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REPORT NO. 87

SEPARATION OF TURBULENT, INCOMPRESSIBLE FLOW FROM A CURVED, BACKWARD-FACING STEP

GEORGE R. NICE
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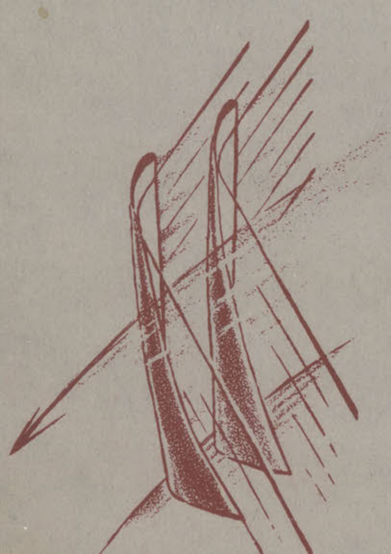
Research was carried out under the Bureau of Ships General Hydromechanics Research Program, S-R009 01 01, Administered by the David Taylor Model Basin

Contract Nonr 1841(91)



OCTOBER 1966

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
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Gas Turbine Laboratory

Massachusetts Institute of Technology

June 1966

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ABSTRACT

An experimental investigation of turbulent, incompressible flow separation over curved and sharp, backward-facing steps is presented with results for various step heights. Mean velocities in the separating boundary layer as well as the downstream shear layer were recorded. The static pressure in the separated region was determined with a spherical probe.

With the curved step, the boundary layer separated at approximately 28 degrees: the reattachment lengths were somewhat less and the base pressures slightly higher than those with the sharp step.

ACKNOWLEDGEMENTS

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The numerical calculations were performed with the aid of the M.I.T. Computation Center, and Mrs. Madelyn Euvrard typed the report.

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NOMENCLATURE

C_P	Pressure Coefficient, $P - P_o/1/2\ \rho U_o^2$
$C_{P_{st.}}$	Static Pressure Coefficient, $P - P_{st.w.}/1/2\ \rho U_o^2$
C_f	Skin Friction Coefficient, $\tau_w/1/2\ \rho U^2$
H	Boundary Layer Shape Factor, δ^*/θ
h	Step height
m	Reattachment Length
n	Distance Normal to Curved Surface
P	Static Pressure
P_o	Static Pressure at Test Section Inlet
$P_{st.w.}$	Wall Static Pressure
$P_{sph.st.}$	Spherical Probe Reading
P_t	Total Pressure
s	Distance Along Curved Surface
U	Free Stream Velocity
U_o	Velocity at Test Section Inlet
u	Velocity in Viscous Region
W_o	Channel Height above Step
x	Distance Parallel to Inlet Flow Direction
y	Distance Normal to Inlet Flow Direction
α	Angular Position on Curved Step
δ	Boundary Layer Thickness
ρ	Density of Air
θ	Boundary Layer Momentum Thickness

SEPARATION OF TURBULENT, INCOMPRESSIBLE FLOW FROM A CURVED, BACKWARD-FACING STEP

by

Nice, Tseng and Moses

I. INTRODUCTION

Perhaps one of the most fundamental contributions in Prandtl's original boundary layer theory was the realization that in many cases the viscous flow near a solid surface could be treated independently of the outer, inviscid flow. This procedure is possible when there is only a slight interaction between the two regions; thus an iterative calculation for the boundary layer and pressure distribution will converge, and often the interaction is so slight that iteration is not necessary.

However, in many other cases of practical importance, such as flows involving separation, there is a strong interaction between the boundary layer and the outer, inviscid region. Thus, as demonstrated in Reference (1), the two regions cannot be treated independently in separated flow. Even when there is danger of separation this procedure is at best questionable, which very severely limits the practical value of present boundary layer and inviscid flow theory.

The concept of treating the two regions independently is not necessary in simplifying the equations of motion; but it is highly advantageous, if not necessary, in solving the equations. Only in a few special cases, such as relatively narrow internal passages where a one-dimensional pressure assumption is reasonable, can the interaction between the boundary layer and free stream be easily approximated in subsonic flow (Ref. (1)). In supersonic flow, the pressure can often be related to the boundary layer growth through the turning angle of the free stream (as in Ref. (2)).

In general, however, the present understanding and ability to predict the behavior of separated flow is far from satisfactory. Consider, for example, the deceptively complicated flow about a circular cylinder, for which an extensive review by Markovin[3] indicates a continued challenge. A number of problems associated with turbulent separation have been discussed by Kline[4].

Perhaps one of the simplest examples of fully separated flow that has practical significance is that about a backward-facing step, or abrupt expansion. In this case, the separation is fixed by a sharp corner - only the behavior of the separated flow itself needs to be considered. A number of investigators have studied this problem, both in supersonic and subsonic flow.

Work in compressible flow includes that of Crocco and Lees[5], Chapman[6], Korst[7], Nash[8], and a number of others. In incompressible flow, which is of primary interest in this report, studies have been made by Abbott and Kline[9], Tani[10], Hsu[11], Abramovich[12], and Mueller, et al[13].

In most applications involving stall, the separation point is not fixed by a sharp corner but occurs on a smooth surface, if at all. Thus, the separation point and resulting separated flow depends on the development of the upstream boundary layer, which in turn depends on the pressure distribution resulting from the separated flow. The difficulty in accounting for this interaction is perhaps the most serious limitation in applying a boundary layer analysis to practical engineering problems.

The present investigation was designed to provide experimental information for a fully separated flow that was as simple as possible, and yet retain the essential features of many practical applications. To this end, a curved step, which could be compared with the simpler sharp step in the same apparatus (and in the literature) was chosen. Curvature in the free stream was an essential feature of the flow, thus eliminating the possibility of a one-dimensional pressure assumption that changes the nature of the free stream equation.

II. EXPERIMENT

A. Experimental Apparatus and Equipment

A 27 inch by 17 inch channel was fitted to a low speed (100 ft/sec maximum) wind tunnel in the Gas Turbine Laboratory. The channel was fabricated with 1/4 inch Plexiglas on the bottom wall and on one side wall, and with 1/4 inch fiber-board on the remaining surfaces (see Fig. 2). The bottom of the channel was divided into two sections: a one foot long section to house the step and another section of bottom wall (4 feet in length) whose position or height between the side walls was adjustable, thus allowing for different step heights. Two sliding panels were inserted in the side walls immediately behind the step to provide support for a cylindrical probe. These panels allowed the probe to maneuver throughout the entire vertical cross-section at two x-positions and rotate about an axis perpendicular to the two-dimensional plane of flow. A 3/16 inch wide slot was cut one inch away from the bottom wall centerline to accommodate probes at any x-wise position. A row of static pressure taps was placed along the centerline of the channel on both the top and bottom walls. The side walls were also provided with static pressure taps. The exit of the channel was equipped with a screen to raise the pressure in the channel, so that the pressure at the beginning of the curved expansion was close to the ambient pressure. Thus, there was no danger of fluid injection at the step due to a pressure differential. An extra length of channel (18 inches) was fabricated to be inserted before the section containing the step to thicken the upstream boundary layer.

The channel was fabricated to accept either a right angle step or one with curvature. The sharp step consisted of two Plexiglas sections which were cemented together at right angles. Static pressure taps were inserted at x = -1/8, -1, -2, -4 1/4, -6 1/2, and -8 3/4 inches with two others on the face of the step. The curved step was fabricated in three parts: 1) an upstream surface, 2) a cylinder of two inch radius to serve as the curved surface, and 3) a step face wall; all were made of Plexiglas. The two plane sections were joined to the cylinder (see Fig. 1) by knife edges which were cemented to the plane section. Care was taken that the joint between the Plexiglas and the metal of the knife edge was smooth. Care was also taken in fitting the knife edge against the cylinder to insure that it was straight in the transverse direction. The cylinder could be rotated through 90° and was mounted on two circular guides outside of the channel to allow it complete freedom to move along its axis. The cylinder was also fitted with one adjustable total pressure probe with a 2 inch travel (see Fig. 2b) and with two banks of static pressure taps (one bank on either side of the total pressure probe). Each bank contained three taps; one was on the same transverse axis as the total pressure probe and the other two were displaced 15° on either side.

The pressure probes included a cylindrical three-hole probe, a spherical static pressure probe and total pressure probes. One total pressure tube was inserted near the upper surface of the duct 8 inches from the step face to measure the free stream total pressure. Another total pressure probe with a tube of 0.060 inches O.D. was used on the bottom wall 8 3/4 inches from the face of the step to determine the initial boundary layer profile. A third total pressure tube (0.040 inches O.D.) was used to study the developing boundary layer profiles on the curved surface. The latter two probes were made with an elliptical cross-sectional opening to achieve the best possible response while allowing the probe to be manipulated closer to the wall. The three-hole directional probe consisted of a 1/4 inch cylindrical tube which was inserted through the sliding panels in both side walls. A shrouded total head probe (Keil probe) was also used to measure total pressures in the shear layer. Static pressures were measured with a Flow Corporation spherical probe.

The static pressure taps consisted of a small tube of 0.072 outer diameter, which was pressed into a hole bored into the wall material. A small drill (0.016 inch diameter) was passed through the tube cutting through the wall material, ensuring a smooth hole with no irregularities in the surrounding wall material to influence the pressure readings. All pressure measurements were made with an inclined manometer board.

B. Experimental Procedure

1. Nature of flow

In order to determine the two-dimensional character of the flow the following procedure was initiated. Early in the investigation small tufts were introduced into the upstream and downstream sections to determine whether any large transverse flow existed. The tufts produced no observable evidence of mean three-dimensional flow. An aspect ratio (channel height to step height) of three was considered necessary to ensure that the flow around the step was essentially unaffected by the position of the top wall. Hence, with the physical limitations imposed by the size of the wind tunnel, a maximum step height of approximately five inches was possible.

At this maximum step height, measurements were made of the total pressures at several transverse positions on the cylinder at zero degrees for $y = 2$ inches and 0.05 inches. Readings were also taken of the static pressures along the cylinder at zero degrees and also of the upstream total pressure distribution in the y-direction. The results of the measurements indicate that in the center of the span there is less than 3% variation in total pressure in the transverse direction at the cylinder and no observable variation in the upstream pressure in the y-direction. The static pressure on the cylinder also indicated a center section with small pressure variation. Therefore, all subsequent readings were taken in the center four inch section of the channel to achieve the best results.

