that mathematics included geometry and astronomy as subset of it.
[19] R.Rashed, 2014.

Celestial Sphere as shape of their Universe; 2. the relation between the sphere and the observer, that is measuring distance of celestial objects on the surface of the sphere (through the angles of spherical triangle) and measuring the distance between the observer and the sky (through the solid angle); 3. What geometrically meant calculating an angle (sine); 4. How these concepts live behind the bi-dimensional surface of the astrolabe (the stereographic projection).

The design is an attempt of bringing the proof from the two dimension of the paper to the three-dimension of the visitor in order to provide him an experience that is the spatial experience of a proof brought in his three-dimension. The possibility of visualizing the proof improves the understanding of the mathematical concepts the instruments uses and, at the same time, materializes part of the scientific knowledge behind the instrument in a sort of "conceptual explosion" of the object itself.

2.2 CONVEYING MATHEMATICAL PROOFS

The language of the astrolabe, as we have stated above, is mathematics. Its terms and primitive compose the grammar of the axiomatic method and the proof is the syntax of this linguistic system.

The design translates this axiomatic system using architectural objects as means of translation.

Therefore, if we consider that in this language, «each primitive terms is usually declared to be one of several grammatical types: an object, a relation, or a function»[20], then, the primitive term which represents an object of the axiomatic system serve as a noun of our mathematical grammar (points and lines are examples of objects); the one representing a relation is the verb (for example, *lies on, intersect, meet*); the terms representing a function is an «operator» applicable to different objects (for example the *distance* between two points/objects)[21].

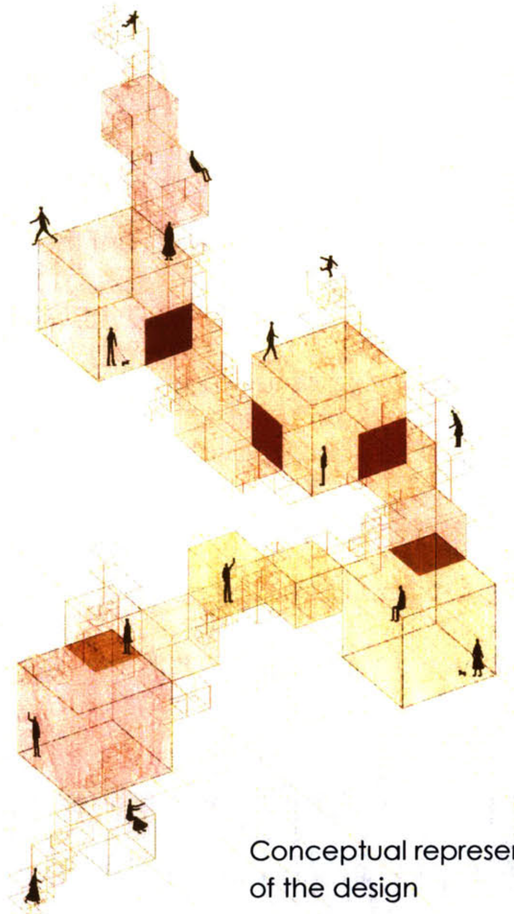
Hence, our design is composed by numbered elements (such as the wooden structure and the connected strings) which represents the terms or nouns of our mathematical grammar. All these elements intersect each other creating relation which are the verbs of our grammar (such as the distance between the vertical pillar *A1*, tangent to the surface of the cylinder circumscribed to the actual sphere, and the center of the sphere is the radius R1 of the sphere itself.

[20] J.M. Lee, *Axiomatic geometry*, p.24
[21] *Ibidem*.

The grammar so structured creates propositions (with noun, verbs and "operator"). The sum of all the proposition describes the syntax of the space as a representation of the mathematical language of the astrolabe.

3. DESIGN

«Sometimes architectures becomes objects out of scale, they acquire the size of a coffee maker or the size of a jalopy because they are represented as ideas and ideas do not have dimension, they are not measurable, they do not obey to quantitative laws».[22]



Conceptual representation of the design

Dealing with the representation of a mathematical concept means being able to visualize an abstraction, an idea or something that does not have a visible or direct correspondent in the reality. Any attempt of visualize mathematics involves the use of geometry.

Geometry is the connection between abstractions and tangible and it is the tool of architects use for shaping the space.

However, the design of the space we are proposing combine the abstraction of mathematics to the rigor of geometry. The four space we are designing are

[22] P.Portoghesi in Introduction to *Aldo Rossi, Disegni. 1990-1997*, Milano, 1999.

not only a simple representation of a geometry, but they are a mathematical proof of a geometrical concept. Namely, what a visitor is going to see around him, his space, is a strict geometrical space design by using the grammar of a mathematical proof.

Nevertheless, we stated that we want the visitors to have what we have called so far an historical experience and that this experience consists in re-living the process of reasoning of the mathematicians which is developing the theorems at the base of the construction of the astrolabe.

Namely, we have to represent the abstraction of a mind process which is reasoning on the abstract concept of mathematics.

Therefore, the following question has to be: how can we visualize a mind process?

In this specific case it is not only a matter of applying mathematics to geometry in order to have a visual correspondent of it, but it is a matter of representing the moment in which this happened. Then, how can we visualize a process of reasoning? What the visual correspondent of this process?

The answer to this question is probably trivial for an architect: sketching.

Whenever our mind (architect or mathematicians) is thinking, our hand print the product of this quick but consistent process on a piece of paper.

The design we are proposing let the visitors experience the cerebral ferment of the mathematicians by leading him in a sort of "sketch architecture" where any architectural element of the structure is visible and all the elements are arranged together according to the rule of the grammar of our space. The grammar of our space is a direct derivative of the grammar of the mathematical proof.

To conclude, the visitors live in a representation of a working progress drawing, he lives in the mind of the mathematicians.

1.1.1 6 MEMOS: LIGHTNESS, QUICKNESS, EXACTITUDE, VISIBILITY, MULTIPLICITY, CONSISTENCY

In order to explain all the parameters and rules that our project has to follow, I tried to analyze each features singularly. I would like to borrow the 6 memos Italo Calvino used in his so-called American lectures in the 1985[23]. These 6 memos were used By Calvino as leading principle for discussing the future of literature in the technological era.

I would like to analyze my design following these principle because paradoxically they are some of the principle that architects try to involve in their design.

[23] I.Calvino, *Lezioni Americane*, Milano, 1993.

3.1.1 QUICKNESS: SKETCHING. LIVING IN THE PROCESS OF REASONING

As we have stated in the introduction of the Design chapter, leaving in the process of reasoning means leaving in a space which is a translation of a sketch. The sketch is the visualization of the thinking of the mathematicians as it is also the visualization of the creative process of the architect.

The main characteristic of a sketch is its quickness from which derives its conciseness: the sketch follow the speed of reasoning and automatically select the main information useful to summarize an idea in few lines.

How can we translate this idea into architecture?

In the case of our design the quickness it is not in the way we represent our mathematical proofs in the space but it is a pivotal pint in the design process.

All the design has been realized by sketching on the paper and constructing physical models which are not a direct representation of the architectural design but a concise synthesis of how can the mathematical concept be represented into the three-dimensional space of the visitors, which are the main elements that constitute the architecture and at the same time highlight the main lines' combination of the concept itself.

The construction of the physical model is the very first step of the design process and the reason lays in the fact that in order to translate the process of reasoning of the mathematicians into an architectural space I needed to be able to visualize only the essential part of the mathematical proof and then quickly translate it into architecture.

Modeling by hand push you to reason about connection and construction: the choice of the wood and strings or paper as a main material is also based on the necessity of representing the sketch translation. The wood can be only cut and intersect as the lines of a sketch intersects on the paper. The cotton strings connected from point to point perfectly simulate the projection lines used in any geometrical drawing.

The manual skills of making the physical model by hand without waiting the time of a cutting machine or without previously plan what the part of the design were going to look like, encourages the architect as a maker to be quick as if he were sketching on the paper; he develop a progressive construction which building methodology is based on a continuous problem-solving process.

Finally, the models developed individually as representation of the four concept, are not perfectly finish not only in order to suggest to the observer the idea of the sketch but also because in the process of representing a mathematical concept and at the same time doing architecture is not necessary to complete the space but just to build enough of it in order to have a suggestion of what the visitors are going to experience.

3.1.2 EXACTITUDE+VISIBILITY: FROM SKETCHES TO ARCHITECTURE. TRANSLATION OF A DRAWINGS AND THE GRAMMAR OF THE SPACE

The exactitude is the precision of the construction and the thorough correspondence between architectural objects and lines of the bi-dimensional geometrical proof.

The exactitude and the rigor is a requirement of any mathematical proof. Hence, it needs to be a principle of the architectural translation of the mathematical proof.

Moreover, whereas in the mathematical demonstration every step has to be clear and consequential and each word has to be defined at the beginning according to a set of preliminary definitions, in the architectural design every element has to be visible in order to create a "structural clarity" where every element of the design is legible.

The visibility becomes a theoretical correspondent of the clarity of the mathematical proof. Each element, joint or intersection of the structure of the project has been designed in order to be visible not only for proving the exactitude of the construction (built as a correspondent of the geometrical drawing used for developing the proof) but also for highlighting a "structural clarity" which resemble the wireframe profile of a sketch.

The architecture of the building is mainly a discrete structure composed by pillars designed as a system of steel beams and wooden timbers.

The pillars itself is composed by two wooden timbers flanked by a two separate "L" steel beams. This double pillar system is then additionally supported by two "C" steel beams located behind the main pillar. These last "C" beams connects the main pillars which works horizontally by sustaining the roof with the vertical system composed by intersection of thinner wooden timbers and glass surfaces.

The goal is representing the base of a geometrical drawing where the pillars creates a grid of points from which the lines of the geometrical construction originate.

Therefore, each architectural objects correspond to a line or to a points of the sketch the mathematicians is hypothetically developing during his process of reasoning. The thicker lines becomes steel beams, the thinner ones wooden timbers, the planes become glass surfaces and so on.

In this way each architectural element end up to be part of a grammar of the space. Each elements works as a term of the grammas which meets other terms composing a syntax that is the syntax of the space and at the same time the syntax of the mathematical proof.

3.1.3 CONSISTENCY AND MULTIPLICITY: DESIGN METHOD

The multiplicity of the project lays in his volumetric composition.

Each room that explain a mathematical concept is seen as independent, conceptually and structurally.

The design I proposed is only one of the possible arrangement of these rooms or individual boxes in the space.

Assuming that we change our urban setting, we are free to change the arrangement of our boxes and also add other boxes creating a growing architecture. The goal is representing not only the endless knowledge that has been accumulated and developed in order to design the astrolabe, but also more generally the essence of knowledge itself (scientific knowledge or not) as an infinite addition of information that can be combined together in order to generate other knowledge.

The connection between the different boxes which is the path the visitors is invited to follow in the museum is aimed to be a slow process of acquisition of knowledge that does not have neither an end nor a beginning.

The choice of representing only four mathematical concept is only a necessary constraint I needed to have in order to do a sort of experiment on the possible representation of the process of reasoning and developing knowledge of a mathematicians.

The experience of this space has to happen over the day. Since the astrolabe was an instrument for calculating the altitude of celestial objects and therefore it was an instrument strictly related with the changing of the sky, the design has to be consistent with this idea.

The visitors who arrive in the morning visits a static space made of pure geometrical constructions. The appearance of the building change over time until the night when a set of optical fibers lights up showing the line of the construction of the mathematical proof. Namely, the architecture of the building is the basic geometrical drawing on which the light strings which appear at the end of the day construct the mathematical proof.

These strings are visible but transparent during the day in order to invite the visitor to ask to himself what they represent and encourage him to try to develop his own proof waiting for the solution.

3.1.4 LIGHTNESS: TRANSLATING A SKETCH USING CONSTRUCTION MATERIALS

The lightness is the lightness of a drawing. The transparency becomes a necessary requirement for the construction not only for addressing what we have defined as "structural clarity" but also for representing the lightness of the lines on a piece of paper.

The double system of glass used for some of the walls and the roofs reacts differently according to the time of the day and the inclination and the intensity of the light that hits it. Therefore, the space seems to change his consistence and his lightness over the day from an opaque combination of volumes to a transparent intersection of lines.

The lightness is also obtained by using wooden and steel structure in order to have the wood as a main material for representing both a specific thickness of lines (intended as translation of the sketch) and also the intersection of those lines. The wood can be cut and intersect and the points of intersection as joints or simple connections are always visible as it occurs in a pencil sketch.

The steel, also visible (in order to respect the "structural clarity" we discuss above) allows us to use thinner wooden timber and obtain a visual effect of lightness.

4. CONCLUSION

The goal of this research was re-creating a historical experience in order to change our approach to historical scientific instruments. Architecture has been used as a tool for achieving this goal.

As I stated in the last section of this thesis, architeture can create an experience and archiecture can visualize ideas. However, the design proposed does not want to be the solution to the issue we are discussing, but only one of the possible solution. The reason why this research cannot define a set of rules or claim to have found a definitive solution is that we are deling with abstraction: we are proposing a visualization/materialization of a representation of an abstract idea.

We could state that in order to have a dialectic approach with a scientific instrument we can re-live its process of creation and that its process of creation is (in this case) the mathemathical framework, namely the process of reasoning. We could also state that the connection between thinking and seeing is sketching and hence our architecture has to resemble a sketch. We should then follow the 6 memos proposed at the end of the thesis as a guide for designing an architecture that meets the requirements and the principles of mathemetics.

The design proposed is an experiment realized by choosing a case of study for proving that architecture can materialize a mathematical concept and that architecture can make out of this concept an experience.

The power of this discipline is the moving factor of this research.

Accordingly, if the main question of the thesis is: how can I create an historical experience? The only certain answer we have found is: using design because it is the only tool we have for make the visitors having an experience and at the same time visualizing an abstraction.

This thesis want to open a discussion on the way we exhibit nowadays historical scientific instruments which affect the way we approach to them. The analysis and the design developed does not want to state a solution but to propose a new way to deal with this issue. We claiming that, as architects, we do not know the solution but we have the privilege to know how to use the tool that embedd the solution in itself becuase architecture is the combination of the abstraction and intuitive language of Art and the exactitude and the practicality of Science.

5. DRAWINGS

5.1 THE GEOMETRY OF THE ASTROLABE

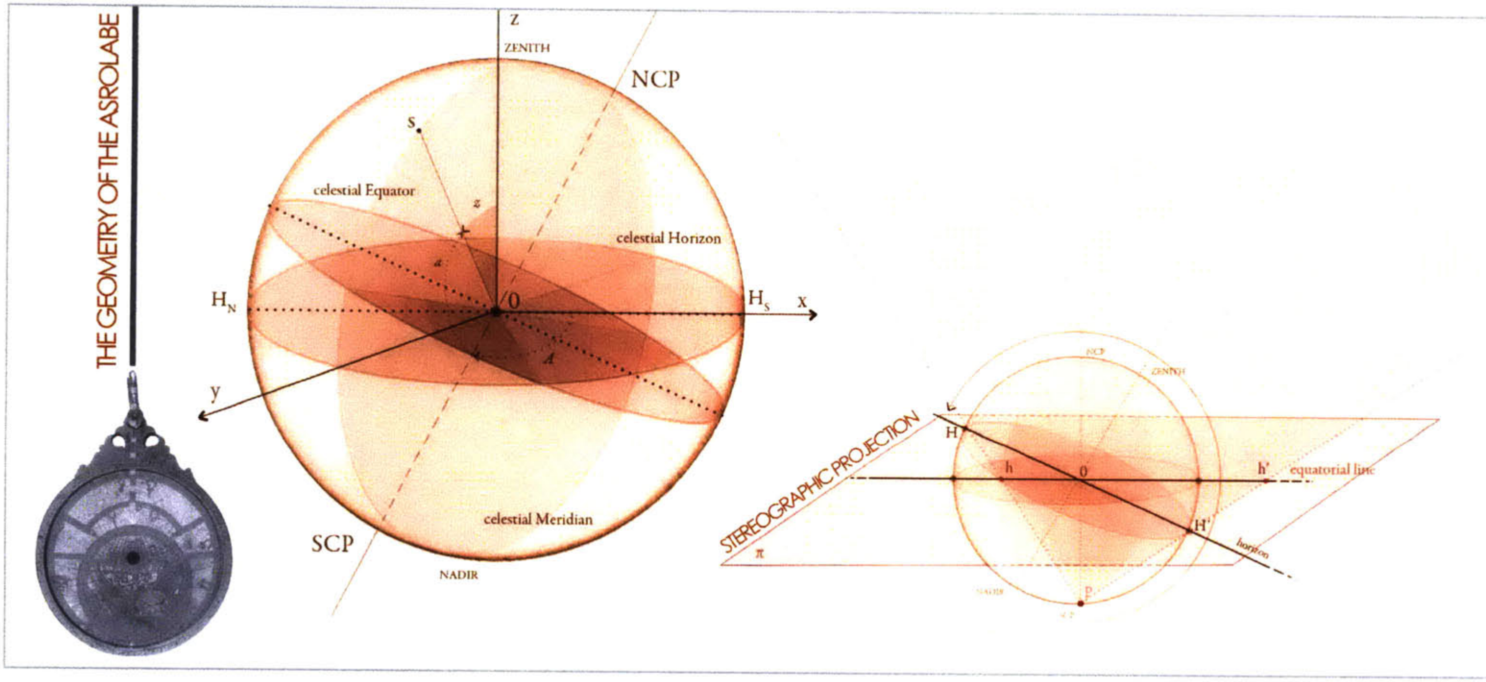

THE GEOMETRY OF THE ASROLABE
z
ZENITH
NCP
S
celestial Equator
celestial Horizon
H_N
0
H_S
x
y
SCP
celestial Meridian
NADIR
STEREOGRAPHIC PROJECTION
H
h
0
h'
equatorial line
H'
horizon
π
P

