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ON MODELING TEAMS OF INTERACTING DECISIONMAKERS WITH BOUNDED RATIONALITY

by

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ABSTRACT

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ON MODELING TEAMS OF INTERACTING DECISIONMAKERS WITH BOUNDED RATIONALITY

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Abstract. An analytical model of a team of well-trained human decisionmakers executing well-defined decisionmaking tasks is presented. Each team member is described by a two-stage model in which received information is first assessed and then responses are selected. An information theoretic framework is used in which bounded rationality is modeled as a constraint on the total rate of internal processing by each decisionmaker. Optimizing and satisficing strategies are derived and their properties analyzed in terms of organizational performance and individual workload. The relevance of this approach to the design and evaluation of command control and communications (C^3) systems is discussed.

Keywords. Decisionmaking, information theory, man-machine systems, organization theory, optimization.

INTRODUCTION

A command control and communications (C^3) system is defined as the collection of equipment and procedures used by commanders and their staff to process information, arrive at decisions, and communicate these decisions to the appropriate units in the organization in a timely manner. Implicit in this definition is the notion that the role of the human decisionmaker is central to the design of organizations and of the C^3 systems that support them. Therefore, in order to study the properties of alternative designs, it is necessary to develop a basic model of an interacting decisionmaker. Such a model, appropriate for a narrow but important class of problems was introduced by Boettcher and Levis (1982). In this paper, the work is extended to consider organizations consisting of several decisionmakers that form a team.

The basic assumption is that a given task, or set of tasks, cannot be carried out by a single decisionmaker because of the large amount of information processing required and because of the fast tempo of operations in a tactical situation. In designing an organizational structure for a team of decisionmakers, two issues need to be resolved: who receives what information and who is assigned to carry out which decisions. The resolution of these issues depends on the limited information processing rate of individual decisionmakers and the tempo of operations. The latter reflects the rate at which tasks are assigned to the organization for execution.

An information theoretic framework is used for both the modeling of the individual decisionmaker and of the organization. Information theoretic approaches to modeling human decisionmakers have a long history (Sheridan and Ferrell, 1974). The basic departure from previous models is in the modeling of the internal processing of the inputs to produce outputs. This processing includes not only transmission (or throughput) but also internal coordination, blockage, and internally generated information. Consequently, the limitations of humans as processors of information and problem solvers, are modeled as a constraint on the total processing activity. This constraint represents one interpretation of the hypothesis that decisionmakers exhibit bounded rationality (March, 1978).

The task of the organization is to receive signals from one or many sources, process them, and produce outputs. The outputs could be signals or actions. Implicit in this model of the organization's function is the hypothesis that decisionmaking is a two-stage process. The first is the assessment of the situation (SA) of the environment, while the second is the selection of a response (RS) appropriate to the situation.

The input signals that describe the environment may come from different sources and, in general, portions of the signals may be received by different members of the organization. It has been shown by Stabile, Levis and Hall (1982) that the general case can be modeled by a single vector source and a set of partitioning matrices that distribute components of the vector signal to the appropriate decisionmakers within the organization. This model is shown in Fig. 1, where the input vector is denoted by X and takes values from a finite alphabet $\mathscr{X}$. The partitions x^i may be disjoint, overlapping or, on occasion, identical.

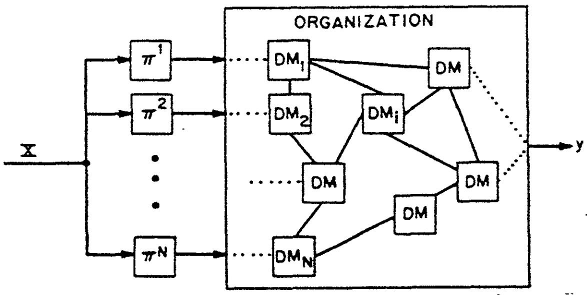

Fig. 1 The problem of information structures for organizations.