2. Curved step

Measurements were made for three step heights with the same upstream boundary layer thickness at two different inlet velocities (95 and 70 fps), and one run was repeated for a thicker upstream δ. It was noticed that the free stream velocity varied slightly due to the back pressure of the screens on the exit plane, but the variation was not significant enough to warrant question.

In each of the above runs the velocity profiles were determined for the upstream boundary layer and the developing boundary layer over the curved surface. Traverses were taken at $\alpha = 0°$, $5°$, $20°$, $25°$ and $30°$ until separation was indicated. The approximate separation point was determined by rotating the cylinder until the total pressure just balanced the static pressure with the total head probe pressed against the cylinder surface. Readings of the streamline direction and total head were taken at two positions downstream of the step with the three-hole cylindrical probe, but these became inaccurate as the probe moved down into the shear layer. Total and static pressure profiles were measured at several x-positions downstream of the step, including the reattachment point. In the recirculation region, the direction giving the maximum total head was assumed to be the flow direction. The static pressure in the flow was determined with a Flow Corporation spherical static pressure probe using the calibration equation,

$$P = \frac{(P_{sph.st.}) + 0.508(P_t)}{1 + 0.508}$$

The wall static pressure measurements were recorded for each step height, these included the top and bottom wall taps and the taps on the cylinder rotated to various angular positions.

3. Sharp step

Essentially the same procedure was followed for the sharp step except that the boundary layer measurement on the step was omitted.

III. EXPERIMENTAL RESULTS

Complete experiments were carried out at two speeds (U_o = 90 and 70 fps.), but since there was negligible difference between the two results only those for the higher speed are included. For the main tests, the boundary layer thickness was approximately 0.125 inches at a position 8 3/4 inches upstream from the face of the step. With the additional constant area section in place, the boundary layer at this point was approximately 0.400 inches thick.

As was mentioned previously, the separation point for the sharp step is predetermined. Naturally, the logical objective of an experimental consideration of the curved step should include an inference of the separation region. The classic definition of stall was developed by Prandtl as a region of back flow, and the separation point is defined as the point on the surface at which $(\frac{\partial u}{\partial y})_{y=0} = 0$. In practical cases a consideration of the velocity profiles (see Figure 3) shows such a definition to be unwieldy. Physical limitations, such as the diameter of the probe and the turbulent nature of the flow disallow much accuracy in determining the velocity profile near the wall. In Figure 3 it is evident that separation has occurred at approximately 28°, if only because the flow is attached at 25° and separated at 30°. Separation can also be defined as the point where $C_f = 0$, which can be determined by balancing the total pressure for a probe on the surface (Preston tube) with the static pressure. By this method the separation point was found to occur between 28° and 32° for step heights of 3 5/8 and 5 1/4 inches. However, for a step height of 2 1/4 inches separation was found to occur betwen 24° and 28°. Increasing the initial boundary layer thickness does not appreciably affect the separation point.

Another method of indicating the separation point can be achieved through a consideration of the shape factor, H. For turbulent flow, separation usually occurs at a value of the shape factor, $H \approx 2.5$. For a step height of 5 1/4 inches (see Figure 4) the shape factor on the curved step reaches the separation value in the vicinity of 27°. This figure was included as typical of the shape factor trend in the other tests. Figure 4 (θ vs. arc length also indicates the growth of the boundary layer which is seen to increase considerably near separation.

Figure 5 shows the pressure distribution for the curved step before and on the step as well as on the downstream surface. The static pressure P has been presented in the form of the pressure coefficient $C_P = P - P_o/1/2\ \rho U_o^2$ where ρ is the air density and P_o and U_o are the static pressure and free stream velocity determined at the top wall eight inches upstream of the step. Positive values of x correspond to locations on the downstream surface and values of x such that $-2 < x < 0$ lie on the curved surface of the step. This figure presents results for step heights of h = 5 1/4", 3 1/4", 2 1/4".

In all cases before the step there exists a pressure drop which reaches a minimum on the curved surface at $\alpha \approx 10°$; since the points were very close one curve was drawn through all of the data points in Figure 5. The pressure distribution on the curved step is shown in more detail in Figures 6 and 7. Throughout the separated region there is a slight negative pressure which is followed by a rapid pressure rise indicating the reattachment of the separated flow. Figure 5 indicates that the pressure rise by reattachment and the reattachment length increase as the step height is increased (see Figure 8). However, the base pressure and the pressure history over the step are essentially the same for different step heights. Figure 5 also contains the measurements for the case of a thicker boundary layer for a step height of 3 5/8 inches. No appreciable change is observed due to the thicker boundary layer (except that the pressure variation is not as large) until the reattachment is encountered. The thicker boundary layer is seen to reattach later than its thinner counterpart. Tani's graphs indicate that the thicker boundary layer attaches somewhat sooner for an abrupt expansion[10].

Figure 9 shows the pressure distribution for the sharp step. The same step heights were used in taking these measurements. The nature of the results agree very well with those taken by Tani[10]. Upstream of the step the pressure distribution experiences a negative pressure change to achieve the base pressure on the step face. After the step there is a slight decrease in pressure followed by a region of high positive pressure gradient which indicates reattachment. The results of measurements taken for a thicker boundary layer reveal that the magnitude of the pressure variation is not as great as for thin boundary layers and that reattachment length is greater. In summation, it is seen that, except for the reattachment length, the pressure distribution is rather insensitive to the changes in step height as well as to changes in the boundary layer thickness.

Figure 10 shows the static pressure distribution in the y-direction as determined by the spherical probe. The static pressure, P, is represented in the form of the pressure coefficient $C_{P_{st.}} = P - P_{st.w.}/1/2\ \rho U_o^2$ where P and $P_{st.w.}$ are at the same position in the x-direction. As can be seen the static pressure is not constant in the y-direction; the maximum absolute value of $C_{P_{st}}$ is about 0.1

Figures 11, 12, and 13 represent the velocity, u, after the curved expansion. The figures show the dimensionless velocity, u/U_o versus the distance from the wall at different values of x, downstream of the step. The zero velocity of u is approximately on the straight line tangent to the curved surface at $25° < \alpha < 30°$. For the abrupt expansion (see Fig. 14) the zero velocity is also on the straight line, $y \approx Cx$ where $C = 1.8$ to 2.2. The maximum back flow velocity, u_b, in the stalled region is about 0.3 U_o for both the abrupt expansion and the curved step. For the step height of 2 1/4 inches u_b/U_o is only about 0.2. The maximum back velocity occurs at approximately 3/4 of the reattachment length from the step face.

IV. DISCUSSION

The separation point was seen to occur at approximately $\alpha \approx 28°$ for all step heights (thin or thick boundary layers) except for a step height of 2 1/4 inches. Figure 7 presents the distribution of the pressure coefficient over the curved surface of the step. The zero mark corresponds to the zero degree position on the step ($x = -2$ inches). The boundary layer for a step height of 2 1/4 inches is seen to undergo a slightly more severe positive pressure gradient than the boundary layer for any other step height; hence, it separates earlier. Thus, although the step height seems to have little importance in determining the separation point for large values of step height, it becomes important when small step heights are considered. Figure 7 also indicates that the points of minimum pressure (corresponding to points of maximum velocity) occur downstream of the point where the area ratio begins to increase and the values are much different from those on the top wall. Hence, it appears that the upstream curvature of the streamlines anticipating the curved expansion is important.

Figures 5 and 9 present the variation of the pressure coefficient for the curved and sharp step. The primary difference between the two pressure distributions occurs on the step surface. For an abrupt expansion, as corroborated by Tani[10], there is a pressure drop on the step surface resulting in the base pressure. However, for a curved step there is a severe initial negative pressure gradient followed by an equally drastic positive gradient after the effective throat to achieve the base pressure which is approximately the upstream pressure. From Figures 5 and 9 it is evident that the reattachment length and the length of the free shear layer are smaller for the curved expansion. This could be due to the intrinsic curvature of the streamlines in rounding the curved expansion. Since a negative pressure gradient exists in the shear layer region for the curved step, the streamline cannot be regarded as straight (as in the abrupt expansion). This tends to decrease the reattachment length. The pressure distribution along the top wall is also presented in Figures 5 and 9, and it is nearly constant in the upstream section for both configurations. At the reattachment point the curved geometry has experienced greater pressure rise on the top wall. This may also be due to the effect of the curved streamlines.

Abbott and Kline[9] have presented an approximate expression for the reattachment length for an abrupt step. They state that separating the expansions into three regimes by geometry ($0 < \frac{h}{W_o} < 0.2$, $0.2 < \frac{h}{W_o} < 1.7$, $1.7 < \frac{h}{W_o}$, where W_o is the upstream channel height) the data follows essentially three straight lines each with a different slope (0.167, 0.173, 0.255, respectively) when plotting h/W_o versus m/W_o. But they advance the premise that $h/W_o = 0.173\ m/W_o$ is a good approximation in most problems. They also conclude that the two-dimensional reattachment length, but not the other aspects of the flow is independent of W_o. Figure 8 shows the variation of the present data with the empirical relation of Kline and Abbott and the effect of curved expansions and thick boundary layers. In both curved and abrupt expansions, thickening the boundary layer varied the reattachment length measurably, and the reattachment length for curved steps differs from the empirical relation of Abbott and Kline. Hence, it is evident that any expression for the reattachment length must be a function of step height, curvature and boundary layer thickness.

Comparison of the shear layer pressure profiles for abrupt and curved expansions reveals that the thickness of the shear layer is greater by perhaps a factor of 1/3 for the curved expansion. The profiles approach the free stream total pressure at the same distance from the wall, but the curved step shear layer profile extends deeper into the cavity region.

V. CONCLUSIONS

The experimental investigation of the flow over a curved backward-facing step leads to the following conclusions:

1) For large step heights, the flow separates at approximately 28 degrees. When the boundary layer thickness is much smaller than the step radius, separation can only result from a rapid increase in pressure. Since the base pressure is not very different from the upstream value, the increase in pressure must be preceded by a rapid decrease. The variations in pressure are accompanied by curvature of the free streamline near the step.

2) Compared to the abrupt expansion, the curved step results in a slightly higher base pressure and a somewhat shorter reattachment length. The shear layer downstream of the step is approximately 1/3 thicker for the curved step.

3) As determined with a spherical probe, the static pressure in the separated region is not constant for either step, but the variation is only about 10% of the free stream dynamic pressure.