PLATE 2

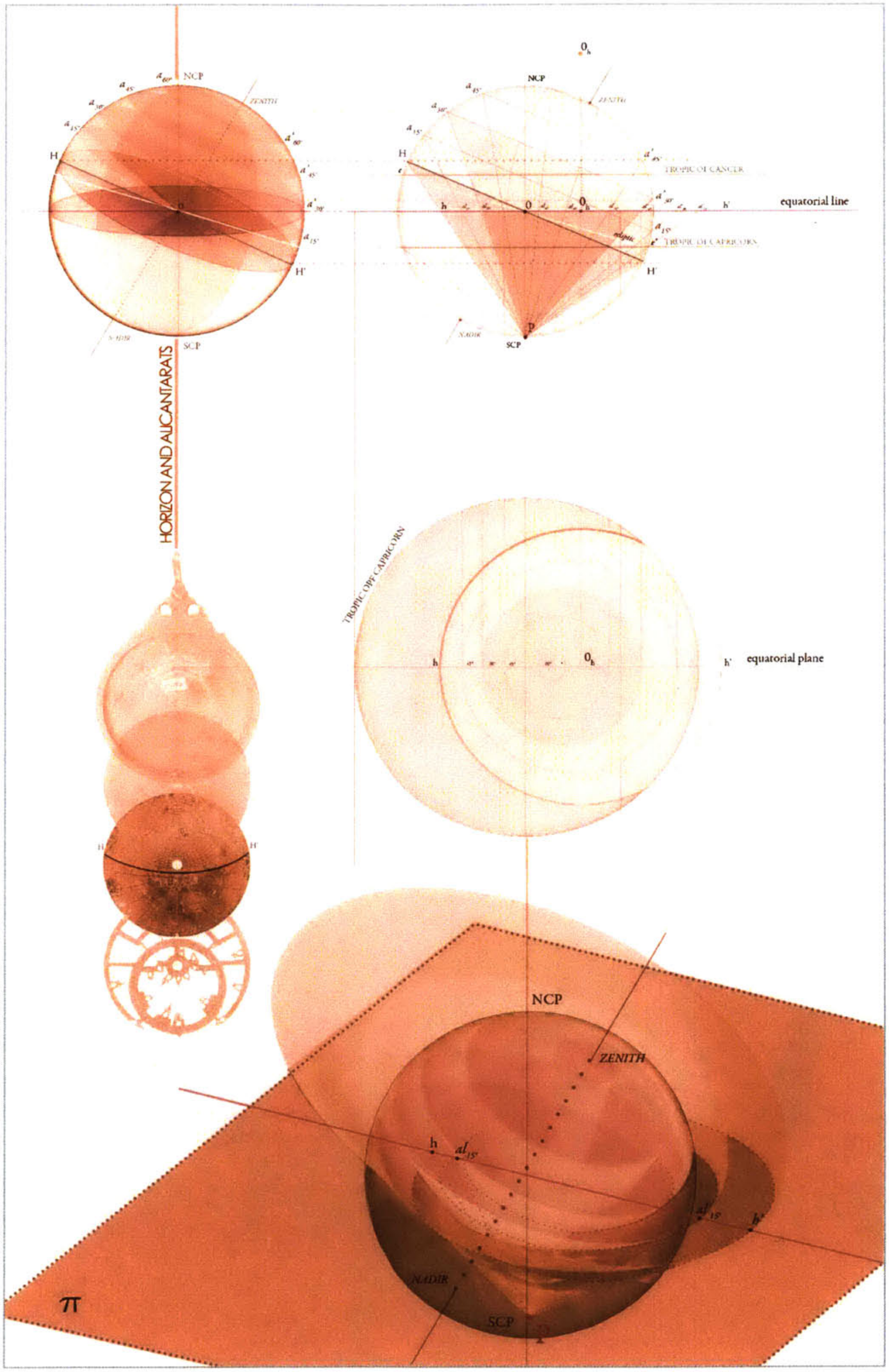
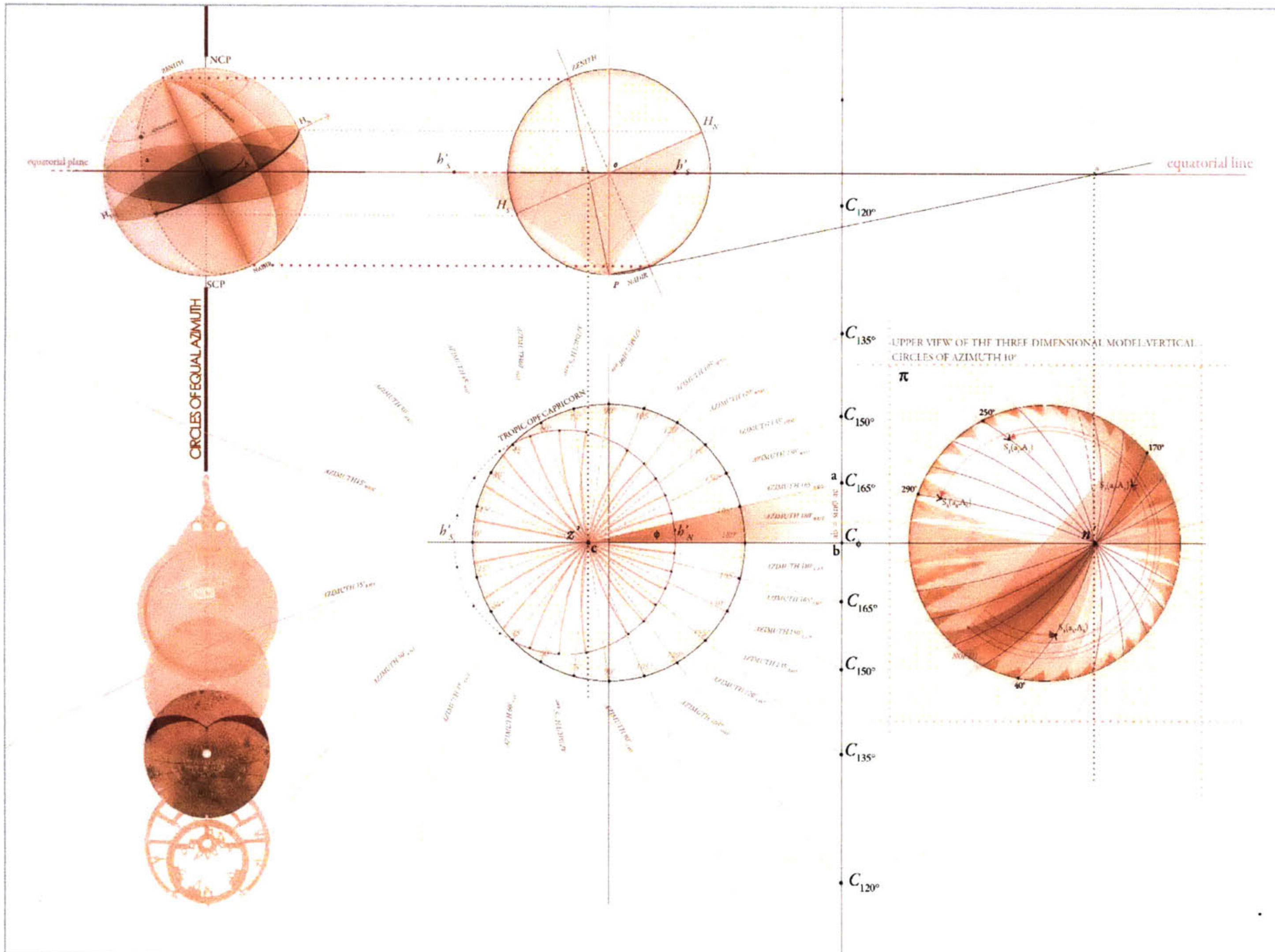
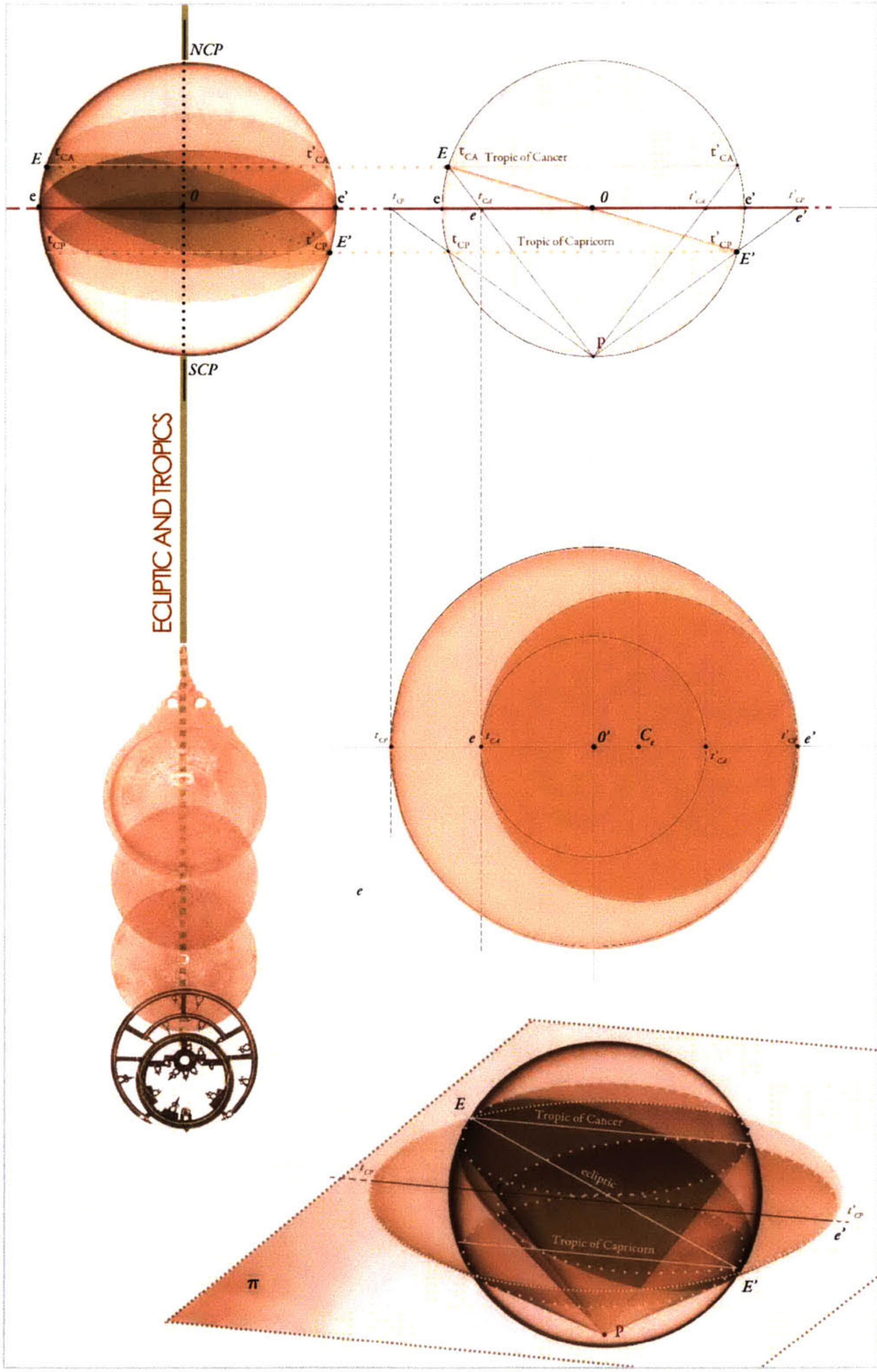

NCP
ZENITH
H
H'
SCP
TROPIC OF CANCER
TROPIC OF CAPRICORN
equatorial line
ecliptic
NADIR
HORIZON AND ALICANTARATS
TROPIC OF CAPRICORN
equatorial plane
NCP
ZENITH
NADIR
SCP
π

PLATE 3



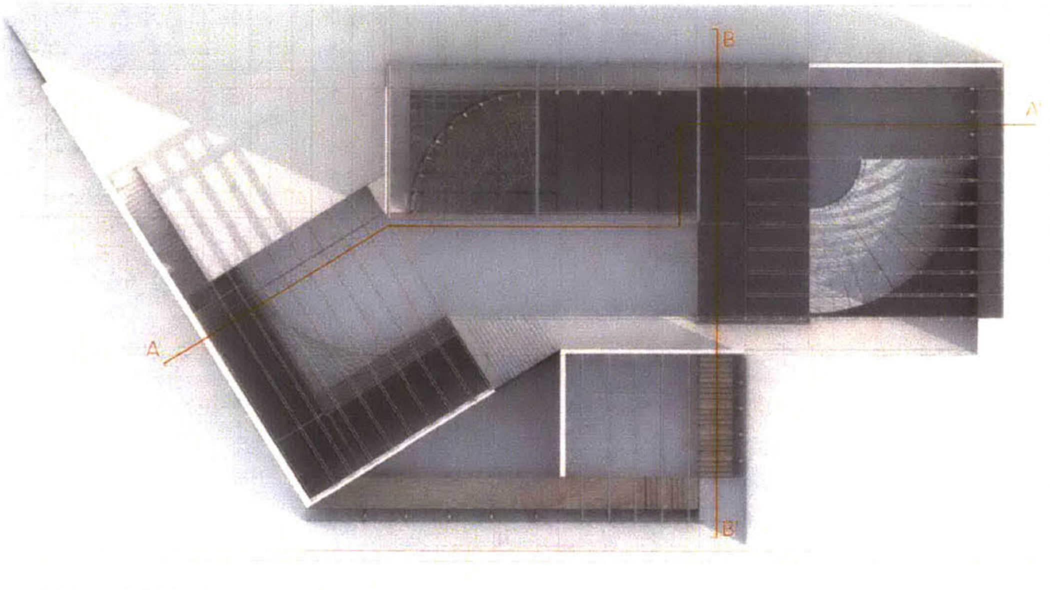
NCP
SCP
equatorial plane
equatorial line
ZENITH
NADIR
H_N
H_S
b'_S
b'_N
P
CIRCLES OF EQUAL AZIMUTH
TROPIC OF CAPRICORN
z'
c
φ
a
b
$C_{120°}$
$C_{135°}$
$C_{150°}$
$C_{165°}$
C_{ϕ}
$C_{165°}$
$C_{150°}$
$C_{135°}$
$C_{120°}$
UPPER VIEW OF THE THREE DIMENSIONAL MODEL: VERTICAL CIRCLES OF AZIMUTH 10°
π
250°
170°
290°
40°
n'