Many classes of organizational structures can be represented by Fig. 1. Consideration in this paper will be restricted to structures that result when a specific set of interactions are allowed between team members, as shown in Fig. 2. In this case, each team member is assigned a specific task, whether it consists of processing inputs received from the external environment or from other team members, for which he is well trained and which he performs again and again for successively arriving inputs. In general, a member of the organization can be represented by a two-stage model as shown in Fig. 2. First, he may receive signals from the environment that he processes in the situation assessment (SA) stage to determine or select a particular value of the variable z that denotes the situation. He may communicate his assessment of the situation to other members and he may receive their assessments in return. This supplementary information may be used to modify his assessment, i.e., it may lead to a different value of z. Possible alternatives of action are evaluated in the response selection (RS) stage. The outcome of this process in the selection of a local action or decision response y that may be communicated to other team members or may form all or part of the organization's response. A command input from other decisionmakers may affect the selection process.

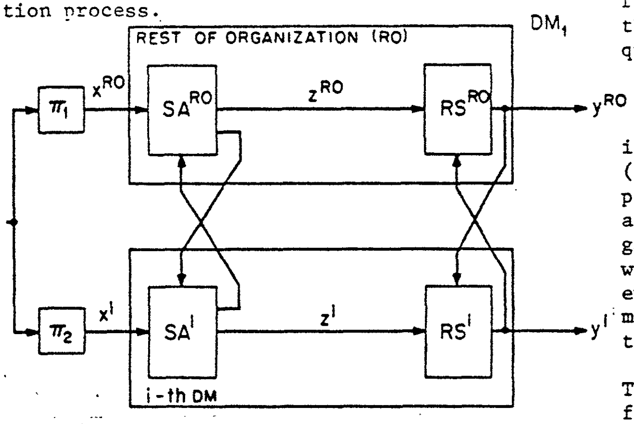

Fig. 2 Allowable team interactions

In the model of the organization developed in the following sections, internal decision strategies for each decisionmaker are introduced that determine the overall mapping between the stimulus (input) to the organization and its response (output). The total activity of each DM as well as the performance measure for the organization as a whole are expressed then in terms of the internal decision strategies. The locus of admissible strategies is shown in the performance-workload space. It is then possible to analyze the effects of the bounded rationality constraints on the organization's performance when either optimizing or satisficing behavior is assumed. The results indicate that the proposed model exhibits useful properties from the point of view of studying the information structure of decisionmaking organizations.

The paper is organized as follows. In the next section, the model of the interacting organization member is developed. In the third section an organization consisting of a team of two decisionmakers is described analytically. In the fourth section, the optimal and the satisficing decision strategies are obtained and analyzed.

MODEL OF THE INTERACTING ORGANIZATION MEMBER

The complete realization of the model for a single decisionmaker (DM) who is interacting with other organization members and with the environment is shown in Fig. 3. The detailed description and analysis of this model, as well as its relationship to previous work, notably that of Drenick (1976) and Froyd and Bailey (1980), has been presented in Boettcher and Levis (1982). Therefore, only the concepts and results needed to formulate the model of the organization will be described in this section.

Let the environment generate a vector symbol x'. The DM receives x which is a noisy measurement of x'. The vector x takes values from a known finite alphabet $\mathscr{X}$ according to the probability distribution p(x). The quantity

$$H(x) = - \sum_x p(x) \log_2 p(x) \tag{1}$$

is defined to be the entropy of the input (Shannon and Weaver, 1949) measured in bits per symbol generated. The quantity H(x) can also be interpreted as the uncertainty regarding which value the random variable x will take. If input symbols are generated every τ seconds on the average, then τ, the mean symbol interarrival time, is a description of the tempo of operations (Lawson, 1981)

The situation assessment stage consists of a finite number of algorithms that the DM can choose from to process the measurement x and obtain the assessed situation z. The internal decisionmaking in this stage is the

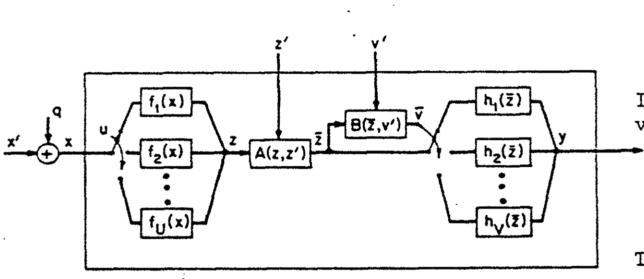

Fig. 3 Single interacting decisionmaker Model

choice of algorithm f_i to process x. Therefore, each algorithm is considered to be active or inactive, depending on the internal decision u. In this paper, it is assumed that the algorithms f_i are deterministic. This implies that once the input is known and the algorithm choice is made, all other variables in the first part of the SA stage are known. Furthermore, because no learning takes place *during* the performance of a sequence of tasks, the successive values taken by the variables of the model are uncorrelated, i.e., the model is memoryless. Hence, all information theoretic expressions appearing in this paper are on a per symbol basis.