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APPENDIX - Boundary Layer on Curved Step

As pointed out in the introductory remarks, the determination of the pressure field is an essential part of any practical problem involving separated flow. Furthermore, near and downstream of the separation point, the velocity must be determined simultaneously with the pressure(1). This interaction between the pressure and boundary layer is the essential difference between the abrupt expansion and the curved step.

Although no attempt will be made to solve the entire problem here, it is of interest to determine if the usual boundary layer approach would be useful upstream of the separation point, using the experimental pressure distribution. A number of approximate boundary layer methods were tried, but since the results were all similar, only the well-known method of Truckenbrôdt (see Ref. 14) will be discussed.

This method, as used here, consists of the momentum integral and kinetic energy (or velocity moment) equations.

$$\frac{d\theta}{dx} + (H + 2)\,\frac{\theta}{U}\,\frac{dU}{dx} = \frac{C_f}{2} \tag{A-1}$$

$$\theta\,\frac{d\overline{H}}{dH}\,\frac{dH}{dx} = (H - 1)\,\overline{H}\,\frac{\theta}{U}\,\frac{dU}{dx} - \overline{H}\,\frac{C_f}{2} + 2d \tag{A-2}$$

where:

$$\begin{aligned} \theta &= \int_0^\delta \left(1 - \frac{u}{U}\right)\frac{u}{U}\,dy \\ \delta^* &= \int_0^\delta \left(1 - \frac{u}{U}\right)dy \\ \delta^{**} &= \int_0^\delta \left(1 - \frac{u^2}{U^2}\right)\frac{u}{U}\,dy \\ H &= \frac{\delta^*}{\theta} \qquad \overline{H} = \frac{\delta^{**}}{\theta} \end{aligned} \tag{A-3}$$

The dissipation integral has been determined empirically:

$$d = \int_0^\delta \frac{\tau}{\rho U^2}\,\frac{\partial u}{\partial y}\,dy \quad 0.0056\,R_\theta^{\,1/6} \tag{A-4}$$

The skin friction is determined from the Ludweig-Tillman equation[16]:

$$C_f = 0.246\ R_\theta^{-.268}\ 10^{-.678H} \tag{A-5}$$

and the two shape factors are related by the approximate relation (which assumes a one-parameter family velocity profile)[14].

$$\overline{H} = \frac{1.269H}{H - 0.379} \tag{A-6}$$

The results of the calculations based on the above equations are shown in Figure 4 for a step height of 5 1/4 inches. The calculations were started with initial values of θ and H, 8 1/2 inches upstream of the step and continued to the separation point. As can be seen, the results are reasonably good even though the ratio of boundary layer thickness to step radius was approximately .25 to 1. This is somewhat surprising, since both the assumptions of a one-parameter family and constant pressure across the boundary layer are questionable under this condition. Thus it might be concluded that although the pressure variation in the y-direction is important in the free stream, it can be neglected in the boundary layer in most cases. This approximation would have some noticeable effect near the separation point, but it is very likely to be insignificant in determining the gross features of the flow.