PLATE 4

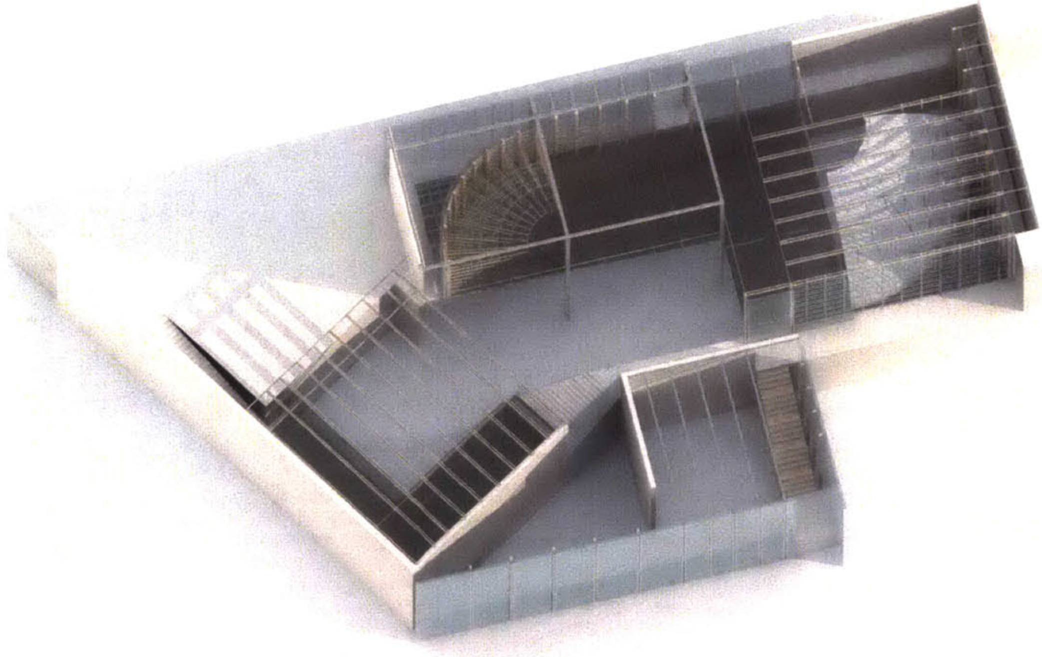

ECLIPTIC AND TROPICS
NCP
SCP
Tropic of Cancer
Tropic of Capricorn
ecliptic
π

5.1 ARCHITECTURE

PLATE 5

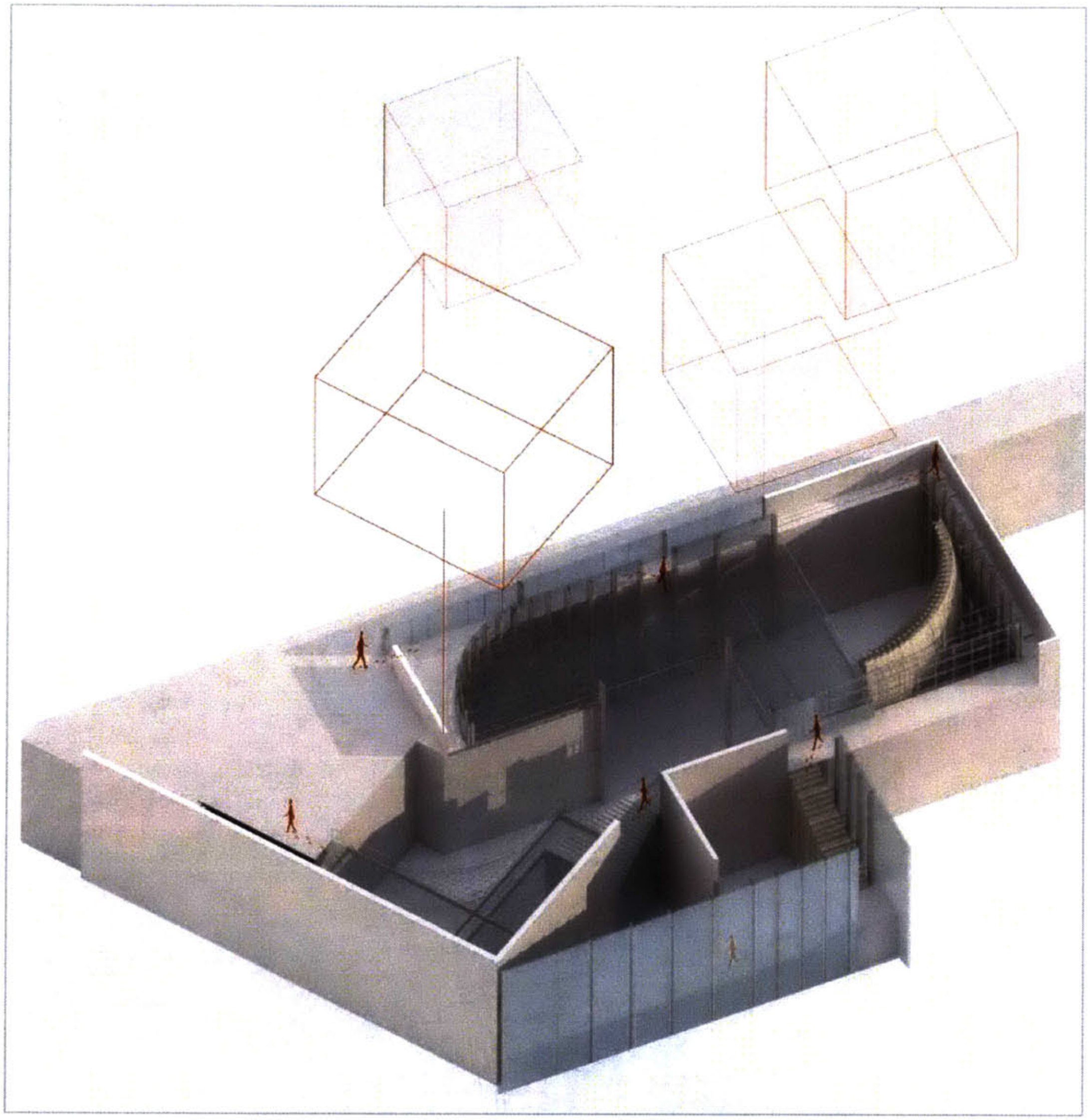


PLAN



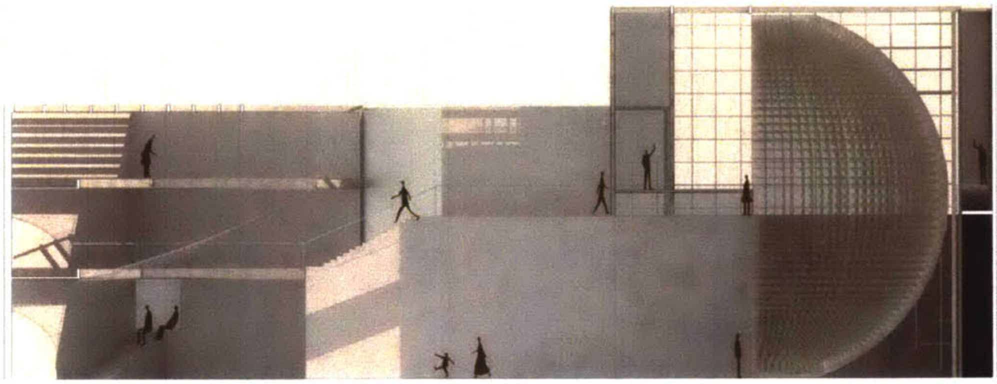
AXONOMETRICAL VIEN OF THE STRUCTURE

PLATE 6

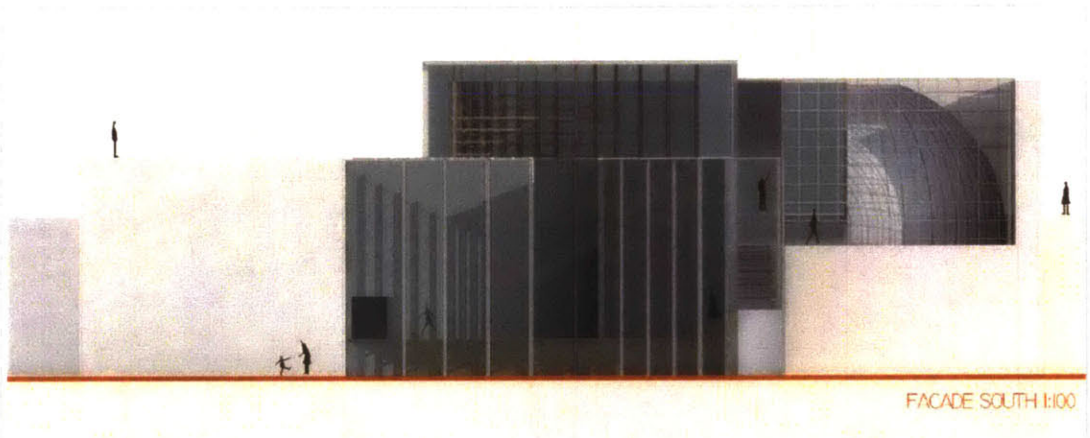


SECTIONED PLAN AND SCHEME OF THE FOUR BOXES WHCIH COMPOSE THE SPACE

PLATE 7

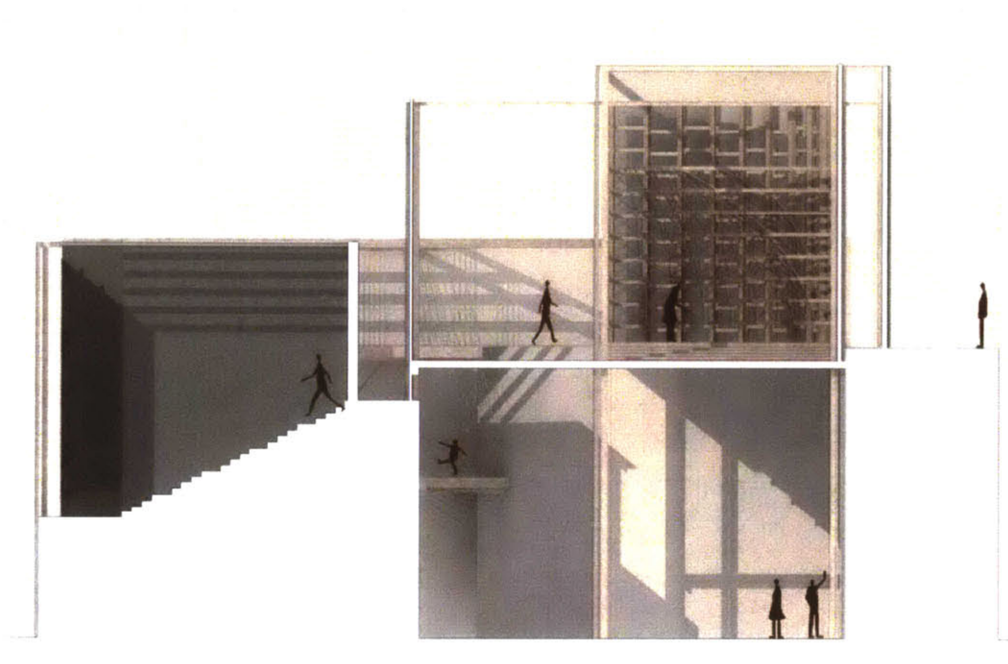


SECTION A-A'



FACADE SOUTH

PLATE 8



SECTION B-B'

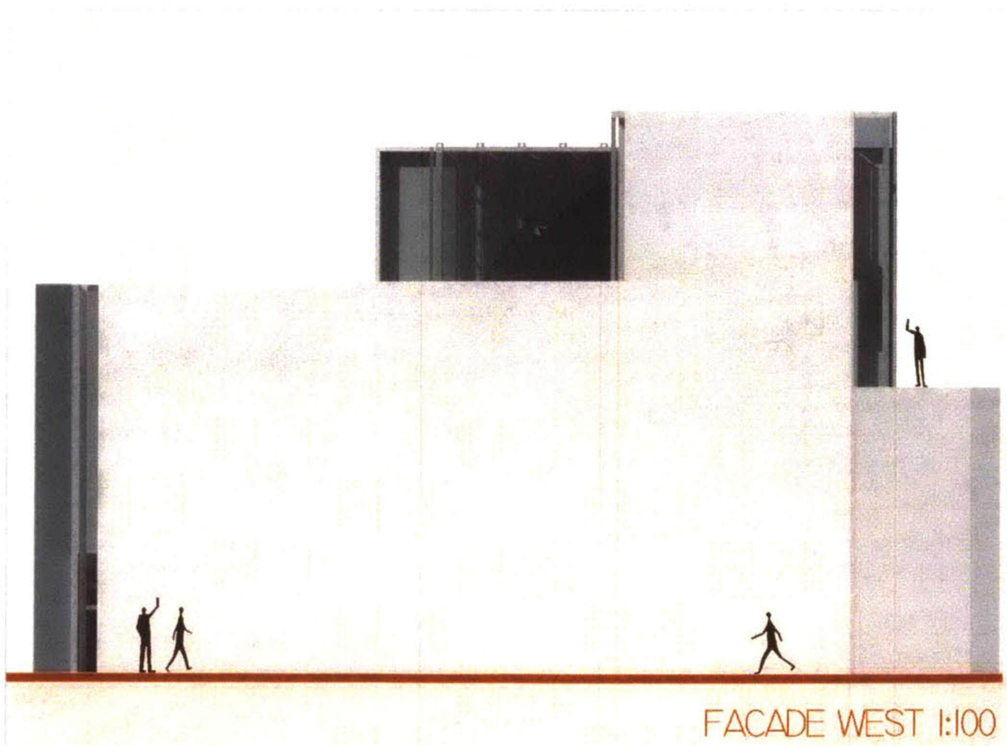


PLATE 9

ROOM 1

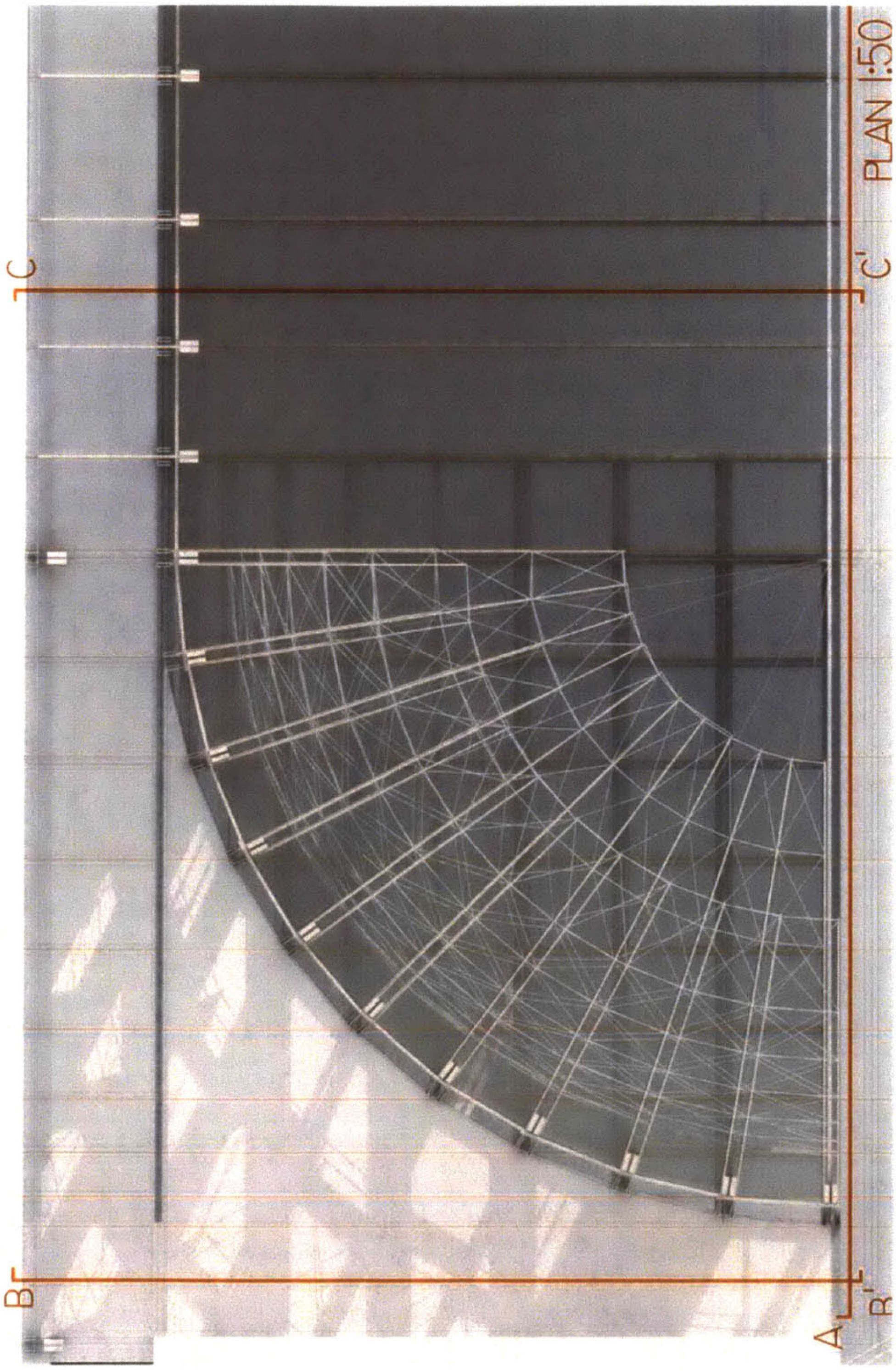


PLATE 10

PLATE 11

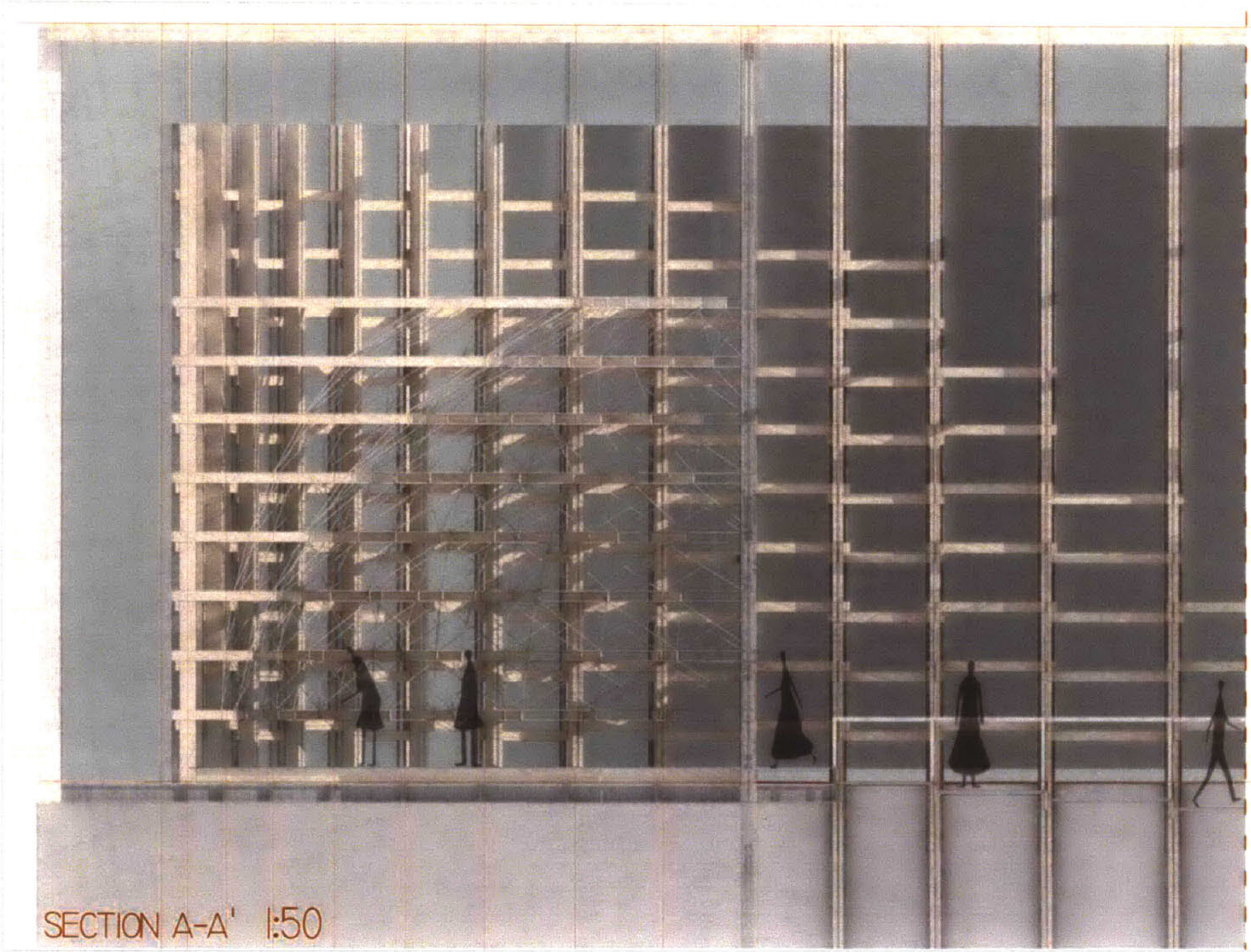
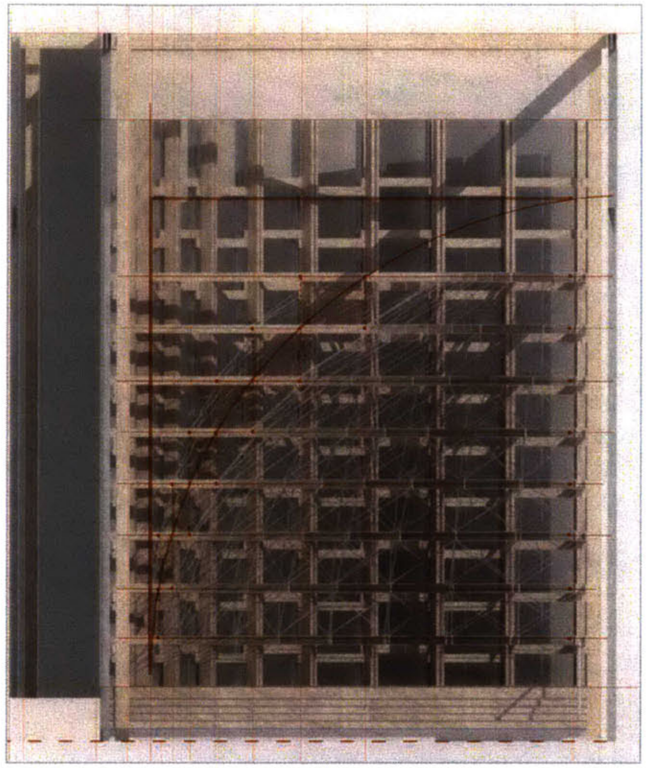


