

**Social Security Reforms, Retirement Plans, and Saving  
under Labor Income Uncertainty**

by

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Licenciado en Economía  
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Doctor of Philosophy

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**Abstract**

This thesis contains four chapters. After a brief introduction, chapter two estimates the equivalent variation of extending the averaging horizon in defined benefit retirement plans. I show that the welfare effect of extending horizons differs across demographic groups, in a way that depends on the degree of persistence of income shocks and the growth rate of earnings. In chapter three I use the expected utility model with precautionary saving to analyze the consumption and saving behavior of individuals with different retirement plans. I show that defined benefit retirement plans may generate higher lifecycle saving than defined contribution plans. Finally chapter four deals with a specific episode of reform to social security. I examine the social security reform of 1994 in Argentina. After the reform individuals can choose between a defined benefit plan administered by the state and a defined contribution plan privately managed. I compare expected utility for a worker under each plan and show how the optimal choice depends on the nature of uncertainty he faces.

Thesis Supervisor: Peter Diamond  
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**To Chongo, Nelly, and Natalio**



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# Chapter 1

## Introduction

The golden age of social security is long gone. Demographic changes, and a huge increase in the share of social security on national expenditures in many countries have raised concern about the long term solvency of these systems. Social security has become one of the most important sources of income for the elderly, and it has been a powerful tool for income redistribution. Sooner or later, most countries will have to implement some changes to the current systems, to make them sustainable in the long run. In this thesis I look at the effect of some of those changes on individuals' welfare, from a microeconomic perspective.

The response to these problems has varied across countries; developed economies have adopted minor changes to their original systems, like increasing the contribution rates, retirement age, and extending coverage to additional groups in the population. Developing economies on the other hand have taken a more radical approach. A wave of structural reforms was started in the 1980s with Chile, and after that many countries have implemented similar changes to their social security systems.

Social security systems differ in many dimensions. In this thesis I concentrate on the way the retirement benefit is calculated. According to this, retirement plans are organized into two main categories: defined benefit plans are those in which the benefit is a function of a worker's characteristics, like retirement age, earnings history, and years of covered work. Defined contribution plans do not specify benefits but contributions; the retirement benefit depends on the amount a worker has contributed over his working life. Although most public pension systems were originally defined benefit, the recent wave of reforms has favored the adoption of defined contribution plans. The claim is that defined contribution plans dominate over defined benefit plans in a number of dimensions: they reduce labor market distortions by creating a closer link between contribution and benefits, they help in the develop-

ment of capital markets, and increase national savings. The empirical evidence on this respect is not very conclusive yet, maybe because the reform episodes are relatively recent. And theoretical models have generally focused on general equilibrium considerations, while very little attention has been given to the effect labor income uncertainty plays in either type of plan.

This thesis compares the behavior of an individual holding different types of retirement plans, either defined benefit or defined contribution, when labor income is uncertain. In chapter 2 I study defined benefit plans. I estimate the welfare effect of changing some features in defined benefit retirement plans. The particular feature I study is an extension in the averaging horizon, the number of years of earnings history entering the formula. Extending the averaging horizon in social security systems has been done in some countries, like the US and France, as a way to reduce the fiscal burden of social security. Assuming that earnings grow over time, a longer averaging horizon will reduce the expected retirement benefit. For a risk neutral individual this will be reflected in a welfare loss, and that is the way most studies in the area have been performed. However, a longer averaging horizon results also in a lower variance of the retirement benefit. This means that a risk averse individual will not necessarily be worse off when the averaging horizon increases. Using a simple model I show the effect of labor income uncertainty on the uncertainty of income at retirement, and the effect of different types of shocks to earnings. Then I estimate an earnings profile in which the variance of the earnings shocks can change over time and over the life cycle and use those estimates to calculate the equivalent variation of extending the averaging horizon for individuals in different demographic groups. I analyze different possible horizons, and find that the welfare effect can be quite different across groups. Even though the expected retirement benefit goes down, most groups are actually better off with a longer horizon, due to the reduction in risk. But I identified some cases in which extending the horizon will result in a welfare loss. This happens to individuals with a very steep earnings profile and with a relatively low variance in earnings.

The third and fourth chapters of this thesis compare defined contribution with defined benefit plans, also in the presence of labor income uncertainty. In chapter three I look at the behavioral response of a representative agent holding a defined benefit or a defined contribution retirement plan. When income is uncertain and the utility function exhibits prudence, people will hold precautionary saving. In this

paper I show that the amount of saving individuals accumulate over the life cycle depends on the type of retirement plan they hold. In order to do this I compare the life cycle consumption and saving behavior of an individual under either plan. I normalize expected benefits and use the same history of shocks so that the only difference in saving behavior comes from precautionary motives associated with holding different retirement plans. I find that the net of social security wealth is higher in the defined benefit case. The explanation lies in the way the benefits are calculated. In one case the benefit is a percentage of the last year of earnings, which has a much higher variance than when the benefit is calculated as a weighted average of the entire earnings history. This type of analysis is particularly relevant for analyzing the effect on individual saving of a social security reform. There is very little evidence so far about the change in individual saving behavior following a reform, and this chapter may help to answer that question.

Chapter four is an evaluation of an actual reform episode. I look at the 1994 social security reform in Argentina. After the reform every single worker is allowed to choose between a defined benefit plan administered by the state and a defined contribution plan privately managed. I compare the expected utility of workers with different demographic characteristics, under each retirement plan. I conclude that for individuals with an income process subject to mainly transitory shocks the defined benefit plan dominates over the defined contribution plan, while the opposite is true when shocks become more permanent.



## Chapter 2

# Income Uncertainty and Averaging Horizons in Defined Benefit Retirement Plans

### 1. Introduction

In spite of the increasing popularity of defined contribution retirement plans, defined benefit plans are still widely used in the US and the rest of the world. According to the Social Security Administration (1995), no country in the OECD has yet adopted a public defined contribution retirement system, and very few countries overall have a social security system organized around a defined contribution scheme. As for private pension plans, the Bureau of Labor Statistics reports that 52 percent of full time workers in the US were covered by defined benefit plans in 1995. Although this is a decline from the 63 percent who had defined benefit plans in 1988, it is still a significant proportion. Most defined benefit plans calculate retirement benefits as a function of an individual's earnings history. The benefit is a percentage of an earnings base which in turn is calculated as the average of the last or highest  $n$  years of earnings. A striking feature of these earnings-related defined benefit plans, and the main fact motivating this paper, is the diversity in averaging horizons used to calculate the earnings base:<sup>1</sup> of the 22 OECD countries using earnings-related formulas in their social security systems, five use the entire earnings history to calculate the earnings base, two rely on 20 years or more, two use 11 to 15 years, five consider 6 to 10 years of earnings, and eight countries make the benefit formula a function of the last 1 to 5 years of earnings history. A natural question to ask when confronted with this fact is what portion of earnings history should be included in the retirement formula? Is there an optimal averaging horizon? How is this

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<sup>1</sup>By averaging horizon I mean the number of years of labor income entering the calculation of the earnings base. Throughout the paper I use the terms averaging horizon, averaging length or averaging period interchangeably.

averaging length related to the stochastic process of earnings? This paper provides a framework to analyze the optimal design of retirement benefits in earnings related defined benefit plans, when labor income is subject to non-insurable risk.

Besides the obvious question of whether shorter or longer average horizons are preferred, there is a second set of questions I address, based on a related fact. It is becoming common practice to extend the averaging horizon used in social security formulas. A longer horizon will tend to reduce retirement benefits given that earnings grow over time, and as such it is a “covered” way to reduce the fiscal burden on social security. Countries like the United States and France have modified their social security formulas by extending the averaging length used to calculate the earnings base.<sup>2</sup> In this context, what are the distributional consequences of extending the averaging horizon? How are different groups in the population affected? How are the gains and losses related to the features of each individual’s earnings profiles?

I use a standard expected utility maximization setting to relate earnings uncertainty to averaging horizons in defined benefit plans. Then I estimate a stochastic process of earnings and calibrate the model using those estimates. The calibration exercise has two goals: first, I look at the effect of changing averaging horizons on the expected utility of different groups in the population. Then I use the same model to find the “optimal” averaging horizon.

I begin with a simple model in which risk averse agents maximize the expected utility of terminal wealth. Assuming that they hold no other assets beyond social security (or pension) wealth, the problem is equivalent to maximizing the expected utility of retirement benefits, conditional on information available at the beginning of the individual’s working life. In the model, retirement benefits are a weighted average of previous earnings. The number of years of working income included in the retirement benefit formula is the variable over which utility is being maximized. If earnings are random, part of the risk associated with the uncertainty in earnings will be translated into the retirement benefit, and the effect of this uncertainty will depend on how much of the earnings history is used in the formula, and on the properties of the income process.

If earnings do not grow over time and earnings shocks are assumed to be *iid*,

---

<sup>2</sup>Currently Austria is debating a similar change.



then it is apparent that agents will prefer plans with long averaging horizons, as the expected benefit is constant but the variance declines as the sample size (the averaging horizon) increases. However this is not a very realistic case since individuals' earnings tend to grow over time. A positive drift in earnings imposes a trade-off between the expected retirement benefit and its variance, as the expected retirement benefit and the variance both increase when the averaging horizon shortens. One of the contributions of this paper is to show that the preferred scheme depends on the degree of persistence of the earnings shocks. In general, for a given drift, longer horizons will be optimal when shocks are mostly permanent while shorter horizons dominate when shocks exhibit some degree of mean reversion. This will be true even in the case in which benefits of different averaging horizons are normalized so that the expected retirement benefit is invariant to how much earnings history is used to calculate them (in this case however the effect is of second order of magnitude).

The model abstracts from additional complications arising from labor supply responses. The effects of social security and private pensions on retirement decisions has been widely explored, and here I want to isolate the effect of earnings uncertainty.

Recent empirical work on the structure of earnings has stressed the importance of both transitory and permanent shocks as sources of uncertainty, and progress has been made to disentangle one from the other. I show here that the optimal retirement benefit depends not only on earnings risk, but also on the type of risk that agents face in their labor income.

If the assumption of *iid* shocks is removed, it is possible that short horizon formulas dominate over formulas that use long horizons even if earnings do not grow over time. For this result the variance of the transitory shocks must decrease over the life cycle, and the variance of permanent shocks has to be negligible. This is due to the fact that, conditional on information available at the beginning of working life, permanent shocks will accumulate period after period while transitory shocks will not, resulting in a small total variance for a short average.

The natural next step is to match the features of the model with actual earnings processes. I estimate a structural model of earnings dynamics using data from the PSID. The starting point is an empirical specification that has been widely explored in the labor literature: earnings are subject to both permanent and transitory shocks. Initially I estimate a process in which the variance of transitory shocks changes over time and the variance of the permanent shocks remains constant. I

use a minimum distance estimator to fit the moment conditions to the sample moments, and find that the variance of transitory shocks is not constant over time, being in general higher during recession than during expansion years. I also find important differences in the ratio of permanent to transitory variances by industry and education levels.

Then I use a second specification that allows for age-related heteroskedasticity. I calculate variances for different age groups, assuming that the variance of permanent shocks changes with time and the variance of transitory shocks changes over age. This time I find that the variance of the permanent shock is lower than before and relatively constant. The variance of the transitory shocks has a U-shaped pattern, but is much higher at the beginning of working life than at the end. Finally I estimate a specification in which permanent shocks are age-heteroskedastic, and find that the best fit for the standard deviation of permanent shocks over the life cycle is a polynomial of third degree on age.

Once earnings profiles have been calculated for different agents in the economy, I proceed to calibrate the model. To get an estimate of gains and losses when the averaging length changes I calculate the equivalent variation of extending the averaging horizon from 20 to 30 years, 30 to 35, and 35 to 38 years. I find that individuals whose labor income is characterized by a high degree of permanent shocks are willing to pay up to 15 percent of their initial income to go to a longer average. However, if permanent shocks are not very important, workers have to be compensated to go to a longer average. The cost of increasing the averaging horizon is no more than 2 percent of initial income. I also find that transitory shocks will matter only if they are very large, and if the averaging horizon is relatively short.

I also use the model to look at the optimal averaging horizon. I find that the preferred horizon is about 26 years for a small standard deviation of earnings, and increases sharply as soon as the standard deviation of permanent shocks goes up.

The diversity in averaging horizons is a feature also shared by pension plans. Kotlikoff and Smith (1984) report that the most common average lengths in US private pension plans are final 3, final 5, final 10, or career average. Even more interesting, the proportion of plans (and of participants in each of these plans) varies significantly by industry. For example while career average plans are used by 7.5 percent of participants in the finance industry, the same type of plan is used by 53 percent in the construction industry. This may provide enough variation

to test the hypothesis of risk sharing between firms and workers: if the volatility of earnings is different across industries and has been increasing over time, then industries where earnings are more volatile should have a higher proportion of long averaging horizons plans. I discuss these issues as extensions for future research, because this will require to include the supply side of pension benefits, as well as issues concerning job mobility across firms or industries.

The paper proceeds as follows: section two discusses related literature, section three presents the model and an illustrative example; section four describes the empirical specification of the structural model for earnings; and the calibration results are presented in section five. Section six concludes and discusses future research.

## **2. Related Literature: Defined Benefit Plans and Earnings Uncertainty**

Most of the papers discussing defined benefit retirement plans and earnings uncertainty do so in the context of a social security reform, i.e. what happens to savings or retirement when a social security system is introduced. Very few papers have analyzed the role of non-insurable earnings risk on earnings related retirement benefits from a microeconomic perspective, and none has looked at the effect of different types of shocks on the expected utility of different retirement benefit plans. Balcer and Diamond (1978) calculate the effect on expected benefits of lengthening averaging horizons, using the provisions of the US social security system. Their analysis concentrates on the expected value of retirement benefits under a 19 year horizon and a 35 year horizon, and disregards second moments and expected utility considerations. Samwick (1993) compares expected utility from different pension arrangements using an intertemporal optimization setting. The purpose of his paper is to test the hypothesis of risk sharing between firms and workers, that is, if firms tend to compensate workers income uncertainty by adopting specific retirement plans. He uses a three period model to show that a flat retirement benefit offers the highest expected utility in the presence of earnings uncertainty, followed in descending order by the highest earnings plan, career average and lowest earnings plan. His model is richer than mine in one dimension, in that it allows for behavioral

responses of consumers through higher or lower precautionary savings depending on the retirement formula, but his earnings process is very simple, and as such it does not recognize the heterogeneity of the population arising from different degrees of persistence in earnings shocks. His model assumes only permanent shocks; transitory shocks are measurement error and as such can be neglected. I show later that it is important to include both types of shocks into the analysis.

### 3. The Model

I use a very simple setting that illustrates the relationship between the length of the averaging period and the nature of earnings uncertainty faced by different individuals. Risk averse agents maximize expected utility of retirement subject to an intertemporal budget constraint. Assuming that consumption is equal to income in every period, the only source of income for retirement comes from social security wealth.<sup>3</sup> Then the problem reduces to maximizing expected utility of income at retirement, conditional on the information available in the first period of work. This can be stated as:

$$\begin{aligned} \max_n \quad & E[U(y_{T+1})|I_0] \\ \text{s.t.} \quad & y_{T+1} = \beta \left( \frac{1}{n} \sum_{i=0}^{n-1} y_{T-i} \right) \end{aligned} \quad (1)$$

where  $T$  is the last working period,  $\beta$  is the replacement rate (percentage of earnings base that is paid at retirement), and  $y_{T+1}$  is the retirement benefit, calculated as a percentage of some average earnings.<sup>4</sup> A different way to rationalize this

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<sup>3</sup>This may seem to be a rather unrealistic assumption, but it is not implausible for the US, where half of the population holds almost no savings. In the last section of the paper I speculate about what would happen in a model where agents have behavioral responses, i.e. they save some portion of income every period.

<sup>4</sup>The coefficient  $\beta$  can potentially be a function of time. There are some retirement benefits that give different weights to different portions of earnings history. Since earnings are uncertain, such weighting would most likely have an effect on the optimal averaging length. I am adopting a constant coefficient for simplicity.

approach is to state the problem as one in which agents maximize an indirect utility function that depends on income (see Stock and Wise (1989)).

I assume that income is measured in real terms, given that most countries index earnings of different periods in order to calculate retirement benefits.<sup>5</sup> The absence of indexing would introduce an additional source of uncertainty, due to the risk of inflation, but the same type of analysis can be performed with nominal earnings instead of real earnings. The stochastic process for inflation can be modeled in a similar way as real earnings, i.e. a drift and a variance, and will be reflected in a higher drift and a higher transitory variance of earnings.

Expected utility is maximized by choice of  $n$ , i.e. the averaging horizon, and there is an additional constraint given by the stochastic process followed by income. Using the standard specification found in the consumption literature (Carroll (1992)), I assume that the earnings process can be represented as the sum of a permanent component that follows a random walk with drift  $\alpha$ , and a transitory component that is white noise:

$$y_t = p_t + \eta_t; \quad p_t = \alpha + p_{t-1} + \epsilon_t; \quad (2)$$

$$\epsilon_t \sim N(0, \sigma_\epsilon^2), \quad \eta_t \sim N(0, \sigma_\eta^2), \quad cov(\epsilon, \eta) = 0$$

which can be rewritten as

$$\Delta y_t = \alpha + \epsilon_t + \eta_t - \eta_{t-1} \quad (3)$$

The optimal averaging horizon will be a function of the drift  $\alpha$ , the variances of permanent and transitory shocks, and the number of working year  $T$ . A simple example will help to illustrate these interactions.

### 3.1. An illustrative example

Assume that utility is CARA:

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<sup>5</sup>The same is true for pension plans with long averaging horizons.

$$U(y_{T+1}) = -\frac{1}{\theta} \exp\{-\theta y_{T+1}\}.$$

If shocks to earnings are normally distributed, then the expected utility of a given retirement benefit can be expressed in mean-variance form:

$$E[U(y_{T+1})|I_0] = -\frac{1}{\theta} \exp\{-\theta E[y_{T+1}|I_0] + \frac{\theta^2}{2} \text{Var}(y_{T+1}|I_0)\}$$

To find the optimal averaging horizon, I compare the expected utility of retirement benefits with different horizons  $n$ . If the two retirement benefits to be considered are  $y_{T+1}^a$  and  $y_{T+1}^b$ , then

$$E[U(y_{T+1}^a)|I_0] > E[U(y_{T+1}^b)|I_0] \iff$$

$$-\frac{1}{\theta} \exp\{-\theta E[y_{T+1}^a|I_0] + \frac{\theta^2}{2} \text{Var}(y_{T+1}^a|I_0)\} > -\frac{1}{\theta} \exp\{-\theta E[y_{T+1}^b|I_0] + \frac{\theta^2}{2} \text{Var}(y_{T+1}^b|I_0)\}$$

which is equivalent to

$$\theta(E[y_{T+1}^b|I_0] - E[y_{T+1}^a|I_0]) + \frac{\theta^2}{2}(\text{Var}(y_{T+1}^a|I_0) - \text{Var}(y_{T+1}^b|I_0)) < 0$$

General expressions for the conditional mean and variances can be found by replacing equation 3 in 1 recursively. Then

$$y_{T+1} = \frac{\beta}{n} \left( (n-1)T\alpha - \alpha \frac{n(n-1)}{2} + ny_0 + \sum_{i=0}^{n-1} \eta_{T-i} - n\eta_0 + \sum_{i=0}^{n-1} (1+i)\epsilon_{T-i} + n \sum_{i=n}^{T-1} \epsilon_{T-i} \right). \quad (4)$$

The conditional expectation and variance of the retirement benefits are

$$E[y_{T+1}|I_0] = \frac{\beta}{n} \left( \alpha((n-1)T) - \frac{n(n-1)}{2} + ny_0 \right) \quad (5)$$

and

$$Var(y_{T+1}|I_0) = \left(\frac{\beta}{n}\right)^2 \left( \sum_{i=0}^{n-1} \sigma_{\eta T-i} + \sum_{i=0}^{n-1} (1+i)^2 \sigma_{\epsilon T-i}^2 + n^2 \left( \sum_{i=n}^{T-1} \sigma_{\epsilon T-i}^2 \right) \right). \quad (6)$$

If shocks are *iid* over time, the previous formula simplifies to

$$Var(y_{T+1}|I_0) = \left(\frac{\beta}{n}\right)^2 \left( \sigma_{\eta}^2 n + \sigma_{\epsilon}^2 \left( n^2 \left( \sum_{i=n}^{T-1} (T-i) \right) + \sum_{i=0}^{n-1} (1+i)^2 \right) \right) \quad (7)$$

I want to find the optimal averaging length, i.e. the value of  $n$  that maximizes expected utility given the process for earnings. When earnings do not grow over time, the expected value of retirement benefits of different averaging horizons is always the same. Then only the variance matters, and the variance will be lower the longer the averaging horizon, meaning that the optimal averaging horizon will be the entire career length. However the presence of permanent and transitory shocks and the well documented fact that earnings have a positive drift during most of an individual's working life implies that the maximization problem may instead have an internal solution. The first and most obvious trade-off is between means and variances: a longer averaging horizon with a positive drift implies a lower expected value, but a lower conditional variance too. Risk averse individuals will be trading-off lower variance with lower expected retirement benefits. A second trade-off is present even if retirement benefits of different averages are normalized to have the same expected value, and it works through the replacement rate  $\beta$ . Depending on the value of  $\beta$ , and on the ratio of permanent to transitory shocks variances, the expected utility of short horizons may turn out to be higher than the expected utility of a longer horizon. However in this case the conditions imposed on the earnings process are much more stringent.

Consider a simple example in which an agent works three periods and retires at the end of the third, receiving her first retirement benefit in period four. Assume there are two possible retirement benefit formulas:

$$y_4^a = \beta y_3; \quad y_4^b = \frac{\gamma}{3}(y_1 + y_2 + y_3).$$

The parameters  $\beta$  and  $\gamma$  are replacement rates that could take any value. Plan  $a$  is a short horizon plan, while plan  $b$  is a career average plan. Replacing earnings of different periods, the retirement benefits can be reexpressed in terms of the drift and shocks of different periods:

$$y_4^a = \beta(3\alpha + y_0 + \epsilon_3 + \epsilon_2 + \epsilon_1 + \eta_3 - \eta_0),$$

$$y_4^b = \frac{\gamma}{3}(6\alpha + 3y_0 + \epsilon_3 + 2\epsilon_2 + 3\epsilon_1 + \eta_3 + \eta_2 + \eta_1 - 3\eta_0)$$

I calculate conditional expectations and variances of each of these retirement benefits, and then compare expected utilities. The benchmark case is an earnings process with zero drift, where as I argued before, the retirement benefit based on the entire earnings history is optimal.<sup>6</sup>

In this example I show how the presence of a drift and the degree of persistence of the shocks affect the optimal averaging length. I look for a range of values of  $\alpha$ ,  $\sigma_\epsilon^2$ , and  $\sigma_\eta^2$  such that shorter or longer averaging horizons are preferred, depending on the relative importance of permanent and transitory shocks.

### 3.2. Positive drift and variance of shocks constant over time

I calculate expected utilities of retirement benefits  $a$  and  $b$  for two different cases depending on the value of  $\beta$ , the replacement rate of retirement benefit  $a$ , and  $\gamma$ , the replacement rate of retirement benefit  $b$ .