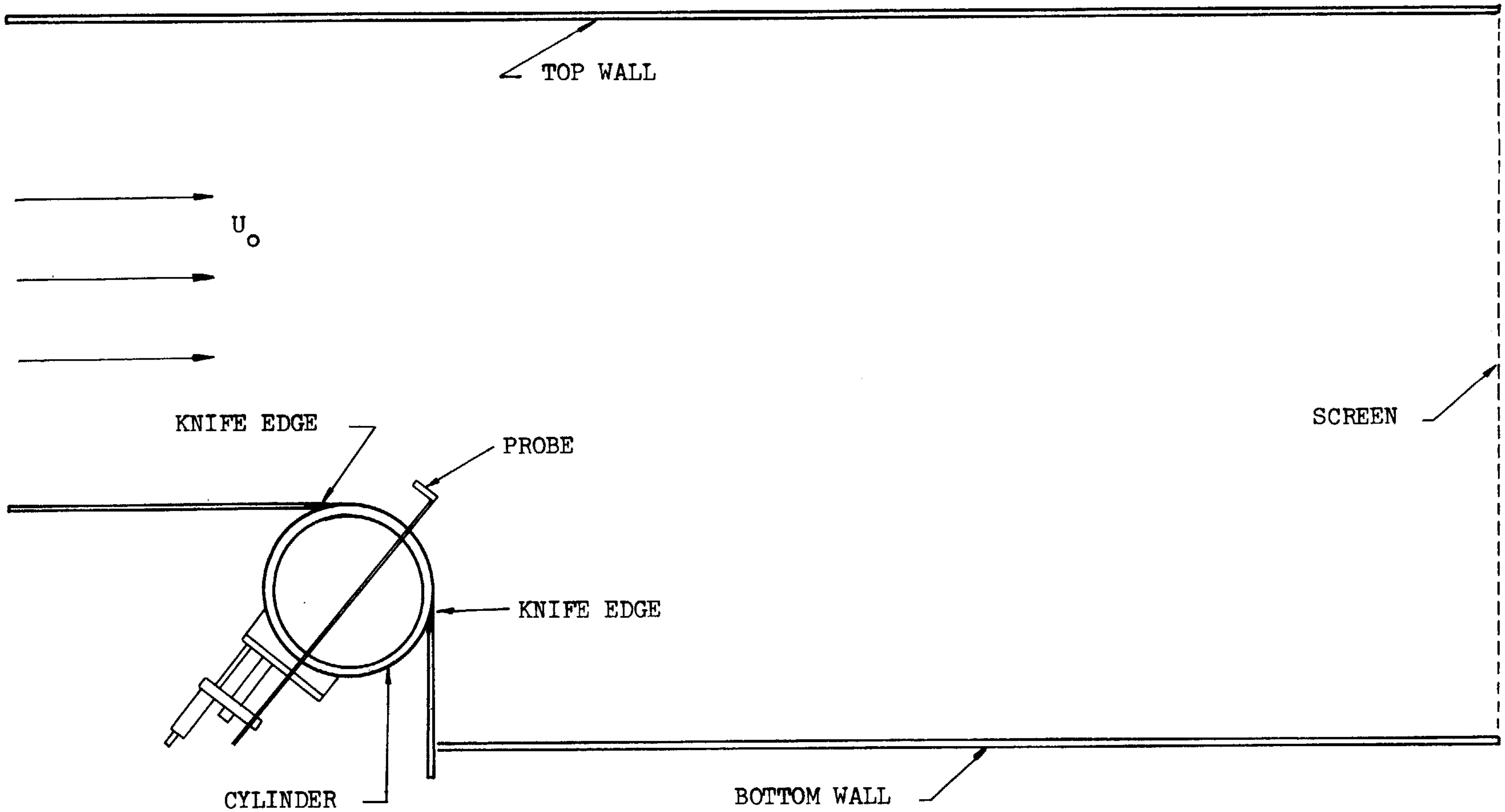

FIGURE 1. STEP CONSTRUCTION

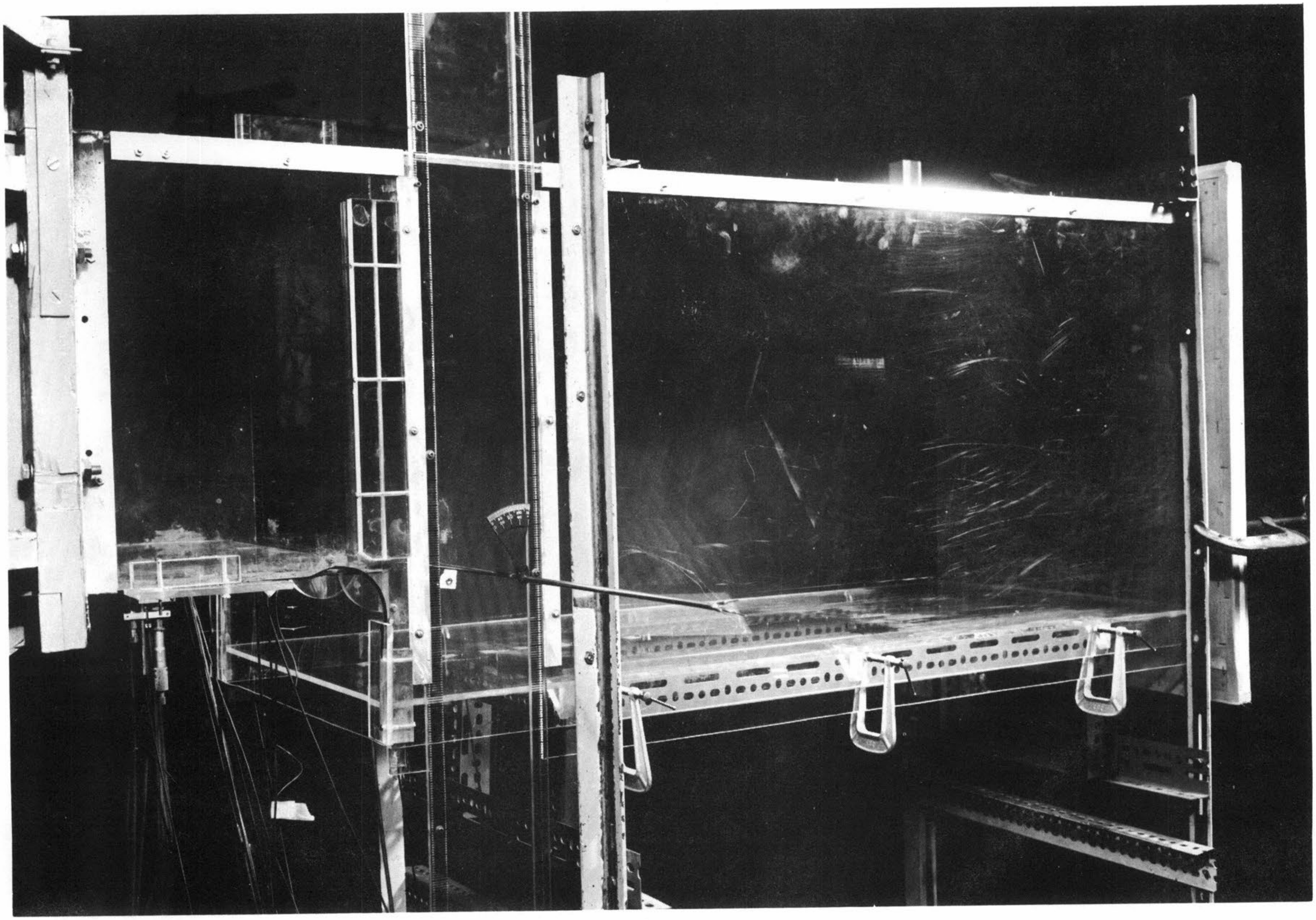

FIGURE 2a. TEST SECTION

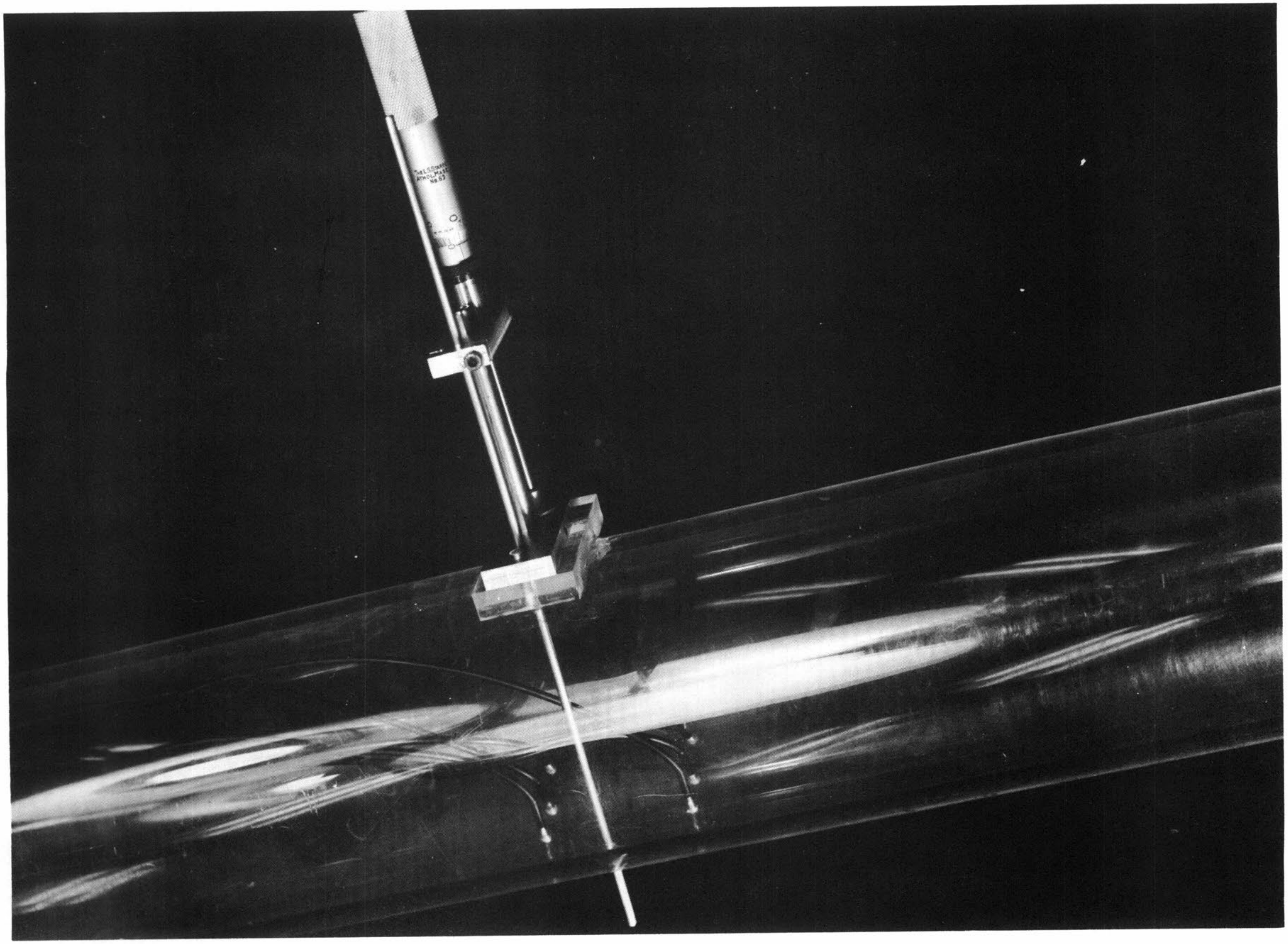

FIGURE 2b. CYLINDER WITH PROBE

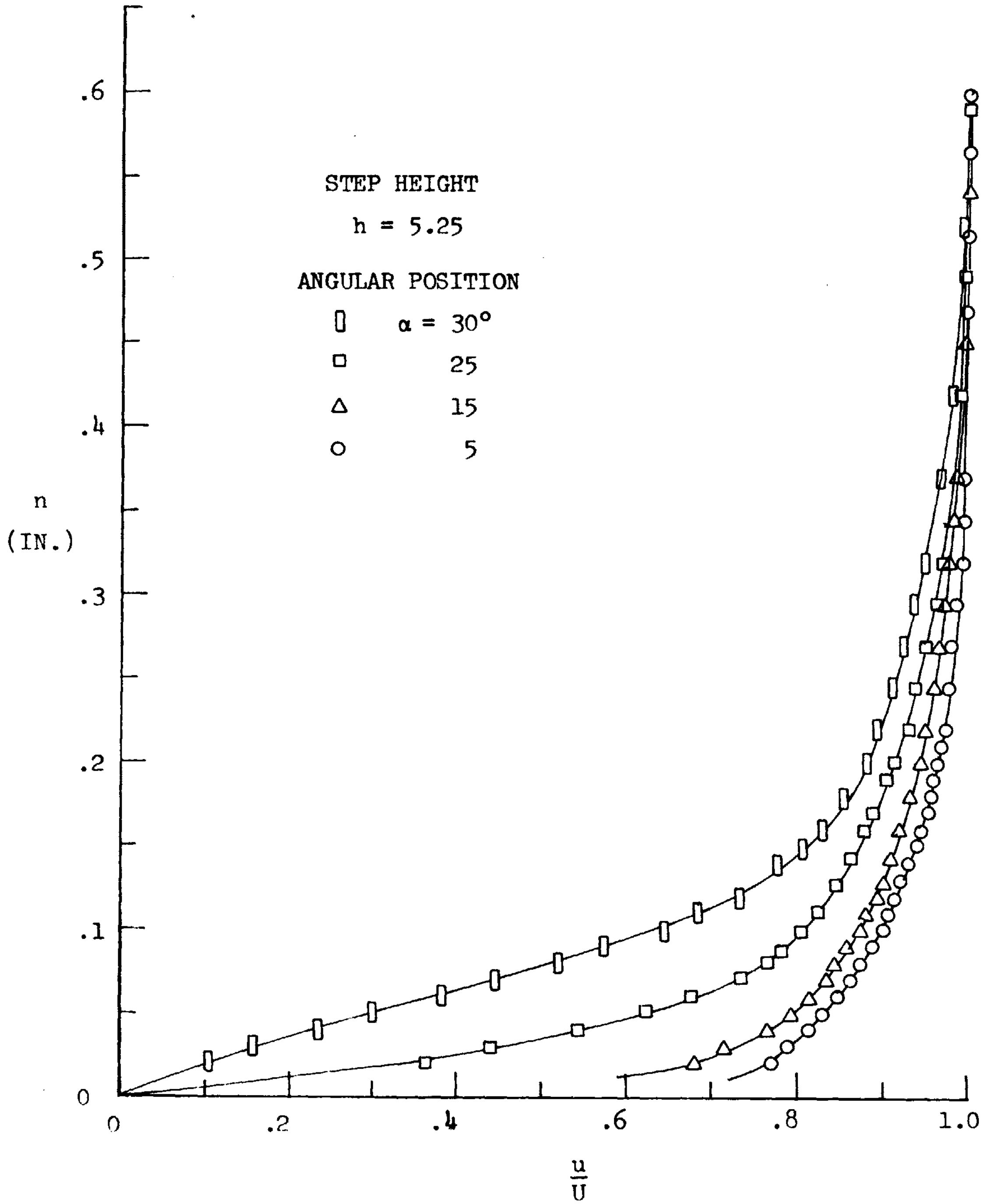

FIGURE 3. BOUNDARY LAYER ON CURVED SURFACE

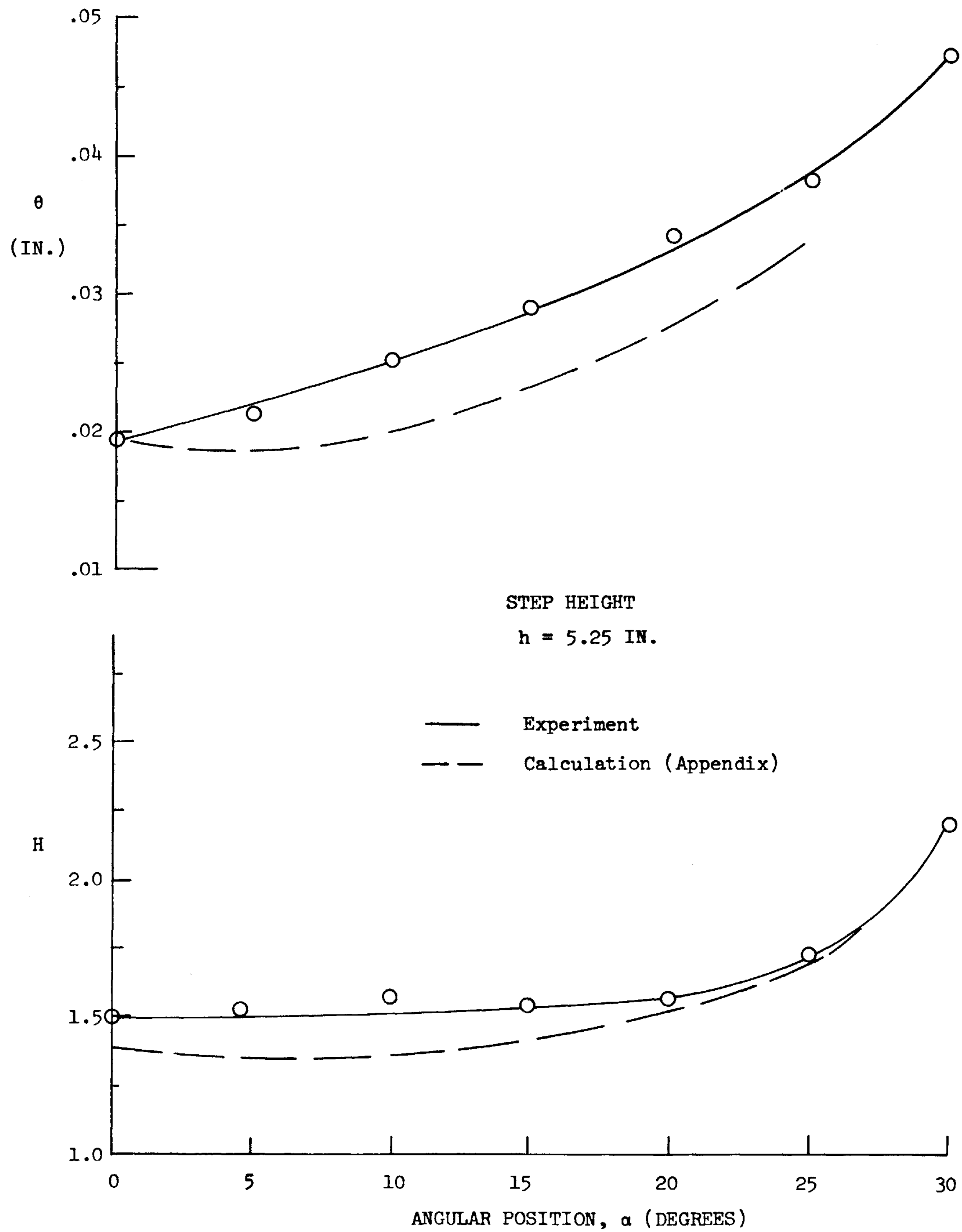

FIGURE 4. BOUNDARY LAYER CURVED SURFACE