The variable z', the supplementary situation assessment received from other members of the organization, combines with the elements of z to produce $\bar{z}$. The variables z and $\bar{z}$ are of the same dimension and take values from the same alphabet. The integration of the situation assessments is accomplished by the subsystem S^A which contains the deterministic algorithm A.

If there is no command input v' from other organization members, then the response selection strategy $p(v|\bar{z})$ specifies the selection of one of the algorithms h_j that map $\bar{z}$ into the output y. The existence of command input v' modifies the decisionmaker's choice v. A final choice $\bar{v}$ is obtained from the function b(v,v'). The latter defines a protocol according to which the command is used, i.e., the values of $\bar{v}$ determined by b(v,v') reflect the degree of option restriction effected by the command. The overall process of mapping the assessed situation $\bar{z}$ and the command input v' into the final choice $\bar{v}$ is depicted by subsystem S^B in Fig. 3. The result of this process is a response selection strategy $p(\bar{v}|\bar{z}v')$ in place of $p(v|\bar{z})$.

The model of the decisionmaking process shown in Fig. 3 may be viewed as a system S consisting of four subsystems: S', the first part of the SA stage; S^A; S^B; and S", the second part of the RS stage. The inputs to this system S are x, z', and v' and the output is y. Furthermore, let each algorithm f_i contain α_i variables denoted by

$$W^i = \{w_1^i, w_2^i, \ldots, w_{\alpha_i}^i\} \qquad i = 1,2,\ldots,U \quad (2)$$

and let each algorithm h_j contain α_j' variables denoted by

$$W^{U+j} = \{w_1^{U+j}, \ldots, w_{\alpha_j'}^{U+j}\} \qquad j = 1,2,\ldots,V \quad (3)$$

It is assumed that the algorithms have no variables in common:

$$W^i \cap W^j = \emptyset \quad \text{for } i \neq j$$
$$\forall i,j \in \{1,2,\ldots,U\} \text{ or } \{1,2,\ldots,V\} \quad (4)$$

The subsystem S' is described by a set of variables

$$S' = \{u, W^1, \ldots, W^U, z\};$$

subsystem S^A by

$$S^A = \{W^A, \bar{z}\};$$

subsystem S^B by

$$S^B = \{W^B, \bar{v}\};$$

subsystem S" by

$$S'' = \{W^{U+1}, \ldots, W^{U+V}, y\}.$$

The mutual information or transmission or throughput (Shannon and Weaver, 1979) between the inputs x, z', and v' and the output y, denoted by T(x,z',v':y) is a description of the input-output relationship of the DM model and expresses the amount by which the output y is related to the inputs x,z',and v':

$$\begin{aligned} G_t &= T(x,z',v':y) \\ &= H(x,z',v') + H(y) - H(x,z',v',y) \\ &= H(y) - H_{x,z',v'}(y) \end{aligned} \quad (5)$$

A quantity complementary to the throughput G_t is that part of the input information which is not transmitted by the system S. It is called blockage and is defined as

$$G_b = H(x,z',v') - G_t \quad (6)$$

In this case, inputs not received or rejected by the system are not taken into account. Blockage can also be expressed as the mutual information between the inputs and all the internal variables of S conditioned on the output y, i.e.,

$$G_b = T_y(x,z',v':u,W^1,\ldots,W^{U+V},W^A,W^B,z,\bar{z}) \quad (7)$$

In contrast to blockage is a quantity that describes the uncertainty in the output when the input is known. It may represent noise in the output generated within S or it may represent information in the output produced by the system. It is defined as the entropy of the system variables conditioned[1] on x,

1 The conditional entropy is defined as

$$H_x(z) = -\sum_x p(x) \sum_z p(z|x)\log_2 p(z|x)$$

that is,

$$G_n = H_x(u,W^1,\ldots,W^{U+V},W^A,W^B,z,\bar{z},\bar{v},y) \quad (8)$$

The final quantity to be considered is the mutual information of all the internal and output variables of the system S. It reflects all system variable interactions and can be interpreted as the coordination required among the system variables to accomplish the processing of the inputs to obtain the output y. It is defined by

$$G_c = T(u:w_1^1:\ldots:w_{\alpha_V'}^{U+V}:w_1^A:\ldots:w_{\alpha_B}^B:z:\bar{z}:\bar{v}:y) \quad (9)$$