#### A. Case 1: $\beta = 1$ , $\gamma = 1$

When  $\beta$  and  $\gamma$  are equal to one, an increase in the averaging horizon decreases the expected value of retirement benefits. A positive drift in earnings means that the longer the averaging horizon, the lower the retirement benefit. This is assuming that the benefit formula remains unchanged, i.e. that  $\beta$  and  $\gamma$  are not adjusted to compensate for lower/higher expected benefits. One can think of several instances in which such changes in averaging lengths have occurred without compensation for the loss in expected benefits. One example of this can be found in the recent proposal

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<sup>6</sup>If the drift is zero, the expected retirement benefit of plan  $a$  is equal to the expected retirement benefit of plan  $b$ . The variance of plan  $a$  is  $\beta^2(4\sigma_\epsilon^2 + \sigma_\eta^2)$  and the variance of plan  $b$  is  $\gamma^2(9\sigma_\epsilon^2 + \sigma_\eta^2)$ , which is always higher, no matter how persistent shocks are (assuming  $\beta = \gamma$ ).



for social security reform in the US, which suggests among other things increasing the averaging horizon from 35 to 38 years without any type of correction in the benefits. France has instituted similar reforms. According to the Social security administration (1995), France's old age benefit formula in 1995 was equivalent to 50 percent of the average of earnings in the 10 highest years after 1947. But for those born in 1934, the number of years for which earnings are averaged was raised to 11, and this increase was supposed to continue one year at a time until year 2008, when the averaging length is going to be 25 years.

Comparing the expected utility of the short horizon plan (plan *a*) and the long horizon plan (plan *b*) I find that

$$E[U(y_4^a)|I_0] > E[U(y_4^b)|I_0] \iff \alpha > \frac{13}{18}\sigma_\epsilon^2 + \frac{1}{3}\sigma_\eta^2$$

If the only source of uncertainty comes from permanent shocks, i.e. if  $\sigma_\eta = 0$ , a short horizon is preferable if  $\alpha > (13/18)\sigma_\epsilon^2$ ; when shocks are mostly transitory, the short horizon is preferred if  $\alpha > (1/3)\sigma_\eta^2$ . Thus it is possible to have a setting in which short horizons are better if shocks are transitory, and long horizons are better if shocks are permanent. It is also possible that the values of  $\alpha$ ,  $\sigma_\epsilon$ , and  $\sigma_\eta$  are such that short horizons will dominate when transitory shocks are very volatile and long horizons will dominate when permanent shocks become more important. I illustrate with an example in table 1.

	$\alpha = 0.2$			$\alpha = 0.3$		
$\sigma_\epsilon^2$ (permanent)	0.1	0.2	0.3	0.1	0.2	0.3
$\sigma_\eta^2$ (transitory)	0.3	0.2	0.1	0.3	0.2	0.1
$n^*$ (optimal length)	1	3	3	1	1	1

**Table 1:** Optimal averaging length for different values of  $\alpha$  (drift) and variances of permanent and transitory shocks.

*B. Case 2:  $\beta \neq 1, \gamma = 1$*

Case 1 illustrates a trade-off between expected benefit and variance, with different results depending on the composition of the shocks. To look exclusively at the effect of transitory and permanent shocks, retirement benefits calculated using different earnings bases can be normalized so that the expected retirement benefit is always the same. To do this I set  $\gamma$  equal to 1 and choose  $\beta$  so that expected retirement benefits are equal to each other. The coefficient  $\beta$  is inversely related to the drift of the earnings process. In this case, the comparison of the expected utilities of different averaging horizons yields

$$E[U(y_4^a)] > E[U(y_4^b)] \iff \frac{1}{2}(\beta^2(3\sigma_\epsilon^2 + 2\sigma_\eta^2) - \frac{14}{9}\sigma_\epsilon^2 - \frac{12}{9}\sigma_\eta^2) < 0$$

$$\iff \beta^2 < \frac{\frac{14}{9}\sigma_\epsilon^2 + \frac{12}{9}\sigma_\eta^2}{3\sigma_\epsilon^2 + 2\sigma_\eta^2}$$

Again, if all shocks are permanent ( $\sigma_\eta^2 = 0$ ), the short average plan (plan *a*) is preferred if  $\beta^2 < (14/27)$ ; if all shocks are transitory, plan *a* will be better if  $\beta^2 < (12/18)$ . It is possible to have a value of  $\beta$  such that plan *a* is preferred when shocks are transitory and plan *b* is better when shocks are permanent, and then when both types of shocks are present the optimal retirement plan will depend on their relative sizes.

	$\beta = 0.75$			$\beta = 0.9$		
$\sigma_\epsilon^2$ (permanent)	0.1	0.2	0.3	0.1	0.2	0.3
$\sigma_\eta^2$ (transitory)	0.3	0.2	0.1	0.3	0.2	0.1
$n^*$ (optimal length)	1	1	3	3	3	3

**Table 2:** Optimal averaging length for different values of  $\beta$  (replacement rate) and variances of permanent and transitory shocks.

Table 2 shows an example in which two values of  $\beta$  are chosen, one that belongs to the interval  $((14/27), (12/18))$  and another outside that range. The total variance of the shocks is fixed, and retirement plans are compared when different types of shocks dominate. When  $\beta$  is in that range, long averages become preferable for more permanent shocks. When  $\beta$  is above that range, long horizons are always preferred. Since the variance of  $y_4^a$  is a function of  $\beta^2$ , it rises at an increasing rate as  $\beta$  increases, so for large  $\beta$ , the longer horizon is preferred regardless of the source of shocks. When  $\beta$  is very low, the variance of the short horizon benefit is very low too so that this plan is preferred regardless of the relative size of permanent and transitory shocks. When  $\beta$  lies within a certain range, the variance of the short horizon retirement benefit is low, but relatively large permanent shocks, which accumulate over time, can make the long horizon preferable. If shocks are transitory, the effect of a lower  $\beta$  dominates, so that the short average plan is preferred.

### 3.3. Zero drift and age-dependent variance

This is a more subtle case. Even if earnings do not grow over time, short horizons may still be optimal if the variance of the transitory shocks is decreasing over the lifetime. In this case the comparison of expected utilities reduces also to comparing variances:

$$E[U(y_4^a)|I_0] > E[U(y_4^b)|I_0]_{\sigma_\eta^2=0} \iff \text{Var}(y_4^a|I_0) < \text{Var}(y_4^b|I_0)_{\sigma_\eta^2=0}$$

and

$$E[U(y_4^a)|I_0] < E[U(y_4^b)|I_0]_{\sigma_\eta^2=0} \iff \text{Var}(y_4^a|I_0) > \text{Var}(y_4^b|I_0)_{\sigma_\eta^2=0}$$

The variance of  $y_4^a$  and  $y_4^b$  are given by

$$\text{Var}(y_4^a|I_0) = \sigma_{\epsilon_1}^2 + \sigma_{\epsilon_2}^2 + \sigma_{\epsilon_3}^2 + \sigma_{\eta_3}^2$$

$$\text{Var}(y_4^b|I_0) = \frac{1}{9}(\sigma_{\epsilon_3}^2 + 4\sigma_{\epsilon_2}^2 + 9\sigma_{\epsilon_1}^2 + \sigma_{\eta_3}^2 + \sigma_{\eta_2}^2 + \sigma_{\eta_1}^2)$$

where now I allow the variances to be different over time. In this case

$$\text{Var}(y_4^a|I_0) < \text{Var}(y_4^b|I_0)|_{\sigma^2=0} \iff 8\sigma_{\eta 3} < \sigma_{\eta 2} + \sigma_{\eta 1}$$

If for example the second period transitory variance is half the first period, the third period variance must be less than 0.4 times the second period variance for the short average to be preferred. A positive drift in this case would reinforce the results in favor of a shorter averaging horizon. This result will not hold if the variance of permanent shocks is the one that decreases over time.

This example highlights the importance of including both transitory and permanent shocks in the analysis, as well as considering the properties of earnings over the lifecycle.<sup>7</sup> The next section looks at these properties more carefully.

#### 4. A structural model of earnings dynamics

Calibrating the model requires estimates of the growth rate of earnings and the variance of permanent and transitory shocks over the life cycle. It would also be useful to have estimates for different industries and occupations, or education levels, because the optimal averaging length may vary across demographic groups. There are very few papers that calculate both the variance of shocks, and the growth rate of earnings. Gourinchas and Parker (1996) estimate a life-cycle profile of earnings using data from the Consumer Expenditure Survey (CEX), but in order to calibrate their model they use the variance of transitory and permanent shocks as estimated by Carroll and Samwick (1997). Another source for the growth rate of earnings can be found in Hubbard, Skinner, and Zeldes (1995), but the model they use is in levels, and the variance of shocks is assumed to be constant over the life-cycle.

Lacking information about age-dependent variances, and to avoid inconsistencies that could arise using estimates from different studies, here I report my own. First I estimate a model in which transitory shocks have a time-varying variance. This shows that the volatility of transitory shocks is different across years, and that it varies by industries and demographic groups. Then I estimate a model that isolates the variance of transitory or permanent shocks by age groups.

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<sup>7</sup>Even though the importance of transitory shocks decreases when  $n$  goes to infinity, it will have an effect if the variance of transitory shocks is sufficiently large.

The literature on earnings dynamics is vast. Error component models are used to disentangle the effect of different types of shocks to earnings. The focus of most papers is on the evolution of earnings instability and inequality over time, i.e. the change in the volatility of transitory and permanent shocks during the last 25 years. The general conclusion is that earnings inequality and earnings volatility have increased during the 1980s, and that changes in the volatility of permanent and transitory shocks have contributed equally to those increases.

Only two papers have dealt with age-related heteroskedasticity. Moffitt and Gottschalk (1993) find that a specification that fits the data well is one in which the log of earnings is the sum of a random walk and a transitory component following an ARMA(1,1) process. My work builds on theirs in various respects: first, I assume changes across ages as well as over time in the variance of the shocks; second, I take a unified approach and look at both changes in volatility and growth rates of earnings; finally, I use smaller cohorts which makes it easier to see how the volatility of shocks evolves over the life cycle. Baker and Solon (1997) also estimate an error component model, using Canadian data. They calculate the variance of permanent and transitory shocks by cohort and time period. The main focus of the paper is not on the change of the variance over the life-cycle, but on measuring the extent to which earnings inequality and volatility differ by cohorts and have changed over the years. They do not report estimates for the growth rate of earnings either. And they identify shocks by cohorts instead of age.

#### 4.1. Data

The data have been extracted from individual and family files in the Panel Study of Income Dynamics (PSID) for the period 1981-1989.<sup>8</sup> The sample consists of males aged 25 to 65 who were heads of household during the entire period.<sup>9</sup> I constrain the

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<sup>8</sup>I use individual files for the entire period 1968-1992 and family files from 1981 to 1990. After selecting the variables of interest I merge each family file with the individual file according to the corresponding wave's interview number, and then merge those files together using the 1968 interview number and person number.

<sup>9</sup>There is a trade-off between the number of years entering the sample and the number of observations. Given that information on earnings is much more complete for heads of household, every single study on earnings dynamics selects individuals who were heads of households during the entire period. Hence the greater the number of years taken into account, the fewer the observations. To identify variances by age group I need the most possible observations, which is why I constrain the sample to only 9 years.

sample only to those heads who were not in school and who experienced positive earnings and hours of work for every single year in the sample.<sup>10</sup> Following the example of other papers (Abowd and Card 1989, Baker 1997) I drop all individuals reporting hours greater than 4680. The sample size is 1434 observations.

## 4.2. Specification

The earnings process I estimate resembles as closely as possible the formulation of the model in section 3. I work with two different specifications, one that incorporates time effects and another one including both time and age effects.

### A. Specification 1

I start by specifying earnings as:

$$y_{it} = p_{it} + \eta_{it}; \quad p_{it} = p_{it-1} + \epsilon_{it}, \quad (8)$$

that is, earnings for individual  $i$  in period  $t$  have a permanent component  $p_{it}$  that follows a random walk, and a transitory component. In this setting  $y_{it}$  should be interpreted as the residuals from a regression of income or log income on a number of demographic explanatory variables, where the predictable component (drift or growth rate) has been already removed. To start I assume that both  $\eta_{it}$  and  $\epsilon_{it}$  are white noise. This specification is equivalent to one in which the difference in earnings follows an MA(1) process. Equation 8 can be rewritten as

$$\Delta y_{it} = \epsilon_{it} + \eta_{it} - \eta_{it-1}. \quad (9)$$

Since it is not possible to identify time effects in both the permanent and transitory variance, I follow Pischke (1995) and assume that the variance of the random walk component is constant over time, while the variance of the transitory shocks

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<sup>10</sup>This last restriction will tend to underestimate the variance because someone who experiences positive earnings over the entire period, has a more stable career path than somebody who experiences episodes of zero earnings. I discuss this issue later.

changes over time:

$$\text{var}(\epsilon_{it}) = \sigma_{\epsilon}^2; \quad \text{var}(\eta_{it}) = \sigma_{\eta t}^2; \quad \text{cov}(\epsilon_{it}, \eta_{it}) = 0. \quad (10)$$

Equations 9 and 10 impose the following moment constraints, used later in the estimation:

$$\begin{aligned} \text{var}(\Delta y_{it}) &= \sigma_{\epsilon}^2 + \sigma_{\eta t}^2 + \sigma_{\eta t-1}^2 \\ \text{cov}(\Delta y_{it}, \Delta y_{it-1}) &= -\sigma_{\eta t-1}^2 \\ \text{cov}(\Delta y_{it}, \Delta y_{it-s}) &= 0, \quad \forall s \geq 2, \end{aligned} \quad (11)$$

if there are  $T$  years in the original sample, there are  $T - 1$  variance conditions in the differenced sample, and  $T - 2$  one-lag covariance conditions. The repeated moment conditions allow me to identify the time variation in transitory shocks.

Although the model I am going to calibrate is “atemporal” in nature,<sup>11</sup> I present estimates of a model in which the variance of shocks changes over time, first to separate age from time effects, and second because the source of variation introduced through changes over time will prove useful in the calibration section.

An alternative specification that incorporates serial correlation in the transitory shocks is the following:

$$\eta_{it} = \xi_{it} + \rho \xi_{it-1}$$

in which case the difference in earnings residuals can be expressed as

$$\Delta y_{it} = \epsilon_{it} + \xi_{it} + (\rho - 1)\xi_{it-1} + \rho \xi_{it-2}.$$

The moment conditions become

$$\text{var}(\Delta y_{it}) = \sigma_{\epsilon}^2 + \sigma_{\xi t}^2 + (\rho - 1)^2 \sigma_{\xi t-1}^2 + \rho^2 \sigma_{\xi t-2}^2$$

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<sup>11</sup>Time enters in the analysis as long as people age over time, but calendar years play no role in the model.

$$\text{cov}(\Delta y_{it}, \Delta y_{it-1}) = (\rho - 1)\sigma_{\xi t-1}^2$$

$$\text{cov}(\Delta y_{it}, \Delta y_{it-2}) = -\rho\sigma_{\xi t-2}^2.$$

### B. Specification 2

In order to isolate age effects, I use the following specification, which differs from the first in the way individuals are grouped:

$$y_{iat} = p_{iat} + \eta_{iat}; \quad p_{iat} = p_{ia-1t-1} + \epsilon_{iat} \quad (12)$$

Now observations are indexed by age groups. For example one cell will be composed of all the individuals who were between 25 and 29 years old in 1982, who will be between 26 and 30 years old in 1983, and so on. Again, it is not possible to identify age and time effects for both transitory and permanent shocks, so I assume that the variance of the transitory shocks changes with age, and the permanent shocks variance changes over time:

$$\text{var}(\epsilon_{iat}) = \sigma_{\epsilon t}^2; \quad \text{var}(\eta_{iat}) = \sigma_{\eta a}^2, \quad (13)$$

Therefore, all workers who are between 25 and 29 years old face the same transitory variance, which is different for different age groups. Also, every individual in 1982 faces the same permanent variance. Formally this is equivalent to

$$\epsilon_{iat} \sim N(0, \sigma_t) \quad iid \quad \forall i, a, \quad \text{and} \quad \eta_{iat} \sim N(0, \sigma_a) \quad iid \quad \forall i, t.$$

This rules out the possibility of correlation of shocks between individuals of different age groups for the same calendar year, which renders the estimation procedure much easier.

Now the moment conditions are given by



$$\begin{aligned} \text{var}(\Delta y_{iat}) &= \sigma_{\epsilon t}^2 + \sigma_{\eta a}^2 + \sigma_{\eta a-1}^2 \\ \text{cov}(\Delta y_{iat}, \Delta y_{iat-1}) &= -\sigma_{\eta a-1}^2 \end{aligned} \quad (14)$$

### 4.3. Estimation procedure

I use a minimum distance estimator to find the values of the variances. The procedure minimizes the difference between sample moments estimated from the data and the moment conditions implied by the model:

$$b_T = \text{argmin}(c_T - f(b))' W_T (c_T - f(b)) \quad (15)$$

where  $T$  is the sample size,  $c_T$  is the vector of sample covariances,  $f(b)$  is a function of the parameters of interest (in this case the moment conditions implied by equations 11 or 14, and  $W_T$  is a weighting matrix. The first step in the estimation is to calculate the sample variance-covariance matrix calculated from the residuals of the change in earnings:

$$\text{cov} = \begin{bmatrix} C_{11} & & & & \\ C_{12} & C_{22} & & & \\ C_{13} & C_{23} & C_{33} & & \\ \dots & & & \ddots & \\ C_{1t} & \dots & & & C_{tt} \end{bmatrix}$$

The elements of the sample covariance matrix are stacked into a vector in the following way:

$$c = \begin{bmatrix} C_{11} \\ C_{12} \\ C_{22} \\ C_{13} \\ \vdots \\ C_{tt} \end{bmatrix}$$

where  $t$  is the number of periods. In the particular case considered here the uncon-

strained sample covariance matrix has a total of 36 elements, which are explained by 15 non zero moment conditions.

When the weighting matrix is the inverse of the matrix of covariances of the estimates of the sample moments, minimum distance gives efficient estimates. However this procedure, called optimal minimum distance (OMD) tends to bias the results downwards in small samples (Altonji and Segal (1996)). An alternative approach is to use the identity matrix, in which case the procedure is termed equally weighted minimum distance estimation (EWMD). I estimate the different parameters using EWMD. I also try a different weighting matrix (WMD) suggested by Pischke (1995) and Baker and Solon (1997), where optimal weights are used in the main diagonal, and zeros everywhere else. These estimates are consistent but not efficient.<sup>12</sup>

The asymptotic distribution of the estimator in 15 is

$$\sqrt{T}(\hat{b}_T - b) \xrightarrow{D} N(0, (F'WF)^{-1}F'WVWF(F'WF))$$

where  $F$  is the Jacobian of the moment function  $f$  evaluated at  $\hat{b}_t$ , and  $V$  is the estimated covariance matrix (which has been calculated using the sample fourth moments).

The moment conditions impose some constraints on the sample, which can be tested using Newey's test (1985). The quadratic form

$$T(c_T - f(\hat{b}_T))'Q_T^-(c_T - f(\hat{b}_T))$$

is distributed according to a  $\chi^2$  distribution with degrees of freedom equal to the difference between the number of elements in  $c_T$  and the rank of the Jacobian matrix of  $f$  evaluated at  $\hat{b}_T$ . The matrix  $Q_T^-$  is the generalized inverse of  $Q$ , and

$$Q_T = P_T V_T P_T'; \quad P_T = I - F_T(F_T' W_T F_T)^{-1} F_T' W_T.$$

In order to identify age and time effects I modify the standard procedure as follows: I divide the entire panel into subsamples according to individuals' age at

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<sup>12</sup>The choice of a weighting matrix other than the asymptotically optimal one is largely an ad-hoc procedure. There is no formal theory about the properties of WMD.

the beginning of the sample period. I classify people into seven age groups: 25-29 years old, 30-34, 35-39, and so on up to 60-64.

I can identify age effects here because most of the age groups are observed twice, in different samples and calendar years (for example people in the 30-34 year old group are observed in 1981 (second subsample) and in 1986 (first subsample)). The next diagram shows how the subsamples are organized:

t	1981	1982	1983	...				1988	1989
Cohort 1	25-29	26-30	27-31	...	30-34	31-35	...		33-37
Cohort 2	30-34	31-35	...	...					38-42
⋮									
Cohort 6	50-54	51-55	...			55-59	...		58-62
Cohort 7	55-59	...					61-65		

Each of the rows represents a subsample that is followed for 9 years, except for the last cohort, who turn 65 before the last year in the sample. Baker and Solon (1997) use a similar approach to identify cohort effects, and their sample has many more observations, which allows them to use two-year cohorts, and to follow them for 9 to 17 years. I cannot follow each cohort for a period longer than 9 years without losing too many observations, and I cannot include younger cohorts in late years for the same reason. This means I will not have an age group that was 25 to 29 years old in say 1986, because that would imply following them over a spell of just three years. And I cannot have individuals aged 25 to 29 in earlier years, like in 1984, because that would create correlations across age groups for a given year. This is due to the fact that some of the individuals who belong in the 25-29 group in 1984 are present in the age cohort who is 25-29 years old in 1981. However since I observe each age group twice (except for the very young and very old), it is possible to separate age from time effects.

Now, for each subsample I calculate the sample covariance matrix, and stack the elements into a vector  $C_s$ , where  $s$  represents an age cohort according to age in 1982. Then I pool all the vectors together into a single vector  $C$  and estimate the parameters in the same way as before. The standard errors are calculated using the same procedure as in the first specification, taking care now of the fact that  $V$  includes information from different panels. Given my assumptions about

correlations across cohorts,  $V$  is a block diagonal matrix, where each block is equal to the estimated covariance matrix for each subsample.

If the estimated covariance matrix is block diagonal, so is the matrix  $Q$  and its inverse. Then it is possible to modify the original specification test in the following way:

$$\sum_{s=1}^n T_s (c_{sT} - f_s(\hat{b}_T))' Q_{sT}^{-1} (c_{sT} - f_s(\hat{b}_T)),$$

where  $s$  refers to the subsample, and  $n$  is the number of subsamples I have.

#### 4.4. Results

In order to evaluate the optimal averaging horizon for different types of workers I need to get estimates of both the first and second moments of the earnings process. Table 3 shows the results from regressing the log of earnings on age and school characteristics.

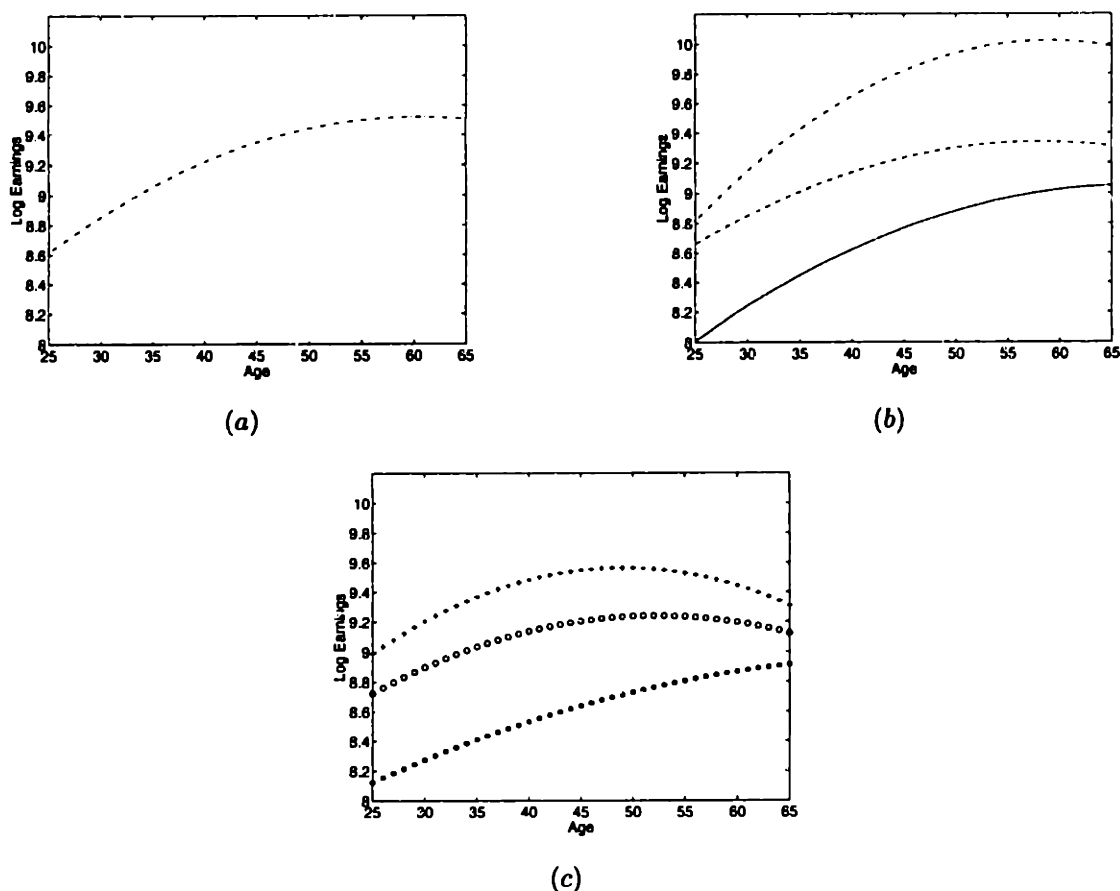
Dependent variable	$\ln(y_t)$
Age	0.0868 (0.0052)
Age <sup>2</sup> /100	-0.086 (0.006)
Schooling	0.1104 (0.0018)
Constant	5.47 (0.1076)
R <sup>2</sup>	0.248

Note: Standard errors in parenthesis.