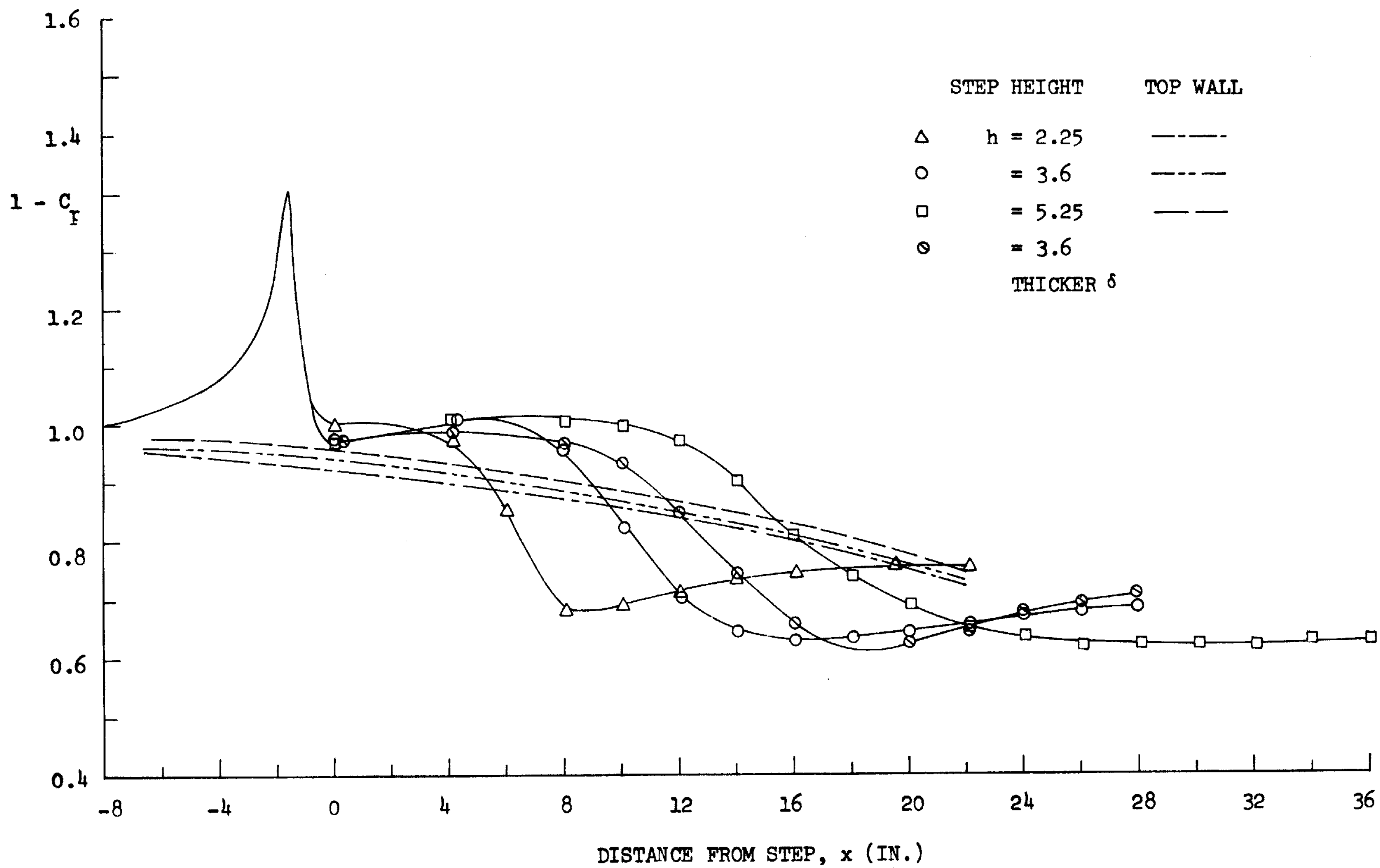

FIGURE 5. PRESSURE DISTRIBUTION ON BOTTOM WALL FOR CURVED STEP

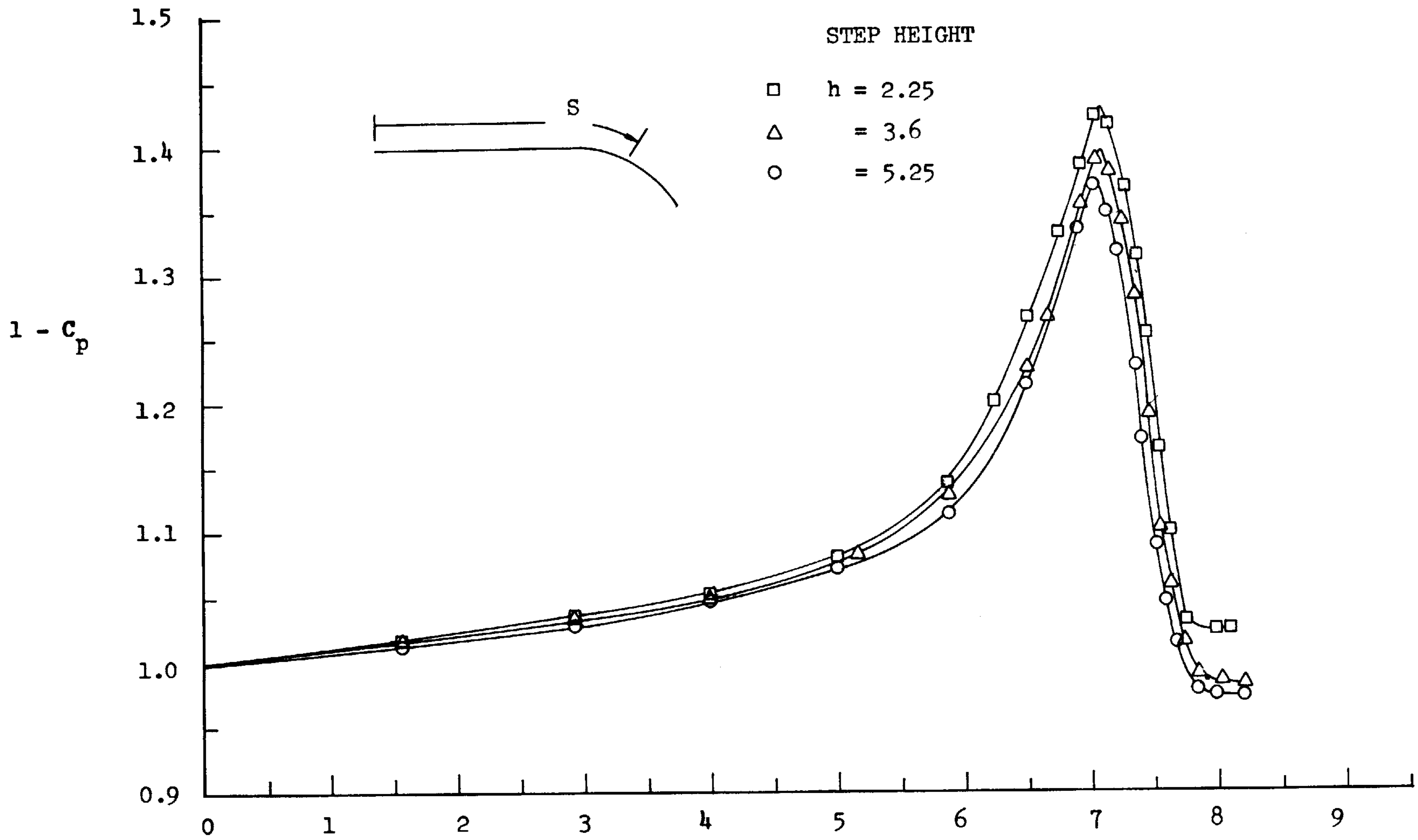

FIGURE 6. WALL PRESSURE DISTRIBUTION ON CURVED STEP

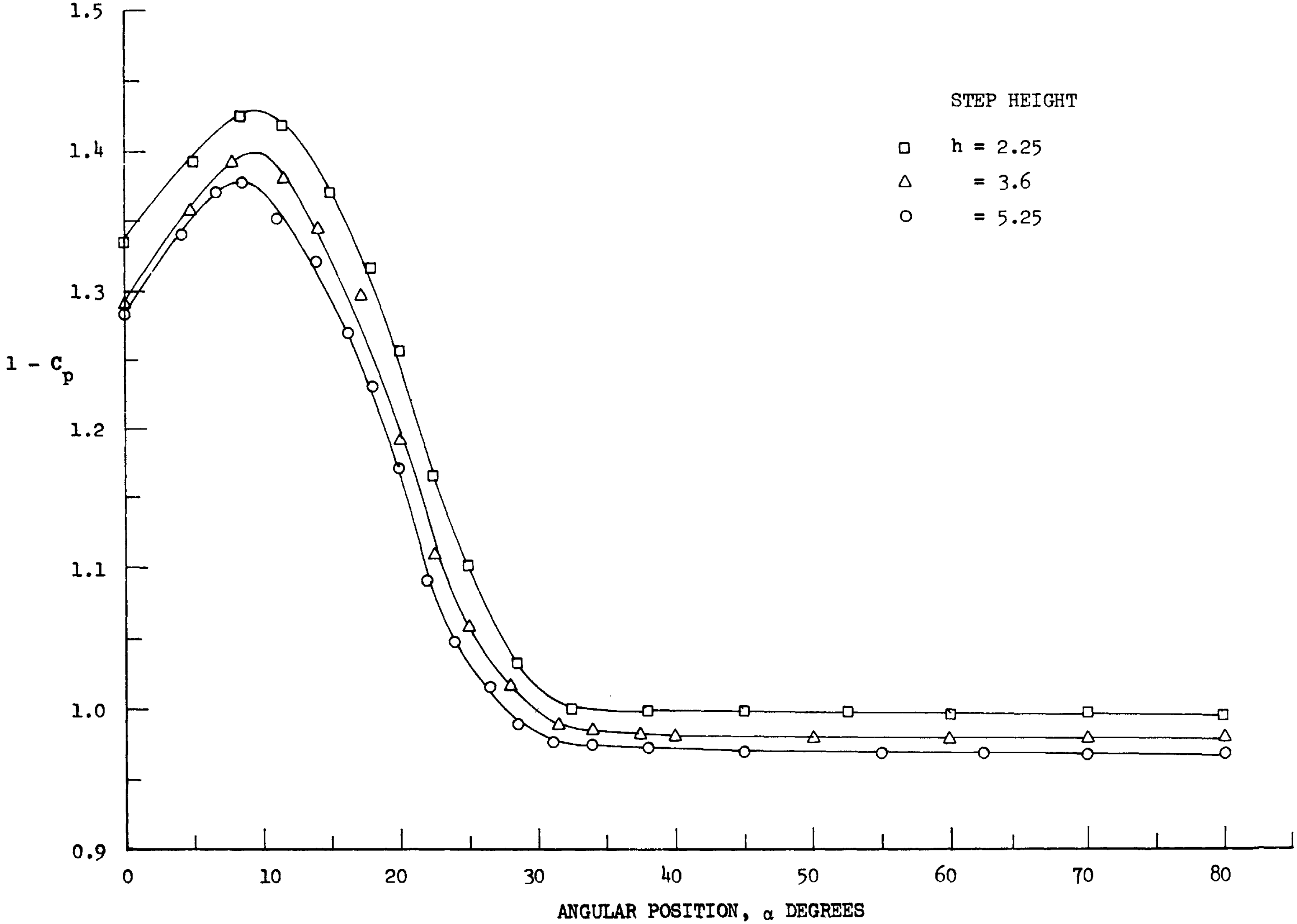

FIGURE 7. PRESSURE DISTRIBUTION ON CURVED SURFACE

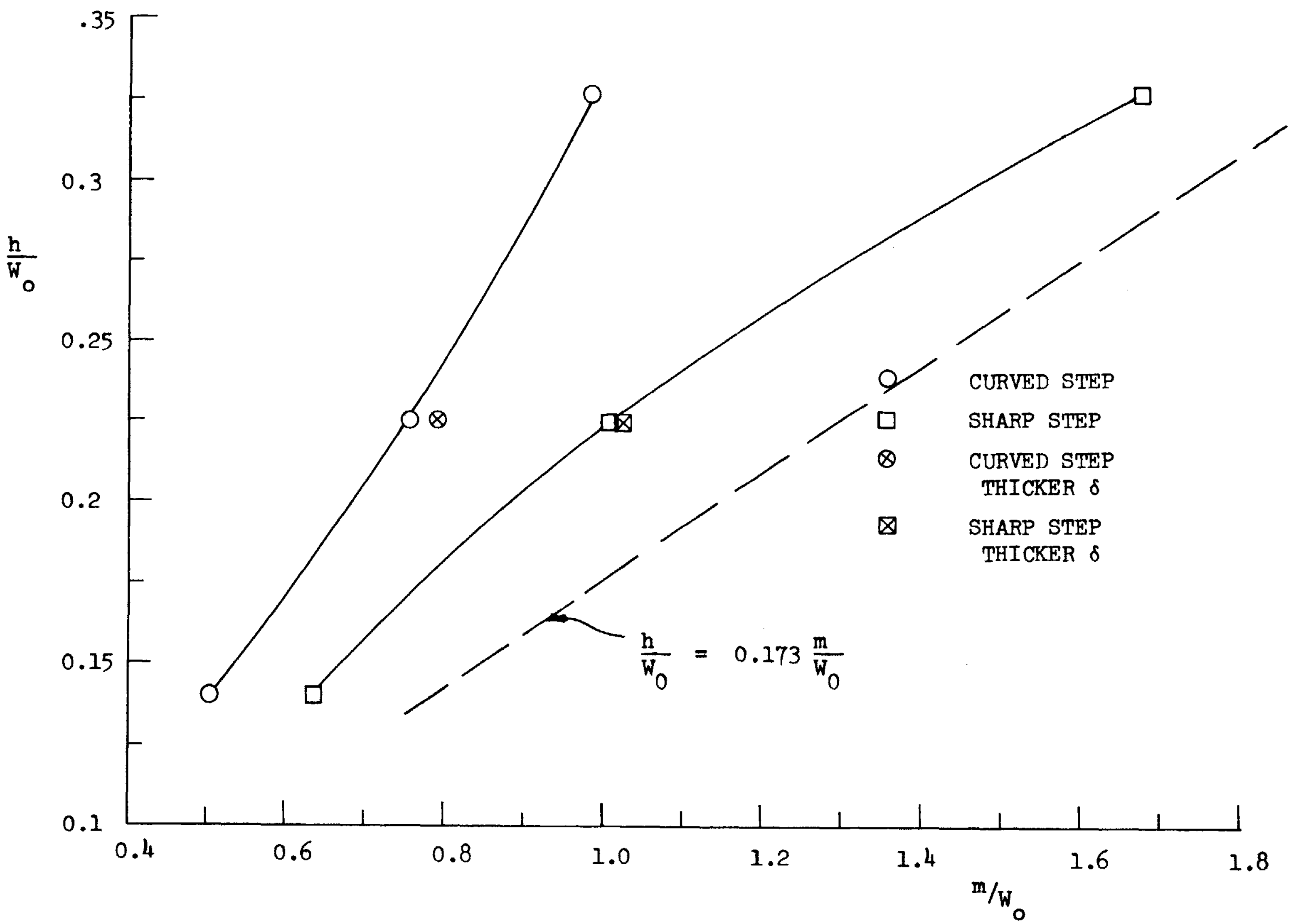

FIGURE 8. REATTACHMENT LENGTH

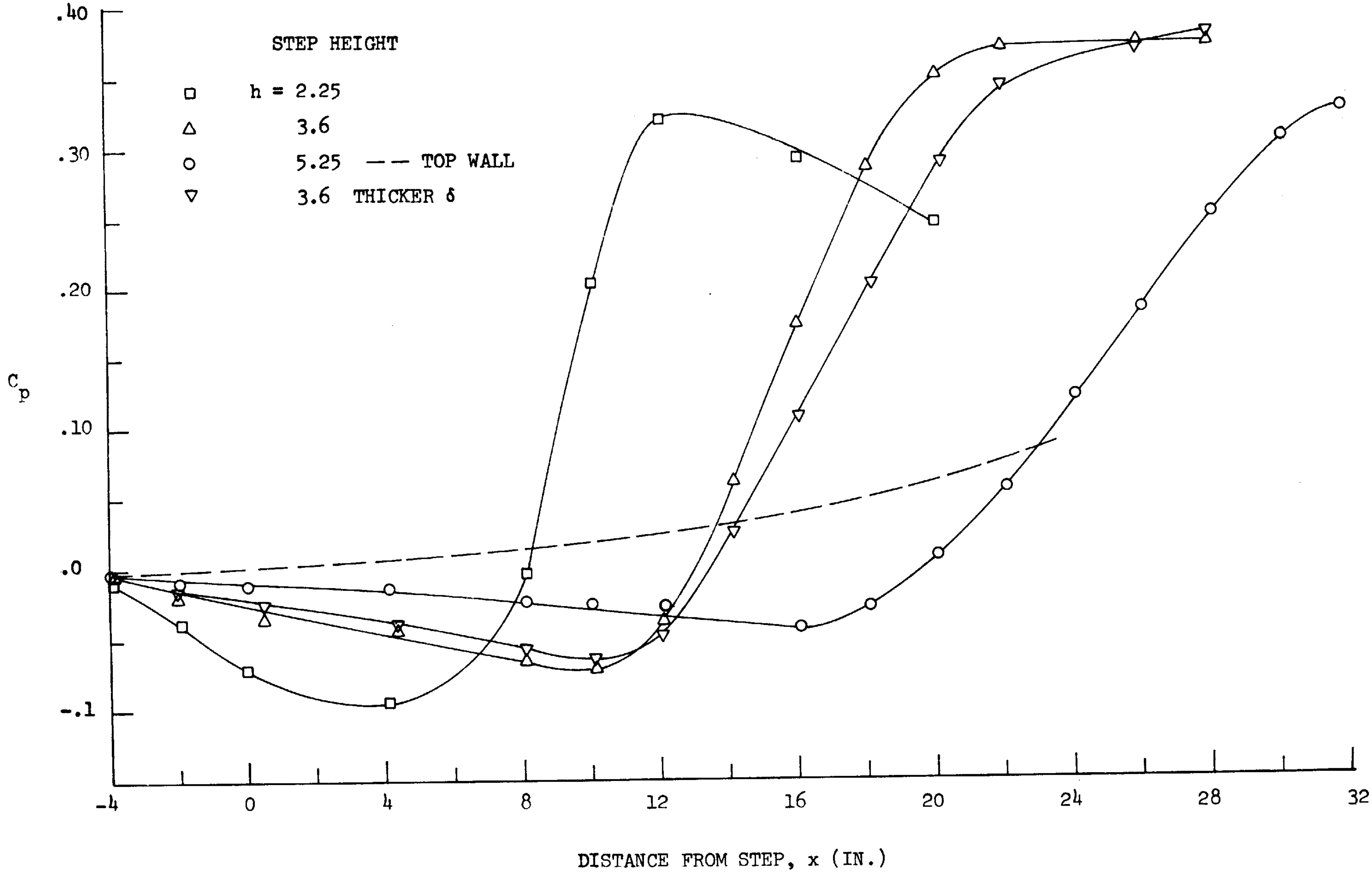