PLATE 12



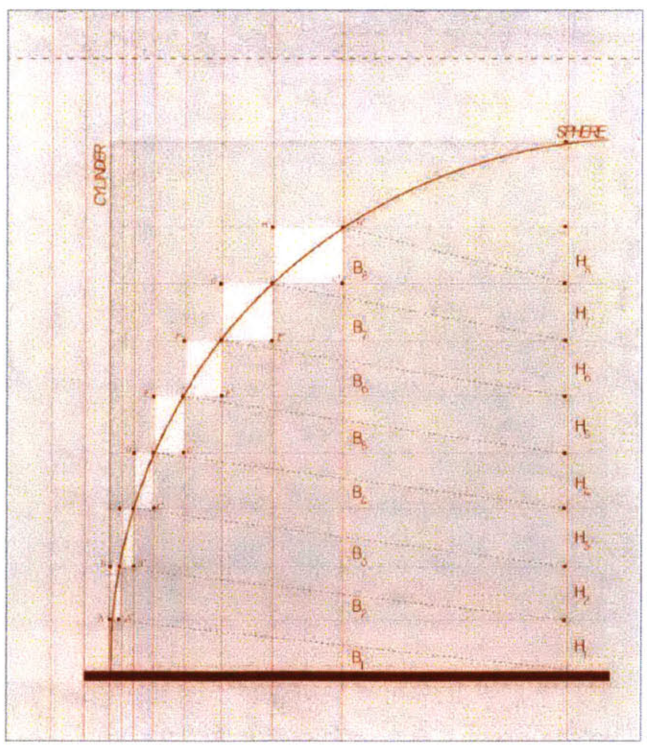


PLATE 13

ROOM 2

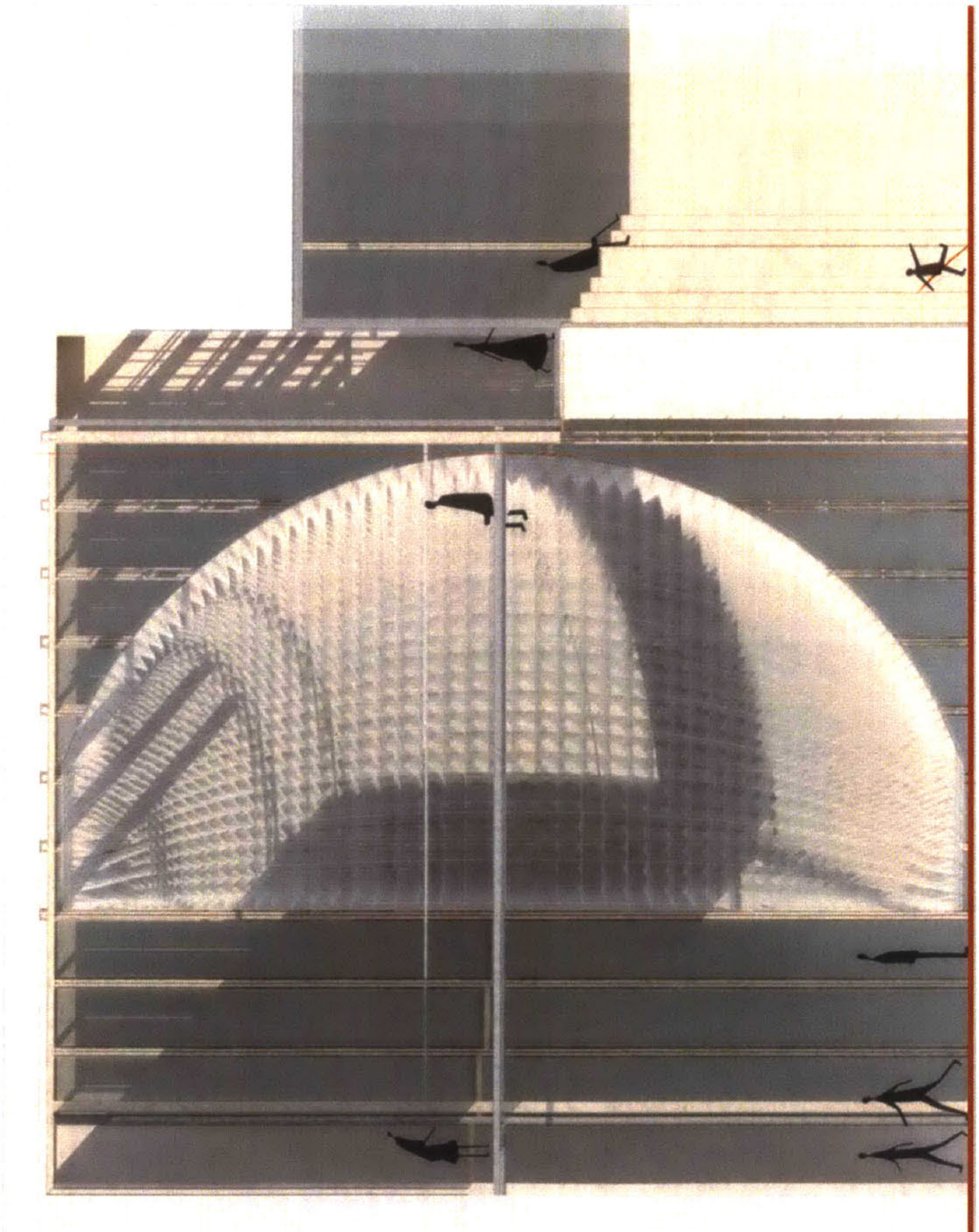
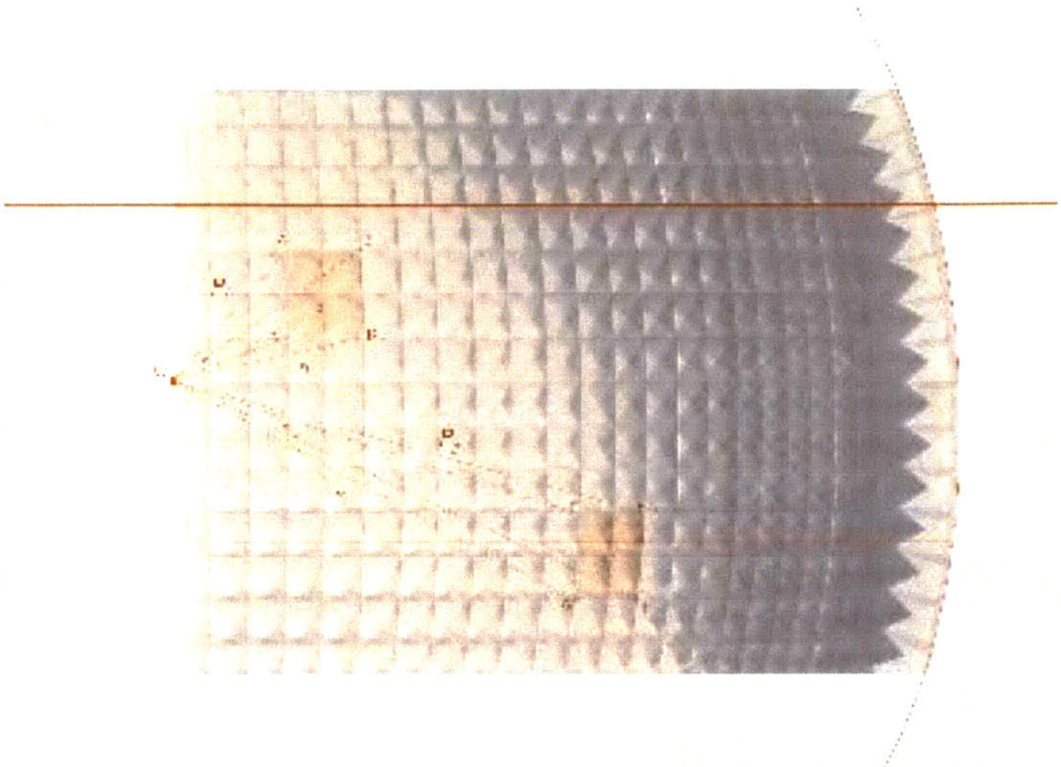