**Table 3:** Regression of log earnings for the entire sample.

The results are in line with previous studies: earnings increase with years of schooling, and increase with age until age 57 for the entire sample. These variables

do not have much predictive power for the growth rate of earnings. The  $R^2$  for the regression in differences (not reported) is approximately 0.01, a result also found in previous studies.<sup>13</sup>



**Figure 1:** Log earnings as a function of age: (a) Entire sample; (b) By education group, less than 12 years of schooling (—), high school graduates (---), and more than 12 years of schooling (- · - ·); (c) By industry group, agriculture and construction (\*), professional (o), and public administration (+).

Figure 1 shows the evolution of log earnings over the lifecycle for the entire sample and for different demographic groups. Depending on the group of the population considered, the rate of growth of earnings will range from 3 to 7 percent at the beginning of working life, and will decline over the life cycle. During the last years of work the growth rate tends to be negative, although some groups experience

<sup>13</sup>Pischke (1995) reports similar results using data from the Survey of Income and Program Participation (SIPP).

positive growth during the entire period (basic industries like agriculture, mining, and construction, and less educated individuals). Even though these figures are noisy, it is apparent that earnings grow over most of the life cycle, and that the growth rate varies by education and occupation groups.

Table 4 shows the sample covariance matrix of the residuals of (log) income changes for the period 1981-1989. There are three interesting things to note: first, the variances (main diagonal) change over time, and the difference is statistically significant. The values reported here are similar to other estimates, like Baker (1997).

	1981-82	1982-83	1983-84	1984-85	1985-86	1986-87	1987-88	1988-89
1981-82	0.1484 (0.0152)	-0.3298	-0.2046	0.0072	-0.0591	-0.0389	0.0131	-0.0454
1982-83	-0.0551 (0.0121)	0.1885 (0.0226)	-0.3829	-0.1200	-0.0778	-0.0111	-0.0078	0.0148
1983-84	-0.0296 (0.0068)	-0.0624 (0.0152)	0.1408 (0.0169)	-0.2868	0.0055	0.0358	-0.0033	0.0112
1984-85	0.0010 (0.0056)	-0.0192 (0.0069)	-0.0397 (0.0084)	0.1358 (0.0137)	-0.2988	-0.0953	-0.0088	-0.0197
1985-86	-0.0082 (0.0055)	-0.0122 (0.0059)	0.0007 (0.0051)	-0.0397 (0.0080)	0.1300 (0.0155)	-0.4037	-0.0684	-0.0495
1986-87	-0.0052 (0.0053)	-0.0017 (0.0049)	0.0047 (0.0036)	-0.0122 (0.0066)	-0.0506 (0.0107)	0.1208 (0.0138)	-0.2278	-0.0776
1987-88	0.0019 (0.0047)	-0.0013 (0.0042)	-0.0005 (0.0051)	-0.0012 (0.0063)	-0.0095 (0.0080)	-0.0304 (0.0087)	0.1477 (0.0204)	-0.3551
1988-89	-0.0080 (0.0050)	0.0029 (0.0044)	0.0019 (0.0040)	-0.0033 (0.0048)	-0.0082 (0.0052)	-0.0124 (0.0056)	-0.0626 (0.0193)	0.2107 (0.0295)

Note: correlations above main diagonal. Standard errors in parenthesis.

**Table 4:** Sample Variance-Covariance matrix for whole sample

Second, the variance of the shocks is larger for the years 1982-83 and especially 1989. This suggests a higher sample variance during or close to recession years. Third, previous studies have found an increase in earnings inequality (as measured from the variance of earnings), during the 1980s and 1990s compared to the 1970s. Given

that my sample does not have many years, I cannot make any observations regarding that trend.<sup>14</sup> Finally, I find that the second moments decrease significantly for lags of order three or higher.

Parameter	WMD		EWMD	
	Estimate	Std Error	Estimate	Std Error
$\sigma_{\epsilon}$	0.2280	0.0109	0.2215	0.0113
$\sigma_{\eta 81}$	0.1779	0.0323	0.1979	0.0272
$\sigma_{\eta 82}$	0.2544	0.0242	0.2451	0.0245
$\sigma_{\eta 83}$	0.2494	0.0307	0.2492	0.0305
$\sigma_{\eta 84}$	0.1907	0.0229	0.1991	0.0208
$\sigma_{\eta 85}$	0.2023	0.0196	0.2012	0.0194
$\sigma_{\eta 86}$	0.2060	0.0265	0.2141	0.0251
$\sigma_{\eta 87}$	0.1713	0.0261	0.1720	0.0253
$\sigma_{\eta 88}$	0.2539	0.0372	0.2562	0.0370
$\sigma_{\eta 89}$	0.3070	0.0350	0.3098	0.0344
Observations	1434			
<i>p</i> - value	7.2017(10) <sup>-9</sup>		6.7240(10) <sup>-9</sup>	

**Table 5:** Parameter estimates for the entire sample.

Tables 5 to 7 show the estimated standard deviations of permanent and transitory shocks using different specifications. Table 5 shows the results from the first specification, using weighted minimum distance and equally weighted minimum distance. Both methods yield similar results. The standard deviation of the permanent shock is around 0.22, and the standard deviation of the transitory shock ranges between 0.17 and 0.30 depending on the year. Transitory shocks are more important at the beginning and the end of the sample. These results can be compared with a specification in which permanent and transitory shocks are *iid*. In that case the

<sup>14</sup>In any event the purpose of this paper is not to analyze trends in earnings inequality and the sources of it. Baker (1997), and Moffitt and Gottschalk (1994) analyze those issues extensively.

estimates using WMD are 0.23 and 0.20 for  $\sigma_\epsilon$  and  $\sigma_\eta$  respectively, with standard deviations of 0.01 and 0.009.

Parameter	WMD		EWMD	
	Estimate	Std Error	Estimate	Std Error
$\rho$	0.1885	0.0304	0.2027	0.0289
$\sigma_\epsilon$	0.1534	0.0133	0.1471	0.0149
$\sigma_{\xi 81}$	0.2420	0.0254	0.2479	0.0299
$\sigma_{\xi 82}$	0.2996	0.0242	0.2997	0.0249
$\sigma_{\xi 83}$	0.3060	0.0284	0.3151	0.0287
$\sigma_{\xi 84}$	0.2420	0.0196	0.2453	0.0218
$\sigma_{\xi 85}$	0.2497	0.0193	0.2560	0.0199
$\sigma_{\xi 86}$	0.2614	0.0248	0.2620	0.0252
$\sigma_{\xi 87}$	0.2269	0.0218	0.2299	0.0225
$\sigma_{\xi 88}$	0.2957	0.0362	0.2992	0.0361
$\sigma_{\xi 89}$	0.3575	0.0322	0.3605	0.0324
Observations	1434			
<i>p</i> - value	0.0031		0.0031	

**Table 6:** Parameter estimates for entire sample, with serial correlation in the transitory shock.

The  $\chi^2$  test rejects the specification of the model. One possible explanation is the assumption of no serial correlation in the transitory shocks. I have assumed that transitory shocks last only one period, while the unrestricted covariance matrix shows that covariances of order two are still important. The next step is to estimate the same model including serial correlation in transitory shocks. The new estimates are presented in table 6. The MA(1) parameter in the transitory shocks  $\rho$  is positive and significant. The standard deviation of the permanent shocks  $\sigma_\epsilon$  is now smaller than before, and the difference is statistically significant. Finally the specification test this time cannot reject the restrictions imposed by this new model.



Parameter	Estimate	Std Error	Parameter	Estimate	Std Error
$\sigma_{\epsilon 2529}$	0.3766	0.0496	$\rho$	0.15	0.028
$\sigma_{\epsilon 3034}$	0.3282	0.0348	$\sigma_{\xi 2529}$	0.3033	0.0390
$\sigma_{\epsilon 3539}$	0.2888	0.0378	$\sigma_{\xi 3034}$	0.2757	0.0314
$\sigma_{\epsilon 4044}$	0.1866	0.0355	$\sigma_{\xi 3539}$	0.2272	0.0338
$\sigma_{\epsilon 4549}$	0.3001	0.0536	$\sigma_{\xi 4044}$	0.1856	0.0224
$\sigma_{\epsilon 5054}$	0.2569	0.0564	$\sigma_{\xi 4549}$	0.2106	0.0552
$\sigma_{\epsilon 5559}$	0.2401	0.0615	$\sigma_{\xi 5054}$	0.2126	0.0404
$\sigma_{\epsilon 6064}$	0.1774	0.0813	$\sigma_{\xi 5559}$	0.1805	0.0412
$\sigma_{\epsilon 6165}$	0.2963	0.1327	$\sigma_{\xi 6064}$	0.1350	0.0526
$\sigma_{\eta 81}$	0.1368	0.0641	$\sigma_{\xi 6165}$	0.3148	0.1272
$\sigma_{\eta 82}$	0.1775	0.0217	$\sigma_{\epsilon 82}$	0.1624	0.0506
$\sigma_{\eta 83}$	0.1429	0.0215	$\sigma_{\epsilon 83}$	0.1582	0.0365
$\sigma_{\eta 84}$	0.1480	0.0238	$\sigma_{\epsilon 84}$	0.1550	0.0255
$\sigma_{\eta 85}$	0.1699	0.0172	$\sigma_{\epsilon 85}$	0.1655	0.0219
$\sigma_{\eta 86}$	0.1283	0.0200	$\sigma_{\epsilon 86}$	0.1500	0.0277
$\sigma_{\eta 87}$	0.1545	0.0182	$\sigma_{\epsilon 87}$	0.1512	0.0267
$\sigma_{\eta 88}$	0.1710	0.0223	$\sigma_{\epsilon 88}$	0.1685	0.0374
$\sigma_{\eta 89}$	0.2844	0.0512	$\sigma_{\epsilon 89}$	0.3129	0.0437

**Table 7:** WMD estimates of standard deviations of permanent and transitory shocks. Shocks vary over time and life cycle.

The last set of results corresponds to the second specification, where transitory shocks are heteroskedastic across age groups. Table 7 shows two different identification strategies: the first 3 columns present estimates of a model in which permanent shocks are age-heteroskedastic and the variance of transitory shocks changes over time. The standard deviation of transitory shocks is now lower than in table 5, but has a similar pattern, high for 1982, 1988 and 1989, and significantly different from one year to the next. Permanent shocks are non stationary over the life cycle, and a polynomial of third degree seems to be the best fit (not shown).

The last two columns of table 7 show estimates of a very similar model, except that now transitory shocks are age-dependent while permanent shocks are time dependent. Also, I added serial correlation to the transitory shocks. The standard

deviation of transitory shocks exhibits even more variation over the life cycle than over time (compare with values in table 5), with  $\sigma_\eta$  being equal to 0.13 for the age group 60-64, and 0.3 for the 25-29 group. The volatility of permanent shocks is almost the same over the entire period, not higher than 0.16, except for 1989, when it goes up to 0.31.<sup>15</sup>

The standard deviation of transitory shocks decreases over most of the life cycle before increasing again between ages 50 and 55. There is a lot of noise during the second half of the life cycle, probably due to sampling error. One way to reduce this noise is to fit a polynomial of second degree on age directly into the moment conditions, but constraining the standard deviations in such a way may result in inaccurate results too, especially when the permanent shocks are the ones changing over age.

There are two potential problems in the analysis of age-dependent variances. First, some misspecification of the model remains. The  $\chi^2$  tests reject the restrictions implied by each of the models specified above. This is not surprising given the set of simplifying assumptions I have made about the structure of correlations in the model. The rejection clearly implies that the structure of earnings has to incorporate additional elements, maybe in the form of age-dependent permanent shocks or time-dependent transitory shocks. Given that the test rejects both types of specifications (age-heteroskedastic transitory shocks or age-heteroskedastic permanent shocks), it is not possible to assess which specification is better. Nevertheless the patterns that emerge from the estimation are interesting and can serve as a guide to calibrate the model.

The second problem is more serious because it casts some doubt on the underlying variances for age groups. As mentioned above the sample only contains information about individuals with positive earnings during the entire sample. This sample selection will underestimate the true variance of earnings, because people with a continuous stream of positive earnings have a more stable earnings path than individuals with zero earnings in some periods. This problem gets even worse if the variance is underestimated more for some age groups than for others. For example if old people are more likely to have zero earnings than young people because they

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<sup>15</sup>Additional tests have to be performed before choosing between these two specifications of age-related heteroskedasticity.

retire early or for some other reason, then I would be underestimating the variance of the elderly more than the variance of the other age groups. I believe the problem is not that serious. Gakidis (1997) estimates that the probability of zero earnings is approximately equal (and even lower) for older people. If that is the case, then it is likely that the variance may be underestimated more for the younger members of the sample, reinforcing the results I have found.

## 5. Calibration Results

In this section I use the estimates of the stochastic process for labor income to evaluate the distributional effects of changing the averaging horizon, and to calculate the optimal length of the averaging period. I calculate the expected utility of an individual who works for a certain number of years, retires on a pre-specified date, and chooses the best averaging horizon depending on the nature of his income process. I relax some of the simplifying assumptions made in section 3. The equations for the retirement benefit are the same as before, but now I will assume that utility is CRRA:

$$U(y_{T+1}) = \frac{y_{T+1}^{1-\rho}}{1-\rho},$$

and that the distribution of earnings is log-normal. I am using CRRA because, if earnings are distributed log-normally, the retirement benefit is a weighted sum of log normal distributions, and a CARA utility function does not simplify the problem further.

There is no simple way to deal with the fact that earnings are distributed log-normally. The sum of log normals is not stable under addition, which means that the retirement benefit is not a log-normal random variable. One possibility is to approximate the real distribution with some other distribution, and then evaluate the expected utility using the approximation.<sup>16</sup> Another approach, the one used

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<sup>16</sup>This is the approach followed in a number of papers on exotic options, i.e. path dependent options. One particular case is Asian options, in which the strike price is a function of the average of previous prices. Given that the prices are assumed to be log-normally distributed, pricing this type of option leads to the same problem as here.

here, is to evaluate the expected utility numerically using Montecarlo methods. The procedure is the following: using the estimates for the variance of transitory and permanent shocks from the previous section, I generate a sequence of shocks from a log-normal distribution and estimate the value of earnings during working life. With that sequence of earnings for each working year I can calculate the retirement benefit for different averaging horizons and the corresponding utility. I perform 10000 replications, taking the expectation over all the replications to find the expected utility. I use an antithetic variate method to reduce the variance of the estimates calculated in this way.

There are at least three sets of questions one might be interested in answering: first, what is the welfare/utility loss of a worker in a certain industry, when the prevalent averaging horizon is different from the optimal, given the characteristics of the earnings process in that industry? Second, what is the welfare loss associated with a social security reform that extends averaging horizons? Does it differ across demographic groups? Finally, what is the optimal averaging period for a representative individual? How does it depend on the degree of persistence of earnings shocks?

The first question raised is difficult to assess using the stylized model presented here. Private pension plans have many dimensions, of which the averaging period is but one.<sup>17</sup> In light of this I start by evaluating the distributional consequences of changing averaging horizons. Assuming that a retirement benefit formula with an averaging period of  $n$  years exists, I calculate the expected utility that individuals with different earnings processes derive from that retirement benefit.<sup>18</sup> Then I compare it with the expected utility of a retirement benefit formula based on a longer averaging period, and estimate the percentage change in initial income these individuals would be willing to accept so that the expected utility under the new averaging horizon is equal to the original expected utility level. For the calibration I assume that the individual starts working at age 25, and retires at age 65. The coefficient of risk aversion is equal to 2, the replacement rates  $\beta$  and  $\gamma$  are both

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<sup>17</sup>This is also true for social security systems, but as I explain in the last section it is easier to justify abstracting from those issues in the context of social security than in the private pension market.

<sup>18</sup>The reader should keep in mind that I do this conditional on information available in the first working period. It seems a straightforward exercise to extend this analysis to individuals who are at different stages in the lifecycle.

equal to 1, and the growth rate of labor income and the variances of permanent and transitory shocks are as estimated in section 4. I calculate the change in initial income from extending the averaging horizon 10 years, from 20 to 30 years, then from 30 to 35, and finally from 35 to 38 years.

Tables 8 to 10 present the results. I use three different sources of variation in earnings uncertainty: table 8 shows the percentage change in initial income that will keep an individual indifferent between retirement benefits of different averaging lengths (equivalent variations). The first two columns have the same values of  $\sigma_\eta$ , the standard deviation of transitory shocks to income, while for the standard deviation of permanent shocks  $\sigma_\epsilon$ , I chose the values corresponding to the years 1987 and an average of the values for 1981-1989 as presented in table 7.

$\sigma_\epsilon$ (Std. dev. of perm. shock)	0.1534	0.2142	Age-dep	Age-dep
$\sigma_\eta$ (Std. dev. of trans. shock)	Age-dep	Age-dep	0.1775	0.2844
$\Delta y_0/y_0$ , 20 vs 30 avging period	-0.016	-0.094	-0.154	-0.189
$\Delta y_0/y_0$ , 30 vs 35 avging period	0.01	-0.03	-0.08	-0.093
$\Delta y_0/y_0$ , 35 vs 38 avging period	0.012	-0.014	-0.066	-0.068

**Table 8:** Equivalent variation from extending averaging horizons: 20 to 30, 30 to 35, and 35 to 38 years, for different values of permanent and transitory shocks variances. Calibration performed for the entire sample.

Negative entries in table 8 indicate that the individual would be willing to sacrifice part of his initial income to move to a longer averaging horizon. This means that extending the horizon will result in a welfare gain. For example the first entry in table 8 shows that a 25 year old worker would be willing to give up 1.6 percent of his initial income to move from a 20-year averaging horizon to a 30-year average. A positive entry means that the individual would have to be paid in order to accept a longer horizon, because he would be worse off if the horizon was extended. The

results suggest that only when the variance of permanent shocks is low, and the averaging horizon very long, is there a cost to extending the horizon. when the variance of income is relatively low, extending the averaging horizon will reduce the conditional variance of the retirement benefit, but not enough to outweigh the decrease in expected retirement benefits. However for most of the cases examined the reduction in risk due to a longer horizons seems to be higher than the decrease in expected benefits.

Schooling	less 12 yrs	12 yrs	more 12 yrs
$\sigma_\epsilon$ (Std. dev. of perm. shock)	0.2683	0.2039	0.2166
$\sigma_\eta$ (Std. dev. of trans. shock)	0.2356	0.1994	0.1136
$\Delta y_0/y_0$ , 20 vs 30 avging period	-0.186	-0.115	-0.069
$\Delta y_0/y_0$ , 30 vs 35 avging period	-0.077	-0.037	-0.008
$\Delta y_0/y_0$ , 35 vs 38 avging period	-0.04	-0.017	0.002

**Table 9:** Equivalent variation from extending averaging horizons: 20 to 30, 30 to 35, and 35 to 38 years, for different values of permanent and transitory shocks variances. Sample divided according to three different levels of schooling.

The last two columns of table 8 use age-dependent estimates for the standard deviation of permanent shocks, which are much higher than the ones in the other 2 columns (a maximum of 0.37 for age group 25-29, and a minimum of 0.17 for age group 60-64). The only difference between columns 3 and 4 is the size of the standard deviation of transitory shocks. I use the values corresponding to years 1982 and 1989 in table 7. It is apparent that transitory shocks matter, especially when the average horizon is not too long. The larger the value of  $\sigma_\eta$ , the more the percentage of initial income that an agent is willing to sacrifice in order to move to a longer horizon. For long enough horizons the importance of transitory shocks tends to disappear (going from 35 to 38 years results in approximately the same gain in

both columns).

I also estimate equivalent variations for different groups in the population. I split the initial sample into 3 different groups according to educational attainment: those with less than high school, those with 12 years of schooling (high school diploma), and those with more than 12 years of schooling. Then I estimate the standard deviation of permanent and transitory shocks, both using *iid* and time-varying specifications.<sup>19</sup> Table 9 presents the results for each of the education groups. As expected less educated people have a higher volatility of earnings, and the difference with more educated people is statistically significant. Individuals with high school diploma and more than high school are very similar in terms of the volatility of permanent shocks, while the standard deviation of transitory shocks is much higher for the more educated people. It turns out that less educated people would benefit the most from extending the averaging horizon, while only people with more than 12 years of schooling would lose from an increase in the averaging horizon from 35 to 38 years.

Industry	Agriculture	Professional	Public Adm.
$\sigma_\epsilon$ (Std. dev. of perm. shock)	0.1795	0.1543	0.1045
$\sigma_\eta$ (Std. dev. of trans. shock)	0.163	0.1926	0.1240
$\Delta y_0/y_0$ , 20 vs 30 avging period	-0.04	-0.071	-0.033
$\Delta y_0/y_0$ , 30 vs 35 avging period	-0.012	-0.016	0.009
$\Delta y_0/y_0$ , 35 vs 38 avging period	-0.006	-0.003	0.012

**Table 10:** Equivalent variation from extending averaging horizons: 20 to 30, 30 to 35, and 35 to 38 years, for different values of permanent and transitory shocks variances. Sample divided by industry groups.

<sup>19</sup>I use the *iid* estimates in the calibration.

Table 10 shows the same experiment but for three different industry groups. Notice first that the volatility for each of the groups is much lower than the previous estimates, because I used only those individuals that were in the same industry during the entire period. Eliminating individuals who switch jobs across industries eliminates an important source of income volatility. The highest volatility among these three groups corresponds to basic industries like mining and agriculture, and the lowest to public administration employees. The middle column shows estimates for professionals.

Even though workers in basic industries are subject to higher permanent shocks than professionals, this last group is willing to sacrifice a higher portion of its initial income to go to a longer average. The reason is likely related to the very different earnings profiles: the profile in agriculture is much steeper, which means that extending the horizon reduces expected benefit substantially. Workers in the public administration are the ones hurt by an increase in the averaging length. Going from 35 to 38 years will leave them indifferent only if they get paid about 1.2 percent of their initial income.

$\sigma_\epsilon$ (Std. dev. of perm. shock)	0.15	0.2	0.25	0.3	0.35
$\sigma_\eta$ (Std. dev. of trans. shock)	0.35	0.3	0.25	0.2	0.15
$n^*$ (optimal avg horizon)	26	36	38	38	38

**Table 11:** Optimal averaging horizons for different degrees of persistence in income shocks. From first to last column, the standard deviation of permanent shocks  $\sigma_\epsilon$  increases while the standard deviation of transitory shocks  $\sigma_\eta$  decreases. Entire sample.