The Partition Law of Information (Conant, 1976) states that the sum of the four quantities G_t, G_b, G_n, and G_c is equal to the sum on the marginal entropies of all the system variables (internal and output variables):

$$G = G_t + G_b + G_n + G_c \quad (10)$$

where

$$G = \sum_{i,j} H(w_i^j) + H(u) + H(z) + H(\bar{z}) + H(\bar{v}) + H(y) \quad (11)$$

When the definitions for internally generated information G_n and coordination G_c are applied to the specific model of the decisionmaking process shown in Fig. 3 they become

$$G_n = H(u) + H_{\bar{z}}(v) \quad (12)$$

and

$$G_c = G_c' + G_c^A + G_c^B + G_c'' + T(S':S^A:S^B:S'') \quad (13)$$

where

$$G_c' = \sum_{i=1}^{U} [p_i g_c^i(p(x)) + \alpha_i \mathbf{H}(p_i)] + H(z) \quad (14)$$

$$G_c'' = \sum_{j=1}^{V} [p_j g_c^{U+j}(p(\bar{z}|\bar{v}=j)) + \alpha_j \mathbf{H}(p_j)] + H(y) \quad (15)$$

$$G_c^A = g_c^A\ (p(z)) \quad (16)$$

$$G_c^B = g_c^B\ (p(\bar{z})) \quad (17)$$

$$T(S':S^A:S^B:S'') = H(z) + H(\bar{z}) + H(\bar{v},\bar{z}) + T_z(x':z') + T_{\bar{z}}(x',z':v') \quad (18)$$

The expression for G_n shows that it depends on the two internal strategies p(u) and $p(v|\bar{z})$ even though a command input may exist. This implies that the command input v' modifies the DM's internal decision after $p(v|\bar{z})$ has been determined.

In the expressions defining the system coordination p_i is the probability that algorithm f_i has been selected for processing the input x and p_j is the probability that algorithm h_j has been selected, i.e., $p_i = p(u{=}i)$ and $p_j = p(\bar{v}{=}j)$. The quantities g_c^k represent the internal coordination of the corresponding algorithms and depend on the distribution of their respective inputs. The quantity $\mathbf{H}$ is the entropy of a random variable that can take one of two values with probability p:

$$\mathbf{H}(p) = -\ p\ \log p - (1-p)\ \log(1-p) \quad (18)$$

Relation (13) states then that the total coordination in system S can be decomposed into the sum of the internal coordination within each subsystem and the coordination due to the interaction among the subsystems. The subsystem coordinations are given by Eqs. (14) to (17) while the coordination among them is given by Eq. (18). The coordination terms for subsystems S' and S'' reflect the presence of switching due to the internal decision strategies p(u) and $p(\bar{v}|\bar{z}v')$. If there is no switching, i.e., if for example p(u=i)=1 for some i, then $\mathbf{H}$ will be identically zero to all p_i and Eq. (14) will reduce to:

$$G_c' = g_c^i(p(x)) + H(z)$$

and, similarly, Eq. (15) will reduce to:

$$G_c'' = g_c^{U+j}(p(\bar{z}|\bar{v}=j)) + H(y).$$

Finally, the quantity G may be interpreted as the total information processing activity of system S and therefore, it can serve as a measure of the workload of the organization member in carrying out his decisionmaling task.

A TEAM OF TWO DECISIONMAKERS

In the previous section, the information theoretic model of a decisionmaker interacting with other members of his organization was presented. In order to define an organizational structure, it is necessary to specify exactly the interactions of each DM with every other DM (if any) and the interactions with the environment. Then the Partition Law of Information can be applied to each DM. The expressions for total processing activity G and for its components can be derived then either from basic principles, or by specializing the expressions developed in the previous section. To demonstrate the procedure and, at the same time, keep the exposition brief, an organization consisting of two interacting decisionmakers will be analyzed.