PLATE14



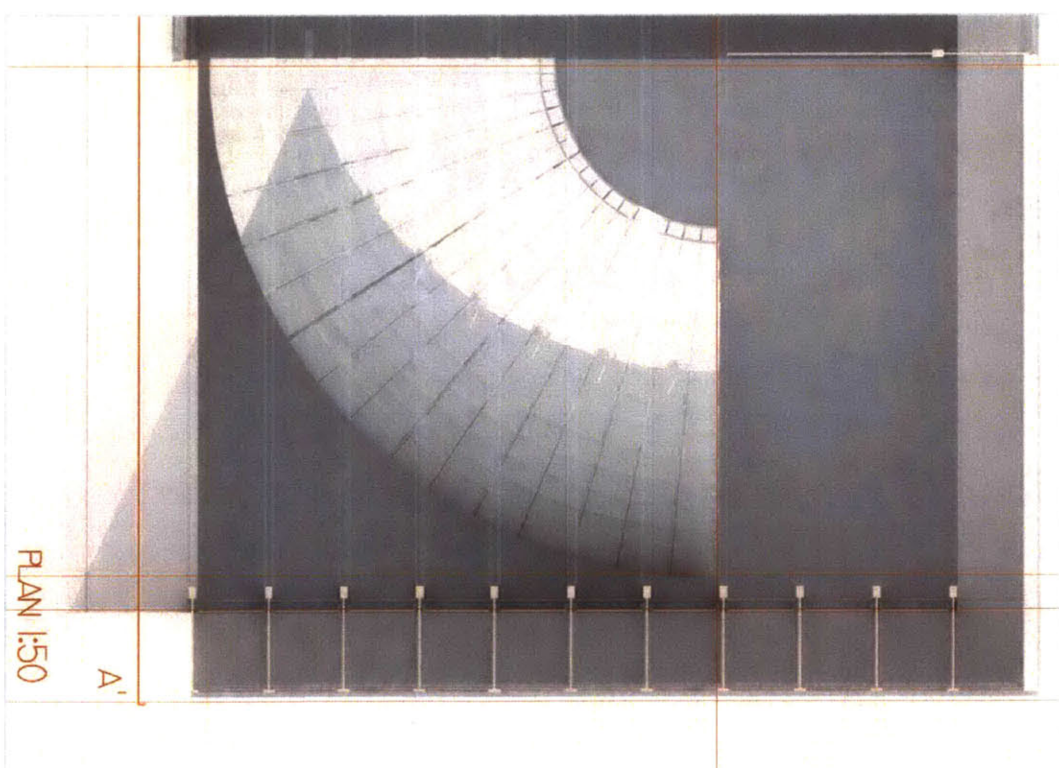


PLATE 15

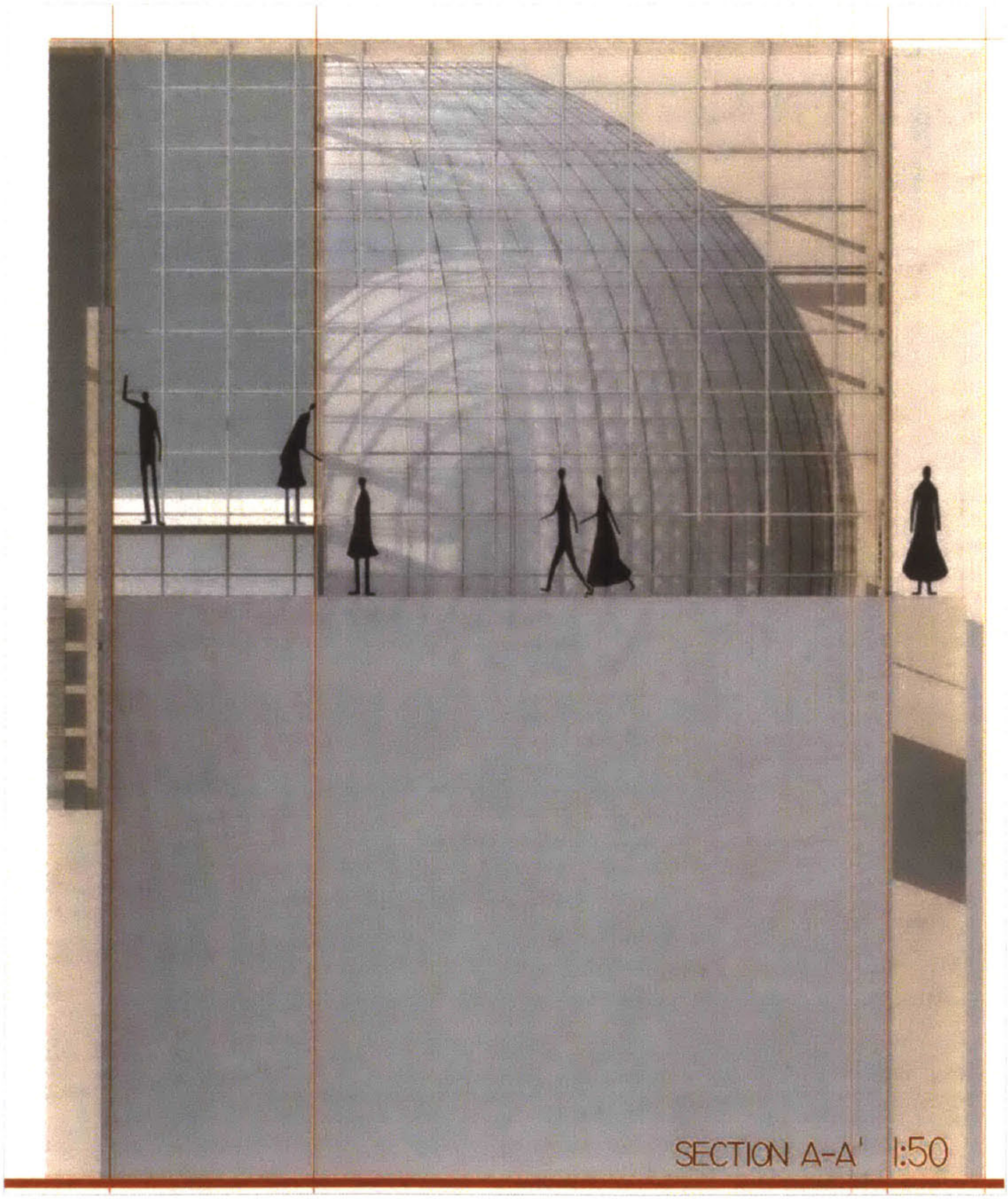


PLATE 16

ROOM 3

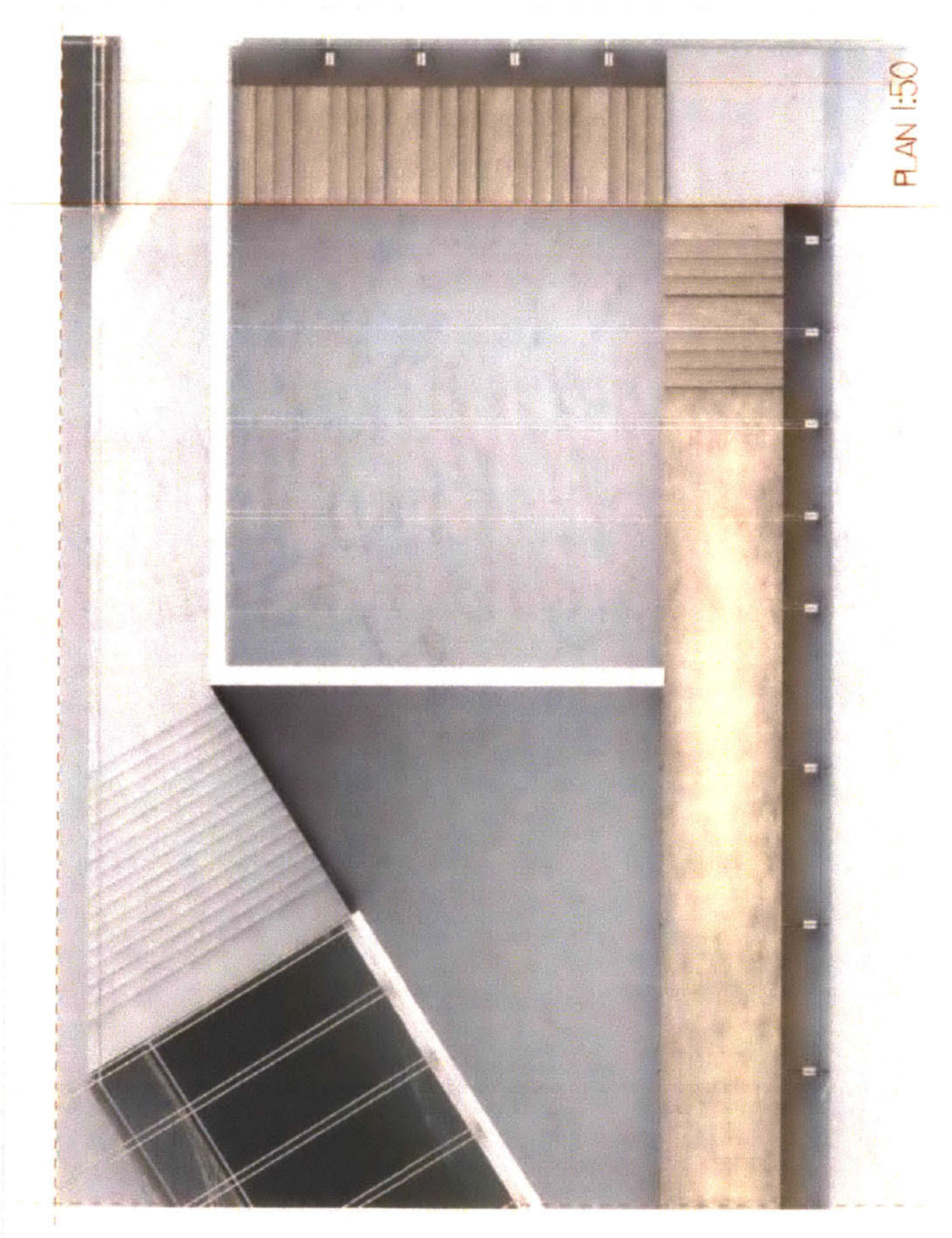


PLATE 17

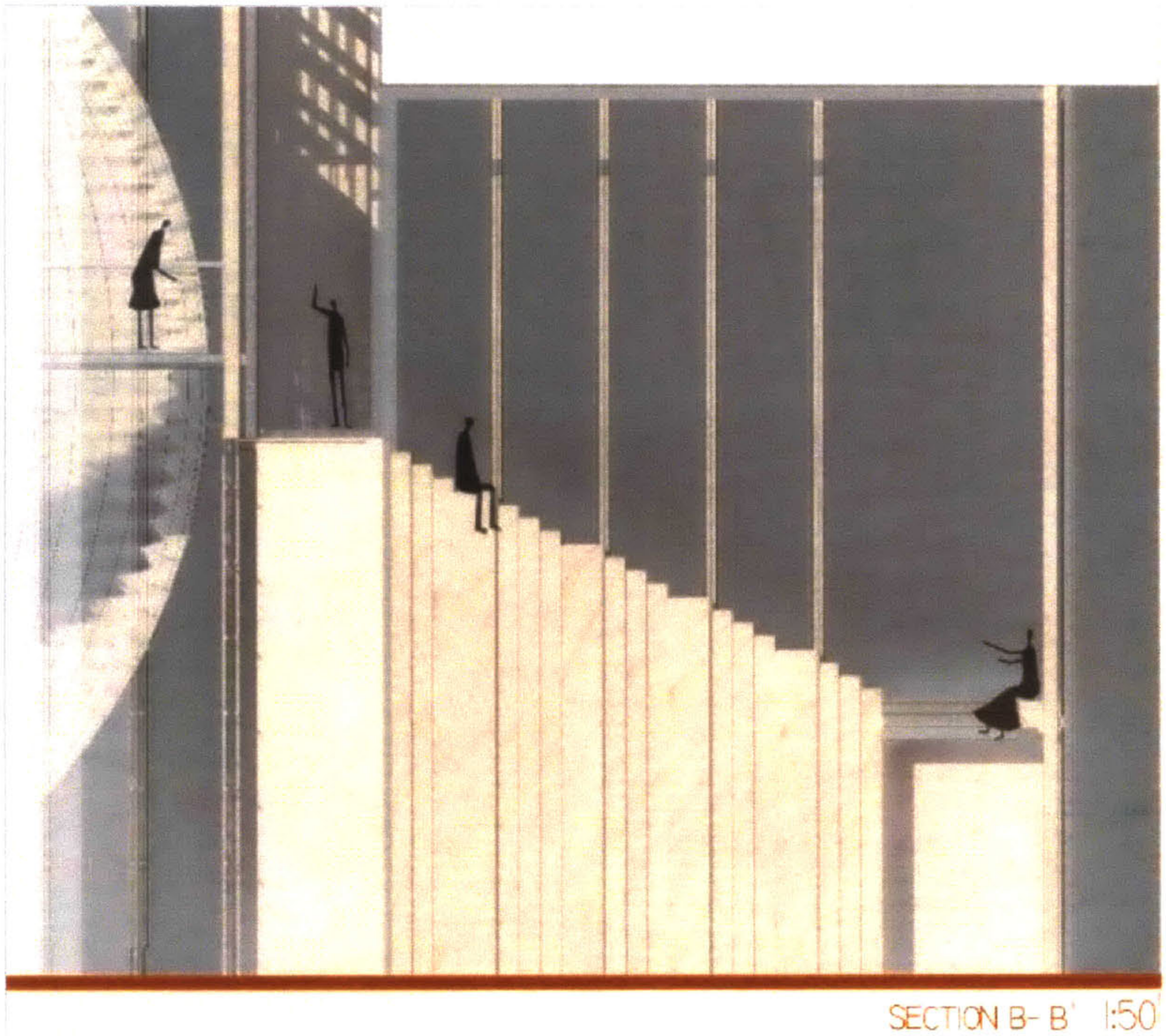


PLATE 18

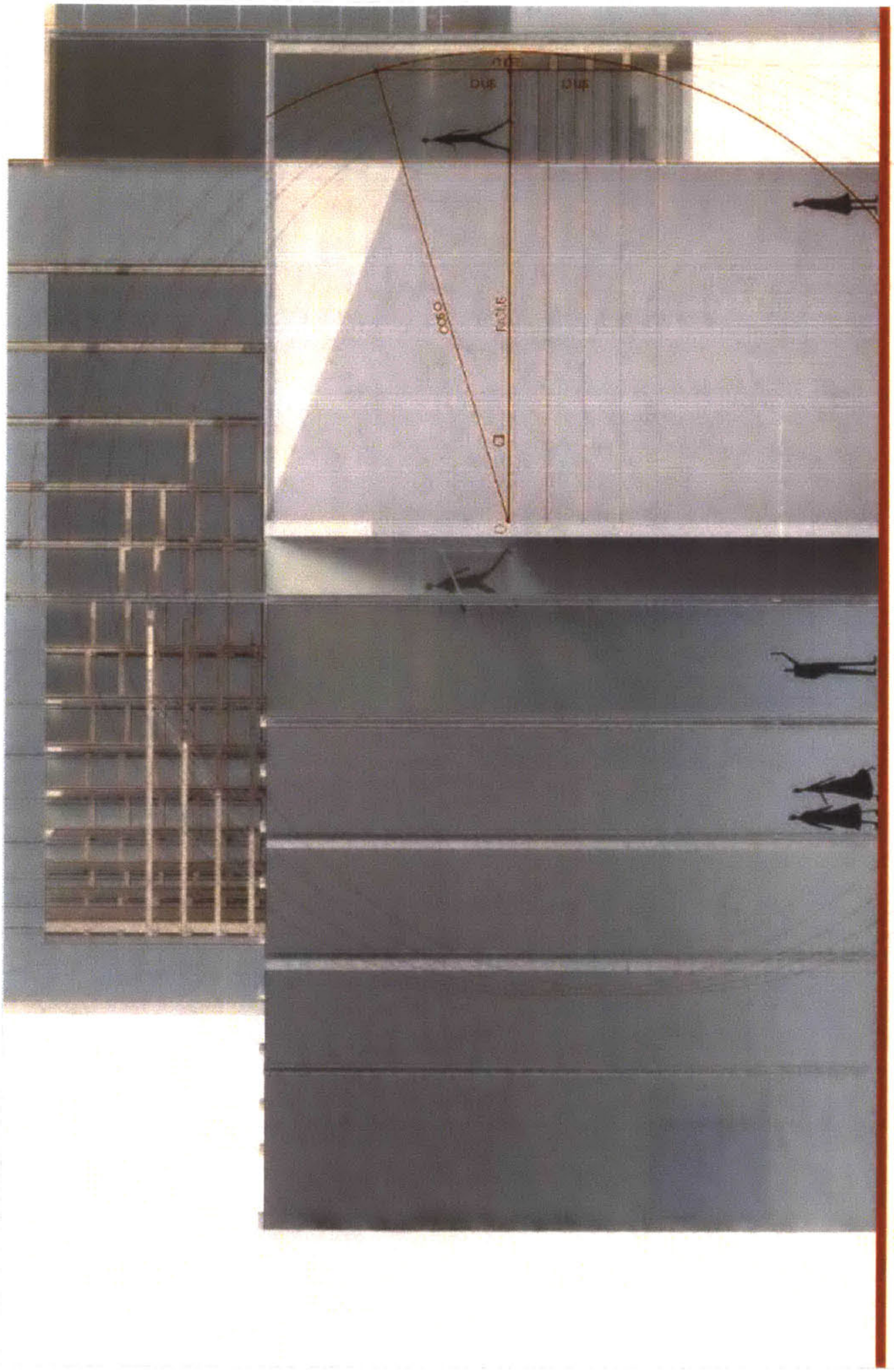


PLATE 19

ROOM 4

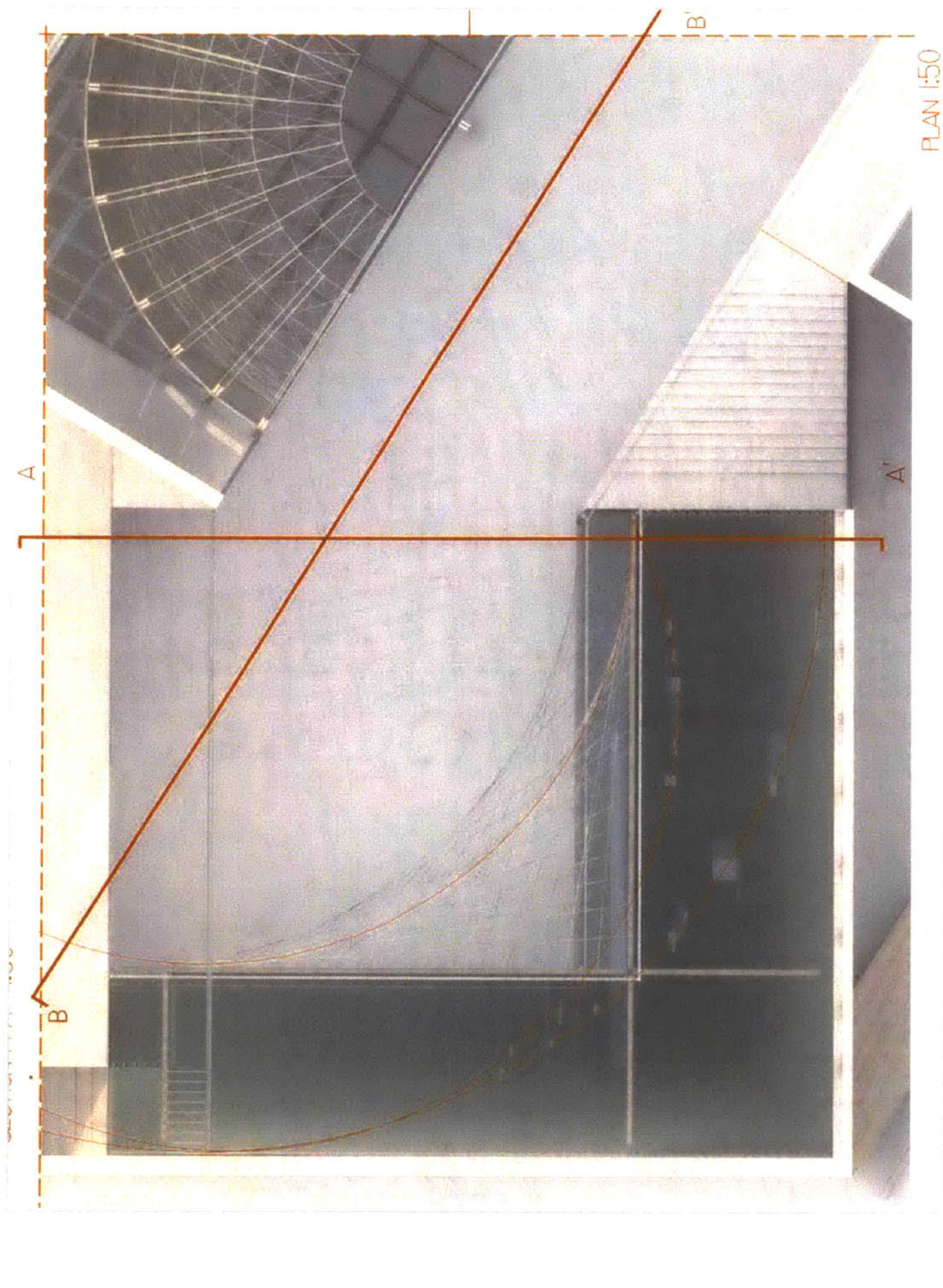
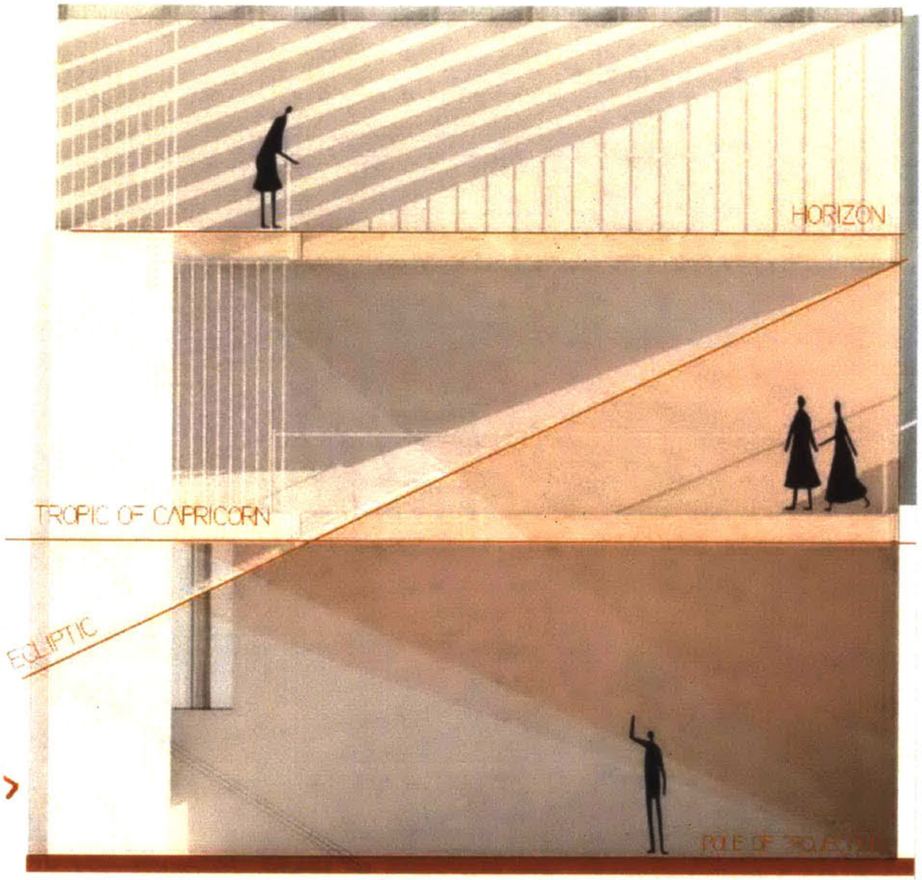