In general most groups in the population would benefit from increasing the length of the averaging horizons, and this will be true in most circumstances (i.e. going from 20 to 30 years or further). This means that in most cases the optimal averaging horizon should be relatively long.

Table 11 reports the optimal averaging horizon for different degrees of persis-



tence of shocks, for the entire sample. The optimal averaging horizon is the averaging horizon that maximizes the expected utility of retirement benefits. The optimal averaging length shortens when the transitory shocks become more important and the volatility of permanent shocks decreases. When the standard deviation of permanent shocks is 9.15 and the transitory shock is 0.35, the preferred horizon is 26 years.

It seems that transitory shocks are relatively unimportant in deciding the optimal averaging length. They become negligible as the time horizon expands.<sup>20</sup> As I showed in the simple example of section 3, it is theoretically possible to have a case in which changing the importance of transitory shocks changes the optimal averaging length. But this is true only when variances and growth rates take on particular values. In the calibration exercise, the growth rate of earnings may be too high or too low for transitory shocks to affect choice, or alternatively, in order to change the optimal horizon in one year, the change in variance of the transitory shocks may have to be too high. However the equivalent variation exercises showed that transitory shocks will matter if the averaging horizon is not too long.

## 6. Conclusion and Extensions

In this paper I analyze the effect of earnings uncertainty on earnings related defined benefit plans when the averaging horizon changes. I find that the distributional effects of extending averaging horizons are important. A change in the averaging length will benefit some people more than others, and will also hurt some groups of the population. I also find that the optimal averaging length for a very stylized defined benefit retirement plan would range between 26 and 38 years, given the estimated characteristics of the stochastic process for earnings.

The goal of this paper is not to provide a positive explanation as to why averaging horizons vary so much across countries and industries. However it is reasonable to conclude that demand factors like the degree of earnings uncertainty of the workers in a particular industry or country should be taken into account. It might be possible to use time variation in earnings uncertainty across industries to find out how much

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<sup>20</sup>At this stage I should remind the reader that a transitory shock may be up to two years length, because given the assumption of serial correlation in transitory shocks, the change in income can be modeled as an MA(2) process.

of the variation in averaging horizons can be explained through this mechanism. A number of obstacles make it difficult to test such a hypothesis: first, workers move across industries; second, there are supply side factors in the private pension market; third, many features of private pension plans have been disregarded here, like vesting and portability of pensions, indexing, etc.

There are a number of natural extensions for this paper. First, the use of actual data for specific countries or industries would make the paper a more specific exercise, and it would be possible to incorporate in the analysis all the subtleties involved in a particular formula. Second, introducing retirement decisions should be straightforward. A model which incorporates an option value of retirement could be used to analyze how different averaging horizons change the incentives to retire. Finally, it would be interesting to extend this into an intertemporal setting in which people make decisions about consumption and savings. Deaton (1991) shows that the degree of persistence of income shocks affects individuals' behavior with regards to precautionary savings. If the averaging horizon changes, the variance of the retirement benefit will change too, which will affect people's behavior before retirement (depending on the degree of persistence of shocks). If individuals hold other forms of savings, the covariance between the returns on those assets and earnings uncertainty needs to be taken into account too.

## 7. Appendix

The variables used in the regression analysis are the following:

- Earnings: head's income from wage and salaries in previous year
- Hours: head's annual hours worked on main job in previous year
- Schooling: years of completed education
- Age
- Sex
- Industry: head's main occupation 3-digit industry

Table 12 shows the mean values for some of the variables used in the analysis, for each year in the sample.

Year	Hours	Earnings	Age	Schooling
1981	2154	7975	35.49151	
1982	2126	7952	36.49843	
1983	2110	8440	37.49591	
1984	2129	8827	38.50786	
1985	2186	9059	39.56254	
1986	2171	9401	40.49277	
1987	2190	9545	41.51351	
1988	2202	9597	42.53237	
1989	2192	9439	43.52608	13.1

Note: Real earnings in 1967 dollars.

**Table 12:** Average values for selected variables in different years

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## Chapter 3

### Life Cycle Behavior under Alternative Retirement Plans

#### 1. Introduction

In this paper I analyze the consumption and saving behavior of individuals over the life cycle, when labor income is uncertain and retirement income depends on earnings history. I will answer the following question: in an environment where labor income is uncertain, and uncertainty matters, do the patterns of consumption and saving differ for agents holding different types of pension plans?

The theoretical predictions about the effect of income uncertainty on the intertemporal patterns of consumption and saving are clear. If the utility function is such that the third derivative is positive, the individual will hold precautionary saving. This means that the characteristics of the earnings process (expected growth rate and variance) will affect the consumer's behavior. There seems to be some consensus on the idea that people save not only for retirement, but also because they do not know with certainty what their income is going to be later in life. The empirical literature is somewhat divided on the amount of precautionary saving people hold over the life cycle, with results varying from rejecting the presence of a precautionary motive to stating that precautionary saving is a significant part of individuals' asset holdings.

Most of this literature has concentrated on the effect of labor income uncertainty, and not much attention has been paid to the effect of retirement income on asset accumulation over the life cycle. However if labor income is uncertain and retirement income is a function of earnings history, then, conditional on the information available at the beginning of working life, retirement income is also uncertain and as such it will have an effect on saving. I develop this point by solving numerically an intertemporal model of consumption and saving that incorporates explicitly uncertainty in retirement income. In the model agents hold precautionary saving and are subject to uninsurable labor income uncertainty. In addition to labor income

uncertainty I assume that there are two types of earnings related retirement plans: defined benefit plans, in which the benefit is specified by a formula that depends on the last years of working income, and defined contribution plans, where the benefit is a weighted average of lifetime earnings. I compare the expected consumption and saving profiles for a representative agent whose benefit at retirement is based on either plan.

Models of precautionary saving are important because they help in explaining some life cycle patterns which cannot be captured by a standard life cycle hypothesis model. The fact that consumption and income track each other closely during the early part of the life cycle goes against the predictions of the standard lifecycle model, in which consumption evolves independently of current resources. If precautionary motives are present, people may prefer not to borrow to finance current consumption even if they think income is going to increase in the future. Using a similar argument, agents will save more when young to protect themselves against uncertainty.

Incorporating explicitly the effect of uncertainty of income at retirement in such a setting is important for a variety of reasons: first, retirement income uncertainty provides an additional source of risk. This implies that the amount of precautionary saving may be larger than the one generated by the standard channels, and it will differ across people depending on the type of pension plan they hold. Second, the issue of the effect of pension plans on saving has become more and more important with the recent attempts in developing and developed economies to reform their social security systems. There seems to be a trend towards defined contribution plans, and it is still unclear what effect these plans have on individuals' behavior.<sup>21</sup>. While the macroeconomic effect of fully funded and pay-as-you-go plans has been investigated extensively, it is still necessary to look into the effect of such changes at the household level.

The structure of the paper is as follows: section 2 summarizes the existing literature on these issues. Section 3 discusses the characteristics of interest in defined benefit and defined contribution plans, as well as some basic facts for the US pen-

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<sup>21</sup>This trend is more pronounced in developing economies, where defined contribution plans have been adopted by governments as their main social security system. In developed economies the trend has been towards private pension defined contribution plans as an additional source of income for retirement, and not as a substitute for social security



sion system. A model of consumer optimization in the presence of labor income uncertainty is presented in section 4. finally I present the calibration results and conclude.

## 2. Previous Literature

There are two strands of the literature that are of interest for this paper: one is related to the implications of income uncertainty for life-cycle consumption; the other has to do with the trade-offs between defined benefit and defined contribution pension plans. The literature on precautionary saving is vast. There is a variety of papers solving intertemporal optimization problems under different sets of assumptions, and empirical papers trying to quantify the amount of and the extent to which people hold precautionary saving. The main assumption necessary to obtain precautionary saving is that the marginal utility of consumption is convex, a behavior known as "prudence". When utility is CARA, as in Caballero (1990), it is possible to derive a closed form solution for the consumption profile. Assuming that income follows a random walk and that life is finite, optimal consumption is shown to be a function of the stock of assets, current income, the remaining lifespan, and the variance of income.

Although using CARA utility functions allows for a closed form solution, it has an important drawback; it does not rule out negative values of consumption at the optimum. That is why most papers in this area rely on CRRA utility functions. Using CRRA utility functions comes at a cost, because there is no closed form solution when income is uncertain, and hence the need to use numerical methods to solve for the optimal consumption rule. This is the approach followed by Deaton (1991), Carroll (1992), and Hubbard, Skinner, and Zeldes (1995) among others.

Deaton solves a model in which there is precautionary saving as well as liquidity constraints. He shows that the evolution of assets and consumption over the life cycle is strongly related to the characteristics of the earnings process, and in particular to the degree of persistence of the shocks. Uncertainty in that model comes exclusively from income and there is no retirement period. Carroll's paper is similar in spirit, and assumes that individuals are impatient and that there is a probability of zero income. Even if agents are impatient so that they do not want to postpone consumption, the positive probability of zero earnings makes them hold positive

assets in every period except in the last one. Finally Hubbard et.al. build a life cycle model in which there are three sources of uncertainty: labor income, lifetime, and medical expenses. These sources of uncertainty are able to explain a number of regularities observed in US consumption and saving data. Gourinchas and Parker (1997) go one step beyond and estimate structural parameters by matching the predictions of their model to the empirical consumption and income profiles estimated with US data.

The papers mentioned previously differ in the treatment of income at retirement. Deaton ignores it completely, Gourinchas and Parker truncate their problem at retirement and assume some continuation payoff from then on, while the others assume that income at retirement is going to be some fixed amount or some percentage of labor income. In doing so they miss the link between labor income and retirement income, and the effect of different retirement formulas on optimal behavior.

The role of retirement income has been incorporated explicitly in a few papers. Bodie, Marcus, and Merton (1988) look at the welfare implications of holding a defined benefit or a defined contribution pension plan, in the presence of different sources of uncertainty. The case of income uncertainty and asset returns uncertainty is analyzed separately in a three period model setting, where a representative agent works during the first two and receives a retirement income in the third period. They do not solve explicitly for consumption rules, and limit to highlight the trade-offs involved in holding either plan. When utility is logarithmic, defined contribution plan dominates both under income and asset return uncertainty. The result is driven by a very short averaging horizon in the defined benefit plan, by the nature of the utility function chosen, and by the assumption that there is no uncertainty during the annuitization period. Incorporating uncertainty during annuitization and extending the average horizon (as well as using a different utility function) would probably change the results in favor of defined benefit plans or at least making them equally good (when only income is uncertain). Samwick (1993) presents a model in which retirement income is uncertain, but his goal is not to analyze the difference in the patterns of saving over the life cycle. The goal of that paper is to test for the degree of risk sharing between workers and firms when labor income is uncertain. The main idea is to show that in industries where labor income risk is higher firms provide workers with retirement plans that compensate for that additional risk.

In order to highlight the link between uncertainty in labor income and income at retirement a three period model is presented. The model does not compare defined benefit with defined contribution plans, but defined benefit plans that differ in the way the benefit is calculated: flat, percentage of highest wage, percentage of lowest wage, and average of the two working periods' income. Assuming that earnings shocks can take on only two values, high and low, Samwick shows that the flat benefit dominates over the others and generates the least amount of precautionary saving. The second best plan is highest wage, then average, and finally lowest wage retirement benefit.

### 3. Defined Benefit, Defined Contribution, Risks and Facts

#### 3.1. Pension Plans and Risks

In this paper I will be comparing the life cycle patterns of consumption and saving for individuals with two different types of retirement plans: defined benefit and defined contribution. Defined benefit plans are those in which the benefit at retirement is known in advance, based on a formula that takes into account a number of factors like retirement age, number of working years, number of years of covered work, labor income, etc.<sup>22</sup> Defined contribution plans on the other hand are those in which there is no preset formula for the benefit calculation. however a contribution rate is set, and the benefit will depend on the effective contributions made until the time of retirement.

A typical defined benefit retirement plan can be represented in the following way:

$$Y_{db} = \alpha \frac{1}{n} \sum_{t=T-n}^T Y_t, \quad (1)$$

where  $\alpha$  is a replacement rate (percentage of earnings base that individuals receive at retirement),  $n$  is the averaging horizon, i.e. the number of periods of earnings history used to calculate benefits,  $Y$  is labor income, and  $T$  is the last working

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<sup>22</sup>By years of covered work I mean number of years the person has worked and made contributions.

period. As for the defined contribution plan, the benefit results from assuming that a worker deposits a proportion  $\tau$  of earnings every period in a personal account. This funds earn a real gross return  $R$ , and the pension fund charges a proportion  $\gamma$  of earnings to administer those funds. At the time of retirement the amount accumulated in the personal account is:

$$(\tau - \gamma) \sum_{t=1}^T Y_t R^{T+1-t} \quad (2)$$

When the worker retires those funds are used to buy an annuity. The insurance company charges a proportion  $\xi$  of the funds, and the remainder is used to pay  $Y_{dc}$  every period until the time of death. Assuming that the remaining funds accumulated in the account earn some interest during the annuitization phase, the retirement benefit each period is equal to

$$Y_{dc} = (1 - \xi)(\tau - \gamma) \frac{\sum_{t=1}^T Y_t R^{T+1-t}}{\sum_{t=0}^{D-T-1} (1/R^t)} \quad (3)$$

where  $D$  is time of death.

By examining the formulas it is apparent that a number of factors may affect the riskiness associated with each plan: asset returns, income, political, inflation, and lifespan. Asset returns uncertainty will affect only the defined contribution plan, both during the accumulation and the annuitization stages.<sup>23</sup> Inflation is more likely to affect defined benefit plans because in most private pension schemes the labor income entering the formula is not indexed to inflation. This is especially true for defined benefit plans with long averaging horizons, that base the benefit in a relatively long earnings history. However this is not a problem in social security systems -most of them index earnings history before computing the retirement benefit-. By political risk I refer to the possibility of insolvency. In the defined benefit plan such a risk can be modeled by assuming that the replacement rate  $\beta$  is stochastic. In the

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<sup>23</sup>If the defined benefit plan is organized in such a way that there is a surplus in the system, and the surplus is invested in the same way the funds in the defined contribution plan are, then the risk associated to asset returns will affect both retirement plans.

defined contribution plan, a factor like political risk could be incorporated into the administrative cost  $\gamma$  charged by the pension funds.

In this paper I will concentrate on the effect of labor income uncertainty. It should be clear from the formulas that the volatility of earnings will affect both retirement plans. Even though defined benefit plans are explicitly linked to earnings history, defined contribution plans are related to income in a much subtler way. In fact defined contribution plans are a weighted average of lifetime earnings, where contributions at the beginning of working life have a higher weight. Two identical individuals may end up with a different consumption profile only because the retirement plan they have is different. Even if the rate of return on defined contribution assets, or the contribution rate is normalized so that the expected retirement benefit is equal across plans, nothing prevents the variances from being different across plans, and if individuals care about uncertainty they will adjust their behavior. I expand on these issues in the next section.

### 3.2. Pension Plans: Trends and Facts in the US

A good source for data on pensions and wealth is the Survey of Consumer Finances (SCF). This survey is conducted every three years and contains very detailed information on assets, income, and demographic characteristics. I chose three different years: 1989, 1992, and 1995, and restrict the sample to households whose head is between 25 and 65 years of age, who is not retired nor a student.

Main Pension	Covered	IRA	1989	1992	1995
None	56	40	53	59	57
DB	18	44	26	17	13
DC	23	51	19	21	27

**Table 1:** Percentage of people with different types of pension plan, by main pension type.

table 1 shows the percentage of people with coverage by pension plan for different years and for the entire sample. The SCF reports coverage by main pension plan

and by additional pension plans, which may be secondary or pension rights acquired in previous jobs. There is a significant portion of the population not covered by any pension plan, and this percentage seems to have increased over time. It is also apparent that defined contribution plans have become more popular over time; while 19 percent of the people held defined contribution plans in 1989, the percentage has increased to 27 percent in 1995. The same table shows that a good percentage of people without a pension plan coverage hold some type of retirement saving: 40 percent of those not covered hold IRAs, and this percentage increases to 51 percent for individuals covered with defined contribution plans.

Table 2 shows the percentage of people covered by type of pension plan and age.

Main Pension	25-34	35-44	45-54	55-65
None	58	52	51	67
DB	14	20	22	13
DC	25	24	24	17

**Table 2:** Percentage of people with different types of pension plan, by main type and age group.

Coverage increases with age, except for the very old, which may reflect the fact that on the job pension coverage has become more common over time and the very old were not offered pension coverage when they started to work. Defined contribution plans seems to be more popular among young people, while defined benefit plans are preferred by middle age individuals, which may reflect industry effects (for example high-tech companies employ very young people and at the same time offer defined contribution plans). The fact that the very old hold mostly defined contribution plans is somewhat puzzling, and may also reflect industry effects (small, family run businesses may also prefer to offer defined contribution plans and may employ relatively older workers).

The next set of tables compare non-retirement assets and income for individuals in different pension plans. Table 3 shows median values of financial and non-financial assets for the entire sample, by year, and by pension type.

	Financial Assets excluding retirement (NORETFIN)	NORETFIN + IRAs	Non-Financial Assets	Primary Residence	Net Worth excluding Retirement (NORETNW)	NORETNW + IRAs
All	5.3	6.8	11.5	54	46.6	49.4
By year:						
1989	5.9	7.3	12	53.3	54.2	56.6
1992	5.3	6.6	10	48.8	42.4	45.8
1995	4.9	6.6	13	55	44.5	46.9
By pension plan type:						
none	2.7	3.5	8.8	24.6	32.2	34.4
DB	8.6	11	13.5	75	68.9	71.2
DC	9.3	12.3	14.4	76.2	55.8	59
Mix	11.6	19.8	11.2	76	62	75.7

**Table 3:** Median real assets and wealth for entire sample, by year, and by pension type plan (thousands of dollars 1995).

The value of assets is not very informative because it does not include an estimate of retirement wealth. But it is useful as a benchmark to detect any particular pattern for individuals in different pension plans. The first thing to notice is that non retirement assets are much lower for people with no pension coverage. This is true across different asset categories, including primary residence. Second, although the median values of financial and non-financial assets do not differ much across types of pension plans, total net worth does; the median net worth excluding retirement assets is about sixty nine thousand dollars for individuals whose main plan is defined benefit, and fifty six for individuals holding defined contribution plans. The reason is to be found in debt holdings by each category. The median debt holdings are shown in table 4. While the median debt holdings for individuals with defined contribution plans is forty thousand dollars, the median debt for individuals with defined benefit plans is ten thousand dollars less. This could be explained for the different age profile of holders of defined benefit and defined contribution plans. Finally table 4 shows no significant difference in median income by pension type, except for the case of individuals with no coverage, for whom income is much lower.

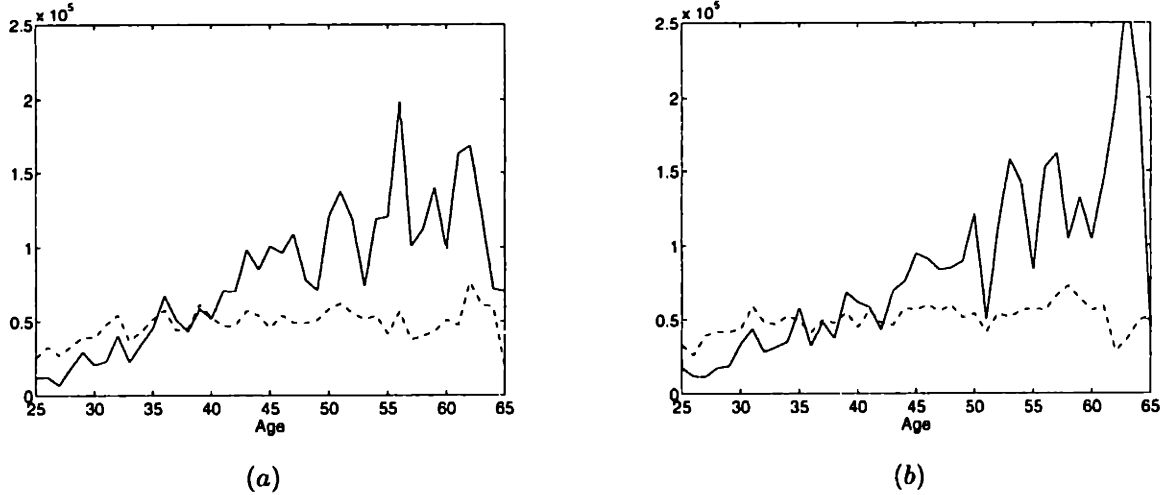
	Income	Debt
All	38	17.6
By year:		
1989	39.3	16.3
1992	37.4	17
1995	37	20
By pension plan type:		
none	27.1	9.2
DB	47.8	31.7
DC	49.2	39.9
Mix	49.1	19.6

**Table 4:** Median real income and total debt for entire sample, by year, and by pension type plan (thousands of dollars 1995).

Figure 1 shows real total income and net worth excluding retirement, by pension type. The evolution of both variables over the life cycle are relatively similar, although they start to diverge late in life. Income for people in defined benefit plans is slightly higher after age 60 or so, while net worth is higher for people in defined contribution plans around the same age.

There are many reasons why the life cycle profiles could differ across holders of different pension plans, many of which cannot be captured with the data gathered. Leaving aside the possibility that expected retirement benefits are different or uncertainty in income differs across groups, it is possible that households holding defined benefit plans are inherently different from people holding defined contribution plans. The fact that they have a different pension plan may be reflecting different attitudes toward risk. For example if all the “high savers” are in a defined contribution plan, then defined contribution plans may look as if they generate higher saving while the higher saving are due to individual characteristics.





**Figure 1:** Real total net worth excluding retirement (—) and real total income (---) by age and main pension type: (a) DB; (b) DC.

In the model that follows I will exploit one channel through which the profiles will differ; assuming a representative agent setting, I will assume that the risk associated to each plan differs, and then I will compare the expected profiles for the defined benefit and the defined contribution case.

## 4. The Model

### 4.1. Standard Model with Labor Income Uncertainty

I will use a formulation similar to Gourinchas and Parker (1997) to highlight the differences when retirement income is incorporated. In the standard life cycle model with uncertain income, individuals are assumed to live for  $D$  periods and work for  $T < D$ . A representative consumer maximizes the expected net present discounted value of lifetime utility, subject to a dynamic budget constraint. Lifetime utility is then equal to

$$E \left[ \sum_{t=1}^D \beta^t u(C_t) + \beta^{D+1} V_{D+1}(W_{D+1}) | I_0 \right]$$

where  $C_t$  is consumption in period  $t$ ,  $\beta$  is the discount factor,  $I_0$  is the information set available at the beginning of working life, and  $V_{D+1}$  represents the value to the consumer of assets left at the time of death.

The dynamic budget constraint is given by

$$A_{t+1} = R(A_t + Y_t - C_t).$$

As I explained in section 2 most models now assume that instantaneous utility is *CRRA*:

$$u(C) = \frac{C^{1-\rho}}{1-\rho}.$$

I will concentrate on the effect of labor income uncertainty which means that both  $T$  and  $D$  are known and fixed. Traditionally income uncertainty has been decomposed into permanent and transitory shocks. More precisely the log of income is assumed to be equal to

$$y_t = p_t + u_t$$

$$p_t = g_t + p_{t-1} + \epsilon_t$$

where  $u_t$  and  $\epsilon_t$  are *iid* shocks,  $p_t$  represents the permanent component, and  $g_t$  is the expected growth rate.