The specific organizational structure is shown in Fig. 4. Both decisionmakers #1 and #2 receive synchronized signals from the environment -- they receive different partitions of the input X to the organization. Each member processes the external input through one of his algorithms f_i to obtain

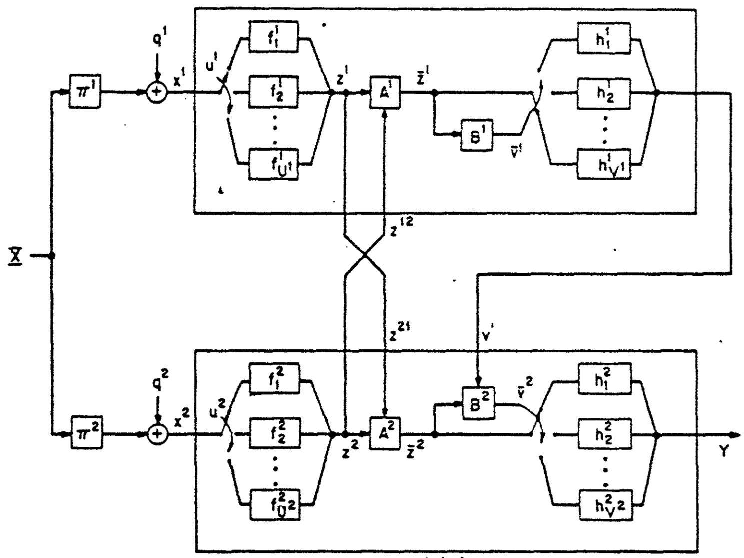

Fig. 4 A two decisionmaker team

his partial assessment z of the external situation. The partial assessments are then communicated to each other (variables z^{12} and z^{21} in Fig. 4). The first DM obtains his modified assessment $\bar{z}^1$ and then selects a response which is, in this case, a command input to the second DM. The latter receives the command input v' and, on the basis of that and his modified situation assessment $\bar{z}^2$, selects a response. The result is the output Y of the organization.

This particular configuration can be interpreted as follows: The second DM receives detailed observations about a small portion of the environment on which he has to act. He sends his estimate of the situation to the first DM who has a broad view of the situation. (DM_1 may be receiving situation assessments from other DMs that interact with him in the same way as DM_2). Then DM_1 selects an overall strategy and communicates that to DM_2 (and to all others). This signal, v', restricts the option selection of DM_2 to be consistent with the overall strategy determined by DM_1. Finally, DM_2 generates a response to his (local) situation input that has been improved by the information he has received from DM_1.

The five quantities that characterize the information processing and decisionmaking activity of each decisionmaker are obtained directly by specializing Eqs. (5), (6), (12)-(18). The basic assumption that allows the derivations of the various expressions is that the graph showing the interactions between the DMs is acyclical.

Decisionmaker #1

$$G_t^1 = T(x^1, z^{12} : z^{21}, v') \tag{20}$$

$$G_b^1 = H(x^1, z^{12}) - G_t^1 \tag{21}$$

$$G_n^1 = H(u^1) + H_{\bar{z}^1}(v^1) \tag{22}$$

$$G_c^1 = \sum_{i=1}^{U^1} [p_i g_c^i(p(x^1)) + \alpha_i \mathbf{H}(p_i)] + H(z^1, z^{21}) + g_c^{A^1}(p(z^1), p(z^{12})) + \sum_{j=1}^{V^1} [p_j g_c^j(p(\bar{z}^1|\bar{v}^1) + \alpha_j \mathbf{H}(p_j)] + H(v') + (H(z^1) + H(\bar{z}^1) + T_{z^1}(x^1 : z^{12}) \tag{23}$$

Decisionmaker # 2

$$G_t^2 = T(x^2, z^{21}, v' : z^{12}, y) \tag{24}$$

$$G_b^2 = H(x^2, z^{21}, v') - G_t^2 \tag{25}$$

$$G_n^2 = H(u^2) + H_{\bar{z}^2}(v^2) \tag{26}$$

$$G_c^2 = \sum_{i=1}^{U^2} [p_i g_c^i(p(x^2)) + \alpha_i \mathbf{H}(p_i)] + H(z^2, z^{12}) + g_c^{A^2}(p(z^2), p(z^{21})) + g_c^{B^2}(p(\bar{z}^2), p(v')) + \sum_{j=1}^{V^2} [p_j g_c^j(p(\bar{z}^2|\bar{v}^2)) + \alpha_j \mathbf{H}(p_j)]$$

$$+ H(Y) + H(z^2) + H(\bar{z}^2) + H(\bar{z}^2,\bar{v}^2)$$

$$+ T_{z^2}(x^2 : z^{21}) + T_{\bar{z}^2}(x^2, z^{21} : v') \qquad (27)$$

It follows from expressions (20) to (27) that the interactions affect the total activity G of each DM. At the same time, these interactions model the control that is exerted by the DMs on each other. These controls are exerted either directly through the command inputs v' or indirectly through z^{12} and z^{21}.