PLATE 20

PLATE

17



SECTION B- B' 1:50

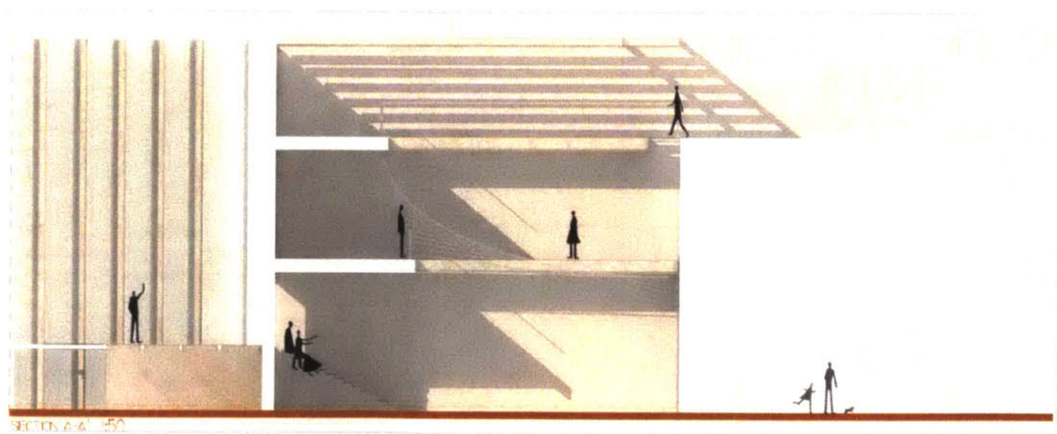


PLATE 21

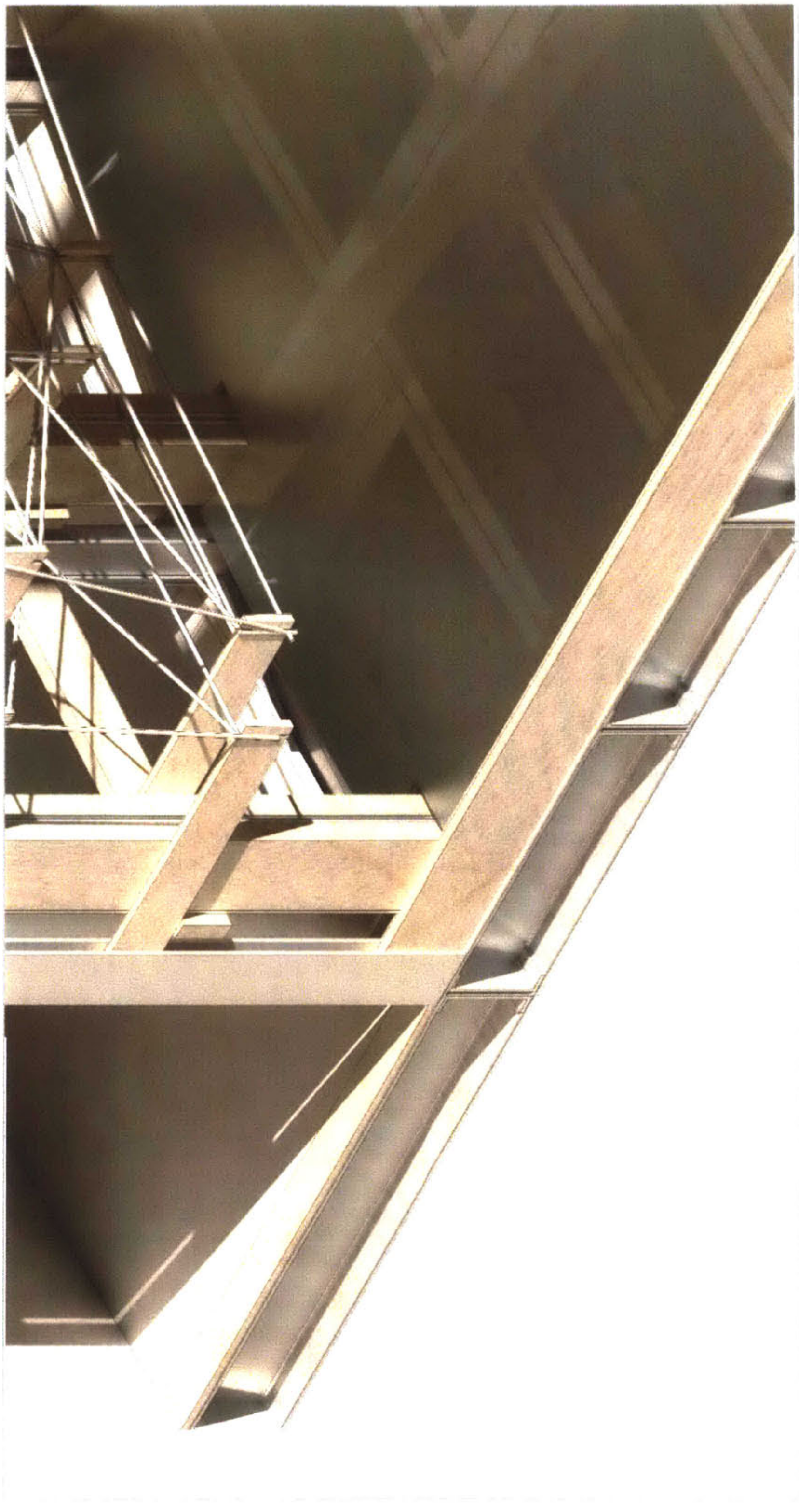


PLATE 22

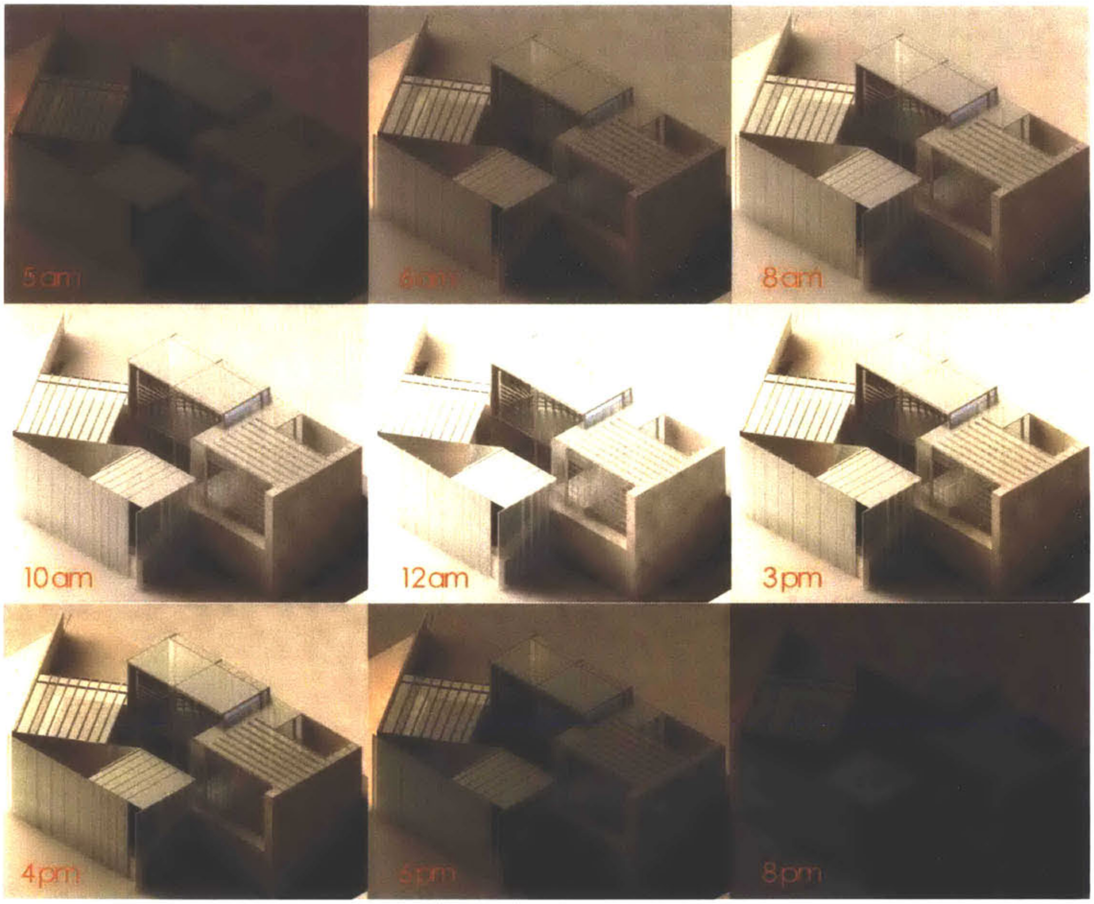


PLATE 23

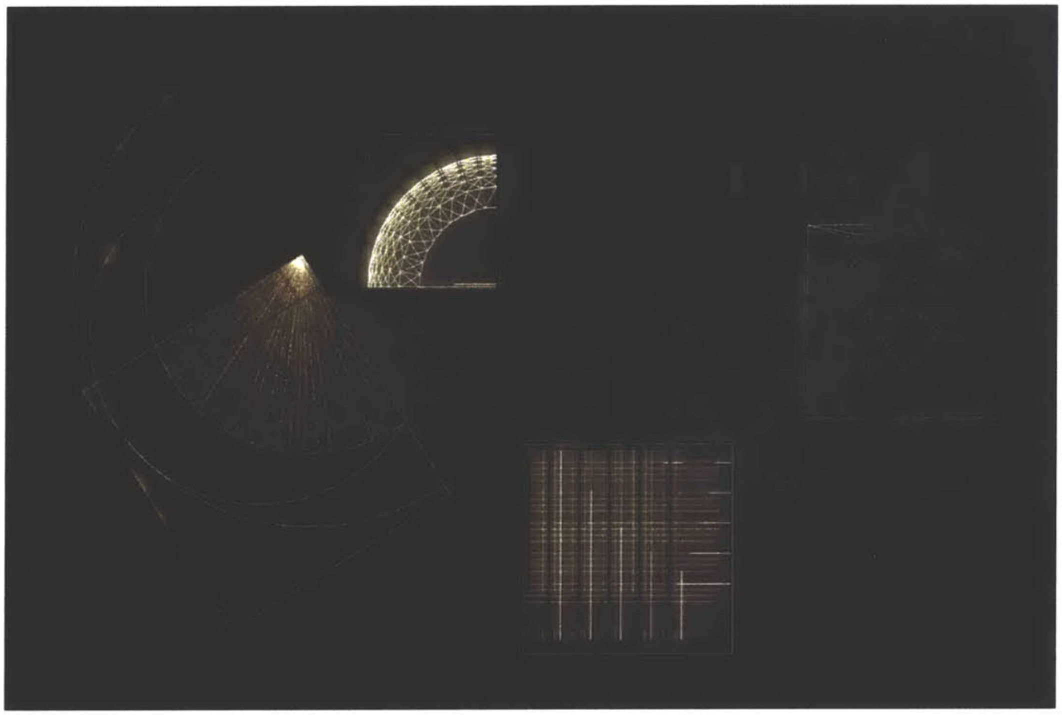


PLATE 24

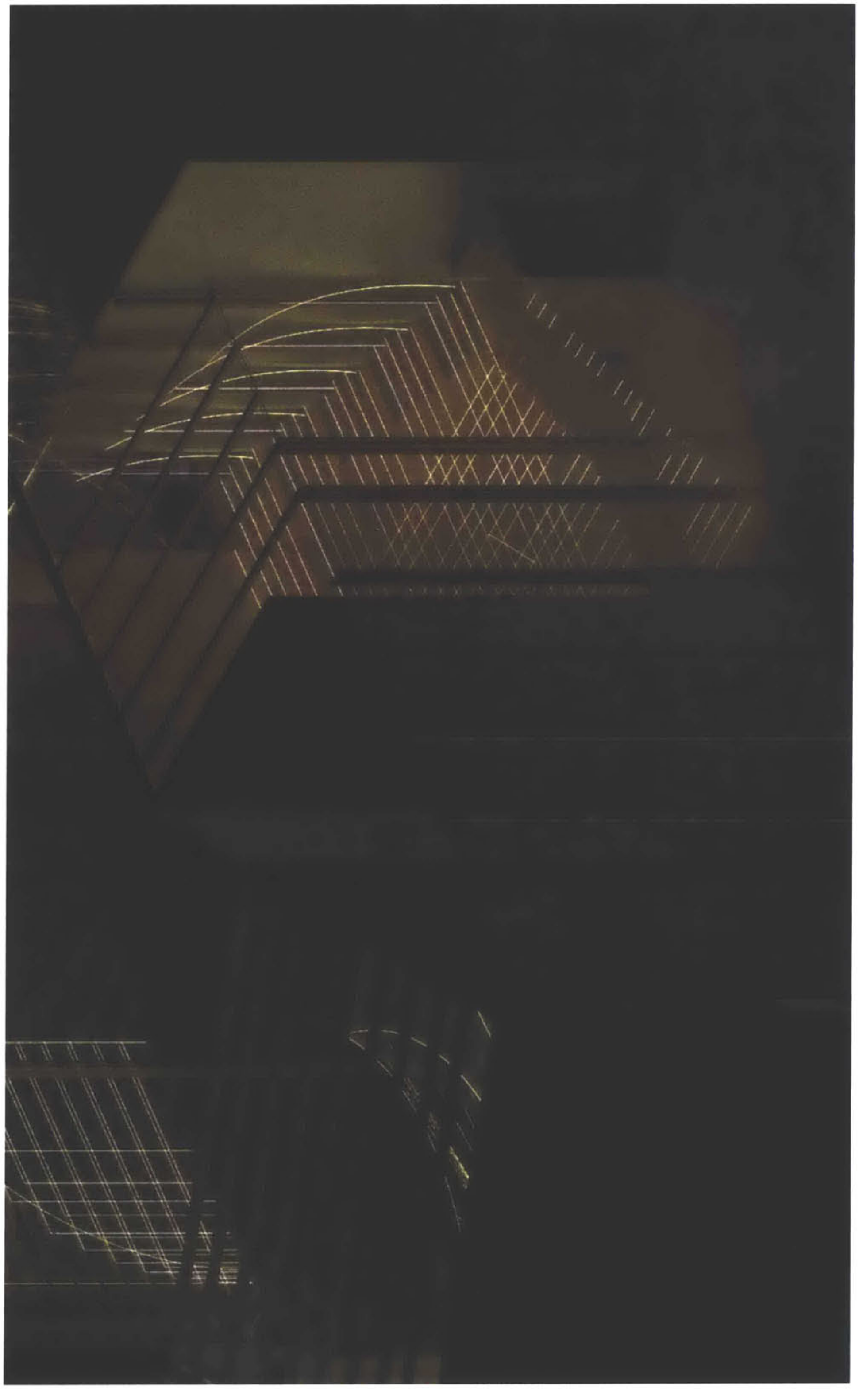
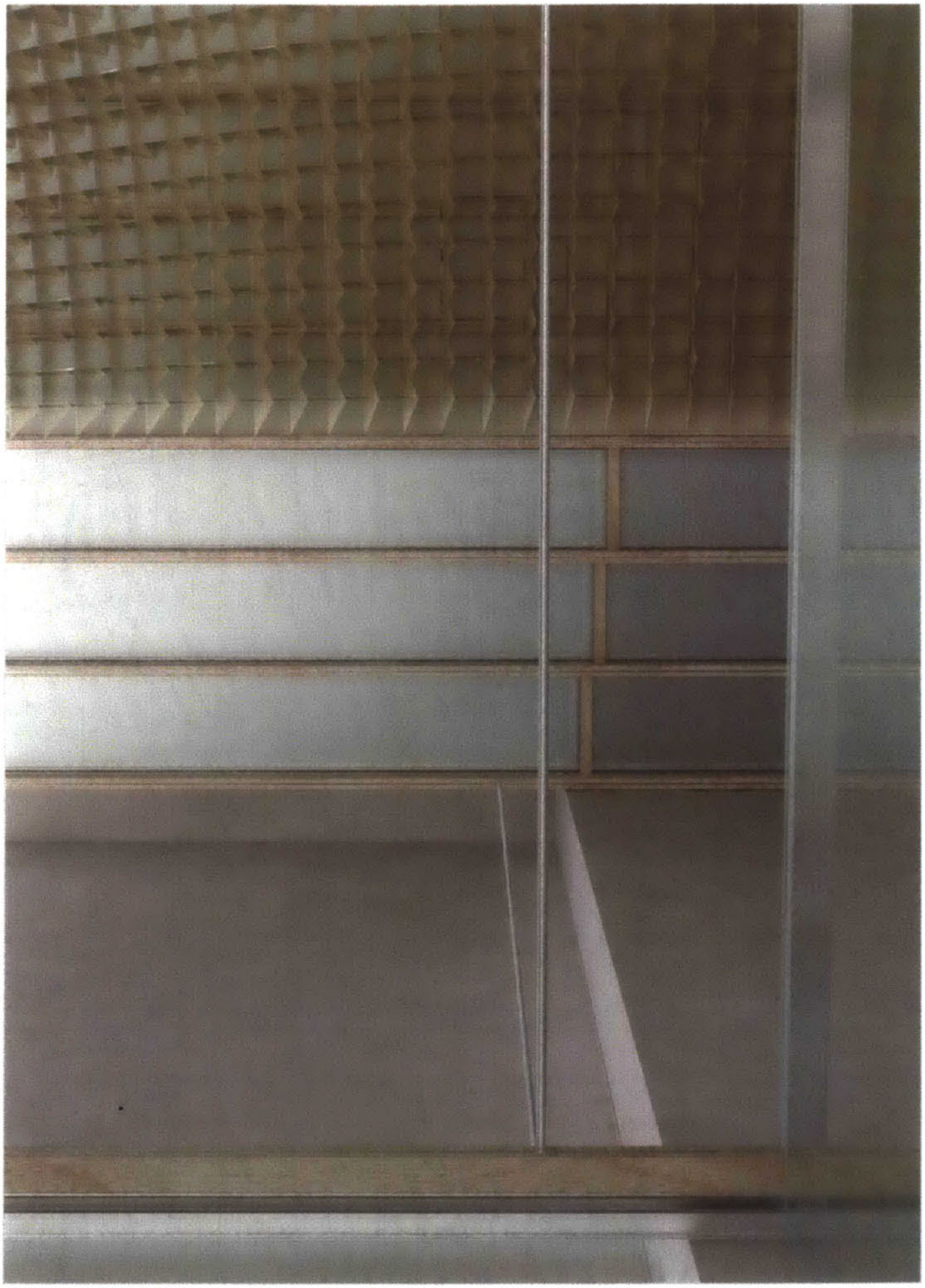


PLATE 25



5.3. SKETCHING

PLATE 26

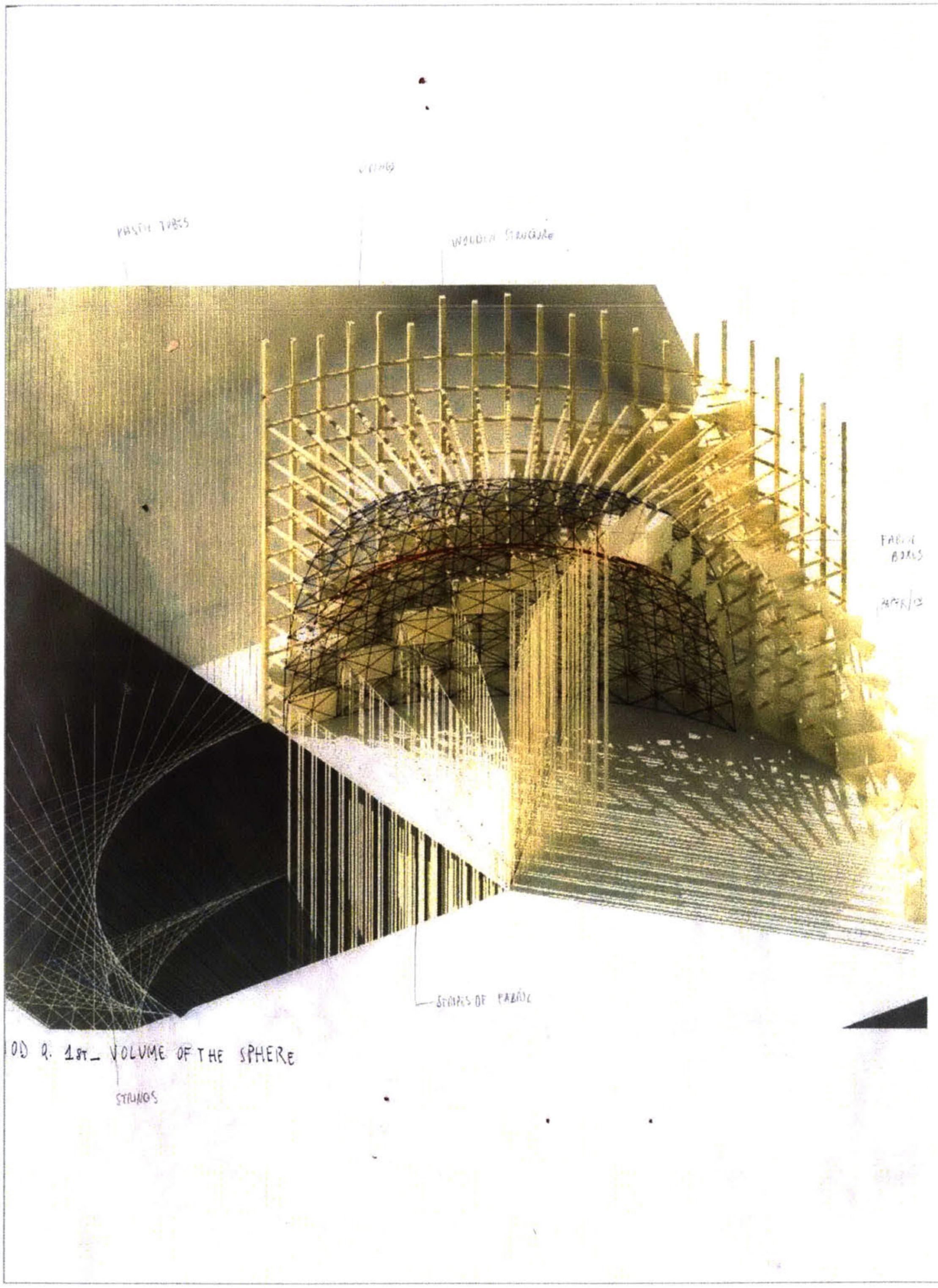

WOODEN STRUCTURE
STRIPS OF FABRIC
VOLUME OF THE SPHERE
STRINGS

PLATE 27

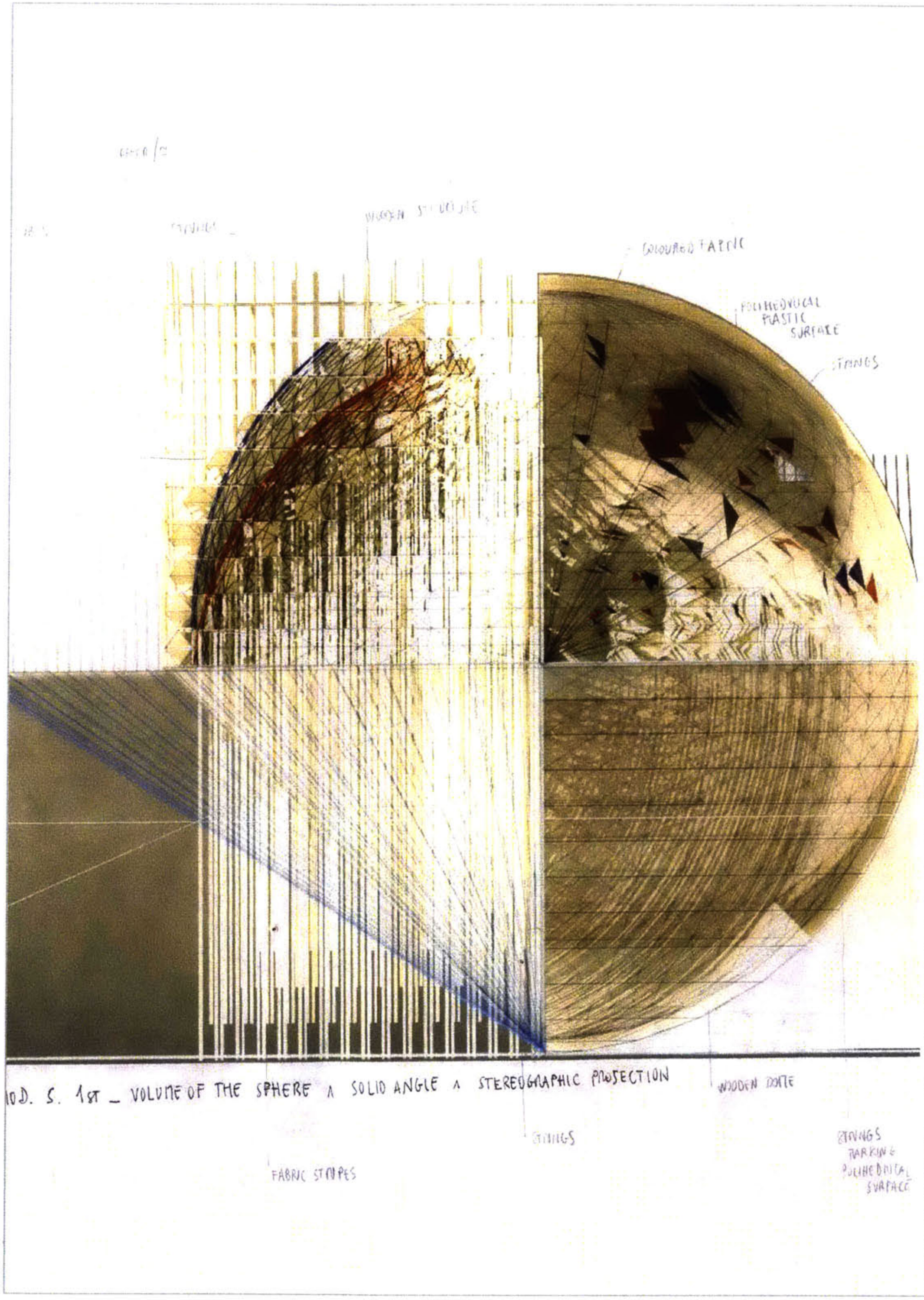

COLOURED FABRIC
POLIHEDRICAL PLASTIC SURFACE
OD. S. 1st _ VOLUME OF THE SPHERE ∧ SOLID ANGLE ∧ STEREOGRAPHIC PROJECTION
WOODEN DOME
FABRIC STRIPES