Previous papers have dealt with retirement in different ways. Gourinchas and Parker for example decide to truncate the problem at retirement, and to assume some functional form for the utility at retirement. Their problem can be expressed as

$$V_t(A_t, y_t) = \max_{c_t, \dots, c_T} E \left[ \sum_{i=t}^T \beta^{i-t} \frac{C_i^{1-\rho}}{1-\rho} + \beta^{T-t} V_T(A_T, Y_T) | I_t \right]$$

$$s.t. \quad A_{t+1} = R_t (A_t + Y_t - c_t),$$

$$A_t \geq 0 \quad \forall t.$$

Deaton has no retirement either, assumes that the individual works every single period of his life.

## 4.2. Incorporating Retirement Income

In addition to the assumptions I made previously, I have to specify the relationship between labor income and retirement income. I will express the dynamic budget constraint as

$$A_{t+1} = R_t (A_t + YD_t - C_t),$$

$$YD_t = \begin{cases} Y_t(1 - \tau) & t \leq T \\ Y_{T+1} & T + 1 \leq t \leq D. \end{cases} \quad (4)$$

where  $\tau$  is the contribution individuals make to personal accounts or to the government to finance a social security system, and  $Y_{T+1}$  is retirement income.

As I explained before retirement income can take on two different values, represented by equations 1 and 2 which I repeat next:

$$Y_{db} = \alpha \frac{1}{n} \sum_{t=T-n}^T Y_t,$$

and

$$Y_{dc} = (1 - \xi)(\tau - \gamma) \frac{\sum_{t=1}^T Y_t R^{T+1-t}}{\sum_{t=0}^{D-T-1} (1/R^t)}$$

Although it is not strictly necessary I will assume that there are liquidity constraints, so that the value of assets has to be non-negative every period.:

$$A_t \geq 0 \quad \forall t.$$

I will also assume that retirement assets are illiquid in that they can only be used after retirement. This will eliminate the additional complication introduced by

having a choice between two different types of assets. And, although many defined contribution plans offer the possibility of increasing contributions voluntarily, I will assume that contributions are fixed, constant, and mandatory.

The individual's problem is a discrete and finite time, stochastic dynamic programming problem. In the presence of labor income uncertainty, and with *CRRA* preferences there is no closed form solution and the problem has to be solved numerically. The solution to the problem is a set of policy functions that specify the optimal value of consumption for different values of the state variables. The number of state variables will depend on the type of retirement benefit. For the defined benefit plan I will start by assuming that the average horizon is equal to the last period of working income ( $n$  in equation 1 is equal to 1). This will be an important factor in explaining the difference in the life cycle patterns of consumption and saving. This retirement benefit specification implies that I want to keep track of income as a separate state variable, and this is the reason why, contrary to the standard in the literature I will treat income and assets as two separate state variables, instead of defining a unique state variable, "cash on hand" that contains both. In the defined benefit plan then the state variables are assets and current income. As for the defined contribution plan, I will define a new state variable to keep track of the amount accumulated in the individual's personal account over time. So this problem has three state variables besides time. after retirement the problem becomes simpler because there is no uncertainty in income, and then it is only necessary to keep track of assets and income, which then is retirement income.

applying the Bellman's optimality principle the problem can be restated as

$$V_t(A_t, Y_t, Z_t) = \max_{C_t} u(C_t) + \beta E [V_{t+1}(A_{t+1}, Y_{t+1}, Z_{t+1})] \quad (5)$$

subject to the dynamic budget constraint, the nonnegativity of assets, initial values for income and assets, the definitions of retirement income, and the income process.

There are two ways to solve this type of problem: to solve recursively starting from last period's Euler equation, or to iterate on the value function instead. In either case the finite nature of the problem is invoked, and a backward induction solution is implemented. I chose the second procedure. Assuming no bequests, in the last period an individual will consume the total value of his assets, which means

that the Bellman equation at time  $D$  is

$$V_T(A_T, Y_T) = \max_{C_T} u(C_T) + \beta E [V_{T+1}(A_{T+1}, Y_{T+1})]$$

$$s.t. \quad 0 = R(A_T + Y_T - C_T)$$

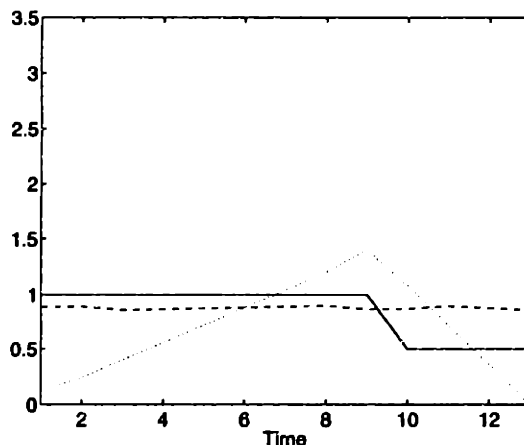
where  $V_{T+1}(A_{T+1}, Y_{T+1})$  is equal to zero. A solution for consumption at time  $T$  can then be found. After that consumption in period  $T - 1$  can be found for all the different possible values of the state variables, and the same process is repeated until period 1. Once optimal consumption is obtained for different values of assets and income, it is enough to specify an initial value of the state variables to retrieve the optimal consumption function.

## 5. Calibration and Results

In this section I present preliminary results regarding the evolution of assets and consumption over the life cycle. As a first stage I have simplified the model further by assuming that only permanent shocks matter. I have done this because using both transitory and permanent shocks would involve introducing an additional state variable, increasing the computation time of the numerical solution significantly, particularly for the defined contribution case. Neglecting transitory shocks implies that it will not be possible to see how the behavior of the agents changes with the degree of persistence of the shocks. As I show elsewhere (Dulitzky 1997), changing the persistence of shocks to income might change individuals' preferences for either retirement plan.

The process for income is then a logarithmic random walk with drift. I have also assumed that the growth rate of income is zero (so  $g_t = 0$ ). I will assume that there are only 13 time periods, 9 of which the individual spends working, and the other 4 retired. Each of these periods corresponds roughly to five years of life, so this agent would start working at 25 years of age, retire when he or she is 65, and die at 85. This means that in the defined benefit plan the averaging horizon will be

five years.<sup>24</sup>



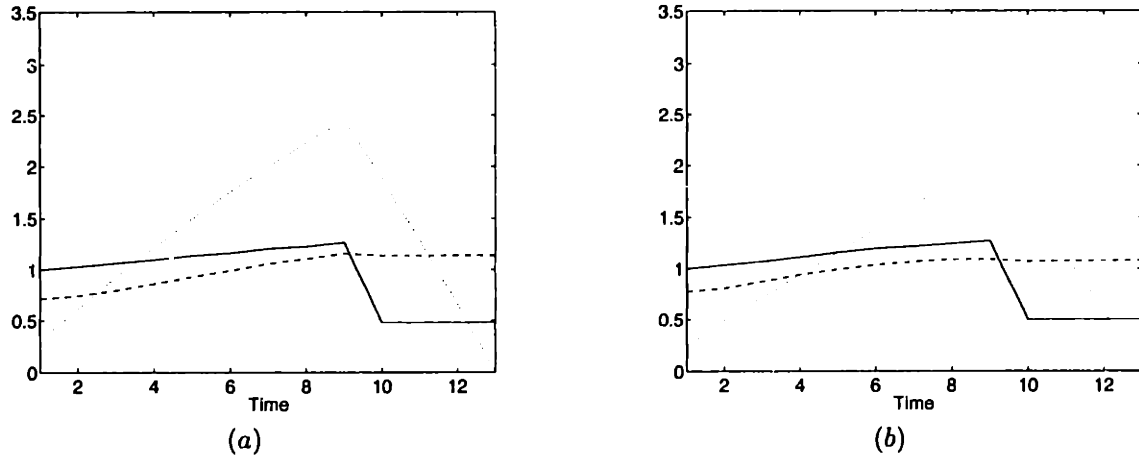
**Figure 2:** Assets(...), consumption(- - -) and income(—) throughout the lifecycle. No uncertainty. Additional parameter values:  $\rho = 3$ ,  $R = 1 + r = 1.05$ ,  $\delta = 0.05$ .

Figure 2 presents the case in which income is known with certainty. As expected the consumption profile is flat. This result is independent of the type of retirement plan, because the contribution rate to the defined contribution plan has been chosen so that retirement benefits are equal.<sup>25</sup> Instead of normalizing benefits I could have chosen the contribution rate so that the liabilities to the firms or the government are the same under each plan. I preferred to normalize benefits because I want to isolate the effect of income risk. Normalizing liabilities does not guarantee that ex-ante or ex-post retirement benefits are going to be equal to each other.

The fluctuations of consumption around the optimal level are due to the solution method employed. To compute the optimal consumption rule I discretized the state space. I use a 96 point grid for assets, between 0 and 9.5, and a 50 point grid for income, between 0.1 and 5. When a value for income or assets falls in between these points, the value is approximated to the immediate next point, creating the fluctuations observed in the graph.

<sup>24</sup> Alternatively a new state variable could have been created to keep track of benefits in the defined benefit plan, in the same fashion as for the defined contribution system.

<sup>25</sup> An alternative would have been to choose the rate of return on the defined contribution plan so that benefits are equal.



**Figure 3:** Expected profiles of income(—), consumption(- - -), and assets(...) over the life cycle, for (a) defined benefit and (b) defined contribution plans. Additional parameters values:  $\rho = 3$ ,  $R = 1 + r = 1.05$ ,  $\delta = 0.05$ ,  $\sigma = 0.25$ .

When income is uncertain individuals will hold precautionary saving, which means that the total value of assets should be higher than in the certainty case. This is indeed the case. Figure 3 shows the life cycle profiles for expected income, consumption, and assets for the defined benefit and defined contribution plans. The paths for income are identical by construction. This means that any difference in the life-cycle profiles will have to come from income risk.

The expected consumption path tracks income closely early in life and after retirement, when uncertainty is resolved it becomes flat. Consumers in the defined benefit plan have a steeper consumption profile before retirement than the counterpart in the defined contribution plan. Consumption for the former starts at a lower level and ends up being higher by the time of retirement.

Even though the process for income has no drift, the expected value of income increases over time because income follows a logarithmic random walk. Specifically if

$$y_t = y_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma^2),$$

then

$$E[Y_{t+1}] = \exp\left(\frac{\sigma^2}{2}\right) Y_{t-1},$$

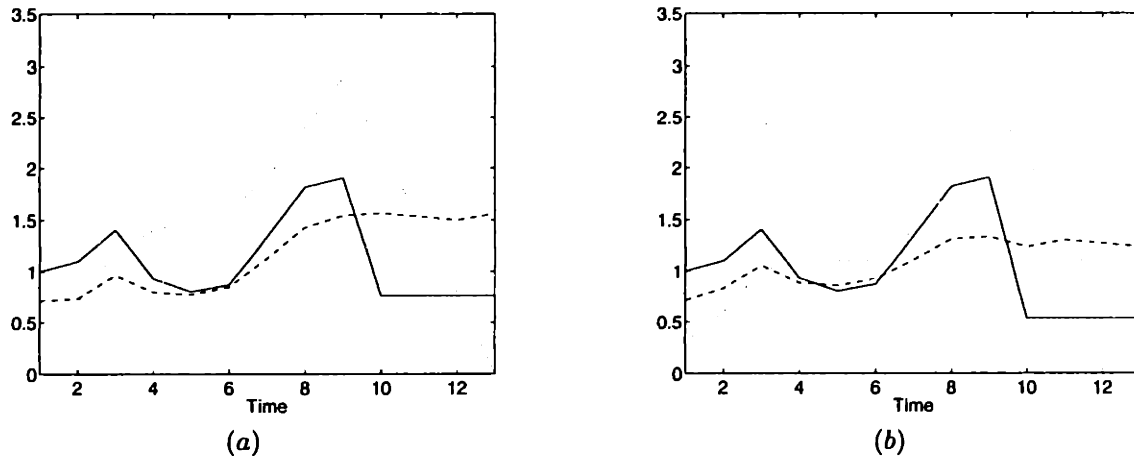
which grows over time.

The patterns for asset accumulation are approximately equal for individuals in defined benefit or defined contribution plans. The difference lies in the level. The maximum expected level of assets for a defined benefit agent is about 2.5 times initial income, while it is about twice the level of initial income for a representative agent enrolled in a defined contribution plan. Does this mean that defined contribution agents save less than defined benefit ones? Well, one has to be clear about what kind of saving are being consider here. By construction retirement benefit is the same for both. This means that social security wealth is the same, no matter what plan individuals are in. Assets in this context are net of social security wealth assets, which means that I am not including the amount accumulated in the accounts as part of individuals private saving. What the graph shows is that if expected retirement benefits are equal across plans, then net of social security wealth should be higher for individuals in the defined benefit plan, because conditional on information available at the beginning of working life, the variance of the defined benefit retirement plan is higher than the variance of the defined contribution retirement plan.

A heated discussion has been going on these days about the return of defined contribution plans compared to defined benefit plans, in light of different “privatization” of social security projects. In most cases, the debate over the rate of return of a defined benefit or a defined contribution plan has been made without paying much attention to the risk in either system. Here instead I abstract from the difference in returns and concentrate on the difference in risk arising from the presence of labor income risk.

There are many different reasons why people in defined contribution plans could end up saving more that people in defined benefit plans; different returns, different lifetime earnings, different earnings profiles. But the conclusions of this paper are based on a comparison where all these differences have been eliminated. Figure 4 shows a particular realization of income, consumption, and assets over the life cycle. As explained before, I am normalizing the contributions to the defined contribution plan so that the expected retirement benefit is equal across plans. However nothing prevents a particular realization to give a higher retirement benefit to the agents in either plan. In the case of figure 4 even though the shocks are the same, the defined benefit retirement plan is more generous.





**Figure 4:** Particular realization of income(—), consumption(- - -), and assets(...). Earnings shocks are the same for (a) defined benefit and (b) defined contribution plans.

## 6. Extensions and Conclusion

This paper examines the life cycle behavior of individuals under defined benefit or defined contribution plans. The question answered can be exemplified in the following experiment: imagine a country with a social security system where the benefits are calculated using a defined benefit formula. Then a social security reform is implemented, where the defined benefit plan is replaced by a defined contribution plan, where contributions are mandatory, and the amount accumulated in the accounts can be used only after retirement. Also the expected retirement benefits are equal under each plan. Then an individual's private saving is going to be lower under the new system. The result is due to the fact that both systems benefits are linked to earnings history, and the variance of the defined contribution plan is lower than the variance of the defined benefit plan.

There are many different ways to extend this exercise, a number of them suggested in the previous paragraph: to allow for voluntary contributions, and for early withdrawals of the funds, to calibrate the model in order to match some real world examples, where the (expected) rate of return may be different across plans, etc. It would also be interesting to make the model more realistic by incorporating more time periods, and changing the earnings process. As I said before adding a transitory shock would imply the need of an additional state variable (or some kind of normalization). This is straightforward to do, except for the increase in computational time.

If the prediction of the model is that agents with defined contribution plans should save less than their defined benefit counterparts, for the same social security (or expected retirement benefit) wealth, it might be possible to get some empirical evidence to test this result. If information on the expected value of pensions were available, it would be possible to control for it and then find the effect of income uncertainty on asset accumulation. Of course, information about the specific features of the pension plans should be necessary. An additional problem would be the fact that the model assumes that individuals are identical in every other respect. This ignores the possibility that agents in defined contribution plans are more or less risk averse than individuals in defined benefit plans, which would contaminate the results too. This seems a somewhat difficult problem to solve.

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## **Chapter 4**

# **Defined Benefit Vs. Defined Contribution Plans under Earnings and Asset Returns Uncertainty. The Case of Argentina**

### **1. Introduction**

There is an enormous variety in the way social security systems are organized across countries. Among other things these systems vary with respect to the requirements necessary to qualify for retirement benefits, and how those benefits are computed. I exploit this variety by considering the following particular case: imagine an individual is given the possibility of choosing between a defined benefit (DB) or a defined contribution (DC) retirement plan; enrollment is mandatory and it is a once in a lifetime decision, with no other options available. Then what are the factors and economic incentives that govern this choice? How is it affected by the presence of uncertainty in earnings and asset returns? Does one system strictly dominate the other? This may sound as a rather unrealistic experiment because most countries have only one social security regime and use other instruments as complements to the basic retirement benefit, but it is happening already in a number of developing economies like Argentina and Colombia, and it is likely to extend to other places with somewhat different rules.

The motivation for this paper stems from a number of observations. First, long term solvency of social security systems seems to be a pervasive problem. There is a growing concern in developed economies about the future of their public pension systems. Demographic changes and a strong dependence of the income of the elderly on social security are among the factors that have contributed to this concern. According to the World Bank (1994), Social security has now on average the highest share of expenditures as a percentage of GDP of all social programs in OECD. And in the US social security benefit payments in 1993 were about 18% of federal budget and 4% of GDP. Also public pensions in the US represent more than

50% of household income for the median retired person. Offering people a choice between different retirement plans might be a plausible alternative to alleviate this problem.

Second, this concern about the future of pensions is shared by developing economies as well, and in fact the biggest changes have been taking place in those countries. The first mandatory saving plan was introduced in Malaysia in 1951, and after that a number of countries have implemented structural reforms to their social security systems. These reforms seem to show a strong preference for defined contribution plans, which are replacing old systems completely, as in Chile, or partially as in Argentina. The specific feature of the reforms I want to study is the possibility of different retirement plans coexisting, so people can enroll in the one they prefer.

Third, both developing and industrialized economies seem to show strong preferences towards defined contribution plans, either as complements or substitutes for defined benefit plans. The changes introduced by developed economies consist on increasing contribution rates, and mainly creating alternative sources of saving for retirement so that social security will become a less important source of income for retirees in the future. The individual retirement accounts and 401(k) plans, for example, provide incentives for US workers to save for their retirement (see Poterba et. al. (1996)). These are defined contribution in nature, because of the way funds are accumulated. European countries are trying to reduce the burden on public pensions through a shift to a multipillar system, which consists of a basic pay-as-you-go-defined benefit plan that provides a limited amount of income for retirement and works as a tool to redistribute income to the poor, an occupational pension scheme, and a voluntary personal saving scheme. I take a closer look at defined contribution plans, and try to see if the strong preference governments exhibit for this type of plans has a counterpart on individual's preferences.

The fourth and final observation that motivates this paper is related to the nature of uncertainty observed in households earnings. A number of studies have documented the importance of different kinds of shocks affecting earnings.<sup>26</sup> I am particularly interested in studying how the different degrees of persistence in earnings shocks affect the optimal choice of retirement plan.<sup>27</sup>

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<sup>26</sup>See Macurdy (1982), Abowd and Card (1989), and Pischke (1995).

<sup>27</sup>Simple examples can illustrate what I mean by different degrees of persistence: the shocks affecting a laid-off worker's earnings in the US is very different from, say, a similar situation for

I concentrate on the case of Argentina. After a social security reform implemented in 1994 people can choose between a pay-as-you-go (PAYG) defined benefit plan administered by the state and a fully funded (FF) defined contribution scheme privately managed. This is true not only for old workers (as in Chile), but also for every new worker entering the labor force, which means that even after the transition period both systems could potentially coexist forever.<sup>28</sup> I compare expected retirement benefits under each plan for a number of simulated earnings profiles and alternative assumptions regarding lifespan, years of covered work, and administrative costs. Working first with risk neutral agents, I estimate the gross return on assets in the defined contribution plan that makes people indifferent between the two regimes, so that for a return higher than the break-even an individual will prefer the defined contribution system. I find that for very reasonable rates consumers will end up choosing the defined benefit plan. Then I perform a series of comparative statics exercises to calculate how that break-even rate of return changes when other variables entering the retirement formulas change.

Next I turn to the case in which agents are risk averse. Using the same profiles as before, I calculate the variance of the retirement benefits for each plan, conditional on the information available at the time of choosing the retirement plan, that is the first working period, replacing the rate of return on the defined contribution assets by the value that makes expected retirement benefits equal to each other. In that way I can base the comparison only in terms of variances and can abstract from differences in expected values. I show that the more permanent earnings shocks are, the more likely it is that people will choose the defined contribution plan, while when transitory shocks become more important the variance of the defined contribution plan is higher than the variance of the defined benefit (for the same expected value), implying individuals will pick the defined benefit retirement scheme. The intuition for these results is clear once the retirement formulas are compared: the defined benefit plan is based on the last ten years of earnings, while the defined contribution scheme is based on the entire lifetime earnings stream, and hence the

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a european worker; a worker that has no long term employment will experience a different mix of short term/long term risk than a person with a stable job, etc.

<sup>28</sup>The US equivalent of this would be allowing workers to deposit their SS contributions into a personal account instead of diverting them to the SS fund, and then use them, along with accumulated interest to purchase an annuity at the time of retirement.

mix of transitory/permanent shocks will be reflected in the choice of system. This will be true whenever the time span of earnings used to calculate benefits is different. It is not only that the number of periods used in the computation of benefits differs across formulas, but also the particular periods chosen (last 10 working years in the defined benefit case, working lifetime in the defined contribution plan).<sup>29</sup> Maybe even more important is the effect of uncertain asset returns. Although there is not much information about the type of process returns follow, it is apparent that the expected return is relatively high but so is its variance.<sup>30</sup> When asset returns are random, the results are reinforced in favor of defined benefit plans. Even when the variance of the asset returns is very small (less than 0.01), the defined contribution plan has a variance that is larger than the variance of the defined benefit retirement benefit for every degree of persistence of the shocks to earnings.

The structure of the paper is as follows: section 2 presents some institutional details regarding the retirement benefits structure in Argentina, and explains the differences between the two plans in terms of variables entering the computations, and risks involved. Section 3 presents the model, and the results are presented in sections 4 and 5 for the risk neutral and risk averse consumers respectively. Section 6 presents some simulations for the case of random asset returns, and section 7 concludes.

Before proceeding with the rest of the paper, I would like to call your attention to the following: the funded nature of the systems, i.e. the coexistence of a pay-as-you-go and a fully funded regime is not the relevant issue here, but the fact that one system is defined benefit and the other is defined contribution. It is perfectly possible to have a PAYG system partially or totally funded, and at the level I am working (individual) it is not important if the current retirees are financed by current workers or not.

## 2. Institutional Setting

The public pension system in place in Argentina before the reform was a pay-as-you-go (PAYG), defined benefit (DB) state administered scheme. It was very

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<sup>29</sup>For example in the US benefits are calculated taking an average of best 35 years of earnings.

<sup>30</sup>Average real returns in Chile have been around 13% per year, but during the last few years returns have gone down dramatically, even below zero.



decentralized in the sense that there were different funds for different occupations and each province had its own pension fund too.

The new system is a mix of PAYG and FF plans, but the contributions and benefits for the PAYG are different from the original. Contributions to the system are 11 percent of payroll for workers, 16 percent for employers and 27 percent for self-employed. Contributions are tax-deductible.<sup>31</sup> The minimum earnings subject to social security contributions is equivalent to 3 *ampos* and earnings are capped at 60 *ampos*. An *ampo*,<sup>32</sup> is defined as the average employee's contributions into the system, equivalent to 11 percent of the average wage in the economy, given that contributions are obligatory:

$$ampo = \frac{\text{Total employee's contributions}}{n} = 0.11 \times \sum \frac{y_n}{n} = 0.11\bar{y}$$

Affiliation to the system is mandatory, but every worker is given the choice between contributing to the pay-as-you-go or fully funded regimes. This choice is not limited to older workers who have already made contributions to the old system and as such are entitled to at least partial retirement benefits.

The structure of the social security system is fairly simple: every worker will receive at retirement a universal flat benefit (*pbu*), and an additional benefit that will depend on the retirement plan chosen. Table 1 summarizes the main features of each system.

## 2.1. State Administered Regime

The state administered regime is organized as a PAYG, defined benefit system. This means that the current working population finances the current retirees, and that benefits are determined by a formula that considers a number of factors like years of covered work and earnings, but remain independent of the contributions made into the system. This plan is financed with contributions from workers who have chosen the defined benefit plan, contributions of employers (all), 16 out of the 27 percentage points of self-employed, and additional resources specified in the federal budget.