Both decisionmakers in Fig. 4 are subject to indirect control. The supplementary situation assessment z^{12} modifies the assessment z^1 to produce the final assessment $\bar{z}^1$. Since $\bar{z}^1$ affects the choice of output, it follows that DM_2 has influenced the response of DM_1. Similarly, DM_1, influences through z^{21} the response of DM_2.

Direct control is exerted through the command input from DM_1 to DM_2. The variable v' modifies the response selection strategy $p(v^2|\bar{z}^2)$ directly. Both direct and indirect control may improve the performance of a DM; they can also degrade it.

The values of the total processing activities G^1 and G^2 depend on the choice of internal decision strategies adopted by DM_1 and DM_2. Define a pure internal decision strategy to be one for which both the situation assessment strategy p(u) and the response selection strategy $p(v|\bar{z})$ are pure, i.e., an algorithm f_i is selected with probability one, and an algorithm h_j is selected is selected with probability one when the situation is assessed as being some $\bar{z}$:

$$D_i^1 = \{p(u^1=i')=1 \ ; \ p(v^1=j'|\bar{z}^1=\bar{z}^1_m)\} \qquad (28)$$

for some i', for some j' and for each $\bar{z}^1_m$. Similarly,

$$D_j^2 = \{p(u^2=i')=1 \ ; \ p(v^2=j'|\bar{z}^2=\bar{z}^2_m)\} \qquad (29)$$

There are $n_1 = U^1 \cdot (V^1)^{M^1}$ possible pure internal decision strategies for DM_1 and $n_2 = U^2 \cdot (V^2)^{M^2}$ for DM_2. The quantity M is the size of the alphabet of $\bar{z}$.

All other internal decision strategies are mixed (Owen, 1968) and are obtained as a convex combination of pure strategies:

$$D^1(p_k) = \sum_{k=1}^{n_1} p_k D_k^2 \qquad (30)$$

$$D^2(p_\ell) = \sum_{\ell=1}^{n_2} p_\ell D_\ell^2 \qquad (31)$$

where p_k and p_ℓ are probabilities.

A pair of pure strategies, one for DM_1 and one for DM_2, defines a pure strategy for the organization:

$$\Delta_{ij} = \{D_i^1, D_j^2\} \qquad (32)$$

Independent internal decision strategies for each DM, whether pure or mixed, induce a behavioral strategy (Owen, 1968) for the organization

$$\Delta = \{D^1(p_k), D^2(p_\ell)\} \qquad (33)$$

Given such a behavioral strategy, it is then possible to compute the total processing activity G for each DM:

$$G^1 = G^1(\Delta) \ ; \ G^2 = G^2(\Delta) \qquad (34)$$

Alternatively, the distributions on u and v can be specified directly for each decisionmaker. This results in a set of behavioral strategies for the organization.

$$\Delta_b = \{p(u^1), p(v^1|\bar{z}^1) \ ; \ p(u^2), p(v^2|\bar{z}^2)\} \qquad (35)$$

that includes the set specified by Eq.(33) as well as strategies that are not induced by mixed internal decision strategies for each DM. Then, the total activity G can be computed from

$$G^1 = G^1(\Delta_b) \quad ; \ G^2 = G^2(\Delta_b). \qquad (36)$$

These interpretations of the expressions for the total activity are particularly useful in modeling the bounded rationality constraint for each decisionmaker and in analyzing the organization's performance in the performance-workload space.

BOUNDED RATIONALITY AND PERFORMANCE EVALUATION

The qualitative notion that the rationality of a human decisionmaker is not perfect, but is bounded, has been modeled as a constraint on the total activity G:

$$G^i = G_t^i + G_b^i + G_n^i + G_c^i \leq F^i \tau \qquad (37)$$

where τ is the mean symbol interarrival time and F the maximum rate of information processing that characterizes decisionmaker i. This constraint implies that the decisionmaker must process his inputs at a rate that it as least equal to the rate with which they arrive. For a detailed discussion of this particular model of bounded rationality see Boettcher and Levis (1982).