PLATE 28

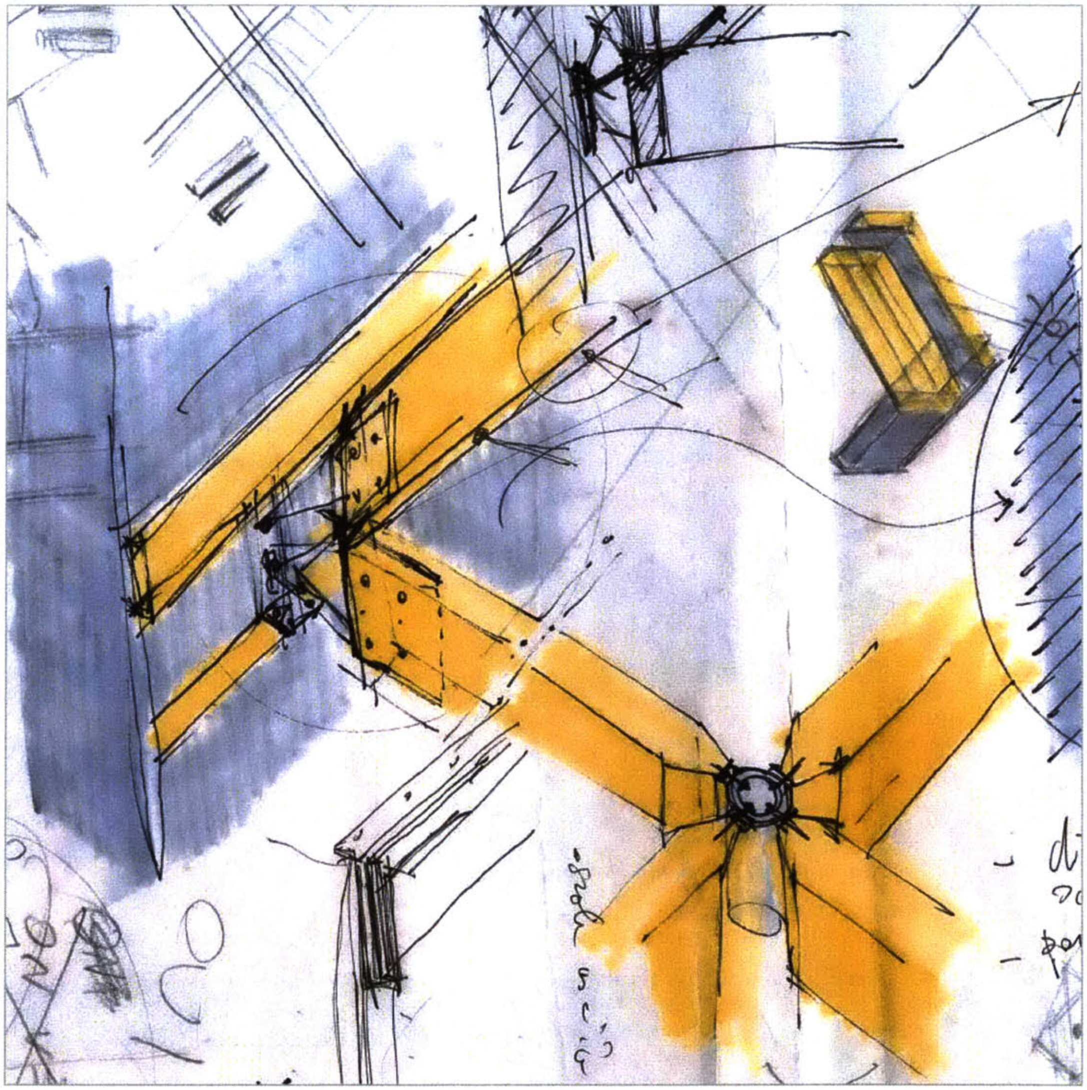
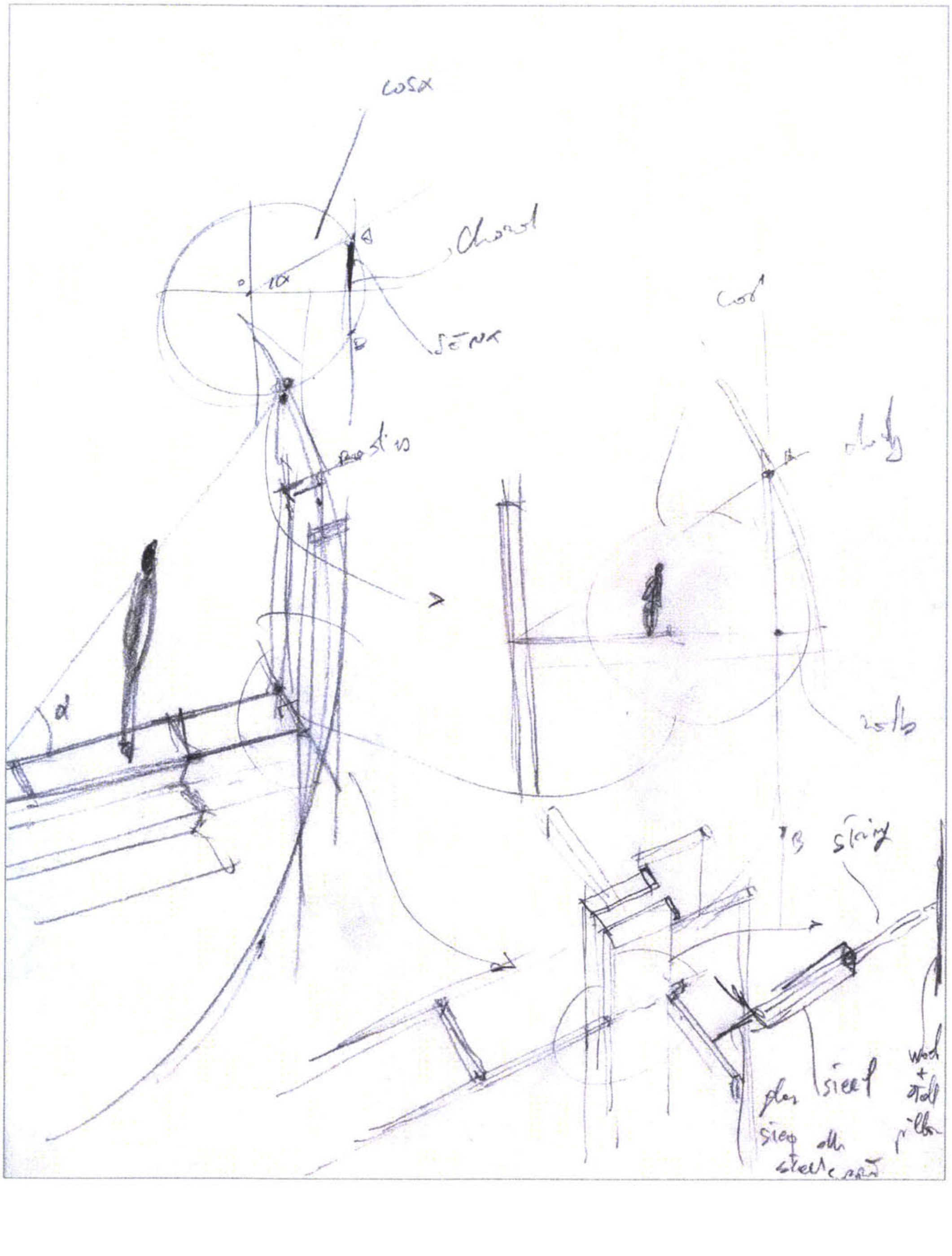


PLATE 29



PALTE 30

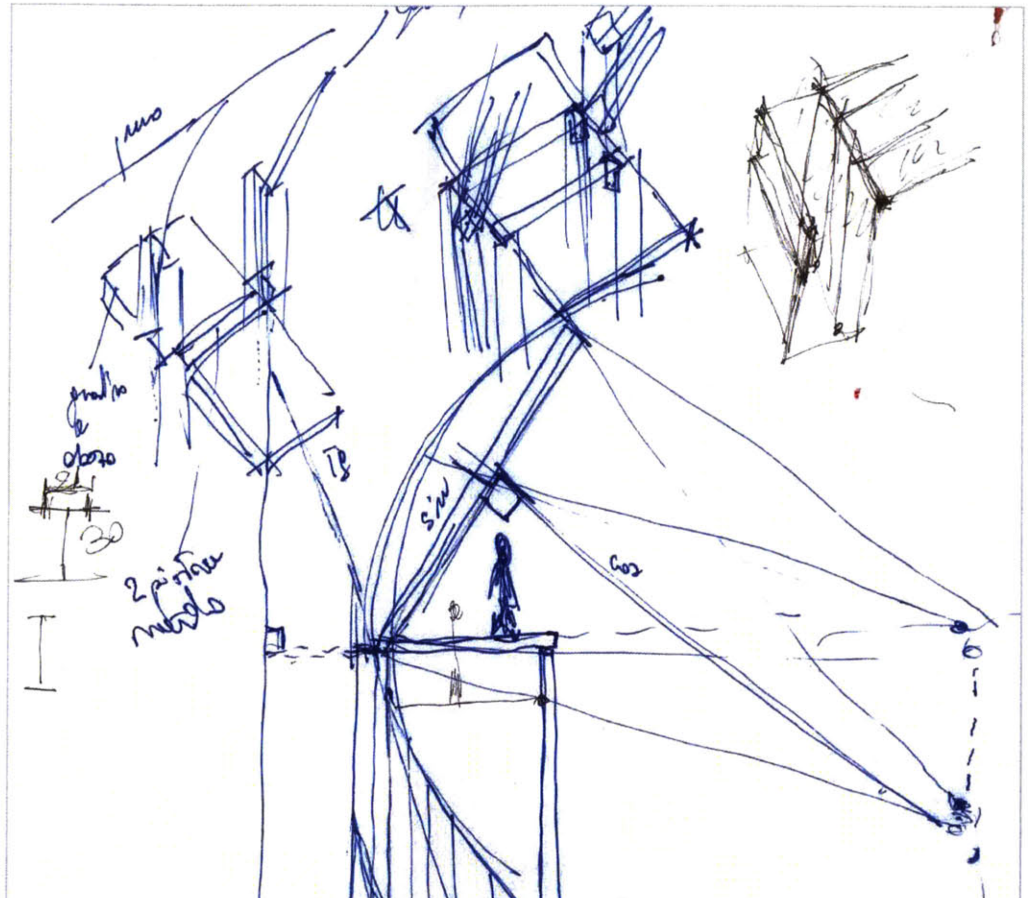
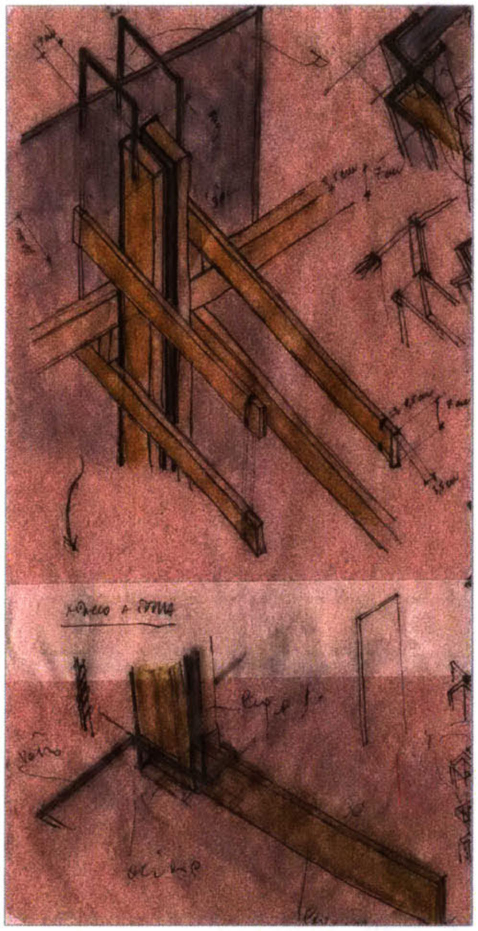
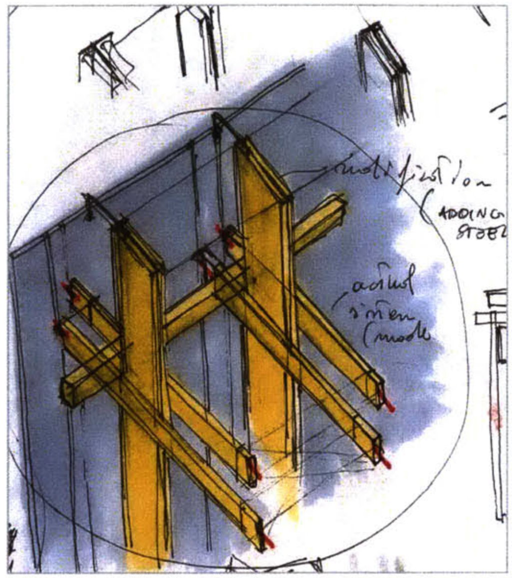
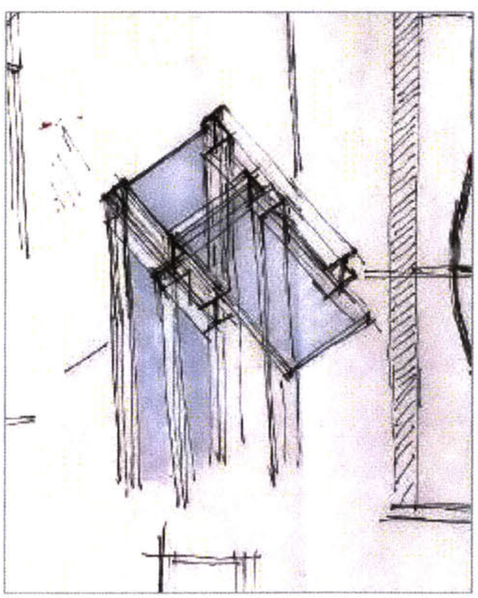