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<sup>31</sup>The law contemplates the possibility of a phased reduction of employers contributions. At July 1996 employers contributions were on average 12 percent (the reduction is different across regions).

<sup>32</sup>*ampo* stands for "Aporte medio previsional obligatorio".

In order to be able to qualify for retirement benefits, workers have to be 65 and 60 years old respectively for men and women. A worker can access retirement only after 30 years of covered work (i.e. 30 years of working with contributions), while a year of lacking contributions can be replaced by 2 years of age over the statutory retirement age.

Features	State administered	Privately managed
Type of system	DB-PAYG	DC-FF
Contribution rate	11% payroll (workers who choose that system); 16% employers (all)	11% payroll (workers who choose that system)
Retirement age	65 for men and 60 for women	same
Years of covered work	30	30
Flat benefit	$2.5 \times 0.11 \times \bar{y}$	same
PC benefit	$0.015 \times T \times \sum_{t=T-9}^T y_t / 10$	same
Additional benefit	$0.085 \times T \times \sum_{t=T-9}^T y_t / 10$	Annuity
Voluntary contributions	No	Only over mandatory ones
Early retirement	No	Only if accumulated enough savings
Late retirement	Yes	Yes

**Table 1:** Features of the two retirement plans in Argentina.

In addition to the *pbu*, workers who qualify for retirement are entitled to receive two other types of retirement benefit, a compensatory benefit (*pc*) and an additional benefit if they choose the defined benefit plan (*pap*). The *pbu* is a basic universal benefit conferred to every worker in the system, both in the state and privately managed regimes, and is equal to 2.5 *ampos* per month, with a slight increase per year contributions over the minimum. This is approximately equivalent to 27.5

percent of the average wage in the economy.<sup>33</sup> The *pbu* is indexed to the average wage of the economy.

The compensatory benefit is intended to recognize contributions made under the old system, which means that it is also paid to individuals in both regimes. The monthly *pc* benefit is equivalent to 1.5 percent per year of covered work (with a maximum of 35 years), this percentage calculated over the average wage of the last 10 years of work prior to retirement. This part of the benefit is also wage-indexed. Finally *pap* is given only to workers in the defined benefit scheme, thus being the only part of the retirement benefit that will affect choice between systems. It represents a monthly payment of 0.85 percent per year of covered work, this percentage calculated in the same way as the *pc*.

The state administered plan also provides disability insurance, old age pensions and survivors benefits. To qualify for old age, an individual has to be at least 70 years old and have 10 years of covered work. The benefit in either case is equivalent to 70 percent of the average earnings of the last five years, plus *pbu* and *pc* whenever applies.

## 2.2. Privately Managed Regime

People who decide to enroll in the privately managed scheme will still receive the *pbu* and the *pc* (in case they were entitled to it) with funds coming from the other regime. The defined contribution scheme is financed through contributions of workers that chose that system and 11 out of the 27 percentage points of self-employed with an additional provision that contributions can be increased voluntarily. Contributions go to an individual account and the funds are invested and administered by private organizations called *Administradoras de fondos de jubilaciones y pensiones* (AFJP). Workers can freely choose their AFJP, and can switch at most twice a year if they have contributed at least 4 months to the AFJP that they want to leave. Each AFJP is in charge of administering only one fund.

To qualify for the retirement benefit a worker has to fulfill the same requirements as in the public regime, that is 30 years of covered work and 65 and 60 years of age for men and women respectively with the addition of early and late retirement schemes.

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<sup>33</sup>Given that it is not income-dependent, the *pbu* is the portion of retirement benefit which can be considered purely redistributive.

Workers can receive benefits before the statutory age if at the time of retirement the amount accumulated in the personal account is relatively high.<sup>34</sup> Early retirees do not receive the defined benefit part of their benefits (i.e. *pbu* and *pc*) until they reach the statutory retirement age, and this option is only available to individuals who chose the defined contribution plan.

Late retirement is open to workers in both regimes. They can receive or postpone the corresponding retirement benefit accumulated in the personal accounts (if any), but in either case they will not receive the defined benefit part of their benefit until they retire, and moreover, they have to keep contributing until effective retirement. This represents an implicit tax on work after the statutory retirement age.

Once an individual decides to retire, he or she can choose among three different modalities:

1. Purchase of an annuity. The AFJP will transfer the worker's funds from his/her account to the insurance company that is going to pay the worker's pension, the State guaranteeing a benefit up to 5 max *pbu*.
2. Purchase an annuity from the AFJP directly.
3. Workers whose retirement benefit is less than 50 percent of the maximum PBU can retrieve from their accounts an amount equivalent to 50 percent of the maximum PBU, either until the balance of the account goes to zero or the retiree dies.<sup>35</sup>

Funds accumulated in personal accounts can be invested by each AFJP in a variety of financial instruments within certain limits provided by the law, in particular up to 50 percent of the fund in government bonds, 30 percent in saving accounts, 50 percent in the national stock market, and 10 percent in foreign stock markets.<sup>36</sup>

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<sup>34</sup>The potential retirement benefit has to be higher than 50 percent of the last five years' average income, and greater or equal than twice the maximum *pbu*. These provisions try to ensure retirees will have enough accumulated savings.

<sup>35</sup>Given that I am not modelling the behavior of the AFJP it is not relevant for the purpose of this paper who is selling the annuity, unless prices are different for these options, which I assume are not. I will also assume that the individual is taking the annuity option instead of the phased withdrawal.

<sup>36</sup>As of september 1996, 49 percent of funds were invested in government bonds, 18 percent in savings accounts, and 14 percent in stocks. Less than 1 percent of funds were invested in foreign stock markets.

The rate of return of each pension fund has to be over a certain minimum that is equivalent to 70 percent of the average rate of return of the entire system. An emergency fund is created to guarantee the minimum return, to be used by the AFJP whose rate of return is below it. Whenever this emergency fund is not enough to cover the difference between the minimum rentability and the rentability of the fund, the State will cover the difference. The privately managed plan also offers disability and survivors benefits. The benefit is equal to the one provided by the state with the difference that the AFJP is responsible for the payment (except the *pbu* and *pc* if they apply).

Disability insurance provides an incentive that my analysis neglects, because the replacement rate for disability insurance is in many cases much more generous than the standard retirement benefit, but as long as it is the same for both systems it will not affect choice. Nor will choice be affected by survivors benefits if the individual has no bequest motives.

### **3. Choice of System under Uncertainty**

In this paper I exploit the different nature of the retirement formulas to highlight the trade-offs involved in the choice of a particular retirement system. The defining characteristic of defined benefit plans is that benefits are based on an explicit formula that uses information about the worker to compute the retirement benefit. Usually benefits are based on the number of years of covered work and on wages history. On the other hand defined contribution plans specify contributions; there is an implicit formula too, but the information used to compute benefits is mainly related to the worker's contributions. Contributions are deposited in a personal account and at the time of retirement the worker receives the amount accumulated plus the interest earned on those funds.

While both plans provide insurance against longevity risk, they are sensitive to other types of risk. Defined contribution plans are particularly vulnerable to asset returns uncertainty, a factor that will not affect defined benefit plans as long as the funds accumulated are not invested in assets with random returns. Uncertainty about earnings will have an effect on both retirement plans, because benefits are somehow based on the worker's history of earnings. There are other types of risks

that might influence choice, but these two seem to be the most important ones. <sup>37</sup>

I start the analysis of optimal choice of retirement plan by using a very simple setting. An individual will choose the retirement plan that maximizes his expected utility at retirement, conditional on the information available at the time of making the decision about what plan to enroll into, in this case the first working period:

$$\max_{Y_{T+1}} E[U(Y_{T+1})|I_1] \quad (1)$$

$$s.t. \quad Y_{T+1} = \begin{cases} Y_{db} & \text{if chose defined benefit} \\ Y_{dc} & \text{if chose defined contribution} \end{cases}$$

where  $T$  is the last working period,  $Y$  represents earnings in each period, and  $I_1$  is the information set available at time 1, the first period of work. Following a standard representation, earnings are assumed to be the sum of a white noise process, the transitory component with variance  $\sigma_u^2$ , and a random walk with drift, the permanent component:

$$\begin{aligned} Y_t &= P_t + u_t \\ P_t &= \alpha + P_{t-1} + \eta_t, \end{aligned} \quad (2)$$

where  $\eta$  is also white noise with variance  $\sigma_\eta^2$ .

The retirement benefit is going to be determined by the formulas for each plan. The DB benefit ( $Y_{db}$ ) is a proportion  $\delta$  of the average earnings of the last ten years prior to retirement:

$$Y_{db} = \delta \frac{\sum_{t=T-9}^T Y_t}{10} + z \quad (3)$$

The defined contribution benefit formula ( $Y_{dc}$ ) results from assuming that a

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<sup>37</sup>Other types include uncertain lifespan, inflation risk, and probability of default. The introduction of lifespan uncertainty introduces a completely different set of issues, like optimal annuitization time or optimal time to claim benefits.

worker deposits a proportion  $\tau$  of earnings every period in a personal account. This funds earn a real gross return  $R$ , and the pension fund charges a proportion  $\gamma$  of earnings to administer those funds. At the time of retirement the amount accumulated in the personal account is:

$$(\tau - \gamma) \sum_{t=1}^T Y_t R^{T+1-t} \quad (4)$$

When the worker retires those funds are used to buy an annuity. The insurance company charges a proportion  $\beta$  of the funds, and the remainder is used to pay  $Y_{dc}$  every period until the time of death. The present discounted values of the annuity has to be equal to the amount accumulated in the account. Thus the retirement benefit each period is equal to

$$Y_{dc} = (1 - \beta)(\tau - \gamma) \frac{\sum_{t=1}^T Y_t R^{T+1-t}}{\sum_{t=0}^{D-T-1} (1/R^t)} + z \quad (5)$$

Both plans provide the same universal flat payment  $z$  independent of contributions and workers characteristics so this element will not affect choice.

This problem can be expressed in mean-variance form, either by assuming that utility is quadratic or that retirement benefits are normally distributed, which is true for the specific earnings process chosen. In particular, if utility is CARA,

$$u(Y) = -\frac{1}{\theta} \exp[-\theta Y],$$

then

$$E[U(Y_{T+1})|I_1] = -\frac{1}{\theta} \exp\{\theta E[Y] + \frac{\theta^2}{2} V(Y)\}.$$

The defined benefit plan will be preferred to the defined contribution if

$$\theta E[Y_{db}] + \frac{\theta^2}{2} V(Y_{db}) > \theta E[Y_{dc}] + \frac{\theta^2}{2} V(Y_{dc})$$

or

$$E[Y_{db}] - E[Y_{dc}] + \frac{\theta}{2}(V(Y_{db}) - V(Y_{dc})) > 0$$

The main results of this paper are based on the differential effect of earnings uncertainty on each retirement plan. Earnings uncertainty will affect defined benefit plans because benefits are calculated taking into account workers earnings histories, or at least some portion of those. And it will also affect defined contribution plans because contributions are a percentage of earnings, which follow a random process. In the particular case of Argentina the defined benefit formula calculates benefits using the average of the last ten years of earnings, while the DC formula uses lifetime earnings, so the random process followed by this variable will have an impact by itself. This will be very different from say, the one it would have in the US, where social security benefits are based on the best 35 years of earnings.<sup>38</sup> And this effect does not only depend on the different time span used in the computation of benefits but on the other variables affecting benefits as well, like years of covered work, administrative costs, etc.

I assume that contributions to the defined contribution plan cannot be increased voluntarily. The existence of voluntary contributions would create the need to consider an intertemporal framework and include an extra variable (how much to deposit in the personal account every period). I also assume that the worker starts contributing and claims benefits at the same time under either system, and that death and retirement age are fixed. All these assumptions eliminate a number of incentives offered by one plan respect to the other, but on the other hand let me concentrate on one particular feature, the effect of earnings uncertainty.

Two more things deserve special attention: first, the use of the information set at time 1 to calculate expected utilities, and second, the use of a static optimization setting instead of an intertemporal model. The explanation for the first is simple: the decision about enrolling in a given retirement plan has to be made at the beginning of the working life, with no chance to switch back and forth between systems. Then it is natural to make choice conditional on the information available at that

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<sup>38</sup>Even in that case, as long as earnings history is longer than 35 years, a defined contribution plan competing with the current SS system would imply a different degree of uncertainty due to earnings.



time. Using different information sets will most likely result in a different optimal choice, for example in the case that people want to make the decision only after they have acquired a certain standard of living, or when individuals do not care about retirement at the beginning of their working lives because retirement is too far away in the future, but those patterns of behavior are ruled out by the initial assumptions regarding preferences.

Regarding the second issue, the model is intertemporal but not dynamic, since it is assumed that individuals consume all their income every period, i.e. there are no savings beyond the ones implied by social security contributions. I chose this setting for a number of reasons: first, I am not interested on the behavioral response that the choice of a particular system generates, but on the choice itself, and how it is affected by a number of variables. Second, even though this is a simplistic view of the world, there is some evidence that people reach old age with very little assets, and that SS benefits are a big proportion of income for the elderly. Third, it is a useful “first pass” analysis that allows me to isolate the financial aspect of the choice from an individual’s point of view, and highlight the different nature of the retirement formulas implied by each plan. Finally, the behavioral response of individuals over lifetime has an interest of its own, and if different plans imply different consumption and savings patterns this opens the possibility of affecting savings rates in the economy by a particular retirement plan “mix”. Other papers analyzing retirement issues have made similar assumptions: Stock and Wise (1990) analyze the effect of pension plans on the option value of work and retirement decisions. In their setting preferences depend on retirement income and then there is no consumption decision. I discuss in a different paper the implications of incorporating consumption and saving decisions.

#### **4. Decision under Risk Neutrality**

I first assume that individuals are risk neutral, so that they care only about the expected value of the retirement benefit. Thus the utility maximizing plan will be the one with the highest expected value, irrespective of the variances. The expected values depend on the number of years of covered work  $T$ , lifespan  $D$ , the price of buying an annuity  $\beta$ , the administrative cost of the pension funds  $\gamma$ , and the evolution of earnings over time.

In order to evaluate the effect of each of those variables on the choice of retirement system, I start by defining a benchmark case and calculating the rate of return on the defined contribution plan that will make agents indifferent between systems. That is I solve for the value of  $R$  that makes expressions 3 and 5 equal to each other. I call this value break-even return ( $R^*$ ). I look at break-even rates instead of expected benefits because I am interested not only in what system a person prefers but also on the rate of return it yields, and the way it changes with other variables. Obviously the expected benefit of the defined contribution plan is increasing with its own return, and if the break-even rate were negative or very low, the DB plan would look very attractive. If on the other hand the required rate of return were extremely high, it would be very difficult for the defined benefit plan to dominate.<sup>39</sup> If the expected rate of return turns out to be higher than the break-even rate, agents will prefer the defined contribution plan. I examine how this break-even rate changes when the variables affecting benefits change, so I can get some additional information about what system is more likely to be chosen in different circumstances. The parameter values for the benchmark simulations are detailed in table 2.

$z$ (Flat benefit)	27.5
$y_1$ (Initial income)	100
$\tau$ (Contribution rate)	0.11
$\beta$ (Cost of annuity)	0.2
$\gamma$ (Administrative cost)	0.034
$\delta$ (Replacement rate)	0.0085 $T$ if $T \leq 35$ 0.2975 if $T > 35$
Earnings drift	5 percent/yr.

**Table 2:** Initial parameters values

The values for  $z$  and  $\tau$  are taken from the argentinian retirement schemes,  $\gamma$  is the administrative cost charged by pension funds in Argentina as a proportion of

<sup>39</sup>The rates of return on pension funds in Argentina have been extremely high during these first years since the reform. The pattern seems to be similar to other countries experiences, like Chile. However in 1996 returns in that country were negative for the first time since the adoption of the current pensions regime.

earnings, and it is somewhat similar to other countries like Chile (where it is even higher); the value of  $\beta$  has been taken from Mitchell, Poterba, and Warshawsky (1990). An earnings drift of 5 per year is roughly equivalent to a growth rate of earnings of 3 percent per year.

The values of the break-even rate of return ( $R^* - 1$ ) turned out to be positive in every case simulated. This means that the defined contribution plan does not dominate unambiguously. Under plausible assumptions about the rate of growth of earnings and other variables individuals will still choose the defined benefit plan. Next I analyze under what circumstances people prefer one plan or the other.

#### 4.1. Comparative Statics

The break-even rate of return for the benchmark case is approximately equal to 3.2 percent. This means that for rates of return lower than 3% per year people will prefer the defined benefit plan. Since the defined contribution plan does not strictly dominate the defined benefit plan, it is worthwhile to analyze which variables are more important determinants of the choice between plans. Therefore I perform a number of comparative statics to determine how sensitive the break-even rate of return on DC assets is to changes in different variables. A change in a given variable will modify one or both expected retirement benefits, which in turn will affect the value of  $R^*$ . If  $R^*$  is very sensitive to that particular variable, the relative attractiveness of one retirement plan over the other will be altered. Then, at the prevailing return more people will end up choosing the system that now offers higher expected benefits.

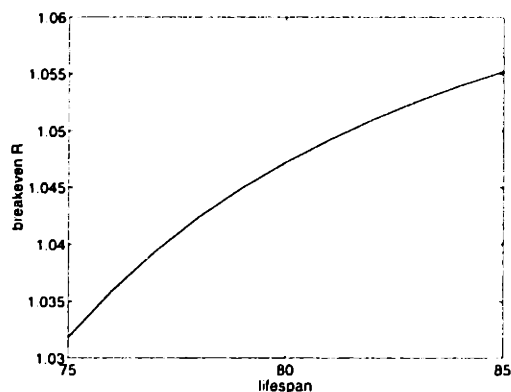
For example let the administrative cost change by a certain amount. If the effect on the required return is not too strong, then the systems are as attractive as they used to be, and not too much people will prefer now the system they did not like before. If the break-even rate is very sensitive to the change in that particular variable, then the likelihood of a given system to be chosen increases because now the expected benefit of one system relative to the other is very different than it used to be, for a given rate of return.

In many cases the direction of the change is obvious, and only the specific amount by which  $R^*$  changes is not known. Other cases however are much more ambiguous. For example changes in years of covered work move both plans retirement benefits

in the same direction, and then it is not clear if  $R^*$  will increase or decrease. I consider the effect of changes in lifespan, years of covered work, price of the annuity, administrative cost of pension funds, income dynamics, and change in the time span on which the DB formula bases its benefits.

### A. Changes in Lifespan

The DB benefit is independent of lifespan so it will be unaffected by that change. On the other hand, for workers that chose the defined contribution plan, the amount accumulated in the personal account has to be spanned across a longer period when lifespan is longer, which means that the annual DC benefit will be lower. Thus, in order to make people indifferent between the two plans the break-even gross return  $R^*$  has to increase.



**Figure 1:** Change in break-even return  $R^*$  due to changes in lifespan

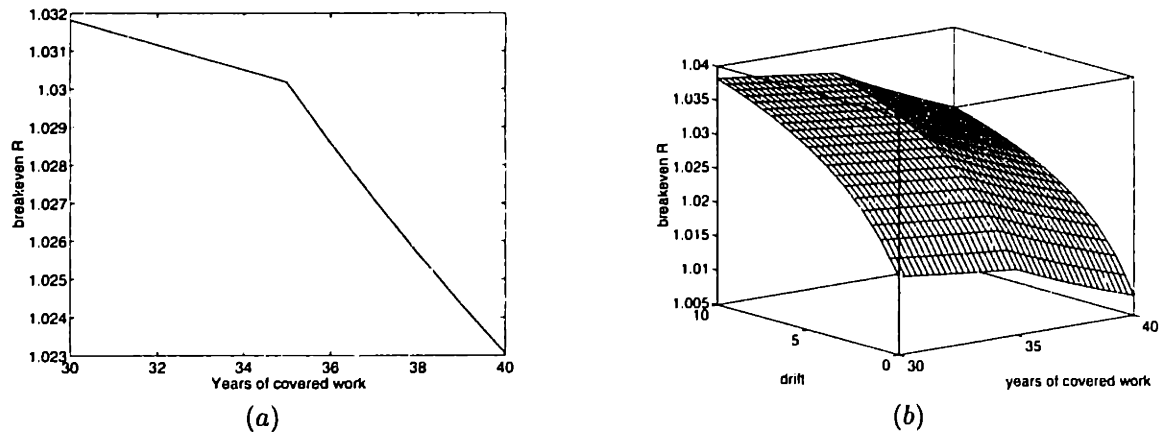
Figure 1 shows the values of the break-even gross return in the defined contribution plan for different lifespans with all the other variables set at their benchmark values.  $R^*$  is concave in lifespan. An increase in lifespan increases the gross return but at a decreasing rate, so the longer people live, the more attractive the defined benefit plan is, but this effect is weaker for very long lived individuals.<sup>40</sup>

<sup>40</sup>This analysis is *ceteris paribus*; it is very likely that if life expectancy changes, the defined benefit plan might also change to reflect the fact that people now live longer.

Since the defined benefit plan is now more attractive at any given return on DC assets, it is likely more people will prefer it and this is more so the lower the starting lifespan.

*B. Changes in Years of Covered Work*

A change in the number of years of covered work (i.e. a change in  $T$ ) will have both direct and indirect effects on expected retirement benefits. The longer the period of covered work, the larger the amount of funds accumulated in the individual's personal account, which means that the DC benefit will be larger. Also, because of the specific rules of the argentinian DB formula, if covered work is less than 35 years, a one year increase increases the expected DB benefit by 0.0085 percent, while no increase will happen after 35 years. The indirect effects stem from the fact that earnings are increasing through time, and then a longer period of covered work means higher earnings and then higher expected benefits. This "drift effect" will affect both plans but in a different way, making the total effect ambiguous. If the break-even rate of return increases after the change in covered work, it means that the DC benefit has increased by less than the defined benefit plan. A decrease in  $R^*$  will indicate that the change in the DB benefit is more than offset by the change in the DC benefit.



**Figure 2:** Change in break-even return  $R^*$  due to changes in years of covered work

Figure 2(a) shows the value of  $R^*$  for different periods of covered work for the benchmark case. First notice that  $R^*$  is always decreasing which means that an

increase in covered work will increase the DC benefit more than the DB benefit, and that the longer the period of covered work, the larger the difference between expected benefits. Also, there is a kink in year 35 due to the way DB benefits are calculated, so that after 35 years of covered work the incentive to choose the DC plan is even higher than before because the “ $\delta$ ” effect has disappeared.

I can isolate the direct and indirect effect of a change in covered work by looking at the change in  $R^*$  for different drifts, including zero. This is done in figure 2(b). I consider first periods of covered work over 35 years long. When the drift is zero and covered work is over 35 years, the DB benefit does not change with covered work; both sources of changes (drifts and changes in  $\delta$ ) have been removed. As the drift goes up, both DB and DC benefits will go up, the first due exclusively to the drift effect and the second due to the combined effect of a higher drift and more years of covered work, so that the increase in DC benefit is relatively higher than before and then the decrease in  $R^*$  is larger.

When the period of covered work is less than 35 years, the additional effect on DB benefits has to be considered. The change in  $R^*$  is not monotonically increasing anymore but exhibits some kind of U-shaped pattern (very small to be noticed in the graph). An increase in  $T$  increases the DC expected benefit relatively more for very low and very high earnings drifts.