FIGURE 9. PRESSURE DISTRIBUTION ON BOTTOM WALL FOR SHARP STEP

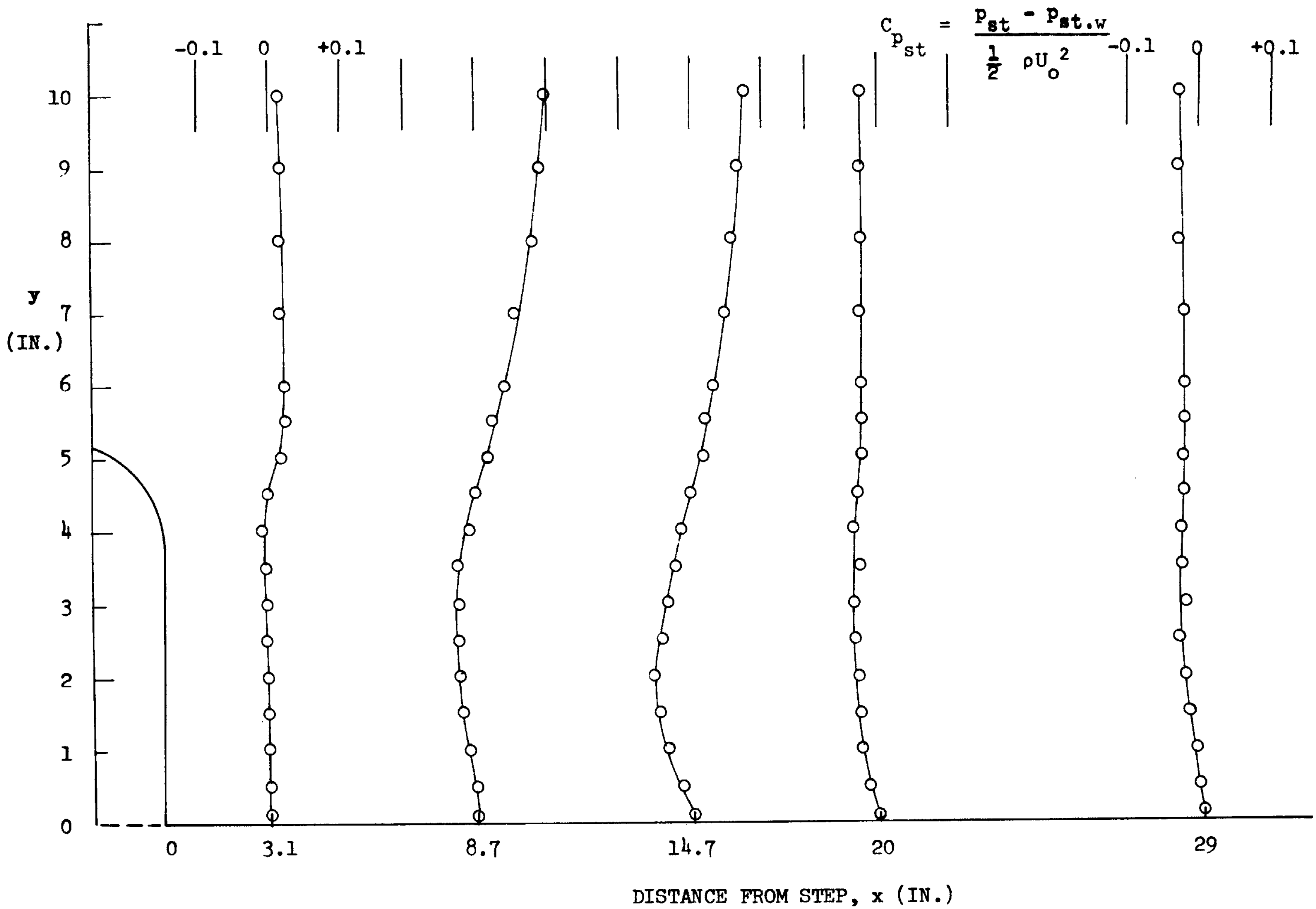

FIGURE 10. STATIC PRESSURE DISTRIBUTION DOWNSTREAM OF CURVED STEP

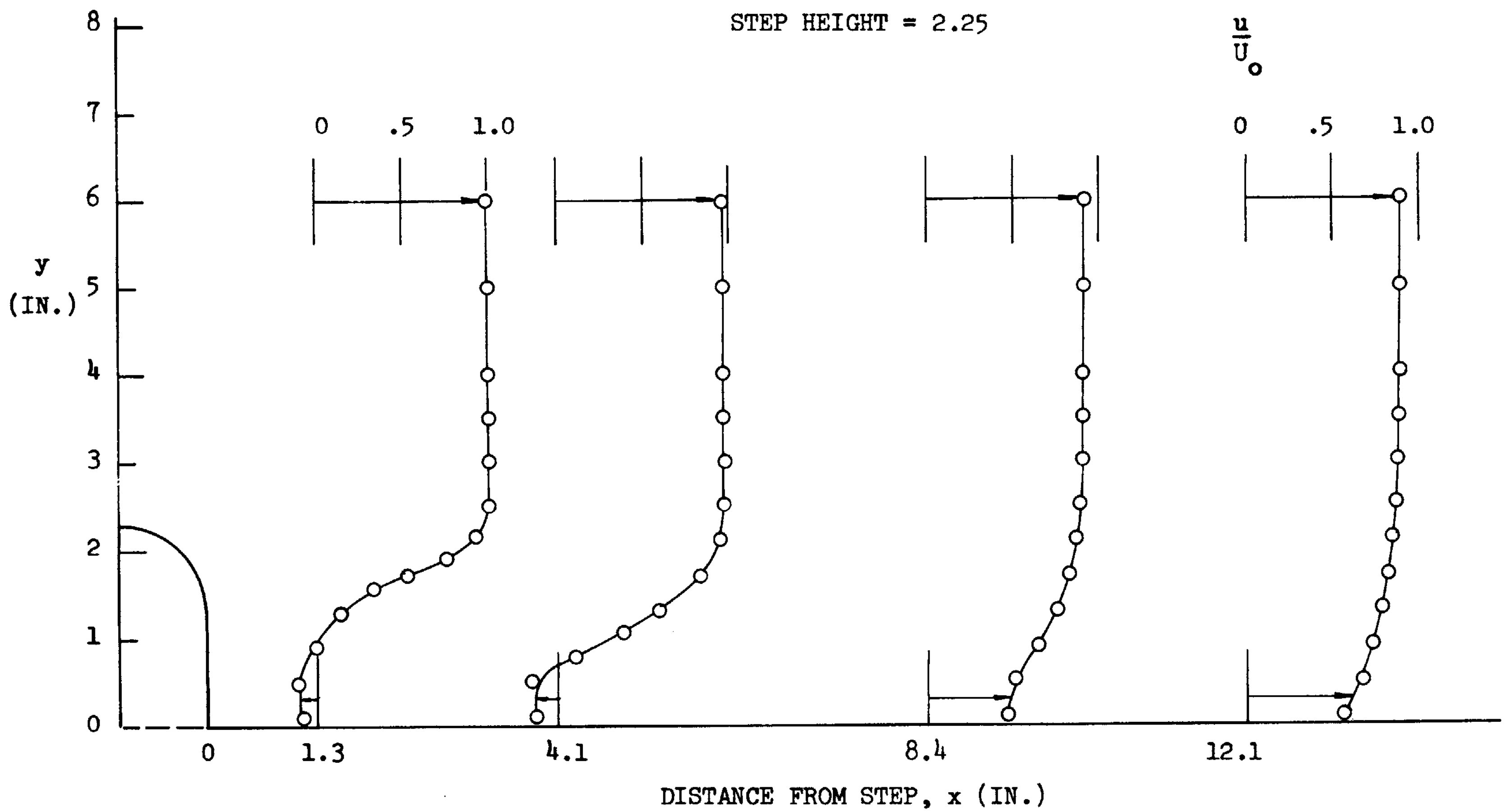

FIGURE 11. VELOCITY DISTRIBUTION OF CURVED STEP

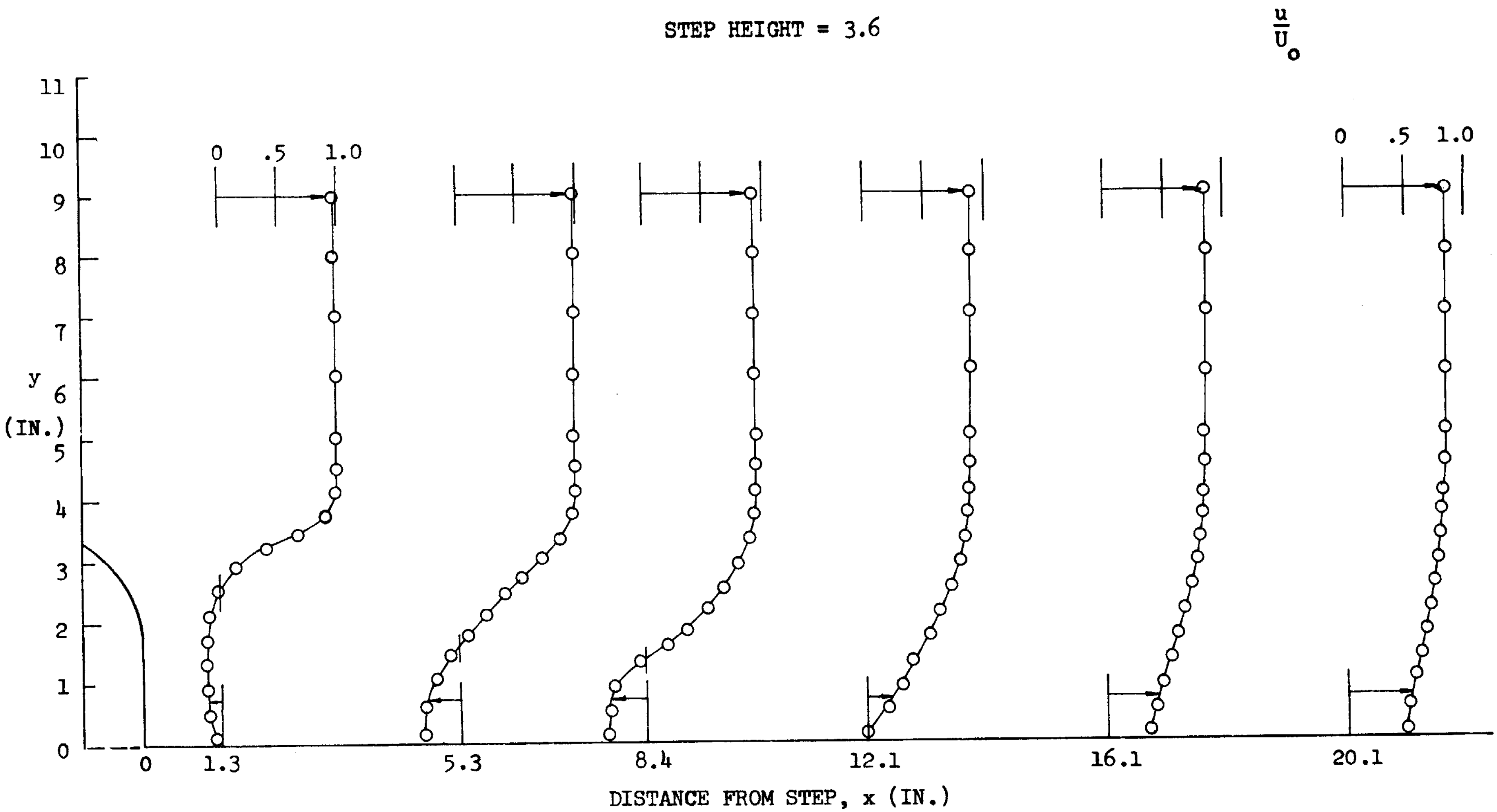

FIGURE 12. VELOCITY DISTRIBUTION DOWNSTREAM OF CURVED STEP

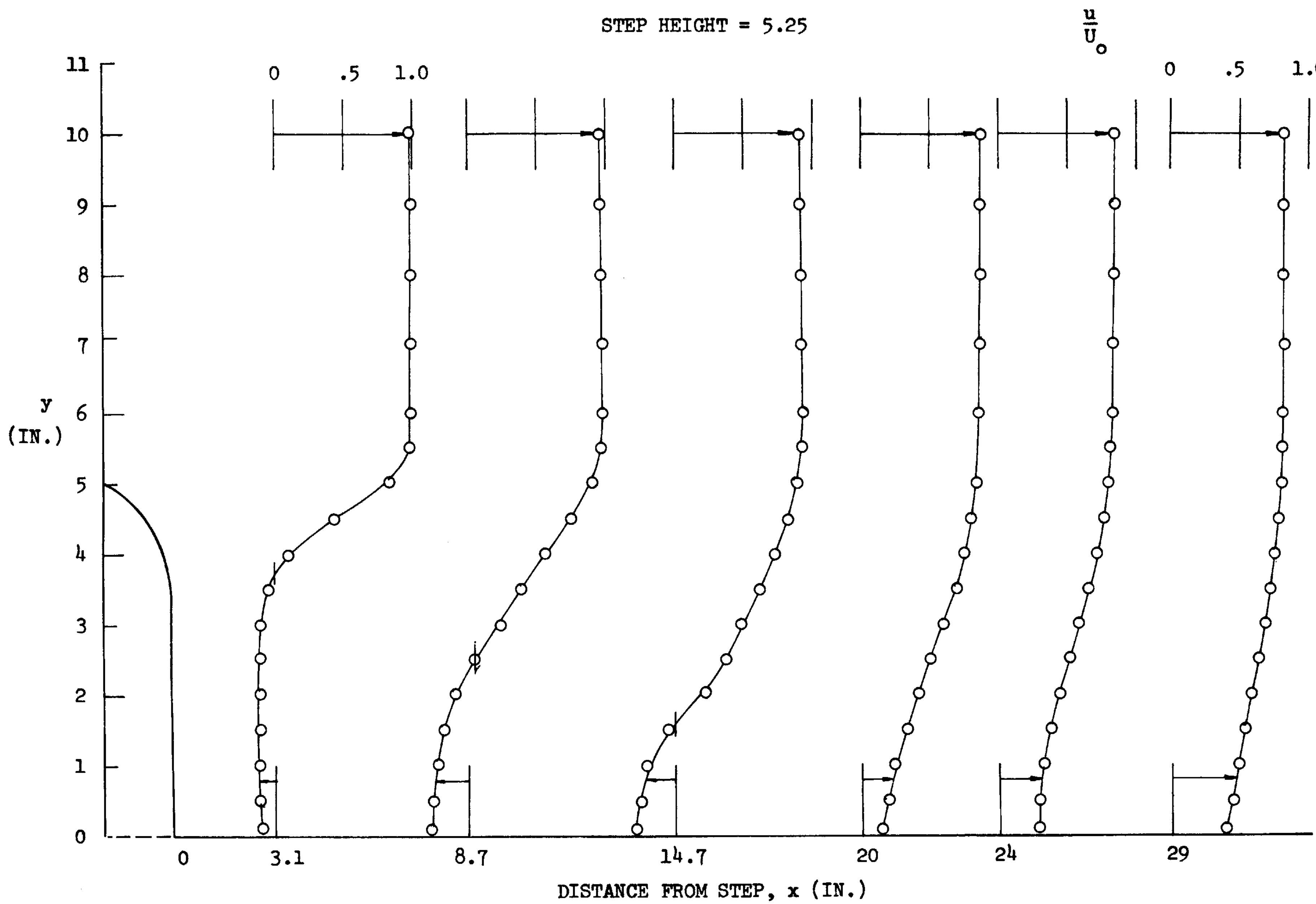