30
sin
cos

PLATE 31







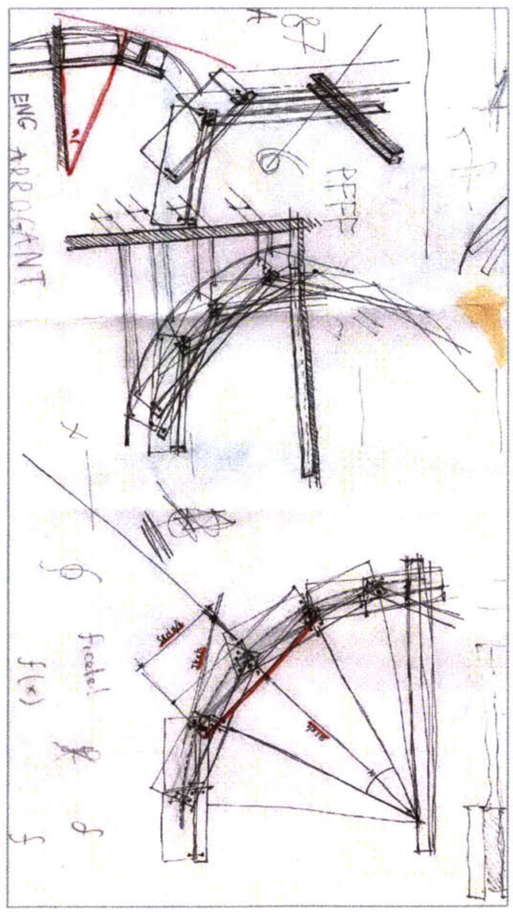


PLATE 32

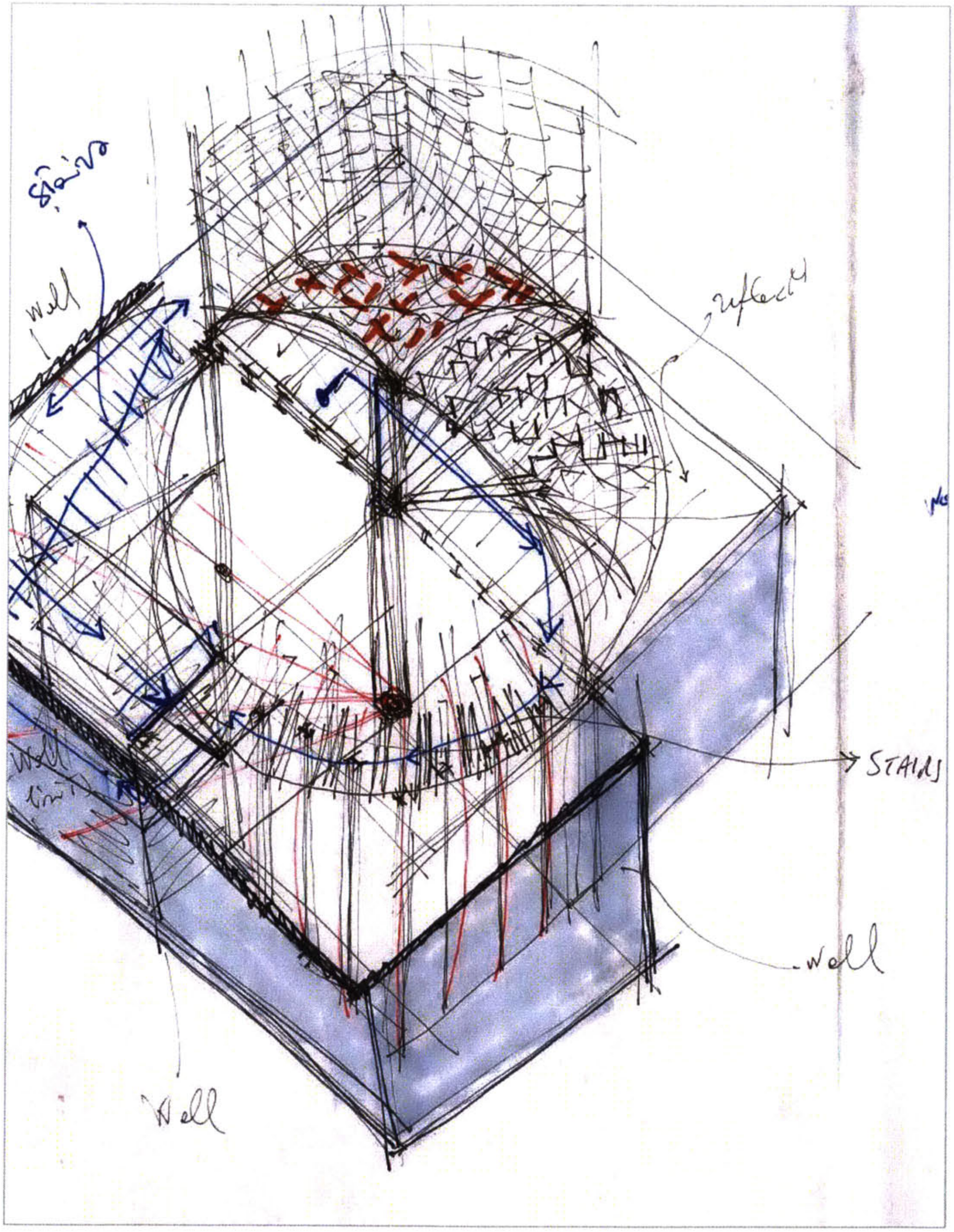


PLATE 33

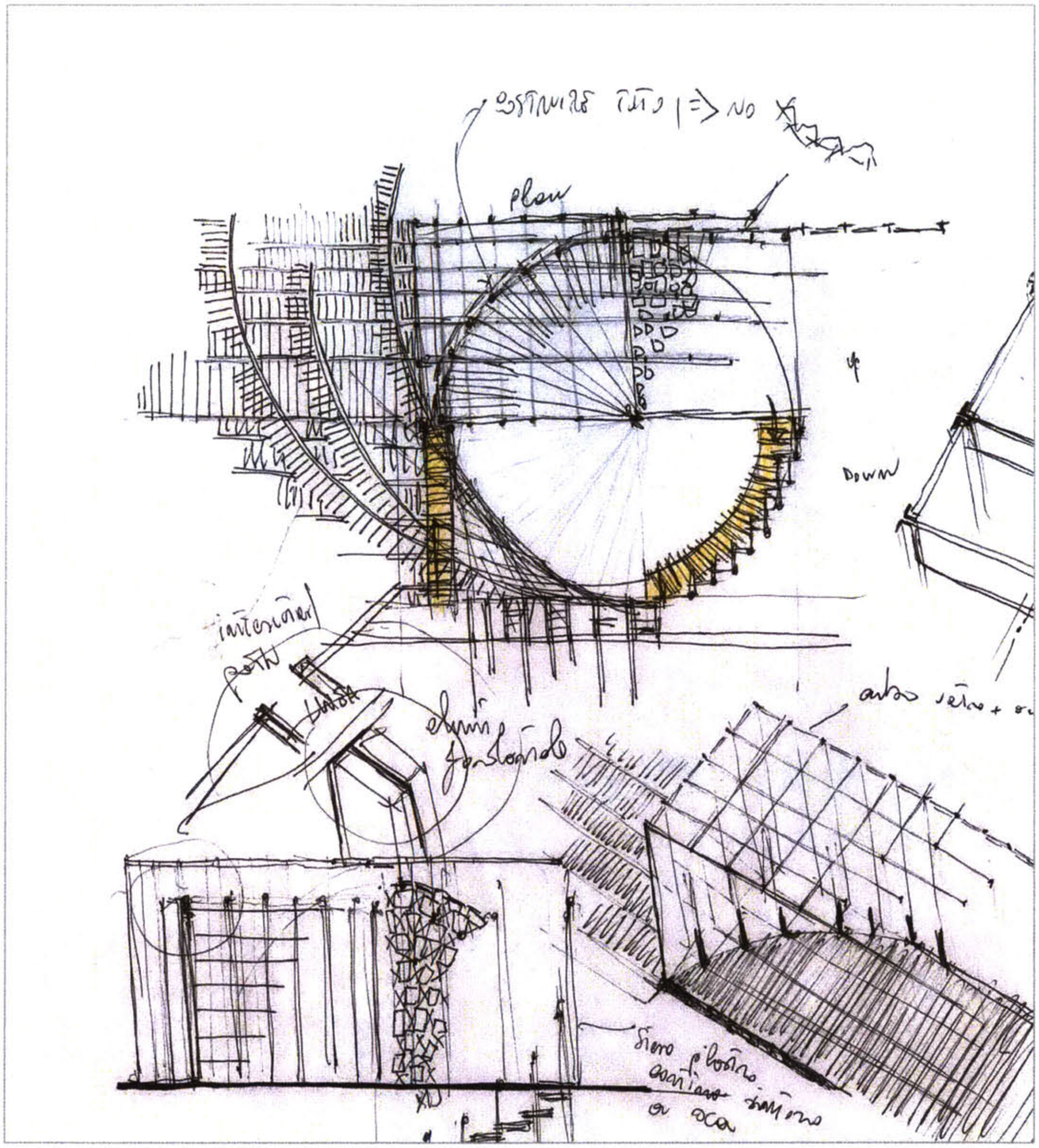


PLATE 34

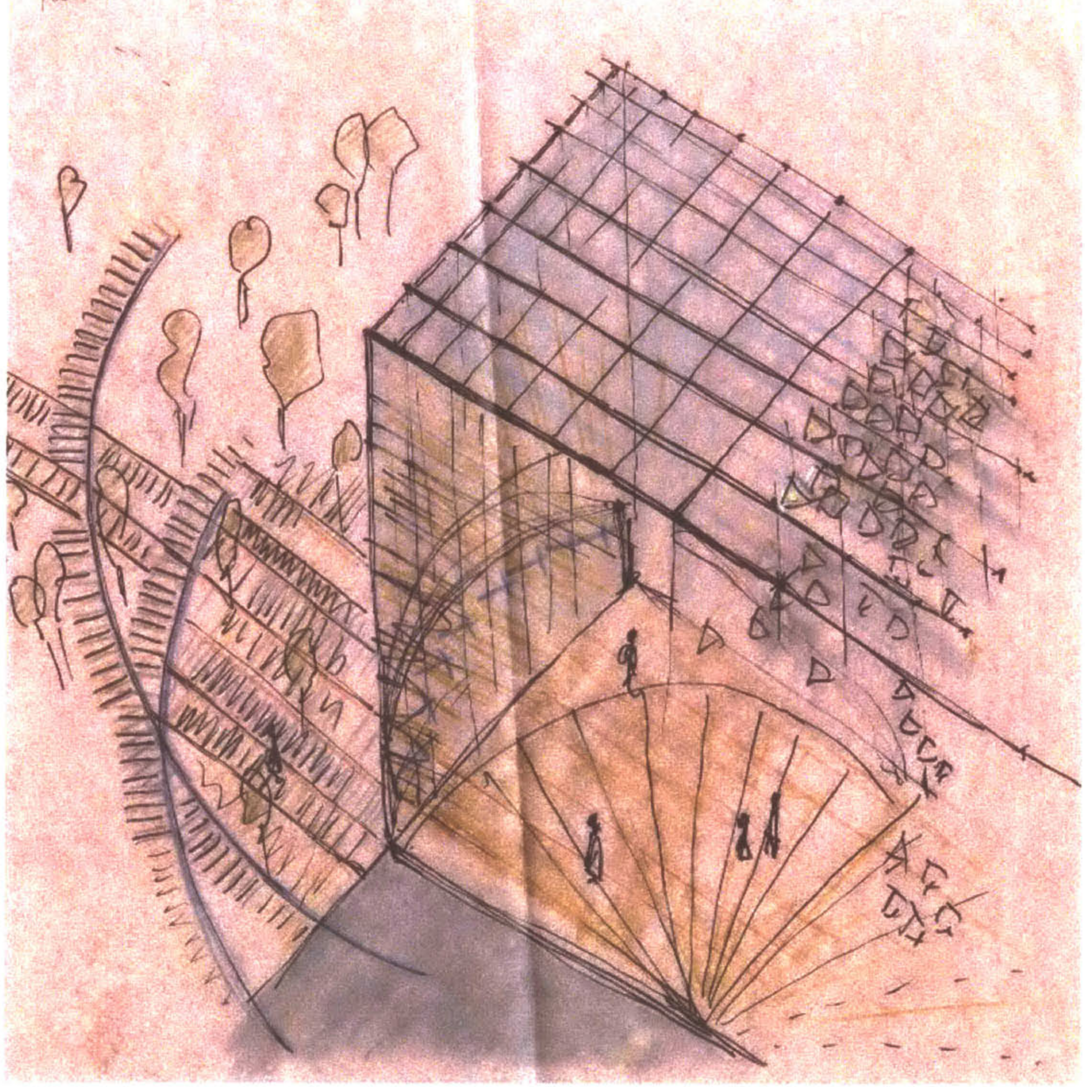
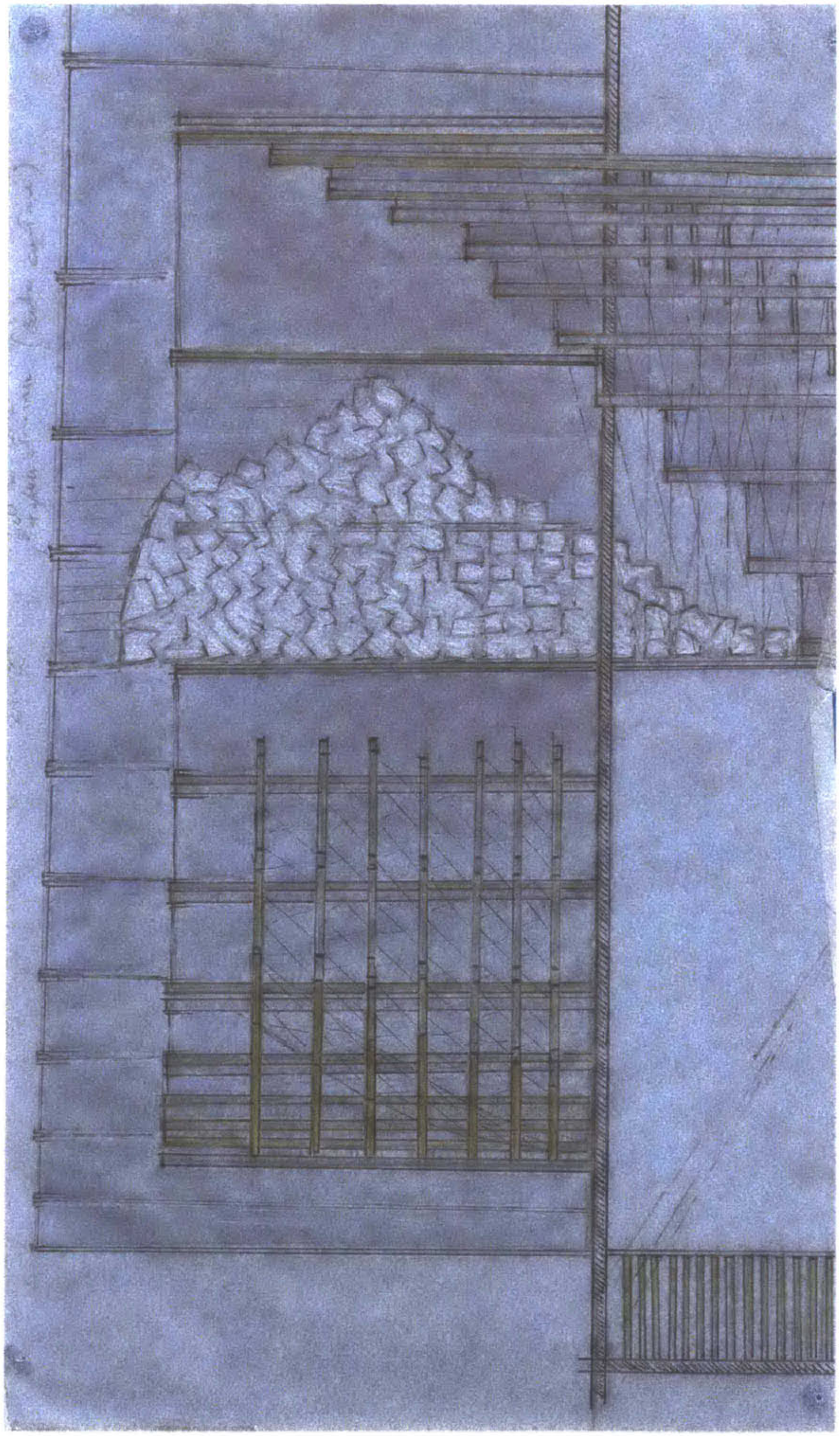
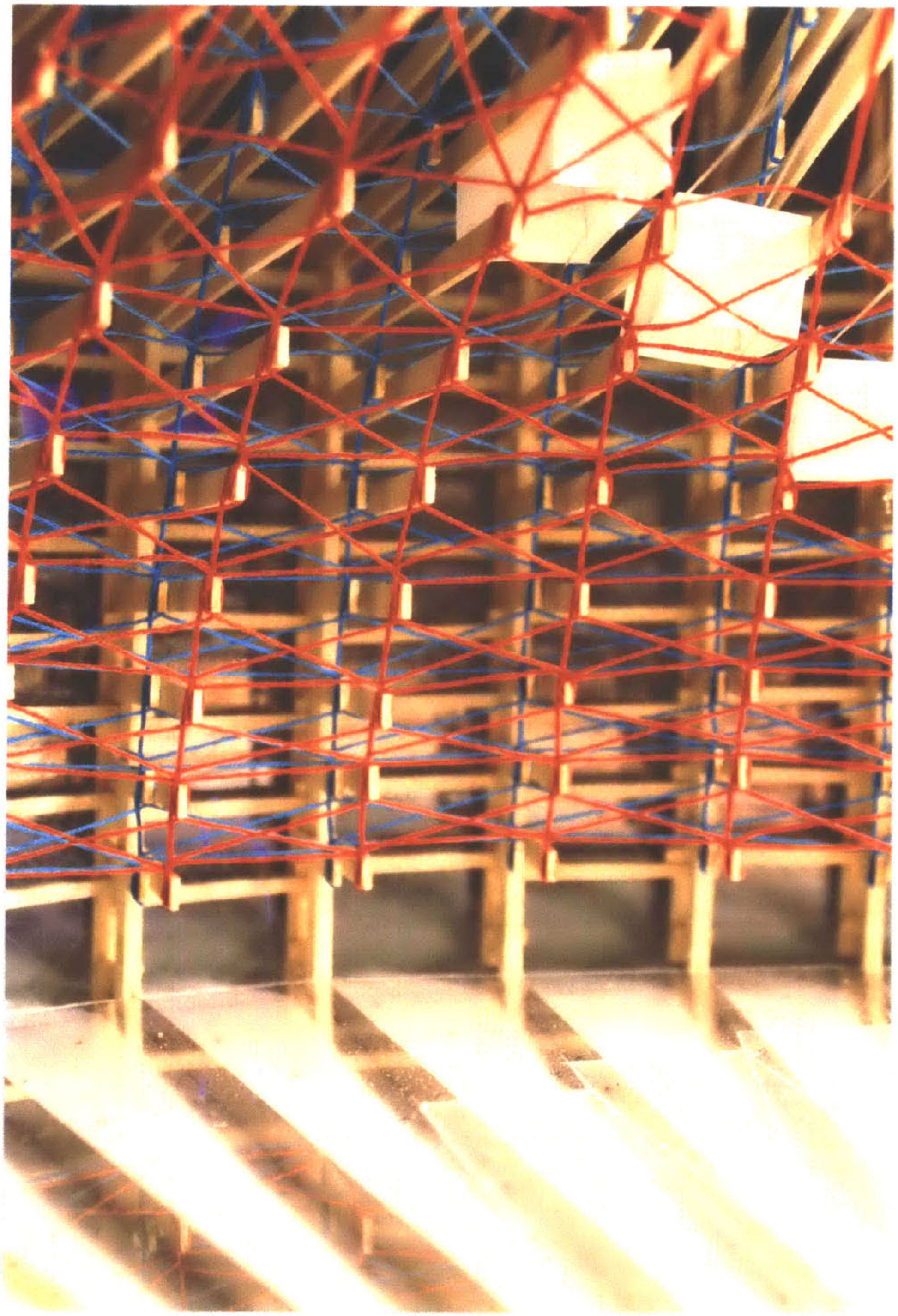


PLATE 35



5.3. PHYSICAL MODEL

PLATE 36



Room 1

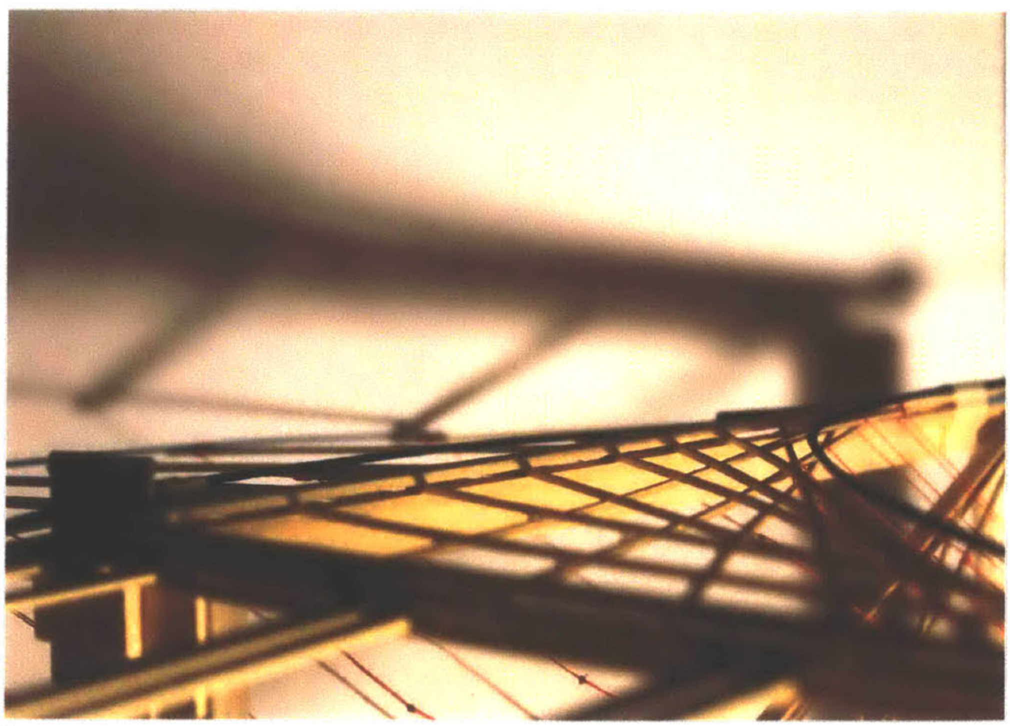
PLATE 37

Room 2

PLATE 38

Room 3

PLATE 39



Room 4

BIBLIOGRAPHY

Albertini B., Bagnoli S., *Carlo Scarpa. Architecture in details*, Milano, 1988

Bahamon A., *Sketch, plan, build*, New York, 2005

Baird D., *Thinking Knowledge. A philosophy of scientific instruments*, University of California press, Berkley, 2004

Baron M., *The origin of the infinitesimal calculus*, Pergamon Press, 1969

Benjamin W., *Sul Concetto di Storia,* Frankfurt, 1974

Berggren J.L., *Episodes in Mathematics of Medieval Islam*, New York, 1986

Calvino I., Six memos for the next millennium, Harvard University Press, Cambridge, 1988

Charette F., *Mathematical instrumentation in fourteenth-century Egypt and Syria*, Leiden, 2003

Chemla K., *The History of Mathematical proofs in ancient traditions*, Cambridge University Press, 2012

P.Cook, *Drawing. The motive force of architecture*, West Sussex, 2008

Curd M., Cover J.A., *Philosphy of Science*, London, 1998

Falmagne J., Doignon J., *Learning spaces*, Berlin, 2011

Frascari M., Hale J., Starkey B. (edited by), *From models to drawings : imagination and representation in architecture*, London, 2007

Frege G., *Conceptual Notations*, Oxford University Press, 1972

Gibbs S.and Saliba G., *Planispheric astrolabes. The national museum of American history*, Washington, 1984

Goldstein B.R., *Theory and Observation in Ancient and Medieval Astronomy*, London, 1985

Grob B., Hooijmaijers H., *Who needs scientific instruments*, Leiden, 2005

Gunther R.T., *The astrolabes of the world*, London, 1976

Hilbert D., *Foundations of Geometry*, La Salle, 1971

Hogendijk J.P. and Sabra A.I., *The enterprise of Science in Islam*, MIT Press, Cambridge, 2003

d'Hollander R., *L'astrolabe:histoire,théorie et pratique*, Paris, 1999

Høyrup J., *In Measure, Number, and Weight. Studies in Mathematics and Culture*, State University of New York Press, 1994

King D.A., *Islamic Astronomical Instruments*, London, 1987

King D.A., *Islamic astronomy and geography*, Farnham, 2012

King D.A., *Astronomy in the service of Islam*, 1993

Klee P., Pedagogical Sketchbook, London, 1968

Kliment S., *Architectural sketching and rendering : techniques for designers and artists*, New York, 1984

Lee J.M., *Axiomatic Geometry*, Providence, 2010

Lorch R. (edited by), *Al Farghani. On the astrolabe*, Stuttgart, 2005

Morelon R., Hasnawi A., *De Zénon d'Élée à Poincaré. Recueil d'études en hommage à Roshdi Rashed*, Louvain-Paris, 2004

Masrour F., *The geometry of visual space and the nature of visual experience*, University of Wisconsin, 2014

Meuser N., *Construction and Design Manual. Architectural Drawings*, Berlin, 2012

Neugebauer O., *A history of Ancient Mathematical Astronomy*, vol. 1-3, Berlin, 1975

Pincock C., *Mathematics and Scientific Representation*, Oxford University Press, 2012

Russell B., *Introduction of Mathematical Philosophy*, New York, 1972

Rashed R., *A history of Arabic sciences and mathematics*, vol.1-4, New York, 2014

Rashed R., *Geometry and Dioptrics in classical Islam*, London, 2005

Rosenthal F., *Knowledge Triumphant: the concept of knowledge in the medieval Islam*, Leiden, 2007

Rossi A., Disegni 1990-1997, Milano, 1999

Saliba G., *A history of Arabic Astronomy. Planetary theories during the Golden Age of Islam*, New York, 1994

Saliba G., *Islamic science and the making of European Renaissance*, MIT Press, 2007

Torretti R., *Philosophy of Geometry from Riemann to Poincaré*, Dodrecth, 1978

Van Brummelen G., *The mathematics of the heavens and Earth*, Princeton University Press, 2009