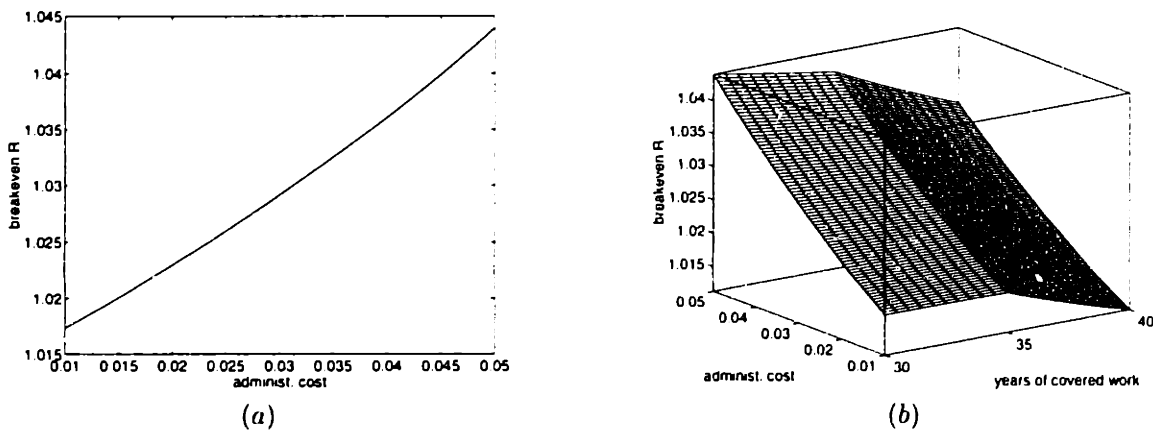
Thus the defined contribution plan offers stronger incentives than the defined benefit plan when the period of covered work increases. The incentive to choose the defined contribution plan is higher the longer the period of covered work, particularly after 35 years. In that case the incentive to choose DC becomes higher with the drift, while for a period shorter than 35 years the incentive is higher for flat and very steep earnings profiles.

### *C. Change in administrative cost*

Administrative costs are an important component of defined contribution plans. These include the cost of administering funds charged by pension funds during working life, and the cost charged by insurance companies at retirement, when the individual uses his funds to buy an annuity. One of the undesirable features of the newly privatized SS systems is the high administrative cost, much higher than the administrative costs of some “efficiently” run public systems. Diamond (1995) discusses the different factors that make the cost of privately run systems higher

than a compulsory public system: economies of scale, the cost of competition to attract more customers, and differences in services provided.

An increase in both kinds of costs will decrease the DC expected retirement benefit and as a consequence of that  $R^*$  will increase. Figure 3(a) shows the change in  $R^*$  due to changes in the administrative cost during the accumulation stage,  $\gamma$ . It is positive and increasing, which means that the incentive to choose the defined benefit plan becomes relatively higher the larger the administrative cost.



**Figure 3:** Change in break-even return  $R^*$  due to changes in administrative cost

Figure (b) plots the break-even values of  $R$  for different years of covered work and administrative costs. The required return will be higher the higher the administrative cost and the shorter the period of covered work. However for very small values of  $\gamma$ ,  $R^*$  will increase with covered work (i.e. the DC benefit increases by less than the DB benefit), only until covered work equals 35 years, and decrease afterwards. So contrary to the result stated in the previous section, an increase in covered work might induce individuals to choose the defined benefit plan, if the rate of return is fixed, and the administrative cost of pension funds is very low.

## 5. Risk Averse Individuals

When consumers are risk averse, they will consider not only the expected value but the uncertainty that each retirement benefit entails. So when maximizing expected

utility, they might not choose the benefit with the highest expected value if they are too risk averse. Assuming joint normality of shocks or mean-variance preferences, a risk averse individual will prefer the system that has a lower variance, for the same expected value. In order to analyze choice for risk averse individuals I use the information collected in the previous sections about break-even returns. For different benchmark cases I calculate the variance of the alternative expected retirement benefits conditional on the first working period, for those rates of return that equalize the expected values of both retirement systems. Given that the expected retirement benefits are calculated using different formulas, and earnings of different periods enter the formulas differently, the nature of the random process governing the evolution of earnings will have a significant role in the variance calculation.

### 5.1. Earnings uncertainty

Equation 2 (repeated below) shows the earnings process I use for the calculations. Earnings are equal to the sum of a permanent and transitory component. The permanent component follows a random walk with drift and the transitory component is white noise:

$$Y_t = P_t + U_t \tag{2}$$

$$P_t = \alpha + P_{t-1} + \eta_t.$$

I am interested in what system people choose depending on the nature of uncertainty. Given the way benefits are calculated people with different earnings processes will choose differently, even if other fundamental variables have the same value.

There are a number of studies that look at the earnings process from an individual perspective. Macurdy (1982) estimates the process for the difference in log earnings and log wages using annual real earnings for a sample taken from the Michigan Panel Study of Income Dynamics, and finds the best fit to be an ARMA(0,2), although other specifications like an ARMA(0,1) or an ARMA(1,1) are good too. Pischke (1995) looks at the difference between aggregate and individual income processes and uses them as a way to explain the failure of life cycle theories of consumption. He estimates an earnings process using monthly and quarterly data from the 1984 Survey of Income and Program Participation. He also finds that the best



specification for the change in earnings is an MA(2) process. Other conclusions are that individual earnings are much more volatile than its aggregate per capita counterpart, that transitory shocks tend to be important, and that the variance seems not to remain constant over time.

Expression 2 can be rewritten as

$$Y_t = \alpha + Y_{t-1} + U_t - U_{t-1} + \eta_t$$

which has the following equivalent representation:

$$\Delta Y_t = \alpha_t + \epsilon_t - \phi \epsilon_{t-1}, \quad (6)$$

where  $\Delta Y_t = Y_t - Y_{t-1}$  and  $\epsilon$  is white noise with variance  $\sigma_\epsilon^2$ . That is earnings follow an ARIMA(0,1,1) with drift. The variance of the transitory and permanent shocks in (2) can be reexpressed in terms of the variance of  $\epsilon$  by equating the moments of both processes:

$$\sigma_u^2 = \phi \sigma_\epsilon^2$$

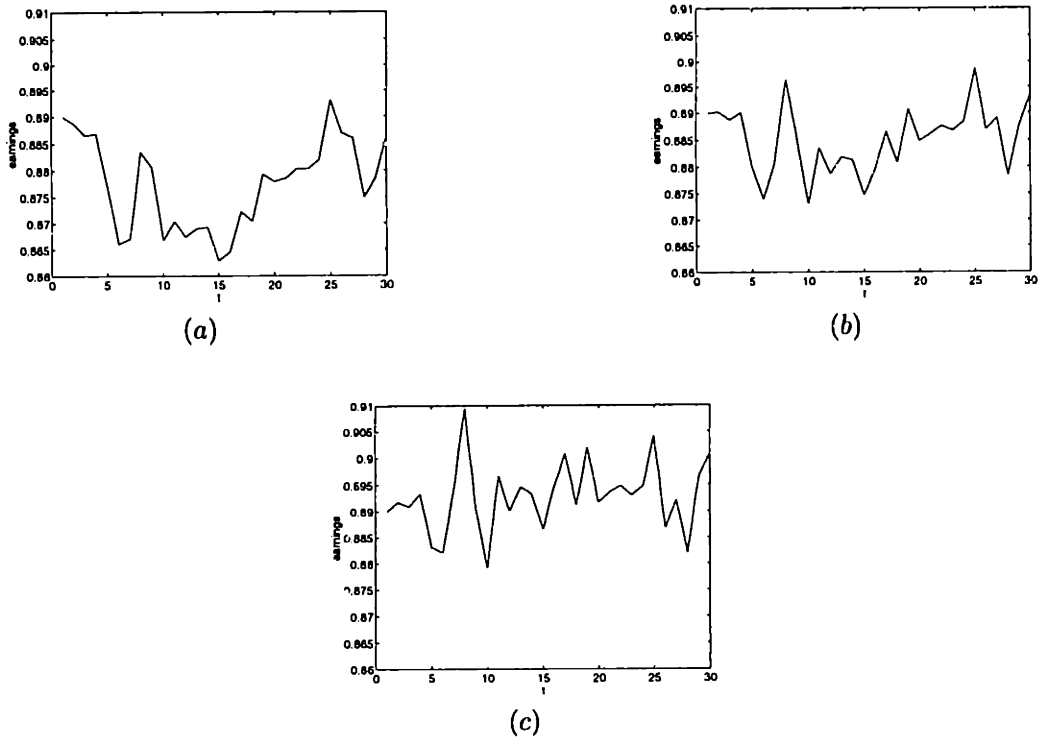
and

$$\sigma_\eta^2 = (1 - \phi)^2 \sigma_\epsilon^2$$

Define  $\lambda = \sigma_\eta^2 / \sigma_u^2$  to be the ratio of the variance of the permanent to the transitory shocks. Then it is possible to show that

$$\phi = 1 + \frac{1}{2} \left( \lambda + (4\lambda + \lambda^2)^{\frac{1}{2}} \right)$$

The relative importance of permanent shocks for different individuals will be reflected in the values of  $\lambda$  and  $\phi$ . The closer to zero  $\phi$  is, the more important permanent shocks are ( $\sigma_\eta^2$  becomes very large compared to  $\sigma_u^2$ ), while values of  $\phi$  closer to one indicate that transitory shocks are more relevant.



**Figure 4:** Simulated earnings profiles for three different degrees of persistence ( $\phi$ ): (a)  $\phi=0$ ; (b)  $\phi=0.5$ ; (c)  $\phi=1$ . The standard deviation of  $\epsilon$  is set to 0.25

The role of transitory and permanent shocks has been studied in the context of consumption and savings behavior, as in Deaton (1991), but little has been said about their role in the choice of retirement plan. It is reasonable to presume that people with different earnings structures will choose differently, and one of the elements that differentiates earnings processes is the relative presence of short and long term risks. Figure 4 shows three simulated earnings profiles for three different values of  $\phi$ , assuming that the drift of the process is equal to zero. The figures are drawn for  $\phi=0$ , 0.5, and 1 respectively from left to right.

It is not obvious which retirement benefit has a higher variance, at the same expected value. The defined benefit plan uses the last periods of working income to compute retirement benefits, while the defined contribution scheme takes into account the entire working lifetime. The interaction between earnings and other variables like rates of return, the replacement rate  $\delta$ , and the contribution rate  $\tau$ , introduces a source of ambiguity in the relationship between the variance of the defined benefit and the defined contribution retirement benefits. The weights

imposed on different periods of working life vary across plans, and that will be reflected on the value of the respective variances. The important question is if the variance of one of the retirement plans will be higher than the other for every possible value of  $\phi$ , in which case one retirement system dominates for every possible mix of risks. And the answer to this question, which is developed further next, is no.

## 5.2. Comparative Statics

When agents are risk neutral it is sufficient to check which retirement plan yields the highest expected benefit to see what system is preferred. However when agents are risk averse, the system they choose will be the one with the lowest variance (at the same expected value). I proceed to calculate the variance of each retirement benefit under different scenarios. In every case I make expected benefits equal to each other by replacing  $R$  with the break-even value  $R^*$ . When  $R$  is equal to  $R^*$  the expected retirement benefit will not matter in the decision, and the individual will choose the plan with the lowest variance.

The formulas for the variance of each retirement plan are calculated from expressions (3) and (4). Since I am conditioning on the information available on the first working period, the variances as of time 1, the number of working years and lifespan are very important in the calculation. Let  $n$  be the first working period's earnings included in the average calculation of the DB benefit (so for example if the number of years of covered work is 30 and the average is based on the last 10 years of working income  $n$  is equal to 20). The variance of the DB and DC retirement benefits is given by

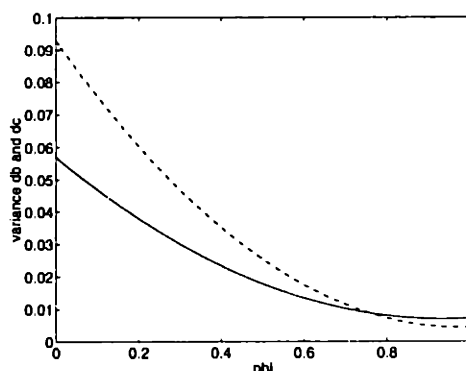
$$V(Y_{db}) = \left( \frac{\delta}{T-n} \right)^2 \sigma_{\epsilon}^2 (1 + (T-n)^2 \phi^2 + (T-n)^2 (n-1)(1-\phi)^2 + \sum_{t=2}^{T-n} (t - (t-1)\phi)^2) \quad (7)$$

$$V(Y_{dc}) = \left( \frac{(1-\beta)(\tau-\gamma)}{\sum_{t=0}^{D-T-1} \left(\frac{1}{R^t}\right)} \right)^2 \sigma_\epsilon^2 \left( R^2 + \phi^2 \left( \sum_{t=1}^{T-1} R^t \right)^2 + \sum_{m=2}^{T-1} \left( R^m + (1-\phi) \sum_{t=1}^{m-1} R^t \right)^2 \right) \quad (8)$$

Both  $V(Y_{dc})$  and  $V(Y_{db})$  depend on  $\phi$ . I want to know how the variances change with the ratio of permanent to transitory shocks, and if the variance of one retirement plan is larger than the other for every possible value of  $\phi$ . It is possible that, given the interaction between the different variables entering the formulas, there exists a value of  $\phi$  such that the variance of the defined benefit retirement plan is greater than the variance of the defined contribution plan for some values of  $\phi$ , and smaller for others. If that is the case, then individuals whose working income is subject to very permanent shocks will prefer one system, and people with transitory processes will prefer the other. The break-even value of  $\phi$  is the one that makes expressions 7 and 8 equal to each other.

The mechanics of this section are very similar to those in section 4. I calculate the variances of each retirement benefit for the benchmark case, find the break-even value of  $\phi$  (if such value exists), and then I analyze how this value changes with changes in the variables. From expressions 7 and 8 it is possible to see that  $V(Y_{db})$  depends on the variance of the earnings shocks  $\sigma_\epsilon^2$ ,  $\delta$ , the years of covered work, and the length of the averaging period used in the formula. On the other hand  $V(Y_{db})$  will be unaffected by changes in lifespan, interest rates and income drifts. As for  $V(Y_{dc})$ , it is necessary to keep in mind that I am comparing variances for the same expected value of retirement benefits. This implies that changes to variables affecting the defined contribution retirement benefit will have two kinds of effects: a direct effect as observed in 8, and an indirect effect that happens when the change in a variable triggers a change in the break-even return  $R^*$  to restore equality of expected benefits. So for a given rate of return the variance of the DC benefit depends positively on  $\tau$ ,  $\sigma_\epsilon$ , and negatively on  $\beta$  and  $\gamma$ . An increase in lifespan decreases the variance for a fixed  $R$  and an increase in  $T$  increases it. Earnings drifts will affect the variance only indirectly through changes in  $R^*$ . These

comparative statics exercises are intended to find out what kind of worker is more likely to choose a given retirement plan. Imagine a person whose earnings process is characterized by a value of  $\phi$  that coincides with the break-even value. If after the change in one variable the variance of the defined contribution plan increases more than the variance of the defined benefit plan, then the break-even value of  $\phi$  will go down (check figure 5), and all the agents with the same characteristics as the original will now prefer the defined benefit plan. If the individual starts at a value of  $\phi$  different than the break-even, a change in a particular variable might be enough to revert the preferences of that type of agent, moving him from the interval where  $\phi < \phi^*$  to the interval  $\phi > \phi^*$  or vice versa. Figure 5 plots  $V(Y_{db})$  and  $V(Y_{dc})$  for different values of  $\phi$ . All the other parameters correspond to the benchmark case, and  $R^*$  has been chosen so that  $E[Y_{db}] = E[Y_{dc}]$ .



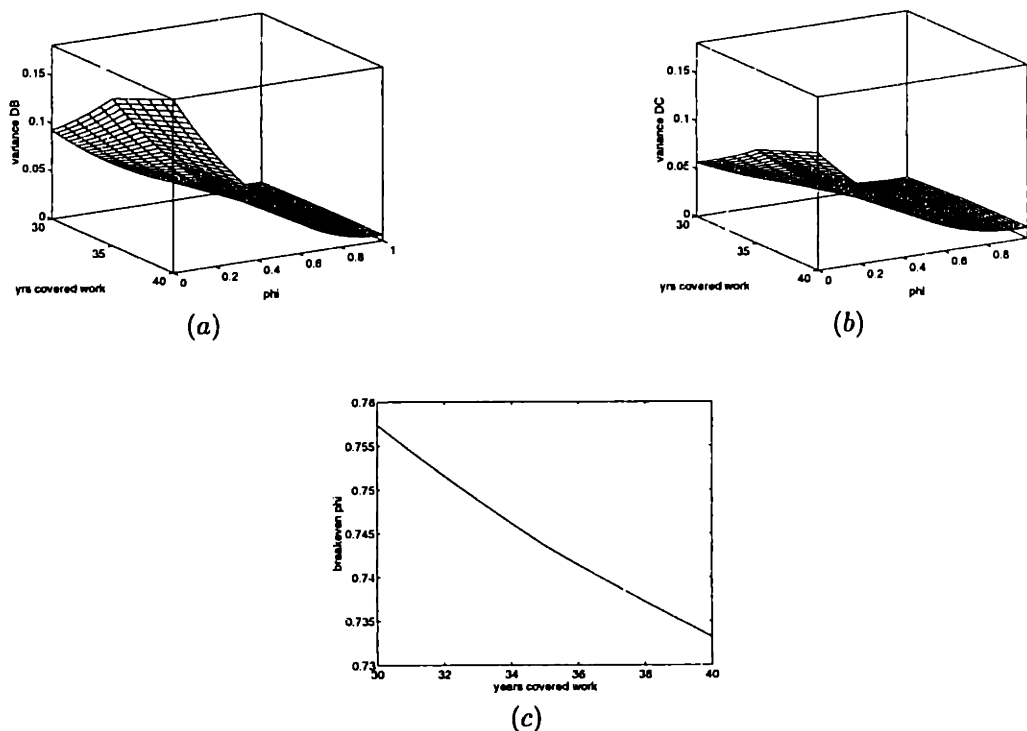
**Figure 5:** Variance of DB (---) and DC plans for different degrees of persistence in income shocks ( $\phi$ ). The break-even rate of return ( $R^*$ ) is set to 1.03182, the standard deviation of the shocks ( $\sigma_c$ ) is 0.25

The variance of both the defined contribution and the defined benefit retirement plans increases with the degree of persistence of the shocks (i.e. the variances are higher for small values of  $\phi$ ). When the earnings growth rate is zero,  $V(Y_{db})$  is larger than  $V(Y_{dc})$  for every value of  $\phi$ , which means that independent of the nature of the shocks the individual will choose the defined contribution plan. However for all the other cases considered in this section, there seems to be a value of  $\phi$  between 0 and 1 such that the variances of both plans are equal to each other, and then the worker is indifferent as to what plan to choose. As shown in figure 5, the break-even value of  $\phi$  for the benchmark case is 0.75741. This result suggests that those individuals

whose earnings process is subject to very transitory shocks will prefer the defined contribution plan.

### A. Changes in covered work

I am still assuming that age of retirement is fixed, so an increase in covered work is equivalent to assume that the individual starts working earlier in life. Both  $V(Y_{db})$  and  $V(Y_{dc})$  are increasing in  $T$ , so longer periods of covered work increase the variance of the retirement benefits.



**Figure 6:** Variance of DB and DC plans for different degrees of persistence  $\phi$  and years of covered work. Figure (c) shows break-even values of  $\phi$  for different periods of covered work.

Figures 6 (a) and (b) show the variance of  $Y_{db}$  and  $Y_{dc}$  respectively, for different values of  $\phi$  and years of covered work. When the number of working years goes up, there is a direct increase in both plans' variances due to the larger period of time left until retirement. Additionally the DB benefit is calculated as a percentage  $\delta$  of an average of earnings, and this percentage increases if  $T$  goes up (only up to 35

years) resulting in an increase in the variance for every value of  $\phi$  (notice the kink at age 35). The increase in  $T$  will trigger a reduction in rates of return on the defined contribution plan's assets, and this moves  $V(Y_{dc})$  down. Thus the final effect of an increase in  $T$  on the DC variance is ambiguous.

The defined contribution plan has much less dispersion than the defined benefit plan for all the range of values of  $\phi$  and covered work, as it is lower for small values of  $\phi$  and higher for values of  $\phi$  closer to 1. This is probably due to the counteracting effect of changes in  $R^*$ . In both cases the variance is increasing in  $\phi$  and decreasing in covered work, although less so for values of  $\phi$  closer to one. The fact that  $V(Y_{dc})$  increases with covered work for every value of  $\phi$  means that the rate of return effect is not enough to compensate for the longer period until retirement. Also the effect of the change in  $\delta$  seems to be larger the more permanent the shocks are.

The break-even values of  $\phi$  for different periods of covered work are shown in figure (c). Each of the points over the line represents a value of  $\phi$  such that  $V(Y_{db}) = V(Y_{dc})$ , and  $E[Y_{db}] = E[Y_{dc}]$ . To the left of the line individuals prefer the defined contribution plan, while to the right the defined benefit plan dominates.

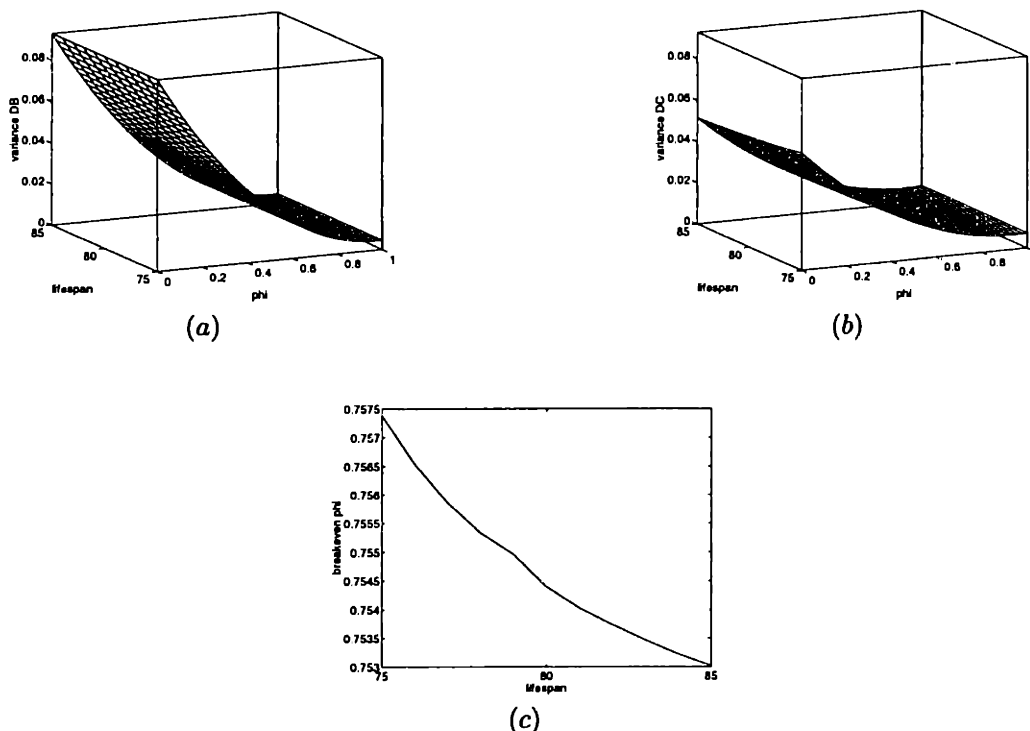
The break-even values of  $\phi$  for different periods of covered work are shown in figure (c). Each of the points over the line represents a value of  $\phi$  such that  $V(Y_{db}) = V(Y_{dc})$ , and  $E[Y_{db}] = E[Y_{dc}]$ . To the left of the line individuals prefer the defined contribution plan, while to the right the defined benefit plan dominates.