FIGURE 13. VELOCITY DISTRIBUTION OF CURVED STEP

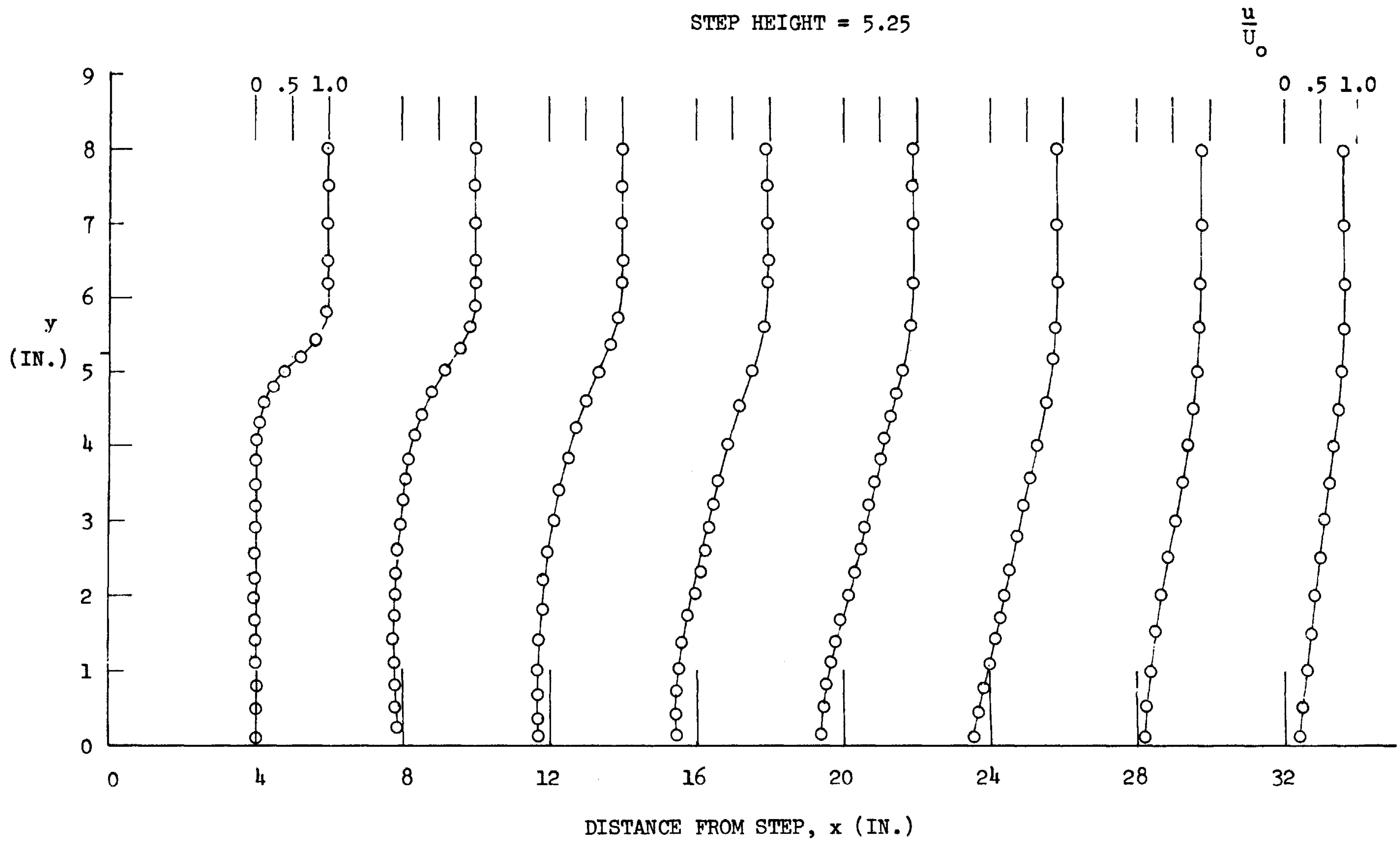

FIGURE 14. VELOCITY DISTRIBUTION OF SHARP STEP

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13. ABSTRACT

An experimental investigation of turbulent, incompressible flow separation over curved and sharp backward-facing steps is presented with results for various step heights. Mean velocities in the separating boundary layer as well as the downstream shear layer were recorded. The static pressure in the separated region was determined with a spherical probe.

With the curved step, the boundary layer separated at approximately 28 degrees: the reattachment lengths were somewhat less and the base pressure slightly higher than those with the sharp step.

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