As stated earlier, the task of the organization has been modeled as receiving inputs X' and producing outputs Y. Now, let Y' be the desired response to the input X' and let L(X') be a function or a table that associates a Y' with each member of the input X'.

The organization's actual response Y can be compared to the desired response Y' using a function d(Y,Y') which assigns a cost to each possible pair (Y,Y'). The expected value of the cost can be obtained by averaging over all possible inputs. This value, computed as a function of the organization's decision strategy , can serve as a performance index J. For example, if the function d(Y,Y') takes the value of zero when the actual response matches the desired response and the value of unity otherwise, then

$$J(\Delta) = E\{d(Y,Y')\} = p(Y \neq Y') \tag{38}$$

which represents the probability of the organization making the wrong decision in response to inputs X.

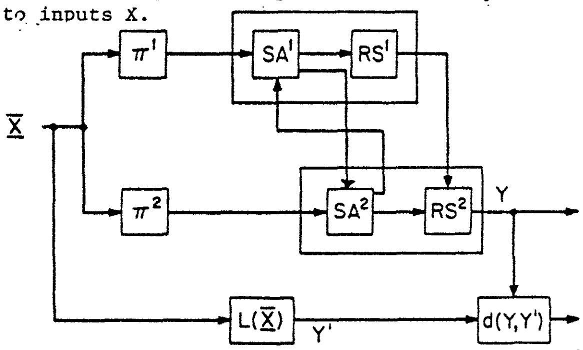

Fig. 5 Performance evaluation of organization

The information obtained from evaluating the performance of a specific structure and the associated decision strategies can be used by the organization designer in defining and allocating tasks (selecting the partitioning matrices π^i) and in changing the number and contents of the situation assessment and response selection algorithms.

The complete model of the team of two decisionmakers with bounded rationality is shown in Fig. 5. Two problems can be defined:

(a) Determine the strategies that minimize J;

(b) Determine the set of strategies for which $J \leq \bar{J}$.

The first is an optimization problem while the latter is formulated so as to obtain satisficing strategies with respect to a performance threshold $\bar{J}$. Since the bounded rationality constraint for both DMs depends on τ, the internal decision strategies of each DM will also depend on the tempo of operations. The unconstrained case can be thought of as the limiting case when $\tau \rightarrow \infty$.

A useful way of describing the properties of the solutions to the two problems is by introducing the performance-workload space (J,G^1,G^2). The locus of the admissible triples (J,G^1,G^2) is determine by analyzing the functional dependence of J, G^1, and G^2 on the organization strategy Δ, Eq. (33).

The total activity G^i of decisionmaker i is a convex function of the Δ in the sense that

$$G^i(\Delta) \geq \sum_{k,\ell} G^i(\Delta_{k\ell})p_k p_\ell \tag{39}$$

where $\Delta_{k\ell}$ is defined in Eq. (32). An equivalent representation of Δ is obtained from Eqs. (32) and (33):

$$\Delta = \sum_{k,\ell} \Delta_{k\ell}\, p_k p_\ell \tag{40}$$

which describes the relative occurence of each pure organization strategy $\Delta_{k\ell}$.

The result in Eq. (39) follows from the definition of G^i as the sum of the marginal entropies of each system variable, Eq. (11), and the fact that the possible distributions p(w), where w is any system variable, are elements of a convex distribution space determined by the organization decision strategies, i.e.,

$$p(w) \in \{p(w) \mid p(w) = \sum_{k,\ell} p(w|\Delta_{k\ell})p_k p_\ell\} \tag{41}$$

The performance index of the organization can also be obtained as a function of Δ. Corresponding to each $\Delta_{k\ell}$ is a value $J_{k\ell}$ of the performance index. Since any organization strategy being considered is a weighted sum of pure strategies, Eq. (40), the organization's performance can be expressed as

$$J(\Delta) = \sum_{k,\ell} J_{k\ell}\, p_k\, p_\ell. \tag{42}$$

Equations (39) and (42) are parametric in the probabilities p_k and p_ℓ. The locus of all admissible (J,G^1,G^2) triples can be obtained by constructing first all binary variations between pure strategies; each binary variation defines a line in the three dimensional space (J,G^1,G^2). These binary variations for a specific realization of the model in Fig.4 are drawn in Fig. 6. Each decisionmaker has only two pure strategies. Therefore, there are four triples that correspond to these pure strategies and four lines that join them under the assumption that one decisionmaker's pure strategy remains fixed while the other one considers variations between his two pure strategies.