There are two ways to interpret a particular break-even value of  $\phi$ . One is in terms of the ratio of the variances of permanent to transitory shocks. It was shown previously that  $\phi$  could be expressed as a function of the ratio of permanent and transitory shocks. So a value of  $\phi$  between 0.73 and 0.75 means that the variance of the transitory shocks is between 10 and 12 times the variance of permanent shocks. This interpretation might be misleading due to the presence of a non stochastic permanent component too (i.e. the drift). The second interpretation is in terms of how much of the shocks are reverted next period, and according to the calculations approximately more than 75 percent of the shocks have to be reverted next period for individuals to prefer the defined benefit plan (when all the other variables are set at their benchmark values). In any case, earnings have to be very transitory, more so the shorter the period of covered work. The fact that the break-even value of  $\phi$  decreases when the period of covered work goes up means that the variance of the defined contribution plan increases more than the variance of the defined benefit

plan, making it more likely for people to choose it.

### B. Changes in Lifespan

An increase in lifespan does not affect  $Y_{db}$ , and hence its variance does not change either. But if lifespan goes up, there is a direct effect that causes  $V(Y_{dc})$  to decrease, while the indirect effect (increase in  $R^*$ ) works in the opposite direction.



**Figure 7:** Variance of DB and DC plans for different degrees of persistence  $\phi$  and lifespan. Figure (c) shows different values of  $\phi^*$  for different lifespans.

As figures 7 (a) and (b) show, the variance is increasing in  $\phi$  for both plans and every value of lifespan. For values of  $\phi$  close to zero,  $V(Y_{dc})$  is decreasing with lifespan, while for  $\phi$  close to one (around 0.7),  $V(Y_{dc})$  increases with lifespan. This means that for earnings processes where long term risk is more important the direct effect is larger than the rate of return effect, while for transitory processes the rate of return effect dominates.

Figure (c) plots the break-even values of  $\phi$  against lifespan. The higher the lifespan the less transitory the income process has to be to induce individuals to



choose the defined benefit plan.

### 5.3. Age-Dependent Variances

It is reasonable and plausible that the size of the shocks to which earnings are subject be different for different ages. An age-dependent variance will be relevant for the analysis as long as the time period used to calculate benefits differs across systems. Then if for example the variance increases over time, the shocks will be higher during the last years of working life, and that will be reflected in the variance of the system that puts more weight on those years.

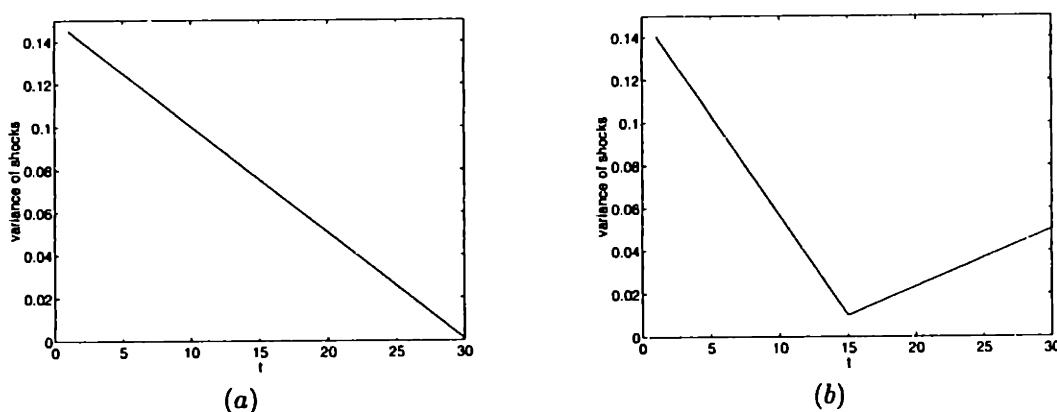


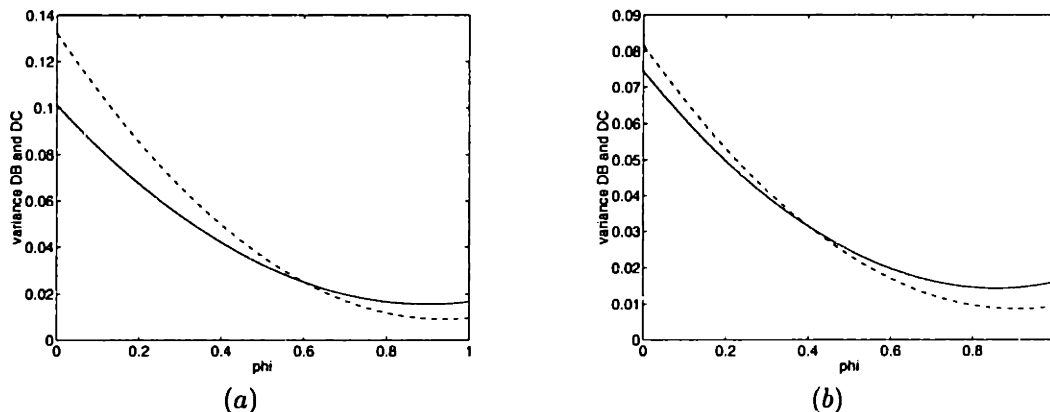
Figure 8: Age-dependent variances: two examples.

There is no prediction about what the evolution of the volatility of earnings should be over the life cycle. One possibility is that young workers switch jobs relatively often, and then the volatility of earnings is higher at the beginning of the life cycle and decreases afterwards. But it is also possible that relatively old workers decide to retire early but keep working, although not full time, and then their income fluctuates too. Based on this I use two different age-dependent variance structures to illustrate this point: the first case consists of a linearly decreasing variance, while for the second the variance decreases linearly until mid career and then starts increasing again but with a lower slope.<sup>41</sup> These two cases are illustrated in figure 8.

There is no case in repeating all the comparative statics. As an illustration I show in figure 9 the value of  $V(Y_{db})$  and  $V(Y_{dc})$  for different values of  $\phi$ . As expected

<sup>41</sup>In Dulitzky (1997) I show that the shocks to earnings are decreasing until age fifty or so and then start increasing again, but never go back to the original level.

the value of  $\phi$  at which individuals will start choosing the defined benefit plan is now lower than before, about 0.601 for the negative sloped variance of the shocks and 0.403 for the v-shaped case.



**Figure 9:** Variance of DB (---) and DC plans for age-dependent variances.

## 6. Uncertain Asset Returns

So far the only source of uncertainty has been through random earnings. The conclusion from previous sections was that this uncertainty will hinge differently on the defined benefit and the defined contribution retirement plans, and that the type of uncertainty (i.e. short term versus long term risk) will affect the choice of retirement plan. In this section I introduce asset returns uncertainty. In particular I assume that the funds accumulated in personal accounts are invested in assets with uncertain returns, which means that individuals choosing this plan are subject to an additional source of uncertainty beyond the one stemming from earnings.<sup>42</sup>

The formula for the defined contribution retirement benefit is the same as before, with the only difference that now returns are time specific because the shocks affecting them will be different in every period. The defined contribution retirement benefit at time  $T + 1$  will be given by:

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<sup>42</sup>I am assuming that the funds in excess of payments in the defined benefit plan (if any) are not invested in uncertain return assets. This means that random returns affect exclusively the defined contribution plan.

$$Y_{dc} = (1 - \beta)(\tau - \gamma) \frac{\sum_{t=1}^T \left( Y_t \prod_{s=t}^T R_s \right)}{1 + \sum_{t=T+2}^D \left( \prod_{i=T+2}^t R_i \right)^{-1}} + z.$$

There are at least three issues I want to consider regarding the effects of returns uncertainty on the optimal choice of a retirement plan: first, how to model the process for asset returns; second, what is the differential impact of uncertainty during the accumulation and the annuitization phases, and finally, how the results are affected by the presence of aggregate shocks that will imply some correlation between returns and earnings.

I follow the literature and assume that returns follow some kind of mean reverting process:

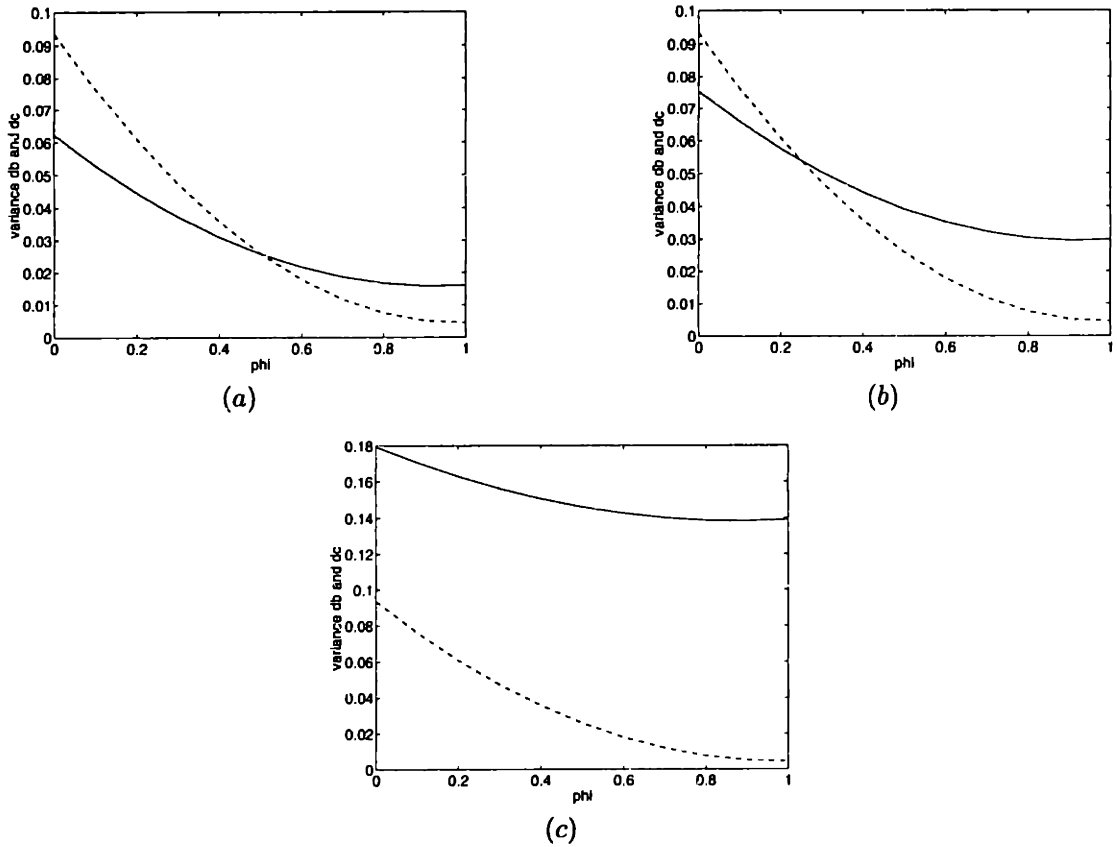
$$R_t = \bar{R} + \iota R_{t-1} + u_t$$

where  $u_t \sim N(0, \sigma_u^2)$ .<sup>43</sup> Regarding earnings, I retain the previous specification. I calculate the variance of the defined benefit and defined contribution retirement benefits for the benchmark case, using the value of the gross return on assets  $R$  that makes expected benefits equal to each other. Again, both variances are going to be functions of the earnings moving average coefficient  $\phi$ , and I look for the value of  $\phi$  that makes the variances equal to each other, if such value indeed exists. The purpose of this is to see if the choice depends on the relative importance of transitory and permanent shocks to which earnings are subject. The existence of a “break-even” value of  $\phi$  ( $\phi^*$ ) would imply that agents prefer one plan for  $\phi < \phi^*$ , when shocks are very permanent (because the variance will be lower at the same expected value) and another plan when the shocks are very transitory.

Closed form solutions for the conditional variances were straightforward when the only random variable was earnings. Now the variance involves the product and ratio of many random variables, some of them correlated with each other. I use Montecarlo methods to estimate them.

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<sup>43</sup>This is the standard way to model asset returns. There is not enough information to estimate the process for the return in defined contribution assets in countries like Chile, because the reform has been relatively recent.

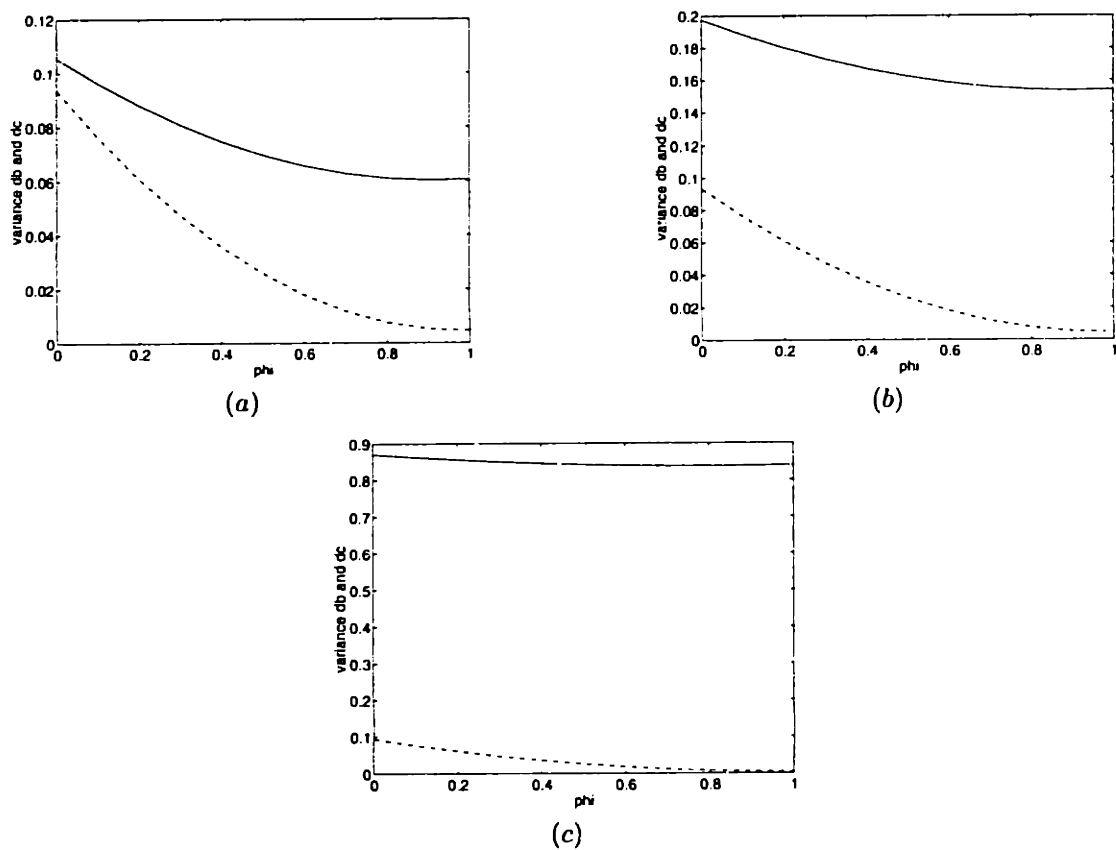


**Figure 10:** Variance of DB (---) and DC plans for different degrees of persistence  $\phi$ . Returns follow an AR(1) process:  $R_t = \bar{R} + \iota R_{t-1} + u_t$ . The value of  $\iota$  is different for each of the figures: (a)  $\iota = 0.1$ ; (b)  $\iota = 0.5$ ; (c)  $\iota = 0.8$ . The process for earnings is the same as before, and  $\sigma_{u_t} = 0.0004$

Figure 10 plots the variances of both retirement benefits for different values of  $\phi$ . I estimate the variances for different degrees of persistence in the autoregressive coefficient  $\iota$ . The pattern in the variances is similar to the uncertain earnings case: they are decreasing with  $\phi$ , and  $V(Y_{db})$  is higher than  $V(Y_{dc})$  for low values of  $\phi$ . By construction the variance of the defined benefit retirement benefit is not affected by stochastic returns. Figure 10(a) has been drawn for a value of  $\iota$  equal to 0.1. The break-even  $\phi$  is approximately equal to 0.5, and as can be seen in figures 10(b) and (c), the higher the value of  $\iota$  the lower  $\phi^*$  is (0.25 in figure (b)). Moreover, when the persistence in the asset returns process is high enough there is no break-even value of  $\phi$ , the variance of the DC benefit being always above the variance of the DB.

The previous figures were drawn for  $\sigma_u$  equal to 0.0004. This is a very small value for the standard deviation of the returns shocks, and in the next set of pictures I

look at the effect of increasing that value. Figure 11 reproduces the previous graph but now the standard deviation of the shocks to the defined contribution's asset returns  $\sigma_u$  is set to 0.001 instead of 0.0004. This time there is no break-even value of  $\phi$ , the variance of the defined contribution retirement benefit is always above that of the defined benefit.

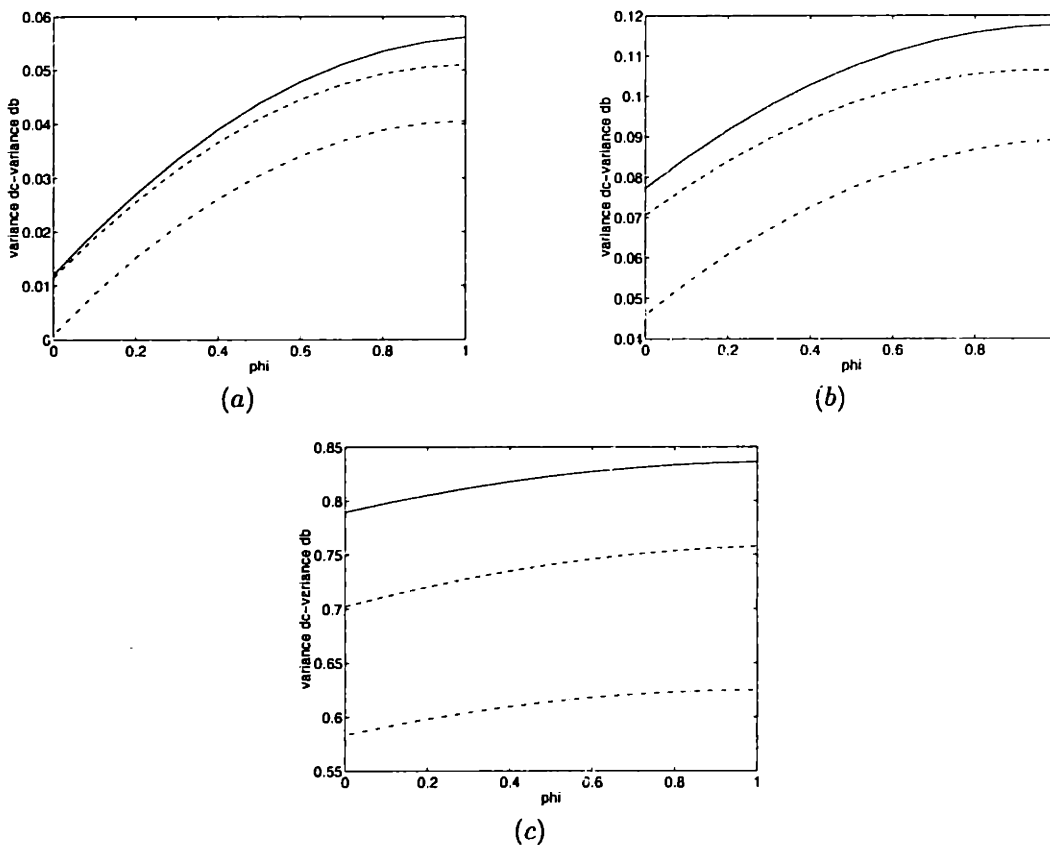


**Figure 11:** Variance of DB (—) and DC plans for different degrees of persistence  $\phi$ . Returns follow an AR(1) process:  $R_t = \bar{R} + \nu R_{t-1} + u_t$ . The value of  $\nu$  is different for each of the figures: (a)  $\nu = 0.1$ ; (b)  $\nu = 0.5$ ; (c)  $\nu = 0.8$ . The process for earnings is the same as before, and  $\sigma_{u_t} = 0.001$

The second issue I want to address has to do with the structure of uncertainty in asset returns. In particular, the variances are calculated conditional on the first working period's information set. That means that the uncertainty on returns during the accumulation period has the same structure as the uncertainty during the annuitization period. This is correct under the assumption that there is no sepa-

ration between accumulation of funds in the personal accounts and annuitization after retirement, or equivalently that the individual “buys” the annuity at the time of choosing the defined contribution plan (first working period).

There are at least two more possibilities: first, the individual buys the annuity at the time of retirement, which means that conditioning on the first working period might not be the right approach. This will tend to overestimate the variance of the defined contribution retirement benefit. The second possibility is that the uncertainty in returns is resolved at the time of buying the annuity, after which returns are set to their break-even values.



**Figure 12:** Variance DC minus variance DB for different degrees of persistence  $\phi$ . Returns follow an AR(1) process:  $R_t = \bar{R} + \iota R_{t-1} + u_t$ . The variance is conditional on first working period (—), on the first working period and at retirement (---), and returns are certain after retirement (- · - · -). Other parameter information:  $\sigma_{u_t} = 0.001$ ,  $\iota$  is equal to: (a) 0.1, (b) 0.4, and (c) 0.8.

Figure 12 shows the difference between the variance of the DC and DB retire-

ment benefits for different values of  $\phi$  and different values of the AR parameter in the returns process, for the three specifications described above. The dashed line corresponds to the case in which accumulation and annuitization are recognized as two separate stages, so that the variance of retirement benefits are computed conditional on the first working period and the first retirement period -at the time of purchasing the annuity-. The dashed-dotted line represents the case in which returns are certain after retirement. The variance of the defined benefit plan is the same for each specification, so differences in the position of the lines indicate differences in the DC variance. Obviously the variance of the defined contribution plan is highest when it is conditioned on the first working period and is lowest when returns are assumed to be known with certainty after retirement. The difference is increasing with  $\phi$  and with  $\iota$  suggesting that the defined contribution plan becomes more unattractive the more permanent the shocks to the asset returns are.

As expected then, once the assumption of deterministic asset returns is abandoned the defined contribution system becomes much more unattractive. This is so even when the standard deviation of the returns is unrealistically small. Since the reform is relatively recent, it is not possible to find actual estimates for the return process, but it is likely that the standard deviation of the process will be much higher than the ones simulated in this paper, casting some shadow on the overall performance of the defined contribution plan.

## 7. Conclusion

This paper looks at the optimal choice of retirement system from an individual perspective, when more than one plan is available. In this particular case, individuals can choose between a defined benefit plan in which the benefit is calculated as an average of the last ten years of working income, and a defined contribution plan where benefits are a weighted average of lifetime earnings. It is shown that when earnings are uncertain, the defined contribution plan will be chosen by individuals whose earnings are subject mostly to permanent shocks. If on the other hand transitory shocks are very important the defined benefit plan is preferred. This analysis suggests then that a compulsory defined contribution system may not be welfare improving, because given the choice many workers would prefer a defined benefit plan. If this is the case, a natural question arises as to why do governments

promote defined contribution plans. A complete answer to this question has to introduce explicitly the government's budget constraint and/or objective functions that may include different arguments than those of individuals.

There are a number of possible ways in which to extend this work. First, it would be interesting to analyze how different retirement plans will affect consumption and saving behavior over the lifecycle. The inclusion of behavioral responses from agents to different retirement formulas seems to be a natural extension, because the choice of a particular retirement plan is very likely going to generate some response in terms of consumption and saving. Second, a more general approach involving other agents in the economy like the government would help to understand why defined contribution plans are so popular all over the world. Finally, I would like to study the interaction between heterogeneity in earnings and choice of retirement plan in a general equilibrium setting. If agents with different earnings processes prefer different systems, then in equilibrium the rate of return on assets will be a function of the proportion of people choosing each plan, and the proportion of people choosing each system will depend on the rate of return on assets. An issue that deserves attention is then under what conditions two different systems will survive over time, in a steady state equilibrium. It would be desirable to find some empirical support to this analysis. Unfortunately the reform is relatively recent and as such it is difficult to find the data necessary to evaluate the reform.



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