For this particular example, the second decisionmaker's strategy does not affect the workload or total processing of the first DM; however, the first DM affects both performance and the total activity of the second decisionmaker through his command or direct control input. These properties are seen more closely if the locus of admissible triples is projected on the (J,G^1) and the (J,G^2) planes. The results for the second decisionmaker are shown in Fig.7; similar results are obtained for DM_1. As expected, these figures are similar to the ones obtained from the analysis of a

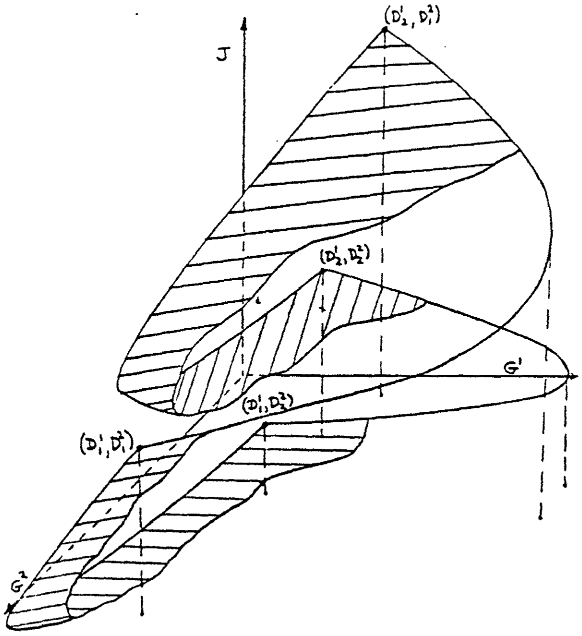

Fig. 6 The locus of binary variations of pure strategies for a team of two decisionmakers with two pure strategies each

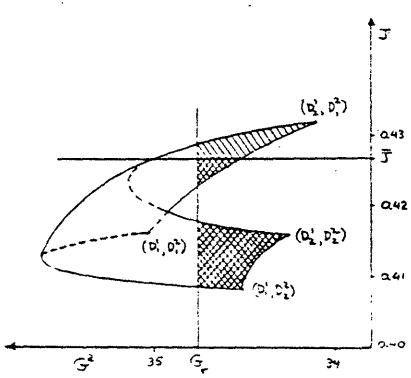

Fig. 7 Region of admissible (J,G^2) pairs

single decisionmaker (Boettcher and Levis, 1982). The complete locus is obtained by considering all combinations of mixed strategy pairs (Eq. (33)); the surface generated in this example is indicated in Fig.6. It is clear from the construction that the minimum value of the performance index is obtained for a pure organizational strategy as are the ones that minimize the workloads of either decisionmaker or of both. Therefore, the minimum error strategy is a pure strategy when there are no bounded rationality constraints.

The bounded rationality constraints can be realized in the form of planes of constant G^i in the three dimensional space (J,G^1,G^2). Thus the constraint for DM_1 is a plane parallel to the G^2 axis and intersecting the G^1 axis at

$$G_r^1 = F^1 \tau \text{ with } G^1 \leq G_2^1$$

Similarly, the constraint for DM_2 is a plane that intersects the G^2 axis at $G_r^2=F^2\tau$. For fixed values of F^i, the bounded rationality constraint is proportional to the tempo of operations. As the tempo of operations increases the G_r^i become smaller and fewer of the potential strategies are feasible.

The solutions of the satisficing problem can be characterized as that subset of feasible solutions for which

$$J(\Delta) = \sum_{k,\ell} J_{k\ell}\, p_k p_\ell \leq \bar{J} \qquad (43)$$

The condition (43) defines a plane in the (J,G^1,G^2) space that is parallel to the (G^1,G^2) plane and intersects the J axis at $\bar{J}$. All points on the locus below this plane, which also satisfy the bounded rationality constraints, are satisficing strategies. While an infinite number of strategies can be satisficing, the difference in total activity between them can be quite large. This is shown in the shaded region in Fig.7. The method of analysis presented in this paper is readily extendable to teams of N decisionmakers whose interconnections can be represented by an acyclical graph.

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